Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.4-u-a+b-arctanh-c-x-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [538]. This is test number [194].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (538)	% 0. (0)
Mathematica	% 99.63 (536)	% 0.37 (2)
Maple	% 94.42 (508)	% 5.58 (30)
Maxima	% 47.96 (258)	% 52.04 (280)
Fricas	% 47.58 (256)	% 52.42 (282)
Sympy	% 25.65 (138)	% 74.35 (400)
Giac	% 31.78 (171)	% 68.22 (367)

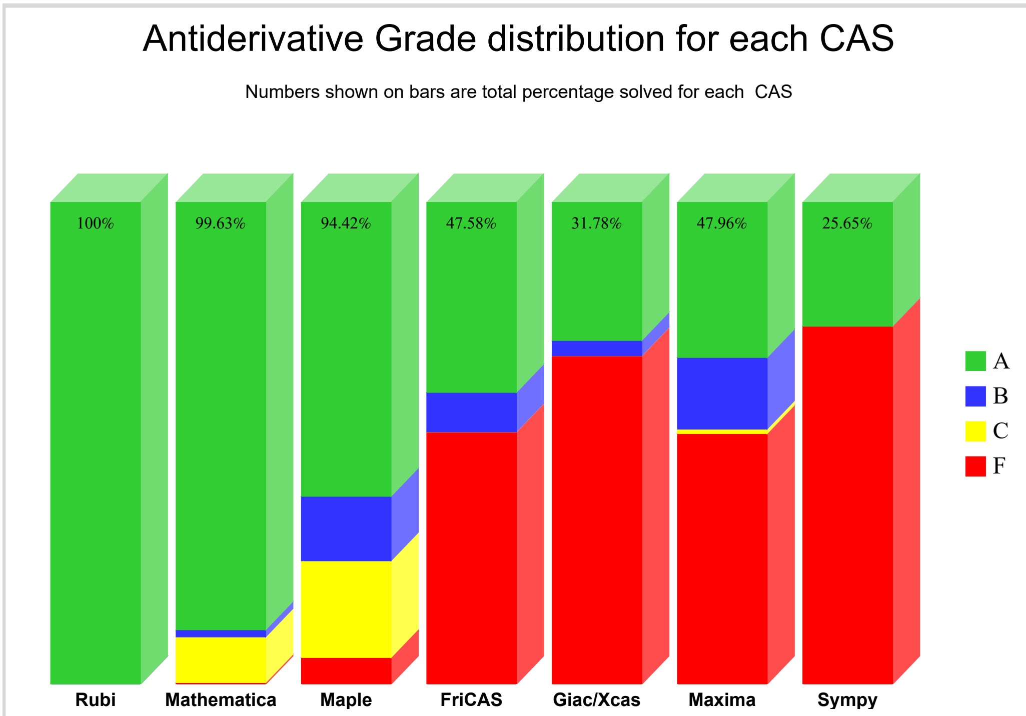
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

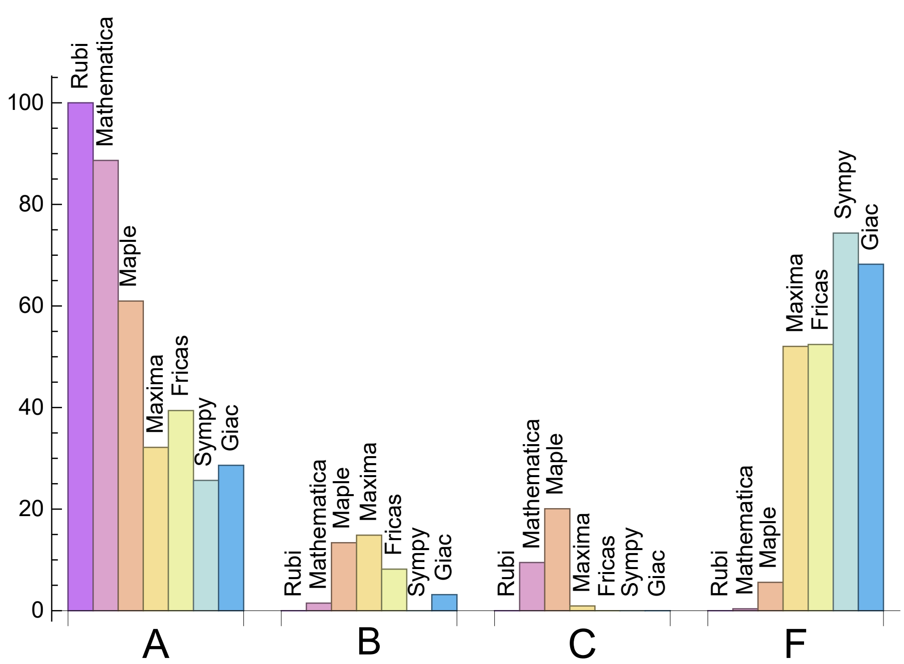
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	88.66	1.49	9.48	0.37
Maple	60.97	13.38	20.07	5.58
Maxima	32.16	14.87	0.93	52.04
Fricas	39.41	8.18	0.	52.42
Sympy	25.65	0.	0.	74.35
Giac	28.62	3.16	0.	68.22

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.27	140.45	0.89	125.	1.
Mathematica	1.03	155.82	0.91	102.	0.89
Maple	0.27	656.8	3.59	181.5	1.4
Maxima	0.87	241.02	1.88	167.	1.69
Fricas	1.54	261.06	2.92	204.5	2.26
Sympy	2.72	88.12	0.86	46.	0.92
Giac	0.79	108.67	1.15	96.	1.37

1.4 list of integrals that has no closed form antiderivative

{141, 142, 143, 144, 145, 146, 160, 185, 186, 187, 188, 189, 190, 191, 218, 219, 220, 221, 222, 223, 249, 251, 252, 254, 255, 257, 281, 282, 286, 287, 291, 292, 296, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 387, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 479, 480, 485, 486, 491, 492, 511, 512, 535}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 178, 180, 182, 183, 184, 197, 201, 203, 205, 207, 209, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 306, 312, 318, 319, 320, 348, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 497, 503, 504, 508, 510, 517, 518, 529, 531, 533, 536, 537}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

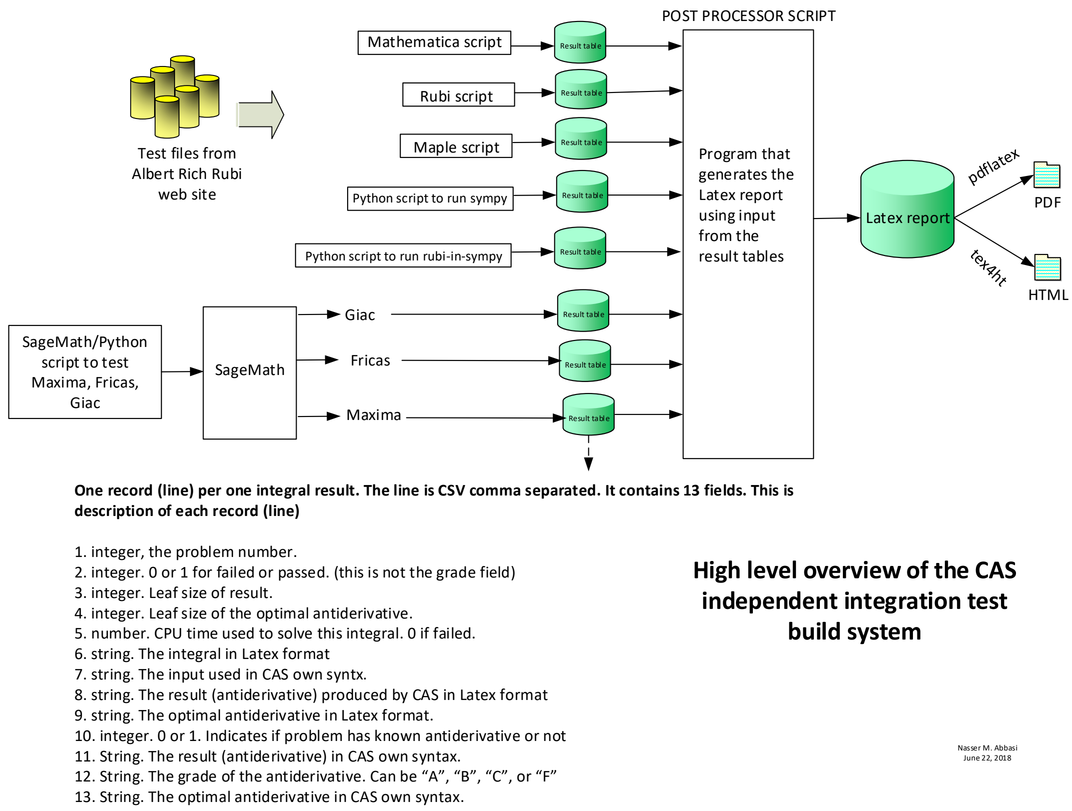
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 98, 103, 104, 105, 106, 107, 111, 112, 113, 114, 115, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 141, 142, 143, 144, 145, 146, 150, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 503, 505, 507, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 529, 531, 535, 538 }

B grade: { 4, 120, 121, 383, 405, 504, 528, 530 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 99, 100, 101, 102, 108, 109, 110, 116, 117, 132, 133, 139, 140, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 212, 240, 246, 270, 272, 278, 312, 319, 502, 506, 508, 509, 510, 533, 534, 536, 537 }

F grade: { 527, 532}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 76, 77, 78, 84, 85, 94, 127, 134, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 176, 178, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 230, 237, 241, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 365, 367, 370, 371, 372, 373, 375, 377, 378, 379, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 419, 420, 421, 422, 424, 425, 426, 428, 430, 431, 432, 433, 434, 435, 436, 437, 439, 441, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 491, 492, 493, 498, 499, 500, 501, 505, 506, 507, 508, 511, 512, 517, 518, 519, 520, 521, 535 }

B grade: { 23, 28, 34, 40, 46, 47, 48, 49, 67, 71, 74, 75, 79, 83, 86, 87, 92, 93, 107, 114, 115, 118, 133, 139, 140, 173, 175, 180, 181, 182, 214, 227, 228, 229, 231, 232, 233, 247, 259, 260, 262, 263, 264, 265, 268, 278, 279, 303, 305, 306, 307, 308, 310, 319, 345, 369, 413, 418, 423, 488, 489, 490, 494, 495, 496, 502, 503, 504, 509, 510, 537, 538 }

C grade: { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 135, 136, 137, 138, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 235, 236, 238, 239, 240, 242, 243, 245, 246, 266, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 309, 311, 312, 313, 314, 315, 316, 317, 318, 346, 347, 364, 366, 368, 388, 427, 429, 448, 450, 497, 522, 523, 524, 525, 526, 527, 533 }

F grade: { 280, 320, 348, 374, 376, 380, 381, 382, 383, 397, 404, 405, 438, 440, 442, 443, 444, 471, 475, 513, 514, 515, 516, 528, 529, 530, 531, 532, 534, 536 }

2.1.4 Maxima

A grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 14, 18, 19, 20, 21, 24, 25, 30, 31, 35, 36, 42, 54, 62, 66, 69, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 211, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 227, 249, 251, 252, 254, 255, 257, 259, 261, 281, 282, 286, 287, 291, 292, 296, 302, 304, 307, 310, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 364, 366, 368, 371, 387, 388, 390, 391, 393, 395, 398, 399, 403, 406, 407, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 469, 470, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 506, 507, 508, 520, 521, 525, 527, 535

B grade: { 4, 13, 17, 22, 23, 28, 29, 32, 33, 34, 37, 40, 41, 61, 67, 71, 75, 76, 77, 78, 79, 83, 84, 85, 86, 87, 92, 93, 94, 107, 114, 115, 118, 124, 125, 126, 166, 181, 228, 229, 230, 231, 232, 233, 235, 237, 239, 244, 250, 253, 256, 260, 262, 263, 264, 265, 267, 268, 269, 271, 274, 275, 276, 303, 305, 306, 308, 309, 311, 313, 314, 315, 316, 317, 345, 346, 347, 472, 473, 474 }

C grade: { 522, 523, 524, 526, 538 }

F grade: { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 68, 70, 72, 73, 74, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 177, 179, 183, 208, 210, 212, 217, 226, 234, 236, 238, 240, 241, 242, 243, 245, 246, 247, 248, 258, 266, 270, 272, 273, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 312, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 468, 471, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 107, 114, 115, 118, 124, 125, 126, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 218, 219, 220, 221, 222, 223, 224, 228, 230, 232, 237, 244, 249, 251, 252, 253, 254, 255, 256, 257, 260, 261, 262, 264, 267, 268, 269, 274, 275, 276, 281, 282, 286, 287, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 314, 315, 316, 317, 321, 322, 328, 329, 335, 341, 344, 345, 346, 347, 349, 350, 358, 359, 364, 366, 368, 371, 387, 388, 390, 391, 393, 395, 398, 399, 403, 406, 407, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 434, 436, 448, 450, 457, 462, 463, 464, 468, 469, 470, 472, 473, 474, 476, 477, 478, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 511, 512, 519, 520, 521, 522, 523, 524, 525, 526, 535, 538 }

B grade: { 248, 250, 258, 283, 284, 285, 288, 289, 290, 293, 294, 295, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 513, 514, 515, 516 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 263, 265, 266, 270, 271, 272, 273, 277, 278, 279, 280, 306, 312, 313, 318, 319, 320, 348, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 400, 401, 402, 404, 405, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 475, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 517, 518, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 54, 61, 62, 66, 141, 142, 143, 144, 145, 146, 161, 162, 163, 164, 165, 167, 169, 170, 171, 173, 175, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 214, 216, 218, 219, 220, 221, 222, 223, 224, 228, 230, 232, 237, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 264, 281, 282, 286, 287, 291, 292, 296, 302, 304, 307, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 412, 415, 417, 420, 422, 425, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 505, 511, 512, 522, 523, 524, 525, 526 }

B grade: { }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 166, 168, 172, 174, 176, 177, 178, 179, 180, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 215, 217, 225, 226, 227, 229, 231, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 414, 416, 418, 419, 421, 423, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

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A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 41, 42, 54, 61, 62, 66, 107, 124, 141, 142, 143, 144, 145, 146, 160, 161, 162, 163, 165, 167, 169, 171, 173, 175, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 206, 218, 219, 220, 221, 222, 223, 224, 228, 230, 237, 244, 249, 251, 252, 253, 254, 255, 256, 257, 258, 261, 281, 282, 286, 287, 291, 292, 296, 302, 304, 321, 322, 328, 329, 335, 341, 349, 350, 358, 359, 364, 366, 368, 387, 388, 390, 391, 395, 403, 411, 412, 415, 416, 417, 420, 421, 422, 425, 427, 429, 448, 450, 462, 463, 464, 468, 469, 470, 479, 480, 485, 486, 491, 492, 498, 499, 500, 501, 511, 512, 513, 514, 515, 516, 519, 520, 521, 522, 524, 535, 538 }

B grade: { 23, 28, 34, 40, 164, 170, 248, 250, 371, 393, 434, 436, 457, 505, 523, 525, 526 }

C grade: { }

F grade: { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 166, 168, 172, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 197, 199, 201, 203, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 225, 226, 227, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 245, 246, 247, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 288, 289, 290, 293, 294, 295, 297, 298, 299, 300, 301, 303, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 365, 367, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 389, 392, 394, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 413, 414, 418, 419, 423, 424, 426, 428, 430, 431, 432, 433, 435, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 451, 452, 453, 454, 455, 456, 458, 459, 460, 461, 465, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 481, 482, 483, 484, 487, 488, 489, 490, 493, 494, 495, 496, 497, 502, 503, 504, 506, 507, 508, 509, 510, 517, 518, 527, 528, 529, 530, 531, 532, 533, 534, 536, 537 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	97	101	163	275	124	147
normalized size	1	1.	0.9	0.94	1.51	2.55	1.15	1.36
time (sec)	N/A	0.097	0.074	0.041	0.957	2.05	3.053	1.248

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	87	91	149	240	112	134
normalized size	1	1.	0.91	0.95	1.55	2.5	1.17	1.4
time (sec)	N/A	0.097	0.063	0.032	0.948	2.049	2.589	1.247

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	79	81	134	219	100	120
normalized size	1	1.	0.94	0.96	1.6	2.61	1.19	1.43
time (sec)	N/A	0.075	0.055	0.031	0.972	1.968	1.613	1.223

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	95	65	115	185	75	103
normalized size	1	1.	2.16	1.48	2.61	4.2	1.7	2.34
time (sec)	N/A	0.03	0.01	0.027	0.955	2.012	0.969	1.185

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	86	0	0	0	0
normalized size	1	1.	0.9	1.43	0.	0.	0.	0.
time (sec)	N/A	0.069	0.075	0.039	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	71	105	0	0	0	0
normalized size	1	1.	1.01	1.5	0.	0.	0.	0.
time (sec)	N/A	0.086	0.064	0.046	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	76	84	120	220	95	115
normalized size	1	1.	1.36	1.5	2.14	3.93	1.7	2.05
time (sec)	N/A	0.054	0.06	0.039	0.957	2.038	4.171	1.189

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	86	95	134	244	117	132
normalized size	1	1.	0.88	0.97	1.37	2.49	1.19	1.35
time (sec)	N/A	0.087	0.064	0.039	0.96	2.225	2.389	1.271

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	94	105	154	267	129	146
normalized size	1	1.	0.85	0.95	1.4	2.43	1.17	1.33
time (sec)	N/A	0.092	0.065	0.04	0.966	2.102	3.096	1.257

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	125	159	284	373	196	220
normalized size	1	1.	0.8	1.01	1.81	2.38	1.25	1.4
time (sec)	N/A	0.168	0.105	0.031	0.964	2.037	8.174	1.255

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	115	147	248	332	177	203
normalized size	1	1.	0.8	1.03	1.73	2.32	1.24	1.42
time (sec)	N/A	0.154	0.094	0.029	0.976	2.054	3.031	1.325

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	107	135	242	302	167	188
normalized size	1	1.	0.83	1.05	1.88	2.34	1.29	1.46
time (sec)	N/A	0.13	0.088	0.029	0.969	1.933	2.457	1.233

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	92	121	198	255	131	163
normalized size	1	1.	1.3	1.7	2.79	3.59	1.85	2.3
time (sec)	N/A	0.043	0.105	0.026	0.959	2.04	1.548	1.218

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	103	142	234	0	0	0
normalized size	1	1.	0.9	1.25	2.05	0.	0.	0.
time (sec)	N/A	0.114	0.103	0.042	1.454	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	80	73	123	0	0	0	0
normalized size	1	1.31	1.2	2.02	0.	0.	0.	0.
time (sec)	N/A	0.126	0.106	0.041	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	143	176	0	0	0	0
normalized size	1	1.	1.04	1.28	0.	0.	0.	0.
time (sec)	N/A	0.14	0.101	0.046	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	103	141	212	293	158	188
normalized size	1	1.	1.27	1.74	2.62	3.62	1.95	2.32
time (sec)	N/A	0.085	0.094	0.037	0.962	2.173	4.616	1.303

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	114	153	240	332	189	205
normalized size	1	1.	0.78	1.04	1.63	2.26	1.29	1.39
time (sec)	N/A	0.149	0.097	0.04	0.975	2.099	5.27	1.251

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	122	165	262	363	199	223
normalized size	1	1.	0.76	1.02	1.63	2.25	1.24	1.39
time (sec)	N/A	0.16	0.095	0.04	0.97	2.36	4.643	1.415

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	151	199	385	440	243	267
normalized size	1	1.	0.79	1.04	2.01	2.29	1.27	1.39
time (sec)	N/A	0.181	0.134	0.028	0.973	2.308	6.683	1.335

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	142	187	358	413	235	251
normalized size	1	1.	0.8	1.05	2.01	2.32	1.32	1.41
time (sec)	N/A	0.176	0.126	0.031	0.966	2.22	4.327	1.256

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	173	329	369	211	235
normalized size	1	1.	0.99	1.28	2.44	2.73	1.56	1.74
time (sec)	N/A	0.101	0.107	0.029	0.968	1.899	4.209	1.218

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	115	162	296	333	182	215
normalized size	1	1.	1.37	1.93	3.52	3.96	2.17	2.56
time (sec)	N/A	0.051	0.145	0.029	0.971	2.015	2.598	1.245

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	148	182	308	0	0	0
normalized size	1	1.	0.97	1.2	2.03	0.	0.	0.
time (sec)	N/A	0.166	0.136	0.042	1.478	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	149	189	309	0	0	0
normalized size	1	1.	0.99	1.26	2.06	0.	0.	0.
time (sec)	N/A	0.158	0.141	0.049	1.448	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	165	200	0	0	0	0
normalized size	1	1.	1.03	1.25	0.	0.	0.	0.
time (sec)	N/A	0.168	0.144	0.052	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	175	216	0	0	0	0
normalized size	1	1.	0.99	1.23	0.	0.	0.	0.
time (sec)	N/A	0.194	0.139	0.049	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	131	181	308	373	207	236
normalized size	1	1.	1.41	1.95	3.31	4.01	2.23	2.54
time (sec)	N/A	0.099	0.119	0.04	0.96	2.151	3.919	1.405

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	140	193	338	398	233	254
normalized size	1	1.	1.02	1.41	2.47	2.91	1.7	1.85
time (sec)	N/A	0.119	0.123	0.039	0.968	2.072	4.861	1.535

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	149	205	369	446	257	271
normalized size	1	1.	0.76	1.05	1.88	2.28	1.31	1.38
time (sec)	N/A	0.177	0.125	0.039	0.974	2.094	8.384	1.77

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	177	237	504	521	294	317
normalized size	1	1.	0.79	1.06	2.25	2.33	1.31	1.42
time (sec)	N/A	0.213	0.158	0.03	0.971	2.245	7.013	1.223

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	168	225	458	490	279	300
normalized size	1	1.	0.98	1.32	2.68	2.87	1.63	1.75
time (sec)	N/A	0.181	0.146	0.027	0.974	2.202	5.841	1.269

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	159	215	440	452	269	285
normalized size	1	1.	1.04	1.41	2.88	2.95	1.76	1.86
time (sec)	N/A	0.116	0.141	0.03	0.979	2.124	4.795	1.299

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	146	202	382	402	226	261
normalized size	1	1.	1.36	1.89	3.57	3.76	2.11	2.44
time (sec)	N/A	0.055	0.178	0.026	0.95	2.046	5.007	1.28

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	179	222	373	0	0	0
normalized size	1	1.	0.97	1.2	2.02	0.	0.	0.
time (sec)	N/A	0.197	0.17	0.044	1.469	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	194	229	379	0	0	0
normalized size	1	1.	1.09	1.29	2.13	0.	0.	0.
time (sec)	N/A	0.2	0.175	0.049	1.463	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	143	210	396	0	0	0
normalized size	1	1.	0.92	1.35	2.54	0.	0.	0.
time (sec)	N/A	0.189	0.166	0.045	1.45	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	197	240	0	0	0	0
normalized size	1	1.	1.04	1.27	0.	0.	0.	0.
time (sec)	N/A	0.217	0.175	0.049	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	209	209	206	256	0	0	0	0
normalized size	1	1.	0.99	1.22	0.	0.	0.	0.
time (sec)	N/A	0.229	0.17	0.049	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	157	221	404	443	253	286
normalized size	1	1.	1.44	2.03	3.71	4.06	2.32	2.62
time (sec)	N/A	0.105	0.147	0.04	0.98	2.157	5.928	1.342

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	166	233	444	483	291	304
normalized size	1	1.	1.1	1.54	2.94	3.2	1.93	2.01
time (sec)	N/A	0.125	0.155	0.04	0.988	2.201	6.818	1.638

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	175	245	477	524	301	320
normalized size	1	1.	0.76	1.07	2.08	2.29	1.31	1.4
time (sec)	N/A	0.197	0.175	0.042	1.015	2.311	17.409	2.606

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	129	253	0	0	0	0
normalized size	1	1.	0.73	1.43	0.	0.	0.	0.
time (sec)	N/A	0.287	0.397	0.045	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	97	213	0	0	0	0
normalized size	1	1.	0.67	1.47	0.	0.	0.	0.
time (sec)	N/A	0.184	0.244	0.045	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	94	94	75	157	0	0	0	0
normalized size	1	1.	0.8	1.67	0.	0.	0.	0.
time (sec)	N/A	0.103	0.154	0.04	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	51	51	52	112	0	0	0	0
normalized size	1	1.	1.02	2.2	0.	0.	0.	0.
time (sec)	N/A	0.048	0.097	0.039	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	46	46	55	156	0	0	0	0
normalized size	1	1.	1.2	3.39	0.	0.	0.	0.
time (sec)	N/A	0.074	0.11	0.046	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	93	225	0	0	0	0
normalized size	1	1.	1.	2.42	0.	0.	0.	0.
time (sec)	N/A	0.153	0.192	0.053	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	133	286	0	0	0	0
normalized size	1	1.	0.91	1.96	0.	0.	0.	0.
time (sec)	N/A	0.234	0.345	0.058	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	172	328	0	0	0	0
normalized size	1	1.	0.93	1.77	0.	0.	0.	0.
time (sec)	N/A	0.346	0.423	0.059	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	142	265	0	0	0	0
normalized size	1	1.	0.78	1.46	0.	0.	0.	0.
time (sec)	N/A	0.222	0.701	0.064	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	121	216	0	0	0	0
normalized size	1	1.	0.81	1.45	0.	0.	0.	0.
time (sec)	N/A	0.185	0.575	0.055	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	99	192	0	0	0	0
normalized size	1	1.	0.93	1.81	0.	0.	0.	0.
time (sec)	N/A	0.143	0.379	0.046	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	64	84	130	104	121	131
normalized size	1	1.	1.12	1.47	2.28	1.82	2.12	2.3
time (sec)	N/A	0.045	0.068	0.035	0.953	2.071	4.215	1.137

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	124	124	101	221	0	0	0	0
normalized size	1	1.	0.81	1.78	0.	0.	0.	0.
time (sec)	N/A	0.174	0.407	0.053	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	140	269	0	0	0	0
normalized size	1	1.	0.82	1.57	0.	0.	0.	0.
time (sec)	N/A	0.221	0.814	0.058	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	212	212	189	338	0	0	0	0
normalized size	1	1.	0.89	1.59	0.	0.	0.	0.
time (sec)	N/A	0.257	1.135	0.06	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	189	319	0	0	0	0
normalized size	1	1.	0.83	1.41	0.	0.	0.	0.
time (sec)	N/A	0.288	0.83	0.049	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	167	270	0	0	0	0
normalized size	1	1.	0.86	1.39	0.	0.	0.	0.
time (sec)	N/A	0.248	0.708	0.053	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	150	145	246	0	0	0	0
normalized size	1	1.	0.97	1.64	0.	0.	0.	0.
time (sec)	N/A	0.219	0.493	0.048	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	99	136	205	180	389	178
normalized size	1	1.	1.29	1.77	2.66	2.34	5.05	2.31
time (sec)	N/A	0.082	0.107	0.037	0.989	2.13	3.763	1.204

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	86	100	181	163	289	157
normalized size	1	1.	1.12	1.3	2.35	2.12	3.75	2.04
time (sec)	N/A	0.055	0.073	0.033	0.976	2.117	4.165	1.25

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	147	264	0	0	0	0
normalized size	1	1.	0.91	1.64	0.	0.	0.	0.
time (sec)	N/A	0.227	0.525	0.057	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	218	218	186	319	0	0	0	0
normalized size	1	1.	0.85	1.46	0.	0.	0.	0.
time (sec)	N/A	0.274	1.239	0.058	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	268	268	220	394	0	0	0	0
normalized size	1	1.	0.82	1.47	0.	0.	0.	0.
time (sec)	N/A	0.317	1.497	0.062	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	75	95	178	209	294	157
normalized size	1	1.	0.94	1.19	2.22	2.61	3.68	1.96
time (sec)	N/A	0.054	0.105	0.034	0.973	2.377	8.15	1.16

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	41	41	39	126	162	0	0	0
normalized size	1	1.	0.95	3.07	3.95	0.	0.	0.
time (sec)	N/A	0.065	0.074	0.045	0.961	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	271	422	0	0	0	0
normalized size	1	1.	1.	1.56	0.	0.	0.	0.
time (sec)	N/A	0.65	0.753	0.054	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	236	234	383	543	0	0	0
normalized size	1	1.	0.99	1.62	2.3	0.	0.	0.
time (sec)	N/A	0.526	0.575	0.061	2.101	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	201	341	0	0	0	0
normalized size	1	1.	1.03	1.74	0.	0.	0.	0.
time (sec)	N/A	0.394	0.48	0.048	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	156	296	392	0	0	0
normalized size	1	1.	1.39	2.64	3.5	0.	0.	0.
time (sec)	N/A	0.119	0.319	0.051	1.741	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	228	3644	0	0	0	0
normalized size	1	1.	1.19	19.08	0.	0.	0.	0.
time (sec)	N/A	0.451	0.53	0.403	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	249	3104	0	0	0	0
normalized size	1	1.	1.24	15.44	0.	0.	0.	0.
time (sec)	N/A	0.484	0.516	0.82	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	206	400	0	0	0	0
normalized size	1	1.	1.36	2.65	0.	0.	0.	0.
time (sec)	N/A	0.365	0.281	0.071	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	246	440	563	0	0	0
normalized size	1	1.	1.19	2.14	2.73	0.	0.	0.
time (sec)	N/A	0.453	0.472	0.07	3.068	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	329	569	1034	0	0	0
normalized size	1	1.	0.92	1.6	2.9	0.	0.	0.
time (sec)	N/A	1.024	1.047	0.054	2.248	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	297	521	815	0	0	0
normalized size	1	1.	0.95	1.67	2.61	0.	0.	0.
time (sec)	N/A	0.884	1.	0.053	2.15	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	263	478	824	0	0	0
normalized size	1	1.	0.94	1.71	2.94	0.	0.	0.
time (sec)	N/A	0.652	0.749	0.049	2.164	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	227	372	626	0	0	0
normalized size	1	1.	1.3	2.13	3.58	0.	0.	0.
time (sec)	N/A	0.164	0.668	0.048	1.775	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	324	1082	0	0	0	0
normalized size	1	1.	1.17	3.89	0.	0.	0.	0.
time (sec)	N/A	0.586	0.705	0.853	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	283	283	341	6039	0	0	0	0
normalized size	1	1.	1.2	21.34	0.	0.	0.	0.
time (sec)	N/A	0.629	0.529	0.839	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	370	1167	0	0	0	0
normalized size	1	1.	1.18	3.73	0.	0.	0.	0.
time (sec)	N/A	0.673	0.75	1.217	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	270	550	749	0	0	0
normalized size	1	1.	1.11	2.25	3.07	0.	0.	0.
time (sec)	N/A	0.256	0.639	0.076	3.078	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	415	415	385	662	1253	0	0	0
normalized size	1	1.	0.93	1.6	3.02	0.	0.	0.
time (sec)	N/A	1.459	1.633	0.057	2.219	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	377	377	356	618	1046	0	0	0
normalized size	1	1.	0.94	1.64	2.77	0.	0.	0.
time (sec)	N/A	1.23	1.289	0.055	2.159	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	286	286	325	570	1053	0	0	0
normalized size	1	1.	1.14	1.99	3.68	0.	0.	0.
time (sec)	N/A	0.597	1.189	0.054	2.155	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	206	206	293	462	846	0	0	0
normalized size	1	1.	1.42	2.24	4.11	0.	0.	0.
time (sec)	N/A	0.214	0.862	0.05	1.76	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	355	355	448	1186	0	0	0	0
normalized size	1	1.	1.26	3.34	0.	0.	0.	0.
time (sec)	N/A	0.813	0.828	1.046	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	361	361	479	1270	0	0	0	0
normalized size	1	1.	1.33	3.52	0.	0.	0.	0.
time (sec)	N/A	0.776	0.635	1.339	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	461	1358	0	0	0	0
normalized size	1	1.	1.2	3.53	0.	0.	0.	0.
time (sec)	N/A	0.788	1.004	1.214	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	396	396	569	1337	0	0	0	0
normalized size	1	1.	1.44	3.38	0.	0.	0.	0.
time (sec)	N/A	0.927	0.721	1.52	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	343	646	1098	0	0	0
normalized size	1	1.	1.27	2.38	4.05	0.	0.	0.
time (sec)	N/A	0.308	0.771	0.075	3.105	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	352	352	372	691	1057	0	0	0
normalized size	1	1.	1.06	1.96	3.	0.	0.	0.
time (sec)	N/A	0.368	1.104	0.083	3.154	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	479	479	402	736	1297	0	0	0
normalized size	1	1.	0.84	1.54	2.71	0.	0.	0.
time (sec)	N/A	0.509	1.377	0.073	3.172	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	329	329	347	1298	0	0	0	0
normalized size	1	1.	1.05	3.95	0.	0.	0.	0.
time (sec)	N/A	0.84	0.863	0.964	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	260	1192	0	0	0	0
normalized size	1	1.	1.05	4.83	0.	0.	0.	0.
time (sec)	N/A	0.533	0.504	0.623	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	172	172	140	5361	0	0	0	0
normalized size	1	1.	0.81	31.17	0.	0.	0.	0.
time (sec)	N/A	0.306	0.483	0.507	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	102	822	0	0	0	0
normalized size	1	1.	1.21	9.79	0.	0.	0.	0.
time (sec)	N/A	0.146	0.204	0.266	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	132	1389	0	0	0	0
normalized size	1	1.	1.71	18.04	0.	0.	0.	0.
time (sec)	N/A	0.166	0.374	0.306	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	225	7232	0	0	0	0
normalized size	1	1.	1.39	44.64	0.	0.	0.	0.
time (sec)	N/A	0.412	0.64	0.685	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	250	250	317	1841	0	0	0	0
normalized size	1	1.	1.27	7.36	0.	0.	0.	0.
time (sec)	N/A	0.634	1.016	1.205	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	388	2010	0	0	0	0
normalized size	1	1.	1.16	6.02	0.	0.	0.	0.
time (sec)	N/A	0.976	1.421	1.181	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	394	394	425	1467	0	0	0	0
normalized size	1	1.	1.08	3.72	0.	0.	0.	0.
time (sec)	N/A	0.841	1.68	0.806	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	331	354	1354	0	0	0	0
normalized size	1	1.	1.07	4.09	0.	0.	0.	0.
time (sec)	N/A	0.625	1.256	0.666	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	260	295	5542	0	0	0	0
normalized size	1	1.	1.13	21.32	0.	0.	0.	0.
time (sec)	N/A	0.476	0.919	0.507	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	233	1030	0	0	0	0
normalized size	1	1.	1.24	5.48	0.	0.	0.	0.
time (sec)	N/A	0.343	0.625	0.277	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	124	341	374	215	0	203
normalized size	1	1.	1.16	3.19	3.5	2.01	0.	1.9
time (sec)	N/A	0.124	0.132	0.056	0.99	2.053	0.	1.179

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	254	1566	0	0	0	0
normalized size	1	1.	0.86	5.31	0.	0.	0.	0.
time (sec)	N/A	0.64	0.848	0.339	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	371	371	347	7397	0	0	0	0
normalized size	1	1.	0.94	19.94	0.	0.	0.	0.
time (sec)	N/A	0.797	1.712	0.682	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	480	480	452	2009	0	0	0	0
normalized size	1	1.	0.94	4.19	0.	0.	0.	0.
time (sec)	N/A	0.953	2.152	1.	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	408	408	420	1565	0	0	0	0
normalized size	1	1.	1.03	3.84	0.	0.	0.	0.
time (sec)	N/A	0.811	2.013	0.816	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	418	5750	0	0	0	0
normalized size	1	1.	1.24	17.06	0.	0.	0.	0.
time (sec)	N/A	0.662	1.307	0.556	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	310	1241	0	0	0	0
normalized size	1	1.	1.17	4.68	0.	0.	0.	0.
time (sec)	N/A	0.544	1.395	0.345	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	150	460	579	354	0	0
normalized size	1	1.	0.96	2.93	3.69	2.25	0.	0.
time (sec)	N/A	0.214	0.308	0.066	1.029	1.977	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	183	398	539	338	0	0
normalized size	1	1.	1.17	2.54	3.43	2.15	0.	0.
time (sec)	N/A	0.178	0.137	0.062	1.033	1.986	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	376	1752	0	0	0	0
normalized size	1	1.	1.04	4.84	0.	0.	0.	0.
time (sec)	N/A	0.804	1.401	0.432	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	448	448	479	7593	0	0	0	0
normalized size	1	1.	1.07	16.95	0.	0.	0.	0.
time (sec)	N/A	0.983	2.26	0.723	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	168	386	601	458	0	0
normalized size	1	1.	0.95	2.19	3.41	2.6	0.	0.
time (sec)	N/A	0.218	0.173	0.062	1.045	1.968	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	59	717	0	0	0	0
normalized size	1	1.	0.88	10.7	0.	0.	0.	0.
time (sec)	N/A	0.138	0.15	0.32	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	644	963	0	0	0	0
normalized size	1	1.	2.1	3.15	0.	0.	0.	0.
time (sec)	N/A	0.658	1.446	0.316	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	488	811	0	0	0	0
normalized size	1	1.	2.03	3.38	0.	0.	0.	0.
time (sec)	N/A	0.443	0.988	0.278	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	334	6440	0	0	0	0
normalized size	1	1.	1.75	33.72	0.	0.	0.	0.
time (sec)	N/A	0.301	0.501	0.58	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	152	1491	0	0	0	0
normalized size	1	1.	1.37	13.43	0.	0.	0.	0.
time (sec)	N/A	0.221	0.287	0.313	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	198	1895	714	354	0	279
normalized size	1	1.	1.42	13.63	5.14	2.55	0.	2.01
time (sec)	N/A	0.193	0.149	0.377	1.061	2.024	0.	1.191

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	215	2752	1075	554	0	0
normalized size	1	1.	1.03	13.23	5.17	2.66	0.	0.
time (sec)	N/A	0.371	0.212	0.432	1.099	1.903	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	279	3637	1465	788	0	0
normalized size	1	1.	1.01	13.23	5.33	2.87	0.	0.
time (sec)	N/A	0.614	0.218	0.454	1.187	1.998	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	172	400	0	0	0	0
normalized size	1	1.	0.56	1.29	0.	0.	0.	0.
time (sec)	N/A	0.636	0.348	0.825	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	126	833	0	0	0	0
normalized size	1	1.	0.61	4.06	0.	0.	0.	0.
time (sec)	N/A	0.374	0.275	0.449	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	104	104	82	703	0	0	0	0
normalized size	1	1.	0.79	6.76	0.	0.	0.	0.
time (sec)	N/A	0.164	0.084	0.191	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	86	1217	0	0	0	0
normalized size	1	1.	0.92	13.09	0.	0.	0.	0.
time (sec)	N/A	0.172	0.148	0.284	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	86	1217	0	0	0	0
normalized size	1	1.	0.92	13.09	0.	0.	0.	0.
time (sec)	N/A	0.176	0.073	0.207	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	154	1451	0	0	0	0
normalized size	1	1.	0.81	7.6	0.	0.	0.	0.
time (sec)	N/A	0.463	0.29	0.536	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	222	664	0	0	0	0
normalized size	1	1.	0.73	2.18	0.	0.	0.	0.
time (sec)	N/A	0.748	0.629	0.984	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	384	384	233	496	0	0	0	0
normalized size	1	1.	0.61	1.29	0.	0.	0.	0.
time (sec)	N/A	0.862	0.407	0.571	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	172	454	0	0	0	0
normalized size	1	1.	0.66	1.74	0.	0.	0.	0.
time (sec)	N/A	0.498	0.27	0.223	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	112	285	0	0	0	0
normalized size	1	1.	0.85	2.18	0.	0.	0.	0.
time (sec)	N/A	0.214	0.109	0.201	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	102	843	0	0	0	0
normalized size	1	1.	0.86	7.14	0.	0.	0.	0.
time (sec)	N/A	0.221	0.169	0.277	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	102	843	0	0	0	0
normalized size	1	1.	0.86	7.14	0.	0.	0.	0.
time (sec)	N/A	0.225	0.078	0.214	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	172	583	0	0	0	0
normalized size	1	1.	0.72	2.44	0.	0.	0.	0.
time (sec)	N/A	0.549	0.501	0.309	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	380	380	250	858	0	0	0	0
normalized size	1	1.	0.66	2.26	0.	0.	0.	0.
time (sec)	N/A	0.956	0.868	0.524	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	2.3	0.274	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.112	0.253	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.133	0.298	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	1.936	0.252	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	1.046	0.115	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	1.216	0.113	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	474	381	0	0	0	0
normalized size	1	1.	1.72	1.39	0.	0.	0.	0.
time (sec)	N/A	0.256	6.675	0.125	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	394	298	0	0	0	0
normalized size	1	1.	1.84	1.39	0.	0.	0.	0.
time (sec)	N/A	0.2	3.085	0.115	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	315	217	0	0	0	0
normalized size	1	1.	2.02	1.39	0.	0.	0.	0.
time (sec)	N/A	0.154	2.442	0.112	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	104	148	0	0	0	0
normalized size	1	1.	0.91	1.3	0.	0.	0.	0.
time (sec)	N/A	0.071	0.196	0.103	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	148	148	294	210	0	0	0	0
normalized size	1	1.	1.99	1.42	0.	0.	0.	0.
time (sec)	N/A	0.164	1.639	0.119	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	360	279	0	0	0	0
normalized size	1	1.	1.8	1.4	0.	0.	0.	0.
time (sec)	N/A	0.204	3.222	0.134	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	261	261	435	367	0	0	0	0
normalized size	1	1.	1.67	1.41	0.	0.	0.	0.
time (sec)	N/A	0.245	6.121	0.125	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	385	385	1297	1656	0	0	0	0
normalized size	1	1.	3.37	4.3	0.	0.	0.	0.
time (sec)	N/A	0.427	16.059	1.298	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	1036	13923	0	0	0	0
normalized size	1	1.	3.71	49.9	0.	0.	0.	0.
time (sec)	N/A	0.258	15.014	1.167	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	188	188	938	1170	0	0	0	0
normalized size	1	1.	4.99	6.22	0.	0.	0.	0.
time (sec)	N/A	0.047	12.686	0.272	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	1034	1799	0	0	0	0
normalized size	1	1.	3.24	5.64	0.	0.	0.	0.
time (sec)	N/A	0.431	12.193	0.44	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	412	412	1188	26776	0	0	0	0
normalized size	1	1.	2.88	64.99	0.	0.	0.	0.
time (sec)	N/A	0.604	13.593	2.493	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	275	275	733	1507	0	0	0	0
normalized size	1	1.	2.67	5.48	0.	0.	0.	0.
time (sec)	N/A	0.336	9.249	0.247	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.51	0.684	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	99	166	71	105
normalized size	1	1.	1.	0.93	1.38	2.31	0.99	1.46
time (sec)	N/A	0.107	0.019	0.03	0.953	2.312	3.305	1.175

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	79	65	97	143	54	113
normalized size	1	1.	1.25	1.03	1.54	2.27	0.86	1.79
time (sec)	N/A	0.081	0.017	0.027	0.95	2.258	2.291	1.173

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	59	88	149	63	95
normalized size	1	1.	1.	0.95	1.42	2.4	1.02	1.53
time (sec)	N/A	0.087	0.015	0.03	0.951	2.19	1.839	1.162

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	69	57	50	117	46	100
normalized size	1	1.	1.72	1.42	1.25	2.92	1.15	2.5
time (sec)	N/A	0.022	0.014	0.027	0.965	2.194	1.366	1.163

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	48	63	115	49	69
normalized size	1	1.	0.73	0.75	0.98	1.8	0.77	1.08
time (sec)	N/A	0.021	0.009	0.026	0.954	2.212	0.996	1.17

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	48	48	60	69	120	0	0	0
normalized size	1	1.	1.25	1.44	2.5	0.	0.	0.
time (sec)	N/A	0.047	0.018	0.046	0.952	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	45	49	122	41	65
normalized size	1	1.	1.	1.18	1.29	3.21	1.08	1.71
time (sec)	N/A	0.051	0.01	0.033	0.943	2.23	1.231	1.155

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	68	87	109	0	0	0
normalized size	1	1.	1.21	1.55	1.95	0.	0.	0.
time (sec)	N/A	0.051	0.031	0.044	0.957	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	59	72	144	63	99
normalized size	1	1.	1.	1.02	1.24	2.48	1.09	1.71
time (sec)	N/A	0.076	0.015	0.038	0.945	2.242	1.732	1.17

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	71	59	82	116	46	97
normalized size	1	1.	1.69	1.4	1.95	2.76	1.1	2.31
time (sec)	N/A	0.03	0.016	0.036	0.956	2.244	1.237	1.236

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	68	84	167	75	109
normalized size	1	1.	1.	0.96	1.18	2.35	1.06	1.54
time (sec)	N/A	0.088	0.016	0.039	0.956	2.245	2.909	1.182

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	113	225	257	0	0	0
normalized size	1	1.	0.7	1.39	1.59	0.	0.	0.
time (sec)	N/A	0.579	0.902	0.049	0.984	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	88	205	197	247	114	139
normalized size	1	1.	0.76	1.77	1.7	2.13	0.98	1.2
time (sec)	N/A	0.44	0.045	0.045	1.019	2.122	3.201	1.181

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	95	205	234	0	0	0
normalized size	1	1.	0.69	1.49	1.7	0.	0.	0.
time (sec)	N/A	0.413	0.242	0.048	1.013	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	66	185	100	203	88	115
normalized size	1	1.	0.69	1.95	1.05	2.14	0.93	1.21
time (sec)	N/A	0.049	0.03	0.045	0.965	2.198	1.823	1.169

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	71	182	194	0	0	0
normalized size	1	1.	0.62	1.58	1.69	0.	0.	0.
time (sec)	N/A	0.103	0.05	0.047	0.98	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	145	663	0	0	0	0
normalized size	1	1.	0.99	4.54	0.	0.	0.	0.
time (sec)	N/A	0.312	0.044	0.836	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	102	170	205	0	0	0
normalized size	1	1.	1.1	1.83	2.2	0.	0.	0.
time (sec)	N/A	0.218	0.132	0.055	0.986	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	174	741	0	0	0	0
normalized size	1	1.	1.01	4.31	0.	0.	0.	0.
time (sec)	N/A	0.334	0.074	1.208	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	93	237	254	0	0	0
normalized size	1	1.	0.8	2.04	2.19	0.	0.	0.
time (sec)	N/A	0.308	0.262	0.06	0.985	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	82	199	221	239	102	0
normalized size	1	1.	0.92	2.24	2.48	2.69	1.15	0.
time (sec)	N/A	0.108	0.033	0.06	0.968	2.519	2.461	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	114	258	308	0	0	0
normalized size	1	1.	0.8	1.8	2.15	0.	0.	0.
time (sec)	N/A	0.445	0.431	0.062	0.998	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	134	829	0	0	0	0
normalized size	1	1.	0.85	5.28	0.	0.	0.	0.
time (sec)	N/A	0.192	0.294	0.551	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	232	251	374	0	0	0
normalized size	1	1.	1.2	1.3	1.94	0.	0.	0.
time (sec)	N/A	0.227	0.247	0.049	1.461	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.745	0.252	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.318	0.233	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	1.091	0.26	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.966	0.25	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.315	0.233	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	1.146	0.263	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.013	1.142	0.237	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	87	120	213	100	127
normalized size	1	1.	1.	0.91	1.25	2.22	1.04	1.32
time (sec)	N/A	0.189	0.027	0.028	0.961	1.985	4.934	1.158

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	103	85	119	177	76	135
normalized size	1	1.	1.18	0.98	1.37	2.03	0.87	1.55
time (sec)	N/A	0.137	0.028	0.027	0.943	1.955	4.001	1.183

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	86	79	109	190	90	116
normalized size	1	1.	1.	0.92	1.27	2.21	1.05	1.35
time (sec)	N/A	0.161	0.021	0.029	0.95	2.08	3.117	1.16

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	93	77	62	154	68	77
normalized size	1	1.	1.86	1.54	1.24	3.08	1.36	1.54
time (sec)	N/A	0.037	0.023	0.028	0.951	1.906	2.185	1.161

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	71	68	89	161	75	103
normalized size	1	1.	0.68	0.65	0.86	1.55	0.72	0.99
time (sec)	N/A	0.044	0.017	0.03	0.975	1.952	1.771	1.168

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	70	70	73	89	143	0	0	0
normalized size	1	1.	1.04	1.27	2.04	0.	0.	0.
time (sec)	N/A	0.097	0.073	0.039	0.955	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	64	65	77	150	68	89
normalized size	1	1.	1.	1.02	1.2	2.34	1.06	1.39
time (sec)	N/A	0.111	0.016	0.039	0.949	2.	2.699	1.216

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	80	111	0	0	0
normalized size	1	1.	1.	1.29	1.79	0.	0.	0.
time (sec)	N/A	0.094	0.061	0.042	0.954	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	69	89	162	75	109
normalized size	1	1.	1.	1.01	1.31	2.38	1.1	1.6
time (sec)	N/A	0.11	0.018	0.036	0.95	2.113	2.423	1.222

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	77	77	84	105	151	0	0	0
normalized size	1	1.	1.09	1.36	1.96	0.	0.	0.
time (sec)	N/A	0.101	0.082	0.047	0.965	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	80	96	192	88	120
normalized size	1	1.	1.	0.96	1.16	2.31	1.06	1.45
time (sec)	N/A	0.138	0.023	0.038	0.949	1.962	3.412	1.175

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	202	202	138	259	289	0	0	0
normalized size	1	1.	0.68	1.28	1.43	0.	0.	0.
time (sec)	N/A	1.025	1.805	0.049	0.981	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	108	239	230	300	153	174
normalized size	1	1.	0.69	1.53	1.47	1.92	0.98	1.12
time (sec)	N/A	0.819	0.063	0.047	0.96	1.883	5.252	1.182

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	178	178	121	239	267	0	0	0
normalized size	1	1.	0.68	1.34	1.5	0.	0.	0.
time (sec)	N/A	0.779	1.099	0.05	0.984	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	82	219	126	261	133	151
normalized size	1	1.	0.59	1.59	0.91	1.89	0.96	1.09
time (sec)	N/A	0.088	0.054	0.049	0.965	2.006	3.571	1.209

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	99	216	236	0	0	0
normalized size	1	1.	0.58	1.26	1.38	0.	0.	0.
time (sec)	N/A	0.133	0.654	0.047	0.963	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	191	728	0	0	0	0
normalized size	1	1.	1.03	3.91	0.	0.	0.	0.
time (sec)	N/A	0.53	0.063	1.165	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	156	156	182	222	270	0	0	0
normalized size	1	1.	1.17	1.42	1.73	0.	0.	0.
time (sec)	N/A	0.418	0.056	0.056	0.977	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	183	774	0	0	0	0
normalized size	1	1.	1.13	4.78	0.	0.	0.	0.
time (sec)	N/A	0.461	0.07	1.15	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	167	167	153	249	274	0	0	0
normalized size	1	1.	0.92	1.49	1.64	0.	0.	0.
time (sec)	N/A	0.43	0.069	0.061	0.989	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	238	927	0	0	0	0
normalized size	1	1.	1.11	4.33	0.	0.	0.	0.
time (sec)	N/A	0.549	0.36	2.052	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	118	272	323	0	0	0
normalized size	1	1.	0.75	1.73	2.06	0.	0.	0.
time (sec)	N/A	0.594	0.762	0.066	0.988	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	99	233	254	300	148	0
normalized size	1	1.	0.88	2.06	2.25	2.65	1.31	0.
time (sec)	N/A	0.194	0.059	0.06	0.964	2.117	4.792	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	140	292	343	0	0	0
normalized size	1	1.	0.77	1.6	1.87	0.	0.	0.
time (sec)	N/A	0.816	1.381	0.064	0.994	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	124	253	275	338	168	0
normalized size	1	1.	0.73	1.49	1.62	1.99	0.99	0.
time (sec)	N/A	0.841	0.065	0.062	0.974	1.995	5.916	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	183	883	0	0	0	0
normalized size	1	1.	0.74	3.56	0.	0.	0.	0.
time (sec)	N/A	0.252	0.602	1.285	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.904	0.32	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.477	0.328	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	1.447	0.38	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.753	0.379	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	1.105	0.313	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.01	0.32	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	79	88	111	198	97	124
normalized size	1	1.	0.55	0.61	0.77	1.38	0.67	0.86
time (sec)	N/A	0.067	0.046	0.029	0.967	2.391	4.303	1.207

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	124	250	269	0	0	0
normalized size	1	1.	0.55	1.1	1.19	0.	0.	0.
time (sec)	N/A	0.171	1.183	0.049	0.977	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	231	932	0	0	0	0
normalized size	1	1.	0.68	2.76	0.	0.	0.	0.
time (sec)	N/A	0.333	1.097	2.487	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	60	165	162	0	0	0
normalized size	1	1.	0.69	1.9	1.86	0.	0.	0.
time (sec)	N/A	0.133	0.127	0.049	0.969	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	145	115	128	41	80
normalized size	1	1.	1.	3.45	2.74	3.05	0.98	1.9
time (sec)	N/A	0.069	0.037	0.049	0.97	2.096	1.62	1.197

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	44	125	169	0	0	0
normalized size	1	1.	0.81	2.31	3.13	0.	0.	0.
time (sec)	N/A	0.071	0.049	0.046	0.967	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	88	47	10	30
normalized size	1	1.	1.	0.92	6.77	3.62	0.77	2.31
time (sec)	N/A	0.016	0.004	0.025	0.976	1.871	1.458	1.135

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	45	45	42	130	178	0	0	0
normalized size	1	1.	0.93	2.89	3.96	0.	0.	0.
time (sec)	N/A	0.087	0.067	0.052	0.97	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	132	111	150	37	0
normalized size	1	1.	1.	3.22	2.71	3.66	0.9	0.
time (sec)	N/A	0.078	0.04	0.056	0.96	2.132	3.383	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	84	84	60	209	219	0	0	0
normalized size	1	1.	0.71	2.49	2.61	0.	0.	0.
time (sec)	N/A	0.155	0.259	0.058	0.97	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	112	812	0	0	0	0
normalized size	1	1.	0.83	6.01	0.	0.	0.	0.
time (sec)	N/A	0.302	0.122	0.292	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	59	5573	270	0	0	0
normalized size	1	1.	0.79	74.31	3.6	0.	0.	0.
time (sec)	N/A	0.169	0.172	0.394	0.99	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	68	741	0	0	0	0
normalized size	1	1.	0.87	9.5	0.	0.	0.	0.
time (sec)	N/A	0.157	0.065	0.278	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	171	49	10	30
normalized size	1	1.	1.	0.92	13.15	3.77	0.77	2.31
time (sec)	N/A	0.026	0.005	0.023	0.975	2.278	2.232	1.163

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	60	1188	0	0	0	0
normalized size	1	1.	0.91	18.	0.	0.	0.	0.
time (sec)	N/A	0.181	0.063	0.268	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	61	4449	320	0	0	0
normalized size	1	1.	0.92	67.41	4.85	0.	0.	0.
time (sec)	N/A	0.215	0.223	0.374	0.993	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	133	1360	0	0	0	0
normalized size	1	1.	0.96	9.86	0.	0.	0.	0.
time (sec)	N/A	0.357	0.341	0.41	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	142	248	0	0	0	0
normalized size	1	1.	0.69	1.21	0.	0.	0.	0.
time (sec)	N/A	0.469	0.269	0.895	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	78	788	0	0	0	0
normalized size	1	1.	0.76	7.65	0.	0.	0.	0.
time (sec)	N/A	0.269	0.242	0.327	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	87	776	0	0	0	0
normalized size	1	1.	0.81	7.19	0.	0.	0.	0.
time (sec)	N/A	0.206	0.064	0.285	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	282	49	10	30
normalized size	1	1.	1.	0.92	21.69	3.77	0.77	2.31
time (sec)	N/A	0.023	0.005	0.021	0.982	2.208	1.121	1.184

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	83	1245	0	0	0	0
normalized size	1	1.	0.91	13.68	0.	0.	0.	0.
time (sec)	N/A	0.221	0.073	0.349	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	93	826	0	0	0	0
normalized size	1	1.	1.03	9.18	0.	0.	0.	0.
time (sec)	N/A	0.273	0.194	0.343	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	165	406	0	0	0	0
normalized size	1	1.	0.82	2.03	0.	0.	0.	0.
time (sec)	N/A	0.496	0.444	1.251	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	63	14	34
normalized size	1	1.	1.	0.8	0.	4.2	0.93	2.27
time (sec)	N/A	0.025	0.008	0.036	0.	2.43	1.766	1.153

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	1.313	0.132	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	28	46	7	28
normalized size	1	1.	1.	1.11	3.11	5.11	0.78	3.11
time (sec)	N/A	0.028	0.029	0.023	0.955	2.268	0.892	1.173

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.227	0.117	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.128	0.117	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	31	46	8	30
normalized size	1	1.	1.	1.09	2.82	4.18	0.73	2.73
time (sec)	N/A	0.025	0.005	0.023	0.97	2.109	1.211	1.14

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.141	0.118	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.562	0.115	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	57	49	12	30
normalized size	1	1.	1.	0.92	4.38	3.77	0.92	2.31
time (sec)	N/A	0.026	0.005	0.024	0.974	2.244	1.5	1.192

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	0.721	0.127	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	212	26	41
normalized size	1	1.	1.	1.06	0.	12.47	1.53	2.41
time (sec)	N/A	0.031	0.011	0.024	0.	2.151	2.971	1.194

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	64	203	239	0	0	0
normalized size	1	1.	0.59	1.86	2.19	0.	0.	0.
time (sec)	N/A	0.16	0.159	0.057	0.969	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	169	170	142	0	0
normalized size	1	1.	0.79	2.96	2.98	2.49	0.	0.
time (sec)	N/A	0.063	0.096	0.053	0.965	2.002	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	66	68	84	96	61	99
normalized size	1	1.	1.2	1.24	1.53	1.75	1.11	1.8
time (sec)	N/A	0.038	0.054	0.034	0.948	1.927	1.916	1.114

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	44	169	165	139	0	0
normalized size	1	1.	0.81	3.13	3.06	2.57	0.	0.
time (sec)	N/A	0.022	0.072	0.052	0.96	1.859	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	63	190	278	0	0	0
normalized size	1	1.	0.69	2.09	3.05	0.	0.	0.
time (sec)	N/A	0.164	0.168	0.064	0.982	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	77	180	203	252	253	0
normalized size	1	1.	0.94	2.2	2.48	3.07	3.09	0.
time (sec)	N/A	0.145	0.141	0.062	0.983	2.129	4.477	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	83	265	315	0	0	0
normalized size	1	1.	0.67	2.15	2.56	0.	0.	0.
time (sec)	N/A	0.358	0.43	0.066	0.99	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	103	907	0	0	0	0
normalized size	1	1.	0.64	5.63	0.	0.	0.	0.
time (sec)	N/A	0.291	0.178	0.348	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	93	1722	369	211	0	0
normalized size	1	1.	0.99	18.32	3.93	2.24	0.	0.
time (sec)	N/A	0.103	0.13	0.406	0.98	1.922	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	43	191	197	140	0	0
normalized size	1	1.	0.52	2.33	2.4	1.71	0.	0.
time (sec)	N/A	0.068	0.051	0.055	0.966	2.097	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	93	1695	362	208	0	0
normalized size	1	1.	1.06	19.26	4.11	2.36	0.	0.
time (sec)	N/A	0.064	0.111	0.413	0.986	2.052	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	106	1290	0	0	0	0
normalized size	1	1.	0.78	9.49	0.	0.	0.	0.
time (sec)	N/A	0.292	0.184	0.431	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	97	4589	548	0	0	0
normalized size	1	1.	0.68	32.32	3.86	0.	0.	0.
time (sec)	N/A	0.315	0.348	0.55	1.015	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	146	3040	0	0	0	0
normalized size	1	1.	0.71	14.83	0.	0.	0.	0.
time (sec)	N/A	0.696	1.024	1.34	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	227	227	139	1015	0	0	0	0
normalized size	1	1.	0.61	4.47	0.	0.	0.	0.
time (sec)	N/A	0.403	0.181	0.348	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	72	1771	628	258	0	0
normalized size	1	1.	0.6	14.64	5.19	2.13	0.	0.
time (sec)	N/A	0.134	0.081	0.442	1.038	1.916	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	91	1708	402	209	0	0
normalized size	1	1.	0.76	14.35	3.38	1.76	0.	0.
time (sec)	N/A	0.112	0.058	0.407	1.003	1.946	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	71	1742	620	255	0	0
normalized size	1	1.	0.62	15.15	5.39	2.22	0.	0.
time (sec)	N/A	0.096	0.058	0.408	1.033	1.924	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	193	135	1387	0	0	0	0
normalized size	1	1.	0.7	7.19	0.	0.	0.	0.
time (sec)	N/A	0.388	0.185	0.501	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	144	442	0	0	0	0
normalized size	1	1.	0.75	2.31	0.	0.	0.	0.
time (sec)	N/A	0.435	0.341	0.871	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	215	672	0	0	0	0
normalized size	1	1.	0.71	2.23	0.	0.	0.	0.
time (sec)	N/A	0.955	0.75	1.29	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	87	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.224	0.356	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	4.573	0.191	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	42	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	1.598	0.237	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	0	161	0	0
normalized size	1	1.	1.	0.89	0.	5.96	0.	0.
time (sec)	N/A	0.103	0.113	0.064	0.	2.012	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	112	0	0
normalized size	1	1.	1.	0.93	0.	8.	0.	0.
time (sec)	N/A	0.07	0.064	0.058	0.	1.97	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	24	0	157	0	0
normalized size	1	1.	0.74	0.89	0.	5.81	0.	0.
time (sec)	N/A	0.066	0.09	0.057	0.	2.028	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.998	0.155	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	60	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.316	3.339	0.209	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	36	36	0	258	0	0
normalized size	1	1.	0.95	0.95	0.	6.79	0.	0.
time (sec)	N/A	0.133	0.14	0.063	0.	2.14	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	32	28	0	252	0	0
normalized size	1	1.	0.89	0.78	0.	7.	0.	0.
time (sec)	N/A	0.207	0.072	0.063	0.	2.01	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	30	36	0	244	0	0
normalized size	1	1.	0.86	1.03	0.	6.97	0.	0.
time (sec)	N/A	0.095	0.091	0.066	0.	1.99	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	3.963	0.162	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.225	10.891	0.227	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	47	51	0	309	0	0
normalized size	1	1.	0.73	0.8	0.	4.83	0.	0.
time (sec)	N/A	0.272	0.113	0.063	0.	2.081	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	43	0	317	0	0
normalized size	1	1.	0.92	0.6	0.	4.4	0.	0.
time (sec)	N/A	0.112	0.061	0.06	0.	1.955	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	51	0	296	0	0
normalized size	1	1.	1.	0.88	0.	5.1	0.	0.
time (sec)	N/A	0.234	0.073	0.064	0.	1.966	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.247	3.424	0.169	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	73	68	0	358	0	0
normalized size	1	1.	0.75	0.7	0.	3.69	0.	0.
time (sec)	N/A	0.143	0.133	0.064	0.	1.97	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	84	83	0	408	0	0
normalized size	1	1.	0.7	0.69	0.	3.4	0.	0.
time (sec)	N/A	0.29	0.092	0.065	0.	2.077	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	101	98	0	473	0	0
normalized size	1	1.	0.66	0.64	0.	3.07	0.	0.
time (sec)	N/A	0.196	0.124	0.067	0.	2.013	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	112	113	0	524	0	0
normalized size	1	1.	0.63	0.64	0.	2.96	0.	0.
time (sec)	N/A	0.344	0.095	0.063	0.	2.078	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	128	128	0	591	0	0
normalized size	1	1.	0.61	0.61	0.	2.8	0.	0.
time (sec)	N/A	0.256	0.125	0.066	0.	2.304	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	136	134	151	158	127
normalized size	1	1.	1.27	1.77	1.74	1.96	2.05	1.65
time (sec)	N/A	0.063	0.083	0.043	0.961	2.023	4.001	1.231

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	61	225	242	201	0	0
normalized size	1	1.	0.61	2.25	2.42	2.01	0.	0.
time (sec)	N/A	0.072	0.084	0.057	0.985	1.939	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	92	111	150	158	113
normalized size	1	1.	1.17	1.23	1.48	2.	2.11	1.51
time (sec)	N/A	0.047	0.057	0.035	0.969	1.993	4.028	1.188

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	65	225	246	213	0	0
normalized size	1	1.	0.69	2.39	2.62	2.27	0.	0.
time (sec)	N/A	0.045	0.063	0.061	0.98	2.03	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	81	234	362	0	0	0
normalized size	1	1.	0.63	1.81	2.81	0.	0.	0.
time (sec)	N/A	0.25	0.2	0.063	0.999	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	94	228	275	358	578	0
normalized size	1	1.	0.76	1.85	2.24	2.91	4.7	0.
time (sec)	N/A	0.235	0.162	0.06	0.997	2.067	8.623	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	71	297	305	217	0	0
normalized size	1	1.	0.56	2.34	2.4	1.71	0.	0.
time (sec)	N/A	0.184	0.093	0.07	0.986	1.932	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	121	2571	524	294	0	0
normalized size	1	1.	0.74	15.77	3.21	1.8	0.	0.
time (sec)	N/A	0.25	0.116	0.441	1.004	2.054	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	71	247	278	219	0	0
normalized size	1	1.	0.57	1.98	2.22	1.75	0.	0.
time (sec)	N/A	0.094	0.07	0.058	0.983	2.075	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	127	2571	529	302	0	0
normalized size	1	1.	0.84	17.03	3.5	2.	0.	0.
time (sec)	N/A	0.11	0.107	0.443	1.023	1.905	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	196	196	129	1392	0	0	0	0
normalized size	1	1.	0.66	7.1	0.	0.	0.	0.
time (sec)	N/A	0.449	0.397	0.506	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	127	4797	721	0	0	0
normalized size	1	1.	0.61	22.95	3.45	0.	0.	0.
time (sec)	N/A	0.492	0.567	0.602	1.038	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	135	2634	590	313	0	0
normalized size	1	1.	0.7	13.72	3.07	1.63	0.	0.
time (sec)	N/A	0.268	0.128	0.457	1.031	2.175	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	107	2646	887	348	0	0
normalized size	1	1.	0.5	12.31	4.13	1.62	0.	0.
time (sec)	N/A	0.359	0.123	0.432	1.06	2.037	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	148	2582	570	312	0	0
normalized size	1	1.	0.79	13.73	3.03	1.66	0.	0.
time (sec)	N/A	0.164	0.086	0.348	1.014	2.06	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	111	2646	895	375	0	0
normalized size	1	1.	0.55	13.03	4.41	1.85	0.	0.
time (sec)	N/A	0.177	0.154	0.452	1.066	2.127	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	189	1533	0	0	0	0
normalized size	1	1.	0.68	5.53	0.	0.	0.	0.
time (sec)	N/A	0.626	0.283	0.477	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	218	842	0	0	0	0
normalized size	1	1.	0.78	3.	0.	0.	0.	0.
time (sec)	N/A	0.69	0.691	0.714	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	152	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	0.485	0.469	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	7.708	0.196	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	14.659	0.189	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	36	0	328	0	0
normalized size	1	1.	0.76	0.88	0.	8.	0.	0.
time (sec)	N/A	0.122	0.12	0.059	0.	2.296	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	281	0	0
normalized size	1	1.	0.83	0.83	0.	9.69	0.	0.
time (sec)	N/A	0.118	0.111	0.062	0.	2.273	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	0	224	0	0
normalized size	1	1.	0.81	0.89	0.	8.3	0.	0.
time (sec)	N/A	0.113	0.097	0.066	0.	2.349	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	281	0	0
normalized size	1	1.	0.83	0.83	0.	9.69	0.	0.
time (sec)	N/A	0.094	0.107	0.06	0.	2.562	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	33	36	0	325	0	0
normalized size	1	1.	0.8	0.88	0.	7.93	0.	0.
time (sec)	N/A	0.083	0.069	0.066	0.	2.271	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	48	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	1.232	0.185	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.905	11.958	0.199	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	49	62	0	545	0	0
normalized size	1	1.	0.92	1.17	0.	10.28	0.	0.
time (sec)	N/A	0.186	0.186	0.069	0.	2.323	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	76	80	54	0	540	0	0
normalized size	1	1.38	1.45	0.98	0.	9.82	0.	0.
time (sec)	N/A	0.521	0.114	0.06	0.	2.359	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	60	56	38	0	370	0	0
normalized size	1	1.46	1.37	0.93	0.	9.02	0.	0.
time (sec)	N/A	0.287	0.202	0.067	0.	2.332	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	75	54	0	535	0	0
normalized size	1	1.	1.42	1.02	0.	10.09	0.	0.
time (sec)	N/A	0.245	0.099	0.062	0.	2.37	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	43	60	0	531	0	0
normalized size	1	1.	0.88	1.22	0.	10.84	0.	0.
time (sec)	N/A	0.121	0.132	0.061	0.	2.018	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.65	4.268	0.249	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	60	90	0	597	0	0
normalized size	1	1.	0.6	0.9	0.	5.97	0.	0.
time (sec)	N/A	0.594	0.21	0.07	0.	2.028	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	160	66	82	0	621	0	0
normalized size	1	1.5	0.62	0.77	0.	5.8	0.	0.
time (sec)	N/A	0.655	0.26	0.072	0.	1.994	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	109	56	51	0	437	0	0
normalized size	1	1.27	0.65	0.59	0.	5.08	0.	0.
time (sec)	N/A	0.586	0.155	0.067	0.	1.983	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	96	82	0	606	0	0
normalized size	1	1.	0.96	0.82	0.	6.06	0.	0.
time (sec)	N/A	0.464	0.19	0.062	0.	1.999	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	88	0	578	0	0
normalized size	1	1.	1.25	1.28	0.	8.38	0.	0.
time (sec)	N/A	0.27	0.178	0.064	0.	2.014	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	175	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.767	5.386	0.221	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	108	122	0	647	0	0
normalized size	1	1.	0.86	0.98	0.	5.18	0.	0.
time (sec)	N/A	0.5	0.247	0.073	0.	2.059	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	132	152	0	720	0	0
normalized size	1	1.	0.78	0.89	0.	4.24	0.	0.
time (sec)	N/A	0.963	0.164	0.071	0.	2.038	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	166	182	0	803	0	0
normalized size	1	1.	0.65	0.71	0.	3.12	0.	0.
time (sec)	N/A	1.349	0.3	0.07	0.	1.999	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	81	281	324	293	0	0
normalized size	1	1.	0.6	2.1	2.42	2.19	0.	0.
time (sec)	N/A	0.074	0.182	0.059	1.003	1.949	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	157	3447	697	412	0	0
normalized size	1	1.	0.73	16.11	3.26	1.93	0.	0.
time (sec)	N/A	0.169	0.289	0.512	1.035	2.036	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	143	3550	1176	506	0	0
normalized size	1	1.	0.49	12.2	4.04	1.74	0.	0.
time (sec)	N/A	0.327	0.153	0.523	1.113	2.061	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	252	252	257	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.301	0.764	0.497	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	8.307	0.237	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	38.73	0.182	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	48	0	568	0	0
normalized size	1	1.	0.73	0.87	0.	10.33	0.	0.
time (sec)	N/A	0.145	0.156	0.072	0.	2.038	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	516	0	0
normalized size	1	1.	0.77	0.77	0.	12.	0.	0.
time (sec)	N/A	0.144	0.162	0.061	0.	2.03	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	48	0	558	0	0
normalized size	1	1.	0.73	0.87	0.	10.15	0.	0.
time (sec)	N/A	0.155	0.135	0.076	0.	1.957	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	348	0	0
normalized size	1	1.	0.83	0.83	0.	12.	0.	0.
time (sec)	N/A	0.125	0.141	0.066	0.	1.95	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	48	0	559	0	0
normalized size	1	1.	1.	0.87	0.	10.16	0.	0.
time (sec)	N/A	0.142	0.159	0.076	0.	1.968	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	33	0	516	0	0
normalized size	1	1.	1.	0.77	0.	12.	0.	0.
time (sec)	N/A	0.112	0.198	0.063	0.	1.91	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	48	0	564	0	0
normalized size	1	1.	0.73	0.87	0.	10.25	0.	0.
time (sec)	N/A	0.103	0.16	0.073	0.	2.022	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	1.442	0.202	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	2.891	0.224	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	56	78	0	965	0	0
normalized size	1	1.	0.84	1.16	0.	14.4	0.	0.
time (sec)	N/A	0.307	0.185	0.069	0.	2.02	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	86	0	954	0	0
normalized size	1	1.	0.85	1.3	0.	14.45	0.	0.
time (sec)	N/A	0.143	0.281	0.068	0.	1.991	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	73	121	0	1037	0	0
normalized size	1	1.	0.64	1.06	0.	9.1	0.	0.
time (sec)	N/A	0.585	0.358	0.076	0.	1.84	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	131	0	1014	0	0
normalized size	1	1.	0.93	1.47	0.	11.39	0.	0.
time (sec)	N/A	0.386	0.187	0.075	0.	1.939	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	79	120	269	212	0	161
normalized size	1	1.	0.57	0.86	1.94	1.53	0.	1.16
time (sec)	N/A	0.235	0.099	0.282	1.458	2.082	0.	1.259

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	160	175	0	0	0	0
normalized size	1	1.	0.81	0.89	0.	0.	0.	0.
time (sec)	N/A	0.217	0.504	0.262	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	60	99	151	166	0	109
normalized size	1	1.	0.69	1.14	1.74	1.91	0.	1.25
time (sec)	N/A	0.124	0.069	0.236	1.448	2.186	0.	1.229

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	125	154	0	0	0	0
normalized size	1	1.	0.86	1.05	0.	0.	0.	0.
time (sec)	N/A	0.105	0.219	0.26	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	29	81	57	135	0	63
normalized size	1	1.	0.91	2.53	1.78	4.22	0.	1.97
time (sec)	N/A	0.047	0.031	0.222	1.444	2.075	0.	1.214

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	76	366	0	0	0	0
normalized size	1	1.	0.8	3.85	0.	0.	0.	0.
time (sec)	N/A	0.026	0.073	0.247	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	57	99	0	0	0	0
normalized size	1	1.	0.76	1.32	0.	0.	0.	0.
time (sec)	N/A	0.065	0.092	0.235	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	48	72	69	127	0	150
normalized size	1	1.	1.14	1.71	1.64	3.02	0.	3.57
time (sec)	N/A	0.076	0.051	0.231	1.44	2.099	0.	1.27

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	126	141	0	0	0	0
normalized size	1	1.	0.92	1.03	0.	0.	0.	0.
time (sec)	N/A	0.136	0.69	0.254	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	160	175	0	0	0	0
normalized size	1	1.	0.78	0.85	0.	0.	0.	0.
time (sec)	N/A	0.314	0.392	0.258	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	161	161	188	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.255	0.763	0.233	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	104	151	0	0	0	0
normalized size	1	1.	0.87	1.26	0.	0.	0.	0.
time (sec)	N/A	0.099	0.215	0.237	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	119	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.099	0.359	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	68	68	100	158	0	0	0	0
normalized size	1	1.	1.47	2.32	0.	0.	0.	0.
time (sec)	N/A	0.141	0.084	0.251	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	89	131	0	0	0	0
normalized size	1	1.	0.85	1.25	0.	0.	0.	0.
time (sec)	N/A	0.167	0.49	0.245	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	152	152	188	231	0	0	0	0
normalized size	1	1.	1.24	1.52	0.	0.	0.	0.
time (sec)	N/A	0.415	1.197	0.283	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	215	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.57	0.887	0.3	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	570	0	0	0	0	0
normalized size	1	1.	1.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.34	4.467	0.285	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	157	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	0.18	0.267	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	451	0	0	0	0	0
normalized size	1	1.	2.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.45	0.377	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	146	215	0	0	0	0
normalized size	1	1.	1.43	2.11	0.	0.	0.	0.
time (sec)	N/A	0.173	0.134	0.275	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	131	190	0	0	0	0
normalized size	1	1.	1.34	1.94	0.	0.	0.	0.
time (sec)	N/A	0.249	0.372	0.279	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	267	267	416	386	0	0	0	0
normalized size	1	1.	1.56	1.45	0.	0.	0.	0.
time (sec)	N/A	0.447	8.749	0.321	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.425	0.826	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	76	144	146	201	0	120
normalized size	1	1.	1.03	1.95	1.97	2.72	0.	1.62
time (sec)	N/A	0.17	0.063	0.249	1.453	2.022	0.	1.229

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	121	190	0	0	0	0
normalized size	1	1.	0.88	1.39	0.	0.	0.	0.
time (sec)	N/A	0.105	0.226	0.346	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	27	66	53	103	0	82
normalized size	1	1.	0.63	1.53	1.23	2.4	0.	1.91
time (sec)	N/A	0.051	0.036	0.205	0.955	2.005	0.	1.239

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	27	38	49	101	0	80
normalized size	1	1.	0.68	0.95	1.22	2.52	0.	2.
time (sec)	N/A	0.025	0.031	0.22	0.957	1.981	0.	1.229

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	97	157	0	0	0	0
normalized size	1	1.	0.87	1.4	0.	0.	0.	0.
time (sec)	N/A	0.186	0.179	0.253	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	89	132	113	223	0	209
normalized size	1	1.	1.09	1.61	1.38	2.72	0.	2.55
time (sec)	N/A	0.171	0.11	0.253	0.957	2.065	0.	1.261

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	182	205	0	0	0	0
normalized size	1	1.	1.02	1.15	0.	0.	0.	0.
time (sec)	N/A	0.402	1.527	0.289	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.49	0.759	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	165	230	0	0	0	0
normalized size	1	1.	0.89	1.24	0.	0.	0.	0.
time (sec)	N/A	0.31	0.372	0.273	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	193	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.312	0.242	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	82	84	146	0	0
normalized size	1	1.	0.5	1.21	1.24	2.15	0.	0.
time (sec)	N/A	0.098	0.055	0.211	0.953	2.002	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	38	49	77	150	0	0
normalized size	1	1.	0.6	0.78	1.22	2.38	0.	0.
time (sec)	N/A	0.04	0.041	0.225	0.956	2.285	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	159	232	0	0	0	0
normalized size	1	1.	1.25	1.83	0.	0.	0.	0.
time (sec)	N/A	0.341	0.241	0.27	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	171	171	215	207	0	0	0	0
normalized size	1	1.	1.26	1.21	0.	0.	0.	0.
time (sec)	N/A	0.313	1.078	0.27	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	221	221	266	313	0	0	0	0
normalized size	1	1.	1.2	1.42	0.	0.	0.	0.
time (sec)	N/A	0.776	2.703	0.299	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.476	0.747	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	249	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.405	0.42	0.291	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	541	0	0	0	0	0
normalized size	1	1.	2.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.936	0.276	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	45	98	119	197	0	0
normalized size	1	1.	0.48	1.04	1.27	2.1	0.	0.
time (sec)	N/A	0.125	0.062	0.212	0.96	2.283	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	45	56	116	196	0	0
normalized size	1	1.	0.51	0.64	1.32	2.23	0.	0.
time (sec)	N/A	0.071	0.05	0.224	0.968	2.248	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	230	305	0	0	0	0
normalized size	1	1.	1.24	1.65	0.	0.	0.	0.
time (sec)	N/A	0.409	0.339	0.287	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	270	282	0	0	0	0
normalized size	1	1.	1.44	1.51	0.	0.	0.	0.
time (sec)	N/A	0.425	1.89	0.292	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	555	482	0	0	0	0
normalized size	1	1.	1.54	1.34	0.	0.	0.	0.
time (sec)	N/A	0.982	7.73	0.346	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.411	0.74	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	3.537	0.27	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	26	0	0	0	0
normalized size	1	1.	1.	2.89	0.	0.	0.	0.
time (sec)	N/A	0.104	0.069	0.21	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0
normalized size	1	1.	1.	1.11	0.	0.	0.	0.
time (sec)	N/A	0.062	0.08	0.219	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	1.037	0.286	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.444	0.738	0.	0.	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	3.062	0.289	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	34	90	0	0	0	0
normalized size	1	1.	0.94	2.5	0.	0.	0.	0.
time (sec)	N/A	0.122	0.032	0.227	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0
normalized size	1	1.	0.91	1.77	0.	0.	0.	0.
time (sec)	N/A	0.126	0.084	0.22	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.417	5.899	0.287	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.49	0.746	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	6.189	0.273	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	43	154	0	0	0	0
normalized size	1	1.	0.63	2.26	0.	0.	0.	0.
time (sec)	N/A	0.196	0.123	0.248	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	86	0	0	0	0
normalized size	1	1.	0.68	1.32	0.	0.	0.	0.
time (sec)	N/A	0.163	0.09	0.228	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	123	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.485	18.073	0.311	0.	0.	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	243	243	178	195	0	0	0	0
normalized size	1	1.	0.73	0.8	0.	0.	0.	0.
time (sec)	N/A	0.308	0.669	0.282	0.	0.	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	79	120	201	205	0	139
normalized size	1	1.	0.58	0.88	1.48	1.51	0.	1.02
time (sec)	N/A	0.203	0.065	0.259	1.475	2.242	0.	1.187

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	160	175	0	0	0	0
normalized size	1	1.	0.82	0.9	0.	0.	0.	0.
time (sec)	N/A	0.193	0.437	0.26	0.	0.	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	99	80	163	0	86
normalized size	1	1.	0.83	1.68	1.36	2.76	0.	1.46
time (sec)	N/A	0.049	0.042	0.233	1.439	2.067	0.	1.213

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	117	152	0	0	0	0
normalized size	1	1.	0.82	1.06	0.	0.	0.	0.
time (sec)	N/A	0.055	0.247	0.231	0.	0.	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	100	100	91	113	0	0	0	0
normalized size	1	1.	0.91	1.13	0.	0.	0.	0.
time (sec)	N/A	0.126	0.115	0.269	0.	0.	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	121	188	0	0	0	0
normalized size	1	1.	0.93	1.45	0.	0.	0.	0.
time (sec)	N/A	0.163	0.346	0.289	0.	0.	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	126	141	0	0	0	0
normalized size	1	1.	0.93	1.04	0.	0.	0.	0.
time (sec)	N/A	0.198	0.662	0.256	0.	0.	0.	0.

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	79	96	122	163	0	263
normalized size	1	1.	1.13	1.37	1.74	2.33	0.	3.76
time (sec)	N/A	0.082	0.072	0.236	1.439	2.042	0.	1.273

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	222	164	0	0	0	0
normalized size	1	1.	1.16	0.86	0.	0.	0.	0.
time (sec)	N/A	0.3	1.621	0.313	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	104	116	275	208	0	377
normalized size	1	1.	0.69	0.77	1.83	1.39	0.	2.51
time (sec)	N/A	0.373	0.122	0.276	1.454	1.98	0.	2.255

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	243	243	307	183	0	0	0	0
normalized size	1	1.	1.26	0.75	0.	0.	0.	0.
time (sec)	N/A	0.413	3.379	0.286	0.	0.	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	336	336	268	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.403	1.564	0.322	0.	0.	0.	0.

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	175	211	0	0	0	0
normalized size	1	1.	0.62	0.75	0.	0.	0.	0.
time (sec)	N/A	1.073	0.641	0.273	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	228	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.853	1.106	0.303	0.	0.	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	175	175	135	175	0	0	0	0
normalized size	1	1.	0.77	1.	0.	0.	0.	0.
time (sec)	N/A	0.125	0.393	0.252	0.	0.	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	187	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.714	0.206	0.	0.	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	203	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.393	0.279	0.262	0.	0.	0.	0.

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	223	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.383	0.726	0.273	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	188	231	0	0	0	0
normalized size	1	1.	1.25	1.53	0.	0.	0.	0.
time (sec)	N/A	0.544	1.217	0.289	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	177	171	0	0	0	0
normalized size	1	1.	1.05	1.01	0.	0.	0.	0.
time (sec)	N/A	0.297	2.09	0.264	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	292	292	272	215	0	0	0	0
normalized size	1	1.	0.93	0.74	0.	0.	0.	0.
time (sec)	N/A	0.818	1.331	0.213	0.	0.	0.	0.

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	79	140	248	247	0	255
normalized size	1	1.	0.42	0.75	1.33	1.33	0.	1.37
time (sec)	N/A	0.571	0.084	0.18	1.465	1.426	0.	1.273

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	243	243	224	195	0	0	0	0
normalized size	1	1.	0.92	0.8	0.	0.	0.	0.
time (sec)	N/A	0.572	0.907	0.195	0.	0.	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	61	120	103	204	0	116
normalized size	1	1.	0.75	1.48	1.27	2.52	0.	1.43
time (sec)	N/A	0.058	0.065	0.195	1.435	1.446	0.	1.181

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	176	173	0	0	0	0
normalized size	1	1.	0.93	0.92	0.	0.	0.	0.
time (sec)	N/A	0.086	0.584	0.253	0.	0.	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	143	132	0	0	0	0
normalized size	1	1.	0.99	0.92	0.	0.	0.	0.
time (sec)	N/A	0.232	0.211	0.2	0.	0.	0.	0.

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	179	168	205	0	0	0	0
normalized size	1	1.	0.94	1.15	0.	0.	0.	0.
time (sec)	N/A	0.284	0.578	0.221	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	158	145	0	0	0	0
normalized size	1	1.	0.94	0.86	0.	0.	0.	0.
time (sec)	N/A	0.387	0.943	0.204	0.	0.	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	199	220	0	0	0	0
normalized size	1	1.	1.05	1.16	0.	0.	0.	0.
time (sec)	N/A	0.312	1.187	0.223	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	282	164	0	0	0	0
normalized size	1	1.	1.48	0.86	0.	0.	0.	0.
time (sec)	N/A	0.566	3.602	0.184	0.	0.	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	104	116	170	204	0	374
normalized size	1	1.	1.11	1.23	1.81	2.17	0.	3.98
time (sec)	N/A	0.101	0.082	0.169	1.451	1.328	0.	1.823

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	243	243	474	184	0	0	0	0
normalized size	1	1.	1.95	0.76	0.	0.	0.	0.
time (sec)	N/A	0.773	6.802	0.203	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	224	193	0	0	0	0
normalized size	1	1.	0.96	0.83	0.	0.	0.	0.
time (sec)	N/A	0.121	1.098	0.281	0.	0.	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	176	173	0	0	0	0
normalized size	1	1.	0.93	0.92	0.	0.	0.	0.
time (sec)	N/A	0.089	0.06	0.251	0.	0.	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	117	152	0	0	0	0
normalized size	1	1.	0.82	1.06	0.	0.	0.	0.
time (sec)	N/A	0.054	0.067	0.21	0.	0.	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	49	59	100	161	0	122
normalized size	1	1.	0.55	0.66	1.12	1.81	0.	1.37
time (sec)	N/A	0.051	0.051	0.208	0.962	1.452	0.	1.33

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	65	79	146	220	0	154
normalized size	1	1.	0.49	0.59	1.1	1.65	0.	1.16
time (sec)	N/A	0.081	0.062	0.172	0.977	1.549	0.	1.272

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	81	99	189	285	0	186
normalized size	1	1.	0.46	0.56	1.07	1.61	0.	1.05
time (sec)	N/A	0.113	0.076	0.174	0.987	1.756	0.	1.291

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	206	345	0	0	0	0
normalized size	1	1.	0.71	1.19	0.	0.	0.	0.
time (sec)	N/A	0.15	0.654	0.319	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	235	235	119	319	0	0	0	0
normalized size	1	1.	0.51	1.36	0.	0.	0.	0.
time (sec)	N/A	0.102	0.283	0.414	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	109	290	0	0	0	0
normalized size	1	1.	0.6	1.59	0.	0.	0.	0.
time (sec)	N/A	0.062	0.119	0.362	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	74	0	115	0	95
normalized size	1	1.	0.9	1.54	0.	2.4	0.	1.98
time (sec)	N/A	0.029	0.055	0.25	0.	1.668	0.	1.271

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	64	160	122	180	0	150
normalized size	1	1.	0.61	1.52	1.16	1.71	0.	1.43
time (sec)	N/A	0.066	0.069	0.256	0.99	1.597	0.	1.291

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	80	250	178	244	0	201
normalized size	1	1.	0.51	1.59	1.13	1.55	0.	1.28
time (sec)	N/A	0.101	0.082	0.268	1.025	1.592	0.	1.312

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	187	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.1	0.026	0.	0.	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	70	84	410	238	0	0
normalized size	1	1.	0.5	0.6	2.95	1.71	0.	0.
time (sec)	N/A	0.093	0.074	0.17	1.695	1.676	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	94	118	694	329	0	0
normalized size	1	1.	0.45	0.57	3.34	1.58	0.	0.
time (sec)	N/A	0.152	0.085	0.178	1.73	1.728	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	120	152	1014	428	0	0
normalized size	1	1.	0.43	0.55	3.66	1.55	0.	0.
time (sec)	N/A	0.228	0.11	0.191	1.794	1.707	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	569	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	3.707	0.245	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	87	105	0	305	0	0
normalized size	1	1.	0.46	0.55	0.	1.6	0.	0.
time (sec)	N/A	0.165	0.093	0.178	0.	1.7	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	119	153	0	433	0	0
normalized size	1	1.	0.41	0.53	0.	1.5	0.	0.
time (sec)	N/A	0.304	0.11	0.211	0.	1.716	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	151	201	0	574	0	0
normalized size	1	1.	0.39	0.52	0.	1.49	0.	0.
time (sec)	N/A	0.484	0.132	0.202	0.	1.712	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	1.148	0.238	0.	0.	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.162	0.37	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0
normalized size	1	1.	1.	1.11	0.	0.	0.	0.
time (sec)	N/A	0.053	0.107	0.233	0.	0.	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	21	0	0	0	0
normalized size	1	1.	0.81	0.78	0.	0.	0.	0.
time (sec)	N/A	0.093	0.06	0.155	0.	0.	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	30	0	0	0	0
normalized size	1	1.	0.76	0.73	0.	0.	0.	0.
time (sec)	N/A	0.109	0.063	0.154	0.	0.	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	39	0	0	0	0
normalized size	1	1.	0.73	0.71	0.	0.	0.	0.
time (sec)	N/A	0.123	0.07	0.168	0.	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	1.463	0.247	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.894	0.385	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	62	0	0	0	0
normalized size	1	1.	0.91	1.77	0.	0.	0.	0.
time (sec)	N/A	0.123	0.11	0.24	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	120	0	0	0	0
normalized size	1	1.	0.87	2.31	0.	0.	0.	0.
time (sec)	N/A	0.16	0.133	0.157	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	56	176	0	0	0	0
normalized size	1	1.	0.85	2.67	0.	0.	0.	0.
time (sec)	N/A	0.177	0.167	0.171	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	65	232	0	0	0	0
normalized size	1	1.	0.81	2.9	0.	0.	0.	0.
time (sec)	N/A	0.191	0.209	0.159	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	1.714	0.27	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	1.072	0.559	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	86	0	0	0	0
normalized size	1	1.	0.68	1.32	0.	0.	0.	0.
time (sec)	N/A	0.167	0.115	0.253	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	56	180	0	0	0	0
normalized size	1	1.	0.71	2.28	0.	0.	0.	0.
time (sec)	N/A	0.362	0.207	0.163	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	79	272	0	0	0	0
normalized size	1	1.	0.85	2.92	0.	0.	0.	0.
time (sec)	N/A	0.405	0.229	0.174	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	99	364	0	0	0	0
normalized size	1	1.	0.93	3.4	0.	0.	0.	0.
time (sec)	N/A	0.439	0.212	0.186	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	193	1871	0	0	0	0
normalized size	1	1.	1.58	15.34	0.	0.	0.	0.
time (sec)	N/A	0.318	0.387	0.444	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	213	334	373	559	372	331
normalized size	1	1.	0.87	1.36	1.52	2.28	1.52	1.35
time (sec)	N/A	0.18	0.112	0.035	0.967	1.945	15.058	1.173

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	150	233	267	383	245	232
normalized size	1	1.	0.89	1.38	1.58	2.27	1.45	1.37
time (sec)	N/A	0.129	0.076	0.035	0.962	1.947	7.839	1.22

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	98	148	177	259	155	153
normalized size	1	1.	0.89	1.35	1.61	2.35	1.41	1.39
time (sec)	N/A	0.135	0.051	0.031	0.984	1.898	3.33	1.176

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	76	88	143	73	82
normalized size	1	1.	1.21	1.33	1.54	2.51	1.28	1.44
time (sec)	N/A	0.067	0.011	0.031	0.953	1.968	1.518	1.203

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	429	662	833	0	0	0	0
normalized size	1	1.	1.54	1.94	0.	0.	0.	0.
time (sec)	N/A	0.442	1.345	0.197	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	590	590	746	2346	0	0	0	0
normalized size	1	1.	1.26	3.98	0.	0.	0.	0.
time (sec)	N/A	0.91	7.171	0.555	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	657	657	1840	4311	0	0	0	0
normalized size	1	1.	2.8	6.56	0.	0.	0.	0.
time (sec)	N/A	0.966	12.902	0.707	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	19	31	58	14	107
normalized size	1	1.	1.	1.12	1.82	3.41	0.82	6.29
time (sec)	N/A	0.044	0.055	0.059	0.97	1.912	1.58	1.166

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	576	176	243	0	0	0
normalized size	1	1.	3.37	1.03	1.42	0.	0.	0.
time (sec)	N/A	0.24	0.885	0.106	0.96	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	203	196	267	0	0	0
normalized size	1	1.	1.	0.97	1.32	0.	0.	0.
time (sec)	N/A	0.258	0.039	0.161	0.972	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	86	86	260	110	161	0	0	0
normalized size	1	1.	3.02	1.28	1.87	0.	0.	0.
time (sec)	N/A	0.061	0.085	0.11	0.974	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	485	606	0	0	0	0
normalized size	1	1.	1.22	1.53	0.	0.	0.	0.
time (sec)	N/A	0.372	1.008	0.186	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	258	258	874	1599	0	0	0	0
normalized size	1	1.	3.39	6.2	0.	0.	0.	0.
time (sec)	N/A	0.328	18.673	0.337	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	3.939	0.772	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.858	0.632	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	119	0	0	763	0	96
normalized size	1	1.	1.92	0.	0.	12.31	0.	1.55
time (sec)	N/A	0.122	0.111	0.497	0.	2.134	0.	1.246

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	226	0	0	1499	0	182
normalized size	1	1.	1.77	0.	0.	11.71	0.	1.42
time (sec)	N/A	0.375	0.305	0.485	0.	2.508	0.	1.225

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	329	0	0	2612	0	294
normalized size	1	1.	1.64	0.	0.	13.06	0.	1.47
time (sec)	N/A	1.082	0.582	0.483	0.	2.889	0.	1.245

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	431	0	0	4143	0	471
normalized size	1	1.	1.52	0.	0.	14.64	0.	1.66
time (sec)	N/A	1.511	0.933	0.497	0.	3.691	0.	1.306

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	97	229	0	0	0	0
normalized size	1	1.	0.52	1.23	0.	0.	0.	0.
time (sec)	N/A	0.088	0.324	0.434	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	90	210	0	0	0	0
normalized size	1	1.	0.62	1.46	0.	0.	0.	0.
time (sec)	N/A	0.054	0.099	0.354	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	30	52	0	93	0	73
normalized size	1	1.	0.81	1.41	0.	2.51	0.	1.97
time (sec)	N/A	0.025	0.042	0.217	0.	2.019	0.	1.198

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	45	112	90	142	0	116
normalized size	1	1.	0.54	1.35	1.08	1.71	0.	1.4
time (sec)	N/A	0.053	0.049	0.237	0.975	1.958	0.	1.212

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	55	176	134	190	0	159
normalized size	1	1.	0.44	1.42	1.08	1.53	0.	1.28
time (sec)	N/A	0.082	0.059	0.26	0.992	1.967	0.	1.211

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	236	4757	428	590	338	614
normalized size	1	1.	0.75	15.1	1.36	1.87	1.07	1.95
time (sec)	N/A	0.776	0.157	1.915	0.983	1.931	31.773	1.508

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	192	3739	366	441	279	555
normalized size	1	1.	0.85	16.62	1.63	1.96	1.24	2.47
time (sec)	N/A	0.269	0.149	1.247	0.988	1.957	19.737	1.456

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	183	3994	340	455	258	477
normalized size	1	1.	0.74	16.17	1.38	1.84	1.04	1.93
time (sec)	N/A	0.634	0.124	1.142	0.998	1.957	12.832	1.306

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	129	2951	231	298	202	412
normalized size	1	1.	0.92	21.08	1.65	2.13	1.44	2.94
time (sec)	N/A	0.119	0.104	0.639	0.977	2.114	6.931	1.258

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	C	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	144	2529	240	306	148	301
normalized size	1	1.	1.38	24.32	2.31	2.94	1.42	2.89
time (sec)	N/A	0.195	0.017	0.611	0.972	2.245	4.484	1.193

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	216	0	1638	205	0	0	0
normalized size	1	1.	0.	7.58	0.95	0.	0.	0.
time (sec)	N/A	0.28	0.237	1.191	1.462	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	94	332	0	0	0	0	0
normalized size	1	0.9	3.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	0.208	2.427	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	152	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.157	3.545	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	191	460	0	0	0	0	0
normalized size	1	0.97	2.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.465	0.384	5.205	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	244	244	299	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.261	0.151	14.305	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	250	0	0	0	0	0	0
normalized size	1	0.98	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.664	0.286	13.339	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	1601	10161	0	0	0	0
normalized size	1	1.	3.13	19.85	0.	0.	0.	0.
time (sec)	N/A	0.763	5.148	1.908	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	599	599	1251	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.797	2.984	1.73	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.252	0.214	0.759	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	613	613	1226	0	0	0	0	0
normalized size	1	1.	2.	0.	0.	0.	0.	0.
time (sec)	N/A	0.739	3.238	0.793	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	470	470	982	961	0	0	0	0
normalized size	1	1.	2.09	2.04	0.	0.	0.	0.
time (sec)	N/A	0.746	4.544	2.897	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	122	70	247	305	166	0	138
normalized size	1	1.56	0.9	3.17	3.91	2.13	0.	1.77
time (sec)	N/A	0.293	0.082	0.063	1.216	1.922	0.	1.183

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [110] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.	18	0.333
2	A	7	6	1.	18	0.333
3	A	7	6	1.	16	0.375
4	A	4	3	1.	15	0.2
5	A	6	4	1.	18	0.222
6	A	8	7	1.	18	0.389
7	A	4	4	1.	18	0.222
8	A	4	4	1.	18	0.222
9	A	4	4	1.	18	0.222
10	A	7	6	1.	20	0.3
11	A	7	6	1.	20	0.3
12	A	7	6	1.	18	0.333
13	A	4	3	1.	17	0.176
14	A	9	7	1.	20	0.35
15	A	11	9	1.31	20	0.45
16	A	11	9	1.	20	0.45
17	A	4	4	1.	20	0.2
18	A	4	4	1.	20	0.2
19	A	4	4	1.	20	0.2
20	A	7	6	1.	20	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
21	A	7	6	1.	20	0.3
22	A	4	4	1.	18	0.222
23	A	4	3	1.	17	0.176
24	A	13	9	1.	20	0.45
25	A	14	11	1.	20	0.55
26	A	14	11	1.	20	0.55
27	A	15	10	1.	20	0.5
28	A	4	4	1.	20	0.2
29	A	4	5	1.	20	0.25
30	A	4	4	1.	20	0.2
31	A	7	6	1.	20	0.3
32	A	4	4	1.	20	0.2
33	A	4	4	1.	18	0.222
34	A	4	3	1.	17	0.176
35	A	17	10	1.	20	0.5
36	A	18	12	1.	20	0.6
37	A	17	12	1.	20	0.6
38	A	18	12	1.	20	0.6
39	A	19	10	1.	20	0.5
40	A	4	4	1.	20	0.2
41	A	4	5	1.	20	0.25
42	A	4	4	1.	20	0.2
43	A	16	11	1.	20	0.55
44	A	11	9	1.	20	0.45
45	A	7	6	1.	18	0.333
46	A	3	3	1.	17	0.176
47	A	2	2	1.	20	0.1
48	A	8	8	1.	20	0.4
49	A	12	10	1.	20	0.5
50	A	17	11	1.	20	0.55
51	A	16	13	1.	20	0.65
52	A	13	10	1.	20	0.5
53	A	10	8	1.	18	0.444
54	A	5	4	1.	17	0.235
55	A	11	9	1.	20	0.45
56	A	16	14	1.	20	0.7
57	A	19	16	1.	20	0.8
58	A	21	13	1.	20	0.65
59	A	18	10	1.	20	0.5
60	A	15	8	1.	20	0.4
61	A	5	5	1.	18	0.278
62	A	5	4	1.	17	0.235
63	A	16	9	1.	20	0.45
64	A	21	14	1.	20	0.7

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	24	16	1.	20	0.8
66	A	5	4	1.	16	0.25
67	A	3	3	1.	17	0.176
68	A	27	15	1.	20	0.75
69	A	22	14	1.	20	0.7
70	A	17	12	1.	18	0.667
71	A	9	7	1.	17	0.412
72	A	13	11	1.	20	0.55
73	A	12	10	1.	20	0.5
74	A	14	11	1.	20	0.55
75	A	18	13	1.	20	0.65
76	A	43	15	1.	22	0.682
77	A	36	15	1.	22	0.682
78	A	28	14	1.	20	0.7
79	A	12	10	1.	19	0.526
80	A	19	14	1.	22	0.636
81	A	17	15	1.	22	0.682
82	A	20	15	1.	22	0.682
83	A	14	13	1.	22	0.591
84	A	62	15	1.	22	0.682
85	A	52	15	1.	22	0.682
86	A	38	14	1.	20	0.7
87	A	16	12	1.	19	0.632
88	A	28	16	1.	22	0.727
89	A	23	17	1.	22	0.773
90	A	25	20	1.	22	0.909
91	A	28	17	1.	22	0.773
92	A	18	14	1.	22	0.636
93	A	22	15	1.	22	0.682
94	A	29	14	1.	22	0.636
95	A	26	14	1.	22	0.636
96	A	16	12	1.	22	0.546
97	A	9	9	1.	20	0.45
98	A	3	4	1.	19	0.21
99	A	3	4	1.	22	0.182
100	A	8	8	1.	22	0.364
101	A	17	13	1.	22	0.591
102	A	26	15	1.	22	0.682
103	A	33	19	1.	22	0.864
104	A	24	17	1.	22	0.773
105	A	18	14	1.	22	0.636
106	A	13	10	1.	20	0.5
107	A	8	6	1.	19	0.316
108	A	19	13	1.	22	0.591

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	23	17	1.	22	0.773
110	A	31	22	1.	22	1.
111	A	37	17	1.	22	0.773
112	A	31	14	1.	22	0.636
113	A	26	10	1.	22	0.454
114	A	13	7	1.	20	0.35
115	A	13	6	1.	19	0.316
116	A	32	13	1.	22	0.591
117	A	36	17	1.	22	0.773
118	A	18	6	1.	18	0.333
119	A	4	5	1.	20	0.25
120	A	26	15	1.	18	0.833
121	A	17	13	1.	18	0.722
122	A	11	10	1.	16	0.625
123	A	4	5	1.	18	0.278
124	A	11	6	1.	18	0.333
125	A	24	6	1.	18	0.333
126	A	42	6	1.	18	0.333
127	A	19	13	1.	18	0.722
128	A	10	9	1.	16	0.562
129	A	4	5	1.	15	0.333
130	A	4	5	1.	18	0.278
131	A	5	6	1.	19	0.316
132	A	10	8	1.	18	0.444
133	A	18	10	1.	18	0.556
134	A	21	10	1.	19	0.526
135	A	12	8	1.	17	0.471
136	A	5	5	1.	16	0.312
137	A	5	5	1.	19	0.263
138	A	6	6	1.	20	0.3
139	A	12	10	1.	19	0.526
140	A	21	11	1.	19	0.579
141	A	0	0	0.	0	0.
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	A	0	0	0.	0	0.
146	A	0	0	0.	0	0.
147	A	16	12	1.	19	0.632
148	A	12	10	1.	19	0.526
149	A	9	7	1.	17	0.412
150	A	4	4	1.	16	0.25
151	A	7	6	1.	19	0.316
152	A	12	11	1.	19	0.579

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
153	A	15	13	1.	19	0.684
154	A	14	11	1.	21	0.524
155	A	8	7	1.	19	0.368
156	A	1	1	1.	18	0.056
157	A	9	7	1.	21	0.333
158	A	13	11	1.	21	0.524
159	A	9	7	1.	17	0.412
160	A	0	0	0.	0	0.
161	A	9	4	1.	18	0.222
162	A	9	4	1.	18	0.222
163	A	9	4	1.	18	0.222
164	A	2	1	1.	16	0.062
165	A	3	3	1.	15	0.2
166	A	5	5	1.	18	0.278
167	A	8	8	1.	18	0.444
168	A	5	5	1.	18	0.278
169	A	10	7	1.	18	0.389
170	A	3	2	1.	18	0.111
171	A	9	4	1.	18	0.222
172	A	34	10	1.	20	0.5
173	A	26	8	1.	20	0.4
174	A	24	10	1.	20	0.5
175	A	4	4	1.	18	0.222
176	A	7	7	1.	17	0.412
177	A	12	10	1.	20	0.5
178	A	10	10	1.	20	0.5
179	A	15	12	1.	20	0.6
180	A	13	8	1.	20	0.4
181	A	11	8	1.	20	0.4
182	A	22	8	1.	20	0.4
183	A	8	8	1.	17	0.471
184	A	10	5	1.	19	0.263
185	A	0	0	0.	0	0.
186	A	0	0	0.	0	0.
187	A	0	0	0.	0	0.
188	A	0	0	0.	0	0.
189	A	0	0	0.	0	0.
190	A	0	0	0.	0	0.
191	A	0	0	0.	0	0.
192	A	14	4	1.	20	0.2
193	A	14	4	1.	20	0.2
194	A	14	4	1.	20	0.2
195	A	3	2	1.	18	0.111
196	A	4	3	1.	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
197	A	10	6	1.	20	0.3
198	A	13	9	1.	20	0.45
199	A	9	6	1.	20	0.3
200	A	13	9	1.	20	0.45
201	A	10	5	1.	20	0.25
202	A	15	7	1.	20	0.35
203	A	59	10	1.	22	0.454
204	A	47	8	1.	22	0.364
205	A	44	10	1.	22	0.454
206	A	5	4	1.	20	0.2
207	A	9	7	1.	19	0.368
208	A	23	12	1.	22	0.546
209	A	20	13	1.	22	0.591
210	A	21	15	1.	22	0.682
211	A	19	13	1.	22	0.591
212	A	29	13	1.	22	0.591
213	A	27	8	1.	22	0.364
214	A	16	8	1.	22	0.364
215	A	42	8	1.	22	0.364
216	A	56	9	1.	22	0.409
217	A	12	9	1.	19	0.474
218	A	0	0	0.	0	0.
219	A	0	0	0.	0	0.
220	A	0	0	0.	0	0.
221	A	0	0	0.	0	0.
222	A	0	0	0.	0	0.
223	A	0	0	0.	0	0.
224	A	5	3	1.	17	0.176
225	A	12	8	1.	19	0.421
226	A	17	9	1.	19	0.474
227	A	8	8	1.	20	0.4
228	A	4	4	1.	20	0.2
229	A	4	4	1.	18	0.222
230	A	1	1	1.	17	0.059
231	A	3	3	1.	20	0.15
232	A	7	7	1.	20	0.35
233	A	7	7	1.	20	0.35
234	A	10	9	1.	22	0.409
235	A	7	7	1.	22	0.318
236	A	4	5	1.	20	0.25
237	A	1	1	1.	19	0.053
238	A	4	5	1.	22	0.227
239	A	6	6	1.	22	0.273
240	A	13	11	1.	22	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
241	A	14	11	1.	22	0.5
242	A	7	7	1.	22	0.318
243	A	5	6	1.	20	0.3
244	A	1	1	1.	19	0.053
245	A	5	6	1.	22	0.273
246	A	7	7	1.	22	0.318
247	A	13	9	1.	22	0.409
248	A	1	1	1.	21	0.048
249	A	0	0	0.	0	0.
250	A	1	1	1.	19	0.053
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	1	1	1.	19	0.053
254	A	0	0	0.	0	0.
255	A	0	0	0.	0	0.
256	A	1	1	1.	19	0.053
257	A	0	0	0.	0	0.
258	A	1	1	1.	19	0.053
259	A	8	8	1.	20	0.4
260	A	2	2	1.	20	0.1
261	A	3	3	1.	18	0.167
262	A	2	2	1.	17	0.118
263	A	7	7	1.	20	0.35
264	A	10	10	1.	20	0.5
265	A	15	10	1.	20	0.5
266	A	8	9	1.	22	0.409
267	A	4	4	1.	22	0.182
268	A	3	3	1.	20	0.15
269	A	4	4	1.	19	0.21
270	A	8	9	1.	22	0.409
271	A	11	11	1.	22	0.5
272	A	22	15	1.	22	0.682
273	A	11	11	1.	22	0.5
274	A	4	4	1.	22	0.182
275	A	5	4	1.	20	0.2
276	A	4	3	1.	19	0.158
277	A	11	11	1.	22	0.5
278	A	12	11	1.	22	0.5
279	A	25	14	1.	22	0.636
280	A	9	8	1.	21	0.381
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	4	3	1.	22	0.136
284	A	4	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
285	A	4	3	1.	19	0.158
286	A	0	0	0.	0	0.
287	A	0	0	0.	0	0.
288	A	5	5	1.	22	0.227
289	A	9	5	1.	20	0.25
290	A	5	5	1.	19	0.263
291	A	0	0	0.	0	0.
292	A	0	0	0.	0	0.
293	A	10	6	1.	22	0.273
294	A	5	5	1.	20	0.25
295	A	10	6	1.	19	0.316
296	A	0	0	0.	0	0.
297	A	6	6	1.	19	0.316
298	A	11	7	1.	19	0.368
299	A	7	6	1.	19	0.316
300	A	12	7	1.	19	0.368
301	A	8	6	1.	19	0.316
302	A	4	3	1.	20	0.15
303	A	3	3	1.	20	0.15
304	A	4	3	1.	18	0.167
305	A	3	3	1.	17	0.176
306	A	12	7	1.	20	0.35
307	A	14	11	1.	20	0.55
308	A	4	4	1.	22	0.182
309	A	13	6	1.	22	0.273
310	A	4	4	1.	20	0.2
311	A	8	5	1.	19	0.263
312	A	13	10	1.	22	0.454
313	A	20	12	1.	22	0.546
314	A	9	7	1.	22	0.318
315	A	13	6	1.	22	0.273
316	A	9	5	1.	20	0.25
317	A	8	5	1.	19	0.263
318	A	21	12	1.	22	0.546
319	A	21	13	1.	22	0.591
320	A	15	7	1.	21	0.333
321	A	0	0	0.	0	0.
322	A	0	0	0.	0	0.
323	A	5	3	1.	22	0.136
324	A	5	3	1.	22	0.136
325	A	4	3	1.	22	0.136
326	A	5	3	1.	20	0.15
327	A	5	3	1.	19	0.158
328	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	0	0	0.	0	0.
330	A	6	4	1.	22	0.182
331	A	20	7	1.38	22	0.318
332	A	12	6	1.46	22	0.273
333	A	10	6	1.	20	0.3
334	A	6	4	1.	19	0.21
335	A	0	0	0.	0	0.
336	A	21	8	1.	22	0.364
337	A	25	8	1.5	22	0.364
338	A	22	8	1.27	22	0.364
339	A	19	7	1.	20	0.35
340	A	11	7	1.	19	0.368
341	A	0	0	0.	0	0.
342	A	20	7	1.	19	0.368
343	A	35	8	1.	19	0.421
344	A	49	8	1.	19	0.421
345	A	4	3	1.	17	0.176
346	A	13	5	1.	19	0.263
347	A	13	5	1.	19	0.263
348	A	21	7	1.	21	0.333
349	A	0	0	0.	0	0.
350	A	0	0	0.	0	0.
351	A	6	3	1.	22	0.136
352	A	6	3	1.	22	0.136
353	A	6	3	1.	22	0.136
354	A	5	3	1.	22	0.136
355	A	6	3	1.	22	0.136
356	A	6	3	1.	20	0.15
357	A	6	3	1.	19	0.158
358	A	0	0	0.	0	0.
359	A	0	0	0.	0	0.
360	A	13	6	1.	20	0.3
361	A	7	4	1.	19	0.21
362	A	22	6	1.	20	0.3
363	A	14	7	1.	19	0.368
364	A	9	4	1.	22	0.182
365	A	7	5	1.	22	0.227
366	A	5	4	1.	22	0.182
367	A	3	3	1.	22	0.136
368	A	2	2	1.	20	0.1
369	A	1	1	1.	19	0.053
370	A	1	1	1.	22	0.045
371	A	4	4	1.	22	0.182
372	A	3	3	1.	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	6	4	1.	24	0.167
374	A	11	8	1.	24	0.333
375	A	2	2	1.	22	0.091
376	A	8	5	1.	21	0.238
377	A	8	5	1.	24	0.208
378	A	2	2	1.	24	0.083
379	A	13	10	1.	24	0.417
380	A	21	8	1.	24	0.333
381	A	13	9	1.	24	0.375
382	A	9	6	1.	22	0.273
383	A	10	6	1.	21	0.286
384	A	10	6	1.	24	0.25
385	A	9	6	1.	24	0.25
386	A	13	9	1.	24	0.375
387	A	0	0	0.	0	0.
388	A	5	4	1.	22	0.182
389	A	2	2	1.	22	0.091
390	A	2	2	1.	20	0.1
391	A	1	1	1.	19	0.053
392	A	4	4	1.	22	0.182
393	A	6	6	1.	22	0.273
394	A	8	6	1.	22	0.273
395	A	0	0	0.	0	0.
396	A	5	4	1.	24	0.167
397	A	11	8	1.	24	0.333
398	A	2	2	1.	22	0.091
399	A	2	2	1.	21	0.095
400	A	11	8	1.	24	0.333
401	A	5	5	1.	24	0.208
402	A	25	13	1.	24	0.542
403	A	0	0	0.	0	0.
404	A	13	9	1.	24	0.375
405	A	13	9	1.	24	0.375
406	A	3	3	1.	22	0.136
407	A	2	2	1.	21	0.095
408	A	14	10	1.	24	0.417
409	A	12	9	1.	24	0.375
410	A	28	13	1.	24	0.542
411	A	0	0	0.	0	0.
412	A	0	0	0.	0	0.
413	A	2	2	1.	22	0.091
414	A	2	2	1.	21	0.095
415	A	0	0	0.	0	0.
416	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
417	A	0	0	0.	0	0.
418	A	3	3	1.	22	0.136
419	A	3	3	1.	21	0.143
420	A	0	0	0.	0	0.
421	A	0	0	0.	0	0.
422	A	0	0	0.	0	0.
423	A	4	4	1.	22	0.182
424	A	4	4	1.	21	0.19
425	A	0	0	0.	0	0.
426	A	11	6	1.	22	0.273
427	A	9	5	1.	22	0.227
428	A	7	6	1.	22	0.273
429	A	3	3	1.	20	0.15
430	A	2	2	1.	19	0.105
431	A	3	3	1.	22	0.136
432	A	6	6	1.	22	0.273
433	A	5	4	1.	22	0.182
434	A	5	5	1.	22	0.227
435	A	9	5	1.	22	0.227
436	A	21	7	1.	22	0.318
437	A	14	5	1.	22	0.227
438	A	45	10	1.	24	0.417
439	A	21	7	1.	24	0.292
440	A	29	10	1.	24	0.417
441	A	3	3	1.	22	0.136
442	A	10	7	1.	21	0.333
443	A	11	8	1.	24	0.333
444	A	11	8	1.	24	0.333
445	A	22	11	1.	24	0.458
446	A	6	5	1.	24	0.208
447	A	27	7	1.	22	0.318
448	A	24	6	1.	22	0.273
449	A	19	7	1.	22	0.318
450	A	4	3	1.	20	0.15
451	A	3	2	1.	19	0.105
452	A	7	6	1.	22	0.273
453	A	9	7	1.	22	0.318
454	A	9	6	1.	22	0.273
455	A	12	7	1.	22	0.318
456	A	15	6	1.	22	0.273
457	A	6	5	1.	22	0.227
458	A	24	6	1.	22	0.273
459	A	4	2	1.	19	0.105
460	A	3	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
461	A	2	2	1.	19	0.105
462	A	2	2	1.	19	0.105
463	A	3	2	1.	19	0.105
464	A	4	2	1.	19	0.105
465	A	4	3	1.	20	0.15
466	A	3	3	1.	20	0.15
467	A	2	2	1.	20	0.1
468	A	1	1	1.	20	0.05
469	A	2	2	1.	20	0.1
470	A	3	2	1.	20	0.1
471	A	10	7	1.	21	0.333
472	A	5	4	1.	21	0.19
473	A	9	4	1.	21	0.19
474	A	14	4	1.	21	0.19
475	A	12	8	1.	21	0.381
476	A	5	4	1.	21	0.19
477	A	9	4	1.	21	0.19
478	A	14	4	1.	21	0.19
479	A	0	0	0.	0	0.
480	A	0	0	0.	0	0.
481	A	2	2	1.	21	0.095
482	A	5	3	1.	21	0.143
483	A	6	3	1.	21	0.143
484	A	7	3	1.	21	0.143
485	A	0	0	0.	0	0.
486	A	0	0	0.	0	0.
487	A	3	3	1.	21	0.143
488	A	6	4	1.	21	0.19
489	A	7	4	1.	21	0.19
490	A	8	4	1.	21	0.19
491	A	0	0	0.	0	0.
492	A	0	0	0.	0	0.
493	A	4	4	1.	21	0.19
494	A	12	7	1.	21	0.333
495	A	14	7	1.	21	0.333
496	A	16	7	1.	21	0.333
497	A	7	6	1.	28	0.214
498	A	4	4	1.	14	0.286
499	A	4	4	1.	14	0.286
500	A	5	5	1.	14	0.357
501	A	5	4	1.	12	0.333
502	A	17	5	1.	14	0.357
503	A	25	13	1.	14	0.929
504	A	23	11	1.	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	1	1	1.	20	0.05
506	A	17	5	1.	14	0.357
507	A	17	5	1.	16	0.312
508	A	4	4	1.	10	0.4
509	A	17	5	1.	12	0.417
510	A	10	7	1.	15	0.467
511	A	0	0	0.	0	0.
512	A	0	0	0.	0	0.
513	A	5	6	1.	16	0.375
514	A	7	9	1.	16	0.562
515	A	8	9	1.	16	0.562
516	A	8	9	1.	16	0.562
517	A	3	3	1.	15	0.2
518	A	2	2	1.	15	0.133
519	A	1	1	1.	15	0.067
520	A	2	2	1.	15	0.133
521	A	3	2	1.	15	0.133
522	A	26	15	1.	27	0.556
523	A	14	11	1.	27	0.407
524	A	21	15	1.	27	0.556
525	A	7	8	1.	25	0.32
526	A	9	8	1.	24	0.333
527	A	14	9	1.	27	0.333
528	A	8	8	0.9	27	0.296
529	A	5	6	1.	27	0.222
530	A	17	16	0.97	27	0.593
531	A	10	7	1.	27	0.259
532	A	26	18	0.98	27	0.667
533	A	22	17	1.	22	0.773
534	A	28	12	1.	21	0.571
535	A	0	0	0.	0	0.
536	A	28	14	1.	24	0.583
537	A	20	17	1.	24	0.708
538	A	16	7	1.56	20	0.35

Chapter 3

Listing of integrals

3.1 $\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=108

$$\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(cx+1)}{40c^4}+\frac{bdx^3}{12c}+\frac{1}{20}b$$

```
[Out] (b*d*x)/(4*c^3) + (b*d*x^2)/(10*c^2) + (b*d*x^3)/(12*c) + (b*d*x^4)/20 + (d
*x^4*(a + b*ArcTanh[c*x]))/4 + (c*d*x^5*(a + b*ArcTanh[c*x]))/5 + (9*b*d*Lo
g[1 - c*x])/(40*c^4) - (b*d*Log[1 + c*x])/(40*c^4)
```

Rubi [A] time = 0.096842, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 5936, 12, 801, 633, 31}

$$\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx^2}{10c^2}+\frac{bdx}{4c^3}+\frac{9bd\log(1-cx)}{40c^4}-\frac{bd\log(cx+1)}{40c^4}+\frac{bdx^3}{12c}+\frac{1}{20}b$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^3) + (b*d*x^2)/(10*c^2) + (b*d*x^3)/(12*c) + (b*d*x^4)/20 + (d
*x^4*(a + b*ArcTanh[c*x]))/4 + (c*d*x^5*(a + b*ArcTanh[c*x]))/5 + (9*b*d*Lo
g[1 - c*x])/(40*c^4) - (b*d*Log[1 + c*x])/(40*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^4(5+4cx)}{20\left(1-c^2x^2\right)}dx$$
$$=\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\frac{x^4(5+4cx)}{1-c^2x^2}dx$$
$$=\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{20}(bcd)\int\left(-\frac{5}{c^4}-\frac{4x}{c^3}-\right.$$
$$=\frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}\right.$$
$$=\frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}\right.$$
$$=\frac{bdx}{4c^3}+\frac{bdx^2}{10c^2}+\frac{bdx^3}{12c}+\frac{1}{20}bdx^4+\frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}\right.$$

Mathematica [A] time = 0.0737976, size = 97, normalized size = 0.9

$$\frac{d\left(24ac^5x^5+30ac^4x^4+6bc^4x^4+10bc^3x^3+12bc^2x^2+6bc^4x^4(4cx+5)\tanh^{-1}(cx)+30bcx+27b\log(1-cx)-3b\log(cx\right.}{120c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(30*b*c*x + 12*b*c^2*x^2 + 10*b*c^3*x^3 + 30*a*c^4*x^4 + 6*b*c^4*x^4 + 2
4*a*c^5*x^5 + 6*b*c^4*x^4*(5 + 4*c*x)*ArcTanh[c*x] + 27*b*Log[1 - c*x] - 3*
b*Log[1 + c*x]))/(120*c^4)
```

Maple [A] time = 0.041, size = 101, normalized size = 0.9

$$\frac{cdax^5}{5}+\frac{dax^4}{4}+\frac{cdb\mathrm{Artanh}\,(cx)\,x^5}{5}+\frac{db\mathrm{Artanh}\,(cx)\,x^4}{4}+\frac{bdx^4}{20}+\frac{bdx^3}{12\,c}+\frac{bdx^2}{10\,c^2}+\frac{bdx}{4\,c^3}+\frac{9\,db\ln(cx-1)}{40\,c^4}-\frac{db\ln(cx+1}{40\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c*d*a*x^5+1/4*d*a*x^4+1/5*c*d*b*arctanh(c*x)*x^5+1/4*d*b*arctanh(c*x)*x
^4+1/20*b*d*x^4+1/12*b*d*x^3/c+1/10*b*d*x^2/c^2+1/4*b*d*x/c^3+9/40/c^4*d*b*
ln(c*x-1)-1/40*b*d*ln(c*x+1)/c^4
```

Maxima [A] time = 0.957459, size = 163, normalized size = 1.51

$$\frac{1}{5}\,acdx^5 + \frac{1}{4}\,adx^4 + \frac{1}{20}\left(4\,x^5\,\operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2 - 1\right)}{c^6}\right)\right)bcd + \frac{1}{24}\left(6\,x^4\,\operatorname{artanh}(cx) + c\left(\frac{2\left(c^2x^3\right.}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^
2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d + 1/24*(6*x^4*arctanh(c*x) + c*(2*(
c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d
```

Fricas [A] time = 2.04982, size = 275, normalized size = 2.55

$$\frac{24\,ac^5dx^5 + 6\,(5\,a + b)c^4dx^4 + 10\,bc^3dx^3 + 12\,bc^2dx^2 + 30\,bcdx - 3\,bd\,\log(cx + 1) + 27\,bd\,\log(cx - 1) + 3\left(4\,bc^5dx^5 +\right.}{120\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*d*x^5 + 6*(5*a + b)*c^4*d*x^4 + 10*b*c^3*d*x^3 + 12*b*c^2*d
*x^2 + 30*b*c*d*x - 3*b*d*log(c*x + 1) + 27*b*d*log(c*x - 1) + 3*(4*b*c^5*d
*x^5 + 5*b*c^4*d*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] time = 3.05266, size = 124, normalized size = 1.15

$$\begin{cases} \frac{acdx^5}{5} + \frac{adx^4}{4} + \frac{bcdx^5\operatorname{atanh}(cx)}{5} + \frac{bdx^4\operatorname{atanh}(cx)}{4} + \frac{bdx^4}{20} + \frac{bdx^3}{12c} + \frac{bdx^2}{10c^2} + \frac{bdx}{4c^3} + \frac{bd\log\left(x - \frac{1}{c}\right)}{5c^4} - \frac{bd\operatorname{atanh}(cx)}{20c^4} & \text{for } c \neq 0 \\ \frac{adx^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**5/5 + a*d*x**4/4 + b*c*d*x**5*atanh(c*x)/5 + b*d*x**4*a
tanh(c*x)/4 + b*d*x**4/20 + b*d*x**3/(12*c) + b*d*x**2/(10*c**2) + b*d*x/(4
*c**3) + b*d*log(x - 1/c)/(5*c**4) - b*d*atanh(c*x)/(20*c**4), Ne(c, 0)), (
a*d*x**4/4, True))
```

Giac [A] time = 1.24833, size = 147, normalized size = 1.36

$$\frac{1}{5}acdx^5 + \frac{1}{20}(5ad + bd)x^4 + \frac{bdx^3}{12c} + \frac{bdx^2}{10c^2} + \frac{1}{40}\left(4bcdx^5 + 5bdx^4\right)\log\left(-\frac{cx+1}{cx-1}\right) + \frac{bdx}{4c^3} - \frac{bd\log(cx+1)}{40c^4} + \frac{9bd\log(cx}{40c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c*d*x^5 + 1/20*(5*a*d + b*d)*x^4 + 1/12*b*d*x^3/c + 1/10*b*d*x^2/c^2
+ 1/40*(4*b*c*d*x^5 + 5*b*d*x^4)*log(-(c*x + 1)/(c*x - 1)) + 1/4*b*d*x/c^3
- 1/40*b*d*log(c*x + 1)/c^4 + 9/40*b*d*log(c*x - 1)/c^4
```

3.2 $\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=96

$$\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx}{4c^2}+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(cx+1)}{24c^3}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3$$

```
[Out] (b*d*x)/(4*c^2) + (b*d*x^2)/(6*c) + (b*d*x^3)/12 + (d*x^3*(a + b*ArcTanh[c*
x]))/3 + (c*d*x^4*(a + b*ArcTanh[c*x]))/4 + (7*b*d*Log[1 - c*x])/(24*c^3) +
 (b*d*Log[1 + c*x])/(24*c^3)
```

Rubi [A] time = 0.0974052, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 5936, 12, 801, 633, 31}

$$\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{bdx}{4c^2}+\frac{7bd\log(1-cx)}{24c^3}+\frac{bd\log(cx+1)}{24c^3}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(4*c^2) + (b*d*x^2)/(6*c) + (b*d*x^3)/12 + (d*x^3*(a + b*ArcTanh[c*
x]))/3 + (c*d*x^4*(a + b*ArcTanh[c*x]))/4 + (7*b*d*Log[1 - c*x])/(24*c^3) +
 (b*d*Log[1 + c*x])/(24*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^3(4+3cx)}{12\left(1-c^2x^2\right)}dx$$
$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x^3(4+3cx)}{1-c^2x^2}\,dx$$
$$=\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\left(-\frac{3}{c^3}-\frac{4x}{c^2}-\right.$$
$$=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-$$
$$=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)-$$
$$=\frac{bdx}{4c^2}+\frac{bdx^2}{6c}+\frac{1}{12}bdx^3+\frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)+$$

Mathematica [A] time = 0.0631771, size = 87, normalized size = 0.91

$$\frac{d\left(6ac^4x^4+8ac^3x^3+2bc^3x^3+4bc^2x^2+2bc^3x^3(3cx+4)\tanh^{-1}(cx)+6bcx+7b\log(1-cx)+b\log(cx+1)\right)}{24c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(6*b*c*x + 4*b*c^2*x^2 + 8*a*c^3*x^3 + 2*b*c^3*x^3 + 6*a*c^4*x^4 + 2*b*c
^3*x^3*(4 + 3*c*x)*ArcTanh[c*x] + 7*b*Log[1 - c*x] + b*Log[1 + c*x]))/(24*c
^3)
```

Maple [A] time = 0.032, size = 91, normalized size = 1.

$$\frac{cdax^4}{4}+\frac{dax^3}{3}+\frac{cdb\mathrm{Artanh}(cx)x^4}{4}+\frac{db\mathrm{Artanh}(cx)x^3}{3}+\frac{bdx^3}{12}+\frac{bdx^2}{6c}+\frac{bdx}{4c^2}+\frac{7db\ln(cx-1)}{24c^3}+\frac{db\ln(cx+1)}{24c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*c*d*a*x^4+1/3*d*a*x^3+1/4*c*d*b*arctanh(c*x)*x^4+1/3*d*b*arctanh(c*x)*x
^3+1/12*b*d*x^3+1/6*b*d*x^2/c+1/4*b*d*x/c^2+7/24/c^3*d*b*ln(c*x-1)+1/24*b*d
*ln(c*x+1)/c^3
```

Maxima [A] time = 0.948211, size = 149, normalized size = 1.55

$$\frac{1}{4}\,acdx^4 + \frac{1}{3}\,adx^3 + \frac{1}{24}\left(6\,x^4\,\mathrm{artanh}\left(cx\right) + c\left(\frac{2\left(c^2x^3 + 3\,x\right)}{c^4} - \frac{3\,\log\left(cx + 1\right)}{c^5} + \frac{3\,\log\left(cx - 1\right)}{c^5}\right)\right)bcd + \frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c*d*x^4 + 1/3*a*d*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*
x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d + 1/6*(2*x^3*arcta
nh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d
```

Fricas [A] time = 2.04877, size = 240, normalized size = 2.5

$$\frac{6\,ac^4dx^4 + 2\left(4\,a + b\right)c^3dx^3 + 4\,bc^2dx^2 + 6\,bcdx + bd\log\left(cx + 1\right) + 7\,bd\log\left(cx - 1\right) + \left(3\,bc^4dx^4 + 4\,bc^3dx^3\right)\log\left(-\frac{cx+1}{cx-1}\right.}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d*x^4 + 2*(4*a + b)*c^3*d*x^3 + 4*b*c^2*d*x^2 + 6*b*c*d*x + b
*d*log(c*x + 1) + 7*b*d*log(c*x - 1) + (3*b*c^4*d*x^4 + 4*b*c^3*d*x^3)*log(
-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] time = 2.58851, size = 112, normalized size = 1.17

$$\begin{cases} \frac{acdx^4}{4} + \frac{adx^3}{3} + \frac{bcdx^4\,\mathrm{atanh}\left(cx\right)}{4} + \frac{bdx^3\,\mathrm{atanh}\left(cx\right)}{3} + \frac{bdx^3}{12} + \frac{bdx^2}{6c} + \frac{bdx}{4c^2} + \frac{bd\log\left(x - \frac{1}{c}\right)}{3c^3} + \frac{bd\,\mathrm{atanh}\left(cx\right)}{12c^3} & \text{for}\ c \neq 0 \\ \frac{adx^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**4/4 + a*d*x**3/3 + b*c*d*x**4*atanh(c*x)/4 + b*d*x**3*a
tanh(c*x)/3 + b*d*x**3/12 + b*d*x**2/(6*c) + b*d*x/(4*c**2) + b*d*log(x - 1
/c)/(3*c**3) + b*d*atanh(c*x)/(12*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

Giac [A] time = 1.24663, size = 134, normalized size = 1.4

$$\frac{1}{4}\,acdx^4 + \frac{1}{12}\left(4\,ad + bd\right)x^3 + \frac{bdx^2}{6\,c} + \frac{1}{24}\left(3\,bcdx^4 + 4\,bdx^3\right)\log\left(-\frac{cx + 1}{cx - 1}\right) + \frac{bdx}{4\,c^2} + \frac{bd\log\left(cx + 1\right)}{24\,c^3} + \frac{7\,bd\log\left(cx - 1\right)}{24\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*c*d*x^4 + 1/12*(4*a*d + b*d)*x^3 + 1/6*b*d*x^2/c + 1/24*(3*b*c*d*x^4
+ 4*b*d*x^3)*log(-(c*x + 1)/(c*x - 1)) + 1/4*b*d*x/c^2 + 1/24*b*d*log(c*x +
 1)/c^3 + 7/24*b*d*log(c*x - 1)/c^3
```

3.3 $\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=84

$$\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{5bd\log(1-cx)}{12c^2}-\frac{bd\log(cx+1)}{12c^2}+\frac{bdx}{2c}+\frac{1}{6}bdx^2$$

```
[Out] (b*d*x)/(2*c) + (b*d*x^2)/6 + (d*x^2*(a + b*ArcTanh[c*x]))/2 + (c*d*x^3*(a
+ b*ArcTanh[c*x]))/3 + (5*b*d*Log[1 - c*x])/(12*c^2) - (b*d*Log[1 + c*x])/(
12*c^2)
```

Rubi [A] time = 0.074793, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {43, 5936, 12, 801, 633, 31}

$$\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{5bd\log(1-cx)}{12c^2}-\frac{bd\log(cx+1)}{12c^2}+\frac{bdx}{2c}+\frac{1}{6}bdx^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/(2*c) + (b*d*x^2)/6 + (d*x^2*(a + b*ArcTanh[c*x]))/2 + (c*d*x^3*(a
+ b*ArcTanh[c*x]))/3 + (5*b*d*Log[1 - c*x])/(12*c^2) - (b*d*Log[1 + c*x])/(
12*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx &= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bc)\int\frac{dx^2(3+2cx)}{6-6c^2x^2}dx \\ &= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bcd)\int\frac{x^2(3+2cx)}{6-6c^2x^2}dx \\ &= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-(bcd)\int\left(-\frac{1}{2c^2}-\frac{x}{3c}+\right. \\ &= \frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)-\frac{(bd)\int}{} \\ &= \frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}(bd) \\ &= \frac{bdx}{2c}+\frac{1}{6}bdx^2+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)+\frac{5bd\log}{1}\end{aligned}$$

Mathematica [A] time = 0.0554376, size = 79, normalized size = 0.94

$$\frac{d\left(4ac^3x^3+6ac^2x^2+2bc^2x^2+2bc^2x^2(2cx+3)\tanh^{-1}(cx)+6bcx+5b\log(1-cx)-b\log(cx+1)\right)}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d*(6*b*c*x + 6*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 2*b*c^2*x^2*(3 + 2*
c*x)*ArcTanh[c*x] + 5*b*Log[1 - c*x] - b*Log[1 + c*x]))/(12*c^2)
```

Maple [A] time = 0.031, size = 81, normalized size = 1.

$$\frac{cdax^3}{3}+\frac{dax^2}{2}+\frac{cdb\mathrm{Artanh}\,(cx)\,x^3}{3}+\frac{db\mathrm{Artanh}\,(cx)\,x^2}{2}+\frac{bdx^2}{6}+\frac{bdx}{2\,c}+\frac{5\,db\ln\,(cx-1)}{12\,c^2}-\frac{db\ln\,(cx+1)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/3*c*d*a*x^3+1/2*d*a*x^2+1/3*c*d*b*arctanh(c*x)*x^3+1/2*d*b*arctanh(c*x)*x
^2+1/6*b*d*x^2+1/2*b*d*x/c+5/12/c^2*d*b*ln(c*x-1)-1/12*b*d*ln(c*x+1)/c^2
```

Maxima [A] time = 0.97236, size = 134, normalized size = 1.6

$$\frac{1}{3}acdx^3+\frac{1}{6}\left(2\,x^3\,\mathrm{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bcd+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\,(cx)+c\left(\frac{2\,x}{c^2}-\frac{\log\,(cx+1)}{c^3}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c*d*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4
))*b*c*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1
)/c^3 + log(c*x - 1)/c^3))*b*d
```

Fricas [A] time = 1.96771, size = 219, normalized size = 2.61

$$\frac{4\,ac^3dx^3 + 2\,(3\,a + b)c^2dx^2 + 6\,bcdx - bd\log\left(cx + 1\right) + 5\,bd\log\left(cx - 1\right) + \left(2\,bc^3dx^3 + 3\,bc^2dx^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*c^3*d*x^3 + 2*(3*a + b)*c^2*d*x^2 + 6*b*c*d*x - b*d*log(c*x + 1)
+ 5*b*d*log(c*x - 1) + (2*b*c^3*d*x^3 + 3*b*c^2*d*x^2)*log(-(c*x + 1)/(c*x
- 1)))/c^2
```

Sympy [A] time = 1.61303, size = 100, normalized size = 1.19

$$\begin{cases} \frac{acdx^3}{3} + \frac{adx^2}{2} + \frac{bcdx^3\operatorname{atanh}\left(cx\right)}{3} + \frac{bdx^2\operatorname{atanh}\left(cx\right)}{2} + \frac{bdx^2}{6} + \frac{bdx}{2c} + \frac{bd\log\left(x-\frac{1}{c}\right)}{3c^2} - \frac{bd\operatorname{atanh}\left(cx\right)}{6c^2} & \text{for}\ c \neq 0 \\ \frac{adx^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**3/3 + a*d*x**2/2 + b*c*d*x**3*atanh(c*x)/3 + b*d*x**2*a
tanh(c*x)/2 + b*d*x**2/6 + b*d*x/(2*c) + b*d*log(x - 1/c)/(3*c**2) - b*d*at
anh(c*x)/(6*c**2), Ne(c, 0)), (a*d*x**2/2, True))
```

Giac [A] time = 1.22319, size = 120, normalized size = 1.43

$$\frac{1}{3}\,acdx^3 + \frac{1}{6}\,(3\,ad + bd)x^2 + \frac{bdx}{2\,c} + \frac{1}{12}\left(2\,bcdx^3 + 3\,bdx^2\right)\log\left(-\frac{cx+1}{cx-1}\right) - \frac{bd\log\left(cx+1\right)}{12\,c^2} + \frac{5\,bd\log\left(cx-1\right)}{12\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*c*d*x^3 + 1/6*(3*a*d + b*d)*x^2 + 1/2*b*d*x/c + 1/12*(2*b*c*d*x^3 + 3
*b*d*x^2)*log(-(c*x + 1)/(c*x - 1)) - 1/12*b*d*log(c*x + 1)/c^2 + 5/12*b*d*
log(c*x - 1)/c^2
```

3.4 $\int (d+cdx) \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=44

$$\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{bd\log(1-cx)}{c}+\frac{bdx}{2}$$

```
[Out] (b*d*x)/2 + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*c) + (b*d*Log[1 - c*x])
/c
```

Rubi [A] time = 0.0299195, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5926, 627, 43}

$$\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{bd\log(1-cx)}{c}+\frac{bdx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d*x)/2 + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*c) + (b*d*Log[1 - c*x])
/c
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)dx = \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} - \frac{b\int \frac{(d+cdx)^2}{1-c^2x^2}dx}{2d}$$
$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} - \frac{b\int \frac{d+cdx}{\frac{1}{d}-\frac{cx}{d}}dx}{2d}$$
$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} - \frac{b\int \left(-d^2-\frac{2d^2}{-1+cx}\right)dx}{2d}$$
$$= \frac{bdx}{2} + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2c} + \frac{bd\log(1-cx)}{c}$$

Mathematica [B] time = 0.0095079, size = 95, normalized size = 2.16

$$\frac{1}{2}acdx^2 + adx + \frac{bd\log\left(1-c^2x^2\right)}{2c} + \frac{1}{2}bcdx^2\tanh^{-1}(cx) + \frac{bd\log(1-cx)}{4c} - \frac{bd\log(cx+1)}{4c} + bdx\tanh^{-1}(cx) + \frac{bdx}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x]),x]
```

```
[Out] a*d*x + (b*d*x)/2 + (a*c*d*x^2)/2 + b*d*x*ArcTanh[c*x] + (b*c*d*x^2*ArcTanh
[c*x])/2 + (b*d*Log[1 - c*x])/(4*c) - (b*d*Log[1 + c*x])/(4*c) + (b*d*Log[1
 - c^2*x^2])/(2*c)
```

Maple [A] time = 0.027, size = 65, normalized size = 1.5

$$\frac{cdax^2}{2} + adx + \frac{cdb\mathrm{Artanh}\,(cx)\,x^2}{2} + db\mathrm{Artanh}\,(cx)\,x + \frac{bdx}{2} + \frac{3\,db\ln(cx-1)}{4\,c} + \frac{db\ln(cx+1)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/2*c*d*a*x^2+a*d*x+1/2*c*d*b*arctanh(c*x)*x^2+d*b*arctanh(c*x)*x+1/2*b*d*x
+3/4/c*d*b*ln(c*x-1)+1/4/c*d*b*ln(c*x+1)
```

Maxima [B] time = 0.955194, size = 115, normalized size = 2.61

$$\frac{1}{2}\,acdx^2 + \frac{1}{4}\left(2\,x^2\,\mathrm{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)bcd + adx + \frac{\left(2\,cx\,\mathrm{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*b*c*d + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 +
1))*b*d/c
```

Fricas [A] time = 2.01151, size = 185, normalized size = 4.2

$$\frac{2\,ac^2dx^2 + 2\,(2\,a + b)cdx + bd\log\left(cx + 1\right) + 3\,bd\log\left(cx - 1\right) + \left(bc^2dx^2 + 2\,bcdx\right)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*c^2*d*x^2 + 2*(2*a + b)*c*d*x + b*d*log(c*x + 1) + 3*b*d*log(c*x -
 1) + (b*c^2*d*x^2 + 2*b*c*d*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [A] time = 0.968841, size = 75, normalized size = 1.7

$$\begin{cases} \frac{acdx^2}{2} + adx + \frac{bcdx^2\operatorname{atanh}\left(cx\right)}{2} + bdx\operatorname{atanh}\left(cx\right) + \frac{bdx}{2} + \frac{bd\log\left(x-\frac{1}{c}\right)}{c} + \frac{bd\operatorname{atanh}\left(cx\right)}{2c} & \text{for}\ c \neq 0 \\ adx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c*d*x**2/2 + a*d*x + b*c*d*x**2*atanh(c*x)/2 + b*d*x*atanh(c*x
) + b*d*x/2 + b*d*log(x - 1/c)/c + b*d*atanh(c*x)/(2*c), Ne(c, 0)), (a*d*x,
 True))
```

Giac [A] time = 1.18465, size = 103, normalized size = 2.34

$$\frac{1}{2}\,acdx^2 + \frac{1}{2}\,(2\,ad + bd)x + \frac{bd\log\left(cx + 1\right)}{4\,c} + \frac{3\,bd\log\left(cx - 1\right)}{4\,c} + \frac{1}{4}\left(bcdx^2 + 2\,bdx\right)\log\left(-\frac{cx + 1}{cx - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*a*c*d*x^2 + 1/2*(2*a*d + b*d)*x + 1/4*b*d*log(c*x + 1)/c + 3/4*b*d*log(
c*x - 1)/c + 1/4*(b*c*d*x^2 + 2*b*d*x)*log(-(c*x + 1)/(c*x - 1))
```

3.5 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}bd\text{PolyLog}(2,cx)+acdx+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)+bcdx\tanh^{-1}(cx)$$

```
[Out] a*c*d*x + b*c*d*x*ArcTanh[c*x] + a*d*Log[x] + (b*d*Log[1 - c^2*x^2])/2 - (b
*d*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.0693332, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5940, 5910, 260, 5912}

$$-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}bd\text{PolyLog}(2,cx)+acdx+ad\log(x)+\frac{1}{2}bd\log\left(1-c^2x^2\right)+bcdx\tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] a*c*d*x + b*c*d*x*ArcTanh[c*x] + a*d*Log[x] + (b*d*Log[1 - c^2*x^2])/2 - (b
*d*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(cd\left(a+b\tanh^{-1}(cx)\right)+\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}\right)dx$$
$$= d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + (cd)\int \left(a+b\tanh^{-1}(cx)\right)dx$$
$$= acdx + ad\log(x) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) + (bcd)\int \tanh^{-1}(cx)\,dx$$
$$= acdx + bcdx\tanh^{-1}(cx) + ad\log(x) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd\text{Li}_2(cx) - \left(bc^2d\right)\int \frac{}{1}$$
$$= acdx + bcdx\tanh^{-1}(cx) + ad\log(x) + \frac{1}{2}bd\log\left(1-c^2x^2\right) - \frac{1}{2}bd\text{Li}_2(-cx) + \frac{1}{2}bd$$

Mathematica [A] time = 0.0754164, size = 54, normalized size = 0.9

$$\frac{1}{2}d\left(-b\text{PolyLog}(2,-cx)+b\text{PolyLog}(2,cx)+2acx+2a\log(x)+b\log\left(1-c^2x^2\right)+2bcx\tanh^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d*(2*a*c*x + 2*b*c*x*ArcTanh[c*x] + 2*a*Log[x] + b*Log[1 - c^2*x^2] - b*Po
lyLog[2, -(c*x)] + b*PolyLog[2, c*x]))/2
```

Maple [A] time = 0.039, size = 86, normalized size = 1.4

$$da\ln(cx) + acdx + db\text{Artanh}(cx)\ln(cx) + bcdx\text{Artanh}(cx) + \frac{db\ln(cx-1)}{2} + \frac{db\ln(cx+1)}{2} - \frac{db\text{dilog}(cx)}{2} - \frac{db\text{dil}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x,x)
```

```
[Out] d*a*ln(c*x)+a*c*d*x+d*b*arctanh(c*x)*ln(c*x)+b*c*d*x*arctanh(c*x)+1/2*d*b*l
n(c*x-1)+1/2*d*b*ln(c*x+1)-1/2*d*b*dilog(c*x)-1/2*d*b*dilog(c*x+1)-1/2*d*b*
ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$acdx + \frac{1}{2}\left(2\,cx\,\text{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)bd + \frac{1}{2}\,bd\int \frac{\log(cx+1)-\log(-cx+1)}{x}\,dx + ad\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] a*c*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d + 1/2*b*d*integr
ate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*d*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{acdx + ad + (bcdx + bd)\operatorname{artanh}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int ac\,dx + \int \frac{a}{x}\,dx + \int bc\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x,x)
```

```
[Out] d*(Integral(a*c, x) + Integral(a/x, x) + Integral(b*c*atanh(c*x), x) + Inte
gral(b*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)(b\operatorname{artanh}(cx) + a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x, x)
```

3.6 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{2}bcd\text{PolyLog}(2,-cx)+\frac{1}{2}bcd\text{PolyLog}(2,cx)-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)+bcd\log(x$$

```
[Out] -((d*(a + b*ArcTanh[c*x]))/x) + a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
- c^2*x^2])/2 - (b*c*d*PolyLog[2, -(c*x)])/2 + (b*c*d*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.0859438, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5940, 5916, 266, 36, 29, 31, 5912}

$$-\frac{1}{2}bcd\text{PolyLog}(2,-cx)+\frac{1}{2}bcd\text{PolyLog}(2,cx)-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)+bcd\log(x$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] -((d*(a + b*ArcTanh[c*x]))/x) + a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[1
- c^2*x^2])/2 - (b*c*d*PolyLog[2, -(c*x)])/2 + (b*c*d*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx &= \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{x}\right)dx \\
&= d\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+(cd)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx \\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+(bcd)\int \frac{}{x(1} \\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+\frac{1}{2}(bcd)\,\text{Subs} \\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)-\frac{1}{2}bcd\text{Li}_2(-cx)+\frac{1}{2}bcd\text{Li}_2(cx)+\frac{1}{2}(bcd)\,\text{Subs} \\
&= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{x}+acd\log(x)+bcd\log(x)-\frac{1}{2}bcd\log\left(1-c^2x^2\right)-\frac{1}{2}bcd\text{Li}_2(
\end{aligned}$$

Mathematica [A] time = 0.0639396, size = 71, normalized size = 1.01

$$\frac{1}{2}bcd(\text{PolyLog}(2,cx)-\text{PolyLog}(2,-cx))+acd\log(x)-\frac{ad}{x}+bcd\left(-\frac{1}{2}\log\left(1-c^2x^2\right)+\log(cx)-\frac{\tanh^{-1}(cx)}{cx}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*c*d*Log[x] + b*c*d*(-(ArcTanh[c*x]/(c*x)) + Log[c*x] - Log[1
 - c^2*x^2]/2) + (b*c*d*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

Maple [A] time = 0.046, size = 105, normalized size = 1.5

$$-\frac{da}{x}+cda\ln(cx)-\frac{db\text{Artanh}(cx)}{x}+cdb\text{Artanh}(cx)\ln(cx)-\frac{cdb\ln(cx-1)}{2}+cdb\ln(cx)-\frac{cdb\ln(cx+1)}{2}-\frac{cdb\text{dilog}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] -a*d/x+c*d*a*ln(c*x)-d*b*arctanh(c*x)/x+c*d*b*arctanh(c*x)*ln(c*x)-1/2*c*d*
b*ln(c*x-1)+c*d*b*ln(c*x)-1/2*c*d*b*ln(c*x+1)-1/2*c*d*b*dilog(c*x)-1/2*c*d*
b*dilog(c*x+1)-1/2*c*d*b*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,bcd\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{x}\,dx+acd\log\left(x\right)-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}\left(cx\right)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*d*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c*d*log(x) - 1
/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - a*d/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{acdx+ad+(bcdx+bd)\operatorname{artanh}\left(cx\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c*d*x + a*d + (b*c*d*x + b*d)*arctanh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a}{x^2}\,dx+\int\frac{ac}{x}\,dx+\int\frac{b\operatorname{atanh}\left(cx\right)}{x^2}\,dx+\int\frac{bc\operatorname{atanh}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d*(Integral(a/x**2, x) + Integral(a*c/x, x) + Integral(b*atanh(c*x)/x**2, x
) + Integral(b*c*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cdx+d)(b\operatorname{artanh}\left(cx\right)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)/x^2, x)
```

3.7 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=56

$$-\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)-\frac{bcd}{2x}$$

```
[Out] -(b*c*d)/(2*x) - (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*x^2) + b*c^2*d*Log
[x] - b*c^2*d*Log[1 - c*x]
```

Rubi [A] time = 0.0540103, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {37, 5936, 12, 77}

$$-\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)-\frac{bcd}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d)/(2*x) - (d*(1 + c*x)^2*(a + b*ArcTanh[c*x]))/(2*x^2) + b*c^2*d*Log
[x] - b*c^2*d*Log[1 - c*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx &= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-(bc)\int \frac{d(-1-cx)}{2x^2(1-cx)}\,dx\\ &= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{1}{2}(bcd)\int \frac{-1-cx}{x^2(1-cx)}\,dx\\ &= -\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{1}{2}(bcd)\int \left(-\frac{1}{x^2}-\frac{2c}{x}+\frac{2c^2}{-1+cx}\right)dx\\ &= -\frac{bcd}{2x}-\frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}+bc^2d\log(x)-bc^2d\log(1-cx)\end{aligned}$$

Mathematica [A] time = 0.0603059, size = 76, normalized size = 1.36

$$-\frac{d\left(4acx+2a-4bc^2x^2\log(x)+3bc^2x^2\log(1-cx)+bc^2x^2\log(cx+1)+2bcx+2(2bcx+b)\tanh^{-1}(cx)\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(d*(2*a + 4*a*c*x + 2*b*c*x + 2*(b + 2*b*c*x)*ArcTanh[c*x] - 4*b*c^2*x^2*L
og[x] + 3*b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x]))/(4*x^2)
```

Maple [A] time = 0.039, size = 84, normalized size = 1.5

$$-\frac{cda}{x}-\frac{da}{2\,x^2}-\frac{cdb\mathrm{Artanh}\,(cx)}{x}-\frac{db\mathrm{Artanh}\,(cx)}{2\,x^2}-\frac{3\,c^2db\ln(cx-1)}{4}+c^2db\ln(cx)-\frac{cdb}{2\,x}-\frac{c^2db\ln(cx+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] -c*d*a/x-1/2*d*a/x^2-c*d*b*arctanh(c*x)/x-1/2*d*b*arctanh(c*x)/x^2-3/4*c^2*
d*b*ln(c*x-1)+c^2*d*b*ln(c*x)-1/2*b*c*d/x-1/4*c^2*d*b*ln(c*x+1)
```

Maxima [A] time = 0.957128, size = 120, normalized size = 2.14

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}\,(cx)}{x}\right)bcd+\frac{1}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)bd\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d + 1/4*((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d - a*c*d/x -
 1/2*a*d/x^2
```

Fricas [A] time = 2.03806, size = 220, normalized size = 3.93

$$-\frac{bc^2dx^2\log(cx+1)+3\,bc^2dx^2\log(cx-1)-4\,bc^2dx^2\log(x)+2\,(2\,a+b)cdx+2\,ad+(2\,bcdx+bd)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c^2*d*x^2*log(c*x + 1) + 3*b*c^2*d*x^2*log(c*x - 1) - 4*b*c^2*d*x^2
*log(x) + 2*(2*a + b)*c*d*x + 2*a*d + (2*b*c*d*x + b*d)*log(-(c*x + 1)/(c*x
 - 1)))/x^2
```

Sympy [A] time = 4.1712, size = 95, normalized size = 1.7

$$\begin{cases} -\frac{acd}{x}-\frac{ad}{2x^2}+bc^2d\log(x)-bc^2d\log\left(x-\frac{1}{c}\right)-\frac{bc^2d\operatorname{atanh}(cx)}{2}-\frac{bcd\operatorname{atanh}(cx)}{x}-\frac{bcd}{2x}-\frac{bd\operatorname{atanh}(cx)}{2x^2} & \text{for}\ c\neq 0\\ -\frac{ad}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] Piecewise((-a*c*d/x - a*d/(2*x**2) + b*c**2*d*log(x) - b*c**2*d*log(x - 1/c
) - b*c**2*d*atanh(c*x)/2 - b*c*d*atanh(c*x)/x - b*c*d/(2*x) - b*d*atanh(c*
x)/(2*x**2), Ne(c, 0)), (-a*d/(2*x**2), True))
```

Giac [A] time = 1.1892, size = 115, normalized size = 2.05

$$-\frac{1}{4}\,bc^2d\log(cx+1)-\frac{3}{4}\,bc^2d\log(cx-1)+bc^2d\log(x)-\frac{(2\,bcdx+bd)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,x^2}-\frac{2\,acdx+bcdx+ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*c^2*d*log(c*x + 1) - 3/4*b*c^2*d*log(c*x - 1) + b*c^2*d*log(x) - 1/4
*(2*b*c*d*x + b*d)*log(-(c*x + 1)/(c*x - 1))/x^2 - 1/2*(2*a*c*d*x + b*c*d*x
 + a*d)/x^2
```

3.8 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{bc^2d}{2x}+\frac{1}{3}bc^3d\log(x)-\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(cx+1)-\frac{b\cdots}{6\cdots}$$

```
[Out] -(b*c*d)/(6*x^2) - (b*c^2*d)/(2*x) - (d*(a + b*ArcTanh[c*x]))/(3*x^3) - (c*
d*(a + b*ArcTanh[c*x]))/(2*x^2) + (b*c^3*d*Log[x])/3 - (5*b*c^3*d*Log[1 - c
*x])/12 + (b*c^3*d*Log[1 + c*x])/12
```

Rubi [A] time = 0.0866609, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 801}

$$-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{bc^2d}{2x}+\frac{1}{3}bc^3d\log(x)-\frac{5}{12}bc^3d\log(1-cx)+\frac{1}{12}bc^3d\log(cx+1)-\frac{b\cdots}{6\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d)/(6*x^2) - (b*c^2*d)/(2*x) - (d*(a + b*ArcTanh[c*x]))/(3*x^3) - (c*
d*(a + b*ArcTanh[c*x]))/(2*x^2) + (b*c^3*d*Log[x])/3 - (5*b*c^3*d*Log[1 - c
*x])/12 + (b*c^3*d*Log[1 + c*x])/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^4}dx = -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - (bc)\int \frac{d(-2-3cx)}{6x^3\left(1-c^2x^2\right)}dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{6}(bcd)\int \frac{-2-3cx}{x^3\left(1-c^2x^2\right)}dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{6}(bcd)\int \left(-\frac{2}{x^3} - \frac{3c}{x^2} - \frac{2c^2}{x} + \frac{}{2(-}\right.$$

$$= -\frac{bcd}{6x^2} - \frac{bc^2d}{2x} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{2x^2} + \frac{1}{3}bc^3d\log(x) - \frac{5}{12}b$$

Mathematica [A] time = 0.0640414, size = 86, normalized size = 0.88

$$-\frac{d\left(6acx+4a+6bc^2x^2-4bc^3x^3\log(x)+5bc^3x^3\log(1-cx)-bc^3x^3\log(cx+1)+2bcx+2b(3cx+2)\tanh^{-1}(cx)\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(d*(4*a + 6*a*c*x + 2*b*c*x + 6*b*c^2*x^2 + 2*b*(2 + 3*c*x)*ArcTanh[c*x] -
 4*b*c^3*x^3*Log[x] + 5*b*c^3*x^3*Log[1 - c*x] - b*c^3*x^3*Log[1 + c*x]))/(
12*x^3)
```

Maple [A] time = 0.039, size = 95, normalized size = 1.

$$-\frac{cda}{2\,x^2} - \frac{da}{3\,x^3} - \frac{cdb\mathrm{Artanh}\,(cx)}{2\,x^2} - \frac{db\mathrm{Artanh}\,(cx)}{3\,x^3} - \frac{5\,c^3db\ln(cx-1)}{12} - \frac{cdb}{6\,x^2} - \frac{bc^2d}{2\,x} + \frac{c^3db\ln(cx)}{3} + \frac{bc^3d\ln(cx+1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -1/2*c*d*a/x^2-1/3*d*a/x^3-1/2*c*d*b*arctanh(c*x)/x^2-1/3*d*b*arctanh(c*x)/
x^3-5/12*c^3*d*b*ln(c*x-1)-1/6*b*c*d/x^2-1/2*b*c^2*d/x+1/3*c^3*d*b*ln(c*x)+
1/12*b*c^3*d*ln(c*x+1)
```

Maxima [A] time = 0.960429, size = 134, normalized size = 1.37

$$\frac{1}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\,(cx)}{x^2}\right)bcd-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\,(cx}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
- 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b*d - 1/2*a*c*d/x^2 - 1/3*a*d/x^3
```

Fricas [A] time = 2.22538, size = 244, normalized size = 2.49

$$\frac{bc^3dx^3\log(cx+1) - 5\,bc^3dx^3\log(cx-1) + 4\,bc^3dx^3\log(x) - 6\,bc^2dx^2 - 2\,(3\,a+b)cdx - 4\,ad - (3\,bcdx + 2\,bd)\log\left(-\right.}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*d*x^3*log(c*x + 1) - 5*b*c^3*d*x^3*log(c*x - 1) + 4*b*c^3*d*x^3
*log(x) - 6*b*c^2*d*x^2 - 2*(3*a + b)*c*d*x - 4*a*d - (3*b*c*d*x + 2*b*d)*l
og(-(c*x + 1)/(c*x - 1)))/x^3
```

Sympy [A] time = 2.38852, size = 117, normalized size = 1.19

$$\begin{cases} -\frac{acd}{2x^2} - \frac{ad}{3x^3} + \frac{bc^3d\log(x)}{3} - \frac{bc^3d\log\left(x-\frac{1}{c}\right)}{3} + \frac{bc^3d\operatorname{atanh}(cx)}{6} - \frac{bc^2d}{2x} - \frac{bcd\operatorname{atanh}(cx)}{2x^2} - \frac{bcd}{6x^2} - \frac{bd\operatorname{atanh}(cx)}{3x^3} & \text{for}\ c \neq 0 \\ -\frac{ad}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*c*d/(2*x**2) - a*d/(3*x**3) + b*c**3*d*log(x)/3 - b*c**3*d*lo
g(x - 1/c)/3 + b*c**3*d*atanh(c*x)/6 - b*c**2*d/(2*x) - b*c*d*atanh(c*x)/(2
*x**2) - b*c*d/(6*x**2) - b*d*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d/(3*x**3
), True))
```

Giac [A] time = 1.27117, size = 132, normalized size = 1.35

$$\frac{1}{12}\,bc^3d\log(cx+1) - \frac{5}{12}\,bc^3d\log(cx-1) + \frac{1}{3}\,bc^3d\log(x) - \frac{(3\,bcdx + 2\,bd)\log\left(-\frac{cx+1}{cx-1}\right)}{12\,x^3} - \frac{3\,bc^2dx^2 + 3\,acdx + bcdx +}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b*c^3*d*log(c*x + 1) - 5/12*b*c^3*d*log(c*x - 1) + 1/3*b*c^3*d*log(x)
- 1/12*(3*b*c*d*x + 2*b*d)*log(-(c*x + 1)/(c*x - 1))/x^3 - 1/6*(3*b*c^2*d*x
^2 + 3*a*c*d*x + b*c*d*x + 2*a*d)/x^3
```

3.9 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=110

$$-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d}{6x^2}-\frac{bc^3d}{4x}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4d\log(cx+1)$$

```
[Out] -(b*c*d)/(12*x^3) - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4*x) - (d*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c*d*(a + b*ArcTanh[c*x]))/(3*x^3) + (b*c^4*d*Log[x])/3 -
 (7*b*c^4*d*Log[1 - c*x])/24 - (b*c^4*d*Log[1 + c*x])/24
```

Rubi [A] time = 0.0924434, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 801}

$$-\frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d}{6x^2}-\frac{bc^3d}{4x}+\frac{1}{3}bc^4d\log(x)-\frac{7}{24}bc^4d\log(1-cx)-\frac{1}{24}bc^4d\log(cx+1)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d)/(12*x^3) - (b*c^2*d)/(6*x^2) - (b*c^3*d)/(4*x) - (d*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c*d*(a + b*ArcTanh[c*x]))/(3*x^3) + (b*c^4*d*Log[x])/3 -
 (7*b*c^4*d*Log[1 - c*x])/24 - (b*c^4*d*Log[1 + c*x])/24
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)}{x^5}dx = -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - (bc)\int \frac{d(-3-4cx)}{12x^4\left(1-c^2x^2\right)}dx$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{12}(bcd)\int \frac{-3-4cx}{x^4\left(1-c^2x^2\right)}dx$$
$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{12}(bcd)\int \left(-\frac{3}{x^4} - \frac{4c}{x^3} - \frac{3c^2}{x^2} -$$
$$= -\frac{bcd}{12x^3} - \frac{bc^2d}{6x^2} - \frac{bc^3d}{4x} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)}{3x^3} + \frac{1}{3}bc^4d$$

Mathematica [A] time = 0.0649337, size = 94, normalized size = 0.85

$$-\frac{d\left(8acx+6a+6bc^3x^3+4bc^2x^2-8bc^4x^4\log(x)+7bc^4x^4\log(1-cx)+bc^4x^4\log(cx+1)+2bcx+2b(4cx+3)\tanh\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(d*(6*a + 8*a*c*x + 2*b*c*x + 4*b*c^2*x^2 + 6*b*c^3*x^3 + 2*b*(3 + 4*c*x)*
ArcTanh[c*x] - 8*b*c^4*x^4*Log[x] + 7*b*c^4*x^4*Log[1 - c*x] + b*c^4*x^4*Lo
g[1 + c*x]))/(24*x^4)
```

Maple [A] time = 0.04, size = 105, normalized size = 1.

$$-\frac{da}{4\,x^4} - \frac{cda}{3\,x^3} - \frac{db\mathrm{Artanh}\,(cx)}{4\,x^4} - \frac{cdb\mathrm{Artanh}\,(cx)}{3\,x^3} - \frac{7\,c^4db\ln(cx-1)}{24} - \frac{cdb}{12\,x^3} - \frac{bc^2d}{6\,x^2} - \frac{bc^3d}{4\,x} + \frac{c^4db\ln(cx)}{3} - \frac{bc^4d\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/4*d*a/x^4-1/3*c*d*a/x^3-1/4*d*b*arctanh(c*x)/x^4-1/3*c*d*b*arctanh(c*x)/
x^3-7/24*c^4*d*b*ln(c*x-1)-1/12*b*c*d/x^3-1/6*b*c^2*d/x^2-1/4*b*c^3*d/x+1/3
*c^4*d*b*ln(c*x)-1/24*b*c^4*d*ln(c*x+1)
```

Maxima [A] time = 0.965634, size = 154, normalized size = 1.4

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}\,(cx)}{x^3}\right)bcd+\frac{1}{24}\left(\left(3\,c^3\log(cx+1)-3\,c^3\log(cx-1)-\frac{2\left(3\,c^2x^2\right.}{x^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c*d + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)
/x^3)*c - 6*arctanh(c*x)/x^4)*b*d - 1/3*a*c*d/x^3 - 1/4*a*d/x^4
```

Fricas [A] time = 2.10247, size = 267, normalized size = 2.43

$$-\frac{bc^4dx^4\log(cx+1)+7\,bc^4dx^4\log(cx-1)-8\,bc^4dx^4\log(x)+6\,bc^3dx^3+4\,bc^2dx^2+2\,(4\,a+b)cdx+6\,ad+(4\,bcdx+3}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(b*c^4*d*x^4*log(c*x + 1) + 7*b*c^4*d*x^4*log(c*x - 1) - 8*b*c^4*d*x^
4*log(x) + 6*b*c^3*d*x^3 + 4*b*c^2*d*x^2 + 2*(4*a + b)*c*d*x + 6*a*d + (4*b
*c*d*x + 3*b*d)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [A] time = 3.09623, size = 129, normalized size = 1.17

$$\begin{cases} -\frac{acd}{3x^3}-\frac{ad}{4x^4}+\frac{bc^4d\log(x)}{3}-\frac{bc^4d\log\left(x-\frac{1}{c}\right)}{3}-\frac{bc^4d\operatorname{atanh}(cx)}{12}-\frac{bc^3d}{4x}-\frac{bc^2d}{6x^2}-\frac{bcd\operatorname{atanh}(cx)}{3x^3}-\frac{bcd}{12x^3}-\frac{bd\operatorname{atanh}(cx)}{4x^4} & \text{for } c\neq 0 \\ -\frac{ad}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c*d/(3*x**3) - a*d/(4*x**4) + b*c**4*d*log(x)/3 - b*c**4*d*lo
g(x - 1/c)/3 - b*c**4*d*atanh(c*x)/12 - b*c**3*d/(4*x) - b*c**2*d/(6*x**2)
- b*c*d*atanh(c*x)/(3*x**3) - b*c*d/(12*x**3) - b*d*atanh(c*x)/(4*x**4), Ne
(c, 0)), (-a*d/(4*x**4), True))
```

Giac [A] time = 1.25656, size = 146, normalized size = 1.33

$$-\frac{1}{24}\,bc^4d\log(cx+1)-\frac{7}{24}\,bc^4d\log(cx-1)+\frac{1}{3}\,bc^4d\log(x)-\frac{(4\,bcdx+3\,bd)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,x^4}-\frac{3\,bc^3dx^3+2\,bc^2dx^2+4\,acd}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] -1/24*b*c^4*d*log(c*x + 1) - 7/24*b*c^4*d*log(c*x - 1) + 1/3*b*c^4*d*log(x)
 - 1/24*(4*b*c*d*x + 3*b*d)*log(-(c*x + 1)/(c*x - 1))/x^4 - 1/12*(3*b*c^3*d
*x^3 + 2*b*c^2*d*x^2 + 4*a*c*d*x + b*c*d*x + 3*a*d)/x^4
```

3.10 $\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=157

$$\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}+\frac{49bd^2\log(1}{120c^4}$$

```
[Out] (5*b*d^2*x)/(12*c^3) + (b*d^2*x^2)/(5*c^2) + (5*b*d^2*x^3)/(36*c) + (b*d^2*
x^4)/10 + (b*c*d^2*x^5)/30 + (d^2*x^4*(a + b*ArcTanh[c*x]))/4 + (2*c*d^2*x^
5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x]))/6 + (49*b*d^
2*Log[1 - c*x])/(120*c^4) - (b*d^2*Log[1 + c*x])/(120*c^4)
```

Rubi [A] time = 0.167813, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}+\frac{49bd^2\log(1}{120c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```


```
[Out] (5*b*d^2*x)/(12*c^3) + (b*d^2*x^2)/(5*c^2) + (5*b*d^2*x^3)/(36*c) + (b*d^2*
x^4)/10 + (b*c*d^2*x^5)/30 + (d^2*x^4*(a + b*ArcTanh[c*x]))/4 + (2*c*d^2*x^
5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x]))/6 + (49*b*d^
2*Log[1 - c*x])/(120*c^4) - (b*d^2*Log[1 + c*x])/(120*c^4)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+$$

$$=\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+$$

$$=\frac{5bd^2x}{12c^3}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x^3}{36c}+\frac{1}{10}bd^2x^4+\frac{1}{30}bcd^2x^5+\frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)+$$

Mathematica [A] time = 0.104789, size = 125, normalized size = 0.8

$$\frac{d^2\left(60ac^6x^6+144ac^5x^5+90ac^4x^4+12bc^5x^5+36bc^4x^4+50bc^3x^3+72bc^2x^2+6bc^4x^4\left(10c^2x^2+24cx+15\right)\tanh^{-1}(cx)\right.}{360c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(150*b*c*x + 72*b*c^2*x^2 + 50*b*c^3*x^3 + 90*a*c^4*x^4 + 36*b*c^4*x^4
 + 144*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^4*x^4*(15 + 24*c*x +
 10*c^2*x^2)*ArcTanh[c*x] + 147*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(360*c^
4)
```

Maple [A] time = 0.031, size = 159, normalized size = 1.

$$\frac{c^2d^2ax^6}{6}+\frac{2\,cd^2ax^5}{5}+\frac{d^2ax^4}{4}+\frac{c^2d^2b\mathrm{Artanh}\,(cx)\,x^6}{6}+\frac{2\,cd^2b\mathrm{Artanh}\,(cx)\,x^5}{5}+\frac{d^2b\mathrm{Artanh}\,(cx)\,x^4}{4}+\frac{bcd^2x^5}{30}+\frac{bd^2x^4}{10}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*c^2*d^2*a*x^6+2/5*c*d^2*a*x^5+1/4*d^2*a*x^4+1/6*c^2*d^2*b*arctanh(c*x)*
x^6+2/5*c*d^2*b*arctanh(c*x)*x^5+1/4*d^2*b*arctanh(c*x)*x^4+1/30*b*c*d^2*x^
5+1/10*b*d^2*x^4+5/36*b*d^2*x^3/c+1/5*b*d^2*x^2/c^2+5/12*b*d^2*x/c^3+49/120
```

```
/c^4*d^2*b*ln(c*x-1)-1/120*b*d^2*ln(c*x+1)/c^4
```

Maxima [A] time = 0.96363, size = 284, normalized size = 1.81

$$\frac{1}{6}\,ac^2d^2x^6+\frac{2}{5}\,acd^2x^5+\frac{1}{4}\,ad^2x^4+\frac{1}{180}\left(30\,x^6\,\text{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15\,\log\,(\right.\right.}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^2*d^2*x^6 + 2/5*a*c*d^2*x^5 + 1/4*a*d^2*x^4 + 1/180*(30*x^6*arctanh
(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*
log(c*x - 1)/c^7))*b*c^2*d^2 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x
^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c*d^2 + 1/24*(6*x^4*arctanh(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^2
```

Fricas [A] time = 2.037, size = 373, normalized size = 2.38

$$\frac{60\,ac^6d^2x^6+12\,(12\,a+b)c^5d^2x^5+18\,(5\,a+2\,b)c^4d^2x^4+50\,bc^3d^2x^3+72\,bc^2d^2x^2+150\,bcd^2x-3\,bd^2\,\log\,(cx+1)+1}{360\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(60*a*c^6*d^2*x^6 + 12*(12*a + b)*c^5*d^2*x^5 + 18*(5*a + 2*b)*c^4*d^
2*x^4 + 50*b*c^3*d^2*x^3 + 72*b*c^2*d^2*x^2 + 150*b*c*d^2*x - 3*b*d^2*log(c
*x + 1) + 147*b*d^2*log(c*x - 1) + 3*(10*b*c^6*d^2*x^6 + 24*b*c^5*d^2*x^5 +
 15*b*c^4*d^2*x^4)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] time = 8.17384, size = 196, normalized size = 1.25

$$\begin{cases}\frac{ac^2d^2x^6}{6}+\frac{2acd^2x^5}{5}+\frac{ad^2x^4}{4}+\frac{bc^2d^2x^6\,\text{atanh}\,(cx)}{6}+\frac{2bcd^2x^5\,\text{atanh}\,(cx)}{5}+\frac{bcd^2x^5}{30}+\frac{bd^2x^4\,\text{atanh}\,(cx)}{4}+\frac{bd^2x^4}{10}+\frac{5bd^2x^3}{36c}+\frac{bd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}+ \\ \frac{ad^2x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**6/6 + 2*a*c*d**2*x**5/5 + a*d**2*x**4/4 + b*c**2*
d**2*x**6*atanh(c*x)/6 + 2*b*c*d**2*x**5*atanh(c*x)/5 + b*c*d**2*x**5/30 +
b*d**2*x**4*atanh(c*x)/4 + b*d**2*x**4/10 + 5*b*d**2*x**3/(36*c) + b*d**2*x
**2/(5*c**2) + 5*b*d**2*x/(12*c**3) + 2*b*d**2*log(x - 1/c)/(5*c**4) - b*d*
*2*atanh(c*x)/(60*c**4), Ne(c, 0)), (a*d**2*x**4/4, True))
```

Giac [A] time = 1.25513, size = 220, normalized size = 1.4

$$\frac{1}{6}\,ac^2d^2x^6+\frac{1}{30}\left(12\,acd^2+bcd^2\right)x^5+\frac{5\,bd^2x^3}{36\,c}+\frac{1}{20}\left(5\,ad^2+2\,bd^2\right)x^4+\frac{bd^2x^2}{5\,c^2}+\frac{1}{120}\left(10\,bc^2d^2x^6+24\,bcd^2x^5+15\,bd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*a*c^2*d^2*x^6 + 1/30*(12*a*c*d^2 + b*c*d^2)*x^5 + 5/36*b*d^2*x^3/c + 1/
20*(5*a*d^2 + 2*b*d^2)*x^4 + 1/5*b*d^2*x^2/c^2 + 1/120*(10*b*c^2*d^2*x^6 +
24*b*c*d^2*x^5 + 15*b*d^2*x^4)*log(-(c*x + 1)/(c*x - 1)) + 5/12*b*d^2*x/c^3
 - 1/120*b*d^2*log(c*x + 1)/c^4 + 49/120*b*d^2*log(c*x - 1)/c^4
```

3.11 $\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=143

$$\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^2x}{2c^2}+\frac{31bd^2\log(1-cx)}{60c^3}+\frac{b}{}$$

```
[Out] (b*d^2*x)/(2*c^2) + (4*b*d^2*x^2)/(15*c) + (b*d^2*x^3)/6 + (b*c*d^2*x^4)/20
 + (d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c*d^2*x^4*(a + b*ArcTanh[c*x]))/2 +
(c^2*d^2*x^5*(a + b*ArcTanh[c*x]))/5 + (31*b*d^2*Log[1 - c*x])/(60*c^3) + (
b*d^2*Log[1 + c*x])/(60*c^3)
```

Rubi [A] time = 0.154411, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{bd^2x}{2c^2}+\frac{31bd^2\log(1-cx)}{60c^3}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*d^2*x)/(2*c^2) + (4*b*d^2*x^2)/(15*c) + (b*d^2*x^3)/6 + (b*c*d^2*x^4)/20
 + (d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c*d^2*x^4*(a + b*ArcTanh[c*x]))/2 +
(c^2*d^2*x^5*(a + b*ArcTanh[c*x]))/5 + (31*b*d^2*Log[1 - c*x])/(60*c^3) + (
b*d^2*Log[1 + c*x])/(60*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}\right.$$

$$= \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a\right.$$

$$= \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a\right.$$

$$= \frac{bd^2x}{2c^2}+\frac{4bd^2x^2}{15c}+\frac{1}{6}bd^2x^3+\frac{1}{20}bcd^2x^4+\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}cd^2x^4\left(a\right.$$

Mathematica [A] time = 0.0936998, size = 115, normalized size = 0.8

$$\frac{d^2\left(12ac^5x^5+30ac^4x^4+20ac^3x^3+3bc^4x^4+10bc^3x^3+16bc^2x^2+2bc^3x^3\left(6c^2x^2+15cx+10\right)\tanh^{-1}(cx)+30bcx+31b\right.}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(30*b*c*x + 16*b*c^2*x^2 + 20*a*c^3*x^3 + 10*b*c^3*x^3 + 30*a*c^4*x^4
+ 3*b*c^4*x^4 + 12*a*c^5*x^5 + 2*b*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2)*ArcTan
h[c*x] + 31*b*Log[1 - c*x] + b*Log[1 + c*x]))/(60*c^3)
```

Maple [A] time = 0.029, size = 147, normalized size = 1.

$$\frac{c^2d^2ax^5}{5}+\frac{cd^2ax^4}{2}+\frac{d^2ax^3}{3}+\frac{c^2d^2b\text{Artanh}(cx)x^5}{5}+\frac{cd^2b\text{Artanh}(cx)x^4}{2}+\frac{d^2b\text{Artanh}(cx)x^3}{3}+\frac{bcd^2x^4}{20}+\frac{bd^2x^3}{6}+\frac{4b}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c^2*d^2*a*x^5+1/2*c*d^2*a*x^4+1/3*d^2*a*x^3+1/5*c^2*d^2*b*arctanh(c*x)*
x^5+1/2*c*d^2*b*arctanh(c*x)*x^4+1/3*d^2*b*arctanh(c*x)*x^3+1/20*b*c*d^2*x^
4+1/6*b*d^2*x^3+4/15*b*d^2*x^2/c+1/2*b*d^2*x/c^2+31/60/c^3*d^2*b*ln(c*x-1)+
1/60*b*d^2*ln(c*x+1)/c^3
```

Maxima [A] time = 0.975962, size = 248, normalized size = 1.73

$$\frac{1}{5}ac^2d^2x^5 + \frac{1}{2}acd^2x^4 + \frac{1}{20}\left(4x^5\operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2x^2}{c^4} + \frac{2\log(c^2x^2 - 1)}{c^6}\right)\right)bc^2d^2 + \frac{1}{3}ad^2x^3 + \frac{1}{12}\left(6x^4\operatorname{artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^2*d^2*x^5 + 1/2*a*c*d^2*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^
4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/12*
(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log
(c*x - 1)/c^5))*b*c*d^2 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^
2 - 1)/c^4))*b*d^2
```

Fricas [A] time = 2.05371, size = 332, normalized size = 2.32

$$\frac{12\,ac^5d^2x^5 + 3\,(10\,a + b)c^4d^2x^4 + 10\,(2\,a + b)c^3d^2x^3 + 16\,bc^2d^2x^2 + 30\,bcd^2x + bd^2\log(cx + 1) + 31\,bd^2\log(cx - 1) +}{60\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^2*x^5 + 3*(10*a + b)*c^4*d^2*x^4 + 10*(2*a + b)*c^3*d^2*x^
3 + 16*b*c^2*d^2*x^2 + 30*b*c*d^2*x + b*d^2*log(c*x + 1) + 31*b*d^2*log(c*x
 - 1) + (6*b*c^5*d^2*x^5 + 15*b*c^4*d^2*x^4 + 10*b*c^3*d^2*x^3)*log(-(c*x +
 1)/(c*x - 1)))/c^3
```

Sympy [A] time = 3.03088, size = 177, normalized size = 1.24

$$\begin{cases} \frac{ac^2d^2x^5}{5} + \frac{acd^2x^4}{2} + \frac{ad^2x^3}{3} + \frac{bc^2d^2x^5\operatorname{atanh}(cx)}{5} + \frac{bcd^2x^4\operatorname{atanh}(cx)}{2} + \frac{bcd^2x^4}{20} + \frac{bd^2x^3\operatorname{atanh}(cx)}{3} + \frac{bd^2x^3}{6} + \frac{4bd^2x^2}{15c} + \frac{bd^2x}{2c^2} + \frac{8bd^2\log(x-}{15c^3} \\ \frac{ad^2x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**5/5 + a*c*d**2*x**4/2 + a*d**2*x**3/3 + b*c**2*d*
*2*x**5*atanh(c*x)/5 + b*c*d**2*x**4*atanh(c*x)/2 + b*c*d**2*x**4/20 + b*d*
*2*x**3*atanh(c*x)/3 + b*d**2*x**3/6 + 4*b*d**2*x**2/(15*c) + b*d**2*x/(2*c
**2) + 8*b*d**2*log(x - 1/c)/(15*c**3) + b*d**2*atanh(c*x)/(30*c**3), Ne(c,
 0)), (a*d**2*x**3/3, True))
```

Giac [A] time = 1.32498, size = 203, normalized size = 1.42

$$\frac{1}{5}ac^2d^2x^5 + \frac{1}{20}\left(10\,acd^2 + bcd^2\right)x^4 + \frac{4\,bd^2x^2}{15\,c} + \frac{1}{6}\left(2\,ad^2 + bd^2\right)x^3 + \frac{bd^2x}{2\,c^2} + \frac{1}{60}\left(6\,bc^2d^2x^5 + 15\,bcd^2x^4 + 10\,bd^2x^3\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c^2*d^2*x^5 + 1/20*(10*a*c*d^2 + b*c*d^2)*x^4 + 4/15*b*d^2*x^2/c + 1/
6*(2*a*d^2 + b*d^2)*x^3 + 1/2*b*d^2*x/c^2 + 1/60*(6*b*c^2*d^2*x^5 + 15*b*c*
d^2*x^4 + 10*b*d^2*x^3)*log(-(c*x + 1)/(c*x - 1)) + 1/60*b*d^2*log(c*x + 1)
/c^3 + 31/60*b*d^2*log(c*x - 1)/c^3
```

3.12 $\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=129

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{17bd^2\log(1-cx)}{24c^2}-\frac{bd^2\log(c}{24c^2}$$

```
[Out] (3*b*d^2*x)/(4*c) + (b*d^2*x^2)/3 + (b*c*d^2*x^3)/12 + (d^2*x^2*(a + b*ArcT
anh[c*x]))/2 + (2*c*d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c^2*d^2*x^4*(a + b*A
rcTanh[c*x]))/4 + (17*b*d^2*Log[1 - c*x])/(24*c^2) - (b*d^2*Log[1 + c*x])/(
24*c^2)
```

Rubi [A] time = 0.13022, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{17bd^2\log(1-cx)}{24c^2}-\frac{bd^2\log(c}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^2*x)/(4*c) + (b*d^2*x^2)/3 + (b*c*d^2*x^3)/12 + (d^2*x^2*(a + b*ArcT
anh[c*x]))/2 + (2*c*d^2*x^3*(a + b*ArcTanh[c*x]))/3 + (c^2*d^2*x^4*(a + b*A
rcTanh[c*x]))/4 + (17*b*d^2*Log[1 - c*x])/(24*c^2) - (b*d^2*Log[1 + c*x])/(
24*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx = \frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(\right.$$

$$=\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(\right.$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-}\right.$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-}\right.$$

$$=\frac{3bd^2x}{4c}+\frac{1}{3}bd^2x^2+\frac{1}{12}bcd^2x^3+\frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-}\right.$$

Mathematica [A] time = 0.0879692, size = 107, normalized size = 0.83

$$\frac{d^2\left(6ac^4x^4+16ac^3x^3+12ac^2x^2+2bc^3x^3+8bc^2x^2+2bc^2x^2\left(3c^2x^2+8cx+6\right)\tanh^{-1}(cx)+18bcx+17b\log(1-cx)-b\right.}{24c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(18*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 16*a*c^3*x^3 + 2*b*c^3*x^3 +
6*a*c^4*x^4 + 2*b*c^2*x^2*(6 + 8*c*x + 3*c^2*x^2)*ArcTanh[c*x] + 17*b*Log[1
 - c*x] - b*Log[1 + c*x]))/(24*c^2)
```

Maple [A] time = 0.029, size = 135, normalized size = 1.1

$$\frac{c^2d^2ax^4}{4}+\frac{2\,cd^2ax^3}{3}+\frac{d^2ax^2}{2}+\frac{c^2d^2b\mathrm{Artanh}\,(cx)\,x^4}{4}+\frac{2\,cd^2b\mathrm{Artanh}\,(cx)\,x^3}{3}+\frac{d^2b\mathrm{Artanh}\,(cx)\,x^2}{2}+\frac{bcd^2x^3}{12}+\frac{bd^2x^2}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*c^2*d^2*a*x^4+2/3*c*d^2*a*x^3+1/2*d^2*a*x^2+1/4*c^2*d^2*b*arctanh(c*x)*
x^4+2/3*c*d^2*b*arctanh(c*x)*x^3+1/2*d^2*b*arctanh(c*x)*x^2+1/12*b*c*d^2*x^
3+1/3*b*d^2*x^2+3/4*b*d^2*x/c+17/24/c^2*d^2*b*ln(c*x-1)-1/24*b*d^2*ln(c*x+1
)/c^2
```

Maxima [A] time = 0.969073, size = 242, normalized size = 1.88

$$\frac{1}{4}\,ac^2d^2x^4+\frac{2}{3}\,acd^2x^3+\frac{1}{24}\left(6\,x^4\,\mathrm{artanh}\left(cx\right)+c\left(\frac{2\left(c^2x^3+3\,x\right)}{c^4}-\frac{3\,\log\left(cx+1\right)}{c^5}+\frac{3\,\log\left(cx-1\right)}{c^5}\right)\right)bc^2d^2+\frac{1}{3}\left(2\,x^3\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^2*d^2*x^4 + 2/3*a*c*d^2*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*
x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^2*d^2 + 1/3*
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c*d^2 + 1/2*a*d
^2*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x
- 1)/c^3))*b*d^2
```

Fricas [A] time = 1.93336, size = 302, normalized size = 2.34

$$\frac{6\,ac^4d^2x^4+2\left(8\,a+b\right)c^3d^2x^3+4\left(3\,a+2\,b\right)c^2d^2x^2+18\,bcd^2x-bd^2\log\left(cx+1\right)+17\,bd^2\log\left(cx-1\right)+\left(3\,bc^4d^2x^4+8\right.}{24\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d^2*x^4 + 2*(8*a + b)*c^3*d^2*x^3 + 4*(3*a + 2*b)*c^2*d^2*x^2
 + 18*b*c*d^2*x - b*d^2*log(c*x + 1) + 17*b*d^2*log(c*x - 1) + (3*b*c^4*d^2
*x^4 + 8*b*c^3*d^2*x^3 + 6*b*c^2*d^2*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 2.45654, size = 167, normalized size = 1.29

$$\begin{cases}\frac{ac^2d^2x^4}{4}+\frac{2acd^2x^3}{3}+\frac{ad^2x^2}{2}+\frac{bc^2d^2x^4\,\mathrm{atanh}\left(cx\right)}{4}+\frac{2bcd^2x^3\,\mathrm{atanh}\left(cx\right)}{3}+\frac{bcd^2x^3}{12}+\frac{bd^2x^2\,\mathrm{atanh}\left(cx\right)}{2}+\frac{bd^2x^2}{3}+\frac{3bd^2x}{4c}+\frac{2bd^2\log\left(x-\frac{1}{c}\right)}{3c^2}- \\ \frac{ad^2x^2}{2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**4/4 + 2*a*c*d**2*x**3/3 + a*d**2*x**2/2 + b*c**2*
d**2*x**4*atanh(c*x)/4 + 2*b*c*d**2*x**3*atanh(c*x)/3 + b*c*d**2*x**3/12 +
b*d**2*x**2*atanh(c*x)/2 + b*d**2*x**2/3 + 3*b*d**2*x/(4*c) + 2*b*d**2*log(
x - 1/c)/(3*c**2) - b*d**2*atanh(c*x)/(12*c**2), Ne(c, 0)), (a*d**2*x**2/2,
 True))
```

Giac [A] time = 1.23304, size = 188, normalized size = 1.46

$$\frac{1}{4}\,ac^2d^2x^4+\frac{1}{12}\left(8\,acd^2+bcd^2\right)x^3+\frac{3\,bd^2x}{4\,c}+\frac{1}{6}\left(3\,ad^2+2\,bd^2\right)x^2-\frac{bd^2\log\left(cx+1\right)}{24\,c^2}+\frac{17\,bd^2\log\left(cx-1\right)}{24\,c^2}+\frac{1}{24}\left(3\,bc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*c^2*d^2*x^4 + 1/12*(8*a*c*d^2 + b*c*d^2)*x^3 + 3/4*b*d^2*x/c + 1/6*(3
*a*d^2 + 2*b*d^2)*x^2 - 1/24*b*d^2*log(c*x + 1)/c^2 + 17/24*b*d^2*log(c*x -
 1)/c^2 + 1/24*(3*b*c^2*d^2*x^4 + 8*b*c*d^2*x^3 + 6*b*d^2*x^2)*log(-(c*x +
1)/(c*x - 1))
```

3.13 $\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=71

$$\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{bd^2(cx+1)^2}{6c}+\frac{4bd^2\log(1-cx)}{3c}+\frac{2}{3}bd^2x$$

```
[Out] (2*b*d^2*x)/3 + (b*d^2*(1 + c*x)^2)/(6*c) + (d^2*(1 + c*x)^3*(a + b*ArcTanh
[c*x]))/(3*c) + (4*b*d^2*Log[1 - c*x])/(3*c)
```

Rubi [A] time = 0.0425078, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5926, 627, 43}

$$\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{bd^2(cx+1)^2}{6c}+\frac{4bd^2\log(1-cx)}{3c}+\frac{2}{3}bd^2x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (2*b*d^2*x)/3 + (b*d^2*(1 + c*x)^2)/(6*c) + (d^2*(1 + c*x)^3*(a + b*ArcTanh
[c*x]))/(3*c) + (4*b*d^2*Log[1 - c*x])/(3*c)
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)dx = \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c} - \frac{b\int\frac{(d+cdx)^3}{1-c^2x^2}dx}{3d}$$
$$= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c} - \frac{b\int\frac{(d+cdx)^2}{\frac{1}{d}-\frac{cx}{d}}dx}{3d}$$
$$= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c} - \frac{b\int\left(-2d^3+\frac{4d^2}{\frac{1}{d}-\frac{cx}{d}}-d^2(d+cdx)\right)dx}{3d}$$
$$= \frac{2}{3}bd^2x + \frac{bd^2(1+cx)^2}{6c} + \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{4bd^2\log(1-cx)}{3c}$$

Mathematica [A] time = 0.10513, size = 92, normalized size = 1.3

$$\frac{d^2\left(2ac^3x^3+6ac^2x^2+6acx+bc^2x^2+b\log\left(1-c^2x^2\right)+2bcx\left(c^2x^2+3cx+3\right)\tanh^{-1}(cx)+6bcx+6b\log(1-cx)\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^2*(6*a*c*x + 6*b*c*x + 6*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + 2*b*c*x*(
3 + 3*c*x + c^2*x^2)*ArcTanh[c*x] + 6*b*Log[1 - c*x] + b*Log[1 - c^2*x^2]))
/(6*c)
```

Maple [A] time = 0.026, size = 121, normalized size = 1.7

$$\frac{c^2x^3ad^2}{3} + cx^2ad^2 + axd^2 + \frac{d^2a}{3c} + \frac{c^2d^2b\mathrm{Artanh}(cx)\,x^3}{3} + cd^2b\mathrm{Artanh}(cx)\,x^2 + d^2b\mathrm{Artanh}(cx)\,x + \frac{d^2b\mathrm{Artanh}(cx)}{3c} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/3*c^2*x^3*a*d^2+c*x^2*a*d^2+a*x*d^2+1/3/c*d^2*a+1/3*c^2*d^2*b*arctanh(c*x
)*x^3+c*d^2*b*arctanh(c*x)*x^2+d^2*b*arctanh(c*x)*x+1/3/c*d^2*b*arctanh(c*x
)+1/6*c*d^2*b*x^2+b*d^2*x+4/3/c*d^2*b*ln(c*x-1)
```

Maxima [B] time = 0.959171, size = 198, normalized size = 2.79

$$\frac{1}{3}ac^2d^2x^3 + \frac{1}{6}\left(2x^3\,\mathrm{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)bc^2d^2 + acd^2x^2 + \frac{1}{2}\left(2x^2\,\mathrm{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*c^2*d^2*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)
/c^4))*b*c^2*d^2 + a*c*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log
(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctanh(c
```

```
*x) + log(-c^2*x^2 + 1))*b*d^2/c
```

Fricas [A] time = 2.03953, size = 255, normalized size = 3.59

$$\frac{2\,ac^3d^2x^3 + (6\,a + b)c^2d^2x^2 + 6\,(a + b)cd^2x + bd^2\log(cx + 1) + 7\,bd^2\log(cx - 1) + \left(bc^3d^2x^3 + 3\,bc^2d^2x^2 + 3\,bcd^2x\right)\text{lo}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*d^2*x^3 + (6*a + b)*c^2*d^2*x^2 + 6*(a + b)*c*d^2*x + b*d^2*lo
g(c*x + 1) + 7*b*d^2*log(c*x - 1) + (b*c^3*d^2*x^3 + 3*b*c^2*d^2*x^2 + 3*b*
c*d^2*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [A] time = 1.54826, size = 131, normalized size = 1.85

$$\begin{cases} \frac{ac^2d^2x^3}{3} + acd^2x^2 + ad^2x + \frac{bc^2d^2x^3\operatorname{atanh}(cx)}{3} + bcd^2x^2\operatorname{atanh}(cx) + \frac{bcd^2x^2}{6} + bd^2x\operatorname{atanh}(cx) + bd^2x + \frac{4bd^2\log\left(x-\frac{1}{c}\right)}{3c} + \frac{bd}{} \\ ad^2x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**2*d**2*x**3/3 + a*c*d**2*x**2 + a*d**2*x + b*c**2*d**2*x**3
*atanh(c*x)/3 + b*c*d**2*x**2*atanh(c*x) + b*c*d**2*x**2/6 + b*d**2*x*atanh
(c*x) + b*d**2*x + 4*b*d**2*log(x - 1/c)/(3*c) + b*d**2*atanh(c*x)/(3*c), N
e(c, 0)), (a*d**2*x, True))
```

Giac [A] time = 1.21788, size = 163, normalized size = 2.3

$$\frac{1}{3}\,ac^2d^2x^3 + \frac{1}{6}\left(6\,acd^2 + bcd^2\right)x^2 + \frac{bd^2\log(cx + 1)}{6\,c} + \frac{7\,bd^2\log(cx - 1)}{6\,c} + \left(ad^2 + bd^2\right)x + \frac{1}{6}\left(bc^2d^2x^3 + 3\,bcd^2x^2 + 3\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*a*c^2*d^2*x^3 + 1/6*(6*a*c*d^2 + b*c*d^2)*x^2 + 1/6*b*d^2*log(c*x + 1)/
c + 7/6*b*d^2*log(c*x - 1)/c + (a*d^2 + b*d^2)*x + 1/6*(b*c^2*d^2*x^3 + 3*b
*c*d^2*x^2 + 3*b*d^2*x)*log(-(c*x + 1)/(c*x - 1))
```

3.14 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=114

$$-\frac{1}{2}bd^2\text{PolyLog}(2,-cx)+\frac{1}{2}bd^2\text{PolyLog}(2,cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+2acd^2x+ad^2\log(x)+bd^2\log\left(1-c^2x^2\right)$$

```
[Out] 2*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTanh[c*x])/2 + 2*b*c*d^2*x*ArcTanh[
c*x] + (c^2*d^2*x^2*(a + b*ArcTanh[c*x]))/2 + a*d^2*Log[x] + b*d^2*Log[1 -
c^2*x^2] - (b*d^2*PolyLog[2, -(c*x)])/2 + (b*d^2*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.113866, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5940, 5910, 260, 5912, 5916, 321, 206}

$$-\frac{1}{2}bd^2\text{PolyLog}(2,-cx)+\frac{1}{2}bd^2\text{PolyLog}(2,cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+2acd^2x+ad^2\log(x)+bd^2\log\left(1-c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 2*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTanh[c*x])/2 + 2*b*c*d^2*x*ArcTanh[
c*x] + (c^2*d^2*x^2*(a + b*ArcTanh[c*x]))/2 + a*d^2*Log[x] + b*d^2*Log[1 -
c^2*x^2] - (b*d^2*PolyLog[2, -(c*x)])/2 + (b*d^2*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(2cd^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right.\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(2cd^2\right)\int \left(a+b\tanh^{-1}(cx)\right)dx+\left(c^2d^2\right)\int x\left(a+\right.$$

$$= 2acd^2x+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+ad^2\log(x)-\frac{1}{2}bd^2\text{Li}_2(-cx)+\frac{1}{2}bd^2\text{Li}_2($$

$$= 2acd^2x+\frac{1}{2}bcd^2x+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+ad^2\log$$

$$= 2acd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\tanh^{-1}(cx)+2bcd^2x\tanh^{-1}(cx)+\frac{1}{2}c^2d^2x^2\left(a+b\tan\right.$$

Mathematica [A] time = 0.10277, size = 103, normalized size = 0.9

$$\frac{1}{4}d^2\left(-2b\text{PolyLog}(2,-cx)+2b\text{PolyLog}(2,cx)+2ac^2x^2+8acx+4a\log(x)+4b\log\left(1-c^2x^2\right)+2bc^2x^2\tanh^{-1}(cx)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^2*(8*a*c*x + 2*b*c*x + 2*a*c^2*x^2 + 8*b*c*x*ArcTanh[c*x] + 2*b*c^2*x^2*
ArcTanh[c*x] + 4*a*Log[x] + b*Log[1 - c*x] - b*Log[1 + c*x] + 4*b*Log[1 - c
^2*x^2] - 2*b*PolyLog[2, -(c*x)] + 2*b*PolyLog[2, c*x]))/4
```

Maple [A] time = 0.042, size = 142, normalized size = 1.3

$$\frac{d^2ac^2x^2}{2}+2\,acd^2x+d^2a\ln(cx)+\frac{d^2b\text{Artanh}(cx)\,c^2x^2}{2}+2\,bcd^2x\text{Artanh}(cx)+d^2b\text{Artanh}(cx)\ln(cx)-\frac{d^2b\text{dilog}(c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x)
```

```
[Out] 1/2*d^2*a*c^2*x^2+2*a*c*d^2*x+d^2*a*ln(c*x)+1/2*d^2*b*arctanh(c*x)*c^2*x^2+
2*b*c*d^2*x*arctanh(c*x)+d^2*b*arctanh(c*x)*ln(c*x)-1/2*d^2*b*dilog(c*x)-1/
```

```
2*d^2*b*dilog(c*x+1)-1/2*d^2*b*ln(c*x)*ln(c*x+1)+1/2*b*c*d^2*x+5/4*d^2*b*ln
(c*x-1)+3/4*d^2*b*ln(c*x+1)
```

Maxima [A] time = 1.45441, size = 234, normalized size = 2.05

$$\frac{1}{4}bc^2d^2x^2\log(cx+1) - \frac{1}{4}bc^2d^2x^2\log(-cx+1) + \frac{1}{2}ac^2d^2x^2 + 2\,acd^2x + \frac{1}{2}bcd^2x + \left(2\,cx\,\mathrm{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^2*d^2*x^2*log(c*x + 1) - 1/4*b*c^2*d^2*x^2*log(-c*x + 1) + 1/2*a*c^
2*d^2*x^2 + 2*a*c*d^2*x + 1/2*b*c*d^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*b*d^2 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^2 + 1/2*
(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^2 - 1/4*b*d^2*log(c*x + 1) +
1/4*b*d^2*log(c*x - 1) + a*d^2*log(x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ac^2d^2x^2 + 2\,acd^2x + ad^2 + \left(bc^2d^2x^2 + 2\,bcd^2x + bd^2\right)\mathrm{artanh}\,(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int 2ac\,dx + \int \frac{a}{x}\,dx + \int ac^2x\,dx + \int 2bc\,\mathrm{atanh}\,(cx)\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x}\,dx + \int bc^2x\,\mathrm{atanh}\,(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**2*(Integral(2*a*c, x) + Integral(a/x, x) + Integral(a*c**2*x, x) + Integ
ral(2*b*c*atanh(c*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(b*c**2*x*
atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^2(b\,\mathrm{artanh}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x, x)
```

3.15 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=61

$$-bcd^2\text{PolyLog}(2,-cx)+bcd^2\text{PolyLog}(2,cx)+\frac{d^2\left(c^2x^2-1\right)\left(a+b\tanh^{-1}(cx)\right)}{x}+cd^2(2a+b)\log(x)$$

```
[Out] (d^2*(-1 + c^2*x^2)*(a + b*ArcTanh[c*x]))/x + (2*a + b)*c*d^2*Log[x] - b*c*
d^2*PolyLog[2, -(c*x)] + b*c*d^2*PolyLog[2, c*x]
```

Rubi [A] time = 0.125832, antiderivative size = 80, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5940, 5910, 260, 5916, 266, 36, 29, 31, 5912}

$$-bcd^2\text{PolyLog}(2,-cx)+bcd^2\text{PolyLog}(2,cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2x+2acd^2\log(x)+bc^2d^2x\tanh^{-1}(cx)+bc$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] a*c^2*d^2*x + b*c^2*d^2*x*ArcTanh[c*x] - (d^2*(a + b*ArcTanh[c*x]))/x + 2*a
*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, -(c*x)] + b*c*d^2*PolyL
og[2, c*x]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(c^2d^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}\right.$$

$$= d^2\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(2cd^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(c^2d^2\right)\int \left(a+b\,\mathrm{t}\right.$$

$$= ac^2d^2x-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)-bcd^2\mathrm{Li}_2(-cx)+bcd^2\mathrm{Li}_2(cx)+$$

$$= ac^2d^2x+bc^2d^2x\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)-bcd^2\mathrm{Li}_2($$

$$= ac^2d^2x+bc^2d^2x\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)+\frac{1}{2}bcd^2\,\mathrm{lo}$$

$$= ac^2d^2x+bc^2d^2x\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x}+2acd^2\log(x)+bcd^2\log$$

Mathematica [A] time = 0.106028, size = 73, normalized size = 1.2

$$\frac{d^2\left(-bcx\text{PolyLog}(2,-cx)+bcx\text{PolyLog}(2,cx)+ac^2x^2+2acx\log(x)-a+bc^2x^2\tanh^{-1}(cx)+bcx\log(cx)-b\tanh^{-1}\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-a + a*c^2*x^2 - b*ArcTanh[c*x] + b*c^2*x^2*ArcTanh[c*x] + 2*a*c*x*Lo
g[x] + b*c*x*Log[c*x] - b*c*x*PolyLog[2, -(c*x)] + b*c*x*PolyLog[2, c*x]))/
x
```

Maple [A] time = 0.041, size = 123, normalized size = 2.

$$d^2ac^2x - \frac{d^2a}{x} + 2\,cd^2a\ln(cx) + bc^2d^2x\mathrm{Artanh}(cx) - \frac{d^2b\mathrm{Artanh}(cx)}{x} + 2\,cd^2b\mathrm{Artanh}(cx)\ln(cx) + cd^2b\ln(cx) - cd^2bd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] d^2*a*c^2*x-d^2*a/x+2*c*d^2*a*ln(c*x)+b*c^2*d^2*x*arctanh(c*x)-d^2*b*arctan
h(c*x)/x+2*c*d^2*b*arctanh(c*x)*ln(c*x)+c*d^2*b*ln(c*x)-c*d^2*b*dilog(c*x)-
c*d^2*b*dilog(c*x+1)-c*d^2*b*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ac^2d^2x + \frac{1}{2}\left(2\,cx\,\mathrm{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)bcd^2 + bcd^2\int\frac{\log(cx+1) - \log(-cx+1)}{x}\,dx + 2\,acd^2\log(x) - \frac{1}{2}\left(c\left(\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] a*c^2*d^2*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^2 + b*c*d^
2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 2*a*c*d^2*log(x) - 1/2*(
c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^2 - a*d^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ac^2d^2x^2 + 2\,acd^2x + ad^2 + \left(bc^2d^2x^2 + 2\,bcd^2x + bd^2\right)\mathrm{artanh}(cx)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int ac^2\,dx + \int\frac{a}{x^2}\,dx + \int\frac{2ac}{x}\,dx + \int bc^2\,\mathrm{atanh}(cx)\,dx + \int\frac{b\,\mathrm{atanh}(cx)}{x^2}\,dx + \int\frac{2bc\,\mathrm{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(a*c**2, x) + Integral(a/x**2, x) + Integral(2*a*c/x, x) + In
tegral(b*c**2*atanh(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(2*
b*c*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^2 (b\,\mathrm{artanh}\,(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x^2, x)
```

3.16 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^3} dx$

Optimal. Leaf size=137

$$-\frac{1}{2}bc^2d^2\text{PolyLog}(2,-cx)+\frac{1}{2}bc^2d^2\text{PolyLog}(2,cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2\log(x)-b$$

```
[Out] -(b*c*d^2)/(2*x) + (b*c^2*d^2*ArcTanh[c*x])/2 - (d^2*(a + b*ArcTanh[c*x]))/
(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x]))/x + a*c^2*d^2*Log[x] + 2*b*c^2*d^2
*Log[x] - b*c^2*d^2*Log[1 - c^2*x^2] - (b*c^2*d^2*PolyLog[2, -(c*x)])/2 + (
b*c^2*d^2*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.139936, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912}

$$-\frac{1}{2}bc^2d^2\text{PolyLog}(2,-cx)+\frac{1}{2}bc^2d^2\text{PolyLog}(2,cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2\log(x)-b$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2)/(2*x) + (b*c^2*d^2*ArcTanh[c*x])/2 - (d^2*(a + b*ArcTanh[c*x]))/
(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x]))/x + a*c^2*d^2*Log[x] + 2*b*c^2*d^2
*Log[x] - b*c^2*d^2*Log[1 - c^2*x^2] - (b*c^2*d^2*PolyLog[2, -(c*x)])/2 + (
b*c^2*d^2*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int\left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x}\right.$$

$$= d^2\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(2cd^2\right)\int\frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(c^2d^2\right)\int\frac{a+b\text{ta}}{}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2\log(x)-\frac{1}{2}bc^2d^2\mathrm{Li}_2($$

$$= -\frac{bcd^2}{2x}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^2d^2\log(x)-\frac{1}{2}bc$$

$$= -\frac{bcd^2}{2x}+\frac{1}{2}bc^2d^2\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}$$

$$= -\frac{bcd^2}{2x}+\frac{1}{2}bc^2d^2\tanh^{-1}(cx)-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}$$

Mathematica [A] time = 0.100833, size = 143, normalized size = 1.04

$$\frac{d^2\left(-2bc^2x^2\text{PolyLog}(2,-cx)+2bc^2x^2\text{PolyLog}(2,cx)+4ac^2x^2\log(x)-8acx-2a+8bc^2x^2\log(cx)-bc^2x^2\log(1-c\right.}{4x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a - 8*a*c*x - 2*b*c*x - 2*b*ArcTanh[c*x] - 8*b*c*x*ArcTanh[c*x] +
4*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*
x^2*Log[1 + c*x] - 4*b*c^2*x^2*Log[1 - c^2*x^2] - 2*b*c^2*x^2*PolyLog[2, -(
c*x)] + 2*b*c^2*x^2*PolyLog[2, c*x]))/(4*x^2)
```

Maple [A] time = 0.046, size = 176, normalized size = 1.3

$$-2\,\frac{cd^2a}{x}+c^2d^2a\ln(cx)-\frac{d^2a}{2\,x^2}-2\,\frac{cd^2b\mathrm{Artanh}(cx)}{x}+c^2d^2b\mathrm{Artanh}(cx)\ln(cx)-\frac{d^2b\mathrm{Artanh}(cx)}{2\,x^2}-\frac{5\,c^2d^2b\ln(cx-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] -2*c*d^2*a/x+c^2*d^2*a*ln(c*x)-1/2*d^2*a/x^2-2*c*d^2*b*arctanh(c*x)/x+c^2*d
^2*b*arctanh(c*x)*ln(c*x)-1/2*d^2*b*arctanh(c*x)/x^2-5/4*c^2*d^2*b*ln(c*x-1
)-1/2*b*c*d^2/x+2*c^2*d^2*b*ln(c*x)-3/4*c^2*d^2*b*ln(c*x+1)-1/2*c^2*d^2*b*d
ilog(c*x)-1/2*c^2*d^2*b*dilog(c*x+1)-1/2*c^2*d^2*b*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,bc^2d^2\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+ac^2d^2\log(x)-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}(cx)}{x}\right)bcd^2+\frac{1}{4}\left(\left(c\,\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*d^2*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^2*d^2*lo
g(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^2 + 1/4*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^2 - 2*a
*c*d^2/x - 1/2*a*d^2/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ac^2d^2x^2+2\,acd^2x+ad^2+\left(bc^2d^2x^2+2\,bcd^2x+bd^2\right)\mathrm{artanh}(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^2*d^2*x^2 + 2*a*c*d^2*x + a*d^2 + (b*c^2*d^2*x^2 + 2*b*c*d^2*
x + b*d^2)*arctanh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{a}{x^3}\,dx+\int\frac{2ac}{x^2}\,dx+\int\frac{ac^2}{x}\,dx+\int\frac{b\,\mathrm{atanh}(cx)}{x^3}\,dx+\int\frac{2bc\,\mathrm{atanh}(cx)}{x^2}\,dx+\int\frac{bc^2\,\mathrm{atanh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(2*a*c/x**2, x) + Integral(a*c**2/x, x)
 + Integral(b*atanh(c*x)/x**3, x) + Integral(2*b*c*atanh(c*x)/x**2, x) + In
tegral(b*c**2*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^2 (b \operatorname{artanh}(cx) + a)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)/x^3, x)
```

3.17 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{bc^2d^2}{x}+\frac{4}{3}bc^3d^2\log(x)-\frac{4}{3}bc^3d^2\log(1-cx)-\frac{bcd^2}{6x^2}$$

```
[Out] -(b*c*d^2)/(6*x^2) - (b*c^2*d^2)/x - (d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x]))
/(3*x^3) + (4*b*c^3*d^2*Log[x])/3 - (4*b*c^3*d^2*Log[1 - c*x])/3
```

Rubi [A] time = 0.0849651, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {37, 5936, 12, 88}

$$-\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{bc^2d^2}{x}+\frac{4}{3}bc^3d^2\log(x)-\frac{4}{3}bc^3d^2\log(1-cx)-\frac{bcd^2}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2)/(6*x^2) - (b*c^2*d^2)/x - (d^2*(1 + c*x)^3*(a + b*ArcTanh[c*x]))
/(3*x^3) + (4*b*c^3*d^2*Log[x])/3 - (4*b*c^3*d^2*Log[1 - c*x])/3
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)}{x^4} dx = -\frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - (bc)\int \frac{(d+cdx)^2}{3x^3(-1+cx)} dx$$
$$= -\frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{3}(bc)\int \frac{(d+cdx)^2}{x^3(-1+cx)} dx$$
$$= -\frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{1}{3}(bc)\int \left(-\frac{d^2}{x^3} - \frac{3cd^2}{x^2} - \frac{4c^2d^2}{x} + \frac{4c^3d^2}{-1+cx}\right)$$
$$= -\frac{bcd^2}{6x^2} - \frac{bc^2d^2}{x} - \frac{d^2(1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} + \frac{4}{3}bc^3d^2\log(x) - \frac{4}{3}bc^3d^2\log$$

Mathematica [A] time = 0.0938123, size = 103, normalized size = 1.27

$$-\frac{d^2 \left(6ac^2x^2 + 6acx + 2a + 6bc^2x^2 - 8bc^3x^3\log(x) + 7bc^3x^3\log(1-cx) + bc^3x^3\log(cx+1) + 2b\left(3c^2x^2 + 3cx + 1\right)\tan\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(d^2*(2*a + 6*a*c*x + b*c*x + 6*a*c^2*x^2 + 6*b*c^2*x^2 + 2*b*(1 + 3*c*x +
 3*c^2*x^2)*ArcTanh[c*x] - 8*b*c^3*x^3*Log[x] + 7*b*c^3*x^3*Log[1 - c*x] +
b*c^3*x^3*Log[1 + c*x]))/(6*x^3)
```

Maple [A] time = 0.037, size = 141, normalized size = 1.7

$$-\frac{c^2d^2a}{x} - \frac{cd^2a}{x^2} - \frac{d^2a}{3x^3} - \frac{c^2d^2b\mathrm{Artanh}(cx)}{x} - \frac{cd^2b\mathrm{Artanh}(cx)}{x^2} - \frac{d^2b\mathrm{Artanh}(cx)}{3x^3} - \frac{7c^3d^2b\ln(cx-1)}{6} - \frac{cd^2b}{6x^2} - \frac{c^2d^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -c^2*d^2*a/x-c*d^2*a/x^2-1/3*d^2*a/x^3-c^2*d^2*b*arctanh(c*x)/x-c*d^2*b*arc
tanh(c*x)/x^2-1/3*d^2*b*arctanh(c*x)/x^3-7/6*c^3*d^2*b*ln(c*x-1)-1/6*b*c*d^
2/x^2-b*c^2*d^2/x+4/3*c^3*d^2*b*ln(c*x)-1/6*c^3*d^2*b*ln(c*x+1)
```

Maxima [B] time = 0.961842, size = 212, normalized size = 2.62

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\mathrm{artanh}(cx)}{x}\right)bc^2d^2 + \frac{1}{2}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\mathrm{artanh}(cx)}{x^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^2 + 1/2*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^2 - 1
/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b
*d^2 - a*c^2*d^2/x - a*c*d^2/x^2 - 1/3*a*d^2/x^3
```

Fricas [A] time = 2.17284, size = 293, normalized size = 3.62

$$-\frac{bc^3d^2x^3\log(cx+1) + 7\,bc^3d^2x^3\log(cx-1) - 8\,bc^3d^2x^3\log(x) + 6\,(a+b)c^2d^2x^2 + (6\,a+b)cd^2x + 2\,ad^2 + \left(3\,bc^2d^2x^2 + \right.}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c^3*d^2*x^3*log(c*x + 1) + 7*b*c^3*d^2*x^3*log(c*x - 1) - 8*b*c^3*d
^2*x^3*log(x) + 6*(a + b)*c^2*d^2*x^2 + (6*a + b)*c*d^2*x + 2*a*d^2 + (3*b*
c^2*d^2*x^2 + 3*b*c*d^2*x + b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^3
```

Sympy [A] time = 4.61552, size = 158, normalized size = 1.95

$$\begin{cases} -\frac{ac^2d^2}{x} - \frac{acd^2}{x^2} - \frac{ad^2}{3x^3} + \frac{4bc^3d^2\log(x)}{3} - \frac{4bc^3d^2\log\left(x-\frac{1}{c}\right)}{3} - \frac{bc^3d^2\operatorname{atanh}(cx)}{3} - \frac{bc^2d^2\operatorname{atanh}(cx)}{x} - \frac{bc^2d^2}{x} - \frac{bcd^2\operatorname{atanh}(cx)}{x^2} - \frac{bcd^2}{6x^2} - \frac{bd^2\operatorname{atanh}}{3x^3} \\ -\frac{ad^2}{3x^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] Piecewise((-a*c**2*d**2/x - a*c*d**2/x**2 - a*d**2/(3*x**3) + 4*b*c**3*d**2
*log(x)/3 - 4*b*c**3*d**2*log(x - 1/c)/3 - b*c**3*d**2*atanh(c*x)/3 - b*c**
2*d**2*atanh(c*x)/x - b*c**2*d**2/x - b*c*d**2*atanh(c*x)/x**2 - b*c*d**2/(
6*x**2) - b*d**2*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a*d**2/(3*x**3), True))
```

Giac [A] time = 1.30308, size = 188, normalized size = 2.32

$$-\frac{1}{6}\,bc^3d^2\log(cx+1) - \frac{7}{6}\,bc^3d^2\log(cx-1) + \frac{4}{3}\,bc^3d^2\log(x) - \frac{\left(3\,bc^2d^2x^2 + 3\,bcd^2x + bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{6\,x^3} - \frac{6\,ac^2d^2x^2 + 6\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c^3*d^2*log(c*x + 1) - 7/6*b*c^3*d^2*log(c*x - 1) + 4/3*b*c^3*d^2*lo
g(x) - 1/6*(3*b*c^2*d^2*x^2 + 3*b*c*d^2*x + b*d^2)*log(-(c*x + 1)/(c*x - 1)
)/x^3 - 1/6*(6*a*c^2*d^2*x^2 + 6*b*c^2*d^2*x^2 + 6*a*c*d^2*x + b*c*d^2*x +
2*a*d^2)/x^3
```

3.18 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=147

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d^2}{3x^2}-\frac{3bc^3d^2}{4x}+\frac{2}{3}bc^4d^2\log(x)-\frac{17}{24}$$

```
[Out] -(b*c*d^2)/(12*x^3) - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (d^2*(a +
 b*ArcTanh[c*x]))/(4*x^4) - (2*c*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) - (c^2*d
^2*(a + b*ArcTanh[c*x]))/(2*x^2) + (2*b*c^4*d^2*Log[x])/3 - (17*b*c^4*d^2*L
og[1 - c*x])/24 + (b*c^4*d^2*Log[1 + c*x])/24
```

Rubi [A] time = 0.149283, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d^2}{3x^2}-\frac{3bc^3d^2}{4x}+\frac{2}{3}bc^4d^2\log(x)-\frac{17}{24}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^2)/(12*x^3) - (b*c^2*d^2)/(3*x^2) - (3*b*c^3*d^2)/(4*x) - (d^2*(a +
 b*ArcTanh[c*x]))/(4*x^4) - (2*c*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) - (c^2*d
^2*(a + b*ArcTanh[c*x]))/(2*x^2) + (2*b*c^4*d^2*Log[x])/3 - (17*b*c^4*d^2*L
og[1 - c*x])/24 + (b*c^4*d^2*Log[1 + c*x])/24
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^5}dx = -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - (b$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{12}$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{2x^2} - \frac{1}{12}$$

$$= -\frac{bcd^2}{12x^3} - \frac{bc^2d^2}{3x^2} - \frac{3bc^3d^2}{4x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - c^2$$

Mathematica [A] time = 0.0967844, size = 114, normalized size = 0.78

$$-\frac{d^2\left(12ac^2x^2+16acx+6a+18bc^3x^3+8bc^2x^2-16bc^4x^4\log(x)+17bc^4x^4\log(1-cx)-bc^4x^4\log(cx+1)+2b\left(6c^2x^2+\right.\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(d^2*(6*a + 16*a*c*x + 2*b*c*x + 12*a*c^2*x^2 + 8*b*c^2*x^2 + 18*b*c^3*x^3
 + 2*b*(3 + 8*c*x + 6*c^2*x^2)*ArcTanh[c*x] - 16*b*c^4*x^4*Log[x] + 17*b*c^
4*x^4*Log[1 - c*x] - b*c^4*x^4*Log[1 + c*x]))/(24*x^4)
```

Maple [A] time = 0.04, size = 153, normalized size = 1.

$$-\frac{d^2a}{4x^4} - \frac{c^2d^2a}{2x^2} - \frac{2cd^2a}{3x^3} - \frac{d^2b\mathrm{Artanh}(cx)}{4x^4} - \frac{c^2d^2b\mathrm{Artanh}(cx)}{2x^2} - \frac{2cd^2b\mathrm{Artanh}(cx)}{3x^3} - \frac{17c^4d^2b\ln(cx-1)}{24} - \frac{cd^2b}{12x^3} - \frac{c^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/4*d^2*a/x^4-1/2*c^2*d^2*a/x^2-2/3*c*d^2*a/x^3-1/4*d^2*b*arctanh(c*x)/x^4
-1/2*c^2*d^2*b*arctanh(c*x)/x^2-2/3*c*d^2*b*arctanh(c*x)/x^3-17/24*c^4*d^2*
b*ln(c*x-1)-1/12*b*c*d^2/x^3-1/3*b*c^2*d^2/x^2-3/4*b*c^3*d^2/x+2/3*c^4*d^2*
b*ln(c*x)+1/24*b*c^4*d^2*ln(c*x+1)
```

Maxima [A] time = 0.975368, size = 240, normalized size = 1.63

$$\frac{1}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}(cx)}{x^2}\right)bc^2d^2-\frac{1}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}(}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*
d^2 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c*d^2 + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x
```

```
^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^2 - 1/2*a*c^2*d^2/x^2 - 2/3*a*c*d^
2/x^3 - 1/4*a*d^2/x^4
```

Fricas [A] time = 2.09943, size = 332, normalized size = 2.26

$$\frac{bc^4d^2x^4\log(cx+1) - 17\,bc^4d^2x^4\log(cx-1) + 16\,bc^4d^2x^4\log(x) - 18\,bc^3d^2x^3 - 4\,(3\,a+2\,b)c^2d^2x^2 - 2\,(8\,a+b)cd^2x - }{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(b*c^4*d^2*x^4*log(c*x + 1) - 17*b*c^4*d^2*x^4*log(c*x - 1) + 16*b*c^4
*d^2*x^4*log(x) - 18*b*c^3*d^2*x^3 - 4*(3*a + 2*b)*c^2*d^2*x^2 - 2*(8*a + b
)*c*d^2*x - 6*a*d^2 - (6*b*c^2*d^2*x^2 + 8*b*c*d^2*x + 3*b*d^2)*log(-(c*x +
 1)/(c*x - 1)))/x^4
```

Sympy [A] time = 5.26977, size = 189, normalized size = 1.29

$$\begin{cases} -\frac{ac^2d^2}{2x^2} - \frac{2acd^2}{3x^3} - \frac{ad^2}{4x^4} + \frac{2bc^4d^2\log(x)}{3} - \frac{2bc^4d^2\log\left(x-\frac{1}{c}\right)}{3} + \frac{bc^4d^2\operatorname{atanh}(cx)}{12} - \frac{3bc^3d^2}{4x} - \frac{bc^2d^2\operatorname{atanh}(cx)}{2x^2} - \frac{bc^2d^2}{3x^2} - \frac{2bcd^2\operatorname{atanh}(cx)}{3x^3} - \frac{b}{1} \\ -\frac{ad^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c**2*d**2/(2*x**2) - 2*a*c*d**2/(3*x**3) - a*d**2/(4*x**4) +
2*b*c**4*d**2*log(x)/3 - 2*b*c**4*d**2*log(x - 1/c)/3 + b*c**4*d**2*atanh(c
*x)/12 - 3*b*c**3*d**2/(4*x) - b*c**2*d**2*atanh(c*x)/(2*x**2) - b*c**2*d**
2/(3*x**2) - 2*b*c*d**2*atanh(c*x)/(3*x**3) - b*c*d**2/(12*x**3) - b*d**2*a
tanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**2/(4*x**4), True))
```

Giac [A] time = 1.25085, size = 205, normalized size = 1.39

$$\frac{1}{24}\,bc^4d^2\log(cx+1) - \frac{17}{24}\,bc^4d^2\log(cx-1) + \frac{2}{3}\,bc^4d^2\log(x) - \frac{\left(6\,bc^2d^2x^2 + 8\,bcd^2x + 3\,bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,x^4} - \frac{9\,bc^3d^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] 1/24*b*c^4*d^2*log(c*x + 1) - 17/24*b*c^4*d^2*log(c*x - 1) + 2/3*b*c^4*d^2*
log(x) - 1/24*(6*b*c^2*d^2*x^2 + 8*b*c*d^2*x + 3*b*d^2)*log(-(c*x + 1)/(c*x
 - 1))/x^4 - 1/12*(9*b*c^3*d^2*x^3 + 6*a*c^2*d^2*x^2 + 4*b*c^2*d^2*x^2 + 8*
a*c*d^2*x + b*c*d^2*x + 3*a*d^2)/x^4
```

3.19 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=161

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2}{6x^3}-\frac{bc^4d^2}{2x}+\frac{8}{15}bc^5d^2\log(x)$$

```
[Out] -(b*c*d^2)/(20*x^4) - (b*c^2*d^2)/(6*x^3) - (4*b*c^3*d^2)/(15*x^2) - (b*c^4
*d^2)/(2*x) - (d^2*(a + b*ArcTanh[c*x]))/(5*x^5) - (c*d^2*(a + b*ArcTanh[c*
x]))/(2*x^4) - (c^2*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log[x]
)/15 - (31*b*c^5*d^2*Log[1 - c*x])/60 - (b*c^5*d^2*Log[1 + c*x])/60
```

Rubi [A] time = 0.160266, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2}{6x^3}-\frac{bc^4d^2}{2x}+\frac{8}{15}bc^5d^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^2)/(20*x^4) - (b*c^2*d^2)/(6*x^3) - (4*b*c^3*d^2)/(15*x^2) - (b*c^4
*d^2)/(2*x) - (d^2*(a + b*ArcTanh[c*x]))/(5*x^5) - (c*d^2*(a + b*ArcTanh[c*
x]))/(2*x^4) - (c^2*d^2*(a + b*ArcTanh[c*x]))/(3*x^3) + (8*b*c^5*d^2*Log[x]
)/15 - (31*b*c^5*d^2*Log[1 - c*x])/60 - (b*c^5*d^2*Log[1 + c*x])/60
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - ($$
$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} -$$
$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}(cx)\right)}{2x^4} - \frac{c^2d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} -$$
$$= -\frac{bcd^2}{20x^4} - \frac{bc^2d^2}{6x^3} - \frac{4bc^3d^2}{15x^2} - \frac{bc^4d^2}{2x} - \frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{cd^2\left(a+b\tanh^{-1}\right.}{2x^4}$$

Mathematica [A] time = 0.0954493, size = 122, normalized size = 0.76

$$-\frac{d^2\left(20ac^2x^2+30acx+12a+30bc^4x^4+16bc^3x^3+10bc^2x^2-32bc^5x^5\log(x)+31bc^5x^5\log(1-cx)+bc^5x^5\log(cx+\right.}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(d^2*(12*a + 30*a*c*x + 3*b*c*x + 20*a*c^2*x^2 + 10*b*c^2*x^2 + 16*b*c^3*x
^3 + 30*b*c^4*x^4 + 2*b*(6 + 15*c*x + 10*c^2*x^2)*ArcTanh[c*x] - 32*b*c^5*x
^5*Log[x] + 31*b*c^5*x^5*Log[1 - c*x] + b*c^5*x^5*Log[1 + c*x]))/(60*x^5)
```

Maple [A] time = 0.04, size = 165, normalized size = 1.

$$-\frac{cd^2a}{2x^4} - \frac{d^2a}{5x^5} - \frac{c^2d^2a}{3x^3} - \frac{cd^2b\mathrm{Artanh}(cx)}{2x^4} - \frac{d^2b\mathrm{Artanh}(cx)}{5x^5} - \frac{c^2d^2b\mathrm{Artanh}(cx)}{3x^3} - \frac{31c^5d^2b\ln(cx-1)}{60} - \frac{cd^2b}{20x^4} - \frac{c^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -1/2*c*d^2*a/x^4-1/5*d^2*a/x^5-1/3*c^2*d^2*a/x^3-1/2*c*d^2*b*arctanh(c*x)/x
^4-1/5*d^2*b*arctanh(c*x)/x^5-1/3*c^2*d^2*b*arctanh(c*x)/x^3-31/60*c^5*d^2*
b*ln(c*x-1)-1/20*b*c*d^2/x^4-1/6*b*c^2*d^2/x^3-4/15*b*c^3*d^2/x^2-1/2*b*c^4
*d^2/x+8/15*c^5*d^2*b*ln(c*x)-1/60*b*c^5*d^2*ln(c*x+1)
```

Maxima [A] time = 0.969873, size = 262, normalized size = 1.63

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}(cx)}{x^3}\right)bc^2d^2+\frac{1}{12}\left(\left(3\,c^3\log(cx+1)-3\,c^3\log(cx-1)-\frac{2\left(3\,c^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^2*d^2 + 1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2
+ 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 - 1) -
```

```
 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^2 - 1/3*
a*c^2*d^2/x^3 - 1/2*a*c*d^2/x^4 - 1/5*a*d^2/x^5
```

Fricas [A] time = 2.36032, size = 363, normalized size = 2.25

$$-\frac{bc^5d^2x^5\log\left(cx+1\right)+31\,bc^5d^2x^5\log\left(cx-1\right)-32\,bc^5d^2x^5\log\left(x\right)+30\,bc^4d^2x^4+16\,bc^3d^2x^3+10\,(2\,a+b)c^2d^2x^2+3\,(1\cdots}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(b*c^5*d^2*x^5*log(c*x + 1) + 31*b*c^5*d^2*x^5*log(c*x - 1) - 32*b*c^
5*d^2*x^5*log(x) + 30*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a + b)*c^2*d
^2*x^2 + 3*(10*a + b)*c*d^2*x + 12*a*d^2 + (10*b*c^2*d^2*x^2 + 15*b*c*d^2*x
 + 6*b*d^2)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A] time = 4.64253, size = 199, normalized size = 1.24

$$\begin{cases} -\frac{ac^2d^2}{3x^3}-\frac{acd^2}{2x^4}-\frac{ad^2}{5x^5}+\frac{8bc^5d^2\log\left(x\right)}{15}-\frac{8bc^5d^2\log\left(x-\frac{1}{c}\right)}{15}-\frac{bc^5d^2\operatorname{atanh}\left(cx\right)}{30}-\frac{bc^4d^2}{2x}-\frac{4bc^3d^2}{15x^2}-\frac{bc^2d^2\operatorname{atanh}\left(cx\right)}{3x^3}-\frac{bc^2d^2}{6x^3}-\frac{bcd^2\operatorname{atanh}\left(cx\right)}{2x^4}-\cdots \\ -\frac{ad^2}{5x^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**2*d**2/(3*x**3) - a*c*d**2/(2*x**4) - a*d**2/(5*x**5) + 8*
b*c**5*d**2*log(x)/15 - 8*b*c**5*d**2*log(x - 1/c)/15 - b*c**5*d**2*atanh(c
*x)/30 - b*c**4*d**2/(2*x) - 4*b*c**3*d**2/(15*x**2) - b*c**2*d**2*atanh(c*
x)/(3*x**3) - b*c**2*d**2/(6*x**3) - b*c*d**2*atanh(c*x)/(2*x**4) - b*c*d**
2/(20*x**4) - b*d**2*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**2/(5*x**5), Tru
e))
```

Giac [A] time = 1.41537, size = 223, normalized size = 1.39

$$-\frac{1}{60}\,bc^5d^2\log\left(cx+1\right)-\frac{31}{60}\,bc^5d^2\log\left(cx-1\right)+\frac{8}{15}\,bc^5d^2\log\left(x\right)-\frac{\left(10\,bc^2d^2x^2+15\,bcd^2x+6\,bd^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{60\,x^5}-\frac{30\,bc^4\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/60*b*c^5*d^2*log(c*x + 1) - 31/60*b*c^5*d^2*log(c*x - 1) + 8/15*b*c^5*d^
2*log(x) - 1/60*(10*b*c^2*d^2*x^2 + 15*b*c*d^2*x + 6*b*d^2)*log(-(c*x + 1)/
(c*x - 1))/x^5 - 1/60*(30*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 20*a*c^2*d^2*x
^2 + 10*b*c^2*d^2*x^2 + 30*a*c*d^2*x + 3*b*c*d^2*x + 12*a*d^2)/x^5
```

3.20 $\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=192

$$\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+$$

```
[Out] (3*b*d^3*x)/(4*c^3) + (13*b*d^3*x^2)/(35*c^2) + (b*d^3*x^3)/(4*c) + (13*b*d
^3*x^4)/70 + (b*c*d^3*x^5)/10 + (b*c^2*d^3*x^6)/42 + (d^3*x^4*(a + b*ArcTan
h[c*x]))/4 + (3*c*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^3*x^6*(a + b*Arc
Tanh[c*x]))/2 + (c^3*d^3*x^7*(a + b*ArcTanh[c*x]))/7 + (209*b*d^3*Log[1 - c
*x])/(280*c^4) - (b*d^3*Log[1 + c*x])/(280*c^4)
```

Rubi [A] time = 0.18143, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(4*c^3) + (13*b*d^3*x^2)/(35*c^2) + (b*d^3*x^3)/(4*c) + (13*b*d
^3*x^4)/70 + (b*c*d^3*x^5)/10 + (b*c^2*d^3*x^6)/42 + (d^3*x^4*(a + b*ArcTan
h[c*x]))/4 + (3*c*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (c^2*d^3*x^6*(a + b*Arc
Tanh[c*x]))/2 + (c^3*d^3*x^7*(a + b*ArcTanh[c*x]))/7 + (209*b*d^3*Log[1 - c
*x])/(280*c^4) - (b*d^3*Log[1 + c*x])/(280*c^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x^4\left(a+b\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x^4\left(a+b\right.$$

$$=\frac{3bd^3x}{4c^3}+\frac{13bd^3x^2}{35c^2}+\frac{bd^3x^3}{4c}+\frac{13}{70}bd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}bc^2d^3x^6+\frac{1}{4}d^3x^4\left(a+b\right.$$

Mathematica [A] time = 0.133598, size = 151, normalized size = 0.79

$$\frac{d^3\left(120ac^7x^7+420ac^6x^6+504ac^5x^5+210ac^4x^4+20bc^6x^6+84bc^5x^5+156bc^4x^4+210bc^3x^3+312bc^2x^2+6bc^4x^4\left(20c^3\right.\right.}{840c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(630*b*c*x + 312*b*c^2*x^2 + 210*b*c^3*x^3 + 210*a*c^4*x^4 + 156*b*c^4
*x^4 + 504*a*c^5*x^5 + 84*b*c^5*x^5 + 420*a*c^6*x^6 + 20*b*c^6*x^6 + 120*a*
c^7*x^7 + 6*b*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x]
+ 627*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(840*c^4)
```

Maple [A] time = 0.028, size = 199, normalized size = 1.

$$\frac{c^3d^3ax^7}{7}+\frac{c^2d^3ax^6}{2}+\frac{3\,cd^3ax^5}{5}+\frac{d^3ax^4}{4}+\frac{c^3d^3b\mathrm{Artanh}\,(cx)\,x^7}{7}+\frac{c^2d^3b\mathrm{Artanh}\,(cx)\,x^6}{2}+\frac{3\,cd^3b\mathrm{Artanh}\,(cx)\,x^5}{5}+\frac{d^3bA}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/7*c^3*d^3*a*x^7+1/2*c^2*d^3*a*x^6+3/5*c*d^3*a*x^5+1/4*d^3*a*x^4+1/7*c^3*d
^3*b*arctanh(c*x)*x^7+1/2*c^2*d^3*b*arctanh(c*x)*x^6+3/5*c*d^3*b*arctanh(c*
x)*x^5+1/4*d^3*b*arctanh(c*x)*x^4+1/42*b*c^2*d^3*x^6+1/10*b*c*d^3*x^5+13/70
```

```
*b*d^3*x^4+1/4*b*d^3*x^3/c+13/35*b*d^3*x^2/c^2+3/4*b*d^3*x/c^3+209/280/c^4*
d^3*b*ln(c*x-1)-1/280*b*d^3*ln(c*x+1)/c^4
```

Maxima [A] time = 0.973141, size = 385, normalized size = 2.01

$$\frac{1}{7}ac^3d^3x^7+\frac{1}{2}ac^2d^3x^6+\frac{3}{5}acd^3x^5+\frac{1}{84}\left(12x^7\operatorname{artanh}(cx)+c\left(\frac{2c^4x^6+3c^2x^4+6x^2}{c^6}+\frac{6\log\left(c^2x^2-1\right)}{c^8}\right)\right)bc^3d^3+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^3*d^3*x^7 + 1/2*a*c^2*d^3*x^6 + 3/5*a*c*d^3*x^5 + 1/84*(12*x^7*arct
anh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8)
)*b*c^3*d^3 + 1/4*a*d^3*x^4 + 1/60*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 +
 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*c^2*
d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
 1)/c^6))*b*c*d^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^3
```

Fricas [A] time = 2.30813, size = 440, normalized size = 2.29

$$\frac{120\,ac^7d^3x^7+20\,(21\,a+b)c^6d^3x^6+84\,(6\,a+b)c^5d^3x^5+6\,(35\,a+26\,b)c^4d^3x^4+210\,bc^3d^3x^3+312\,bc^2d^3x^2+630\,bcd}{840\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/840*(120*a*c^7*d^3*x^7 + 20*(21*a + b)*c^6*d^3*x^6 + 84*(6*a + b)*c^5*d^3
*x^5 + 6*(35*a + 26*b)*c^4*d^3*x^4 + 210*b*c^3*d^3*x^3 + 312*b*c^2*d^3*x^2
+ 630*b*c*d^3*x - 3*b*d^3*log(c*x + 1) + 627*b*d^3*log(c*x - 1) + 3*(20*b*c
^7*d^3*x^7 + 70*b*c^6*d^3*x^6 + 84*b*c^5*d^3*x^5 + 35*b*c^4*d^3*x^4)*log(-(
c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] time = 6.68272, size = 243, normalized size = 1.27

$$\begin{cases}\frac{ac^3d^3x^7}{7}+\frac{ac^2d^3x^6}{2}+\frac{3acd^3x^5}{5}+\frac{ad^3x^4}{4}+\frac{bc^3d^3x^7\operatorname{atanh}(cx)}{7}+\frac{bc^2d^3x^6\operatorname{atanh}(cx)}{2}+\frac{bc^2d^3x^6}{42}+\frac{3bcd^3x^5\operatorname{atanh}(cx)}{5}+\frac{bcd^3x^5}{10}+\frac{bd^3x^4\operatorname{atanh}}{4} \\ \frac{ad^3x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**7/7 + a*c**2*d**3*x**6/2 + 3*a*c*d**3*x**5/5 + a*
d**3*x**4/4 + b*c**3*d**3*x**7*atanh(c*x)/7 + b*c**2*d**3*x**6*atanh(c*x)/2
 + b*c**2*d**3*x**6/42 + 3*b*c*d**3*x**5*atanh(c*x)/5 + b*c*d**3*x**5/10 +
b*d**3*x**4*atanh(c*x)/4 + 13*b*d**3*x**4/70 + b*d**3*x**3/(4*c) + 13*b*d**
3*x**2/(35*c**2) + 3*b*d**3*x/(4*c**3) + 26*b*d**3*log(x - 1/c)/(35*c**4) -
```

```
b*d**3*atanh(c*x)/(140*c**4), Ne(c, 0)), (a*d**3*x**4/4, True))
```

Giac [A] time = 1.33519, size = 267, normalized size = 1.39

$$\frac{1}{7}ac^3d^3x^7 + \frac{1}{42}\left(21\,ac^2d^3 + bc^2d^3\right)x^6 + \frac{bd^3x^3}{4\,c} + \frac{1}{10}\left(6\,acd^3 + bcd^3\right)x^5 + \frac{1}{140}\left(35\,ad^3 + 26\,bd^3\right)x^4 + \frac{13\,bd^3x^2}{35\,c^2} + \frac{3\,bd^3x}{4\,c^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/7*a*c^3*d^3*x^7 + 1/42*(21*a*c^2*d^3 + b*c^2*d^3)*x^6 + 1/4*b*d^3*x^3/c +
 1/10*(6*a*c*d^3 + b*c*d^3)*x^5 + 1/140*(35*a*d^3 + 26*b*d^3)*x^4 + 13/35*b
*d^3*x^2/c^2 + 3/4*b*d^3*x/c^3 + 1/280*(20*b*c^3*d^3*x^7 + 70*b*c^2*d^3*x^6
 + 84*b*c*d^3*x^5 + 35*b*d^3*x^4)*log(-(c*x + 1)/(c*x - 1)) - 1/280*b*d^3*l
og(c*x + 1)/c^4 + 209/280*b*d^3*log(c*x - 1)/c^4
```

3.21 $\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=178

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+$$

```
[Out] (11*b*d^3*x)/(12*c^2) + (7*b*d^3*x^2)/(15*c) + (11*b*d^3*x^3)/36 + (3*b*c*d
^3*x^4)/20 + (b*c^2*d^3*x^5)/30 + (d^3*x^3*(a + b*ArcTanh[c*x]))/3 + (3*c*d
^3*x^4*(a + b*ArcTanh[c*x]))/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x]))/6 + (37*b*d^3*Log[1 - c*x])/(40*c^3) + (b
*d^3*Log[1 + c*x])/(120*c^3)
```

Rubi [A] time = 0.175664, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```


```
[Out] (11*b*d^3*x)/(12*c^2) + (7*b*d^3*x^2)/(15*c) + (11*b*d^3*x^3)/36 + (3*b*c*d
^3*x^4)/20 + (b*c^2*d^3*x^5)/30 + (d^3*x^3*(a + b*ArcTanh[c*x]))/3 + (3*c*d
^3*x^4*(a + b*ArcTanh[c*x]))/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x]))/6 + (37*b*d^3*Log[1 - c*x])/(40*c^3) + (b
*d^3*Log[1 + c*x])/(120*c^3)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(c\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(c\right.$$

$$=\frac{11bd^3x}{12c^2}+\frac{7bd^3x^2}{15c}+\frac{11}{36}bd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}bc^2d^3x^5+\frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(c\right.$$

Mathematica [A] time = 0.125615, size = 142, normalized size = 0.8

$$\frac{d^3\left(60ac^6x^6+216ac^5x^5+270ac^4x^4+120ac^3x^3+12bc^5x^5+54bc^4x^4+110bc^3x^3+168bc^2x^2+6bc^3x^3\left(10c^3x^3+36c^2x^2+\right.\right.}{360c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(330*b*c*x + 168*b*c^2*x^2 + 120*a*c^3*x^3 + 110*b*c^3*x^3 + 270*a*c^4
*x^4 + 54*b*c^4*x^4 + 216*a*c^5*x^5 + 12*b*c^5*x^5 + 60*a*c^6*x^6 + 6*b*c^3
*x^3*(20 + 45*c*x + 36*c^2*x^2 + 10*c^3*x^3)*ArcTanh[c*x] + 333*b*Log[1 - c
*x] + 3*b*Log[1 + c*x]))/(360*c^3)
```

Maple [A] time = 0.031, size = 187, normalized size = 1.1

$$\frac{c^3d^3ax^6}{6}+\frac{3c^2d^3ax^5}{5}+\frac{3cd^3ax^4}{4}+\frac{d^3ax^3}{3}+\frac{c^3d^3b\mathrm{Artanh}(cx)x^6}{6}+\frac{3c^2d^3b\mathrm{Artanh}(cx)x^5}{5}+\frac{3cd^3b\mathrm{Artanh}(cx)x^4}{4}+\frac{d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*c^3*d^3*a*x^6+3/5*c^2*d^3*a*x^5+3/4*c*d^3*a*x^4+1/3*d^3*a*x^3+1/6*c^3*d
^3*b*arctanh(c*x)*x^6+3/5*c^2*d^3*b*arctanh(c*x)*x^5+3/4*c*d^3*b*arctanh(c*
x)*x^4+1/3*d^3*b*arctanh(c*x)*x^3+1/30*b*c^2*d^3*x^5+3/20*b*c*d^3*x^4+11/36
```

```
*b*d^3*x^3+7/15*b*d^3*x^2/c+11/12*b*d^3*x/c^2+37/40/c^3*d^3*b*ln(c*x-1)+1/1
20*b*d^3*ln(c*x+1)/c^3
```

Maxima [A] time = 0.965809, size = 358, normalized size = 2.01

$$\frac{1}{6}\,ac^3d^3x^6+\frac{3}{5}\,ac^2d^3x^5+\frac{3}{4}\,acd^3x^4+\frac{1}{180}\left(30\,x^6\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(3\,c^4x^5+5\,c^2x^3+15\,x\right)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15\,\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^3*d^3*x^6 + 3/5*a*c^2*d^3*x^5 + 3/4*a*c*d^3*x^4 + 1/180*(30*x^6*arc
tanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 +
 15*log(c*x - 1)/c^7))*b*c^3*d^3 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 +
 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/8*(6*x
^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
 - 1)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
1)/c^4))*b*d^3
```

Fricas [A] time = 2.22022, size = 413, normalized size = 2.32

$$\frac{60\,ac^6d^3x^6+12\,(18\,a+b)c^5d^3x^5+54\,(5\,a+b)c^4d^3x^4+10\,(12\,a+11\,b)c^3d^3x^3+168\,bc^2d^3x^2+330\,bcd^3x+3\,bd^3\log\,(}{360\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/360*(60*a*c^6*d^3*x^6 + 12*(18*a + b)*c^5*d^3*x^5 + 54*(5*a + b)*c^4*d^3*
x^4 + 10*(12*a + 11*b)*c^3*d^3*x^3 + 168*b*c^2*d^3*x^2 + 330*b*c*d^3*x + 3*
b*d^3*log(c*x + 1) + 333*b*d^3*log(c*x - 1) + 3*(10*b*c^6*d^3*x^6 + 36*b*c^
5*d^3*x^5 + 45*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3)*log(-(c*x + 1)/(c*x - 1)))
/c^3
```

Sympy [A] time = 4.32664, size = 235, normalized size = 1.32

$$\begin{cases}\frac{ac^3d^3x^6}{6}+\frac{3ac^2d^3x^5}{5}+\frac{3acd^3x^4}{4}+\frac{ad^3x^3}{3}+\frac{bc^3d^3x^6\,\mathrm{atanh}\,(cx)}{6}+\frac{3bc^2d^3x^5\,\mathrm{atanh}\,(cx)}{5}+\frac{bc^2d^3x^5}{30}+\frac{3bcd^3x^4\,\mathrm{atanh}\,(cx)}{4}+\frac{3bcd^3x^4}{20}+\frac{bd^3x^3\,\mathrm{ata}}{3} \\ \frac{ad^3x^3}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**6/6 + 3*a*c**2*d**3*x**5/5 + 3*a*c*d**3*x**4/4 +
a*d**3*x**3/3 + b*c**3*d**3*x**6*atanh(c*x)/6 + 3*b*c**2*d**3*x**5*atanh(c*
x)/5 + b*c**2*d**3*x**5/30 + 3*b*c*d**3*x**4*atanh(c*x)/4 + 3*b*c*d**3*x**4
/20 + b*d**3*x**3*atanh(c*x)/3 + 11*b*d**3*x**3/36 + 7*b*d**3*x**2/(15*c) +
 11*b*d**3*x/(12*c**2) + 14*b*d**3*log(x - 1/c)/(15*c**3) + b*d**3*atanh(c*
```

```
x)/(60*c**3), Ne(c, 0)), (a*d**3*x**3/3, True))
```

Giac [A] time = 1.25597, size = 251, normalized size = 1.41

$$\frac{1}{6}ac^3d^3x^6+\frac{1}{30}\left(18\,ac^2d^3+bc^2d^3\right)x^5+\frac{7\,bd^3x^2}{15\,c}+\frac{3}{20}\left(5\,acd^3+bcd^3\right)x^4+\frac{1}{36}\left(12\,ad^3+11\,bd^3\right)x^3+\frac{11\,bd^3x}{12\,c^2}+\frac{bd^3\log\left(cx\right.}{120\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*a*c^3*d^3*x^6 + 1/30*(18*a*c^2*d^3 + b*c^2*d^3)*x^5 + 7/15*b*d^3*x^2/c
+ 3/20*(5*a*c*d^3 + b*c*d^3)*x^4 + 1/36*(12*a*d^3 + 11*b*d^3)*x^3 + 11/12*b
*d^3*x/c^2 + 1/120*b*d^3*log(c*x + 1)/c^3 + 37/40*b*d^3*log(c*x - 1)/c^3 +
1/120*(10*b*c^3*d^3*x^6 + 36*b*c^2*d^3*x^5 + 45*b*c*d^3*x^4 + 20*b*d^3*x^3)
*log(-(c*x + 1)/(c*x - 1))
```

3.22 $\int x(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=135

$$\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{bd^3(cx+1)^4}{20c^2}+\frac{bd^3(cx+1)^3}{20c^2}+\frac{3bd^3(cx+1)^2}{20c^2}+6b$$

```
[Out] (3*b*d^3*x)/(5*c) + (3*b*d^3*(1 + c*x)^2)/(20*c^2) + (b*d^3*(1 + c*x)^3)/(2
0*c^2) + (b*d^3*(1 + c*x)^4)/(20*c^2) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x
]))/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (6*b*d^3*Log
[1 - c*x])/(5*c^2)
```

Rubi [A] time = 0.10111, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 77}

$$\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{bd^3(cx+1)^4}{20c^2}+\frac{bd^3(cx+1)^3}{20c^2}+\frac{3bd^3(cx+1)^2}{20c^2}+6b$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (3*b*d^3*x)/(5*c) + (3*b*d^3*(1 + c*x)^2)/(20*c^2) + (b*d^3*(1 + c*x)^3)/(2
0*c^2) + (b*d^3*(1 + c*x)^4)/(20*c^2) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x
]))/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (6*b*d^3*Log
[1 - c*x])/(5*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)dx=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-(bc)\int\frac{(-1+$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{b\int\frac{(-1+4cx)(}{1-c}}{20c}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}-\frac{b\int\left(-12d^3\right.}{}$$

$$=\frac{3bd^3x}{5c}+\frac{3bd^3(1+cx)^2}{20c^2}+\frac{bd^3(1+cx)^3}{20c^2}+\frac{bd^3(1+cx)^4}{20c^2}-\frac{d^3(1+cx)^4\left(a+b\tanh\right.}{4c^2}$$

Mathematica [A] time = 0.10686, size = 133, normalized size = 0.99

$$\frac{d^3\left(8ac^5x^5+30ac^4x^4+40ac^3x^3+20ac^2x^2+2bc^4x^4+10bc^3x^3+24bc^2x^2+2bc^2x^2\left(4c^3x^3+15c^2x^2+20cx+10\right)\tanh^{-1}\right.}{40c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(50*b*c*x + 20*a*c^2*x^2 + 24*b*c^2*x^2 + 40*a*c^3*x^3 + 10*b*c^3*x^3
+ 30*a*c^4*x^4 + 2*b*c^4*x^4 + 8*a*c^5*x^5 + 2*b*c^2*x^2*(10 + 20*c*x + 15*
c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x] + 49*b*Log[1 - c*x] - b*Log[1 + c*x]))/(4
0*c^2)
```

Maple [A] time = 0.029, size = 173, normalized size = 1.3

$$\frac{c^3d^3ax^5}{5}+\frac{3c^2d^3ax^4}{4}+cd^3ax^3+\frac{d^3ax^2}{2}+\frac{c^3d^3b\text{Artanh}(cx)x^5}{5}+\frac{3c^2d^3b\text{Artanh}(cx)x^4}{4}+cd^3b\text{Artanh}(cx)x^3+\frac{d^3b\text{Ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c^3*d^3*a*x^5+3/4*c^2*d^3*a*x^4+c*d^3*a*x^3+1/2*d^3*a*x^2+1/5*c^3*d^3*b
*arctanh(c*x)*x^5+3/4*c^2*d^3*b*arctanh(c*x)*x^4+c*d^3*b*arctanh(c*x)*x^3+1
/2*d^3*b*arctanh(c*x)*x^2+1/20*c^2*d^3*b*x^4+1/4*c*d^3*b*x^3+3/5*d^3*b*x^2+
5/4*b*d^3*x/c+49/40/c^2*d^3*b*ln(c*x-1)-1/40/c^2*d^3*b*ln(c*x+1)
```

Maxima [B] time = 0.968479, size = 329, normalized size = 2.44

$$\frac{1}{5}ac^3d^3x^5+\frac{3}{4}ac^2d^3x^4+\frac{1}{20}\left(4x^5\operatorname{artanh}(cx)+c\left(\frac{c^2x^4+2x^2}{c^4}+\frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^3d^3+acd^3x^3+\frac{1}{8}\left(6x^4\operatorname{artanh}(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^3*d^3*x^5 + 3/4*a*c^2*d^3*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*
x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^3 + a*c*d^3*x^3 + 1/8*(
```

```
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^2*d^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c*d^3 + 1/2*a*d^3*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^3
```

Fricas [A] time = 1.89864, size = 369, normalized size = 2.73

$$\frac{8\,ac^5d^3x^5 + 2\,(15\,a+b)c^4d^3x^4 + 10\,(4\,a+b)c^3d^3x^3 + 4\,(5\,a+6\,b)c^2d^3x^2 + 50\,bcd^3x - bd^3\log(cx+1) + 49\,bd^3\log(cx}{40\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*d^3*x^5 + 2*(15*a + b)*c^4*d^3*x^4 + 10*(4*a + b)*c^3*d^3*x^3
 + 4*(5*a + 6*b)*c^2*d^3*x^2 + 50*b*c*d^3*x - b*d^3*log(c*x + 1) + 49*b*d^3
*log(c*x - 1) + (4*b*c^5*d^3*x^5 + 15*b*c^4*d^3*x^4 + 20*b*c^3*d^3*x^3 + 10
*b*c^2*d^3*x^2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 4.20865, size = 211, normalized size = 1.56

$$\begin{cases} \frac{ac^3d^3x^5}{5} + \frac{3ac^2d^3x^4}{4} + acd^3x^3 + \frac{ad^3x^2}{2} + \frac{bc^3d^3x^5\operatorname{atanh}(cx)}{5} + \frac{3bc^2d^3x^4\operatorname{atanh}(cx)}{4} + \frac{bc^2d^3x^4}{20} + bcd^3x^3\operatorname{atanh}(cx) + \frac{bcd^3x^3}{4} + \frac{bd^3x^2 \dots \\ \frac{ad^3x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**5/5 + 3*a*c**2*d**3*x**4/4 + a*c*d**3*x**3 + a*d*
*3*x**2/2 + b*c**3*d**3*x**5*atanh(c*x)/5 + 3*b*c**2*d**3*x**4*atanh(c*x)/4
 + b*c**2*d**3*x**4/20 + b*c*d**3*x**3*atanh(c*x) + b*c*d**3*x**3/4 + b*d**
3*x**2*atanh(c*x)/2 + 3*b*d**3*x**2/5 + 5*b*d**3*x/(4*c) + 6*b*d**3*log(x -
 1/c)/(5*c**2) - b*d**3*atanh(c*x)/(20*c**2), Ne(c, 0)), (a*d**3*x**2/2, Tr
ue))
```

Giac [A] time = 1.21847, size = 235, normalized size = 1.74

$$\frac{1}{5}\,ac^3d^3x^5 + \frac{1}{20}\left(15\,ac^2d^3 + bc^2d^3\right)x^4 + \frac{5\,bd^3x}{4\,c} + \frac{1}{4}\left(4\,acd^3 + bcd^3\right)x^3 + \frac{1}{10}\left(5\,ad^3 + 6\,bd^3\right)x^2 - \frac{bd^3\log(cx+1)}{40\,c^2} + \frac{49\,b \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c^3*d^3*x^5 + 1/20*(15*a*c^2*d^3 + b*c^2*d^3)*x^4 + 5/4*b*d^3*x/c + 1
/4*(4*a*c*d^3 + b*c*d^3)*x^3 + 1/10*(5*a*d^3 + 6*b*d^3)*x^2 - 1/40*b*d^3*lo
g(c*x + 1)/c^2 + 49/40*b*d^3*log(c*x - 1)/c^2 + 1/40*(4*b*c^3*d^3*x^5 + 15*
b*c^2*d^3*x^4 + 20*b*c*d^3*x^3 + 10*b*d^3*x^2)*log(-(c*x + 1)/(c*x - 1))
```

3.23 $\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=84

$$\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+\frac{bd^3(cx+1)^3}{12c}+\frac{bd^3(cx+1)^2}{4c}+\frac{2bd^3\log(1-cx)}{c}+bd^3x$$

```
[Out] b*d^3*x + (b*d^3*(1 + c*x)^2)/(4*c) + (b*d^3*(1 + c*x)^3)/(12*c) + (d^3*(1
+ c*x)^4*(a + b*ArcTanh[c*x]))/(4*c) + (2*b*d^3*Log[1 - c*x])/c
```

Rubi [A] time = 0.051169, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5926, 627, 43}

$$\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4c}+\frac{bd^3(cx+1)^3}{12c}+\frac{bd^3(cx+1)^2}{4c}+\frac{2bd^3\log(1-cx)}{c}+bd^3x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] b*d^3*x + (b*d^3*(1 + c*x)^2)/(4*c) + (b*d^3*(1 + c*x)^3)/(12*c) + (d^3*(1
+ c*x)^4*(a + b*ArcTanh[c*x]))/(4*c) + (2*b*d^3*Log[1 - c*x])/c
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right) dx = \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} - \frac{b\int \frac{(d+cdx)^4}{1-c^2x^2} dx}{4d}$$
$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} - \frac{b\int \frac{(d+cdx)^3}{\frac{1}{d}-\frac{cx}{d}} dx}{4d}$$
$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} - \frac{b\int \left(-4d^4 + \frac{8d^3}{\frac{1}{d}-\frac{cx}{d}} - 2d^3(d+cdx) - d^2(d+cdx\right.}{4d}$$
$$= bd^3x + \frac{bd^3(1+cx)^2}{4c} + \frac{bd^3(1+cx)^3}{12c} + \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)}{4c} + \frac{2bd^3 \text{lo}}{}$$

Mathematica [A] time = 0.144953, size = 115, normalized size = 1.37

$$\frac{d^3 \left(6ac^4x^4 + 24ac^3x^3 + 36ac^2x^2 + 24acx + 2bc^3x^3 + 12bc^2x^2 + 6bcx \left(c^3x^3 + 4c^2x^2 + 6cx + 4\right) \tanh^{-1}(cx) + 42bcx + 4\right.}{24c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^3*(24*a*c*x + 42*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c^3*x^3 + 2*
b*c^3*x^3 + 6*a*c^4*x^4 + 6*b*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3)*ArcTanh
[c*x] + 45*b*Log[1 - c*x] + 3*b*Log[1 + c*x]))/(24*c)
```

Maple [B] time = 0.029, size = 162, normalized size = 1.9

$$\frac{c^3x^4ad^3}{4} + c^2x^3ad^3 + \frac{3\,cx^2ad^3}{2} + axd^3 + \frac{d^3a}{4\,c} + \frac{c^3d^3b\text{Artanh}\,(cx)\,x^4}{4} + c^2d^3b\text{Artanh}\,(cx)\,x^3 + \frac{3\,cd^3b\text{Artanh}\,(cx)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/4*c^3*x^4*a*d^3+c^2*x^3*a*d^3+3/2*c*x^2*a*d^3+a*x*d^3+1/4/c*d^3*a+1/4*c^3
*d^3*b*arctanh(c*x)*x^4+c^2*d^3*b*arctanh(c*x)*x^3+3/2*c*d^3*b*arctanh(c*x)
*x^2+d^3*b*arctanh(c*x)*x+1/4/c*d^3*b*arctanh(c*x)+1/12*c^2*d^3*b*x^3+1/2*c
*d^3*b*x^2+7/4*b*d^3*x+2/c*d^3*b*ln(c*x-1)
```

Maxima [B] time = 0.970664, size = 296, normalized size = 3.52

$$\frac{1}{4}\,ac^3d^3x^4 + ac^2d^3x^3 + \frac{1}{24}\left(6\,x^4 \text{artanh}\,(cx) + c\left(\frac{2\left(c^2x^3 + 3\,x\right)}{c^4} - \frac{3\,\log\,(cx+1)}{c^5} + \frac{3\,\log\,(cx-1)}{c^5}\right)\right)bc^3d^3 + \frac{1}{2}\left(2\,x^3 \text{art}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*c^3*d^3*x^4 + a*c^2*d^3*x^3 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^
3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c^3*d^3 + 1/2*(2
*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + 3/2*a*c
```

```
*d^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*
x - 1)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1
))*b*d^3/c
```

Fricas [A] time = 2.01528, size = 333, normalized size = 3.96

$$\frac{6\,ac^4d^3x^4 + 2\,(12\,a+b)c^3d^3x^3 + 12\,(3\,a+b)c^2d^3x^2 + 6\,(4\,a+7\,b)cd^3x + 3\,bd^3\log\left(cx+1\right) + 45\,bd^3\log\left(cx-1\right) + 3\,\left(bc^4d^3\right.}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*a*c^4*d^3*x^4 + 2*(12*a + b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2
 + 6*(4*a + 7*b)*c*d^3*x + 3*b*d^3*log(c*x + 1) + 45*b*d^3*log(c*x - 1) + 3
*(b*c^4*d^3*x^4 + 4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c*d^3*x)*log(-(c*
x + 1)/(c*x - 1)))/c
```

Sympy [A] time = 2.59779, size = 182, normalized size = 2.17

$$\begin{cases} \frac{ac^3d^3x^4}{4} + ac^2d^3x^3 + \frac{3acd^3x^2}{2} + ad^3x + \frac{bc^3d^3x^4\operatorname{atanh}(cx)}{4} + bc^2d^3x^3\operatorname{atanh}(cx) + \frac{bc^2d^3x^3}{12} + \frac{3bcd^3x^2\operatorname{atanh}(cx)}{2} + \frac{bcd^3x^2}{2} + bd^3x\operatorname{ata} \\ ad^3x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**3*d**3*x**4/4 + a*c**2*d**3*x**3 + 3*a*c*d**3*x**2/2 + a*d*
*3*x + b*c**3*d**3*x**4*atanh(c*x)/4 + b*c**2*d**3*x**3*atanh(c*x) + b*c**2
*d**3*x**3/12 + 3*b*c*d**3*x**2*atanh(c*x)/2 + b*c*d**3*x**2/2 + b*d**3*x*a
tanh(c*x) + 7*b*d**3*x/4 + 2*b*d**3*log(x - 1/c)/c + b*d**3*atanh(c*x)/(4*c
), Ne(c, 0)), (a*d**3*x, True))
```

Giac [B] time = 1.2448, size = 215, normalized size = 2.56

$$\frac{1}{4}\,ac^3d^3x^4 + \frac{1}{12}\left(12\,ac^2d^3 + bc^2d^3\right)x^3 + \frac{bd^3\log\left(cx+1\right)}{8\,c} + \frac{15\,bd^3\log\left(cx-1\right)}{8\,c} + \frac{1}{2}\left(3\,acd^3 + bcd^3\right)x^2 + \frac{1}{4}\left(4\,ad^3 + 7\,bd^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*a*c^3*d^3*x^4 + 1/12*(12*a*c^2*d^3 + b*c^2*d^3)*x^3 + 1/8*b*d^3*log(c*x
 + 1)/c + 15/8*b*d^3*log(c*x - 1)/c + 1/2*(3*a*c*d^3 + b*c*d^3)*x^2 + 1/4*(
4*a*d^3 + 7*b*d^3)*x + 1/8*(b*c^3*d^3*x^4 + 4*b*c^2*d^3*x^3 + 6*b*c*d^3*x^2
 + 4*b*d^3*x)*log(-(c*x + 1)/(c*x - 1))
```

3.24 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=152

$$-\frac{1}{2}bd^3\text{PolyLog}(2,-cx)+\frac{1}{2}bd^3\text{PolyLog}(2,cx)+\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3x$$

```
[Out] 3*a*c*d^3*x + (3*b*c*d^3*x)/2 + (b*c^2*d^3*x^2)/6 - (3*b*d^3*ArcTanh[c*x])/
2 + 3*b*c*d^3*x*ArcTanh[c*x] + (3*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/2 + (c^
3*d^3*x^3*(a + b*ArcTanh[c*x]))/3 + a*d^3*Log[x] + (5*b*d^3*Log[1 - c^2*x^2
])/3 - (b*d^3*PolyLog[2, -(c*x)])/2 + (b*d^3*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.165891, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5940, 5910, 260, 5912, 5916, 321, 206, 266, 43}

$$-\frac{1}{2}bd^3\text{PolyLog}(2,-cx)+\frac{1}{2}bd^3\text{PolyLog}(2,cx)+\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3x$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 3*a*c*d^3*x + (3*b*c*d^3*x)/2 + (b*c^2*d^3*x^2)/6 - (3*b*d^3*ArcTanh[c*x])/
2 + 3*b*c*d^3*x*ArcTanh[c*x] + (3*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/2 + (c^
3*d^3*x^3*(a + b*ArcTanh[c*x]))/3 + a*d^3*Log[x] + (5*b*d^3*Log[1 - c^2*x^2
])/3 - (b*d^3*PolyLog[2, -(c*x)])/2 + (b*d^3*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(3cd^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3c^2d^3x\left(a+b\tanh^{-1}(cx)\right)\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(3cd^3\right)\int \left(a+b\tanh^{-1}(cx)\right)dx+\left(3c^2d^3\right)\int x\left(a+b\right.$$

$$= 3acd^3x+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^3d^3x^3\left(a+b\tanh^{-1}(cx)\right)+ad^3\log(x)-$$

$$= 3acd^3x+\frac{3}{2}bcd^3x+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^3d^3x^3\left(a\right.$$

$$= 3acd^3x+\frac{3}{2}bcd^3x-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2\left(a+b\tanh\right.$$

$$= 3acd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}bc^2d^3x^2-\frac{3}{2}bd^3\tanh^{-1}(cx)+3bcd^3x\tanh^{-1}(cx)+\frac{3}{2}c^2d^3x^2$$

Mathematica [A] time = 0.13589, size = 148, normalized size = 0.97

$$\frac{1}{12}d^3\left(-6b\text{PolyLog}(2,-cx)+6b\text{PolyLog}(2,cx)+4ac^3x^3+18ac^2x^2+36acx+12a\log(x)+2bc^2x^2+18b\log\left(1-c^2x^2\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^3*(36*a*c*x + 18*b*c*x + 18*a*c^2*x^2 + 2*b*c^2*x^2 + 4*a*c^3*x^3 + 36*b
*c*x*ArcTanh[c*x] + 18*b*c^2*x^2*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] +
12*a*Log[x] + 9*b*Log[1 - c*x] - 9*b*Log[1 + c*x] + 18*b*Log[1 - c^2*x^2] +
 2*b*Log[-1 + c^2*x^2] - 6*b*PolyLog[2, -(c*x)] + 6*b*PolyLog[2, c*x]))/12
```

Maple [A] time = 0.042, size = 182, normalized size = 1.2

$$\frac{d^3ac^3x^3}{3}+\frac{3\,d^3ac^2x^2}{2}+3\,acd^3x+d^3a\ln(cx)+\frac{d^3b\mathrm{Artanh}(cx)\,c^3x^3}{3}+\frac{3\,d^3b\mathrm{Artanh}(cx)\,c^2x^2}{2}+3\,bcd^3x\mathrm{Artanh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x)
```

```
[Out] 1/3*d^3*a*c^3*x^3+3/2*d^3*a*c^2*x^2+3*a*c*d^3*x+d^3*a*ln(c*x)+1/3*d^3*b*arc
tanh(c*x)*c^3*x^3+3/2*d^3*b*arctanh(c*x)*c^2*x^2+3*b*c*d^3*x*arctanh(c*x)+d
^3*b*arctanh(c*x)*ln(c*x)-1/2*d^3*b*dilog(c*x)-1/2*d^3*b*dilog(c*x+1)-1/2*d
^3*b*ln(c*x)*ln(c*x+1)+1/6*b*c^2*d^3*x^2+3/2*b*c*d^3*x+29/12*d^3*b*ln(c*x-1
)+11/12*d^3*b*ln(c*x+1)
```

Maxima [A] time = 1.47818, size = 308, normalized size = 2.03

$$\frac{1}{3}\,ac^3d^3x^3+\frac{3}{2}\,ac^2d^3x^2+\frac{1}{6}\,bc^2d^3x^2+3\,acd^3x+\frac{3}{2}\,bcd^3x+\frac{3}{2}\left(2\,cx\,\mathrm{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)bd^3-\frac{1}{2}\,(\log(cx)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^3*d^3*x^3 + 3/2*a*c^2*d^3*x^2 + 1/6*b*c^2*d^3*x^2 + 3*a*c*d^3*x + 3
/2*b*c*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*d^3 - 1/2*(lo
g(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*d^3 + 1/2*(log(c*x + 1)*log(-c*x)
 + dilog(c*x + 1))*b*d^3 - 7/12*b*d^3*log(c*x + 1) + 11/12*b*d^3*log(c*x -
1) + a*d^3*log(x) + 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(c*x + 1) -
 1/12*(2*b*c^3*d^3*x^3 + 9*b*c^2*d^3*x^2)*log(-c*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{ac^3d^3x^3+3\,ac^2d^3x^2+3\,acd^3x+ad^3+\left(bc^3d^3x^3+3\,bc^2d^3x^2+3\,bcd^3x+bd^3\right)\mathrm{artanh}(cx)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3ac\,dx+\int\frac{a}{x}\,dx+\int 3ac^2x\,dx+\int ac^3x^2\,dx+\int 3bc\,\mathrm{atanh}(cx)\,dx+\int\frac{b\,\mathrm{atanh}(cx)}{x}\,dx+\int 3bc^2x\,\mathrm{atanh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**3*(Integral(3*a*c, x) + Integral(a/x, x) + Integral(3*a*c**2*x, x) + Int
egral(a*c**3*x**2, x) + Integral(3*b*c*atanh(c*x), x) + Integral(b*atanh(c*
x)/x, x) + Integral(3*b*c**2*x*atanh(c*x), x) + Integral(b*c**3*x**2*atanh(
c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^3(b\,\mathrm{artanh}\,(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x, x)
```

3.25 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{3}{2}bcd^3\text{PolyLog}(2,-cx)+\frac{3}{2}bcd^3\text{PolyLog}(2,cx)+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3x+$$

```
[Out] 3*a*c^2*d^3*x + (b*c^2*d^3*x)/2 - (b*c*d^3*ArcTanh[c*x])/2 + 3*b*c^2*d^3*x*
ArcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/x + (c^3*d^3*x^2*(a + b*ArcTanh[c
*x]))/2 + 3*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 - c^2*x^2] - (3
*b*c*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c*d^3*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.157628, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5940, 5910, 260, 5916, 266, 36, 29, 31, 5912, 321, 206}

$$-\frac{3}{2}bcd^3\text{PolyLog}(2,-cx)+\frac{3}{2}bcd^3\text{PolyLog}(2,cx)+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3x+$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```


```
[Out] 3*a*c^2*d^3*x + (b*c^2*d^3*x)/2 - (b*c*d^3*ArcTanh[c*x])/2 + 3*b*c^2*d^3*x*
ArcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/x + (c^3*d^3*x^2*(a + b*ArcTanh[c
*x]))/2 + 3*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 - c^2*x^2] - (3
*b*c*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c*d^3*PolyLog[2, c*x])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int \left(3c^2d^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\right.$$

$$= d^3\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(3cd^3\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(3c^2d^3\right)\int \left(a+b\tan\right.$$

$$= 3ac^2d^3x-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^3d^3x^2\left(a+b\tanh^{-1}(cx)\right)+3acd^3\log(x)-\frac{3}{2}$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^3d^3x^2\left(a+\right.$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(\right.}{x}$$

$$= 3ac^2d^3x+\frac{1}{2}bc^2d^3x-\frac{1}{2}bcd^3\tanh^{-1}(cx)+3bc^2d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(\right.}{x}$$

Mathematica [A] time = 0.1409, size = 149, normalized size = 0.99

$$\frac{d^3\left(-6bcx\text{PolyLog}(2,-cx)+6bcx\text{PolyLog}(2,cx)+2ac^3x^3+12ac^2x^2+12acx\log(x)-4a+2bc^2x^2+4bcx\log\left(1-c\right.\right.}{4x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^3*(-4*a + 12*a*c^2*x^2 + 2*b*c^2*x^2 + 2*a*c^3*x^3 - 4*b*ArcTanh[c*x] +
12*b*c^2*x^2*ArcTanh[c*x] + 2*b*c^3*x^3*ArcTanh[c*x] + 12*a*c*x*Log[x] + 4*
b*c*x*Log[c*x] + b*c*x*Log[1 - c*x] - b*c*x*Log[1 + c*x] + 4*b*c*x*Log[1 -
c^2*x^2] - 6*b*c*x*PolyLog[2, -(c*x)] + 6*b*c*x*PolyLog[2, c*x]))/(4*x)
```

Maple [A] time = 0.049, size = 189, normalized size = 1.3

$$\frac{d^3ac^3x^2}{2}+3\,ac^2d^3x-\frac{d^3a}{x}+3\,cd^3a\ln(cx)+\frac{d^3b\text{Artanh}(cx)\,c^3x^2}{2}+3\,bc^2d^3x\text{Artanh}(cx)-\frac{d^3b\text{Artanh}(cx)}{x}+3\,cd^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] 1/2*d^3*a*c^3*x^2+3*a*c^2*d^3*x-d^3*a/x+3*c*d^3*a*ln(c*x)+1/2*d^3*b*arctanh
(c*x)*c^3*x^2+3*b*c^2*d^3*x*arctanh(c*x)-d^3*b*arctanh(c*x)/x+3*c*d^3*b*arc
tanh(c*x)*ln(c*x)-3/2*c*d^3*b*dilog(c*x)-3/2*c*d^3*b*dilog(c*x+1)-3/2*c*d^3
*b*ln(c*x)*ln(c*x+1)+1/2*b*c^2*d^3*x+5/4*c*d^3*b*ln(c*x-1)+c*d^3*b*ln(c*x)+
3/4*c*d^3*b*ln(c*x+1)
```

Maxima [A] time = 1.44768, size = 309, normalized size = 2.06

$$\frac{1}{4}\,bc^3d^3x^2\log(cx+1)-\frac{1}{4}\,bc^3d^3x^2\log(-cx+1)+\frac{1}{2}\,ac^3d^3x^2+3\,ac^2d^3x+\frac{1}{2}\,bc^2d^3x+\frac{3}{2}\left(2\,cx\,\text{artanh}(cx)+\log\left(-c^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^3*d^3*x^2*log(c*x + 1) - 1/4*b*c^3*d^3*x^2*log(-c*x + 1) + 1/2*a*c^
3*d^3*x^2 + 3*a*c^2*d^3*x + 1/2*b*c^2*d^3*x + 3/2*(2*c*x*arctanh(c*x) + log
(-c^2*x^2 + 1))*b*c*d^3 - 3/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*
c*d^3 + 3/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c*d^3 - 1/4*b*c*d^3
*log(c*x + 1) + 1/4*b*c*d^3*log(c*x - 1) + 3*a*c*d^3*log(x) - 1/2*(c*(log(c
^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d^3 - a*d^3/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^3d^3x^3+3\,ac^2d^3x^2+3\,acd^3x+ad^3+\left(bc^3d^3x^3+3\,bc^2d^3x^2+3\,bcd^3x+bd^3\right)\text{artanh}(cx)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3ac^2\,dx + \int \frac{a}{x^2}\,dx + \int \frac{3ac}{x}\,dx + \int ac^3x\,dx + \int 3bc^2\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x^2}\,dx + \int \frac{3bc\operatorname{atanh}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**3*(Integral(3*a*c**2, x) + Integral(a/x**2, x) + Integral(3*a*c/x, x) +
Integral(a*c**3*x, x) + Integral(3*b*c**2*atanh(c*x), x) + Integral(b*atanh
(c*x)/x**2, x) + Integral(3*b*c*atanh(c*x)/x, x) + Integral(b*c**3*x*atanh(
c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^2, x)
```

3.26 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=160

$$-\frac{3}{2}bc^2d^3\text{PolyLog}(2,-cx)+\frac{3}{2}bc^2d^3\text{PolyLog}(2,cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^3d^3x+3a$$

```
[Out] -(b*c*d^3)/(2*x) + a*c^3*d^3*x + (b*c^2*d^3*ArcTanh[c*x])/2 + b*c^3*d^3*x*A
rcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[
c*x]))/x + 3*a*c^2*d^3*Log[x] + 3*b*c^2*d^3*Log[x] - b*c^2*d^3*Log[1 - c^2*
x^2] - (3*b*c^2*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c^2*d^3*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.167967, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5940, 5910, 260, 5916, 325, 206, 266, 36, 29, 31, 5912}

$$-\frac{3}{2}bc^2d^3\text{PolyLog}(2,-cx)+\frac{3}{2}bc^2d^3\text{PolyLog}(2,cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+ac^3d^3x+3a$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```


```
[Out] -(b*c*d^3)/(2*x) + a*c^3*d^3*x + (b*c^2*d^3*ArcTanh[c*x])/2 + b*c^3*d^3*x*A
rcTanh[c*x] - (d^3*(a + b*ArcTanh[c*x]))/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[
c*x]))/x + 3*a*c^2*d^3*Log[x] + 3*b*c^2*d^3*Log[x] - b*c^2*d^3*Log[1 - c^2*
x^2] - (3*b*c^2*d^3*PolyLog[2, -(c*x)])/2 + (3*b*c^2*d^3*PolyLog[2, c*x])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int\left(c^3d^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\right.$$

$$= d^3\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(3cd^3\right)\int\frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(3c^2d^3\right)\int\frac{a+b\tan}{x}$$

$$= ac^3d^3x-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x}+3ac^2d^3\log(x)-\frac{3}{2}bc^2$$

$$= -\frac{bcd^3}{2x}+ac^3d^3x+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3cd^3\left(a+b\tanh^{-}\right.}{x}$$

$$= -\frac{bcd^3}{2x}+ac^3d^3x+\frac{1}{2}bc^2d^3\tanh^{-1}(cx)+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx\right.}{2x^2}$$

$$= -\frac{bcd^3}{2x}+ac^3d^3x+\frac{1}{2}bc^2d^3\tanh^{-1}(cx)+bc^3d^3x\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx\right.}{2x^2}$$

Mathematica [A] time = 0.143838, size = 165, normalized size = 1.03

$$\frac{d^3\left(-6bc^2x^2\text{PolyLog}(2,-cx)+6bc^2x^2\text{PolyLog}(2,cx)+4ac^3x^3+12ac^2x^2\log(x)-12acx-2a+12bc^2x^2\log(cx)-bc\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^3*(-2*a - 12*a*c*x - 2*b*c*x + 4*a*c^3*x^3 - 2*b*ArcTanh[c*x] - 12*b*c*x
*ArcTanh[c*x] + 4*b*c^3*x^3*ArcTanh[c*x] + 12*a*c^2*x^2*Log[x] + 12*b*c^2*x
^2*Log[c*x] - b*c^2*x^2*Log[1 - c*x] + b*c^2*x^2*Log[1 + c*x] - 4*b*c^2*x^2
*Log[1 - c^2*x^2] - 6*b*c^2*x^2*PolyLog[2, -(c*x)] + 6*b*c^2*x^2*PolyLog[2,
 c*x]))/(4*x^2)
```

Maple [A] time = 0.052, size = 200, normalized size = 1.3

$$ac^3d^3x-3\,\frac{acd^3}{x}+3\,c^2d^3a\ln(cx)-\frac{d^3a}{2\,x^2}+bc^3d^3x\text{Artanh}(cx)-3\,\frac{d^3bc\text{Artanh}(cx)}{x}+3\,c^2d^3b\text{Artanh}(cx)\ln(cx)-\frac{d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] a*c^3*d^3*x-3*c*d^3*a/x+3*c^2*d^3*a*ln(c*x)-1/2*d^3*a/x^2+b*c^3*d^3*x*arcta
nh(c*x)-3*c*d^3*b*arctanh(c*x)/x+3*c^2*d^3*b*arctanh(c*x)*ln(c*x)-1/2*d^3*b
*arctanh(c*x)/x^2-3/2*c^2*d^3*b*dilog(c*x)-3/2*c^2*d^3*b*dilog(c*x+1)-3/2*c
^2*d^3*b*ln(c*x)*ln(c*x+1)-5/4*c^2*d^3*b*ln(c*x-1)-1/2*b*c*d^3/x+3*c^2*d^3*
b*ln(c*x)-3/4*c^2*d^3*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ac^3d^3x+\frac{1}{2}\left(2\,cx\,\text{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)bc^2d^3+\frac{3}{2}\,bc^2d^3\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+3\,ac^2d^3\log(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] a*c^3*d^3*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^2*d^3 + 3/2*
b*c^2*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 3*a*c^2*d^3*log(
x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^3 + 1/4
*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^3 - 3
*a*c*d^3/x - 1/2*a*d^3/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^3d^3x^3+3\,ac^2d^3x^2+3\,acd^3x+ad^3+\left(bc^3d^3x^3+3\,bc^2d^3x^2+3\,bcd^3x+bd^3\right)\text{artanh}(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int ac^3\,dx + \int \frac{a}{x^3}\,dx + \int \frac{3ac}{x^2}\,dx + \int \frac{3ac^2}{x}\,dx + \int bc^3\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x^3}\,dx + \int \frac{3bc\operatorname{atanh}(cx)}{x^2}\,dx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**3*(Integral(a*c**3, x) + Integral(a/x**3, x) + Integral(3*a*c/x**2, x) +
 Integral(3*a*c**2/x, x) + Integral(b*c**3*atanh(c*x), x) + Integral(b*atan
h(c*x)/x**3, x) + Integral(3*b*c*atanh(c*x)/x**2, x) + Integral(3*b*c**2*at
anh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3(b\operatorname{artanh}(cx) + a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^3, x)
```

3.27 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{2}bc^3d^3\text{PolyLog}(2,-cx)+\frac{1}{2}bc^3d^3\text{PolyLog}(2,cx)-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\right.}{}$$

```
[Out] -(b*c*d^3)/(6*x^2) - (3*b*c^2*d^3)/(2*x) + (3*b*c^3*d^3*ArcTanh[c*x])/2 - (
d^3*(a + b*ArcTanh[c*x]))/(3*x^3) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(2*x^2)
- (3*c^2*d^3*(a + b*ArcTanh[c*x]))/x + a*c^3*d^3*Log[x] + (10*b*c^3*d^3*Log
[x])/3 - (5*b*c^3*d^3*Log[1 - c^2*x^2])/3 - (b*c^3*d^3*PolyLog[2, -(c*x)])/
2 + (b*c^3*d^3*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.194253, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5916, 266, 44, 325, 206, 36, 29, 31, 5912}

$$-\frac{1}{2}bc^3d^3\text{PolyLog}(2,-cx)+\frac{1}{2}bc^3d^3\text{PolyLog}(2,cx)-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{d^3\left(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^3)/(6*x^2) - (3*b*c^2*d^3)/(2*x) + (3*b*c^3*d^3*ArcTanh[c*x])/2 - (
d^3*(a + b*ArcTanh[c*x]))/(3*x^3) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(2*x^2)
- (3*c^2*d^3*(a + b*ArcTanh[c*x]))/x + a*c^3*d^3*Log[x] + (10*b*c^3*d^3*Log
[x])/3 - (5*b*c^3*d^3*Log[1 - c^2*x^2])/3 - (b*c^3*d^3*PolyLog[2, -(c*x)])/
2 + (b*c^3*d^3*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx = \int\left(\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{x^4}+\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}\right.$$

$$= d^3\int\frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\left(3cd^3\right)\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(3c^2d^3\right)\int\frac{a+b\tan}{x^2}$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+a$$

$$= -\frac{3bc^2d^3}{2x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{3c^2d^3\left(a+b\tanh^{-}\right.}{x}$$

$$= -\frac{3bc^2d^3}{2x}+\frac{3}{2}bc^3d^3\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd^3}{6x^2}-\frac{3bc^2d^3}{2x}+\frac{3}{2}bc^3d^3\tanh^{-1}(cx)-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3cd^3\left(a+b\tanh\right.}{2x^2}$$

Mathematica [A] time = 0.139269, size = 175, normalized size = 0.99

$$\frac{d^3\left(-6bc^3x^3\text{PolyLog}(2,-cx)+6bc^3x^3\text{PolyLog}(2,cx)-36ac^2x^2+12ac^3x^3\log(x)-18acx-4a-18bc^2x^2+40bc^3x^3\log\ldots\right)}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-4*a - 18*a*c*x - 2*b*c*x - 36*a*c^2*x^2 - 18*b*c^2*x^2 - 4*b*ArcTanh
[c*x] - 18*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 12*a*c^3*x^3*Lo
g[x] + 40*b*c^3*x^3*Log[c*x] - 9*b*c^3*x^3*Log[1 - c*x] + 9*b*c^3*x^3*Log[1
 + c*x] - 20*b*c^3*x^3*Log[1 - c^2*x^2] - 6*b*c^3*x^3*PolyLog[2, -(c*x)] +
6*b*c^3*x^3*PolyLog[2, c*x]))/(12*x^3)
```

Maple [A] time = 0.049, size = 216, normalized size = 1.2

$$-3\,\frac{c^2d^3a}{x}+c^3d^3a\ln(cx)-\frac{3\,cd^3a}{2\,x^2}-\frac{d^3a}{3\,x^3}-3\,\frac{d^3bc^2\text{Artanh}(cx)}{x}+c^3d^3b\text{Artanh}(cx)\ln(cx)-\frac{3\,cd^3b\text{Artanh}(cx)}{2\,x^2}-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] -3*c^2*d^3*a/x+c^3*d^3*a*ln(c*x)-3/2*c*d^3*a/x^2-1/3*d^3*a/x^3-3*c^2*d^3*b*
arctanh(c*x)/x+c^3*d^3*b*arctanh(c*x)*ln(c*x)-3/2*c*d^3*b*arctanh(c*x)/x^2-
1/3*d^3*b*arctanh(c*x)/x^3-29/12*c^3*d^3*b*ln(c*x-1)-1/6*b*c*d^3/x^2-3/2*b*
c^2*d^3/x+10/3*c^3*d^3*b*ln(c*x)-11/12*c^3*d^3*b*ln(c*x+1)-1/2*c^3*d^3*b*di
log(c*x)-1/2*c^3*d^3*b*dilog(c*x+1)-1/2*c^3*d^3*b*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,bc^3d^3\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+ac^3d^3\log(x)-\frac{3}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}(cx)}{x}\right)bc^2d^3+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^3*d^3*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^3*d^3*lo
g(x) - 3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^3 +
 3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d
^3 - 1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/
x^3)*b*d^3 - 3*a*c^2*d^3/x - 3/2*a*c*d^3/x^2 - 1/3*a*d^3/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^3d^3x^3+3\,ac^2d^3x^2+3\,acd^3x+ad^3+\left(bc^3d^3x^3+3\,bc^2d^3x^2+3\,bcd^3x+bd^3\right)\text{artanh}(cx)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^3*d^3*x^3 + 3*a*c^2*d^3*x^2 + 3*a*c*d^3*x + a*d^3 + (b*c^3*d^
3*x^3 + 3*b*c^2*d^3*x^2 + 3*b*c*d^3*x + b*d^3)*arctanh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a}{x^4}\,dx + \int \frac{3ac}{x^3}\,dx + \int \frac{3ac^2}{x^2}\,dx + \int \frac{ac^3}{x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{x^4}\,dx + \int \frac{3bc\,\mathrm{atanh}\,(cx)}{x^3}\,dx + \int \frac{3bc^2\,\mathrm{atanh}\,(cx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] d**3*(Integral(a/x**4, x) + Integral(3*a*c/x**3, x) + Integral(3*a*c**2/x**
2, x) + Integral(a*c**3/x, x) + Integral(b*atanh(c*x)/x**4, x) + Integral(3
*b*c*atanh(c*x)/x**3, x) + Integral(3*b*c**2*atanh(c*x)/x**2, x) + Integral
(b*c**3*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3 (b\,\mathrm{artanh}\,(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)/x^4, x)
```

3.28 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=93

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d^3}{2x^2}-\frac{7bc^3d^3}{4x}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)-\frac{bcd^3}{12x^3}$$

```
[Out] -(b*c*d^3)/(12*x^3) - (b*c^2*d^3)/(2*x^2) - (7*b*c^3*d^3)/(4*x) - (d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(4*x^4) + 2*b*c^4*d^3*Log[x] - 2*b*c^4*d^3*Lo
g[1 - c*x]
```

Rubi [A] time = 0.0985638, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {37, 5936, 12, 88}

$$-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{bc^2d^3}{2x^2}-\frac{7bc^3d^3}{4x}+2bc^4d^3\log(x)-2bc^4d^3\log(1-cx)-\frac{bcd^3}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^3)/(12*x^3) - (b*c^2*d^3)/(2*x^2) - (7*b*c^3*d^3)/(4*x) - (d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(4*x^4) + 2*b*c^4*d^3*Log[x] - 2*b*c^4*d^3*Lo
g[1 - c*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - (bc)\int \frac{(d+cdx)^3}{4x^4(-1+cx)}\,dx$$
$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{1}{4}(bc)\int \frac{(d+cdx)^3}{x^4(-1+cx)}\,dx$$
$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{1}{4}(bc)\int \left(-\frac{d^3}{x^4} - \frac{4cd^3}{x^3} - \frac{7c^2d^3}{x^2} - \frac{8c^3d^3}{x} + \frac{8c^4}{-1+}\right.$$
$$= -\frac{bcd^3}{12x^3} - \frac{bc^2d^3}{2x^2} - \frac{7bc^3d^3}{4x} - \frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4} + 2bc^4d^3\log(x) - 2bc^4$$

Mathematica [A] time = 0.119112, size = 131, normalized size = 1.41

$$-\frac{d^3\left(24ac^3x^3+36ac^2x^2+24acx+6a+42bc^3x^3+12bc^2x^2-48bc^4x^4\log(x)+45bc^4x^4\log(1-cx)+3bc^4x^4\log(cx+1)\right.}{24x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] -(d^3*(6*a + 24*a*c*x + 2*b*c*x + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 24*a*c^3*x^
3 + 42*b*c^3*x^3 + 6*b*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3)*ArcTanh[c*x] - 4
8*b*c^4*x^4*Log[x] + 45*b*c^4*x^4*Log[1 - c*x] + 3*b*c^4*x^4*Log[1 + c*x]))
/(24*x^4)
```

Maple [B] time = 0.04, size = 181, normalized size = 2.

$$-\frac{d^3a}{4x^4} - \frac{c^3d^3a}{x} - \frac{3c^2d^3a}{2x^2} - \frac{cd^3a}{x^3} - \frac{d^3b\mathrm{Artanh}(cx)}{4x^4} - \frac{c^3d^3b\mathrm{Artanh}(cx)}{x} - \frac{3d^3bc^2\mathrm{Artanh}(cx)}{2x^2} - \frac{cd^3b\mathrm{Artanh}(cx)}{x^3} - 15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/4*d^3*a/x^4-c^3*d^3*a/x-3/2*c^2*d^3*a/x^2-c*d^3*a/x^3-1/4*d^3*b*arctanh(
c*x)/x^4-c^3*d^3*b*arctanh(c*x)/x-3/2*c^2*d^3*b*arctanh(c*x)/x^2-c*d^3*b*ar
ctanh(c*x)/x^3-15/8*c^4*d^3*b*ln(c*x-1)-1/12*b*c*d^3/x^3-1/2*b*c^2*d^3/x^2-
7/4*b*c^3*d^3/x+2*c^4*d^3*b*ln(c*x)-1/8*c^4*d^3*b*ln(c*x+1)
```

Maxima [B] time = 0.960071, size = 308, normalized size = 3.31

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}(cx)}{x}\right)bc^3d^3+\frac{3}{4}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}(cx)}{x^2}\right)bc^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^3 + 3/4*(
(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d^3 -
 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
```

```
*b*c*d^3 - a*c^3*d^3/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2
*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^3 - 3/2*a*c^2*d^3/x^2 - a
*c*d^3/x^3 - 1/4*a*d^3/x^4
```

Fricas [A] time = 2.15089, size = 373, normalized size = 4.01

$$-\frac{3\,bc^4d^3x^4\log\left(cx+1\right)+45\,bc^4d^3x^4\log\left(cx-1\right)-48\,bc^4d^3x^4\log\left(x\right)+6\left(4\,a+7\,b\right)c^3d^3x^3+12\left(3\,a+b\right)c^2d^3x^2+2\,(12}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*b*c^4*d^3*x^4*log(c*x + 1) + 45*b*c^4*d^3*x^4*log(c*x - 1) - 48*b*
c^4*d^3*x^4*log(x) + 6*(4*a + 7*b)*c^3*d^3*x^3 + 12*(3*a + b)*c^2*d^3*x^2 +
 2*(12*a + b)*c*d^3*x + 6*a*d^3 + 3*(4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*
b*c*d^3*x + b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

Sympy [A] time = 3.91854, size = 207, normalized size = 2.23

$$\begin{cases} -\frac{ac^3d^3}{x}-\frac{3ac^2d^3}{2x^2}-\frac{acd^3}{x^3}-\frac{ad^3}{4x^4}+2bc^4d^3\log\left(x\right)-2bc^4d^3\log\left(x-\frac{1}{c}\right)-\frac{bc^4d^3\operatorname{atanh}\left(cx\right)}{4}-\frac{bc^3d^3\operatorname{atanh}\left(cx\right)}{x}-\frac{7bc^3d^3}{4x}-\frac{3bc^2d^3\operatorname{atan}}{2x^2} \\ -\frac{ad^3}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] Piecewise((-a*c**3*d**3/x - 3*a*c**2*d**3/(2*x**2) - a*c*d**3/x**3 - a*d**3
/(4*x**4) + 2*b*c**4*d**3*log(x) - 2*b*c**4*d**3*log(x - 1/c) - b*c**4*d**3
*atanh(c*x)/4 - b*c**3*d**3*atanh(c*x)/x - 7*b*c**3*d**3/(4*x) - 3*b*c**2*d
**3*atanh(c*x)/(2*x**2) - b*c**2*d**3/(2*x**2) - b*c*d**3*atanh(c*x)/x**3 -
 b*c*d**3/(12*x**3) - b*d**3*atanh(c*x)/(4*x**4), Ne(c, 0)), (-a*d**3/(4*x*
*4), True))
```

Giac [B] time = 1.40483, size = 236, normalized size = 2.54

$$-\frac{1}{8}\,bc^4d^3\log\left(cx+1\right)-\frac{15}{8}\,bc^4d^3\log\left(cx-1\right)+2\,bc^4d^3\log\left(x\right)-\frac{\left(4\,bc^3d^3x^3+6\,bc^2d^3x^2+4\,bcd^3x+bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right)}{8\,x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] -1/8*b*c^4*d^3*log(c*x + 1) - 15/8*b*c^4*d^3*log(c*x - 1) + 2*b*c^4*d^3*log
(x) - 1/8*(4*b*c^3*d^3*x^3 + 6*b*c^2*d^3*x^2 + 4*b*c*d^3*x + b*d^3)*log(-(c
*x + 1)/(c*x - 1))/x^4 - 1/12*(12*a*c^3*d^3*x^3 + 21*b*c^3*d^3*x^3 + 18*a*c
^2*d^3*x^2 + 6*b*c^2*d^3*x^2 + 12*a*c*d^3*x + b*c*d^3*x + 3*a*d^3)/x^4
```

3.29 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=137

$$\frac{cd^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3bc^3d^3}{5x^2}-\frac{bc^2d^3}{4x^3}-\frac{5bc^4d^3}{4x}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\,\mathrm{l}$$

```
[Out] -(b*c*d^3)/(20*x^4) - (b*c^2*d^3)/(4*x^3) - (3*b*c^3*d^3)/(5*x^2) - (5*b*c^
4*d^3)/(4*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/(5*x^5) + (c*d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(20*x^4) + (6*b*c^5*d^3*Log[x])/5 - (6*b*c^5*
d^3*Log[1 - c*x])/5
```

Rubi [A] time = 0.118844, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {45, 37, 5936, 12, 148}

$$\frac{cd^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3bc^3d^3}{5x^2}-\frac{bc^2d^3}{4x^3}-\frac{5bc^4d^3}{4x}+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^3)/(20*x^4) - (b*c^2*d^3)/(4*x^3) - (3*b*c^3*d^3)/(5*x^2) - (5*b*c^
4*d^3)/(4*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x]))/(5*x^5) + (c*d^3*(1 +
 c*x)^4*(a + b*ArcTanh[c*x]))/(20*x^4) + (6*b*c^5*d^3*Log[x])/5 - (6*b*c^5*
d^3*Log[1 - c*x])/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-(bc)\int \frac{(-\\ &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{1}{20}(bc)\int\\ &= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{20x^4}-\frac{1}{20}(bc)\int\\ &= -\frac{bcd^3}{20x^4}-\frac{bc^2d^3}{4x^3}-\frac{3bc^3d^3}{5x^2}-\frac{5bc^4d^3}{4x}-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}+\frac{cd^3(1+\end{aligned}$$

Mathematica [A] time = 0.123228, size = 140, normalized size = 1.02

$$-\frac{d^3\left(20ac^3x^3+40ac^2x^2+30acx+8a+50bc^4x^4+24bc^3x^3+10bc^2x^2-48bc^5x^5\log(x)+49bc^5x^5\log(1-cx)-bc^5x^5}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(d^3*(8*a + 30*a*c*x + 2*b*c*x + 40*a*c^2*x^2 + 10*b*c^2*x^2 + 20*a*c^3*x^
3 + 24*b*c^3*x^3 + 50*b*c^4*x^4 + 2*b*(4 + 15*c*x + 20*c^2*x^2 + 10*c^3*x^3
)*ArcTanh[c*x] - 48*b*c^5*x^5*Log[x] + 49*b*c^5*x^5*Log[1 - c*x] - b*c^5*x^
5*Log[1 + c*x]))/(40*x^5)
```

Maple [A] time = 0.039, size = 193, normalized size = 1.4

$$-\frac{3\,cd^3a}{4\,x^4}-\frac{d^3a}{5\,x^5}-\frac{c^3d^3a}{2\,x^2}-\frac{c^2d^3a}{x^3}-\frac{3\,cd^3b\mathrm{Artanh}\,(cx)}{4\,x^4}-\frac{d^3b\mathrm{Artanh}\,(cx)}{5\,x^5}-\frac{c^3d^3b\mathrm{Artanh}\,(cx)}{2\,x^2}-\frac{d^3bc^2\mathrm{Artanh}\,(cx)}{x^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -3/4*c*d^3*a/x^4-1/5*d^3*a/x^5-1/2*c^3*d^3*a/x^2-c^2*d^3*a/x^3-3/4*c*d^3*b*
arctanh(c*x)/x^4-1/5*d^3*b*arctanh(c*x)/x^5-1/2*c^3*d^3*b*arctanh(c*x)/x^2-
c^2*d^3*b*arctanh(c*x)/x^3-49/40*c^5*d^3*b*ln(c*x-1)-1/20*b*c*d^3/x^4-1/4*b
*c^2*d^3/x^3-3/5*b*c^3*d^3/x^2-5/4*b*c^4*d^3/x+6/5*c^5*d^3*b*ln(c*x)+1/40*c
^5*d^3*b*ln(c*x+1)
```

Maxima [B] time = 0.96778, size = 338, normalized size = 2.47

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\left(cx\right)}{x^2}\right)bc^3d^3-\frac{1}{2}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}(}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*
d^3 - 1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c^2*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*
x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^3 - 1/20*((2*c^4*log(c^2*x^2 -
1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^3 -
1/2*a*c^3*d^3/x^2 - a*c^2*d^3/x^3 - 3/4*a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

Fricas [A] time = 2.07202, size = 398, normalized size = 2.91

$$\frac{bc^5d^3x^5\log\left(cx+1\right)-49\,bc^5d^3x^5\log\left(cx-1\right)+48\,bc^5d^3x^5\log\left(x\right)-50\,bc^4d^3x^4-4\left(5\,a+6\,b\right)c^3d^3x^3-10\left(4\,a+b\right)c^2d^3x^2}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(b*c^5*d^3*x^5*log(c*x + 1) - 49*b*c^5*d^3*x^5*log(c*x - 1) + 48*b*c^5
*d^3*x^5*log(x) - 50*b*c^4*d^3*x^4 - 4*(5*a + 6*b)*c^3*d^3*x^3 - 10*(4*a +
b)*c^2*d^3*x^2 - 2*(15*a + b)*c*d^3*x - 8*a*d^3 - (10*b*c^3*d^3*x^3 + 20*b*
c^2*d^3*x^2 + 15*b*c*d^3*x + 4*b*d^3)*log(-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A] time = 4.86091, size = 233, normalized size = 1.7

$$\begin{cases}-\frac{ac^3d^3}{2x^2}-\frac{ac^2d^3}{x^3}-\frac{3acd^3}{4x^4}-\frac{ad^3}{5x^5}+\frac{6bc^5d^3\log\left(x\right)}{5}-\frac{6bc^5d^3\log\left(x-\frac{1}{c}\right)}{5}+\frac{bc^5d^3\,\mathrm{atanh}\left(cx\right)}{20}-\frac{5bc^4d^3}{4x}-\frac{bc^3d^3\,\mathrm{atanh}\left(cx\right)}{2x^2}-\frac{3bc^3d^3}{5x^2}-\frac{bc^2d^3\,\mathrm{atanh}\left(cx\right.}{x^3}\\-\frac{ad^3}{5x^5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**3*d**3/(2*x**2) - a*c**2*d**3/x**3 - 3*a*c*d**3/(4*x**4) -
 a*d**3/(5*x**5) + 6*b*c**5*d**3*log(x)/5 - 6*b*c**5*d**3*log(x - 1/c)/5 +
b*c**5*d**3*atanh(c*x)/20 - 5*b*c**4*d**3/(4*x) - b*c**3*d**3*atanh(c*x)/(2
*x**2) - 3*b*c**3*d**3/(5*x**2) - b*c**2*d**3*atanh(c*x)/x**3 - b*c**2*d**3
/(4*x**3) - 3*b*c*d**3*atanh(c*x)/(4*x**4) - b*c*d**3/(20*x**4) - b*d**3*at
anh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**3/(5*x**5), True))
```

Giac [A] time = 1.53538, size = 254, normalized size = 1.85

$$\frac{1}{40}\,bc^5d^3\log\left(cx+1\right)-\frac{49}{40}\,bc^5d^3\log\left(cx-1\right)+\frac{6}{5}\,bc^5d^3\log\left(x\right)-\frac{\left(10\,bc^3d^3x^3+20\,bc^2d^3x^2+15\,bcd^3x+4\,bd^3\right)\log\left(-\frac{cx+1}{cx-1}\right.}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] 1/40*b*c^5*d^3*log(c*x + 1) - 49/40*b*c^5*d^3*log(c*x - 1) + 6/5*b*c^5*d^3*
log(x) - 1/40*(10*b*c^3*d^3*x^3 + 20*b*c^2*d^3*x^2 + 15*b*c*d^3*x + 4*b*d^3
)*log(-(c*x + 1)/(c*x - 1))/x^5 - 1/20*(25*b*c^4*d^3*x^4 + 10*a*c^3*d^3*x^3
 + 12*b*c^3*d^3*x^3 + 20*a*c^2*d^3*x^2 + 5*b*c^2*d^3*x^2 + 15*a*c*d^3*x + b
*c*d^3*x + 4*a*d^3)/x^5
```

3.30 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx$

Optimal. Leaf size=196

$$-\frac{c^3d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{7bc^4d^3}{15x^2}-\frac{11b}{36}$$

```
[Out] -(b*c*d^3)/(30*x^5) - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)/(36*x^3) - (7
*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTanh[c*x]))/(
6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(5*x^5) - (3*c^2*d^3*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x]))/(3*x^3) + (14*b*c^6*d^3*Lo
g[x])/15 - (37*b*c^6*d^3*Log[1 - c*x])/40 - (b*c^6*d^3*Log[1 + c*x])/120
```

Rubi [A] time = 0.177313, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^3d^3\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{7bc^4d^3}{15x^2}-\frac{11b}{36}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```


```
[Out] -(b*c*d^3)/(30*x^5) - (3*b*c^2*d^3)/(20*x^4) - (11*b*c^3*d^3)/(36*x^3) - (7
*b*c^4*d^3)/(15*x^2) - (11*b*c^5*d^3)/(12*x) - (d^3*(a + b*ArcTanh[c*x]))/(
6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x]))/(5*x^5) - (3*c^2*d^3*(a + b*ArcTanh
[c*x]))/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x]))/(3*x^3) + (14*b*c^6*d^3*Lo
g[x])/15 - (37*b*c^6*d^3*Log[1 - c*x])/40 - (b*c^6*d^3*Log[1 + c*x])/120
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}$$
$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}$$
$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4}$$
$$= -\frac{bcd^3}{30x^5}-\frac{3bc^2d^3}{20x^4}-\frac{11bc^3d^3}{36x^3}-\frac{7bc^4d^3}{15x^2}-\frac{11bc^5d^3}{12x}-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{3cd}{}$$

Mathematica [A] time = 0.125461, size = 149, normalized size = 0.76

$$-\frac{d^3\left(120ac^3x^3+270ac^2x^2+216acx+60a+330bc^5x^5+168bc^4x^4+110bc^3x^3+54bc^2x^2-336bc^6x^6\log(x)+333bc^6x\right.}{360x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -(d^3*(60*a + 216*a*c*x + 12*b*c*x + 270*a*c^2*x^2 + 54*b*c^2*x^2 + 120*a*c
^3*x^3 + 110*b*c^3*x^3 + 168*b*c^4*x^4 + 330*b*c^5*x^5 + 6*b*(10 + 36*c*x +
 45*c^2*x^2 + 20*c^3*x^3)*ArcTanh[c*x] - 336*b*c^6*x^6*Log[x] + 333*b*c^6*x
^6*Log[1 - c*x] + 3*b*c^6*x^6*Log[1 + c*x]))/(360*x^6)
```

Maple [A] time = 0.039, size = 205, normalized size = 1.1

$$-\frac{3c^2d^3a}{4x^4}-\frac{3cd^3a}{5x^5}-\frac{d^3a}{6x^6}-\frac{c^3d^3a}{3x^3}-\frac{3d^3bc^2\mathrm{Artanh}(cx)}{4x^4}-\frac{3cd^3b\mathrm{Artanh}(cx)}{5x^5}-\frac{d^3b\mathrm{Artanh}(cx)}{6x^6}-\frac{c^3d^3b\mathrm{Artanh}(cx}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x)
```

```
[Out] -3/4*c^2*d^3*a/x^4-3/5*c*d^3*a/x^5-1/6*d^3*a/x^6-1/3*c^3*d^3*a/x^3-3/4*c^2*
d^3*b*arctanh(c*x)/x^4-3/5*c*d^3*b*arctanh(c*x)/x^5-1/6*d^3*b*arctanh(c*x)/
x^6-1/3*c^3*d^3*b*arctanh(c*x)/x^3-37/40*c^6*d^3*b*ln(c*x-1)-1/30*b*c*d^3/x
^5-3/20*b*c^2*d^3/x^4-11/36*b*c^3*d^3/x^3-7/15*b*c^4*d^3/x^2-11/12*b*c^5*d^
3/x+14/15*c^6*d^3*b*ln(c*x)-1/120*b*c^6*d^3*ln(c*x+1)
```

Maxima [A] time = 0.973585, size = 369, normalized size = 1.88

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artanh}(cx)}{x^3}\right)bc^3d^3+\frac{1}{8}\left(\left(3c^3\log(cx+1)-3c^3\log(cx-1)-\frac{2\left(3c^2x\right.}{x^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^3*d^3 + 1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^3 - 3/20*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^3 + 1
/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x^
2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^3 - 1/3*a*c^3*d^3/x^3 - 3/4*a*c^2*
d^3/x^4 - 3/5*a*c*d^3/x^5 - 1/6*a*d^3/x^6
```

Fricas [A] time = 2.09426, size = 446, normalized size = 2.28

$$-\frac{3\,bc^6d^3x^6\log(cx+1)+333\,bc^6d^3x^6\log(cx-1)-336\,bc^6d^3x^6\log(x)+330\,bc^5d^3x^5+168\,bc^4d^3x^4+10\,(12\,a+11\,b)c^3d}{360\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/360*(3*b*c^6*d^3*x^6*log(c*x + 1) + 333*b*c^6*d^3*x^6*log(c*x - 1) - 336
*b*c^6*d^3*x^6*log(x) + 330*b*c^5*d^3*x^5 + 168*b*c^4*d^3*x^4 + 10*(12*a +
11*b)*c^3*d^3*x^3 + 54*(5*a + b)*c^2*d^3*x^2 + 12*(18*a + b)*c*d^3*x + 60*a
*d^3 + 3*(20*b*c^3*d^3*x^3 + 45*b*c^2*d^3*x^2 + 36*b*c*d^3*x + 10*b*d^3)*lo
g(-(c*x + 1)/(c*x - 1)))/x^6
```

Sympy [A] time = 8.38368, size = 257, normalized size = 1.31

$$\begin{cases} -\frac{ac^3d^3}{3x^3}-\frac{3ac^2d^3}{4x^4}-\frac{3acd^3}{5x^5}-\frac{ad^3}{6x^6}+\frac{14bc^6d^3\log(x)}{15}-\frac{14bc^6d^3\log\left(x-\frac{1}{c}\right)}{15}-\frac{bc^6d^3\operatorname{atanh}(cx)}{60}-\frac{11bc^5d^3}{12x}-\frac{7bc^4d^3}{15x^2}-\frac{bc^3d^3\operatorname{atanh}(cx)}{3x^3}-\frac{11bc^3d^3}{36x^3}- \\ -\frac{ad^3}{6x^6} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))/x**7,x)
```

```
[Out] Piecewise((-a*c**3*d**3/(3*x**3) - 3*a*c**2*d**3/(4*x**4) - 3*a*c*d**3/(5*x
**5) - a*d**3/(6*x**6) + 14*b*c**6*d**3*log(x)/15 - 14*b*c**6*d**3*log(x -
1/c)/15 - b*c**6*d**3*atanh(c*x)/60 - 11*b*c**5*d**3/(12*x) - 7*b*c**4*d**3
/(15*x**2) - b*c**3*d**3*atanh(c*x)/(3*x**3) - 11*b*c**3*d**3/(36*x**3) - 3
*b*c**2*d**3*atanh(c*x)/(4*x**4) - 3*b*c**2*d**3/(20*x**4) - 3*b*c*d**3*ata
nh(c*x)/(5*x**5) - b*c*d**3/(30*x**5) - b*d**3*atanh(c*x)/(6*x**6), Ne(c, 0
)), (-a*d**3/(6*x**6), True))
```

Giac [A] time = 1.76978, size = 271, normalized size = 1.38

$$-\frac{1}{120}\,bc^6d^3\log(cx+1)-\frac{37}{40}\,bc^6d^3\log(cx-1)+\frac{14}{15}\,bc^6d^3\log(x)-\frac{\left(20\,bc^3d^3x^3+45\,bc^2d^3x^2+36\,bcd^3x+10\,bd^3\right)\log\left(-\right.}{120\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

```
[Out] -1/120*b*c^6*d^3*log(c*x + 1) - 37/40*b*c^6*d^3*log(c*x - 1) + 14/15*b*c^6*
d^3*log(x) - 1/120*(20*b*c^3*d^3*x^3 + 45*b*c^2*d^3*x^2 + 36*b*c*d^3*x + 10
*b*d^3)*log(-(c*x + 1)/(c*x - 1))/x^6 - 1/180*(165*b*c^5*d^3*x^5 + 84*b*c^4
*d^3*x^4 + 60*a*c^3*d^3*x^3 + 55*b*c^3*d^3*x^3 + 135*a*c^2*d^3*x^2 + 27*b*c
^2*d^3*x^2 + 108*a*c*d^3*x + 6*b*c*d^3*x + 30*a*d^3)/x^6
```

3.31 $\int x^3(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=224

$$\frac{1}{8}c^4d^4x^8\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{7}c^3d^4x^7\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c$$

```
[Out] (11*b*d^4*x)/(8*c^3) + (24*b*d^4*x^2)/(35*c^2) + (11*b*d^4*x^3)/(24*c) + (1
2*b*d^4*x^4)/35 + (9*b*c*d^4*x^5)/40 + (2*b*c^2*d^4*x^6)/21 + (b*c^3*d^4*x^
7)/56 + (d^4*x^4*(a + b*ArcTanh[c*x]))/4 + (4*c*d^4*x^5*(a + b*ArcTanh[c*x]
))/5 + c^2*d^4*x^6*(a + b*ArcTanh[c*x]) + (4*c^3*d^4*x^7*(a + b*ArcTanh[c*x
]))/7 + (c^4*d^4*x^8*(a + b*ArcTanh[c*x]))/8 + (769*b*d^4*Log[1 - c*x])/(56
0*c^4) - (b*d^4*Log[1 + c*x])/(560*c^4)
```

Rubi [A] time = 0.213476, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {43, 5936, 12, 1802, 633, 31}

$$\frac{1}{8}c^4d^4x^8\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{7}c^3d^4x^7\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```


```
[Out] (11*b*d^4*x)/(8*c^3) + (24*b*d^4*x^2)/(35*c^2) + (11*b*d^4*x^3)/(24*c) + (1
2*b*d^4*x^4)/35 + (9*b*c*d^4*x^5)/40 + (2*b*c^2*d^4*x^6)/21 + (b*c^3*d^4*x^
7)/56 + (d^4*x^4*(a + b*ArcTanh[c*x]))/4 + (4*c*d^4*x^5*(a + b*ArcTanh[c*x]
))/5 + c^2*d^4*x^6*(a + b*ArcTanh[c*x]) + (4*c^3*d^4*x^7*(a + b*ArcTanh[c*x
]))/7 + (c^4*d^4*x^8*(a + b*ArcTanh[c*x]))/8 + (769*b*d^4*Log[1 - c*x])/(56
0*c^4) - (b*d^4*Log[1 + c*x])/(560*c^4)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int x^3(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx=\frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{1}{4}d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{5}cd^4x^5\left(a+b\tanh^{-1}(cx)\right)+c^2d^4x^6\left(a+b\tanh^{-1}\right.$$

$$=\frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}bc^3$$

$$=\frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}bc^3$$

$$=\frac{11bd^4x}{8c^3}+\frac{24bd^4x^2}{35c^2}+\frac{11bd^4x^3}{24c}+\frac{12}{35}bd^4x^4+\frac{9}{40}bcd^4x^5+\frac{2}{21}bc^2d^4x^6+\frac{1}{56}bc^3$$

Mathematica [A] time = 0.157504, size = 177, normalized size = 0.79

$$\frac{d^4\left(210ac^8x^8+960ac^7x^7+1680ac^6x^6+1344ac^5x^5+420ac^4x^4+30bc^7x^7+160bc^6x^6+378bc^5x^5+576bc^4x^4+770b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(2310*b*c*x + 1152*b*c^2*x^2 + 770*b*c^3*x^3 + 420*a*c^4*x^4 + 576*b*c
^4*x^4 + 1344*a*c^5*x^5 + 378*b*c^5*x^5 + 1680*a*c^6*x^6 + 160*b*c^6*x^6 +
960*a*c^7*x^7 + 30*b*c^7*x^7 + 210*a*c^8*x^8 + 6*b*c^4*x^4*(70 + 224*c*x +
280*c^2*x^2 + 160*c^3*x^3 + 35*c^4*x^4)*ArcTanh[c*x] + 2307*b*Log[1 - c*x]
- 3*b*Log[1 + c*x]))/(1680*c^4)
```

Maple [A] time = 0.03, size = 237, normalized size = 1.1

$$\frac{c^4d^4ax^8}{8}+\frac{4c^3d^4ax^7}{7}+c^2d^4ax^6+\frac{4cd^4ax^5}{5}+\frac{d^4ax^4}{4}+\frac{c^4d^4b\mathrm{Artanh}(cx)x^8}{8}+\frac{4c^3d^4b\mathrm{Artanh}(cx)x^7}{7}+c^2d^4b\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/8*c^4*d^4*a*x^8+4/7*c^3*d^4*a*x^7+c^2*d^4*a*x^6+4/5*c*d^4*a*x^5+1/4*d^4*a
*x^4+1/8*c^4*d^4*b*arctanh(c*x)*x^8+4/7*c^3*d^4*b*arctanh(c*x)*x^7+c^2*d^4*
b*arctanh(c*x)*x^6+4/5*c*d^4*b*arctanh(c*x)*x^5+1/4*d^4*b*arctanh(c*x)*x^4+
1/56*b*c^3*d^4*x^7+2/21*b*c^2*d^4*x^6+9/40*b*c*d^4*x^5+12/35*b*d^4*x^4+11/2
4*b*d^4*x^3/c+24/35*b*d^4*x^2/c^2+11/8*b*d^4*x/c^3+769/560/c^4*d^4*b*ln(c*x
-1)-1/560*b*d^4*ln(c*x+1)/c^4
```

Maxima [A] time = 0.971307, size = 504, normalized size = 2.25

$$\frac{1}{8}\,ac^4d^4x^8+\frac{4}{7}\,ac^3d^4x^7+ac^2d^4x^6+\frac{4}{5}\,acd^4x^5+\frac{1}{1680}\left(210\,x^8\,\mathrm{artanh}\,(cx)+c\left(\frac{2\left(15\,c^6x^7+21\,c^4x^5+35\,c^2x^3+105\,x\right)}{c^8}-\frac{1}{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^4*x^8 + 4/7*a*c^3*d^4*x^7 + a*c^2*d^4*x^6 + 4/5*a*c*d^4*x^5 + 1
/1680*(210*x^8*arctanh(c*x) + c*(2*(15*c^6*x^7 + 21*c^4*x^5 + 35*c^2*x^3 +
105*x)/c^8 - 105*log(c*x + 1)/c^9 + 105*log(c*x - 1)/c^9))*b*c^4*d^4 + 1/21
*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*
x^2 - 1)/c^8))*b*c^3*d^4 + 1/4*a*d^4*x^4 + 1/30*(30*x^6*arctanh(c*x) + c*(2
*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)
/c^7))*b*c^2*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*l
og(c^2*x^2 - 1)/c^6))*b*c*d^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 +
3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d^4
```

Fricas [A] time = 2.24534, size = 521, normalized size = 2.33

$$\frac{210\,ac^8d^4x^8+30\,(32\,a+b)c^7d^4x^7+80\,(21\,a+2\,b)c^6d^4x^6+42\,(32\,a+9\,b)c^5d^4x^5+12\,(35\,a+48\,b)c^4d^4x^4+770\,bc^3d^4x^3\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(210*a*c^8*d^4*x^8 + 30*(32*a + b)*c^7*d^4*x^7 + 80*(21*a + 2*b)*c^6
*d^4*x^6 + 42*(32*a + 9*b)*c^5*d^4*x^5 + 12*(35*a + 48*b)*c^4*d^4*x^4 + 770
*b*c^3*d^4*x^3 + 1152*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 3*b*d^4*log(c*x + 1)
 + 2307*b*d^4*log(c*x - 1) + 3*(35*b*c^8*d^4*x^8 + 160*b*c^7*d^4*x^7 + 280*
b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 + 70*b*c^4*d^4*x^4)*log(-(c*x + 1)/(c*x -
 1)))/c^4
```

Sympy [A] time = 7.01342, size = 294, normalized size = 1.31

$$\begin{cases}\frac{ac^4d^4x^8}{8}+\frac{4ac^3d^4x^7}{7}+ac^2d^4x^6+\frac{4acd^4x^5}{5}+\frac{ad^4x^4}{4}+\frac{bc^4d^4x^8\,\mathrm{atanh}\,(cx)}{8}+\frac{4bc^3d^4x^7\,\mathrm{atanh}\,(cx)}{7}+\frac{bc^3d^4x^7}{56}+bc^2d^4x^6\,\mathrm{atanh}\,(cx)+\frac{2bc^2d^4\ldots}{21} \\ \frac{ad^4x^4}{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**8/8 + 4*a*c**3*d**4*x**7/7 + a*c**2*d**4*x**6 + 4
*a*c*d**4*x**5/5 + a*d**4*x**4/4 + b*c**4*d**4*x**8*atanh(c*x)/8 + 4*b*c**3
*d**4*x**7*atanh(c*x)/7 + b*c**3*d**4*x**7/56 + b*c**2*d**4*x**6*atanh(c*x)
 + 2*b*c**2*d**4*x**6/21 + 4*b*c*d**4*x**5*atanh(c*x)/5 + 9*b*c*d**4*x**5/4
0 + b*d**4*x**4*atanh(c*x)/4 + 12*b*d**4*x**4/35 + 11*b*d**4*x**3/(24*c) +
24*b*d**4*x**2/(35*c**2) + 11*b*d**4*x/(8*c**3) + 48*b*d**4*log(x - 1/c)/(3
5*c**4) - b*d**4*atanh(c*x)/(280*c**4), Ne(c, 0)), (a*d**4*x**4/4, True))
```

Giac [A] time = 1.22285, size = 317, normalized size = 1.42

$$\frac{1}{8}ac^4d^4x^8 + \frac{1}{56}\left(32\,ac^3d^4 + bc^3d^4\right)x^7 + \frac{11\,bd^4x^3}{24\,c} + \frac{1}{21}\left(21\,ac^2d^4 + 2\,bc^2d^4\right)x^6 + \frac{1}{40}\left(32\,acd^4 + 9\,bcd^4\right)x^5 + \frac{24\,bd^4x^2}{35\,c^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/8*a*c^4*d^4*x^8 + 1/56*(32*a*c^3*d^4 + b*c^3*d^4)*x^7 + 11/24*b*d^4*x^3/c
 + 1/21*(21*a*c^2*d^4 + 2*b*c^2*d^4)*x^6 + 1/40*(32*a*c*d^4 + 9*b*c*d^4)*x^
5 + 24/35*b*d^4*x^2/c^2 + 1/140*(35*a*d^4 + 48*b*d^4)*x^4 + 11/8*b*d^4*x/c^
3 - 1/560*b*d^4*log(c*x + 1)/c^4 + 769/560*b*d^4*log(c*x - 1)/c^4 + 1/560*(
35*b*c^4*d^4*x^8 + 160*b*c^3*d^4*x^7 + 280*b*c^2*d^4*x^6 + 224*b*c*d^4*x^5
+ 70*b*d^4*x^4)*log(-(c*x + 1)/(c*x - 1))
```

3.32 $\int x^2(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx$

Optimal. Leaf size=171

$$\frac{d^4(cx+1)^7\left(a+b\tanh^{-1}(cx)\right)}{7c^3}-\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{42}bc^3d^4x^6+\frac{2}{15}bc^2d$$

```
[Out] (5*b*d^4*x)/(3*c^2) + (88*b*d^4*x^2)/(105*c) + (5*b*d^4*x^3)/9 + (47*b*c*d^
4*x^4)/140 + (2*b*c^2*d^4*x^5)/15 + (b*c^3*d^4*x^6)/42 + (d^4*(1 + c*x)^5*(
a + b*ArcTanh[c*x]))/(5*c^3) - (d^4*(1 + c*x)^6*(a + b*ArcTanh[c*x]))/(3*c^
3) + (d^4*(1 + c*x)^7*(a + b*ArcTanh[c*x]))/(7*c^3) + (176*b*d^4*Log[1 - c*
x])/(105*c^3)
```

Rubi [A] time = 0.180548, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {43, 5936, 12, 893}

$$\frac{d^4(cx+1)^7\left(a+b\tanh^{-1}(cx)\right)}{7c^3}-\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3}+\frac{1}{42}bc^3d^4x^6+\frac{2}{15}bc^2d$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (5*b*d^4*x)/(3*c^2) + (88*b*d^4*x^2)/(105*c) + (5*b*d^4*x^3)/9 + (47*b*c*d^
4*x^4)/140 + (2*b*c^2*d^4*x^5)/15 + (b*c^3*d^4*x^6)/42 + (d^4*(1 + c*x)^5*(
a + b*ArcTanh[c*x]))/(5*c^3) - (d^4*(1 + c*x)^6*(a + b*ArcTanh[c*x]))/(3*c^
3) + (d^4*(1 + c*x)^7*(a + b*ArcTanh[c*x]))/(7*c^3) + (176*b*d^4*Log[1 - c*
x])/(105*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
```

```
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int x^2(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx = \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3} - \frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3} + \frac{d^4(1+cx}{}$$

$$= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3} - \frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3} + \frac{d^4(1+cx}{}$$

$$= \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^3} - \frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{3c^3} + \frac{d^4(1+cx}{}$$

$$= \frac{5bd^4x}{3c^2} + \frac{88bd^4x^2}{105c} + \frac{5}{9}bd^4x^3 + \frac{47}{140}bcd^4x^4 + \frac{2}{15}bc^2d^4x^5 + \frac{1}{42}bc^3d^4x^6 + \frac{d^4(1}{}$$

Mathematica [A] time = 0.146447, size = 168, normalized size = 0.98

$$\frac{d^4\left(180ac^7x^7 + 840ac^6x^6 + 1512ac^5x^5 + 1260ac^4x^4 + 420ac^3x^3 + 30bc^6x^6 + 168bc^5x^5 + 423bc^4x^4 + 700bc^3x^3 + 1056b\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(2100*b*c*x + 1056*b*c^2*x^2 + 420*a*c^3*x^3 + 700*b*c^3*x^3 + 1260*a*
c^4*x^4 + 423*b*c^4*x^4 + 1512*a*c^5*x^5 + 168*b*c^5*x^5 + 840*a*c^6*x^6 +
30*b*c^6*x^6 + 180*a*c^7*x^7 + 12*b*c^3*x^3*(35 + 105*c*x + 126*c^2*x^2 + 7
0*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] + 2106*b*Log[1 - c*x] + 6*b*Log[1 + c*
x]))/(1260*c^3)
```

Maple [A] time = 0.027, size = 225, normalized size = 1.3

$$\frac{c^4d^4ax^7}{7} + \frac{2c^3d^4ax^6}{3} + \frac{6c^2d^4ax^5}{5} + cd^4ax^4 + \frac{d^4ax^3}{3} + \frac{c^4d^4b\mathrm{Artanh}(cx)x^7}{7} + \frac{2c^3d^4b\mathrm{Artanh}(cx)x^6}{3} + \frac{6c^2d^4b\mathrm{Artar}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/7*c^4*d^4*a*x^7+2/3*c^3*d^4*a*x^6+6/5*c^2*d^4*a*x^5+c*d^4*a*x^4+1/3*d^4*a
*x^3+1/7*c^4*d^4*b*arctanh(c*x)*x^7+2/3*c^3*d^4*b*arctanh(c*x)*x^6+6/5*c^2*
d^4*b*arctanh(c*x)*x^5+c*d^4*b*arctanh(c*x)*x^4+1/3*d^4*b*arctanh(c*x)*x^3+
1/42*b*c^3*d^4*x^6+2/15*b*c^2*d^4*x^5+47/140*b*c*d^4*x^4+5/9*b*d^4*x^3+88/1
05*b*d^4*x^2/c+5/3*b*d^4*x/c^2+117/70/c^3*d^4*b*ln(c*x-1)+1/210/c^3*d^4*b*l
n(c*x+1)
```

Maxima [B] time = 0.97381, size = 458, normalized size = 2.68

$$\frac{1}{7}ac^4d^4x^7 + \frac{2}{3}ac^3d^4x^6 + \frac{6}{5}ac^2d^4x^5 + \frac{1}{84}\left(12x^7\,\mathrm{artanh}(cx) + c\left(\frac{2c^4x^6 + 3c^2x^4 + 6x^2}{c^6} + \frac{6\log\left(c^2x^2-1\right)}{c^8}\right)\right)bc^4d^4 + a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^4*x^7 + 2/3*a*c^3*d^4*x^6 + 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7*ar
ctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^
8))*b*c^4*d^4 + a*c*d^4*x^4 + 1/45*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 +
 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*c^3*
d^4 + 3/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 -
 1)/c^6))*b*c^2*d^4 + 1/3*a*d^4*x^3 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x
^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*c*d^4 + 1/6*(2*
x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d^4
```

Fricas [A] time = 2.20207, size = 490, normalized size = 2.87

$$\frac{180\,ac^7d^4x^7 + 30\,(28\,a + b)c^6d^4x^6 + 168\,(9\,a + b)c^5d^4x^5 + 9\,(140\,a + 47\,b)c^4d^4x^4 + 140\,(3\,a + 5\,b)c^3d^4x^3 + 1056\,bc^2d^4x^2 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1260*(180*a*c^7*d^4*x^7 + 30*(28*a + b)*c^6*d^4*x^6 + 168*(9*a + b)*c^5*d
^4*x^5 + 9*(140*a + 47*b)*c^4*d^4*x^4 + 140*(3*a + 5*b)*c^3*d^4*x^3 + 1056*
b*c^2*d^4*x^2 + 2100*b*c*d^4*x + 6*b*d^4*log(c*x + 1) + 2106*b*d^4*log(c*x
- 1) + 6*(15*b*c^7*d^4*x^7 + 70*b*c^6*d^4*x^6 + 126*b*c^5*d^4*x^5 + 105*b*c
^4*d^4*x^4 + 35*b*c^3*d^4*x^3)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] time = 5.84074, size = 279, normalized size = 1.63

$$\begin{cases} \frac{ac^4d^4x^7}{7} + \frac{2ac^3d^4x^6}{3} + \frac{6ac^2d^4x^5}{5} + acd^4x^4 + \frac{ad^4x^3}{3} + \frac{bc^4d^4x^7\operatorname{atanh}(cx)}{7} + \frac{2bc^3d^4x^6\operatorname{atanh}(cx)}{3} + \frac{bc^3d^4x^6}{42} + \frac{6bc^2d^4x^5\operatorname{atanh}(cx)}{5} + \frac{2bc^2d^4x^5}{15} + \cdots \\ \frac{ad^4x^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**7/7 + 2*a*c**3*d**4*x**6/3 + 6*a*c**2*d**4*x**5/5
 + a*c*d**4*x**4 + a*d**4*x**3/3 + b*c**4*d**4*x**7*atanh(c*x)/7 + 2*b*c**3
*d**4*x**6*atanh(c*x)/3 + b*c**3*d**4*x**6/42 + 6*b*c**2*d**4*x**5*atanh(c*
x)/5 + 2*b*c**2*d**4*x**5/15 + b*c*d**4*x**4*atanh(c*x) + 47*b*c*d**4*x**4/
140 + b*d**4*x**3*atanh(c*x)/3 + 5*b*d**4*x**3/9 + 88*b*d**4*x**2/(105*c) +
 5*b*d**4*x/(3*c**2) + 176*b*d**4*log(x - 1/c)/(105*c**3) + b*d**4*atanh(c*
x)/(105*c**3), Ne(c, 0)), (a*d**4*x**3/3, True))
```

Giac [A] time = 1.26854, size = 300, normalized size = 1.75

$$\frac{1}{7}\,ac^4d^4x^7 + \frac{1}{42}\left(28\,ac^3d^4 + bc^3d^4\right)x^6 + \frac{88\,bd^4x^2}{105\,c} + \frac{2}{15}\left(9\,ac^2d^4 + bc^2d^4\right)x^5 + \frac{1}{140}\left(140\,acd^4 + 47\,bcd^4\right)x^4 + \frac{5\,bd^4x}{3\,c^2} + \frac{1}{9}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/7*a*c^4*d^4*x^7 + 1/42*(28*a*c^3*d^4 + b*c^3*d^4)*x^6 + 88/105*b*d^4*x^2/
c + 2/15*(9*a*c^2*d^4 + b*c^2*d^4)*x^5 + 1/140*(140*a*c*d^4 + 47*b*c*d^4)*x
^4 + 5/3*b*d^4*x/c^2 + 1/9*(3*a*d^4 + 5*b*d^4)*x^3 + 1/210*b*d^4*log(c*x +
1)/c^3 + 117/70*b*d^4*log(c*x - 1)/c^3 + 1/210*(15*b*c^4*d^4*x^7 + 70*b*c^3
*d^4*x^6 + 126*b*c^2*d^4*x^5 + 105*b*c*d^4*x^4 + 35*b*d^4*x^3)*log(-(c*x +
1)/(c*x - 1))
```

3.33 $\int x(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=153

$$\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^4(cx+1)^5}{30c^2}+\frac{bd^4(cx+1)^4}{30c^2}+\frac{4bd^4(cx+1)^3}{45c^2}+\frac{4bd^4(}{1}$$

```
[Out] (16*b*d^4*x)/(15*c) + (4*b*d^4*(1 + c*x)^2)/(15*c^2) + (4*b*d^4*(1 + c*x)^3
)/(45*c^2) + (b*d^4*(1 + c*x)^4)/(30*c^2) + (b*d^4*(1 + c*x)^5)/(30*c^2) -
(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (d^4*(1 + c*x)^6*(a + b*Ar
cTanh[c*x]))/(6*c^2) + (32*b*d^4*Log[1 - c*x])/(15*c^2)
```

Rubi [A] time = 0.116179, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {43, 5936, 12, 77}

$$\frac{d^4(cx+1)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^4(cx+1)^5}{30c^2}+\frac{bd^4(cx+1)^4}{30c^2}+\frac{4bd^4(cx+1)^3}{45c^2}+\frac{4bd^4(}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```


```
[Out] (16*b*d^4*x)/(15*c) + (4*b*d^4*(1 + c*x)^2)/(15*c^2) + (4*b*d^4*(1 + c*x)^3
)/(45*c^2) + (b*d^4*(1 + c*x)^4)/(30*c^2) + (b*d^4*(1 + c*x)^5)/(30*c^2) -
(d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c^2) + (d^4*(1 + c*x)^6*(a + b*Ar
cTanh[c*x]))/(6*c^2) + (32*b*d^4*Log[1 - c*x])/(15*c^2)
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int x(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-(bc)\int \frac{(-}{}$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{b\int \frac{(-1+5}{}}{}$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{d^4(1+cx)^6\left(a+b\tanh^{-1}(cx)\right)}{6c^2}-\frac{b\int\left(-32\right.}{}$$

$$= \frac{16bd^4x}{15c}+\frac{4bd^4(1+cx)^2}{15c^2}+\frac{4bd^4(1+cx)^3}{45c^2}+\frac{bd^4(1+cx)^4}{30c^2}+\frac{bd^4(1+cx)^5}{30c^2}-\frac{d^4(}{}$$

Mathematica [A] time = 0.141229, size = 159, normalized size = 1.04

$$\frac{d^4\left(30ac^6x^6+144ac^5x^5+270ac^4x^4+240ac^3x^3+90ac^2x^2+6bc^5x^5+36bc^4x^4+100bc^3x^3+192bc^2x^2+6bc^2x^2\left(5c^4x\right.\right.}{180c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(390*b*c*x + 90*a*c^2*x^2 + 192*b*c^2*x^2 + 240*a*c^3*x^3 + 100*b*c^3*
x^3 + 270*a*c^4*x^4 + 36*b*c^4*x^4 + 144*a*c^5*x^5 + 6*b*c^5*x^5 + 30*a*c^6
*x^6 + 6*b*c^2*x^2*(15 + 40*c*x + 45*c^2*x^2 + 24*c^3*x^3 + 5*c^4*x^4)*ArcT
anh[c*x] + 387*b*Log[1 - c*x] - 3*b*Log[1 + c*x]))/(180*c^2)
```

Maple [A] time = 0.03, size = 215, normalized size = 1.4

$$\frac{c^4d^4ax^6}{6}+\frac{4c^3d^4ax^5}{5}+\frac{3c^2d^4ax^4}{2}+\frac{4cd^4ax^3}{3}+\frac{d^4ax^2}{2}+\frac{c^4d^4b\mathrm{Artanh}(cx)x^6}{6}+\frac{4c^3d^4b\mathrm{Artanh}(cx)x^5}{5}+\frac{3c^2d^4b\mathrm{Art}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/6*c^4*d^4*a*x^6+4/5*c^3*d^4*a*x^5+3/2*c^2*d^4*a*x^4+4/3*c*d^4*a*x^3+1/2*d
^4*a*x^2+1/6*c^4*d^4*b*arctanh(c*x)*x^6+4/5*c^3*d^4*b*arctanh(c*x)*x^5+3/2*
c^2*d^4*b*arctanh(c*x)*x^4+4/3*c*d^4*b*arctanh(c*x)*x^3+1/2*d^4*b*arctanh(c
*x)*x^2+1/30*c^3*d^4*b*x^5+1/5*c^2*d^4*b*x^4+5/9*c*d^4*b*x^3+16/15*d^4*b*x^
2+13/6*b*d^4*x/c+43/20/c^2*d^4*b*ln(c*x-1)-1/60/c^2*d^4*b*ln(c*x+1)
```

Maxima [B] time = 0.978578, size = 440, normalized size = 2.88

$$\frac{1}{6}ac^4d^4x^6+\frac{4}{5}ac^3d^4x^5+\frac{3}{2}ac^2d^4x^4+\frac{1}{180}\left(30x^6\,\mathrm{artanh}(cx)+c\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6}-\frac{15\log(cx+1)}{c^7}+\frac{15\log}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^4*x^6 + 4/5*a*c^3*d^4*x^5 + 3/2*a*c^2*d^4*x^4 + 1/180*(30*x^6*a
rctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7
 + 15*log(c*x - 1)/c^7))*b*c^4*d^4 + 1/5*(4*x^5*arctanh(c*x) + c*((c^2*x^4
+ 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^3*d^4 + 4/3*a*c*d^4*x^3 + 1/4*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^2*d^4 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x
^2 - 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^
2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*d^4
```

Fricas [A] time = 2.12382, size = 452, normalized size = 2.95

$$\frac{30\,ac^6d^4x^6 + 6\,(24\,a + b)c^5d^4x^5 + 18\,(15\,a + 2\,b)c^4d^4x^4 + 20\,(12\,a + 5\,b)c^3d^4x^3 + 6\,(15\,a + 32\,b)c^2d^4x^2 + 390\,bcd^4x - 3\,bd}{180\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^6*d^4*x^6 + 6*(24*a + b)*c^5*d^4*x^5 + 18*(15*a + 2*b)*c^4*d^
4*x^4 + 20*(12*a + 5*b)*c^3*d^4*x^3 + 6*(15*a + 32*b)*c^2*d^4*x^2 + 390*b*c
*d^4*x - 3*b*d^4*log(c*x + 1) + 387*b*d^4*log(c*x - 1) + 3*(5*b*c^6*d^4*x^6
 + 24*b*c^5*d^4*x^5 + 45*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 15*b*c^2*d^4*x^
2)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 4.79549, size = 269, normalized size = 1.76

$$\begin{cases} \frac{ac^4d^4x^6}{6} + \frac{4ac^3d^4x^5}{5} + \frac{3ac^2d^4x^4}{2} + \frac{4acd^4x^3}{3} + \frac{ad^4x^2}{2} + \frac{bc^4d^4x^6\operatorname{atanh}(cx)}{6} + \frac{4bc^3d^4x^5\operatorname{atanh}(cx)}{5} + \frac{bc^3d^4x^5}{30} + \frac{3bc^2d^4x^4\operatorname{atanh}(cx)}{2} + \frac{bc^2d^4x^4}{5} + \\ \frac{ad^4x^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**6/6 + 4*a*c**3*d**4*x**5/5 + 3*a*c**2*d**4*x**4/2
 + 4*a*c*d**4*x**3/3 + a*d**4*x**2/2 + b*c**4*d**4*x**6*atanh(c*x)/6 + 4*b*
c**3*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**5/30 + 3*b*c**2*d**4*x**4*atan
h(c*x)/2 + b*c**2*d**4*x**4/5 + 4*b*c*d**4*x**3*atanh(c*x)/3 + 5*b*c*d**4*x
**3/9 + b*d**4*x**2*atanh(c*x)/2 + 16*b*d**4*x**2/15 + 13*b*d**4*x/(6*c) +
32*b*d**4*log(x - 1/c)/(15*c**2) - b*d**4*atanh(c*x)/(30*c**2), Ne(c, 0)),
(a*d**4*x**2/2, True))
```

Giac [A] time = 1.29912, size = 285, normalized size = 1.86

$$\frac{1}{6}\,ac^4d^4x^6 + \frac{1}{30}\left(24\,ac^3d^4 + bc^3d^4\right)x^5 + \frac{13\,bd^4x}{6\,c} + \frac{1}{10}\left(15\,ac^2d^4 + 2\,bc^2d^4\right)x^4 + \frac{1}{9}\left(12\,acd^4 + 5\,bcd^4\right)x^3 - \frac{bd^4\log(cx + 1)}{60\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*a*c^4*d^4*x^6 + 1/30*(24*a*c^3*d^4 + b*c^3*d^4)*x^5 + 13/6*b*d^4*x/c +
1/10*(15*a*c^2*d^4 + 2*b*c^2*d^4)*x^4 + 1/9*(12*a*c*d^4 + 5*b*c*d^4)*x^3 -
1/60*b*d^4*log(c*x + 1)/c^2 + 43/20*b*d^4*log(c*x - 1)/c^2 + 1/30*(15*a*d^4
 + 32*b*d^4)*x^2 + 1/60*(5*b*c^4*d^4*x^6 + 24*b*c^3*d^4*x^5 + 45*b*c^2*d^4*
x^4 + 40*b*c*d^4*x^3 + 15*b*d^4*x^2)*log(-(c*x + 1)/(c*x - 1))
```

3.34 $\int (d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right) dx$

Optimal. Leaf size=107

$$\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}+\frac{bd^4(cx+1)^4}{20c}+\frac{2bd^4(cx+1)^3}{15c}+\frac{2bd^4(cx+1)^2}{5c}+\frac{16bd^4\log(1-cx)}{5c}+\frac{8}{5}bd^4x$$

```
[Out] (8*b*d^4*x)/5 + (2*b*d^4*(1 + c*x)^2)/(5*c) + (2*b*d^4*(1 + c*x)^3)/(15*c)
+ (b*d^4*(1 + c*x)^4)/(20*c) + (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c)
 + (16*b*d^4*Log[1 - c*x])/(5*c)
```

Rubi [A] time = 0.0548806, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5926, 627, 43}

$$\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5c}+\frac{bd^4(cx+1)^4}{20c}+\frac{2bd^4(cx+1)^3}{15c}+\frac{2bd^4(cx+1)^2}{5c}+\frac{16bd^4\log(1-cx)}{5c}+\frac{8}{5}bd^4x$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (8*b*d^4*x)/5 + (2*b*d^4*(1 + c*x)^2)/(5*c) + (2*b*d^4*(1 + c*x)^3)/(15*c)
+ (b*d^4*(1 + c*x)^4)/(20*c) + (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*c)
 + (16*b*d^4*Log[1 - c*x])/(5*c)
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right) dx = \frac{d^4(1+cx)^5 \left(a+b\tanh^{-1}(cx)\right)}{5c} - \frac{b\int \frac{(d+cdx)^5}{1-c^2x^2} dx}{5d}$$
$$= \frac{d^4(1+cx)^5 \left(a+b\tanh^{-1}(cx)\right)}{5c} - \frac{b\int \frac{(d+cdx)^4}{\frac{1}{d}-\frac{cx}{d}} dx}{5d}$$
$$= \frac{d^4(1+cx)^5 \left(a+b\tanh^{-1}(cx)\right)}{5c} - \frac{b\int \left(-8d^5 + \frac{16d^4}{\frac{1}{d}-\frac{cx}{d}} - 4d^4(d+cdx) - 2d^3(d+cd\right.}{5d}$$
$$= \frac{8}{5}bd^4x + \frac{2bd^4(1+cx)^2}{5c} + \frac{2bd^4(1+cx)^3}{15c} + \frac{bd^4(1+cx)^4}{20c} + \frac{d^4(1+cx)^5 \left(a+b\tan\right.}{5c}$$

Mathematica [A] time = 0.178214, size = 146, normalized size = 1.36

$$\frac{d^4 \left(12ac^5x^5 + 60ac^4x^4 + 120ac^3x^3 + 120ac^2x^2 + 60acx + 3bc^4x^4 + 20bc^3x^3 + 66bc^2x^2 + 6b\log\left(1-c^2x^2\right) + 12bcx\left(c^4x\right.\right.}{60c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*d*x)^4*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (d^4*(60*a*c*x + 180*b*c*x + 120*a*c^2*x^2 + 66*b*c^2*x^2 + 120*a*c^3*x^3 +
 20*b*c^3*x^3 + 60*a*c^4*x^4 + 3*b*c^4*x^4 + 12*a*c^5*x^5 + 12*b*c*x*(5 + 1
0*c*x + 10*c^2*x^2 + 5*c^3*x^3 + c^4*x^4)*ArcTanh[c*x] + 180*b*Log[1 - c*x]
 + 6*b*Log[1 - c^2*x^2]))/(60*c)
```

Maple [B] time = 0.026, size = 202, normalized size = 1.9

$$\frac{c^4x^5ad^4}{5} + c^3x^4ad^4 + 2c^2x^3ad^4 + 2cx^2ad^4 + xad^4 + \frac{d^4a}{5c} + \frac{c^4d^4b\mathrm{Artanh}(cx)x^5}{5} + c^3d^4b\mathrm{Artanh}(cx)x^4 + 2c^2d^4b\mathrm{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x)),x)
```

```
[Out] 1/5*c^4*x^5*a*d^4+c^3*x^4*a*d^4+2*c^2*x^3*a*d^4+2*c*x^2*a*d^4+x*a*d^4+1/5/c
*d^4*a+1/5*c^4*d^4*b*arctanh(c*x)*x^5+c^3*d^4*b*arctanh(c*x)*x^4+2*c^2*d^4*
b*arctanh(c*x)*x^3+2*c*d^4*b*arctanh(c*x)*x^2+d^4*b*arctanh(c*x)*x+1/5/c*d^
4*b*arctanh(c*x)+1/20*c^3*d^4*b*x^4+1/3*c^2*d^4*b*x^3+11/10*c*d^4*b*x^2+3*b
*d^4*x+16/5/c*d^4*b*ln(c*x-1)
```

Maxima [B] time = 0.949735, size = 382, normalized size = 3.57

$$\frac{1}{5}ac^4d^4x^5 + ac^3d^4x^4 + \frac{1}{20}\left(4x^5\operatorname{artanh}(cx) + c\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)bc^4d^4 + 2ac^2d^4x^3 + \frac{1}{6}\left(6x^4\operatorname{artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^4*x^5 + a*c^3*d^4*x^4 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4
+ 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*b*c^4*d^4 + 2*a*c^2*d^4*x^3 + 1/6*(
6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(
c*x - 1)/c^5))*b*c^3*d^4 + (2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
 1)/c^4))*b*c^2*d^4 + 2*a*c*d^4*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - lo
g(c*x + 1)/c^3 + log(c*x - 1)/c^3))*b*c*d^4 + a*d^4*x + 1/2*(2*c*x*arctanh(
c*x) + log(-c^2*x^2 + 1))*b*d^4/c
```

Fricas [A] time = 2.04621, size = 402, normalized size = 3.76

$$\frac{12\,ac^5d^4x^5 + 3\,(20\,a + b)c^4d^4x^4 + 20\,(6\,a + b)c^3d^4x^3 + 6\,(20\,a + 11\,b)c^2d^4x^2 + 60\,(a + 3\,b)cd^4x + 6\,bd^4\log(cx + 1) + 186\,}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*a*c^5*d^4*x^5 + 3*(20*a + b)*c^4*d^4*x^4 + 20*(6*a + b)*c^3*d^4*x^
3 + 6*(20*a + 11*b)*c^2*d^4*x^2 + 60*(a + 3*b)*c*d^4*x + 6*b*d^4*log(c*x +
1) + 186*b*d^4*log(c*x - 1) + 6*(b*c^5*d^4*x^5 + 5*b*c^4*d^4*x^4 + 10*b*c^3
*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [A] time = 5.00716, size = 226, normalized size = 2.11

$$\begin{cases} \frac{ac^4d^4x^5}{5} + ac^3d^4x^4 + 2ac^2d^4x^3 + 2acd^4x^2 + ad^4x + \frac{bc^4d^4x^5\operatorname{atanh}(cx)}{5} + bc^3d^4x^4\operatorname{atanh}(cx) + \frac{bc^3d^4x^4}{20} + 2bc^2d^4x^3\operatorname{atanh}(cx) \\ ad^4x \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**4*x**5/5 + a*c**3*d**4*x**4 + 2*a*c**2*d**4*x**3 + 2*a
*c*d**4*x**2 + a*d**4*x + b*c**4*d**4*x**5*atanh(c*x)/5 + b*c**3*d**4*x**4*
atanh(c*x) + b*c**3*d**4*x**4/20 + 2*b*c**2*d**4*x**3*atanh(c*x) + b*c**2*d
**4*x**3/3 + 2*b*c*d**4*x**2*atanh(c*x) + 11*b*c*d**4*x**2/10 + b*d**4*x*at
anh(c*x) + 3*b*d**4*x + 16*b*d**4*log(x - 1/c)/(5*c) + b*d**4*atanh(c*x)/(5
*c), Ne(c, 0)), (a*d**4*x, True))
```

Giac [B] time = 1.28042, size = 261, normalized size = 2.44

$$\frac{1}{5}\,ac^4d^4x^5 + \frac{1}{20}\left(20\,ac^3d^4 + bc^3d^4\right)x^4 + \frac{bd^4\log(cx + 1)}{10\,c} + \frac{31\,bd^4\log(cx - 1)}{10\,c} + \frac{1}{3}\left(6\,ac^2d^4 + bc^2d^4\right)x^3 + \frac{1}{10}\left(20\,acd^4 + 11\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/5*a*c^4*d^4*x^5 + 1/20*(20*a*c^3*d^4 + b*c^3*d^4)*x^4 + 1/10*b*d^4*log(c*
x + 1)/c + 31/10*b*d^4*log(c*x - 1)/c + 1/3*(6*a*c^2*d^4 + b*c^2*d^4)*x^3 +
 1/10*(20*a*c*d^4 + 11*b*c*d^4)*x^2 + (a*d^4 + 3*b*d^4)*x + 1/10*(b*c^4*d^4
*x^5 + 5*b*c^3*d^4*x^4 + 10*b*c^2*d^4*x^3 + 10*b*c*d^4*x^2 + 5*b*d^4*x)*log
(-(c*x + 1)/(c*x - 1))
```

3.35 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=185

$$-\frac{1}{2}bd^4\text{PolyLog}(2,-cx)+\frac{1}{2}bd^4\text{PolyLog}(2,cx)+\frac{1}{4}c^4d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+3c^2d^4x^2$$

```
[Out] 4*a*c*d^4*x + (13*b*c*d^4*x)/4 + (2*b*c^2*d^4*x^2)/3 + (b*c^3*d^4*x^3)/12 -
 (13*b*d^4*ArcTanh[c*x])/4 + 4*b*c*d^4*x*ArcTanh[c*x] + 3*c^2*d^4*x^2*(a +
b*ArcTanh[c*x]) + (4*c^3*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + (c^4*d^4*x^4*(a
+ b*ArcTanh[c*x]))/4 + a*d^4*Log[x] + (8*b*d^4*Log[1 - c^2*x^2])/3 - (b*d^4
*PolyLog[2, -(c*x)])/2 + (b*d^4*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.197096, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5910, 260, 5912, 5916, 321, 206, 266, 43, 302}

$$-\frac{1}{2}bd^4\text{PolyLog}(2,-cx)+\frac{1}{2}bd^4\text{PolyLog}(2,cx)+\frac{1}{4}c^4d^4x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+3c^2d^4x^2$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] 4*a*c*d^4*x + (13*b*c*d^4*x)/4 + (2*b*c^2*d^4*x^2)/3 + (b*c^3*d^4*x^3)/12 -
 (13*b*d^4*ArcTanh[c*x])/4 + 4*b*c*d^4*x*ArcTanh[c*x] + 3*c^2*d^4*x^2*(a +
b*ArcTanh[c*x]) + (4*c^3*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + (c^4*d^4*x^4*(a
+ b*ArcTanh[c*x]))/4 + a*d^4*Log[x] + (8*b*d^4*Log[1 - c^2*x^2])/3 - (b*d^4
*PolyLog[2, -(c*x)])/2 + (b*d^4*PolyLog[2, c*x])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x}\,dx = \int \left(4cd^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+6c^2d^4x\left(a+b\tanh^{-1}(cx)\right)\right.$$

$$= d^4\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(4cd^4\right)\int \left(a+b\tanh^{-1}(cx)\right)dx+\left(6c^2d^4\right)\int x\left(a+b\right.$$

$$= 4acd^4x+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}c^4d^4x^4\left(a\right.$$

$$= 4acd^4x+3bcd^4x+4bcd^4x\tanh^{-1}(cx)+3c^2d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{4}{3}c^3d^4x^3\left(a\right.$$

$$= 4acd^4x+\frac{13}{4}bcd^4x+\frac{1}{12}bc^3d^4x^3-3bd^4\tanh^{-1}(cx)+4bcd^4x\tanh^{-1}(cx)+3c^2d^4x^2$$

$$= 4acd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}bc^2d^4x^2+\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\tanh^{-1}(cx)+4bcd^4x\tanh^{-1}$$

Mathematica [A] time = 0.170134, size = 179, normalized size = 0.97

$$\frac{1}{24}d^4\left(-12b\text{PolyLog}(2,-cx)+12b\text{PolyLog}(2,cx)+6ac^4x^4+32ac^3x^3+72ac^2x^2+96acx+24a\log(x)+2bc^3x^3+16\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x,x]
```

```
[Out] (d^4*(96*a*c*x + 78*b*c*x + 72*a*c^2*x^2 + 16*b*c^2*x^2 + 32*a*c^3*x^3 + 2*
b*c^3*x^3 + 6*a*c^4*x^4 + 96*b*c*x*ArcTanh[c*x] + 72*b*c^2*x^2*ArcTanh[c*x]
 + 32*b*c^3*x^3*ArcTanh[c*x] + 6*b*c^4*x^4*ArcTanh[c*x] + 24*a*Log[x] + 39*
b*Log[1 - c*x] - 39*b*Log[1 + c*x] + 48*b*Log[1 - c^2*x^2] + 16*b*Log[-1 +
c^2*x^2] - 12*b*PolyLog[2, -(c*x)] + 12*b*PolyLog[2, c*x]))/24
```

Maple [A] time = 0.044, size = 222, normalized size = 1.2

$$\frac{d^4ac^4x^4}{4}+\frac{4\,d^4ac^3x^3}{3}+3\,d^4ac^2x^2+4\,acd^4x+d^4a\ln(cx)+\frac{d^4b\text{Artanh}\,(cx)\,c^4x^4}{4}+\frac{4\,d^4b\text{Artanh}\,(cx)\,c^3x^3}{3}+3\,d^4b\text{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x)
```

```
[Out] 1/4*d^4*a*c^4*x^4+4/3*d^4*a*c^3*x^3+3*d^4*a*c^2*x^2+4*a*c*d^4*x+d^4*a*ln(c*
x)+1/4*d^4*b*arctanh(c*x)*c^4*x^4+4/3*d^4*b*arctanh(c*x)*c^3*x^3+3*d^4*b*ar
ctanh(c*x)*c^2*x^2+4*b*c*d^4*x*arctanh(c*x)+d^4*b*arctanh(c*x)*ln(c*x)-1/2*
d^4*b*dilog(c*x)-1/2*d^4*b*dilog(c*x+1)-1/2*d^4*b*ln(c*x)*ln(c*x+1)+1/12*b*
c^3*d^4*x^3+2/3*b*c^2*d^4*x^2+13/4*b*c*d^4*x+103/24*d^4*b*ln(c*x-1)+25/24*d
^4*b*ln(c*x+1)
```

Maxima [A] time = 1.46915, size = 373, normalized size = 2.02

$$\frac{1}{4}\,ac^4d^4x^4+\frac{4}{3}\,ac^3d^4x^3+\frac{1}{12}\,bc^3d^4x^3+3\,ac^2d^4x^2+\frac{2}{3}\,bc^2d^4x^2+4\,acd^4x+\frac{13}{4}\,bcd^4x+2\left(2\,cx\,\text{artanh}\,(cx)+\log\left(-c^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^4*x^4 + 4/3*a*c^3*d^4*x^3 + 1/12*b*c^3*d^4*x^3 + 3*a*c^2*d^4*x^
2 + 2/3*b*c^2*d^4*x^2 + 4*a*c*d^4*x + 13/4*b*c*d^4*x + 2*(2*c*x*arctanh(c*x
) + log(-c^2*x^2 + 1))*b*d^4 - 1/2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1
))*b*d^4 + 1/2*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*d^4 - 23/24*b*d^
4*log(c*x + 1) + 55/24*b*d^4*log(c*x - 1) + a*d^4*log(x) + 1/24*(3*b*c^4*d^
4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(c*x + 1) - 1/24*(3*b*c^4*d
^4*x^4 + 16*b*c^3*d^4*x^3 + 36*b*c^2*d^4*x^2)*log(-c*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^4x^4+4\,ac^3d^4x^3+6\,ac^2d^4x^2+4\,acd^4x+ad^4+\left(bc^4d^4x^4+4\,bc^3d^4x^3+6\,bc^2d^4x^2+4\,bcd^4x+bd^4\right)\text{artan}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int 4ac\,dx + \int \frac{a}{x}\,dx + \int 6ac^2x\,dx + \int 4ac^3x^2\,dx + \int ac^4x^3\,dx + \int 4bc\operatorname{atanh}(cx)\,dx + \int \frac{b\operatorname{atanh}(cx)}{x}\,dx + \int 6bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x,x)
```

```
[Out] d**4*(Integral(4*a*c, x) + Integral(a/x, x) + Integral(6*a*c**2*x, x) + Int
egral(4*a*c**3*x**2, x) + Integral(a*c**4*x**3, x) + Integral(4*b*c*atanh(c
*x), x) + Integral(b*atanh(c*x)/x, x) + Integral(6*b*c**2*x*atanh(c*x), x)
+ Integral(4*b*c**3*x**2*atanh(c*x), x) + Integral(b*c**4*x**3*atanh(c*x),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^4(b\operatorname{artanh}(cx)+a)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x, x)
```

3.36 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=178

$$-2bcd^4\text{PolyLog}(2,-cx)+2bcd^4\text{PolyLog}(2,cx)+\frac{1}{3}c^4d^4x^3\left(a+b\tanh^{-1}(cx)\right)+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+\right.}{}$$

```
[Out] 6*a*c^2*d^4*x + 2*b*c^2*d^4*x + (b*c^3*d^4*x^2)/6 - 2*b*c*d^4*ArcTanh[c*x]
+ 6*b*c^2*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/x + 2*c^3*d^4*x^2
*(a + b*ArcTanh[c*x]) + (c^4*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + 4*a*c*d^4*Lo
g[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 - c^2*x^2])/3 - 2*b*c*d^4*PolyLog[
2, -(c*x)] + 2*b*c*d^4*PolyLog[2, c*x]
```

Rubi [A] time = 0.199811, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5940, 5910, 260, 5916, 266, 36, 29, 31, 5912, 321, 206, 43}

$$-2bcd^4\text{PolyLog}(2,-cx)+2bcd^4\text{PolyLog}(2,cx)+\frac{1}{3}c^4d^4x^3\left(a+b\tanh^{-1}(cx)\right)+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```



```
[Out] 6*a*c^2*d^4*x + 2*b*c^2*d^4*x + (b*c^3*d^4*x^2)/6 - 2*b*c*d^4*ArcTanh[c*x]
+ 6*b*c^2*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/x + 2*c^3*d^4*x^2
*(a + b*ArcTanh[c*x]) + (c^4*d^4*x^3*(a + b*ArcTanh[c*x]))/3 + 4*a*c*d^4*Lo
g[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 - c^2*x^2])/3 - 2*b*c*d^4*PolyLog[
2, -(c*x)] + 2*b*c*d^4*PolyLog[2, c*x]
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}\,dx = \int\left(6c^2d^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}+\frac{4cd^4\left(a+b\tanh^{-1}(cx\right.}{x}\right.$$

$$= d^4\int\frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(4cd^4\right)\int\frac{a+b\tanh^{-1}(cx)}{x}\,dx+\left(6c^2d^4\right)\int\left(a+b\right.$$

$$= 6ac^2d^4x-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+2c^3d^4x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{3}c^4d^4x^3\left(a+\right.$$

$$= 6ac^2d^4x+2bc^2d^4x+6bc^2d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x}+2c^3d^4x^2\left(a\right.$$

$$= 6ac^2d^4x+2bc^2d^4x-2bcd^4\tanh^{-1}(cx)+6bc^2d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh\right.}{x}$$

$$= 6ac^2d^4x+2bc^2d^4x+\frac{1}{6}bc^3d^4x^2-2bcd^4\tanh^{-1}(cx)+6bc^2d^4x\tanh^{-1}(cx)-\frac{d^4}{}$$

Mathematica [A] time = 0.175252, size = 194, normalized size = 1.09

$$d^4\left(-12bcx\text{PolyLog}(2,-cx)+12bcx\text{PolyLog}(2,cx)+2ac^4x^4+12ac^3x^3+36ac^2x^2+24acx\log(x)-6a+bc^3x^3+12b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^2,x]
```

```
[Out] (d^4*(-6*a + 36*a*c^2*x^2 + 12*b*c^2*x^2 + 12*a*c^3*x^3 + b*c^3*x^3 + 2*a*c
^4*x^4 - 6*b*ArcTanh[c*x] + 36*b*c^2*x^2*ArcTanh[c*x] + 12*b*c^3*x^3*ArcTan
h[c*x] + 2*b*c^4*x^4*ArcTanh[c*x] + 24*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 6*
b*c*x*Log[1 - c*x] - 6*b*c*x*Log[1 + c*x] + 15*b*c*x*Log[1 - c^2*x^2] + b*c
*x*Log[-1 + c^2*x^2] - 12*b*c*x*PolyLog[2, -(c*x)] + 12*b*c*x*PolyLog[2, c*
x]))/(6*x)
```

Maple [A] time = 0.049, size = 229, normalized size = 1.3

$$\frac{d^4ac^4x^3}{3}+2\,d^4ac^3x^2+6\,ac^2d^4x-\frac{d^4a}{x}+4\,cd^4a\ln(cx)+\frac{d^4b\text{Artanh}(cx)\,c^4x^3}{3}+2\,d^4b\text{Artanh}(cx)\,c^3x^2+6\,bc^2d^4x\text{Art}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x)
```

```
[Out] 1/3*d^4*a*c^4*x^3+2*d^4*a*c^3*x^2+6*a*c^2*d^4*x-d^4*a/x+4*c*d^4*a*ln(c*x)+1
/3*d^4*b*arctanh(c*x)*c^4*x^3+2*d^4*b*arctanh(c*x)*c^3*x^2+6*b*c^2*d^4*x*ar
ctanh(c*x)-d^4*b*arctanh(c*x)/x+4*c*d^4*b*arctanh(c*x)*ln(c*x)-2*c*d^4*b*di
log(c*x)-2*c*d^4*b*dilog(c*x+1)-2*c*d^4*b*ln(c*x)*ln(c*x+1)+1/6*b*c^3*d^4*x
^2+2*b*c^2*d^4*x+11/3*c*d^4*b*ln(c*x-1)+c*d^4*b*ln(c*x)+5/3*c*d^4*b*ln(c*x+
1)
```

Maxima [A] time = 1.46349, size = 379, normalized size = 2.13

$$\frac{1}{3}\,ac^4d^4x^3+2\,ac^3d^4x^2+\frac{1}{6}\,bc^3d^4x^2+6\,ac^2d^4x+2\,bc^2d^4x+3\left(2\,cx\,\text{artanh}(cx)+\log\left(-c^2x^2+1\right)\right)bcd^4-2\left(\log(cx)\,\text{lo}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^4*x^3 + 2*a*c^3*d^4*x^2 + 1/6*b*c^3*d^4*x^2 + 6*a*c^2*d^4*x + 2
*b*c^2*d^4*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c*d^4 - 2*(log(
c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c*d^4 + 2*(log(c*x + 1)*log(-c*x) +
 dilog(c*x + 1))*b*c*d^4 - 5/6*b*c*d^4*log(c*x + 1) + 7/6*b*c*d^4*log(c*x -
 1) + 4*a*c*d^4*log(x) - 1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c
*x)/x)*b*d^4 - a*d^4/x + 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(c*x + 1)
 - 1/6*(b*c^4*d^4*x^3 + 6*b*c^3*d^4*x^2)*log(-c*x + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right)\text{artanh}\,(}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int 6ac^2\,dx + \int \frac{a}{x^2}\,dx + \int \frac{4ac}{x}\,dx + \int 4ac^3x\,dx + \int ac^4x^2\,dx + \int 6bc^2\,\text{atanh}\,(cx)\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^2}\,dx + \int \frac{4b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**2,x)
```

```
[Out] d**4*(Integral(6*a*c**2, x) + Integral(a/x**2, x) + Integral(4*a*c/x, x) +
Integral(4*a*c**3*x, x) + Integral(a*c**4*x**2, x) + Integral(6*b*c**2*atan
h(c*x), x) + Integral(b*atanh(c*x)/x**2, x) + Integral(4*b*c*atanh(c*x)/x,
x) + Integral(4*b*c**3*x*atanh(c*x), x) + Integral(b*c**4*x**2*atanh(c*x),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^4 (b\,\text{artanh}\,(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^2, x)
```

3.37 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=156

$$-3bc^2d^4\text{PolyLog}(2,-cx)+3bc^2d^4\text{PolyLog}(2,cx)+\frac{1}{2}c^4d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+\right.}{}$$

```
[Out] -(b*c*d^4)/(2*x) + 4*a*c^3*d^4*x + (b*c^3*d^4*x)/2 + 4*b*c^3*d^4*x*ArcTanh[
c*x] - (d^4*(a + b*ArcTanh[c*x]))/(2*x^2) - (4*c*d^4*(a + b*ArcTanh[c*x]))/
x + (c^4*d^4*x^2*(a + b*ArcTanh[c*x]))/2 + 6*a*c^2*d^4*Log[x] + 4*b*c^2*d^4
*Log[x] - 3*b*c^2*d^4*PolyLog[2, -(c*x)] + 3*b*c^2*d^4*PolyLog[2, c*x]
```

Rubi [A] time = 0.189288, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5940, 5910, 260, 5916, 325, 206, 266, 36, 29, 31, 5912, 321}

$$-3bc^2d^4\text{PolyLog}(2,-cx)+3bc^2d^4\text{PolyLog}(2,cx)+\frac{1}{2}c^4d^4x^2\left(a+b\tanh^{-1}(cx)\right)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^4)/(2*x) + 4*a*c^3*d^4*x + (b*c^3*d^4*x)/2 + 4*b*c^3*d^4*x*ArcTanh[
c*x] - (d^4*(a + b*ArcTanh[c*x]))/(2*x^2) - (4*c*d^4*(a + b*ArcTanh[c*x]))/
x + (c^4*d^4*x^2*(a + b*ArcTanh[c*x]))/2 + 6*a*c^2*d^4*Log[x] + 4*b*c^2*d^4
*Log[x] - 3*b*c^2*d^4*PolyLog[2, -(c*x)] + 3*b*c^2*d^4*PolyLog[2, c*x]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}\,dx = \int\left(4c^3d^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}+\frac{4cd^4\left(a+b\tanh^{-1}(cx\right.}{x^2}\right.$$

$$= d^4\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx+\left(4cd^4\right)\int\frac{a+b\tanh^{-1}(cx)}{x^2}\,dx+\left(6c^2d^4\right)\int\frac{a+b\,\mathrm{t}}{}$$

$$= 4ac^3d^4x-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{1}{2}c^4d^4x^2\left(a+b\,\mathrm{t}\right.$$

$$= -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-$$

$$= -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-$$

$$= -\frac{bcd^4}{2x}+4ac^3d^4x+\frac{1}{2}bc^3d^4x+4bc^3d^4x\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{2x^2}-$$

Mathematica [A] time = 0.166052, size = 143, normalized size = 0.92

$$\frac{d^4\left(-6bc^2x^2\mathrm{PolyLog}(2,-cx)+6bc^2x^2\mathrm{PolyLog}(2,cx)+ac^4x^4+8ac^3x^3+12ac^2x^2\log(x)-8acx-a+bc^3x^3+8bc^2x^2\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^3,x]
```

```
[Out] (d^4*(-a - 8*a*c*x - b*c*x + 8*a*c^3*x^3 + b*c^3*x^3 + a*c^4*x^4 - b*ArcTan
h[c*x] - 8*b*c*x*ArcTanh[c*x] + 8*b*c^3*x^3*ArcTanh[c*x] + b*c^4*x^4*ArcTan
h[c*x] + 12*a*c^2*x^2*Log[x] + 8*b*c^2*x^2*Log[c*x] - 6*b*c^2*x^2*PolyLog[2
, -(c*x)] + 6*b*c^2*x^2*PolyLog[2, c*x]))/(2*x^2)
```

Maple [A] time = 0.045, size = 210, normalized size = 1.4

$$\frac{c^4d^4ax^2}{2}+4\,ac^3d^4x-4\,\frac{cad^4}{x}+6\,c^2d^4a\ln(cx)-\frac{d^4a}{2\,x^2}+\frac{c^4d^4b\mathrm{Artanh}(cx)\,x^2}{2}+4\,bc^3d^4x\mathrm{Artanh}(cx)-4\,\frac{d^4bc\mathrm{Artanh}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x)
```

```
[Out] 1/2*c^4*d^4*a*x^2+4*a*c^3*d^4*x-4*c*d^4*a/x+6*c^2*d^4*a*ln(c*x)-1/2*d^4*a/x
^2+1/2*c^4*d^4*b*arctanh(c*x)*x^2+4*b*c^3*d^4*x*arctanh(c*x)-4*c*d^4*b*arct
anh(c*x)/x+6*c^2*d^4*b*arctanh(c*x)*ln(c*x)-1/2*d^4*b*arctanh(c*x)/x^2-3*c^
2*d^4*b*dilog(c*x)-3*c^2*d^4*b*dilog(c*x+1)-3*c^2*d^4*b*ln(c*x)*ln(c*x+1)+1
/2*b*c^3*d^4*x-1/2*b*c*d^4/x+4*c^2*d^4*b*ln(c*x)
```

Maxima [B] time = 1.44967, size = 396, normalized size = 2.54

$$\frac{1}{4}\,bc^4d^4x^2\log(cx+1)-\frac{1}{4}\,bc^4d^4x^2\log(-cx+1)+\frac{1}{2}\,ac^4d^4x^2+4\,ac^3d^4x+\frac{1}{2}\,bc^3d^4x+2\left(2\,cx\,\mathrm{artanh}(cx)+\log\left(-c^2x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*c^4*d^4*x^2*log(c*x + 1) - 1/4*b*c^4*d^4*x^2*log(-c*x + 1) + 1/2*a*c^
4*d^4*x^2 + 4*a*c^3*d^4*x + 1/2*b*c^3*d^4*x + 2*(2*c*x*arctanh(c*x) + log(-
c^2*x^2 + 1))*b*c^2*d^4 - 3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b*c^
2*d^4 + 3*(log(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b*c^2*d^4 - 1/4*b*c^2*d
^4*log(c*x + 1) + 1/4*b*c^2*d^4*log(c*x - 1) + 6*a*c^2*d^4*log(x) - 2*(c*(l
og(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c*d^4 + 1/4*((c*log(c*x +
 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d^4 - 4*a*c*d^4/x - 1
/2*a*d^4/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right)\text{artanh}(}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int 4ac^3\,dx + \int \frac{a}{x^3}\,dx + \int \frac{4ac}{x^2}\,dx + \int \frac{6ac^2}{x}\,dx + \int ac^4x\,dx + \int 4bc^3\,\text{atanh}\,(cx)\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^3}\,dx + \int \frac{4bc}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**3,x)
```

```
[Out] d**4*(Integral(4*a*c**3, x) + Integral(a/x**3, x) + Integral(4*a*c/x**2, x)
 + Integral(6*a*c**2/x, x) + Integral(a*c**4*x, x) + Integral(4*b*c**3*atan
h(c*x), x) + Integral(b*atanh(c*x)/x**3, x) + Integral(4*b*c*atanh(c*x)/x**
2, x) + Integral(6*b*c**2*atanh(c*x)/x, x) + Integral(b*c**4*x*atanh(c*x),
x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^4(b\,\text{artanh}\,(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^3, x)
```

3.38 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=189

$$-2bc^3d^4\text{PolyLog}(2,-cx)+2bc^3d^4\text{PolyLog}(2,cx)-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{3\ldots}$$

```
[Out] -(b*c*d^4)/(6*x^2) - (2*b*c^2*d^4)/x + a*c^4*d^4*x + 2*b*c^3*d^4*ArcTanh[c*
x] + b*c^4*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/(3*x^3) - (2*c*d
^4*(a + b*ArcTanh[c*x]))/x^2 - (6*c^2*d^4*(a + b*ArcTanh[c*x]))/x + 4*a*c^3
*d^4*Log[x] + (19*b*c^3*d^4*Log[x])/3 - (8*b*c^3*d^4*Log[1 - c^2*x^2])/3 -
2*b*c^3*d^4*PolyLog[2, -(c*x)] + 2*b*c^3*d^4*PolyLog[2, c*x]
```

Rubi [A] time = 0.216642, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5940, 5910, 260, 5916, 266, 44, 325, 206, 36, 29, 31, 5912}

$$-2bc^3d^4\text{PolyLog}(2,-cx)+2bc^3d^4\text{PolyLog}(2,cx)-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```


```
[Out] -(b*c*d^4)/(6*x^2) - (2*b*c^2*d^4)/x + a*c^4*d^4*x + 2*b*c^3*d^4*ArcTanh[c*
x] + b*c^4*d^4*x*ArcTanh[c*x] - (d^4*(a + b*ArcTanh[c*x]))/(3*x^3) - (2*c*d
^4*(a + b*ArcTanh[c*x]))/x^2 - (6*c^2*d^4*(a + b*ArcTanh[c*x]))/x + 4*a*c^3
*d^4*Log[x] + (19*b*c^3*d^4*Log[x])/3 - (8*b*c^3*d^4*Log[1 - c^2*x^2])/3 -
2*b*c^3*d^4*PolyLog[2, -(c*x)] + 2*b*c^3*d^4*PolyLog[2, c*x]
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(d+cdx)^4 \left(a+b\tanh^{-1}(cx)\right)}{x^4} dx = \int \left(c^4d^4 \left(a+b\tanh^{-1}(cx)\right) + \frac{d^4 \left(a+b\tanh^{-1}(cx)\right)}{x^4} + \frac{4cd^4 \left(a+b\tanh^{-1}(cx)\right)}{x^3}\right.$$

$$= d^4 \int \frac{a+b\tanh^{-1}(cx)}{x^4} dx + \left(4cd^4\right) \int \frac{a+b\tanh^{-1}(cx)}{x^3} dx + \left(6c^2d^4\right) \int \frac{a+b\,\mathrm{t}}{}$$

$$= ac^4d^4x - \frac{d^4 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{2cd^4 \left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{6c^2d^4 \left(a+b\tanh\right.}{x}$$

$$= -\frac{2bc^2d^4}{x} + ac^4d^4x + bc^4d^4x\tanh^{-1}(cx) - \frac{d^4 \left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{2cd^4 \left(a+b\,\mathrm{t}\right.}{x}$$

$$= -\frac{2bc^2d^4}{x} + ac^4d^4x + 2bc^3d^4\tanh^{-1}(cx) + bc^4d^4x\tanh^{-1}(cx) - \frac{d^4 \left(a+b\tanh\right.}{3x^3}$$

$$= -\frac{bcd^4}{6x^2} - \frac{2bc^2d^4}{x} + ac^4d^4x + 2bc^3d^4\tanh^{-1}(cx) + bc^4d^4x\tanh^{-1}(cx) - \frac{d^4 \left(a+\right.}{}$$

Mathematica [A] time = 0.174583, size = 197, normalized size = 1.04

$$\frac{d^4 \left(-12bc^3x^3\mathrm{PolyLog}(2,-cx) + 12bc^3x^3\mathrm{PolyLog}(2,cx) + 6ac^4x^4 - 36ac^2x^2 + 24ac^3x^3\log(x) - 12acx - 2a - 12bc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^4,x]
```

```
[Out] (d^4*(-2*a - 12*a*c*x - b*c*x - 36*a*c^2*x^2 - 12*b*c^2*x^2 + 6*a*c^4*x^4 -
 2*b*ArcTanh[c*x] - 12*b*c*x*ArcTanh[c*x] - 36*b*c^2*x^2*ArcTanh[c*x] + 6*b
*c^4*x^4*ArcTanh[c*x] + 24*a*c^3*x^3*Log[x] + 38*b*c^3*x^3*Log[c*x] - 6*b*c
^3*x^3*Log[1 - c*x] + 6*b*c^3*x^3*Log[1 + c*x] - 16*b*c^3*x^3*Log[1 - c^2*x
^2] - 12*b*c^3*x^3*PolyLog[2, -(c*x)] + 12*b*c^3*x^3*PolyLog[2, c*x]))/(6*x
^3)
```

Maple [A] time = 0.049, size = 240, normalized size = 1.3

$$ac^4d^4x - 6\frac{c^2d^4a}{x} + 4c^3d^4a\ln(cx) - 2\frac{cad^4}{x^2} - \frac{d^4a}{3x^3} + bc^4d^4x\mathrm{Artanh}(cx) - 6\frac{c^2d^4b\mathrm{Artanh}(cx)}{x} + 4c^3d^4b\mathrm{Artanh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x)
```

```
[Out] a*c^4*d^4*x-6*c^2*d^4*a/x+4*c^3*d^4*a*ln(c*x)-2*c*d^4*a/x^2-1/3*d^4*a/x^3+b
*c^4*d^4*x*arctanh(c*x)-6*c^2*d^4*b*arctanh(c*x)/x+4*c^3*d^4*b*arctanh(c*x)
*ln(c*x)-2*c*d^4*b*arctanh(c*x)/x^2-1/3*d^4*b*arctanh(c*x)/x^3-2*c^3*d^4*b*
dilog(c*x)-2*c^3*d^4*b*dilog(c*x+1)-2*c^3*d^4*b*ln(c*x)*ln(c*x+1)-11/3*c^3*
d^4*b*ln(c*x-1)-1/6*b*c*d^4/x^2-2*b*c^2*d^4/x+19/3*c^3*d^4*b*ln(c*x)-5/3*c^
3*d^4*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$ac^4d^4x + \frac{1}{2}\left(2cx\,\mathrm{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)bc^3d^4 + 2bc^3d^4 \int \frac{\log(cx+1) - \log(-cx+1)}{x} dx + 4ac^3d^4\log(x) - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^4*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b*c^3*d^4 + 2*b*
c^3*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + 4*a*c^3*d^4*log(x)
 - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^2*d^4 + ((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c*d^4 - 1/6*(
(c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*d^4
 - 6*a*c^2*d^4/x - 2*a*c*d^4/x^2 - 1/3*a*d^4/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^4x^4 + 4\,ac^3d^4x^3 + 6\,ac^2d^4x^2 + 4\,acd^4x + ad^4 + \left(bc^4d^4x^4 + 4\,bc^3d^4x^3 + 6\,bc^2d^4x^2 + 4\,bcd^4x + bd^4\right)\text{artanh}\left(\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int ac^4\,dx + \int \frac{a}{x^4}\,dx + \int \frac{4ac}{x^3}\,dx + \int \frac{6ac^2}{x^2}\,dx + \int \frac{4ac^3}{x}\,dx + \int bc^4\,\text{atanh}\,(cx)\,dx + \int \frac{b\,\text{atanh}\,(cx)}{x^4}\,dx + \int \frac{4bc\,\text{at}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**4,x)
```

```
[Out] d**4*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(4*a*c/x**3, x) +
 Integral(6*a*c**2/x**2, x) + Integral(4*a*c**3/x, x) + Integral(b*c**4*ata
nh(c*x), x) + Integral(b*atanh(c*x)/x**4, x) + Integral(4*b*c*atanh(c*x)/x*
*3, x) + Integral(6*b*c**2*atanh(c*x)/x**2, x) + Integral(4*b*c**3*atanh(c*
x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^4 (b\,\text{artanh}\,(cx) + a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^4, x)
```

3.39 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=209

$$-\frac{1}{2}bc^4d^4\text{PolyLog}(2,-cx)+\frac{1}{2}bc^4d^4\text{PolyLog}(2,cx)-\frac{3c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{4c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{4cd^4\left(a\right.}{}$$

```
[Out] -(b*c*d^4)/(12*x^3) - (2*b*c^2*d^4)/(3*x^2) - (13*b*c^3*d^4)/(4*x) + (13*b*
c^4*d^4*ArcTanh[c*x])/4 - (d^4*(a + b*ArcTanh[c*x]))/(4*x^4) - (4*c*d^4*(a
+ b*ArcTanh[c*x]))/(3*x^3) - (3*c^2*d^4*(a + b*ArcTanh[c*x]))/x^2 - (4*c^3*
d^4*(a + b*ArcTanh[c*x]))/x + a*c^4*d^4*Log[x] + (16*b*c^4*d^4*Log[x])/3 -
(8*b*c^4*d^4*Log[1 - c^2*x^2])/3 - (b*c^4*d^4*PolyLog[2, -(c*x)])/2 + (b*c^
4*d^4*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.229174, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5916, 325, 206, 266, 44, 36, 29, 31, 5912}

$$-\frac{1}{2}bc^4d^4\text{PolyLog}(2,-cx)+\frac{1}{2}bc^4d^4\text{PolyLog}(2,cx)-\frac{3c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-\frac{4c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x}-\frac{4cd^4\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```


```
[Out] -(b*c*d^4)/(12*x^3) - (2*b*c^2*d^4)/(3*x^2) - (13*b*c^3*d^4)/(4*x) + (13*b*
c^4*d^4*ArcTanh[c*x])/4 - (d^4*(a + b*ArcTanh[c*x]))/(4*x^4) - (4*c*d^4*(a
+ b*ArcTanh[c*x]))/(3*x^3) - (3*c^2*d^4*(a + b*ArcTanh[c*x]))/x^2 - (4*c^3*
d^4*(a + b*ArcTanh[c*x]))/x + a*c^4*d^4*Log[x] + (16*b*c^4*d^4*Log[x])/3 -
(8*b*c^4*d^4*Log[1 - c^2*x^2])/3 - (b*c^4*d^4*PolyLog[2, -(c*x)])/2 + (b*c^
4*d^4*PolyLog[2, c*x])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}\,dx &= \int \left(\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{x^5}+\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}+\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^3}\right. \\
&= d^4\int \frac{a+b\tanh^{-1}(cx)}{x^5}\,dx+\left(4cd^4\right)\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx+\left(6c^2d^4\right)\int \frac{a+b\tan}{x^3} \\
&= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{x^2}-4 \\
&= -\frac{bcd^4}{12x^3}-\frac{3bc^3d^4}{x}-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{3c^2d^4\left(a+\right.}{} \\
&= -\frac{bcd^4}{12x^3}-\frac{13bc^3d^4}{4x}+3bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4\left(a+b\tanh\right.}{3x^3} \\
&= -\frac{bcd^4}{12x^3}-\frac{2bc^2d^4}{3x^2}-\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\tanh^{-1}(cx)-\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{4x^4}-\frac{4cd^4}{}
\end{aligned}
$$

Mathematica [A] time = 0.170063, size = 206, normalized size = 0.99

$$d^4\left(-12bc^4x^4\text{PolyLog}(2,-cx)+12bc^4x^4\text{PolyLog}(2,cx)-96ac^3x^3-72ac^2x^2+24ac^4x^4\log(x)-32acx-6a-78bc^3x\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^5,x]
```

```
[Out] (d^4*(-6*a - 32*a*c*x - 2*b*c*x - 72*a*c^2*x^2 - 16*b*c^2*x^2 - 96*a*c^3*x^
3 - 78*b*c^3*x^3 - 6*b*ArcTanh[c*x] - 32*b*c*x*ArcTanh[c*x] - 72*b*c^2*x^2*
ArcTanh[c*x] - 96*b*c^3*x^3*ArcTanh[c*x] + 24*a*c^4*x^4*Log[x] + 128*b*c^4*
x^4*Log[c*x] - 39*b*c^4*x^4*Log[1 - c*x] + 39*b*c^4*x^4*Log[1 + c*x] - 64*b
*c^4*x^4*Log[1 - c^2*x^2] - 12*b*c^4*x^4*PolyLog[2, -(c*x)] + 12*b*c^4*x^4*
PolyLog[2, c*x]))/(24*x^4)
```

Maple [A] time = 0.049, size = 256, normalized size = 1.2

$$-\frac{d^4a}{4x^4}-4\frac{c^3d^4a}{x}+c^4d^4a\ln(cx)-3\frac{c^2d^4a}{x^2}-\frac{4cd^4a}{3x^3}-\frac{d^4b\text{Artanh}(cx)}{4x^4}-4\frac{c^3d^4b\text{Artanh}(cx)}{x}+c^4d^4b\text{Artanh}(cx)\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x)
```

```
[Out] -1/4*d^4*a/x^4-4*c^3*d^4*a/x+c^4*d^4*a*ln(c*x)-3*c^2*d^4*a/x^2-4/3*c*d^4*a/
x^3-1/4*d^4*b*arctanh(c*x)/x^4-4*c^3*d^4*b*arctanh(c*x)/x+c^4*d^4*b*arctanh
(c*x)*ln(c*x)-3*c^2*d^4*b*arctanh(c*x)/x^2-4/3*c*d^4*b*arctanh(c*x)/x^3-103
/24*c^4*d^4*b*ln(c*x-1)-1/12*b*c*d^4/x^3-2/3*b*c^2*d^4/x^2-13/4*b*c^3*d^4/x
+16/3*c^4*d^4*b*ln(c*x)-25/24*c^4*d^4*b*ln(c*x+1)-1/2*c^4*d^4*b*dilog(c*x)-
1/2*c^4*d^4*b*dilog(c*x+1)-1/2*c^4*d^4*b*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}bc^4d^4\int\frac{\log(cx+1)-\log(-cx+1)}{x}\,dx+ac^4d^4\log(x)-2\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}(cx)}{x}\right)bc^3d^4+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*b*c^4*d^4*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*c^4*d^4*lo
g(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^3*d^4 + 3
/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^2*d
^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/
x^3)*b*c*d^4 - 4*a*c^3*d^4/x + 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x -
1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*d^4 - 3*a*c^2*d^4/x^2
 - 4/3*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ac^4d^4x^4+4\,ac^3d^4x^3+6\,ac^2d^4x^2+4\,acd^4x+ad^4+\left(bc^4d^4x^4+4\,bc^3d^4x^3+6\,bc^2d^4x^2+4\,bcd^4x+bd^4\right)\text{artan}}{x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^4*x^4 + 4*a*c^3*d^4*x^3 + 6*a*c^2*d^4*x^2 + 4*a*c*d^4*x +
 a*d^4 + (b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 + 6*b*c^2*d^4*x^2 + 4*b*c*d^4*x +
 b*d^4)*arctanh(c*x))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^4\left(\int \frac{a}{x^5}\,dx + \int \frac{4ac}{x^4}\,dx + \int \frac{6ac^2}{x^3}\,dx + \int \frac{4ac^3}{x^2}\,dx + \int \frac{ac^4}{x}\,dx + \int \frac{b\operatorname{atanh}(cx)}{x^5}\,dx + \int \frac{4bc\operatorname{atanh}(cx)}{x^4}\,dx + \int \frac{6bc^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**5,x)
```

```
[Out] d**4*(Integral(a/x**5, x) + Integral(4*a*c/x**4, x) + Integral(6*a*c**2/x**
3, x) + Integral(4*a*c**3/x**2, x) + Integral(a*c**4/x, x) + Integral(b*ata
nh(c*x)/x**5, x) + Integral(4*b*c*atanh(c*x)/x**4, x) + Integral(6*b*c**2*a
tanh(c*x)/x**3, x) + Integral(4*b*c**3*atanh(c*x)/x**2, x) + Integral(b*c**
4*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^4(b\operatorname{artanh}(cx)+a)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^4*(b*arctanh(c*x) + a)/x^5, x)
```

3.40 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=109

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{11bc^3d^4}{10x^2}-\frac{bc^2d^4}{3x^3}-\frac{3bc^4d^4}{x}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)-\frac{bcd^4}{20x^4}$$

```
[Out] -(b*c*d^4)/(20*x^4) - (b*c^2*d^4)/(3*x^3) - (11*b*c^3*d^4)/(10*x^2) - (3*b*
c^4*d^4)/x - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*x^5) + (16*b*c^5*d^4
*Log[x])/5 - (16*b*c^5*d^4*Log[1 - c*x])/5
```

Rubi [A] time = 0.104959, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {37, 5936, 12, 88}

$$-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{11bc^3d^4}{10x^2}-\frac{bc^2d^4}{3x^3}-\frac{3bc^4d^4}{x}+\frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(1-cx)-\frac{bcd^4}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^4)/(20*x^4) - (b*c^2*d^4)/(3*x^3) - (11*b*c^3*d^4)/(10*x^2) - (3*b*
c^4*d^4)/x - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(5*x^5) + (16*b*c^5*d^4
*Log[x])/5 - (16*b*c^5*d^4*Log[1 - c*x])/5
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - (bc)\int \frac{(d+cdx)^4}{5x^5(-1+cx)}\,dx$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{1}{5}(bc)\int \frac{(d+cdx)^4}{x^5(-1+cx)}\,dx$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - \frac{1}{5}(bc)\int \left(-\frac{d^4}{x^5} - \frac{5cd^4}{x^4} - \frac{11c^2d^4}{x^3} - \frac{15c^3d^4}{x^2} - 16c$$

$$= -\frac{bcd^4}{20x^4} - \frac{bc^2d^4}{3x^3} - \frac{11bc^3d^4}{10x^2} - \frac{3bc^4d^4}{x} - \frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{5x^5} + \frac{16}{5}bc^5d^4 \mathrm{l}$$

Mathematica [A] time = 0.147057, size = 157, normalized size = 1.44

$$-\frac{d^4\left(60ac^4x^4+120ac^3x^3+120ac^2x^2+60acx+12a+180bc^4x^4+66bc^3x^3+20bc^2x^2-192bc^5x^5\log(x)+186bc^5x^5\log(1\right.}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^6,x]
```

```
[Out] -(d^4*(12*a + 60*a*c*x + 3*b*c*x + 120*a*c^2*x^2 + 20*b*c^2*x^2 + 120*a*c^3
*x^3 + 66*b*c^3*x^3 + 60*a*c^4*x^4 + 180*b*c^4*x^4 + 12*b*(1 + 5*c*x + 10*c
^2*x^2 + 10*c^3*x^3 + 5*c^4*x^4)*ArcTanh[c*x] - 192*b*c^5*x^5*Log[x] + 186*
b*c^5*x^5*Log[1 - c*x] + 6*b*c^5*x^5*Log[1 + c*x]))/(60*x^5)
```

Maple [B] time = 0.04, size = 221, normalized size = 2.

$$-\frac{cd^4a}{x^4} - \frac{c^4d^4a}{x} - \frac{d^4a}{5x^5} - 2\frac{c^3d^4a}{x^2} - 2\frac{c^2d^4a}{x^3} - \frac{cd^4b\mathrm{Artanh}(cx)}{x^4} - \frac{c^4d^4b\mathrm{Artanh}(cx)}{x} - \frac{d^4b\mathrm{Artanh}(cx)}{5x^5} - 2\frac{c^3d^4b\mathrm{Artanh}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x)
```

```
[Out] -c*d^4*a/x^4-c^4*d^4*a/x-1/5*d^4*a/x^5-2*c^3*d^4*a/x^2-2*c^2*d^4*a/x^3-c*d^
4*b*arctanh(c*x)/x^4-c^4*d^4*b*arctanh(c*x)/x-1/5*d^4*b*arctanh(c*x)/x^5-2*
c^3*d^4*b*arctanh(c*x)/x^2-2*c^2*d^4*b*arctanh(c*x)/x^3-31/10*c^5*d^4*b*ln(
c*x-1)-1/20*b*c*d^4/x^4-1/3*b*c^2*d^4/x^3-11/10*b*c^3*d^4/x^2-3*b*c^4*d^4/x
+16/5*c^5*d^4*b*ln(c*x)-1/10*c^5*d^4*b*ln(c*x+1)
```

Maxima [B] time = 0.980416, size = 404, normalized size = 3.71

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}(cx)}{x}\right)bc^4d^4+\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}(cx)}{x^2}\right)bc^3d^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*c^4*d^4 + ((c*l
og(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^3*d^4 - ((c
```

```
^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*b*c^2*d
^4 - a*c^4*d^4/x + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2
*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c*d^4 - 1/20*((2*c^4*log(c^2*x^2 -
 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*d^4 -
 2*a*c^3*d^4/x^2 - 2*a*c^2*d^4/x^3 - a*c*d^4/x^4 - 1/5*a*d^4/x^5
```

Fricas [A] time = 2.15719, size = 443, normalized size = 4.06

$$-\frac{6\,bc^5d^4x^5\log\left(cx+1\right)+186\,bc^5d^4x^5\log\left(cx-1\right)-192\,bc^5d^4x^5\log\left(x\right)+60\,(a+3\,b)c^4d^4x^4+6\,(20\,a+11\,b)c^3d^4x^3+}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(6*b*c^5*d^4*x^5*log(c*x + 1) + 186*b*c^5*d^4*x^5*log(c*x - 1) - 192*
b*c^5*d^4*x^5*log(x) + 60*(a + 3*b)*c^4*d^4*x^4 + 6*(20*a + 11*b)*c^3*d^4*x
^3 + 20*(6*a + b)*c^2*d^4*x^2 + 3*(20*a + b)*c*d^4*x + 12*a*d^4 + 6*(5*b*c^
4*d^4*x^4 + 10*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*b*c*d^4*x + b*d^4)*log(
-(c*x + 1)/(c*x - 1)))/x^5
```

Sympy [A] time = 5.92803, size = 253, normalized size = 2.32

$$\begin{cases} -\frac{ac^4d^4}{x}-\frac{2ac^3d^4}{x^2}-\frac{2ac^2d^4}{x^3}-\frac{acd^4}{x^4}-\frac{ad^4}{5x^5}+\frac{16bc^5d^4\log\left(x\right)}{5}-\frac{16bc^5d^4\log\left(x-\frac{1}{c}\right)}{5}-\frac{bc^5d^4\operatorname{atanh}\left(cx\right)}{5}-\frac{bc^4d^4\operatorname{atanh}\left(cx\right)}{x}-\frac{3bc^4d^4}{x}-\frac{2bc^3d^4\operatorname{at}}{x} \\ -\frac{ad^4}{5x^5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**6,x)
```

```
[Out] Piecewise((-a*c**4*d**4/x - 2*a*c**3*d**4/x**2 - 2*a*c**2*d**4/x**3 - a*c*d
**4/x**4 - a*d**4/(5*x**5) + 16*b*c**5*d**4*log(x)/5 - 16*b*c**5*d**4*log(x
 - 1/c)/5 - b*c**5*d**4*atanh(c*x)/5 - b*c**4*d**4*atanh(c*x)/x - 3*b*c**4*
d**4/x - 2*b*c**3*d**4*atanh(c*x)/x**2 - 11*b*c**3*d**4/(10*x**2) - 2*b*c**
2*d**4*atanh(c*x)/x**3 - b*c**2*d**4/(3*x**3) - b*c*d**4*atanh(c*x)/x**4 -
b*c*d**4/(20*x**4) - b*d**4*atanh(c*x)/(5*x**5), Ne(c, 0)), (-a*d**4/(5*x**
5), True))
```

Giac [B] time = 1.34178, size = 286, normalized size = 2.62

$$-\frac{1}{10}\,bc^5d^4\log\left(cx+1\right)-\frac{31}{10}\,bc^5d^4\log\left(cx-1\right)+\frac{16}{5}\,bc^5d^4\log\left(x\right)-\frac{\left(5\,bc^4d^4x^4+10\,bc^3d^4x^3+10\,bc^2d^4x^2+5\,bcd^4x+b\right.}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] -1/10*b*c^5*d^4*log(c*x + 1) - 31/10*b*c^5*d^4*log(c*x - 1) + 16/5*b*c^5*d^
4*log(x) - 1/10*(5*b*c^4*d^4*x^4 + 10*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 + 5*
b*c*d^4*x + b*d^4)*log(-(c*x + 1)/(c*x - 1))/x^5 - 1/60*(60*a*c^4*d^4*x^4 +
```

```
180*b*c^4*d^4*x^4 + 120*a*c^3*d^4*x^3 + 66*b*c^3*d^4*x^3 + 120*a*c^2*d^4*x
^2 + 20*b*c^2*d^4*x^2 + 60*a*c*d^4*x + 3*b*c*d^4*x + 12*a*d^4)/x^5
```

3.41 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx$

Optimal. Leaf size=151

$$\frac{cd^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{16bc^4d^4}{15x^2}-\frac{5bc^3d^4}{9x^3}-\frac{bc^2d^4}{5x^4}-\frac{13bc^5d^4}{6x}+\frac{32}{15}bc^6d^4\,\mathrm{l}$$

```
[Out] -(b*c*d^4)/(30*x^5) - (b*c^2*d^4)/(5*x^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c
^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c
*x]))/(6*x^6) + (c*d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(30*x^5) + (32*b*c
^6*d^4*Log[x])/15 - (32*b*c^6*d^4*Log[1 - c*x])/15
```

Rubi [A] time = 0.125417, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {45, 37, 5936, 12, 148}

$$\frac{cd^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{d^4(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}-\frac{16bc^4d^4}{15x^2}-\frac{5bc^3d^4}{9x^3}-\frac{bc^2d^4}{5x^4}-\frac{13bc^5d^4}{6x}+\frac{32}{15}bc^6d^4\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -(b*c*d^4)/(30*x^5) - (b*c^2*d^4)/(5*x^4) - (5*b*c^3*d^4)/(9*x^3) - (16*b*c
^4*d^4)/(15*x^2) - (13*b*c^5*d^4)/(6*x) - (d^4*(1 + c*x)^5*(a + b*ArcTanh[c
*x]))/(6*x^6) + (c*d^4*(1 + c*x)^5*(a + b*ArcTanh[c*x]))/(30*x^5) + (32*b*c
^6*d^4*Log[x])/15 - (32*b*c^6*d^4*Log[1 - c*x])/15
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^7}\,dx = -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-(bc)\int \frac{(-5+}{3}$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{1}{30}(bc)\int \frac{(-}{}$$

$$= -\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+\frac{cd^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{30x^5}-\frac{1}{30}(bc)\int \Big(-$$

$$= -\frac{bcd^4}{30x^5}-\frac{bc^2d^4}{5x^4}-\frac{5bc^3d^4}{9x^3}-\frac{16bc^4d^4}{15x^2}-\frac{13bc^5d^4}{6x}-\frac{d^4(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)}{6x^6}+$$

Mathematica [A] time = 0.154882, size = 166, normalized size = 1.1

$$-\frac{d^4\left(90ac^4x^4+240ac^3x^3+270ac^2x^2+144acx+30a+390bc^5x^5+192bc^4x^4+100bc^3x^3+36bc^2x^2-384bc^6x^6\log(x)+\right.}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^7,x]
```

```
[Out] -(d^4*(30*a + 144*a*c*x + 6*b*c*x + 270*a*c^2*x^2 + 36*b*c^2*x^2 + 240*a*c^
3*x^3 + 100*b*c^3*x^3 + 90*a*c^4*x^4 + 192*b*c^4*x^4 + 390*b*c^5*x^5 + 6*b*
(5 + 24*c*x + 45*c^2*x^2 + 40*c^3*x^3 + 15*c^4*x^4)*ArcTanh[c*x] - 384*b*c^
6*x^6*Log[x] + 387*b*c^6*x^6*Log[1 - c*x] - 3*b*c^6*x^6*Log[1 + c*x]))/(180
*x^6)
```

Maple [A] time = 0.04, size = 233, normalized size = 1.5

$$-\frac{3c^2d^4a}{2x^4}-\frac{4cd^4a}{5x^5}-\frac{c^4d^4a}{2x^2}-\frac{d^4a}{6x^6}-\frac{4c^3d^4a}{3x^3}-\frac{3c^2d^4b\mathrm{Artanh}(cx)}{2x^4}-\frac{4cd^4b\mathrm{Artanh}(cx)}{5x^5}-\frac{c^4d^4b\mathrm{Artanh}(cx)}{2x^2}-\frac{d^4b\mathrm{Art}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x)
```

```
[Out] -3/2*c^2*d^4*a/x^4-4/5*c*d^4*a/x^5-1/2*c^4*d^4*a/x^2-1/6*d^4*a/x^6-4/3*c^3*
d^4*a/x^3-3/2*c^2*d^4*b*arctanh(c*x)/x^4-4/5*c*d^4*b*arctanh(c*x)/x^5-1/2*c
^4*d^4*b*arctanh(c*x)/x^2-1/6*d^4*b*arctanh(c*x)/x^6-4/3*c^3*d^4*b*arctanh(
c*x)/x^3-43/20*c^6*d^4*b*ln(c*x-1)-1/30*b*c*d^4/x^5-1/5*b*c^2*d^4/x^4-5/9*b
*c^3*d^4/x^3-16/15*b*c^4*d^4/x^2-13/6*b*c^5*d^4/x+32/15*c^6*d^4*b*ln(c*x)+1
/60*c^6*d^4*b*ln(c*x+1)
```

Maxima [B] time = 0.9885, size = 444, normalized size = 2.94

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\,\mathrm{artanh}\left(cx\right)}{x^2}\right)bc^4d^4-\frac{2}{3}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\mathrm{artan}}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*c^4*
d^4 - 2/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)
/x^3)*b*c^3*d^4 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*
x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^2*d^4 - 1/5*((2*c^4*log(c^2*x^2 -
 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c*d^4
 - 1/2*a*c^4*d^4/x^2 + 1/180*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) -
2*(15*c^4*x^4 + 5*c^2*x^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*d^4 - 4/3*a*
c^3*d^4/x^3 - 3/2*a*c^2*d^4/x^4 - 4/5*a*c*d^4/x^5 - 1/6*a*d^4/x^6
```

Fricas [A] time = 2.20086, size = 483, normalized size = 3.2

$$\frac{3\,bc^6d^4x^6\log\left(cx+1\right)-387\,bc^6d^4x^6\log\left(cx-1\right)+384\,bc^6d^4x^6\log\left(x\right)-390\,bc^5d^4x^5-6\left(15\,a+32\,b\right)c^4d^4x^4-20\left(12\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] 1/180*(3*b*c^6*d^4*x^6*log(c*x + 1) - 387*b*c^6*d^4*x^6*log(c*x - 1) + 384*
b*c^6*d^4*x^6*log(x) - 390*b*c^5*d^4*x^5 - 6*(15*a + 32*b)*c^4*d^4*x^4 - 20
*(12*a + 5*b)*c^3*d^4*x^3 - 18*(15*a + 2*b)*c^2*d^4*x^2 - 6*(24*a + b)*c*d^
4*x - 30*a*d^4 - 3*(15*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 45*b*c^2*d^4*x^2
+ 24*b*c*d^4*x + 5*b*d^4)*log(-(c*x + 1)/(c*x - 1)))/x^6
```

Sympy [A] time = 6.81763, size = 291, normalized size = 1.93

$$\begin{cases}-\frac{ac^4d^4}{2x^2}-\frac{4ac^3d^4}{3x^3}-\frac{3ac^2d^4}{2x^4}-\frac{4acd^4}{5x^5}-\frac{ad^4}{6x^6}+\frac{32bc^6d^4\log\left(x\right)}{15}-\frac{32bc^6d^4\log\left(x-\frac{1}{c}\right)}{15}+\frac{bc^6d^4\,\mathrm{atanh}\left(cx\right)}{30}-\frac{13bc^5d^4}{6x}-\frac{bc^4d^4\,\mathrm{atanh}\left(cx\right)}{2x^2}-\frac{16bc^4d}{15x^2} \\ -\frac{ad^4}{6x^6}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**7,x)
```

```
[Out] Piecewise((-a*c**4*d**4/(2*x**2) - 4*a*c**3*d**4/(3*x**3) - 3*a*c**2*d**4/(
2*x**4) - 4*a*c*d**4/(5*x**5) - a*d**4/(6*x**6) + 32*b*c**6*d**4*log(x)/15
- 32*b*c**6*d**4*log(x - 1/c)/15 + b*c**6*d**4*atanh(c*x)/30 - 13*b*c**5*d*
*4/(6*x) - b*c**4*d**4*atanh(c*x)/(2*x**2) - 16*b*c**4*d**4/(15*x**2) - 4*b
*c**3*d**4*atanh(c*x)/(3*x**3) - 5*b*c**3*d**4/(9*x**3) - 3*b*c**2*d**4*ata
nh(c*x)/(2*x**4) - b*c**2*d**4/(5*x**4) - 4*b*c*d**4*atanh(c*x)/(5*x**5) -
b*c*d**4/(30*x**5) - b*d**4*atanh(c*x)/(6*x**6), Ne(c, 0)), (-a*d**4/(6*x**
6), True))
```

Giac [A] time = 1.63756, size = 304, normalized size = 2.01

$$\frac{1}{60}\,bc^6d^4\log\left(cx+1\right)-\frac{43}{20}\,bc^6d^4\log\left(cx-1\right)+\frac{32}{15}\,bc^6d^4\log\left(x\right)-\frac{\left(15\,bc^4d^4x^4+40\,bc^3d^4x^3+45\,bc^2d^4x^2+24\,bcd^4x+5\,b\right.}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^7,x, algorithm="giac")
```

```
[Out] 1/60*b*c^6*d^4*log(c*x + 1) - 43/20*b*c^6*d^4*log(c*x - 1) + 32/15*b*c^6*d^
4*log(x) - 1/60*(15*b*c^4*d^4*x^4 + 40*b*c^3*d^4*x^3 + 45*b*c^2*d^4*x^2 + 2
4*b*c*d^4*x + 5*b*d^4)*log(-(c*x + 1)/(c*x - 1))/x^6 - 1/90*(195*b*c^5*d^4*
x^5 + 45*a*c^4*d^4*x^4 + 96*b*c^4*d^4*x^4 + 120*a*c^3*d^4*x^3 + 50*b*c^3*d^
4*x^3 + 135*a*c^2*d^4*x^2 + 18*b*c^2*d^4*x^2 + 72*a*c*d^4*x + 3*b*c*d^4*x +
 15*a*d^4)/x^6
```

3.42 $\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=229

$$-\frac{c^4d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-\frac{d^4\left(a+\right.}{}$$

```
[Out] -(b*c*d^4)/(42*x^6) - (2*b*c^2*d^4)/(15*x^5) - (47*b*c^3*d^4)/(140*x^4) - (
5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/(105*x^2) - (5*b*c^6*d^4)/(3*x) - (d^
4*(a + b*ArcTanh[c*x]))/(7*x^7) - (2*c*d^4*(a + b*ArcTanh[c*x]))/(3*x^6) -
(6*c^2*d^4*(a + b*ArcTanh[c*x]))/(5*x^5) - (c^3*d^4*(a + b*ArcTanh[c*x]))/x
^4 - (c^4*d^4*(a + b*ArcTanh[c*x]))/(3*x^3) + (176*b*c^7*d^4*Log[x])/105 -
(117*b*c^7*d^4*Log[1 - c*x])/70 - (b*c^7*d^4*Log[1 + c*x])/210
```

Rubi [A] time = 0.196895, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {43, 5936, 12, 1802}

$$-\frac{c^4d^4\left(a+b\tanh^{-1}(cx)\right)}{3x^3}-\frac{c^3d^4\left(a+b\tanh^{-1}(cx)\right)}{x^4}-\frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5}-\frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6}-\frac{d^4\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

```
[Out] -(b*c*d^4)/(42*x^6) - (2*b*c^2*d^4)/(15*x^5) - (47*b*c^3*d^4)/(140*x^4) - (
5*b*c^4*d^4)/(9*x^3) - (88*b*c^5*d^4)/(105*x^2) - (5*b*c^6*d^4)/(3*x) - (d^
4*(a + b*ArcTanh[c*x]))/(7*x^7) - (2*c*d^4*(a + b*ArcTanh[c*x]))/(3*x^6) -
(6*c^2*d^4*(a + b*ArcTanh[c*x]))/(5*x^5) - (c^3*d^4*(a + b*ArcTanh[c*x]))/x
^4 - (c^4*d^4*(a + b*ArcTanh[c*x]))/(3*x^3) + (176*b*c^7*d^4*Log[x])/105 -
(117*b*c^7*d^4*Log[1 - c*x])/70 - (b*c^7*d^4*Log[1 + c*x])/210
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7} - \frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6} - \frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - c$$

$$= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7} - \frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6} - \frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - c$$

$$= -\frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7} - \frac{2cd^4\left(a+b\tanh^{-1}(cx)\right)}{3x^6} - \frac{6c^2d^4\left(a+b\tanh^{-1}(cx)\right)}{5x^5} - c$$

$$= -\frac{bcd^4}{42x^6} - \frac{2bc^2d^4}{15x^5} - \frac{47bc^3d^4}{140x^4} - \frac{5bc^4d^4}{9x^3} - \frac{88bc^5d^4}{105x^2} - \frac{5bc^6d^4}{3x} - \frac{d^4\left(a+b\tanh^{-1}(cx)\right)}{7x^7}$$

Mathematica [A] time = 0.174554, size = 175, normalized size = 0.76

$$-\frac{d^4\left(420ac^4x^4+1260ac^3x^3+1512ac^2x^2+840acx+180a+2100bc^6x^6+1056bc^5x^5+700bc^4x^4+423bc^3x^3+168bc^2x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + c*d*x)^4*(a + b*ArcTanh[c*x]))/x^8,x]
```

```
[Out] -(d^4*(180*a + 840*a*c*x + 30*b*c*x + 1512*a*c^2*x^2 + 168*b*c^2*x^2 + 1260
*a*c^3*x^3 + 423*b*c^3*x^3 + 420*a*c^4*x^4 + 700*b*c^4*x^4 + 1056*b*c^5*x^5
 + 2100*b*c^6*x^6 + 12*b*(15 + 70*c*x + 126*c^2*x^2 + 105*c^3*x^3 + 35*c^4*
x^4)*ArcTanh[c*x] - 2112*b*c^7*x^7*Log[x] + 2106*b*c^7*x^7*Log[1 - c*x] + 6
*b*c^7*x^7*Log[1 + c*x]))/(1260*x^7)
```

Maple [A] time = 0.042, size = 245, normalized size = 1.1

$$-\frac{c^3d^4a}{x^4} - \frac{d^4a}{7x^7} - \frac{6c^2d^4a}{5x^5} - \frac{2cd^4a}{3x^6} - \frac{c^4d^4a}{3x^3} - \frac{c^3d^4b\text{Artanh}(cx)}{x^4} - \frac{d^4b\text{Artanh}(cx)}{7x^7} - \frac{6c^2d^4b\text{Artanh}(cx)}{5x^5} - \frac{2cd^4b\text{Artan}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x)
```

```
[Out] -c^3*d^4*a/x^4-1/7*d^4*a/x^7-6/5*c^2*d^4*a/x^5-2/3*c*d^4*a/x^6-1/3*c^4*d^4*
a/x^3-c^3*d^4*b*arctanh(c*x)/x^4-1/7*d^4*b*arctanh(c*x)/x^7-6/5*c^2*d^4*b*a
rctanh(c*x)/x^5-2/3*c*d^4*b*arctanh(c*x)/x^6-1/3*c^4*d^4*b*arctanh(c*x)/x^3
-117/70*c^7*d^4*b*ln(c*x-1)-1/42*b*c*d^4/x^6-2/15*b*c^2*d^4/x^5-47/140*b*c^
3*d^4/x^4-5/9*b*c^4*d^4/x^3-88/105*b*c^5*d^4/x^2-5/3*b*c^6*d^4/x+176/105*c^
7*d^4*b*ln(c*x)-1/210*b*c^7*d^4*ln(c*x+1)
```

Maxima [A] time = 1.01472, size = 477, normalized size = 2.08

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\,\text{artanh}(cx)}{x^3}\right)bc^4d^4+\frac{1}{6}\left(\left(3c^3\log(cx+1)-3c^3\log(cx-1)-\frac{2\left(3c^2x^2+\right.}{x^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*c^4*d^4 + 1/6*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 +
 1)/x^3)*c - 6*arctanh(c*x)/x^4)*b*c^3*d^4 - 3/10*((2*c^4*log(c^2*x^2 - 1)
- 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*b*c^2*d^4 +
 1/45*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x
^2 + 3)/x^5)*c - 30*arctanh(c*x)/x^6)*b*c*d^4 - 1/84*((6*c^6*log(c^2*x^2 -
1) - 6*c^6*log(x^2) + (6*c^4*x^4 + 3*c^2*x^2 + 2)/x^6)*c + 12*arctanh(c*x)/
x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 - a*c^3*d^4/x^4 - 6/5*a*c^2*d^4/x^5 - 2/3*a*
c*d^4/x^6 - 1/7*a*d^4/x^7
```

Fricas [A] time = 2.31061, size = 524, normalized size = 2.29

$$-\frac{6\,bc^7d^4x^7\log(cx+1)+2106\,bc^7d^4x^7\log(cx-1)-2112\,bc^7d^4x^7\log(x)+2100\,bc^6d^4x^6+1056\,bc^5d^4x^5+140\,(3\,a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/1260*(6*b*c^7*d^4*x^7*log(c*x + 1) + 2106*b*c^7*d^4*x^7*log(c*x - 1) - 2
112*b*c^7*d^4*x^7*log(x) + 2100*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^5 + 140*(3
*a + 5*b)*c^4*d^4*x^4 + 9*(140*a + 47*b)*c^3*d^4*x^3 + 168*(9*a + b)*c^2*d^
4*x^2 + 30*(28*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*b*c^4*d^4*x^4 + 105*b*c^3
*d^4*x^3 + 126*b*c^2*d^4*x^2 + 70*b*c*d^4*x + 15*b*d^4)*log(-(c*x + 1)/(c*x
 - 1)))/x^7
```

Sympy [A] time = 17.4093, size = 301, normalized size = 1.31

$$\begin{cases} -\frac{ac^4d^4}{3x^3}-\frac{ac^3d^4}{x^4}-\frac{6ac^2d^4}{5x^5}-\frac{2acd^4}{3x^6}-\frac{ad^4}{7x^7}+\frac{176bc^7d^4\log(x)}{105}-\frac{176bc^7d^4\log\left(x-\frac{1}{c}\right)}{105}-\frac{bc^7d^4\operatorname{atanh}(cx)}{105}-\frac{5bc^6d^4}{3x}-\frac{88bc^5d^4}{105x^2}-\frac{bc^4d^4\operatorname{atanh}(cx}{3x^3} \\ -\frac{ad^4}{7x^7} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**4*(a+b*atanh(c*x))/x**8,x)
```

```
[Out] Piecewise((-a*c**4*d**4/(3*x**3) - a*c**3*d**4/x**4 - 6*a*c**2*d**4/(5*x**5
) - 2*a*c*d**4/(3*x**6) - a*d**4/(7*x**7) + 176*b*c**7*d**4*log(x)/105 - 17
6*b*c**7*d**4*log(x - 1/c)/105 - b*c**7*d**4*atanh(c*x)/105 - 5*b*c**6*d**4
/(3*x) - 88*b*c**5*d**4/(105*x**2) - b*c**4*d**4*atanh(c*x)/(3*x**3) - 5*b*
c**4*d**4/(9*x**3) - b*c**3*d**4*atanh(c*x)/x**4 - 47*b*c**3*d**4/(140*x**4
) - 6*b*c**2*d**4*atanh(c*x)/(5*x**5) - 2*b*c**2*d**4/(15*x**5) - 2*b*c*d**
4*atanh(c*x)/(3*x**6) - b*c*d**4/(42*x**6) - b*d**4*atanh(c*x)/(7*x**7), Ne
(c, 0)), (-a*d**4/(7*x**7), True))
```

Giac [A] time = 2.60587, size = 320, normalized size = 1.4

$$-\frac{1}{210}\,bc^7d^4\log(cx+1)-\frac{117}{70}\,bc^7d^4\log(cx-1)+\frac{176}{105}\,bc^7d^4\log(x)-\frac{\left(35\,bc^4d^4x^4+105\,bc^3d^4x^3+126\,bc^2d^4x^2+70\,b\right.}{210\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^4*(a+b*arctanh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] -1/210*b*c^7*d^4*log(c*x + 1) - 117/70*b*c^7*d^4*log(c*x - 1) + 176/105*b*c
^7*d^4*log(x) - 1/210*(35*b*c^4*d^4*x^4 + 105*b*c^3*d^4*x^3 + 126*b*c^2*d^4
*x^2 + 70*b*c*d^4*x + 15*b*d^4)*log(-(c*x + 1)/(c*x - 1))/x^7 - 1/1260*(210
0*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*x^5 + 420*a*c^4*d^4*x^4 + 700*b*c^4*d^4*x^
4 + 1260*a*c^3*d^4*x^3 + 423*b*c^3*d^4*x^3 + 1512*a*c^2*d^4*x^2 + 168*b*c^2
*d^4*x^2 + 840*a*c*d^4*x + 30*b*c*d^4*x + 180*a*d^4)/x^7
```

3.43 $\int \frac{x^3(a+b\tanh^{-1}(cx))}{d+cdx}\,dx$

Optimal. Leaf size=177

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd}+\frac{ax}{c^3d}+$$

```
[Out] (a*x)/(c^3*d) - (b*x)/(2*c^3*d) + (b*x^2)/(6*c^2*d) + (b*ArcTanh[c*x])/(2*c
^4*d) + (b*x*ArcTanh[c*x])/(c^3*d) - (x^2*(a + b*ArcTanh[c*x]))/(2*c^2*d) +
 (x^3*(a + b*ArcTanh[c*x]))/(3*c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)
])/(c^4*d) + (2*b*Log[1 - c^2*x^2])/(3*c^4*d) - (b*PolyLog[2, 1 - 2/(1 + c*
x)])/(2*c^4*d)
```

Rubi [A] time = 0.287424, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5930, 5916, 266, 43, 321, 206, 5910, 260, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd}+\frac{ax}{c^3d}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```


```
[Out] (a*x)/(c^3*d) - (b*x)/(2*c^3*d) + (b*x^2)/(6*c^2*d) + (b*ArcTanh[c*x])/(2*c
^4*d) + (b*x*ArcTanh[c*x])/(c^3*d) - (x^2*(a + b*ArcTanh[c*x]))/(2*c^2*d) +
 (x^3*(a + b*ArcTanh[c*x]))/(3*c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)
])/(c^4*d) + (2*b*Log[1 - c^2*x^2])/(3*c^4*d) - (b*PolyLog[2, 1 - 2/(1 + c*
x)])/(2*c^4*d)
```


Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx = -\frac{\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx}{c} + \frac{\int x^2\left(a+b\tanh^{-1}(cx)\right)dx}{cd}$$

$$= \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd} + \frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx}{c^2} - \frac{b\int \frac{x^3}{1-c^2x^2}\,dx}{3d} - \frac{\int x\left(a+b\tanh^{-1}(cx)\right)}{c^2d}$$

$$= -\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd} - \frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c^3} - \frac{b\,\mathrm{Subst}\left(\int \frac{x}{1-c^2}\right.}{6d}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3cd} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log}{c^4d}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} + \frac{bx^2}{6c^2d} + \frac{b\tanh^{-1}(cx)}{2c^4d} + \frac{bx\tanh^{-1}(cx)}{c^3d} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3\left(a+\right.}{}$$

$$= \frac{ax}{c^3d} - \frac{bx}{2c^3d} + \frac{bx^2}{6c^2d} + \frac{b\tanh^{-1}(cx)}{2c^4d} + \frac{bx\tanh^{-1}(cx)}{c^3d} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d} + \frac{x^3\left(a+\right.}{}$$

Mathematica [A] time = 0.396859, size = 129, normalized size = 0.73

$$\frac{-3b\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 2ac^3x^3 - 3ac^2x^2 + 6acx - 6a\log(cx+1) + bc^2x^2 + 4b\log\left(1-c^2x^2\right) + b\tanh^{-1}(cx)}{6c^4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (-b + 6*a*c*x - 3*b*c*x - 3*a*c^2*x^2 + b*c^2*x^2 + 2*a*c^3*x^3 + b*ArcTanh
[c*x]*(3 + 6*c*x - 3*c^2*x^2 + 2*c^3*x^3 + 6*Log[1 + E^(-2*ArcTanh[c*x])])
- 6*a*Log[1 + c*x] + 4*b*Log[1 - c^2*x^2] - 3*b*PolyLog[2, -E^(-2*ArcTanh[c
*x])])/(6*c^4*d)
```

Maple [A] time = 0.045, size = 253, normalized size = 1.4

$$\frac{x^3a}{3\,cd} - \frac{ax^2}{2\,c^2d} + \frac{ax}{c^3d} - \frac{a\ln(cx+1)}{dc^4} + \frac{bx^3\mathrm{Artanh}(cx)}{3\,cd} - \frac{b\mathrm{Artanh}(cx)\,x^2}{2\,c^2d} + \frac{bx\mathrm{Artanh}(cx)}{c^3d} - \frac{b\mathrm{Artanh}(cx)\ln(cx+1}{dc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] 1/3/c*a/d*x^3-1/2/c^2*a/d*x^2+a*x/c^3/d-1/c^4*a/d*ln(c*x+1)+1/3/c*b/d*x^3*a
rctanh(c*x)-1/2/c^2*b/d*arctanh(c*x)*x^2+b*x*arctanh(c*x)/c^3/d-1/c^4*b/d*a
rctanh(c*x)*ln(c*x+1)-1/2/c^4*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2/c^4*b/d*ln
(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2/c^4*b/d*dilog(1/2+1/2*c*x)+1/4/c^4*b/d*l
n(c*x+1)^2+1/6*b*x^2/c^2/d-1/2*b*x/c^3/d-2/3/c^4*b/d+5/12/c^4*b/d*ln(c*x-1)
+11/12/c^4*b/d*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{72}\left(2c^4\left(\frac{2\left(c^2x^3+3x\right)}{c^7d} - \frac{3\log(cx+1)}{c^8d} + \frac{3\log(cx-1)}{c^8d}\right) + 216c^4\int \frac{x^4\log(cx+1)}{6\left(c^5dx^2-c^3d\right)}\,dx - 3c^3\left(\frac{x^2}{c^5d} + \frac{\log\left(c^2x^2-1\right)}{c^7d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/72*(2*c^4*(2*(c^2*x^3 + 3*x)/(c^7*d) - 3*log(c*x + 1)/(c^8*d) + 3*log(c*x
 - 1)/(c^8*d)) + 216*c^4*integrate(1/6*x^4*log(c*x + 1)/(c^5*d*x^2 - c^3*d)
, x) - 3*c^3*(x^2/(c^5*d) + log(c^2*x^2 - 1)/(c^7*d)) - 216*c^3*integrate(1
/6*x^3*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x) + 9*c^2*(2*x/(c^5*d) - log(c*x
+ 1)/(c^6*d) + log(c*x - 1)/(c^6*d)) - 216*c*integrate(1/6*x*log(c*x + 1)/(
c^5*d*x^2 - c^3*d), x) - 6*(2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*log(c*x + 1))
*log(-c*x + 1)/(c^4*d) + 18*log(6*c^5*d*x^2 - 6*c^3*d)/(c^4*d) - 216*integr
ate(1/6*log(c*x + 1)/(c^5*d*x^2 - c^3*d), x))*b + 1/6*a*((2*c^2*x^3 - 3*c*x
^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3 \operatorname{artanh}(cx) + ax^3}{cdx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c*d*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{cx+1}\, dx + \int \frac{bx^3 \operatorname{atanh}(cx)}{cx+1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x**3/(c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(c*x + 1), x))/
d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)x^3}{cdx + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d), x)
```

3.44 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx$

Optimal. Leaf size=145

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{ax}{c^2d}-\frac{b\log\left(1-c^2x^2\right)}{2c^3d}+\frac{bx}{2c^2d}-b$$

```
[Out] -((a*x)/(c^2*d)) + (b*x)/(2*c^2*d) - (b*ArcTanh[c*x])/(2*c^3*d) - (b*x*ArcT
anh[c*x])/(c^2*d) + (x^2*(a + b*ArcTanh[c*x]))/(2*c*d) - ((a + b*ArcTanh[c*
x])*Log[2/(1 + c*x)])/(c^3*d) - (b*Log[1 - c^2*x^2])/(2*c^3*d) + (b*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*c^3*d)
```

Rubi [A] time = 0.183807, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5930, 5916, 321, 206, 5910, 260, 5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{ax}{c^2d}-\frac{b\log\left(1-c^2x^2\right)}{2c^3d}+\frac{bx}{2c^2d}-b$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] -((a*x)/(c^2*d)) + (b*x)/(2*c^2*d) - (b*ArcTanh[c*x])/(2*c^3*d) - (b*x*ArcT
anh[c*x])/(c^2*d) + (x^2*(a + b*ArcTanh[c*x]))/(2*c*d) - ((a + b*ArcTanh[c*
x])*Log[2/(1 + c*x)])/(c^3*d) - (b*Log[1 - c^2*x^2])/(2*c^3*d) + (b*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*c^3*d)
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx &= -\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx}{c}+\frac{\int x\left(a+b\tanh^{-1}(cx)\right)\,dx}{cd}\\
&= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}+\frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c^2}-\frac{b\int \frac{x^2}{1-c^2x^2}\,dx}{2d}-\frac{\int \left(a+b\tanh^{-1}(cx)\right)\,dx}{c^2d}\\
&= -\frac{ax}{c^2d}+\frac{bx}{2c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d}-\frac{b\int \frac{1}{1-c^2x^2}\,dx}{2c^2d}-\ldots\\
&= -\frac{ax}{c^2d}+\frac{bx}{2c^2d}-\frac{b\tanh^{-1}(cx)}{2c^3d}-\frac{bx\tanh^{-1}(cx)}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{\left(a+b\tanh^{-1}(c\ldots\right.}{c^3d}\\
&= -\frac{ax}{c^2d}+\frac{bx}{2c^2d}-\frac{b\tanh^{-1}(cx)}{2c^3d}-\frac{bx\tanh^{-1}(cx)}{c^2d}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2cd}-\frac{\left(a+b\tanh^{-1}(c\ldots\right.}{c^3d}
\end{aligned}$$

Mathematica [A] time = 0.243741, size = 97, normalized size = 0.67

$$\frac{b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+ac^2x^2-2acx+2a\log(cx+1)-b\log\left(1-c^2x^2\right)+b\tanh^{-1}(cx)\left(c^2x^2-2cx-2\log\left(e^{-2\tanh\ldots}\right.\right.}{2c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (-2*a*c*x + b*c*x + a*c^2*x^2 + b*ArcTanh[c*x]*(-1 - 2*c*x + c^2*x^2 - 2*Lo
g[1 + E^(-2*ArcTanh[c*x])]) + 2*a*Log[1 + c*x] - b*Log[1 - c^2*x^2] + b*Pol
yLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^3*d)
```

Maple [A] time = 0.045, size = 213, normalized size = 1.5

$$\frac{ax^2}{2\,cd}-\frac{ax}{c^2d}+\frac{a\ln(cx+1)}{c^3d}+\frac{b\mathrm{Artanh}(cx)\,x^2}{2\,cd}-\frac{bx\mathrm{Artanh}(cx)}{c^2d}+\frac{b\mathrm{Artanh}(cx)\ln(cx+1)}{c^3d}+\frac{b\ln(cx+1)}{2\,c^3d}\ln\left(-\frac{cx}{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] 1/2/c*a/d*x^2-a*x/c^2/d+1/c^3*a/d*ln(c*x+1)+1/2/c*b/d*arctanh(c*x)*x^2-b*x*
arctanh(c*x)/c^2/d+1/c^3*b/d*arctanh(c*x)*ln(c*x+1)+1/2/c^3*b/d*ln(-1/2*c*x
+1/2)*ln(c*x+1)-1/2/c^3*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2/c^3*b/d*di
log(1/2+1/2*c*x)-1/4/c^3*b/d*ln(c*x+1)^2+1/2*b*x/c^2/d+1/2/c^3*b/d-1/4/c^3*
b/d*ln(c*x-1)-3/4/c^3*b/d*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(c^3\left(\frac{x^2}{c^4d}+\frac{\log\left(c^2x^2-1\right)}{c^6d}\right)+8\,c^3\int\frac{x^3\log(cx+1)}{2\left(c^4dx^2-c^2d\right)}\,dx-c^2\left(\frac{2\,x}{c^4d}-\frac{\log(cx+1)}{c^5d}+\frac{\log(cx-1)}{c^5d}\right)-8\,c^2\int\frac{x^2\log(cx\ldots}{2\left(c^4dx^2-\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/8*(c^3*(x^2/(c^4*d) + log(c^2*x^2 - 1)/(c^6*d)) + 8*c^3*integrate(1/2*x^3
*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x) - c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^
5*d) + log(c*x - 1)/(c^5*d)) - 8*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^4*d*
x^2 - c^2*d), x) + 8*c*integrate(1/2*x*log(c*x + 1)/(c^4*d*x^2 - c^2*d), x)
 - 2*(c^2*x^2 - 2*c*x + 2*log(c*x + 1))*log(-c*x + 1)/(c^3*d) - 2*log(2*c^4
*d*x^2 - 2*c^2*d)/(c^3*d) + 8*integrate(1/2*log(c*x + 1)/(c^4*d*x^2 - c^2*d
), x))*b + 1/2*a*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{artanh}(cx)+ax^2}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c*d*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{cx+1}\,dx + \int \frac{bx^2\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x**2/(c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(c*x + 1), x))/
d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)x^2}{cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d), x)
```

3.45 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx$

Optimal. Leaf size=94

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{ax}{cd}+\frac{b\log\left(1-c^2x^2\right)}{2c^2d}+\frac{bx\tanh^{-1}(cx)}{cd}$$

```
[Out] (a*x)/(c*d) + (b*x*ArcTanh[c*x])/(c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c
*x)])/(c^2*d) + (b*Log[1 - c^2*x^2])/(2*c^2*d) - (b*PolyLog[2, 1 - 2/(1 + c
*x)])/(2*c^2*d)
```

Rubi [A] time = 0.10252, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5930, 5910, 260, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}+\frac{ax}{cd}+\frac{b\log\left(1-c^2x^2\right)}{2c^2d}+\frac{bx\tanh^{-1}(cx)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (a*x)/(c*d) + (b*x*ArcTanh[c*x])/(c*d) + ((a + b*ArcTanh[c*x])*Log[2/(1 + c
*x)])/(c^2*d) + (b*Log[1 - c^2*x^2])/(2*c^2*d) - (b*PolyLog[2, 1 - 2/(1 + c
*x)])/(2*c^2*d)
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+cdx}\,dx &= -\frac{\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx}{c}+\frac{\int \left(a+b\tanh^{-1}(cx)\right)dx}{cd}\\ &= \frac{ax}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d}+\frac{b\int \tanh^{-1}(cx)\,dx}{cd}-\frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{cd}\\ &= \frac{ax}{cd}+\frac{bx\tanh^{-1}(cx)}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d}-\frac{b\int \frac{x}{1-c^2x^2}\,dx}{d}-\frac{b\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,d\right.}{c^2d}\\ &= \frac{ax}{cd}+\frac{bx\tanh^{-1}(cx)}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d}+\frac{b\log\left(1-c^2x^2\right)}{2c^2d}-\frac{b\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c^2d}\end{aligned}$$

Mathematica [A] time = 0.154252, size = 75, normalized size = 0.8

$$\frac{-b\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2acx-2a\log(cx+1)+b\log\left(1-c^2x^2\right)+2b\tanh^{-1}(cx)\left(cx+\log\left(e^{-2\tanh^{-1}(cx)}+1\right)\right)}{2c^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x),x]
```

```
[Out] (2*a*c*x + 2*b*ArcTanh[c*x]*(c*x + Log[1 + E^(-2*ArcTanh[c*x])]) - 2*a*Log[
1 + c*x] + b*Log[1 - c^2*x^2] - b*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(2*c^2*
d)
```

Maple [A] time = 0.04, size = 157, normalized size = 1.7

$$\frac{ax}{cd}-\frac{a\ln(cx+1)}{c^2d}-\frac{b\mathrm{Artanh}(cx)\ln(cx+1)}{c^2d}+\frac{bx\mathrm{Artanh}(cx)}{cd}-\frac{b\ln(cx+1)}{2c^2d}\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)+\frac{b}{2c^2d}\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] a*x/c/d-1/c^2*a/d*ln(c*x+1)-1/c^2*b/d*arctanh(c*x)*ln(c*x+1)+b*x*arctanh(c*
x)/c/d-1/2/c^2*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2/c^2*b/d*ln(-1/2*c*x+1/2)*
ln(1/2+1/2*c*x)+1/2/c^2*b/d*dilog(1/2+1/2*c*x)+1/4/c^2*b/d*ln(c*x+1)^2+1/2/
c^2*b/d*ln((c*x-1)*(c*x+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(c^2\left(\frac{2\,x}{c^3d} - \frac{\log\left(cx+1\right)}{c^4d} + \frac{\log\left(cx-1\right)}{c^4d}\right) + 2\,c^2\int\frac{x^2\log\left(cx+1\right)}{c^3dx^2-cd}\,dx - 4\,c\int\frac{x\log\left(cx+1\right)}{c^3dx^2-cd}\,dx - \frac{2\left(cx-\log\left(cx+1\right)\right)\mathrm{l}}{c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*x/(c^3*d) - log(c*x + 1)/(c^4*d) + log(c*x - 1)/(c^4*d)) + 2*c^
2*integrate(x^2*log(c*x + 1)/(c^3*d*x^2 - c*d), x) - 4*c*integrate(x*log(c*
x + 1)/(c^3*d*x^2 - c*d), x) - 2*(c*x - log(c*x + 1))*log(-c*x + 1)/(c^2*d)
 + log(c^3*d*x^2 - c*d)/(c^2*d) - 2*integrate(log(c*x + 1)/(c^3*d*x^2 - c*d
), x))*b + a*(x/(c*d) - log(c*x + 1)/(c^2*d))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx\,\text{artanh}\left(cx\right)+ax}{cdx+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(c*d*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{ax}{cx+1}\,dx + \int\frac{bx\,\text{atanh}\left(cx\right)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a*x/(c*x + 1), x) + Integral(b*x*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\text{artanh}\left(cx\right)+a\right)x}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(c*d*x + d), x)
```

3.46 $\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx$

Optimal. Leaf size=51

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}$$

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c*d)) + (b*PolyLog[2, 1 - 2/(1 +
 c*x)])/(2*c*d)
```

Rubi [A] time = 0.0477698, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c*d)) + (b*PolyLog[2, 1 - 2/(1 +
 c*x)])/(2*c*d)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{d+cdx}\,dx &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+cx}\right)}{cd}\\ &= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{cd}+\frac{b\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2cd}\end{aligned}$$

Mathematica [A] time = 0.0973929, size = 52, normalized size = 1.02

$$\frac{b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2a\log(cx+1)-2b\tanh^{-1}(cx)\log\left(e^{-2\tanh^{-1}(cx)}+1\right)}{2cd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x),x]
```

```
[Out] (-2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a*Log[1 + c*x] + b*Poly
Log[2, -E^(-2*ArcTanh[c*x])])/(2*c*d)
```

Maple [B] time = 0.039, size = 112, normalized size = 2.2

$$\frac{a\ln(cx+1)}{cd}+\frac{b\text{Artanh}(cx)\ln(cx+1)}{cd}+\frac{b\ln(cx+1)}{2cd}\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)-\frac{b}{2cd}\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}+\frac{cx}{2}\right)-\frac{b}{2cd}\text{dilog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d),x)
```

```
[Out] 1/c*a/d*ln(c*x+1)+1/c*b/d*arctanh(c*x)*ln(c*x+1)+1/2/c*b/d*ln(-1/2*c*x+1/2)
*ln(c*x+1)-1/2/c*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2/c*b/d*dilog(1/2+1
/2*c*x)-1/4/c*b/d*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\left(2c\int\frac{x\log(cx+1)}{c^2dx^2-d}\,dx-\frac{\log(cx+1)\log(-cx+1)}{cd}\right)b+\frac{a\log(cdx+d)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*integrate(x*log(c*x + 1)/(c^2*d*x^2 - d), x) - log(c*x + 1)*log(-c
*x + 1)/(c*d))*b + a*log(c*d*x + d)/(c*d)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}(cx)+a}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{cx+1}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x + 1), x) + Integral(b*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\,(cx) + a}{cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(c*d*x + d), x)
```

3.47 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx$

Optimal. Leaf size=46

$$\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}$$

```
[Out] ((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*PolyLog[2, -1 + 2/(1 + c
*x)])/(2*d)
```

Rubi [A] time = 0.0735335, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5932, 2447}

$$\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

```
[Out] ((a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*PolyLog[2, -1 + 2/(1 + c
*x)])/(2*d)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx &= \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{(bc)\int \frac{\log\left(2-\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx}{d}\\ &= \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}-\frac{b\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.110306, size = 55, normalized size = 1.2

$$\frac{-b\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-2a\log(cx+1)+2a\log(x)+2b\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)),x]
```

```
[Out] (2*b*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + 2*a*Log[x] - 2*a*Log[1 + c
*x] - b*PolyLog[2, E^(-2*ArcTanh[c*x])])/(2*d)
```

Maple [B] time = 0.046, size = 156, normalized size = 3.4

$$\frac{a\ln(cx)}{d}-\frac{a\ln(cx+1)}{d}-\frac{b\mathrm{Artanh}(cx)\ln(cx+1)}{d}+\frac{b\mathrm{Artanh}(cx)\ln(cx)}{d}+\frac{b}{2d}\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}+\frac{cx}{2}\right)-\frac{b\ln(cx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d),x)
```

```
[Out] a/d*ln(c*x)-a/d*ln(c*x+1)-b/d*arctanh(c*x)*ln(c*x+1)+b/d*arctanh(c*x)*ln(c*
x)+1/2*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2*b/d*ln(-1/2*c*x+1/2)*ln(c*x
+1)+1/2*b/d*dilog(1/2+1/2*c*x)+1/4*b/d*ln(c*x+1)^2-1/2*b/d*dilog(c*x)-1/2*b
/d*dilog(c*x+1)-1/2*b/d*ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{\log(cx+1)}{d}-\frac{\log(x)}{d}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{cdx^2+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(c*x + 1)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x +
 1))/(c*d*x^2 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}(cx)+a}{cdx^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^2 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{cx^2+x}\,dx+\int\frac{b\,\mathrm{atanh}(cx)}{cx^2+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**2 + x), x) + Integral(b*atanh(c*x)/(c*x**2 + x), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x), x)
```

3.48 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx$

Optimal. Leaf size=93

$$\frac{bc\mathrm{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}+\frac{bc\log(x)}{d}$$

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 - c^2*x^2])/(2*
d) - (c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c*PolyLog[2, -1 +
 2/(1 + c*x)])/(2*d)
```

Rubi [A] time = 0.153175, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5934, 5916, 266, 36, 29, 31, 5932, 2447}

$$\frac{bc\mathrm{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}-\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}+\frac{bc\log(x)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (b*c*Log[1 - c^2*x^2])/(2*
d) - (c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c*PolyLog[2, -1 +
 2/(1 + c*x)])/(2*d)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx\right)+\frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{(bc)\int \frac{1}{x(1-c^2x^2)}\,dx}{d}+\frac{\left(bc^2\right)\int \frac{\log}{1}}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\mathrm{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}+\frac{(bc)\,\mathrm{Subst}}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{bc\mathrm{Li}_2\left(-1+\frac{2}{1+cx}\right)}{2d}+\frac{(bc)\,\mathrm{Subst}}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log\left(1-c^2x^2\right)}{2d}-\frac{c\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}+$$

Mathematica [A] time = 0.191839, size = 93, normalized size = 1.

$$\frac{bcx\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-2\left(acx\log(x)-acx\log(cx+1)+a-bcx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+b\tanh^{-1}(cx)\left(cx\log\left(1-e^{-2}\right.\right.\right.}{2dx}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)),x]
```

```
[Out] (-2*(a + b*ArcTanh[c*x]*(1 + c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + a*c*x*Log[
x] - a*c*x*Log[1 + c*x] - b*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + b*c*x*PolyL
og[2, E^(-2*ArcTanh[c*x])])/(2*d*x)
```

Maple [B] time = 0.053, size = 225, normalized size = 2.4

$$-\frac{a}{dx}-\frac{ac\ln(cx)}{d}+\frac{ac\ln(cx+1)}{d}-\frac{b\mathrm{Artanh}(cx)}{dx}-\frac{bc\mathrm{Artanh}(cx)\ln(cx)}{d}+\frac{bc\mathrm{Artanh}(cx)\ln(cx+1)}{d}-\frac{bc\ln(cx-1)}{2d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d),x)
```

```
[Out] -a/d/x-c*a/d*ln(c*x)+c*a/d*ln(c*x+1)-b/d*arctanh(c*x)/x-c*b/d*arctanh(c*x)*
ln(c*x)+c*b/d*arctanh(c*x)*ln(c*x+1)-1/2*c*b/d*ln(c*x-1)+c*b/d*ln(c*x)-1/2*
c*b/d*ln(c*x+1)+1/2*c*b/d*dilog(c*x)+1/2*c*b/d*dilog(c*x+1)+1/2*c*b/d*ln(c*
x)*ln(c*x+1)-1/2*c*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2*c*b/d*ln(-1/2*c
*x+1/2)*ln(c*x+1)-1/2*c*b/d*dilog(1/2+1/2*c*x)-1/4*c*b/d*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{c\log(cx+1)}{d}-\frac{c\log(x)}{d}-\frac{1}{dx}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{cdx^3+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/2*b*integrate((log(c*x + 1)
 - log(-c*x + 1))/(c*d*x^3 + d*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}(cx)+a}{cdx^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^3 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{cx^3+x^2}\,dx+\int\frac{b\,\mathrm{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c*x**3 + x**2), x)
)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\,(cx) + a}{(cdx + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^2), x)
```

3.49 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx$

Optimal. Leaf size=146

$$-\frac{bc^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}+\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{bc^2\log(}{2}$$

```
[Out] -(b*c)/(2*d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x^2
) + (c*(a + b*ArcTanh[c*x]))/(d*x) - (b*c^2*Log[x])/d + (b*c^2*Log[1 - c^2*
x^2])/(2*d) + (c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/(2*d)
```

Rubi [A] time = 0.234477, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5934, 5916, 325, 206, 266, 36, 29, 31, 5932, 2447}

$$-\frac{bc^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}+\frac{c^2\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{a+b\tanh^{-1}(cx)}{2dx^2}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{bc^2\log(}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

```
[Out] -(b*c)/(2*d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x^2
) + (c*(a + b*ArcTanh[c*x]))/(d*x) - (b*c^2*Log[x])/d + (b*c^2*Log[1 - c^2*
x^2])/(2*d) + (c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (b*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx\right) + \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2dx^2} + c^2\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d} + \frac{(bc)\int \frac{1}{x^2(1-c^2x^2)}\,dx}{2d}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{(bc}{}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)\log}{d}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)\log}{d}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{bc^2\log(x)}{d} + \frac{bc^2\log\left(1-\right.}{2d}$$

Mathematica [A] time = 0.344723, size = 133, normalized size = 0.91

$$-\frac{bc^2x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) - 2ac^2x^2\log(x) + 2ac^2x^2\log(cx+1) - 2acx + a + 2bc^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) - b\tanh^{-1}(cx)}{2dx^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)),x]
```

```
[Out] -(a - 2*a*c*x + b*c*x - b*ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^2*Lo
g[1 - E^(-2*ArcTanh[c*x])]) - 2*a*c^2*x^2*Log[x] + 2*a*c^2*x^2*Log[1 + c*x]
 + 2*b*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + b*c^2*x^2*PolyLog[2, E^(-2*Ar
cTanh[c*x])])/(2*d*x^2)
```

Maple [B] time = 0.058, size = 286, normalized size = 2.

$$-\frac{a}{2\,dx^2} + \frac{ac^2\ln(cx)}{d} + \frac{ac}{dx} - \frac{ac^2\ln(cx+1)}{d} - \frac{b\text{Artanh}(cx)}{2\,dx^2} + \frac{c^2b\text{Artanh}(cx)\ln(cx)}{d} + \frac{bc\text{Artanh}(cx)}{dx} - \frac{c^2b\text{Artanh}(c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d),x)
```

```
[Out] -1/2*a/d/x^2+c^2*a/d*ln(c*x)+c*a/d/x-c^2*a/d*ln(c*x+1)-1/2*b/d*arctanh(c*x)
/x^2+c^2*b/d*arctanh(c*x)*ln(c*x)+c*b/d*arctanh(c*x)/x-c^2*b/d*arctanh(c*x)
*ln(c*x+1)-1/2*c^2*b/d*dilog(c*x)-1/2*c^2*b/d*dilog(c*x+1)-1/2*c^2*b/d*ln(c
*x)*ln(c*x+1)+1/2*c^2*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2*c^2*b/d*ln(-
1/2*c*x+1/2)*ln(c*x+1)+1/2*c^2*b/d*dilog(1/2+1/2*c*x)+1/4*c^2*b/d*ln(c*x+1)
^2+1/4*c^2*b/d*ln(c*x-1)-1/2*b*c/d/x-c^2*b/d*ln(c*x)+3/4*c^2*b/d*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\frac{2\,c^2\log(cx+1)}{d} - \frac{2\,c^2\log(x)}{d} - \frac{2\,cx-1}{dx^2}\right)a + \frac{1}{2}\,b\int \frac{\log(cx+1)-\log(-cx+1)}{cdx^4+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a + 1/2*
b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^4 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{artanh}(cx) + a}{cdx^4 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^4 + d*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{cx^4+x^3}\,dx + \int \frac{b \operatorname{atanh}(cx)}{cx^4+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c*x**4 + x**3), x)
)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^3), x)
```

3.50 $\int \frac{a+b\tanh^{-1}(cx)}{x^4(d+cdx)}\,dx$

Optimal. Leaf size=185

$$\frac{bc^3\mathrm{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{a+b}{}$$

```
[Out] -(b*c)/(6*d*x^2) + (b*c^2)/(2*d*x) - (b*c^3*ArcTanh[c*x])/(2*d) - (a + b*Ar
cTanh[c*x])/(3*d*x^3) + (c*(a + b*ArcTanh[c*x]))/(2*d*x^2) - (c^2*(a + b*Ar
cTanh[c*x]))/(d*x) + (4*b*c^3*Log[x])/(3*d) - (2*b*c^3*Log[1 - c^2*x^2])/(3
*d) - (c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c^3*PolyLog[2,
 -1 + 2/(1 + c*x)])/(2*d)
```

Rubi [A] time = 0.345908, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5934, 5916, 266, 44, 325, 206, 36, 29, 31, 5932, 2447}

$$\frac{bc^3\mathrm{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{2d}-\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx}-\frac{c^3\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2}-\frac{a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```


```
[Out] -(b*c)/(6*d*x^2) + (b*c^2)/(2*d*x) - (b*c^3*ArcTanh[c*x])/(2*d) - (a + b*Ar
cTanh[c*x])/(3*d*x^3) + (c*(a + b*ArcTanh[c*x]))/(2*d*x^2) - (c^2*(a + b*Ar
cTanh[c*x]))/(d*x) + (4*b*c^3*Log[x])/(3*d) - (2*b*c^3*Log[1 - c^2*x^2])/(3
*d) - (c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d + (b*c^3*PolyLog[2,
 -1 + 2/(1 + c*x)])/(2*d)
```


Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^4(d+cdx)}\,dx = -\left(c\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)}\,dx\right) + \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^4}\,dx}{d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{3dx^3} + c^2\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)}\,dx - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d} + \frac{(bc)\int \frac{1}{x^3(1-c^2x^2)}\,dx}{3d}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - c^3\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)}\,dx + \frac{(bc)\,\text{Subst}\left(\int \frac{1}{x^2(1-c^2}\right.}{6d}$$

$$= \frac{bc^2}{2dx} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{c^3\left(a+b\tanh^{-1}(cx\right.}{d}$$

$$= -\frac{bc}{6dx^2} + \frac{bc^2}{2dx} - \frac{bc^3\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\tanh^{-1}\right.}{dx}$$

$$= -\frac{bc}{6dx^2} + \frac{bc^2}{2dx} - \frac{bc^3\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\tanh^{-1}\right.}{dx}$$

$$= -\frac{bc}{6dx^2} + \frac{bc^2}{2dx} - \frac{bc^3\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{3dx^3} + \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2dx^2} - \frac{c^2\left(a+b\tanh^{-1}\right.}{dx}$$

Mathematica [A] time = 0.42305, size = 172, normalized size = 0.93

$$\frac{3bc^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) - 6ac^2x^2 - 6ac^3x^3\log(x) + 6ac^3x^3\log(cx+1) + 3acx - 2a + bc^3x^3 + 3bc^2x^2 + 8bc^3x^3\,\text{lo}}{6dx^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^4*(d + c*d*x)),x]
```

```
[Out] (-2*a + 3*a*c*x - b*c*x - 6*a*c^2*x^2 + 3*b*c^2*x^2 + b*c^3*x^3 - b*ArcTanh
[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[
c*x])]) - 6*a*c^3*x^3*Log[x] + 6*a*c^3*x^3*Log[1 + c*x] + 8*b*c^3*x^3*Log[(
c*x)/Sqrt[1 - c^2*x^2]] + 3*b*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])])/(6*d
*x^3)
```

Maple [A] time = 0.059, size = 328, normalized size = 1.8

$$-\frac{a}{3\,dx^3} - \frac{ac^2}{dx} + \frac{ac}{2\,dx^2} - \frac{c^3a\ln(cx)}{d} + \frac{c^3a\ln(cx+1)}{d} - \frac{b\text{Artanh}(cx)}{3\,dx^3} - \frac{c^2b\text{Artanh}(cx)}{dx} + \frac{bc\text{Artanh}(cx)}{2\,dx^2} - \frac{c^3b\text{Artanh}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^4/(c*d*x+d),x)
```

```
[Out] -1/3*a/d/x^3-c^2*a/d/x+1/2*c*a/d/x^2-c^3*a/d*ln(c*x)+c^3*a/d*ln(c*x+1)-1/3*
b/d*arctanh(c*x)/x^3-c^2*b/d*arctanh(c*x)/x+1/2*c*b/d*arctanh(c*x)/x^2-c^3*
b/d*arctanh(c*x)*ln(c*x)+c^3*b/d*arctanh(c*x)*ln(c*x+1)-5/12*c^3*b/d*ln(c*x
-1)-1/6*b*c/d/x^2+1/2*b*c^2/d/x+4/3*c^3*b/d*ln(c*x)-11/12*c^3*b/d*ln(c*x+1)
+1/2*c^3*b/d*dilog(c*x)+1/2*c^3*b/d*dilog(c*x+1)+1/2*c^3*b/d*ln(c*x)*ln(c*x
+1)-1/2*c^3*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2*c^3*b/d*ln(-1/2*c*x+1/
2)*ln(c*x+1)-1/2*c^3*b/d*dilog(1/2+1/2*c*x)-1/4*c^3*b/d*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\left(\frac{6\,c^3\log\left(cx+1\right)}{d}-\frac{6\,c^3\log\left(x\right)}{d}-\frac{6\,c^2x^2-3\,cx+2}{dx^3}\right)a+\frac{1}{2}\,b\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{cdx^5+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^3
))*a + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c*d*x^5 + d*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\left(cx\right)+a}{cdx^5+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c*d*x^5 + d*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{cx^5+x^4}\,dx+\int\frac{b\,\mathrm{atanh}\left(cx\right)}{cx^5+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**4/(c*d*x+d),x)
```

```
[Out] (Integral(a/(c*x**5 + x**4), x) + Integral(b*atanh(c*x)/(c*x**5 + x**4), x)
)/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\mathrm{artanh}\left(cx\right)+a}{\left(cdx+d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^4/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)*x^4), x)
```

3.51 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=181

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^4d^2(cx+1)}-\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}-\frac{2ax}{c^3d^2}-\frac{b\log(}{c}$$

```
[Out] (-2*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(1 + c*x)) - (b*ArcTa
nh[c*x])/(c^4*d^2) - (2*b*x*ArcTanh[c*x])/(c^3*d^2) + (x^2*(a + b*ArcTanh[c
*x]))/(2*c^2*d^2) + (a + b*ArcTanh[c*x])/(c^4*d^2*(1 + c*x)) - (3*(a + b*Ar
cTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^2) - (b*Log[1 - c^2*x^2])/(c^4*d^2) +
(3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Rubi [A] time = 0.222001, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {5940, 5910, 260, 5916, 321, 206, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^4d^2(cx+1)}-\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}-\frac{2ax}{c^3d^2}-\frac{b\log(}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```


```
[Out] (-2*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(1 + c*x)) - (b*ArcTa
nh[c*x])/(c^4*d^2) - (2*b*x*ArcTanh[c*x])/(c^3*d^2) + (x^2*(a + b*ArcTanh[c
*x]))/(2*c^2*d^2) + (a + b*ArcTanh[c*x])/(c^4*d^2*(1 + c*x)) - (3*(a + b*Ar
cTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^2) - (b*Log[1 - c^2*x^2])/(c^4*d^2) +
(3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx = \int\Bigg(-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)^2}+\frac{3\left(a+b\tanh^{-1}(\right.}{c^3d^2(1+cx)}$$

$$=-\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^3d^2}-\frac{2\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^3d^2}+\frac{3\int\frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^3d^2}+\frac{\int x\left(a+b\tanh\right.}{c^2d^2}$$

$$=-\frac{2ax}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^4d^2(1+cx)}-\frac{3\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^4d^2}-\frac{b}{}$$

$$=-\frac{2ax}{c^3d^2}+\frac{bx}{2c^3d^2}-\frac{2bx\tanh^{-1}(cx)}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^4d^2(1+cx)}-\frac{3\left(a+b\right.}{}$$

$$=-\frac{2ax}{c^3d^2}+\frac{bx}{2c^3d^2}-\frac{b\tanh^{-1}(cx)}{2c^4d^2}-\frac{2bx\tanh^{-1}(cx)}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}}{c^4d^2(1+c}$$

$$=-\frac{2ax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^4d^2}-\frac{2bx\tanh^{-1}(cx)}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right.}{2c^2d^2}$$

$$=-\frac{2ax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{c^4d^2}-\frac{2bx\tanh^{-1}(cx)}{c^3d^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right.}{2c^2d^2}$$

Mathematica [A] time = 0.701472, size = 142, normalized size = 0.78

$$\frac{b\left(6\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-4\log\left(1-c^2x^2\right)+2\tanh^{-1}(cx)\left(c^2x^2-4cx-6\log\left(e^{-2\tanh^{-1}(cx)}+1\right)-\sinh\left(2\tanh^{-1}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (-8*a*c*x + 2*a*c^2*x^2 + (4*a)/(1 + c*x) + 12*a*Log[1 + c*x] + b*(2*c*x +
Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2*ArcTanh[c*x
])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x]] - 6*Log[1
 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4
*c^4*d^2)
```

Maple [A] time = 0.064, size = 265, normalized size = 1.5

$$\frac{ax^2}{2c^2d^2}-2\frac{ax}{c^3d^2}+\frac{a}{d^2c^4(cx+1)}+3\frac{a\ln(cx+1)}{d^2c^4}+\frac{b\text{Artanh}(cx)x^2}{2c^2d^2}-2\frac{bx\text{Artanh}(cx)}{c^3d^2}+\frac{b\text{Artanh}(cx)}{d^2c^4(cx+1)}+3\frac{b\text{Artanh}(c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] 1/2/c^2*a/d^2*x^2-2*a*x/c^3/d^2+1/c^4*a/d^2/(c*x+1)+3/c^4*a/d^2*ln(c*x+1)+1
/2/c^2*b/d^2*arctanh(c*x)*x^2-2*b*x*arctanh(c*x)/c^3/d^2+1/c^4*b/d^2*arctan
```

```
h(c*x)/(c*x+1)+3/c^4*b/d^2*arctanh(c*x)*ln(c*x+1)-3/2/c^4*b/d^2*ln(-1/2*c*x
+1/2)*ln(1/2+1/2*c*x)+3/2/c^4*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2/c^4*b/d^
2*dilog(1/2+1/2*c*x)-3/4/c^4*b/d^2*ln(c*x+1)^2+1/2*b*x/c^3/d^2+1/2/c^4*b/d^
2-1/2/c^4*b/d^2*ln(c*x-1)+1/2*b/c^4/d^2/(c*x+1)-3/2/c^4*b/d^2*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(c^4*(2/(c^9*d^2*x + c^8*d^2) + 2*(c*x^2 - 2*x)/(c^7*d^2) + 7*log(c*x
+ 1)/(c^8*d^2) + log(c*x - 1)/(c^8*d^2)) + 16*c^4*integrate(1/2*x^4*log(c*x
 + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c^3*(2/(c^8
*d^2*x + c^7*d^2) - 4*x/(c^6*d^2) + 5*log(c*x + 1)/(c^7*d^2) - log(c*x - 1)
/(c^7*d^2)) - 16*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*
x^2 - c^4*d^2*x - c^3*d^2), x) - 7*c^2*(2/(c^7*d^2*x + c^6*d^2) + 3*log(c*x
 + 1)/(c^6*d^2) + log(c*x - 1)/(c^6*d^2)) + 48*c^2*integrate(1/2*x^2*log(c*
x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x) + 2*c*(2/(c^6*
d^2*x + c^5*d^2) + log(c*x + 1)/(c^5*d^2) - log(c*x - 1)/(c^5*d^2)) + 96*c*
integrate(1/2*x*log(c*x + 1)/(c^6*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d
^2), x) - 4*(c^3*x^3 - 3*c^2*x^2 - 4*c*x + 6*(c*x + 1)*log(c*x + 1) + 2)*lo
g(-c*x + 1)/(c^5*d^2*x + c^4*d^2) + 4/(c^5*d^2*x + c^4*d^2) - 2*log(c*x + 1
)/(c^4*d^2) + 2*log(c*x - 1)/(c^4*d^2) + 48*integrate(1/2*log(c*x + 1)/(c^6
*d^2*x^3 + c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x))*b + 1/2*a*(2/(c^5*d^2*x
+ c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)/(c^4*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3 \operatorname{artanh}(cx) + ax^3}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^2x^2+2cx+1}\,dx + \int \frac{bx^3 \operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**3*atanh(c*x)/(
c**2*x**2 + 2*c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)x^3}{(cdx + d)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^2, x)
```

3.52 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=149

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(cx+1)}+\frac{2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{ax}{c^2d^2}+\frac{b\log\left(1-c^2x^2\right)}{2c^3d^2}-\frac{b}{2c^3d^2(cx}$$

```
[Out] (a*x)/(c^2*d^2) - b/(2*c^3*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c^3*d^2) +
(b*x*ArcTanh[c*x])/(c^2*d^2) - (a + b*ArcTanh[c*x])/(c^3*d^2*(1 + c*x)) + (
2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^3*d^2) + (b*Log[1 - c^2*x^2])/(
2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rubi [A] time = 0.185236, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5910, 260, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(cx+1)}+\frac{2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}+\frac{ax}{c^2d^2}+\frac{b\log\left(1-c^2x^2\right)}{2c^3d^2}-\frac{b}{2c^3d^2(cx}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (a*x)/(c^2*d^2) - b/(2*c^3*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c^3*d^2) +
(b*x*ArcTanh[c*x])/(c^2*d^2) - (a + b*ArcTanh[c*x])/(c^3*d^2*(1 + c*x)) + (
2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^3*d^2) + (b*Log[1 - c^2*x^2])/(
2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}dx = \int\left(\frac{a+b\tanh^{-1}(cx)}{c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)^2}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)}\right)dx$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^2d^2}+\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}dx}{c^2d^2}-\frac{2\int\frac{a+b\tanh^{-1}(cx)}{1+cx}dx}{c^2d^2}$$

$$=\frac{ax}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\int\frac{1}{(1+cx)(1-c^2x^2)}dx}{c^2d^2}+\frac{b}{\ldots}$$

$$=\frac{ax}{c^2d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}-\frac{(2b)\,\mathrm{Su}\ldots}{\ldots}$$

$$=\frac{ax}{c^2d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{b\log(1\ldots}{2c\ldots}$$

$$=\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)\log(\ldots}{c^3d^2}$$

$$=\frac{ax}{c^2d^2}-\frac{b}{2c^3d^2(1+cx)}+\frac{b\tanh^{-1}(cx)}{2c^3d^2}+\frac{bx\tanh^{-1}(cx)}{c^2d^2}-\frac{a+b\tanh^{-1}(cx)}{c^3d^2(1+cx)}+\frac{2\left(a+b\,\mathrm{t}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.575486, size = 121, normalized size = 0.81

$$\frac{b\left(-4\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2\log\left(1-c^2x^2\right)+\sinh\left(2\tanh^{-1}(cx)\right)-\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(2cx\ldots\right.\right.}{4c^3d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] (4*a*c*x - (4*a)/(1 + c*x) - 8*a*Log[1 + c*x] + b*(-Cosh[2*ArcTanh[c*x]] +
2*Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*
x]] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*ArcTan
h[c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

Maple [A] time = 0.055, size = 216, normalized size = 1.5

$$\frac{ax}{c^2d^2}-\frac{a}{c^3d^2(cx+1)}-2\frac{a\ln(cx+1)}{c^3d^2}+\frac{bx\mathrm{Artanh}(cx)}{c^2d^2}-\frac{b\mathrm{Artanh}(cx)}{c^3d^2(cx+1)}-2\frac{b\mathrm{Artanh}(cx)\ln(cx+1)}{c^3d^2}+\frac{b}{c^3d^2}\ln\left(-\frac{cx}{2}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] a*x/c^2/d^2-1/c^3*a/d^2/(c*x+1)-2/c^3*a/d^2*ln(c*x+1)+b*x*arctanh(c*x)/c^2/
d^2-1/c^3*b/d^2*arctanh(c*x)/(c*x+1)-2/c^3*b/d^2*arctanh(c*x)*ln(c*x+1)+1/c
^3*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/c^3*b/d^2*ln(-1/2*c*x+1/2)*ln(c
*x+1)+1/c^3*b/d^2*dilog(1/2+1/2*c*x)+1/2/c^3*b/d^2*ln(c*x+1)^2+1/4/c^3*b/d^
2*ln(c*x-1)-1/2*b/c^3/d^2/(c*x+1)+3/4/c^3*b/d^2*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\left(c^3\left(\frac{2}{c^7d^2x+c^6d^2}-\frac{4x}{c^5d^2}+\frac{5\log(cx+1)}{c^6d^2}-\frac{\log(cx-1)}{c^6d^2}\right)-4c^3\int\frac{x^3\log(cx+1)}{c^5d^2x^3+c^4d^2x^2-c^3d^2x-c^2d^2}\,dx-2c^2\left(\frac{2}{c^6d^2x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(c^3*(2/(c^7*d^2*x + c^6*d^2) - 4*x/(c^5*d^2) + 5*log(c*x + 1)/(c^6*d^
2) - log(c*x - 1)/(c^6*d^2)) - 4*c^3*integrate(x^3*log(c*x + 1)/(c^5*d^2*x^
3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) - 2*c^2*(2/(c^6*d^2*x + c^5*d^2)
 + 3*log(c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 12*c^2*integrate(x^
2*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2), x) + 16*c
*integrate(x*log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c^3*d^2*x - c^2*d^2)
, x) + 4*(c^2*x^2 + c*x - 2*(c*x + 1)*log(c*x + 1) - 1)*log(-c*x + 1)/(c^4*
d^2*x + c^3*d^2) + 2/(c^4*d^2*x + c^3*d^2) - log(c*x + 1)/(c^3*d^2) + log(c
*x - 1)/(c^3*d^2) + 8*integrate(log(c*x + 1)/(c^5*d^2*x^3 + c^4*d^2*x^2 - c
^3*d^2*x - c^2*d^2), x))*b - a*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*l
og(c*x + 1)/(c^3*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{artanh}(cx)+ax^2}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{ax^2}{c^2x^2+2cx+1}\,dx+\int\frac{bx^2\,\mathrm{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x**2*atanh(c*x)/(
c**2*x**2 + 2*c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\mathrm{artanh}(cx)+a)x^2}{(cdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^2, x)
```

3.53 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=106

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(cx+1)}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b}{2c^2d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}$$

```
[Out] b/(2*c^2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*c^2*d^2) + (a + b*ArcTanh[c*x
])/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^2*d^2)
+ (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rubi [A] time = 0.143107, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5940, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(cx+1)}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b}{2c^2d^2(cx+1)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] b/(2*c^2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*c^2*d^2) + (a + b*ArcTanh[c*x
])/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^2*d^2)
+ (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^2}\,dx = \int \left(-\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}\right)dx$$
$$= -\frac{\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{cd^2}+\frac{\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{cd^2}$$
$$= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}-\frac{b\int \frac{1}{(1+cx)(1-c^2x^2)}\,dx}{cd^2}+\frac{b\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}$$
$$= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+cx}\right)}{c^2d^2}-\frac{b$$
$$= \frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c^2d^2}-\frac{b\int\left(\frac{1}{2(1+cx)^2}-\frac{}{2(}\right.}{cd^2}$$
$$= \frac{b}{2c^2d^2(1+cx)}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c^2d^2}+$$
$$= \frac{b}{2c^2d^2(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^2d^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^2(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^2d^2}+\frac{b\mathrm{I}$$

Mathematica [A] time = 0.378704, size = 99, normalized size = 0.93

$$\frac{b\left(2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)+\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(-2\log\left(e^{-2\tanh^{-1}(cx)}+\right.\right.}{4c^2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^2,x]
```

```
[Out] ((4*a)/(1 + c*x) + 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]] + 2*PolyLog[2
, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*Log[1 +
E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]))/(4*c^
2*d^2)
```

Maple [A] time = 0.046, size = 192, normalized size = 1.8

$$\frac{a}{c^2d^2(cx+1)}+\frac{a\ln(cx+1)}{c^2d^2}+\frac{b\text{Artanh}(cx)}{c^2d^2(cx+1)}+\frac{b\text{Artanh}(cx)\ln(cx+1)}{c^2d^2}+\frac{b\ln(cx-1)}{4c^2d^2}+\frac{b}{2c^2d^2(cx+1)}-\frac{b\ln(cx+1)}{4c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] 1/c^2*a/d^2/(c*x+1)+1/c^2*a/d^2*ln(c*x+1)+1/c^2*b/d^2*arctanh(c*x)/(c*x+1)+
1/c^2*b/d^2*arctanh(c*x)*ln(c*x+1)+1/4/c^2*b/d^2*ln(c*x-1)+1/2*b/c^2/d^2/(c
*x+1)-1/4/c^2*b/d^2*ln(c*x+1)-1/2/c^2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x
)+1/2/c^2*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2/c^2*b/d^2*dilog(1/2+1/2*c*x)
-1/4/c^2*b/d^2*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}\left(8c^2\int\frac{x^2\log(cx+1)}{c^4d^2x^3+c^3d^2x^2-c^2d^2x-cd^2}\,dx-c\left(\frac{2}{c^4d^2x+c^3d^2}+\frac{\log(cx+1)}{c^3d^2}-\frac{\log(cx-1)}{c^3d^2}\right)+4c\int\frac{x\log(cx+1)}{c^4d^2x^3+c^3d^2x^2-c^2d^2x-cd^2}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*c^2*integrate(x^2*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^2*d^2*
x - c*d^2), x) - c*(2/(c^4*d^2*x + c^3*d^2) + log(c*x + 1)/(c^3*d^2) - log(
c*x - 1)/(c^3*d^2)) + 4*c*integrate(x*log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x
^2 - c^2*d^2*x - c*d^2), x) - 4*((c*x + 1)*log(c*x + 1) + 1)*log(-c*x + 1)/
(c^3*d^2*x + c^2*d^2) + 2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) +
log(c*x - 1)/(c^2*d^2) + 4*integrate(log(c*x + 1)/(c^4*d^2*x^3 + c^3*d^2*x^
2 - c^2*d^2*x - c*d^2), x))*b + a*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(
c^2*d^2))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx\,\text{artanh}(cx)+ax}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(c^2*d^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax}{c^2x^2+2cx+1}\,dx + \int \frac{bx\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b*x*atanh(c*x)/(c**2*x
**2 + 2*c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)x}{(cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(c*d*x + d)^2, x)
```

3.54 $\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{a+b\tanh^{-1}(cx)}{cd^2(cx+1)}-\frac{b}{2cd^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2cd^2}$$

```
[Out] -b/(2*c*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c*d^2) - (a + b*ArcTanh[c*x])/
(c*d^2*(1 + c*x))
```

Rubi [A] time = 0.0453211, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5926, 627, 44, 207}

$$-\frac{a+b\tanh^{-1}(cx)}{cd^2(cx+1)}-\frac{b}{2cd^2(cx+1)}+\frac{b\tanh^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

```
[Out] -b/(2*c*d^2*(1 + c*x)) + (b*ArcTanh[c*x])/(2*c*d^2) - (a + b*ArcTanh[c*x])/
(c*d^2*(1 + c*x))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^2}\,dx = -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} + \frac{b\int \frac{1}{(d+cdx)(1-c^2x^2)}\,dx}{d}$$
$$= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} + \frac{b\int \frac{1}{\left(\frac{1}{d}-\frac{cx}{d}\right)(d+cdx)^2}\,dx}{d}$$
$$= -\frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} + \frac{b\int \left(\frac{1}{2d(1+cx)^2} - \frac{1}{2d(-1+c^2x^2)}\right)dx}{d}$$
$$= -\frac{b}{2cd^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)} - \frac{b\int \frac{1}{-1+c^2x^2}\,dx}{2d^2}$$
$$= -\frac{b}{2cd^2(1+cx)} + \frac{b\tanh^{-1}(cx)}{2cd^2} - \frac{a+b\tanh^{-1}(cx)}{cd^2(1+cx)}$$

Mathematica [A] time = 0.0680515, size = 64, normalized size = 1.12

$$\frac{-4a-(bcx+b)\log(1-cx)+b\log(cx+1)+bcx\log(cx+1)-4b\tanh^{-1}(cx)-2b}{4cd^2(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^2,x]
```

```
[Out] (-4*a - 2*b - 4*b*ArcTanh[c*x] - (b + b*c*x)*Log[1 - c*x] + b*Log[1 + c*x]
+ b*c*x*Log[1 + c*x])/(4*c*d^2*(1 + c*x))
```

Maple [A] time = 0.035, size = 84, normalized size = 1.5

$$-\frac{a}{cd^2(cx+1)} - \frac{b\mathrm{Artanh}(cx)}{cd^2(cx+1)} - \frac{b\ln(cx-1)}{4cd^2} - \frac{b}{2cd^2(cx+1)} + \frac{b\ln(cx+1)}{4cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d)^2,x)
```

```
[Out] -1/c*a/d^2/(c*x+1)-1/c*b/d^2*arctanh(c*x)/(c*x+1)-1/4/c*b/d^2*ln(c*x-1)-1/2
*b/c/d^2/(c*x+1)+1/4/c*b/d^2*ln(c*x+1)
```

Maxima [A] time = 0.952915, size = 130, normalized size = 2.28

$$-\frac{1}{4}\left(c\left(\frac{2}{c^3d^2x+c^2d^2} - \frac{\log(cx+1)}{c^2d^2} + \frac{\log(cx-1)}{c^2d^2}\right) + \frac{4\,\mathrm{artanh}(cx)}{c^2d^2x+cd^2}\right)b - \frac{a}{c^2d^2x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c
^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*b - a/(c^2*d^2*x + c*d^2)
```

Fricas [A] time = 2.07068, size = 104, normalized size = 1.82

$$\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)-4a-2b}{4\left(c^2d^2x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1)) - 4*a - 2*b)/(c^2*d^2*x + c*d^2)
```

Sympy [A] time = 4.21529, size = 121, normalized size = 2.12

$$\begin{cases} -\frac{2a}{2c^2d^2x+2cd^2}+\frac{bcx\operatorname{atanh}(cx)}{2c^2d^2x+2cd^2}-\frac{b\operatorname{atanh}(cx)}{2c^2d^2x+2cd^2}-\frac{b}{2c^2d^2x+2cd^2} & \text{for } d\neq 0 \\ \tilde{\infty}\left(ax+bx\operatorname{atanh}(cx)+\frac{b\log\left(x-\frac{1}{c}\right)}{c}+\frac{b\operatorname{atanh}(cx)}{c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d)**2,x)
```

```
[Out] Piecewise((-2*a/(2*c**2*d**2*x + 2*c*d**2) + b*c*x*atanh(c*x)/(2*c**2*d**2*
x + 2*c*d**2) - b*atanh(c*x)/(2*c**2*d**2*x + 2*c*d**2) - b/(2*c**2*d**2*x
+ 2*c*d**2), Ne(d, 0)), (zoo*(a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*a
tanh(c*x)/c), True))
```

Giac [A] time = 1.13747, size = 131, normalized size = 2.3

$$-\frac{1}{4}\left(cd^2\left(\frac{\log\left(\left|-\frac{2d}{cdx+d}+1\right|\right)}{c^2d^4}+\frac{2}{(cdx+d)c^2d^3}\right)+\frac{2\log\left(-\frac{cx+1}{cx-1}\right)}{(cdx+d)cd}\right)b-\frac{a}{(cdx+d)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/4*(c*d^2*(log(abs(-2*d/(c*d*x + d) + 1))/(c^2*d^4) + 2/((c*d*x + d)*c^2*
d^3)) + 2*log(-(c*x + 1)/(c*x - 1))/((c*d*x + d)*c*d))*b - a/((c*d*x + d)*c
*d)
```

3.55 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{b\text{PolyLog}(2,-cx)}{2d^2}+\frac{b\text{PolyLog}(2,cx)}{2d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{a+b\tanh^{-1}(cx)}{d^2(cx+1)}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx\right.}{d^2}$$

```
[Out] b/(2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*d^2) + (a + b*ArcTanh[c*x])/(d^2*
(1 + c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 -
 (b*PolyLog[2, -(c*x)])/(2*d^2) + (b*PolyLog[2, c*x])/(2*d^2) - (b*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rubi [A] time = 0.174434, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5940, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}(2,-cx)}{2d^2}+\frac{b\text{PolyLog}(2,cx)}{2d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{a+b\tanh^{-1}(cx)}{d^2(cx+1)}+\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

```
[Out] b/(2*d^2*(1 + c*x)) - (b*ArcTanh[c*x])/(2*d^2) + (a + b*ArcTanh[c*x])/(d^2*
(1 + c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 -
 (b*PolyLog[2, -(c*x)])/(2*d^2) + (b*PolyLog[2, c*x])/(2*d^2) - (b*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)^2} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} - \frac{c\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^2}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{b\text{Li}_2(cx)}{2d^2} - \frac{(bc)\int}{}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{b\text{Li}_2(cx)}{2d^2} - \frac{b\,\text{Sub}}{}$$

$$= \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{b\text{Li}_2(cx)}{2d^2} - \frac{b\text{Li}_2\left(\right.}{}$$

$$= \frac{b}{2d^2(1+cx)} + \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} - \frac{b\text{Li}_2(-cx)}{2d^2} + \frac{bL}{}$$

$$= \frac{b}{2d^2(1+cx)} - \frac{b\tanh^{-1}(cx)}{2d^2} + \frac{a+b\tanh^{-1}(cx)}{d^2(1+cx)} + \frac{a\log(x)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} -$$

Mathematica [A] time = 0.407352, size = 101, normalized size = 0.81

$$\frac{b\left(-2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)+\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)\right.}{4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^2),x]
```

```
[Out] ((4*a)/(1 + c*x) + 4*a*Log[x] - 4*a*Log[1 + c*x] + b*(Cosh[2*ArcTanh[c*x]]
- 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]]
+ 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c
*x]]))/(4*d^2)
```

Maple [A] time = 0.053, size = 221, normalized size = 1.8

$$\frac{a\ln(cx)}{d^2}+\frac{a}{d^2(cx+1)}-\frac{a\ln(cx+1)}{d^2}+\frac{b\text{Artanh}(cx)\ln(cx)}{d^2}+\frac{b\text{Artanh}(cx)}{d^2(cx+1)}-\frac{b\text{Artanh}(cx)\ln(cx+1)}{d^2}-\frac{b\ln(cx-}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x)
```

```
[Out] a/d^2*ln(c*x)+a/d^2/(c*x+1)-a/d^2*ln(c*x+1)+b/d^2*arctanh(c*x)*ln(c*x)+b/d^
2*arctanh(c*x)/(c*x+1)-b/d^2*arctanh(c*x)*ln(c*x+1)-1/2*b/d^2*ln(-1/2*c*x+1
/2)*ln(c*x+1)+1/2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2*b/d^2*dilog(1/
2+1/2*c*x)+1/4*b/d^2*ln(c*x+1)^2+1/4*b/d^2*ln(c*x-1)+1/2*b/d^2/(c*x+1)-1/4*
b/d^2*ln(c*x+1)-1/2*b/d^2*dilog(c*x)-1/2*b/d^2*dilog(c*x+1)-1/2*b/d^2*ln(c*
x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{1}{cd^2x+d^2}-\frac{\log(cx+1)}{d^2}+\frac{\log(x)}{d^2}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{c^2d^2x^3+2cd^2x^2+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/2*b*integrate((lo
g(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}(cx)+a}{c^2d^2x^3+2cd^2x^2+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^3 + 2*c*d^2*x^2 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^3+2cx^2+x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b*atanh(c*x)/(c**2*x*
*3 + 2*c*x**2 + x), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\,(cx) + a}{(cdx + d)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x), x)
```

3.56 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^2}\,dx$

Optimal. Leaf size=171

$$\frac{bc\mathrm{PolyLog}(2,-cx)}{d^2}-\frac{bc\mathrm{PolyLog}(2,cx)}{d^2}+\frac{bc\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^2}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{d^2x}-\frac{2c}{}$$

```
[Out] -(b*c)/(2*d^2*(1 + c*x)) + (b*c*ArcTanh[c*x])/(2*d^2) - (a + b*ArcTanh[c*x]
)/(d^2*x) - (c*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*a*c*Log[x])/d^2 +
 (b*c*Log[x])/d^2 - (2*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 - (b*c*
Log[1 - c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, -(c*x)])/d^2 - (b*c*PolyLog[2,
c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + c*x)])/d^2
```

Rubi [A] time = 0.221096, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5940, 5916, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{bc\mathrm{PolyLog}(2,-cx)}{d^2}-\frac{bc\mathrm{PolyLog}(2,cx)}{d^2}+\frac{bc\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^2}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{d^2x}-\frac{2c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```


```
[Out] -(b*c)/(2*d^2*(1 + c*x)) + (b*c*ArcTanh[c*x])/(2*d^2) - (a + b*ArcTanh[c*x]
)/(d^2*x) - (c*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*a*c*Log[x])/d^2 +
 (b*c*Log[x])/d^2 - (2*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 - (b*c*
Log[1 - c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, -(c*x)])/d^2 - (b*c*PolyLog[2,
c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + c*x)])/d^2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)^2} + \frac{2c^2\left(a+b\tanh^{-1}\right.}{d^2(1+cx)}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{(2c)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} + \frac{c^2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} + \frac{\left(2c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2}$$

$$= -\frac{bc}{2d^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} + \frac{bc\log(x)}{d^2} - \frac{2c\left(a+\right.}{}$$

$$= -\frac{bc}{2d^2(1+cx)} + \frac{bc\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2ac\log(x)}{d^2} + \frac{b}{}$$

Mathematica [A] time = 0.813649, size = 140, normalized size = 0.82

$$\frac{bc\left(4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+\sinh\left(2\tanh^{-1}(cx)\right)-\cosh\left(2\tanh^{-1}(cx)\right)+\tanh^{-1}(cx)\left(-\frac{4}{cx}-8\right.\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^2),x]
```

```
[Out] ((-4*a)/x - (4*a*c)/(1 + c*x) - 8*a*c*Log[x] + 8*a*c*Log[1 + c*x] + b*c*(-C
osh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2*A
rcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2*Ar
cTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]])))/(4
*d^2)
```

Maple [A] time = 0.058, size = 269, normalized size = 1.6

$$-\frac{a}{d^2x} - 2\frac{ac\ln(cx)}{d^2} - \frac{ac}{d^2(cx+1)} + 2\frac{ac\ln(cx+1)}{d^2} - \frac{b\text{Artanh}(cx)}{d^2x} - 2\frac{bc\text{Artanh}(cx)\ln(cx)}{d^2} - \frac{bc\text{Artanh}(cx)}{d^2(cx+1)} + 2\frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x)
```

```
[Out] -a/d^2/x-2*c*a/d^2*ln(c*x)-c*a/d^2/(c*x+1)+2*c*a/d^2*ln(c*x+1)-b/d^2*arctan
h(c*x)/x-2*c*b/d^2*arctanh(c*x)*ln(c*x)-c*b/d^2*arctanh(c*x)/(c*x+1)+2*c*b/
```

```
d^2*arctanh(c*x)*ln(c*x+1)-3/4*c*b/d^2*ln(c*x-1)+c*b/d^2*ln(c*x)-1/2*b*c/d^
2/(c*x+1)-1/4*c*b/d^2*ln(c*x+1)+c*b/d^2*dilog(c*x)+c*b/d^2*dilog(c*x+1)+c*b
/d^2*ln(c*x)*ln(c*x+1)-c*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+c*b/d^2*ln(
-1/2*c*x+1/2)*ln(c*x+1)-c*b/d^2*dilog(1/2+1/2*c*x)-1/2*c*b/d^2*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{2\,cx+1}{cd^2x^2+d^2x}-\frac{2\,c\log\left(cx+1\right)}{d^2}+\frac{2\,c\log\left(x\right)}{d^2}\right)+\frac{1}{2}\,b\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{c^2d^2x^4+2\,cd^2x^3+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d^2
) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^2*d^2*x^4 + 2*c*d^2*x
^3 + d^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx\right)+a}{c^2d^2x^4+2\,cd^2x^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^4 + 2*c*d^2*x^3 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^2x^4+2cx^3+x^2}\,dx+\int\frac{b\,\text{atanh}\left(cx\right)}{c^2x^4+2cx^3+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b*atanh(c*x)/(c**2
*x**4 + 2*c*x**3 + x**2), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{artanh}\left(cx\right)+a}{\left(cdx+d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^2), x)
```

3.57 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{3bc^2\text{PolyLog}(2,-cx)}{2d^2}+\frac{3bc^2\text{PolyLog}(2,cx)}{2d^2}-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)}{}$$

```
[Out] -(b*c)/(2*d^2*x) + (b*c^2)/(2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])/(2*d^2*
x^2) + (2*c*(a + b*ArcTanh[c*x]))/(d^2*x) + (c^2*(a + b*ArcTanh[c*x]))/(d^2
*(1 + c*x)) + (3*a*c^2*Log[x])/d^2 - (2*b*c^2*Log[x])/d^2 + (3*c^2*(a + b*A
rcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (b*c^2*Log[1 - c^2*x^2])/d^2 - (3*b*c^
2*PolyLog[2, -(c*x)])/(2*d^2) + (3*b*c^2*PolyLog[2, c*x])/(2*d^2) - (3*b*c^
2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rubi [A] time = 0.257384, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 16, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{3bc^2\text{PolyLog}(2,-cx)}{2d^2}+\frac{3bc^2\text{PolyLog}(2,cx)}{2d^2}-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}+\frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(cx+1)}+\frac{3c^2\log\left(\frac{2}{cx+1}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

```
[Out] -(b*c)/(2*d^2*x) + (b*c^2)/(2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])/(2*d^2*
x^2) + (2*c*(a + b*ArcTanh[c*x]))/(d^2*x) + (c^2*(a + b*ArcTanh[c*x]))/(d^2
*(1 + c*x)) + (3*a*c^2*Log[x])/d^2 - (2*b*c^2*Log[x])/d^2 + (3*c^2*(a + b*A
rcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (b*c^2*Log[1 - c^2*x^2])/d^2 - (3*b*c^
2*PolyLog[2, -(c*x)])/(2*d^2) + (3*b*c^2*PolyLog[2, c*x])/(2*d^2) - (3*b*c^
2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^2}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^2x^3} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x^2} + \frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2x} - \frac{c^3\left(a+b\tanh^{-1}(\right.}{d^2(1+cx)^2}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2c)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{\left(3c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} - \frac{c^3\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3ac^2\log(x)}{d^2} + \frac{3c^2\left(a\right.}{}$$

$$= -\frac{bc}{2d^2x} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3ac^2\log(x)}{d^2}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(c\right.}{d^2(1+cx)}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2}{2d^2(1+cx)} + \frac{bc^2\tanh^{-1}(cx)}{2d^2} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a\right.}{}$$

$$= -\frac{bc}{2d^2x} + \frac{bc^2}{2d^2(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)}$$

Mathematica [A] time = 1.13541, size = 189, normalized size = 0.89

$$-6bc^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) + \frac{4ac^2}{cx+1} + 12ac^2\log(x) - 12ac^2\log(cx+1) + \frac{8ac}{x} - \frac{2a}{x^2} - 8bc^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 2b\tanh^{-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^2),x]
```

```
[Out] ((-2*a)/x^2 + (8*a*c)/x - (2*b*c)/x + (4*a*c^2)/(1 + c*x) + b*c^2*Cosh[2*Ar
cTanh[c*x]] + 12*a*c^2*Log[x] - 12*a*c^2*Log[1 + c*x] - 8*b*c^2*Log[(c*x)/S
qrt[1 - c^2*x^2]] - 6*b*c^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - b*c^2*Sinh[2*
ArcTanh[c*x]] + 2*b*ArcTanh[c*x]*(c^2 - x^(-2) + (4*c)/x + c^2*Cosh[2*ArcTa
nh[c*x]] + 6*c^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*Sinh[2*ArcTanh[c*x]]))/
(4*d^2)
```

Maple [A] time = 0.06, size = 338, normalized size = 1.6

$$-\frac{a}{2\,d^2x^2}+2\,\frac{ac}{d^2x}+3\,\frac{ac^2\ln(cx)}{d^2}+\frac{ac^2}{d^2\,(cx+1)}-3\,\frac{ac^2\ln(cx+1)}{d^2}-\frac{b\mathrm{Artanh}\,(cx)}{2\,d^2x^2}+2\,\frac{bc\mathrm{Artanh}\,(cx)}{d^2x}+3\,\frac{c^2b\mathrm{Artanh}\,(cx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x)
```

```
[Out] -1/2*a/d^2/x^2+2*c*a/d^2/x+3*c^2*a/d^2*ln(c*x)+c^2*a/d^2/(c*x+1)-3*c^2*a/d^
2*ln(c*x+1)-1/2*b/d^2*arctanh(c*x)/x^2+2*c*b/d^2*arctanh(c*x)/x+3*c^2*b/d^2
*arctanh(c*x)*ln(c*x)+c^2*b/d^2*arctanh(c*x)/(c*x+1)-3*c^2*b/d^2*arctanh(c*
x)*ln(c*x+1)-3/2*c^2*b/d^2*dilog(c*x)-3/2*c^2*b/d^2*dilog(c*x+1)-3/2*c^2*b/
d^2*ln(c*x)*ln(c*x+1)+3/2*c^2*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/2*c^
2*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2*c^2*b/d^2*dilog(1/2+1/2*c*x)+3/4*c^2
*b/d^2*ln(c*x+1)^2+c^2*b/d^2*ln(c*x-1)-1/2*b*c/d^2/x-2*c^2*b/d^2*ln(c*x)+1/
2*b*c^2/d^2/(c*x+1)+c^2*b/d^2*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{6\,c^2\log(cx+1)}{d^2}-\frac{6\,c^2\log(x)}{d^2}-\frac{6\,c^2x^2+3\,cx-1}{cd^2x^3+d^2x^2}\right)+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{c^2d^2x^5+2\,cd^2x^4+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x - 1)
/(c*d^2*x^3 + d^2*x^2)) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c
^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}\,(cx)+a}{c^2d^2x^5+2\,cd^2x^4+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^2*d^2*x^5 + 2*c*d^2*x^4 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^2x^5+2cx^4+x^3}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b*atanh(c*x)/(c**2
*x**5 + 2*c*x**4 + x**3), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\,(cx) + a}{(cdx + d)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^2*x^3), x)
```

3.58 $\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=227

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}+\frac{4\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(cx+1)^2}-\frac{6\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh\right.}{c^5d^3}$$

```
[Out] (-3*a*x)/(c^4*d^3) + (b*x)/(2*c^4*d^3) - b/(8*c^5*d^3*(1 + c*x)^2) + (15*b)
/(8*c^5*d^3*(1 + c*x)) - (19*b*ArcTanh[c*x])/(8*c^5*d^3) - (3*b*x*ArcTanh[c
*x])/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*c^3*d^3) - (a + b*ArcTanh[c*
x])/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh[c*x]))/(c^5*d^3*(1 + c*x))
- (6*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^5*d^3) - (3*b*Log[1 - c^2*x^
2])/(2*c^5*d^3) + (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rubi [A] time = 0.287584, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {5940, 5910, 260, 5916, 321, 206, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}+\frac{4\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(cx+1)^2}-\frac{6\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh\right.}{c^5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (-3*a*x)/(c^4*d^3) + (b*x)/(2*c^4*d^3) - b/(8*c^5*d^3*(1 + c*x)^2) + (15*b)
/(8*c^5*d^3*(1 + c*x)) - (19*b*ArcTanh[c*x])/(8*c^5*d^3) - (3*b*x*ArcTanh[c
*x])/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*c^3*d^3) - (a + b*ArcTanh[c*
x])/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh[c*x]))/(c^5*d^3*(1 + c*x))
- (6*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^5*d^3) - (3*b*Log[1 - c^2*x^
2])/(2*c^5*d^3) + (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx = \int\left(-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{x\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{a+b\tanh^{-1}(cx)}{c^4d^3(1+cx)^3}-\frac{4\left(a+b\tanh^{-1}(c\right.}{c^4d^3(1+cx)^2}\right.$$

$$=\frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^4d^3}-\frac{3\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^4d^3}-\frac{4\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^4d^3}+\frac{6\int\frac{a+b\tanh^{-1}(cx)}{1+cx}\,d}{c^4d^3}$$

$$=-\frac{3ax}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(1+cx)^2}+\frac{4\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3(1+cx)}-\frac{6\left(a+b\tan\right.}{}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^5d^3(1+cx)^2}+\frac{4\left(a+b\right.}{c^5d^3}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b\tanh^{-1}(cx)}{2c^5d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2c^3d^3}-\frac{a+b\tanh^{-1}}{2c^5d^3(1+c}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b}{8c^5d^3(1+cx)^2}+\frac{15b}{8c^5d^3(1+cx)}-\frac{b\tanh^{-1}(cx)}{2c^5d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x^2(}{}$$

$$=-\frac{3ax}{c^4d^3}+\frac{bx}{2c^4d^3}-\frac{b}{8c^5d^3(1+cx)^2}+\frac{15b}{8c^5d^3(1+cx)}-\frac{19b\tanh^{-1}(cx)}{8c^5d^3}-\frac{3bx\tanh^{-1}(cx)}{c^4d^3}+\frac{x}{}$$

Mathematica [A] time = 0.829683, size = 189, normalized size = 0.83

$$\frac{b\left(96\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-48\log\left(1-c^2x^2\right)+4\tanh^{-1}(cx)\left(4c^2x^2-24cx-48\log\left(e^{-2\tanh^{-1}(cx)}+1\right)-14\sinh\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (-96*a*c*x + 16*a*c^2*x^2 - (16*a)/(1 + c*x)^2 + (128*a)/(1 + c*x) + 192*a*
Log[1 + c*x] + b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] -
 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcT
anh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*c^2*x^2
+ 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh
[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^5*d^3)
```

Maple [A] time = 0.049, size = 319, normalized size = 1.4

$$\frac{ax^2}{2\,c^3d^3}-3\,\frac{ax}{c^4d^3}-\frac{a}{2\,c^5d^3\,(cx+1)^2}+4\,\frac{a}{c^5d^3\,(cx+1)}+6\,\frac{a\ln(cx+1)}{c^5d^3}+\frac{b\text{Artanh}\,(cx)\,x^2}{2\,c^3d^3}-3\,\frac{bx\text{Artanh}\,(cx)}{c^4d^3}-\frac{b\text{Artanh}}{2\,c^5d^3\,(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] 1/2/c^3*a/d^3*x^2-3*a*x/c^4/d^3-1/2/c^5*a/d^3/(c*x+1)^2+4/c^5*a/d^3/(c*x+1)
+6/c^5*a/d^3*ln(c*x+1)+1/2/c^3*b/d^3*arctanh(c*x)*x^2-3*b*x*arctanh(c*x)/c^
4/d^3-1/2/c^5*b/d^3*arctanh(c*x)/(c*x+1)^2+4/c^5*b/d^3*arctanh(c*x)/(c*x+1)
+6/c^5*b/d^3*arctanh(c*x)*ln(c*x+1)-3/c^5*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2
*c*x)+3/c^5*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/c^5*b/d^3*dilog(1/2+1/2*c*x)
-3/2/c^5*b/d^3*ln(c*x+1)^2+1/2*b*x/c^4/d^3+1/2/c^5*b/d^3-5/16/c^5*b/d^3*ln(
c*x-1)-1/8*b/c^5/d^3/(c*x+1)^2+15/8*b/c^5/d^3/(c*x+1)-43/16/c^5*b/d^3*ln(c*
x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(c^5*(2*(9*c*x + 8)/(c^12*d^3*x^2 + 2*c^11*d^3*x + c^10*d^3) + 4*(c*x^
2 - 4*x)/(c^9*d^3) + 31*log(c*x + 1)/(c^10*d^3) + log(c*x - 1)/(c^10*d^3))
+ 32*c^5*integrate(1/2*x^5*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^
5*d^3*x - c^4*d^3), x) + 3*c^4*(2*(7*c*x + 6)/(c^11*d^3*x^2 + 2*c^10*d^3*x
+ c^9*d^3) - 8*x/(c^8*d^3) + 17*log(c*x + 1)/(c^9*d^3) - log(c*x - 1)/(c^9*
d^3)) - 32*c^4*integrate(1/2*x^4*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3
- 2*c^5*d^3*x - c^4*d^3), x) - 15*c^3*(2*(5*c*x + 4)/(c^10*d^3*x^2 + 2*c^9*
d^3*x + c^8*d^3) + 7*log(c*x + 1)/(c^8*d^3) + log(c*x - 1)/(c^8*d^3)) + 192
*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^
3*x - c^4*d^3), x) + 9*c^2*(2*(3*c*x + 2)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*
d^3) + log(c*x + 1)/(c^7*d^3) - log(c*x - 1)/(c^7*d^3)) + 576*c^2*integrate
(1/2*x^2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3)
, x) + 9*c*(2*x/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - log(c*x + 1)/(c^6*d
^3) + log(c*x - 1)/(c^6*d^3)) + 576*c*integrate(1/2*x*log(c*x + 1)/(c^8*d^3
*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^3), x) - 8*(c^4*x^4 - 4*c^3*x^3
- 11*c^2*x^2 + 2*c*x + 12*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) + 7)*log(-c*x
+ 1)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + 14*(c*x + 2)/(c^7*d^3*x^2 + 2*
c^6*d^3*x + c^5*d^3) - 7*log(c*x + 1)/(c^5*d^3) + 7*log(c*x - 1)/(c^5*d^3)
+ 192*integrate(1/2*log(c*x + 1)/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x
 - c^4*d^3), x))*b + 1/2*a*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^
3) + (c*x^2 - 6*x)/(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^4 \operatorname{artanh}(cx) + ax^4}{c^3d^3x^3 + 3\,c^2d^3x^2 + 3\,cd^3x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arctanh(c*x) + a*x^4)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^4}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^4\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx)+a)x^4}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^4/(c*d*x + d)^3, x)
```

3.59 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=194

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(cx+1)^2}+\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{ax}{c^3d^3}+\frac{b\,\mathrm{l}}{}$$

```
[Out] (a*x)/(c^3*d^3) + b/(8*c^4*d^3*(1 + c*x)^2) - (11*b)/(8*c^4*d^3*(1 + c*x))
+ (11*b*ArcTanh[c*x])/(8*c^4*d^3) + (b*x*ArcTanh[c*x])/(c^3*d^3) + (a + b*A
rcTanh[c*x])/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x]))/(c^4*d^3*(1
 + c*x)) + (3*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^3) + (b*Log[1 -
 c^2*x^2])/(2*c^4*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Rubi [A] time = 0.248, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5910, 260, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2c^4d^3(cx+1)^2}+\frac{3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}+\frac{ax}{c^3d^3}+\frac{b\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```



```
[Out] (a*x)/(c^3*d^3) + b/(8*c^4*d^3*(1 + c*x)^2) - (11*b)/(8*c^4*d^3*(1 + c*x))
+ (11*b*ArcTanh[c*x])/(8*c^4*d^3) + (b*x*ArcTanh[c*x])/(c^3*d^3) + (a + b*A
rcTanh[c*x])/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x]))/(c^4*d^3*(1
 + c*x)) + (3*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/(c^4*d^3) + (b*Log[1 -
 c^2*x^2])/(2*c^4*d^3) - (3*b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx = \int\left(\frac{a+b\tanh^{-1}(cx)}{c^3d^3} - \frac{a+b\tanh^{-1}(cx)}{c^3d^3(1+cx)^3} + \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}\right.$$

$$= \frac{\int\left(a+b\tanh^{-1}(cx)\right)dx}{c^3d^3} - \frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}dx}{c^3d^3} + \frac{3\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}dx}{c^3d^3} - \frac{3\int\frac{a+b\tanh^{-1}(cx)}{1+cx}}{c^3d^3}$$

$$= \frac{ax}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^4d^3} - \ldots$$

$$= \frac{ax}{c^3d^3} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx\right.}{c^4d^3}$$

$$= \frac{ax}{c^3d^3} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3(1+cx)} + \frac{3\left(a+b\tanh^{-1}(cx\right.}{c^4d^3}$$

$$= \frac{ax}{c^3d^3} + \frac{b}{8c^4d^3(1+cx)^2} - \frac{11b}{8c^4d^3(1+cx)} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tanh^{-1}(cx)}{2c^4d^3(1+cx)^2} - \frac{3\left(a+b\right.}{c^4d}$$

$$= \frac{ax}{c^3d^3} + \frac{b}{8c^4d^3(1+cx)^2} - \frac{11b}{8c^4d^3(1+cx)} + \frac{11b\tanh^{-1}(cx)}{8c^4d^3} + \frac{bx\tanh^{-1}(cx)}{c^3d^3} + \frac{a+b\tan}{2c^4d^3(1}$$

Mathematica [A] time = 0.708057, size = 167, normalized size = 0.86

$$b\left(-48\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+16\log\left(1-c^2x^2\right)+20\sinh\left(2\tanh^{-1}(cx)\right)-\sinh\left(4\tanh^{-1}(cx)\right)-20\cosh\left(2\text{ta}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] (32*a*c*x + (16*a)/(1 + c*x)^2 - (96*a)/(1 + c*x) - 96*a*Log[1 + c*x] + b*(
-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 - c^2*x^2] - 48*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]
*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24*Log[1 + E^(-2
*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*
ArcTanh[c*x]]))/(32*c^4*d^3)
```

Maple [A] time = 0.053, size = 270, normalized size = 1.4

$$\frac{ax}{c^3d^3} + \frac{a}{2\,c^4d^3\,(cx+1)^2} - 3\,\frac{a}{c^4d^3\,(cx+1)} - 3\,\frac{a\ln(cx+1)}{c^4d^3} + \frac{bx\text{Artanh}(cx)}{c^3d^3} + \frac{b\text{Artanh}(cx)}{2\,c^4d^3\,(cx+1)^2} - 3\,\frac{b\text{Artanh}(cx)}{c^4d^3\,(cx+1)} - 3\,\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] a*x/c^3/d^3+1/2/c^4*a/d^3/(c*x+1)^2-3/c^4*a/d^3/(c*x+1)-3/c^4*a/d^3*ln(c*x+
1)+b*x*arctanh(c*x)/c^3/d^3+1/2/c^4*b/d^3*arctanh(c*x)/(c*x+1)^2-3/c^4*b/d^
3*arctanh(c*x)/(c*x+1)-3/c^4*b/d^3*arctanh(c*x)*ln(c*x+1)+3/2/c^4*b/d^3*ln(
-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/2/c^4*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2/
c^4*b/d^3*dilog(1/2+1/2*c*x)+3/4/c^4*b/d^3*ln(c*x+1)^2-3/16/c^4*b/d^3*ln(c*
x-1)+1/8*b/c^4/d^3/(c*x+1)^2-11/8*b/c^4/d^3/(c*x+1)+19/16/c^4*b/d^3*ln(c*x+
```

```
1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/32*(2*c^4*(2*(7*c*x + 6)/(c^10*d^3*x^2 + 2*c^9*d^3*x + c^8*d^3) - 8*x/(c
^7*d^3) + 17*log(c*x + 1)/(c^8*d^3) - log(c*x - 1)/(c^8*d^3)) - 32*c^4*inte
grate(1/2*x^4*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3
*d^3), x) - 6*c^3*(2*(5*c*x + 4)/(c^9*d^3*x^2 + 2*c^8*d^3*x + c^7*d^3) + 7*
log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c^7*d^3)) + 128*c^3*integrate(1/2*x^
3*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) +
288*c^2*integrate(1/2*x^2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4
*d^3*x - c^3*d^3), x) + 9*c*(2*x/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - lo
g(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3)) + 288*c*integrate(1/2*x*log(
c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^3*d^3), x) + 8*(2*c
^3*x^3 + 4*c^2*x^2 - 4*c*x - 6*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1) - 5)*log(
-c*x + 1)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) + 10*(c*x + 2)/(c^6*d^3*x^2
 + 2*c^5*d^3*x + c^4*d^3) - 5*log(c*x + 1)/(c^4*d^3) + 5*log(c*x - 1)/(c^4*
d^3) + 96*integrate(1/2*log(c*x + 1)/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d
^3*x - c^3*d^3), x))*b - 1/2*a*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^
4*d^3) - 2*x/(c^3*d^3) + 6*log(c*x + 1)/(c^4*d^3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3\,\text{artanh}\,(cx) + ax^3}{c^3d^3x^3 + 3\,c^2d^3x^2 + 3\,cd^3x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^3\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)x^3}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(c*d*x + d)^3, x)
```

3.60 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=150

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(cx+1)^2}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{7b}{8c^3d^3(cx+1)}-\frac{8}{}$$

```
[Out] -b/(8*c^3*d^3*(1 + c*x)^2) + (7*b)/(8*c^3*d^3*(1 + c*x)) - (7*b*ArcTanh[c*x
])/(8*c^3*d^3) - (a + b*ArcTanh[c*x])/(2*c^3*d^3*(1 + c*x)^2) + (2*(a + b*A
rcTanh[c*x]))/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])
/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rubi [A] time = 0.218594, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5940, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(cx+1)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(cx+1)^2}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{7b}{8c^3d^3(cx+1)}-\frac{8}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```


```
[Out] -b/(8*c^3*d^3*(1 + c*x)^2) + (7*b)/(8*c^3*d^3*(1 + c*x)) - (7*b*ArcTanh[c*x
])/(8*c^3*d^3) - (a + b*ArcTanh[c*x])/(2*c^3*d^3*(1 + c*x)^2) + (2*(a + b*A
rcTanh[c*x]))/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])
/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx = \int\left(\frac{a+b\tanh^{-1}(cx)}{c^2d^3(1+cx)^3}-\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{c^2d^3(1+cx)}\right)dx$$

$$= \frac{\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{c^2d^3}+\frac{\int\frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{c^2d^3}-\frac{2\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{c^2d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b\int\frac{}{(1+cx}}{2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b\,\mathrm{Subst}}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b\mathrm{Li}_2\left(1-\right.}{2c^3}$$

$$= -\frac{b}{8c^3d^3(1+cx)^2}+\frac{7b}{8c^3d^3(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3(1+cx)}-\frac{\left(a+b\right.}{}$$

$$= -\frac{b}{8c^3d^3(1+cx)^2}+\frac{7b}{8c^3d^3(1+cx)}-\frac{7b\tanh^{-1}(cx)}{8c^3d^3}-\frac{a+b\tanh^{-1}(cx)}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh\right.}{c^3d^3(1+}$$

Mathematica [A] time = 0.492713, size = 145, normalized size = 0.97

$$\frac{b\left(16\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-12\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)+12\cosh\left(2\tanh^{-1}(cx)\right)-\cosh\left(4\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] ((-16*a)/(1 + c*x)^2 + (64*a)/(1 + c*x) + 32*a*Log[1 + c*x] + b*(12*Cosh[2*
ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])]
- 12*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2
*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*
Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*c^3*d^3)
```

Maple [A] time = 0.048, size = 246, normalized size = 1.6

$$-\frac{a}{2\,c^3d^3\,(cx+1)^2}+2\,\frac{a}{c^3d^3\,(cx+1)}+\frac{a\ln(cx+1)}{c^3d^3}-\frac{b\mathrm{Artanh}\,(cx)}{2\,c^3d^3\,(cx+1)^2}+2\,\frac{b\mathrm{Artanh}\,(cx)}{c^3d^3\,(cx+1)}+\frac{b\mathrm{Artanh}\,(cx)\ln(cx+1)}{c^3d^3}-\frac{b}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] -1/2/c^3*a/d^3/(c*x+1)^2+2/c^3*a/d^3/(c*x+1)+1/c^3*a/d^3*ln(c*x+1)-1/2/c^3*
b/d^3*arctanh(c*x)/(c*x+1)^2+2/c^3*b/d^3*arctanh(c*x)/(c*x+1)+1/c^3*b/d^3*a
rctanh(c*x)*ln(c*x+1)-1/2/c^3*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/2/c^
3*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/2/c^3*b/d^3*dilog(1/2+1/2*c*x)-1/4/c^3
*b/d^3*ln(c*x+1)^2+7/16/c^3*b/d^3*ln(c*x-1)-1/8*b/c^3/d^3/(c*x+1)^2+7/8*b/c
^3/d^3/(c*x+1)-7/16/c^3*b/d^3*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{32}\left(64\,c^3\int\frac{x^3\log(cx+1)}{2\left(c^6d^3x^4+2\,c^5d^3x^3-2\,c^3d^3x-c^2d^3\right)}\,dx-4\,c^2\left(\frac{2\,(3\,cx+2)}{c^7d^3x^2+2\,c^6d^3x+c^5d^3}+\frac{\log(cx+1)}{c^5d^3}-\frac{\log(cx-1)}{c^5d^3}\right)+64$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(64*c^3*integrate(1/2*x^3*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 -
2*c^3*d^3*x - c^2*d^3), x) - 4*c^2*(2*(3*c*x + 2)/(c^7*d^3*x^2 + 2*c^6*d^3*
x + c^5*d^3) + log(c*x + 1)/(c^5*d^3) - log(c*x - 1)/(c^5*d^3)) + 64*c^2*in
tegrate(1/2*x^2*log(c*x + 1)/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c
^2*d^3), x) + 7*c*(2*x/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - log(c*x + 1)
/(c^4*d^3) + log(c*x - 1)/(c^4*d^3)) + 96*c*integrate(1/2*x*log(c*x + 1)/(c
^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x) - 8*(4*c*x + 2*(c^2
*x^2 + 2*c*x + 1)*log(c*x + 1) + 3)*log(-c*x + 1)/(c^5*d^3*x^2 + 2*c^4*d^3*
x + c^3*d^3) + 6*(c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*
x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3) + 32*integrate(1/2*log(c*x + 1)
/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3), x))*b + 1/2*a*((4*c
*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)/(c^3*d^3))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{artanh}\,(cx)+ax^2}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(c^3*d^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^
3*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ax^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{bx^2\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*x**
2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)x^2}{(cdx + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(c*d*x + d)^3, x)
```

3.61 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=77

$$\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}-\frac{3b}{8c^2d^3(cx+1)}+\frac{b}{8c^2d^3(cx+1)^2}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}$$

```
[Out] b/(8*c^2*d^3*(1 + c*x)^2) - (3*b)/(8*c^2*d^3*(1 + c*x)) - (b*ArcTanh[c*x])/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c*x)^2)
```

Rubi [A] time = 0.0822645, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {37, 5936, 12, 88, 207}

$$\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(cx+1)^2}-\frac{3b}{8c^2d^3(cx+1)}+\frac{b}{8c^2d^3(cx+1)^2}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] b/(8*c^2*d^3*(1 + c*x)^2) - (3*b)/(8*c^2*d^3*(1 + c*x)) - (b*ArcTanh[c*x])/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5936

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Dist[a
 + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(1 - c^2*x^2),
 x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ[2
*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{(d+cdx)^3}\,dx &= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-(bc)\int \frac{x^2}{2(1-cx)(d+cdx)^3}\,dx\\ &= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-\frac{1}{2}(bc)\int \frac{x^2}{(1-cx)(d+cdx)^3}\,dx\\ &= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}-\frac{1}{2}(bc)\int \left(\frac{1}{2c^2d^3(1+cx)^3}-\frac{3}{4c^2d^3(1+cx)^2}-\frac{1}{4c^2d^3\left(-1+c^2x^2\right)}\right.\\ &= \frac{b}{8c^2d^3(1+cx)^2}-\frac{3b}{8c^2d^3(1+cx)}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}+\frac{b\int \frac{1}{-1+c^2x^2}\,dx}{8cd^3}\\ &= \frac{b}{8c^2d^3(1+cx)^2}-\frac{3b}{8c^2d^3(1+cx)}-\frac{b\tanh^{-1}(cx)}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}\end{aligned}$$

Mathematica [A] time = 0.106985, size = 99, normalized size = 1.29

$$-\frac{16acx+8a-3bc^2x^2\log(cx+1)+6bcx-6bcx\log(cx+1)+3b(cx+1)^2\log(1-cx)-3b\log(cx+1)+8(2bcx+b)\,\mathrm{t}}{16c^2d^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + c*d*x)^3,x]
```

```
[Out] -(8*a + 4*b + 16*a*c*x + 6*b*c*x + 8*(b + 2*b*c*x)*ArcTanh[c*x] + 3*b*(1 +
c*x)^2*Log[1 - c*x] - 3*b*Log[1 + c*x] - 6*b*c*x*Log[1 + c*x] - 3*b*c^2*x^2
*Log[1 + c*x])/(16*c^2*d^3*(1 + c*x)^2)
```

Maple [A] time = 0.037, size = 136, normalized size = 1.8

$$\frac{a}{2\,c^2d^3\,(cx+1)^2}-\frac{a}{c^2d^3\,(cx+1)}+\frac{b\mathrm{Artanh}\,(cx)}{2\,c^2d^3\,(cx+1)^2}-\frac{b\mathrm{Artanh}\,(cx)}{c^2d^3\,(cx+1)}-\frac{3\,b\ln(cx-1)}{16\,c^2d^3}+\frac{b}{8\,c^2d^3\,(cx+1)^2}-\frac{3\,b}{8\,c^2d^3\,(cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] 1/2/c^2*a/d^3/(c*x+1)^2-1/c^2*a/d^3/(c*x+1)+1/2/c^2*b/d^3*arctanh(c*x)/(c*x
+1)^2-1/c^2*b/d^3*arctanh(c*x)/(c*x+1)-3/16/c^2*b/d^3*ln(c*x-1)+1/8*b/c^2/d
^3/(c*x+1)^2-3/8*b/c^2/d^3/(c*x+1)+3/16/c^2*b/d^3*ln(c*x+1)
```

Maxima [B] time = 0.988733, size = 205, normalized size = 2.66

$$-\frac{1}{16}\left(c\left(\frac{2\,(3\,cx+2)}{c^5d^3x^2+2\,c^4d^3x+c^3d^3}-\frac{3\,\log(cx+1)}{c^3d^3}+\frac{3\,\log(cx-1)}{c^3d^3}\right)+\frac{8\,(2\,cx+1)\,\mathrm{artanh}\,(cx)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}\right)b-\frac{(2\,cx+1)a}{2\left(c^4d^3x^2+2\,c^3d^3x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x +
 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c^4
*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*b - 1/2*(2*c*x + 1)*a/(c^4*d^3*x^2 + 2*c
^3*d^3*x + c^2*d^3)
```

Fricas [A] time = 2.13021, size = 180, normalized size = 2.34

$$-\frac{2\,(8\,a+3\,b)cx-\left(3\,bc^2x^2-2\,bcx-b\right)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\,\left(c^4d^3x^2+2\,c^3d^3x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(8*a + 3*b)*c*x - (3*b*c^2*x^2 - 2*b*c*x - b)*log(-(c*x + 1)/(c*x
- 1)) + 8*a + 4*b)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Sympy [A] time = 3.76345, size = 389, normalized size = 5.05

$$\begin{cases} \frac{8ac^2x^2}{24c^4d^3x^2+48c^3d^3x+24c^2d^3}-\frac{8acx}{24c^4d^3x^2+48c^3d^3x+24c^2d^3}-\frac{4a}{24c^4d^3x^2+48c^3d^3x+24c^2d^3}+\frac{9bc^2x^2\operatorname{atanh}(cx)}{24c^4d^3x^2+48c^3d^3x+24c^2d^3}+\frac{3bc^2x^2}{24c^4d^3x^2+48c^3d^3x+24c^2d^3}-\ldots \\ \tilde{\infty}\left(\frac{ax^2}{2}+\frac{bx^2\operatorname{atanh}(cx)}{2}+\frac{bx}{2c}-\frac{b\operatorname{atanh}(cx)}{2c^2}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] Piecewise((8*a*c**2*x**2/(24*c**4*d**3*x**2 + 48*c**3*d**3*x + 24*c**2*d**3
) - 8*a*c*x/(24*c**4*d**3*x**2 + 48*c**3*d**3*x + 24*c**2*d**3) - 4*a/(24*c
**4*d**3*x**2 + 48*c**3*d**3*x + 24*c**2*d**3) + 9*b*c**2*x**2*atanh(c*x)/(
24*c**4*d**3*x**2 + 48*c**3*d**3*x + 24*c**2*d**3) + 3*b*c**2*x**2/(24*c**4
*d**3*x**2 + 48*c**3*d**3*x + 24*c**2*d**3) - 6*b*c*x*atanh(c*x)/(24*c**4*d
**3*x**2 + 48*c**3*d**3*x + 24*c**2*d**3) - 3*b*c*x/(24*c**4*d**3*x**2 + 48
*c**3*d**3*x + 24*c**2*d**3) - 3*b*atanh(c*x)/(24*c**4*d**3*x**2 + 48*c**3*
d**3*x + 24*c**2*d**3) - 3*b/(24*c**4*d**3*x**2 + 48*c**3*d**3*x + 24*c**2*
d**3), Ne(d, 0)), (zoo*(a*x**2/2 + b*x**2*atanh(c*x)/2 + b*x/(2*c) - b*atan
h(c*x)/(2*c**2)), True))
```

Giac [A] time = 1.20421, size = 178, normalized size = 2.31

$$-\frac{(2\,bcx+b)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,\left(c^4d^3x^2+2\,c^3d^3x+c^2d^3\right)}-\frac{8\,acx+3\,bcx+4\,a+2\,b}{8\,\left(c^4d^3x^2+2\,c^3d^3x+c^2d^3\right)}+\frac{3\,b\log\left(cx+1\right)}{16\,c^2d^3}-\frac{3\,b\log\left(cx-1\right)}{16\,c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c*x + b)*log(-(c*x + 1)/(c*x - 1))/(c^4*d^3*x^2 + 2*c^3*d^3*x + c
^2*d^3) - 1/8*(8*a*c*x + 3*b*c*x + 4*a + 2*b)/(c^4*d^3*x^2 + 2*c^3*d^3*x +
c^2*d^3) + 3/16*b*log(c*x + 1)/(c^2*d^3) - 3/16*b*log(c*x - 1)/(c^2*d^3)
```

3.62 $\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^3}\,dx$

Optimal. Leaf size=77

$$-\frac{a+b\tanh^{-1}(cx)}{2cd^3(cx+1)^2}-\frac{b}{8cd^3(cx+1)}-\frac{b}{8cd^3(cx+1)^2}+\frac{b\tanh^{-1}(cx)}{8cd^3}$$

```
[Out] -b/(8*c*d^3*(1 + c*x)^2) - b/(8*c*d^3*(1 + c*x)) + (b*ArcTanh[c*x])/(8*c*d^
3) - (a + b*ArcTanh[c*x])/(2*c*d^3*(1 + c*x)^2)
```

Rubi [A] time = 0.0550454, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5926, 627, 44, 207}

$$-\frac{a+b\tanh^{-1}(cx)}{2cd^3(cx+1)^2}-\frac{b}{8cd^3(cx+1)}-\frac{b}{8cd^3(cx+1)^2}+\frac{b\tanh^{-1}(cx)}{8cd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

```
[Out] -b/(8*c*d^3*(1 + c*x)^2) - b/(8*c*d^3*(1 + c*x)) + (b*ArcTanh[c*x])/(8*c*d^
3) - (a + b*ArcTanh[c*x])/(2*c*d^3*(1 + c*x)^2)
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(d+cdx)^3}\,dx = -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} + \frac{b\int \frac{1}{(d+cdx)^2(1-c^2x^2)}\,dx}{2d}$$
$$= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} + \frac{b\int \frac{1}{\left(\frac{1}{d}-\frac{cx}{d}\right)(d+cdx)^3}\,dx}{2d}$$
$$= -\frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} + \frac{b\int \left(\frac{1}{2d^2(1+cx)^3} + \frac{1}{4d^2(1+cx)^2} - \frac{1}{4d^2(-1+c^2x^2)}\right)dx}{2d}$$
$$= -\frac{b}{8cd^3(1+cx)^2} - \frac{b}{8cd^3(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2} - \frac{b\int \frac{1}{-1+c^2x^2}\,dx}{8d^3}$$
$$= -\frac{b}{8cd^3(1+cx)^2} - \frac{b}{8cd^3(1+cx)} + \frac{b\tanh^{-1}(cx)}{8cd^3} - \frac{a+b\tanh^{-1}(cx)}{2cd^3(1+cx)^2}$$

Mathematica [A] time = 0.0730421, size = 86, normalized size = 1.12

$$\frac{-8a+bc^2x^2\log(cx+1)-2bcx+2bcx\log(cx+1)-b(cx+1)^2\log(1-cx)+b\log(cx+1)-8b\tanh^{-1}(cx)-4b}{16cd^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + c*d*x)^3,x]
```

```
[Out] (-8*a - 4*b - 2*b*c*x - 8*b*ArcTanh[c*x] - b*(1 + c*x)^2*Log[1 - c*x] + b*L
og[1 + c*x] + 2*b*c*x*Log[1 + c*x] + b*c^2*x^2*Log[1 + c*x])/(16*c*d^3*(1 +
 c*x)^2)
```

Maple [A] time = 0.033, size = 100, normalized size = 1.3

$$-\frac{a}{2\,cd^3\,(cx+1)^2}-\frac{b\mathrm{Artanh}\,(cx)}{2\,cd^3\,(cx+1)^2}-\frac{b\ln(cx-1)}{16\,cd^3}-\frac{b}{8\,cd^3\,(cx+1)^2}-\frac{b}{8\,cd^3\,(cx+1)}+\frac{b\ln(cx+1)}{16\,cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*d*x+d)^3,x)
```

```
[Out] -1/2/c*a/d^3/(c*x+1)^2-1/2/c*b/d^3*arctanh(c*x)/(c*x+1)^2-1/16/c*b/d^3*ln(c
*x-1)-1/8*b/c/d^3/(c*x+1)^2-1/8*b/c/d^3/(c*x+1)+1/16/c*b/d^3*ln(c*x+1)
```

Maxima [A] time = 0.975595, size = 181, normalized size = 2.35

$$-\frac{1}{16}\left(c\left(\frac{2\,(cx+2)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}-\frac{\log(cx+1)}{c^2d^3}+\frac{\log(cx-1)}{c^2d^3}\right)+\frac{8\,\mathrm{artanh}\,(cx)}{c^3d^3x^2+2\,c^2d^3x+cd^3}\right)b-\frac{a}{2\left(c^3d^3x^2+2\,c^2d^3x+cd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/
(c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*d
```

```
^3*x + c*d^3))*b - 1/2*a/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Fricas [A] time = 2.11686, size = 163, normalized size = 2.12

$$-\frac{2\,bcx-\left(bc^{2}x^{2}+2\,bcx-3\,b\right)\log\left(-\frac{cx+1}{cx-1}\right)+8\,a+4\,b}{16\left(c^{3}d^{3}x^{2}+2\,c^{2}d^{3}x+cd^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*b*c*x - (b*c^2*x^2 + 2*b*c*x - 3*b)*log(-(c*x + 1)/(c*x - 1)) + 8*
a + 4*b)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Sympy [A] time = 4.16484, size = 289, normalized size = 3.75

$$\begin{cases}-\frac{12a}{24c^{3}d^{3}x^{2}+48c^{2}d^{3}x+24cd^{3}}+\frac{3bc^{2}x^{2}\operatorname{atanh}(cx)}{24c^{3}d^{3}x^{2}+48c^{2}d^{3}x+24cd^{3}}+\frac{bc^{2}x^{2}}{24c^{3}d^{3}x^{2}+48c^{2}d^{3}x+24cd^{3}}+\frac{6bcx\operatorname{atanh}(cx)}{24c^{3}d^{3}x^{2}+48c^{2}d^{3}x+24cd^{3}}-\frac{bcx}{24c^{3}d^{3}x^{2}+48c^{2}d^{3}x+24cd^{3}}-\frac{}{24c^{3}} \\ \tilde{\infty}\left(ax+bx\operatorname{atanh}(cx)+\frac{b\log\left(x-\frac{1}{c}\right)}{c}+\frac{b\operatorname{atanh}(cx)}{c}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*d*x+d)**3,x)
```

```
[Out] Piecewise((-12*a/(24*c**3*d**3*x**2 + 48*c**2*d**3*x + 24*c*d**3) + 3*b*c**
2*x**2*atanh(c*x)/(24*c**3*d**3*x**2 + 48*c**2*d**3*x + 24*c*d**3) + b*c**2
*x**2/(24*c**3*d**3*x**2 + 48*c**2*d**3*x + 24*c*d**3) + 6*b*c*x*atanh(c*x)
/(24*c**3*d**3*x**2 + 48*c**2*d**3*x + 24*c*d**3) - b*c*x/(24*c**3*d**3*x**
2 + 48*c**2*d**3*x + 24*c*d**3) - 9*b*atanh(c*x)/(24*c**3*d**3*x**2 + 48*c*
*2*d**3*x + 24*c*d**3) - 5*b/(24*c**3*d**3*x**2 + 48*c**2*d**3*x + 24*c*d**
3), Ne(d, 0)), (zoo*(a*x + b*x*atanh(c*x) + b*log(x - 1/c)/c + b*atanh(c*x)
/c), True))
```

Giac [A] time = 1.24967, size = 157, normalized size = 2.04

$$-\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{4\left(c^{3}d^{3}x^{2}+2\,c^{2}d^{3}x+cd^{3}\right)}-\frac{bcx+4\,a+2\,b}{8\left(c^{3}d^{3}x^{2}+2\,c^{2}d^{3}x+cd^{3}\right)}+\frac{b\log\left(cx+1\right)}{16\,cd^{3}}-\frac{b\log\left(cx-1\right)}{16\,cd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/4*b*log(-(c*x + 1)/(c*x - 1))/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3) - 1/8*
(b*c*x + 4*a + 2*b)/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3) + 1/16*b*log(c*x +
1)/(c*d^3) - 1/16*b*log(c*x - 1)/(c*d^3)
```

3.63 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^3}\,dx$

Optimal. Leaf size=161

$$-\frac{b\text{PolyLog}(2,-cx)}{2d^3}+\frac{b\text{PolyLog}(2,cx)}{2d^3}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{a+b\tanh^{-1}(cx)}{d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2d^3(cx+1)^2}+\frac{\log\left(\frac{2}{cx+1}\right)}{}$$

```
[Out] b/(8*d^3*(1 + c*x)^2) + (5*b)/(8*d^3*(1 + c*x)) - (5*b*ArcTanh[c*x])/(8*d^3
) + (a + b*ArcTanh[c*x])/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])/(d^3*(1
 + c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (
b*PolyLog[2, -(c*x)])/(2*d^3) + (b*PolyLog[2, c*x])/(2*d^3) - (b*PolyLog[2,
 1 - 2/(1 + c*x)])/(2*d^3)
```

Rubi [A] time = 0.227361, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5940, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{b\text{PolyLog}(2,-cx)}{2d^3}+\frac{b\text{PolyLog}(2,cx)}{2d^3}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{a+b\tanh^{-1}(cx)}{d^3(cx+1)}+\frac{a+b\tanh^{-1}(cx)}{2d^3(cx+1)^2}+\frac{\log\left(\frac{2}{cx+1}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```


```
[Out] b/(8*d^3*(1 + c*x)^2) + (5*b)/(8*d^3*(1 + c*x)) - (5*b*ArcTanh[c*x])/(8*d^3
) + (a + b*ArcTanh[c*x])/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])/(d^3*(1
 + c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (
b*PolyLog[2, -(c*x)])/(2*d^3) + (b*PolyLog[2, c*x])/(2*d^3) - (b*PolyLog[2,
 1 - 2/(1 + c*x)])/(2*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^3}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^2}-\frac{c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)}\right) d$$

$$=\frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3}-\frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3}-\frac{c\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^3}-\frac{c\int \frac{a+b\tanh^{-1}(cx)}{1+cx}\,dx}{d^3}$$

$$=\frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3}-\frac{b\mathrm{Li}_2(-cx)}{2d^3}$$

$$=\frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3}-\frac{b\mathrm{Li}_2(-cx)}{2d^3}$$

$$=\frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3}-\frac{b\mathrm{Li}_2(-cx)}{2d^3}$$

$$=\frac{b}{8d^3(1+cx)^2}+\frac{5b}{8d^3(1+cx)}+\frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}+\frac{a\log(x)}{d^3}+\frac{\left(a+b\tanh^{-1}\right.}{}$$

$$=\frac{b}{8d^3(1+cx)^2}+\frac{5b}{8d^3(1+cx)}-\frac{5b\tanh^{-1}(cx)}{8d^3}+\frac{a+b\tanh^{-1}(cx)}{2d^3(1+cx)^2}+\frac{a+b\tanh^{-1}(cx)}{d^3(1+cx)}+\frac{a\log(x}{d^3}$$

Mathematica [A] time = 0.525359, size = 147, normalized size = 0.91

$$b\left(-16\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-12\sinh\left(2\tanh^{-1}(cx)\right)-\sinh\left(4\tanh^{-1}(cx)\right)+12\cosh\left(2\tanh^{-1}(cx)\right)+\cosh\left(4\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + c*d*x)^3),x]
```

```
[Out] ((16*a)/(1 + c*x)^2 + (32*a)/(1 + c*x) + 32*a*Log[x] - 32*a*Log[1 + c*x] +
b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*PolyLog[2, E^(-2*Arc
Tanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*
x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcT
anh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

Maple [A] time = 0.057, size = 264, normalized size = 1.6

$$\frac{a\ln(cx)}{d^3}+\frac{a}{2\,d^3\,(cx+1)^2}+\frac{a}{d^3\,(cx+1)}-\frac{a\ln(cx+1)}{d^3}+\frac{b\text{Artanh}(cx)\ln(cx)}{d^3}+\frac{b\text{Artanh}(cx)}{2\,d^3\,(cx+1)^2}+\frac{b\text{Artanh}(cx)}{d^3\,(cx+1)}-\frac{bA}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x)
```

```
[Out] a/d^3*ln(c*x)+1/2*a/d^3/(c*x+1)^2+a/d^3/(c*x+1)-a/d^3*ln(c*x+1)+b/d^3*arcta
nh(c*x)*ln(c*x)+1/2*b/d^3*arctanh(c*x)/(c*x+1)^2+b/d^3*arctanh(c*x)/(c*x+1)
-b/d^3*arctanh(c*x)*ln(c*x+1)-1/2*b/d^3*dilog(c*x)-1/2*b/d^3*dilog(c*x+1)-1
/2*b/d^3*ln(c*x)*ln(c*x+1)+1/2*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2*b
/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2*b/d^3*dilog(1/2+1/2*c*x)+1/4*b/d^3*ln(c
*x+1)^2+5/16*b/d^3*ln(c*x-1)+1/8*b/d^3/(c*x+1)^2+5/8*b/d^3/(c*x+1)-5/16*b/d
^3*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a\left(\frac{2\,cx+3}{c^2d^3x^2+2\,cd^3x+d^3}-\frac{2\,\log(cx+1)}{d^3}+\frac{2\,\log(x)}{d^3}\right)+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{c^3d^3x^4+3\,c^2d^3x^3+3\,cd^3x^2+d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 + 2
*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^4
+ 3*c^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}(cx)+a}{c^3d^3x^4+3\,c^2d^3x^3+3\,cd^3x^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^4 + 3*c^2*d^3*x^3 + 3*c*d^3*x^2 +
d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^3x^4+3c^2x^3+3cx^2+x}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b*atanh
(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\,(cx) + a}{(cdx + d)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x), x)
```

3.64 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^3}\,dx$

Optimal. Leaf size=218

$$\frac{3bc\mathrm{PolyLog}(2,-cx)}{2d^3}-\frac{3bc\mathrm{PolyLog}(2,cx)}{2d^3}+\frac{3bc\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}-\frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}-\frac{c\left(a+b\tanh^{-1}(cx\right.}{2d^3(cx+1)^2}$$

```
[Out] -(b*c)/(8*d^3*(1 + c*x)^2) - (9*b*c)/(8*d^3*(1 + c*x)) + (9*b*c*ArcTanh[c*x
])/(8*d^3) - (a + b*ArcTanh[c*x])/(d^3*x) - (c*(a + b*ArcTanh[c*x]))/(2*d^3
*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) - (3*a*c*Log[x])
/d^3 + (b*c*Log[x])/d^3 - (3*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 -
 (b*c*Log[1 - c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, -(c*x)])/(2*d^3) - (3*b
*c*PolyLog[2, c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rubi [A] time = 0.273822, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5940, 5916, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$\frac{3bc\mathrm{PolyLog}(2,-cx)}{2d^3}-\frac{3bc\mathrm{PolyLog}(2,cx)}{2d^3}+\frac{3bc\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}-\frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}-\frac{c\left(a+b\tanh^{-1}(cx\right.}{2d^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```


```
[Out] -(b*c)/(8*d^3*(1 + c*x)^2) - (9*b*c)/(8*d^3*(1 + c*x)) + (9*b*c*ArcTanh[c*x
])/(8*d^3) - (a + b*ArcTanh[c*x])/(d^3*x) - (c*(a + b*ArcTanh[c*x]))/(2*d^3
*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) - (3*a*c*Log[x])
/d^3 + (b*c*Log[x])/d^3 - (3*c*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 -
 (b*c*Log[1 - c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2, -(c*x)])/(2*d^3) - (3*b
*c*PolyLog[2, c*x])/(2*d^3) + (3*b*c*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x^2} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^3} + \frac{2c^2\left(a+b\tanh^{-1}\right.}{d^3(1+cx)^2}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^3} - \frac{(3c)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} + \frac{c^2\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3} + \frac{\left(2c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,d}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} - \frac{3c\left(a+b\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} - \frac{3c\left(a+b\right.}{}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)} - \frac{3ac\log(x)}{d^3} - \frac{3c\left(a+b\right.}{}$$

$$= -\frac{bc}{8d^3(1+cx)^2} - \frac{9bc}{8d^3(1+cx)} - \frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}\right.}{d^3(1+cx)}$$

$$= -\frac{bc}{8d^3(1+cx)^2} - \frac{9bc}{8d^3(1+cx)} + \frac{9bc\tanh^{-1}(cx)}{8d^3} - \frac{a+b\tanh^{-1}(cx)}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}$$

Mathematica [A] time = 1.23909, size = 186, normalized size = 0.85

$$bc\left(48\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+32\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+20\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)-20\cosh\left(2\tan\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + c*d*x)^3),x]
```

```
[Out] ((-32*a)/x - (16*a*c)/(1 + c*x)^2 - (64*a*c)/(1 + c*x) - 96*a*c*Log[x] + 96
*a*c*Log[1 + c*x] + b*c*(-20*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] +
32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 48*PolyLog[2, E^(-2*ArcTanh[c*x])] + 20*S
inh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-8/(c*x) - 10*
Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 24*Log[1 - E^(-2*ArcTanh[c*x]
)] + 10*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])))/(32*d^3)
```

Maple [A] time = 0.058, size = 319, normalized size = 1.5

$$-\frac{a}{d^3x} - 3\frac{ac\ln(cx)}{d^3} - \frac{ac}{2d^3(cx+1)^2} - 2\frac{ac}{d^3(cx+1)} + 3\frac{ac\ln(cx+1)}{d^3} - \frac{b\text{Artanh}(cx)}{d^3x} - 3\frac{bc\text{Artanh}(cx)\ln(cx)}{d^3} - \frac{bc}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x)
```

```
[Out] -a/d^3/x-3*c*a/d^3*ln(c*x)-1/2*c*a/d^3/(c*x+1)^2-2*c*a/d^3/(c*x+1)+3*c*a/d^
3*ln(c*x+1)-b/d^3*arctanh(c*x)/x-3*c*b/d^3*arctanh(c*x)*ln(c*x)-1/2*c*b/d^3
*arctanh(c*x)/(c*x+1)^2-2*c*b/d^3*arctanh(c*x)/(c*x+1)+3*c*b/d^3*arctanh(c*
x)*ln(c*x+1)-17/16*c*b/d^3*ln(c*x-1)+c*b/d^3*ln(c*x)-1/8*b*c/d^3/(c*x+1)^2-
9/8*b*c/d^3/(c*x+1)+1/16*c*b/d^3*ln(c*x+1)+3/2*c*b/d^3*dilog(c*x)+3/2*c*b/d
^3*dilog(c*x+1)+3/2*c*b/d^3*ln(c*x)*ln(c*x+1)-3/2*c*b/d^3*ln(-1/2*c*x+1/2)*
ln(1/2+1/2*c*x)+3/2*c*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/2*c*b/d^3*dilog(1/
2+1/2*c*x)-3/4*c*b/d^3*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\,a\left(\frac{6\,c^2x^2+9\,cx+2}{c^2d^3x^3+2\,cd^3x^2+d^3x}-\frac{6\,c\log\left(cx+1\right)}{d^3}+\frac{6\,c\log\left(x\right)}{d^3}\right)+\frac{1}{2}\,b\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{c^3d^3x^5+3\,c^2d^3x^4+3\,cd^3x^3+d^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c*l
og(c*x + 1)/d^3 + 6*c*log(x)/d^3) + 1/2*b*integrate((log(c*x + 1) - log(-c*
x + 1))/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx\right)+a}{c^3d^3x^5+3\,c^2d^3x^4+3\,cd^3x^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^5 + 3*c^2*d^3*x^4 + 3*c*d^3*x^3 +
d^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx+\int\frac{b\,\text{atanh}\,(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b*at
anh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx) + a}{(cdx + d)^3 x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^2), x)
```

3.65 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^3}\,dx$

Optimal. Leaf size=268

$$-\frac{3bc^2\text{PolyLog}(2,-cx)}{d^3}+\frac{3bc^2\text{PolyLog}(2,cx)}{d^3}-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^3}+\frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}+\frac{c^2\left(a+b\tanh^{-1}\right.}{2d^3(cx+1)}$$

```
[Out] -(b*c)/(2*d^3*x) + (b*c^2)/(8*d^3*(1 + c*x)^2) + (13*b*c^2)/(8*d^3*(1 + c*x
)) - (9*b*c^2*ArcTanh[c*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(2*d^3*x^2) + (3
*c*(a + b*ArcTanh[c*x]))/(d^3*x) + (c^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c
*x)^2) + (3*c^2*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) + (6*a*c^2*Log[x])/d^
3 - (3*b*c^2*Log[x])/d^3 + (6*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^
3 + (3*b*c^2*Log[1 - c^2*x^2])/(2*d^3) - (3*b*c^2*PolyLog[2, -(c*x)])/d^3 +
 (3*b*c^2*PolyLog[2, c*x])/d^3 - (3*b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/d^3
```

Rubi [A] time = 0.316984, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 16, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912, 5926, 627, 44, 207, 5918, 2402, 2315}

$$-\frac{3bc^2\text{PolyLog}(2,-cx)}{d^3}+\frac{3bc^2\text{PolyLog}(2,cx)}{d^3}-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{d^3}+\frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(cx+1)}+\frac{c^2\left(a+b\tanh^{-1}\right.}{2d^3(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```


```
[Out] -(b*c)/(2*d^3*x) + (b*c^2)/(8*d^3*(1 + c*x)^2) + (13*b*c^2)/(8*d^3*(1 + c*x
)) - (9*b*c^2*ArcTanh[c*x])/(8*d^3) - (a + b*ArcTanh[c*x])/(2*d^3*x^2) + (3
*c*(a + b*ArcTanh[c*x]))/(d^3*x) + (c^2*(a + b*ArcTanh[c*x]))/(2*d^3*(1 + c
*x)^2) + (3*c^2*(a + b*ArcTanh[c*x]))/(d^3*(1 + c*x)) + (6*a*c^2*Log[x])/d^
3 - (3*b*c^2*Log[x])/d^3 + (6*c^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^
3 + (3*b*c^2*Log[1 - c^2*x^2])/(2*d^3) - (3*b*c^2*PolyLog[2, -(c*x)])/d^3 +
 (3*b*c^2*PolyLog[2, c*x])/d^3 - (3*b*c^2*PolyLog[2, 1 - 2/(1 + c*x)])/d^3
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+cdx)^3}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{d^3x^3} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x^2} + \frac{6c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3x} - \frac{c^3\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)^3}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3c)\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^3} + \frac{\left(6c^2\right)\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{c^3\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{d^3} -$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{3c^2\left(a+b\tanh^{-1}(cx)\right)}{d^3(1+cx)}$$

$$= -\frac{bc}{2d^3x} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2} + \frac{3c^2\left(a+b\tanh\right.}{d^3(1+cx}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2\tanh^{-1}(cx)}{2d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)}{2d^3(1+cx)^2}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2}{8d^3(1+cx)^2} + \frac{13bc^2}{8d^3(1+cx)} + \frac{bc^2\tanh^{-1}(cx)}{2d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-1}\right.}{d^3x}$$

$$= -\frac{bc}{2d^3x} + \frac{bc^2}{8d^3(1+cx)^2} + \frac{13bc^2}{8d^3(1+cx)} - \frac{9bc^2\tanh^{-1}(cx)}{8d^3} - \frac{a+b\tanh^{-1}(cx)}{2d^3x^2} + \frac{3c\left(a+b\tanh^{-}\right.}{d^3x}$$

Mathematica [A] time = 1.49655, size = 220, normalized size = 0.82

$$\frac{bc^2\left(-96\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) - 96\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right) + 4\tanh^{-1}(cx)\left(-\frac{4}{c^2x^2} + \frac{24}{cx} + 48\log\left(1-e^{-2\tanh^{-1}(cx)}\right) - 14\sinh\left(2\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + c*d*x)^3),x]
```

```
[Out] ((-16*a)/x^2 + (96*a*c)/x + (16*a*c^2)/(1 + c*x)^2 + (96*a*c^2)/(1 + c*x) +
 192*a*c^2*Log[x] - 192*a*c^2*Log[1 + c*x] + b*c^2*(-16/(c*x) + 28*Cosh[2*A
rcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 96*Log[(c*x)/Sqrt[1 - c^2*x^2]] - 96*
PolyLog[2, E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*
(4 - 4/(c^2*x^2) + 24/(c*x) + 14*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]
] + 48*Log[1 - E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcT
anh[c*x]]) - Sinh[4*ArcTanh[c*x]]))/(32*d^3)
```

Maple [A] time = 0.062, size = 394, normalized size = 1.5

$$\frac{c^2b\text{Artanh}(cx)}{2d^3(cx+1)^2}-\frac{bc}{2d^3x}+\frac{c^2b}{8d^3(cx+1)^2}+\frac{13c^2b}{8d^3(cx+1)}-\frac{a}{2d^3x^2}+3\frac{bc\text{Artanh}(cx)}{d^3x}+\frac{33c^2b\ln(cx-1)}{16d^3}-3\frac{c^2b\text{dilo}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x)
```

```
[Out] -1/2*b*c/d^3/x+1/8*b*c^2/d^3/(c*x+1)^2+13/8*b*c^2/d^3/(c*x+1)-1/2*a/d^3/x^2
+1/2*c^2*b/d^3*arctanh(c*x)/(c*x+1)^2+3*c*b/d^3*arctanh(c*x)/x+33/16*c^2*b/
d^3*ln(c*x-1)-3*c^2*b/d^3*dilog(c*x)+6*c^2*a/d^3*ln(c*x)+1/2*c^2*a/d^3/(c*x
+1)^2+3*c^2*a/d^3/(c*x+1)-6*c^2*a/d^3*ln(c*x+1)+3/2*c^2*b/d^3*ln(c*x+1)^2-3
*c^2*b/d^3*ln(c*x)+3*c*a/d^3/x+3*c^2*b/d^3*dilog(1/2+1/2*c*x)+6*c^2*b/d^3*a
rctanh(c*x)*ln(c*x)+15/16*c^2*b/d^3*ln(c*x+1)-1/2*b/d^3*arctanh(c*x)/x^2-3*
c^2*b/d^3*dilog(c*x+1)-6*c^2*b/d^3*arctanh(c*x)*ln(c*x+1)+3*c^2*b/d^3*ln(-1
/2*c*x+1/2)*ln(1/2+1/2*c*x)-3*c^2*b/d^3*ln(c*x)*ln(c*x+1)-3*c^2*b/d^3*ln(-1
/2*c*x+1/2)*ln(c*x+1)+3*c^2*b/d^3*arctanh(c*x)/(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{12c^3x^3+18c^2x^2+4cx-1}{c^2d^3x^4+2cd^3x^3+d^3x^2}-\frac{12c^2\log(cx+1)}{d^3}+\frac{12c^2\log(x)}{d^3}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{c^3d^3x^6+3c^2d^3x^5+3cd^3x^4+d^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*((12*c^3*x^3 + 18*c^2*x^2 + 4*c*x - 1)/(c^2*d^3*x^4 + 2*c*d^3*x^3 + d
^3*x^2) - 12*c^2*log(c*x + 1)/d^3 + 12*c^2*log(x)/d^3) + 1/2*b*integrate((l
og(c*x + 1) - log(-c*x + 1))/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 + d
^3*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}(cx)+a}{c^3d^3x^6+3c^2d^3x^5+3cd^3x^4+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(c^3*d^3*x^6 + 3*c^2*d^3*x^5 + 3*c*d^3*x^4 +
d^3*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a}{c^3x^6+3c^2x^5+3cx^4+x^3}\,dx + \int \frac{b\,\mathrm{atanh}\,(cx)}{c^3x^6+3c^2x^5+3cx^4+x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x) + Integral(b*at
anh(c*x)/(c**3*x**6 + 3*c**2*x**5 + 3*c*x**4 + x**3), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\mathrm{artanh}\,(cx) + a}{(cdx + d)^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((c*d*x + d)^3*x^3), x)
```

3.66 $\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^4} dx$

Optimal. Leaf size=80

$$-\frac{a+b\tanh^{-1}(cx)}{3c(cx+1)^3}-\frac{b}{24c(cx+1)}-\frac{b}{24c(cx+1)^2}-\frac{b}{18c(cx+1)^3}+\frac{b\tanh^{-1}(cx)}{24c}$$

```
[Out] -b/(18*c*(1 + c*x)^3) - b/(24*c*(1 + c*x)^2) - b/(24*c*(1 + c*x)) + (b*ArcT
anh[c*x])/(24*c) - (a + b*ArcTanh[c*x])/(3*c*(1 + c*x)^3)
```

Rubi [A] time = 0.053593, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5926, 627, 44, 207}

$$-\frac{a+b\tanh^{-1}(cx)}{3c(cx+1)^3}-\frac{b}{24c(cx+1)}-\frac{b}{24c(cx+1)^2}-\frac{b}{18c(cx+1)^3}+\frac{b\tanh^{-1}(cx)}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

```
[Out] -b/(18*c*(1 + c*x)^3) - b/(24*c*(1 + c*x)^2) - b/(24*c*(1 + c*x)) + (b*ArcT
anh[c*x])/(24*c) - (a + b*ArcTanh[c*x])/(3*c*(1 + c*x)^3)
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^4}\,dx = -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \frac{1}{(1+cx)^3\left(1-c^2x^2\right)}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \frac{1}{(1-cx)(1+cx)^4}\,dx$$
$$= -\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}+\frac{1}{3}b\int \left(\frac{1}{2(1+cx)^4}+\frac{1}{4(1+cx)^3}+\frac{1}{8(1+cx)^2}-\frac{1}{8\left(-1+c^2x^2\right)}\right)dx$$
$$= -\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}-\frac{1}{24}b\int \frac{1}{-1+c^2x^2}\,dx$$
$$= -\frac{b}{18c(1+cx)^3}-\frac{b}{24c(1+cx)^2}-\frac{b}{24c(1+cx)}+\frac{b\tanh^{-1}(cx)}{24c}-\frac{a+b\tanh^{-1}(cx)}{3c(1+cx)^3}$$

Mathematica [A] time = 0.105182, size = 75, normalized size = 0.94

$$-\frac{48a+2b\left(3c^2x^2+9cx+10\right)+3b(cx+1)^3\log(1-cx)-3b(cx+1)^3\log(cx+1)+48b\tanh^{-1}(cx)}{144c(cx+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(1 + c*x)^4,x]
```

```
[Out] -(48*a + 2*b*(10 + 9*c*x + 3*c^2*x^2) + 48*b*ArcTanh[c*x] + 3*b*(1 + c*x)^3
*Log[1 - c*x] - 3*b*(1 + c*x)^3*Log[1 + c*x])/(144*c*(1 + c*x)^3)
```

Maple [A] time = 0.034, size = 95, normalized size = 1.2

$$-\frac{a}{3c(cx+1)^3}-\frac{b\text{Artanh}(cx)}{3c(cx+1)^3}-\frac{b\ln(cx-1)}{48c}-\frac{b}{18c(cx+1)^3}-\frac{b}{24c(cx+1)^2}-\frac{b}{24c(cx+1)}+\frac{b\ln(cx+1)}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(c*x+1)^4,x)
```

```
[Out] -1/3/c*a/(c*x+1)^3-1/3/c*b/(c*x+1)^3*arctanh(c*x)-1/48/c*b*ln(c*x-1)-1/18*b
/c/(c*x+1)^3-1/24*b/c/(c*x+1)^2-1/24*b/c/(c*x+1)+1/48/c*b*ln(c*x+1)
```

Maxima [A] time = 0.973127, size = 178, normalized size = 2.22

$$-\frac{1}{144}\left(c\left(\frac{2\left(3c^2x^2+9cx+10\right)}{c^5x^3+3c^4x^2+3c^3x+c^2}-\frac{3\log(cx+1)}{c^2}+\frac{3\log(cx-1)}{c^2}\right)+\frac{48\,\text{artanh}(cx)}{c^4x^3+3c^3x^2+3c^2x+c}\right)b-\frac{a}{3\left(c^4x^3+3c^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/144*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
 - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*
c^3*x^2 + 3*c^2*x + c))*b - 1/3*a/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Fricas [A] time = 2.37688, size = 209, normalized size = 2.61

$$-\frac{6\,bc^2x^2 + 18\,bcx - 3\left(bc^3x^3 + 3\,bc^2x^2 + 3\,bcx - 7\,b\right)\log\left(-\frac{cx+1}{cx-1}\right) + 48\,a + 20\,b}{144\left(c^4x^3 + 3\,c^3x^2 + 3\,c^2x + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/144*(6*b*c^2*x^2 + 18*b*c*x - 3*(b*c^3*x^3 + 3*b*c^2*x^2 + 3*b*c*x - 7*b
)*log(-(c*x + 1)/(c*x - 1)) + 48*a + 20*b)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x +
 c)
```

Sympy [A] time = 8.14961, size = 294, normalized size = 3.68

$$\begin{cases} -\frac{24a}{72c^4x^3+216c^3x^2+216c^2x+72c} + \frac{3bc^3x^3\operatorname{atanh}(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c} + \frac{9bc^2x^2\operatorname{atanh}(cx)}{72c^4x^3+216c^3x^2+216c^2x+72c} - \frac{3bc^2x^2}{72c^4x^3+216c^3x^2+216c^2x+72c} + \frac{9bcx\operatorname{at}}{72c^4x^3+216c^3x} \\ ax \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(c*x+1)**4,x)
```

```
[Out] Piecewise((-24*a/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 3*b*c
**3*x**3*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*
b*c**2*x**2*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) -
 3*b*c**2*x**2/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) + 9*b*c*x
*atanh(c*x)/(72*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 9*b*c*x/(7
2*c**4*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 21*b*atanh(c*x)/(72*c**4
*x**3 + 216*c**3*x**2 + 216*c**2*x + 72*c) - 10*b/(72*c**4*x**3 + 216*c**3*
x**2 + 216*c**2*x + 72*c), Ne(c, 0)), (a*x, True))
```

Giac [A] time = 1.15987, size = 157, normalized size = 1.96

$$\frac{b\log\left(cx+1\right)}{48\,c} - \frac{b\log\left(cx-1\right)}{48\,c} - \frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{6\left(c^4x^3 + 3\,c^3x^2 + 3\,c^2x + c\right)} - \frac{3\,bc^2x^2 + 9\,bcx + 24\,a + 10\,b}{72\left(c^4x^3 + 3\,c^3x^2 + 3\,c^2x + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] 1/48*b*log(c*x + 1)/c - 1/48*b*log(c*x - 1)/c - 1/6*b*log(-(c*x + 1)/(c*x -
 1))/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/72*(3*b*c^2*x^2 + 9*b*c*x + 24
*a + 10*b)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

3.67 $\int \frac{\tanh^{-1}(ax)}{cx+acx^2}\,dx$

Optimal. Leaf size=41

$$\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{c}-\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}$$

```
[Out] (ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - PolyLog[2, -1 + 2/(1 + a*x)]/(2*c)
```

Rubi [A] time = 0.0646014, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1593, 5932, 2447}

$$\frac{\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)}{c}-\frac{\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - PolyLog[2, -1 + 2/(1 + a*x)]/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{cx+acx^2}\,dx &= \int \frac{\tanh^{-1}(ax)}{x(c+acx)}\,dx\\ &= \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{a\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}\\ &= \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{c}-\frac{\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c}\end{aligned}$$

Mathematica [A] time = 0.0736186, size = 39, normalized size = 0.95

$$\frac{\tanh^{-1}(ax)\log\left(1-e^{-2\tanh^{-1}(ax)}\right)}{c}-\frac{\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])])/c - PolyLog[2, E^(-2*ArcTanh[a*
x])]/(2*c)
```

Maple [B] time = 0.045, size = 126, normalized size = 3.1

$$\frac{\text{Artanh}(ax)\ln(ax)}{c}-\frac{\text{Artanh}(ax)\ln(ax+1)}{c}+\frac{1}{2c}\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)\ln\left(\frac{1}{2}+\frac{ax}{2}\right)-\frac{\ln(ax+1)}{2c}\ln\left(-\frac{ax}{2}+\frac{1}{2}\right)+\frac{1}{2c}\text{dilo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(a*c*x^2+c*x),x)
```

```
[Out] 1/c*arctanh(a*x)*ln(a*x)-1/c*arctanh(a*x)*ln(a*x+1)+1/2/c*ln(-1/2*a*x+1/2)*
ln(1/2+1/2*a*x)-1/2/c*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/2/c*dilog(1/2+1/2*a*x)+1
/4/c*ln(a*x+1)^2-1/2/c*dilog(a*x)-1/2/c*dilog(a*x+1)-1/2/c*ln(a*x)*ln(a*x+1
)
```

Maxima [B] time = 0.961415, size = 162, normalized size = 3.95

$$\frac{1}{4}a\left(\frac{\log(ax+1)^2}{ac}-\frac{2\left(\log(ax+1)\log\left(-\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}ax+\frac{1}{2}\right)\right)}{ac}-\frac{2\left(\log(ax+1)\log(x)+\text{Li}_2(-ax)\right)}{ac}+\frac{2(\log(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] 1/4*a*(log(a*x + 1)^2/(a*c) - 2*(log(a*x + 1)*log(-1/2*a*x + 1/2) + dilog(1
/2*a*x + 1/2))/(a*c) - 2*(log(a*x + 1)*log(x) + dilog(-a*x))/(a*c) + 2*(log
(-a*x + 1)*log(x) + dilog(a*x))/(a*c)) - (log(a*x + 1)/c - log(x)/c)*arctan
h(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(ax)}{acx^2+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atanh}(ax)}{ax^2+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(a*c*x**2+c*x),x)
```

```
[Out] Integral(atanh(a*x)/(a*x**2 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{acx^2 + cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a*c*x^2 + c*x), x)
```

3.68 $\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=270

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{abdx}{2c^3}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{20c^4}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^4}$$

```
[Out] (a*b*d*x)/(2*c^3) + (3*b^2*d*x)/(10*c^3) + (b^2*d*x^2)/(12*c^2) + (b^2*d*x^
3)/(30*c) - (3*b^2*d*ArcTanh[c*x])/(10*c^4) + (b^2*d*x*ArcTanh[c*x])/(2*c^3
) + (b*d*x^2*(a + b*ArcTanh[c*x]))/(5*c^2) + (b*d*x^3*(a + b*ArcTanh[c*x]))
/(6*c) + (b*d*x^4*(a + b*ArcTanh[c*x]))/10 - (d*(a + b*ArcTanh[c*x])^2)/(20
*c^4) + (d*x^4*(a + b*ArcTanh[c*x])^2)/4 + (c*d*x^5*(a + b*ArcTanh[c*x])^2)
/5 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (b^2*d*Log[1 -
 c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Rubi [A] time = 0.649668, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 15, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {5940, 5916, 5980, 266, 43, 5910, 260, 5948, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{abdx}{2c^3}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{20c^4}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{5c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (a*b*d*x)/(2*c^3) + (3*b^2*d*x)/(10*c^3) + (b^2*d*x^2)/(12*c^2) + (b^2*d*x^
3)/(30*c) - (3*b^2*d*ArcTanh[c*x])/(10*c^4) + (b^2*d*x*ArcTanh[c*x])/(2*c^3
) + (b*d*x^2*(a + b*ArcTanh[c*x]))/(5*c^2) + (b*d*x^3*(a + b*ArcTanh[c*x]))
/(6*c) + (b*d*x^4*(a + b*ArcTanh[c*x]))/10 - (d*(a + b*ArcTanh[c*x])^2)/(20
*c^4) + (d*x^4*(a + b*ArcTanh[c*x])^2)/4 + (c*d*x^5*(a + b*ArcTanh[c*x])^2)
/5 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (b^2*d*Log[1 -
 c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
```

```
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int \left(dx^3\left(a+b\tanh^{-1}(cx)\right)^2+cdx^4\left(a+b\tanh^{-1}(cx)\right)^2\right)dx$$

$$= d\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+(cd)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$

$$= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2-\frac{1}{2}(bcd)\int\frac{x^4\left(a+b\right.}{1\cdots}$$

$$= \frac{1}{4}dx^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}cdx^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}(2bd)\int x^3\left(a+b\right.$$

$$= \frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{10}bdx^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\tanh^{-1}\right.$$

$$= \frac{abdx}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bdx^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{10}bdx^4\left(a+b\right.$$

$$= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^3}{30c}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bdx}{\cdots}$$

$$= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{bd}{\cdots}$$

$$= \frac{abdx}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{b^2dx^3}{30c}-\frac{3b^2d\tanh^{-1}(cx)}{10c^4}+\frac{b^2dx\tanh^{-1}(cx)}{2c^3}+\frac{bd}{\cdots}$$

Mathematica [A] time = 0.752875, size = 271, normalized size = 1.

$$\frac{d\left(12b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+12a^2c^5x^5+15a^2c^4x^4+6abc^4x^4+10abc^3x^3+12abc^2x^2+12ab\log\left(c^2x^2-1\right)+\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(-18*a*b - 5*b^2 + 30*a*b*c*x + 18*b^2*c*x + 12*a*b*c^2*x^2 + 5*b^2*c^2*
x^2 + 10*a*b*c^3*x^3 + 2*b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*
a^2*c^5*x^5 + 3*b^2*(-9 + 5*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 2*b*ArcTa
nh[c*x]*(3*a*c^4*x^4*(5 + 4*c*x) + b*(-9 + 15*c*x + 6*c^2*x^2 + 5*c^3*x^3 +
 3*c^4*x^4) - 12*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15
*a*b*Log[1 + c*x] + 20*b^2*Log[1 - c^2*x^2] + 12*a*b*Log[-1 + c^2*x^2] + 12
*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^4)
```

Maple [A] time = 0.054, size = 422, normalized size = 1.6

$$\frac{b^2dx\text{Artanh}(cx)}{2c^3}+\frac{3b^2dx}{10c^3}+\frac{db^2x^2}{12c^2}+\frac{db^2x^3}{30c}+\frac{2cdab\text{Artanh}(cx)x^5}{5}+\frac{a^2dx^4}{4}+\frac{db^2(\ln(cx+1))^2}{80c^4}+\frac{29db^2\ln(cx-1)}{60c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 3/10*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*b^2*d*x^3/c+2/5*c*d*a*b*arctanh(c*
x)*x^5+1/4*a^2*d*x^4+1/10*d*b^2*arctanh(c*x)*x^4+1/4*d*b^2*arctanh(c*x)^2*x
^4+1/80/c^4*d*b^2*ln(c*x+1)^2+29/60/c^4*d*b^2*ln(c*x-1)+9/80/c^4*d*b^2*ln(c
*x-1)^2-1/5/c^4*d*b^2*dilog(1/2+1/2*c*x)+11/60/c^4*d*b^2*ln(c*x+1)+1/5*c*a^
2*d*x^5+1/2*a*b*d*x/c^3+1/2*b^2*d*x*arctanh(c*x)/c^3+1/6/c*d*a*b*x^3+1/5/c^
2*d*a*b*x^2+1/2*d*a*b*arctanh(c*x)*x^4-1/40/c^4*d*b^2*ln(-1/2*c*x+1/2)*ln(c
*x+1)-1/20/c^4*d*a*b*ln(c*x+1)+9/20/c^4*d*a*b*ln(c*x-1)+1/6/c*d*b^2*arctanh
(c*x)*x^3+1/5/c^2*d*b^2*arctanh(c*x)*x^2+1/5*c*d*b^2*arctanh(c*x)^2*x^5+9/2
0/c^4*d*b^2*arctanh(c*x)*ln(c*x-1)-1/20/c^4*d*b^2*arctanh(c*x)*ln(c*x+1)-9/
40/c^4*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/40/c^4*d*b^2*ln(-1/2*c*x+1/2)*ln(1
/2+1/2*c*x)+1/10*d*a*b*x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctanh(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*(4*x^
5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*
d - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^10 - 15*log(c*x + 1
)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 + 2*x^2)/c^8 + 2*log(c^2*
x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*log(c*x + 1)/(c^6*x^2 - c^
4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x + 1)/c^9 + 3*log(c*x - 1
)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8) + 900*c^2*(2*x/c^6 - log(
c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*integrate(1/150*x*log(c*x + 1)/
(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x + 1)^2 + (12*c^5*x^5 - 15*c^4*
x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c^5*x^5 + 1)*log(c*x + 1))*log
(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*x + 1)^2 - 10*log(-c*x + 1) +
2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4*log(-c*x + 1) + 1) + 2000*(c*x
 - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 9000*(c*x - 1)^2*(2*log
(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000*(c*x - 1)*(log(-c*x + 1)^2 - 2*l
og(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6*x^2 - 150*c^4)/c^5 - 540000*integ
rate(1/150*log(c*x + 1)/(c^6*x^2 - c^4), x))*b^2*c*d + 1/12*(6*x^4*arctanh(
c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))
```

```
*a*b*d + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x
- 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) +
 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2*d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2cdx^4 + a^2dx^3 + \left(b^2cdx^4 + b^2dx^3\right)\text{artanh}\,(cx)^2 + 2\left(abcdx^4 + abdx^3\right)\text{artanh}\,(cx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^4 + a^2*d*x^3 + (b^2*c*d*x^4 + b^2*d*x^3)*arctanh(c*x)^2
 + 2*(a*b*c*d*x^4 + a*b*d*x^3)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int a^2x^3\,dx + \int a^2cx^4\,dx + \int b^2x^3\,\text{atanh}^2\,(cx)\,dx + \int 2abx^3\,\text{atanh}\,(cx)\,dx + \int b^2cx^4\,\text{atanh}^2\,(cx)\,dx + \int 2abcx^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x**3, x) + Integral(a**2*c*x**4, x) + Integral(b**2*x**3*a
tanh(c*x)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(b**2*c*x**
4*atanh(c*x)**2, x) + Integral(2*a*b*c*x**4*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)(b\,\text{artanh}\,(cx) + a)^2x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^3, x)
```

3.69 $\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=236

$$-\frac{b^2 d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{abdx}{2c^2}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(\right.$$

```
[Out] (a*b*d*x)/(2*c^2) + (b^2*d*x)/(3*c^2) + (b^2*d*x^2)/(12*c) - (b^2*d*ArcTanh
[c*x])/(3*c^3) + (b^2*d*x*ArcTanh[c*x])/(2*c^2) + (b*d*x^2*(a + b*ArcTanh[c
*x]))/(3*c) + (b*d*x^3*(a + b*ArcTanh[c*x]))/6 + (d*(a + b*ArcTanh[c*x])^2)
/(12*c^3) + (d*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d*x^4*(a + b*ArcTanh[c*x]
)^2)/4 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*d*Log
[1 - c^2*x^2])/(3*c^3) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)
```

Rubi [A] time = 0.52597, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5940, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 266, 43, 5910, 260, 5948}

$$-\frac{b^2 d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}+\frac{abdx}{2c^2}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^3}+\frac{1}{4}cdx^4\left(a+b\tanh^{-1}(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (a*b*d*x)/(2*c^2) + (b^2*d*x)/(3*c^2) + (b^2*d*x^2)/(12*c) - (b^2*d*ArcTanh
[c*x])/(3*c^3) + (b^2*d*x*ArcTanh[c*x])/(2*c^2) + (b*d*x^2*(a + b*ArcTanh[c
*x]))/(3*c) + (b*d*x^3*(a + b*ArcTanh[c*x]))/6 + (d*(a + b*ArcTanh[c*x])^2)
/(12*c^3) + (d*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d*x^4*(a + b*ArcTanh[c*x]
)^2)/4 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^3) + (b^2*d*Log
[1 - c^2*x^2])/(3*c^3) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^2(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2 dx = \int \left(dx^2\left(a+b\tanh^{-1}(cx)\right)^2 + cdx^3\left(a+b\tanh^{-1}(cx)\right)^2\right) dx$$

$$= d\int x^2\left(a+b\tanh^{-1}(cx)\right)^2 dx + (cd)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2 dx$$

$$= \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{1}{3}(2bcd)\int \frac{x^3\left(a+b\,\mathrm{ta}\right.}{1-}$$

$$= \frac{1}{3}dx^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{4}cdx^4\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}(bd)\int x^2\left(a+b\,\mathrm{tan}\right.$$

$$= \frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right) + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3c^3} +$$

$$= \frac{abdx}{2c^2} + \frac{b^2dx}{3c^2} + \frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} + \frac{1}{6}bdx^3\left(a+b\tanh^{-1}(cx)\right) + \frac{d\left(a+b\right.}{}$$

$$= \frac{abdx}{2c^2} + \frac{b^2dx}{3c^2} - \frac{b^2d\tanh^{-1}(cx)}{3c^3} + \frac{b^2dx\tanh^{-1}(cx)}{2c^2} + \frac{bdx^2\left(a+b\tanh^{-1}(cx)\right)}{3c} +$$

$$= \frac{abdx}{2c^2} + \frac{b^2dx}{3c^2} + \frac{b^2dx^2}{12c} - \frac{b^2d\tanh^{-1}(cx)}{3c^3} + \frac{b^2dx\tanh^{-1}(cx)}{2c^2} + \frac{bdx^2\left(a+b\,\mathrm{tanh}\right.}{3c}$$

Mathematica [A] time = 0.574929, size = 234, normalized size = 0.99

$$d\left(4b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 3a^2c^4x^4 + 4a^2c^3x^3 + 2abc^3x^3 + 4abc^2x^2 + 4ab\log\left(c^2x^2-1\right) + 2b\tanh^{-1}(cx)\left(ac^3x^3\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(-b^2 + 6*a*b*c*x + 4*b^2*c*x + 4*a*b*c^2*x^2 + b^2*c^2*x^2 + 4*a^2*c^3*
x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-7 + 4*c^3*x^3 + 3*c^4*x^4)*ArcT
anh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^3*x^3*(4 + 3*c*x) + b*(-2 + 3*c*x + 2*c^
2*x^2 + c^3*x^3) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] -
 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 - c^2*x^2] + 4*a*b*Log[-1 + c^2*x^2] + 4*
b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(12*c^3)
```

Maple [A] time = 0.061, size = 383, normalized size = 1.6

$$\frac{2\,dab\mathrm{Artanh}\,(cx)\,x^3}{3} + \frac{b^2dx}{3\,c^2} + \frac{db^2x^2}{12\,c} + \frac{a^2dx^3}{3} + \frac{b^2dx\mathrm{Artanh}\,(cx)}{2\,c^2} + \frac{cdab\mathrm{Artanh}\,(cx)\,x^4}{2} + \frac{7\,db^2\mathrm{Artanh}\,(cx)\ln(cx-1}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/3*b^2*d*x/c^2+1/12*b^2*d*x^2/c+1/3*a^2*d*x^3+2/3*d*a*b*arctanh(c*x)*x^3-7
/24/c^3*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/24/c^3*d*b^2*ln(-1/2*c*x+1/2)*ln(
1/2+1/2*c*x)+1/24/c^3*d*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+7/12/c^3*d*b^2*arcta
nh(c*x)*ln(c*x-1)+1/12/c^3*d*b^2*arctanh(c*x)*ln(c*x+1)+1/3/c*d*a*b*x^2+7/1
2/c^3*d*a*b*ln(c*x-1)+1/4*c*d*b^2*arctanh(c*x)^2*x^4+1/3/c*d*b^2*arctanh(c*
x)*x^2+1/2*a*b*d*x/c^2+1/2*b^2*d*x*arctanh(c*x)/c^2+1/12/c^3*d*a*b*ln(c*x+1
)+1/2*c*d*a*b*arctanh(c*x)*x^4+1/2/c^3*d*b^2*ln(c*x-1)+1/3*d*b^2*arctanh(c*
x)^2*x^3+1/6*d*a*b*x^3-1/48/c^3*d*b^2*ln(c*x+1)^2+1/6/c^3*d*b^2*ln(c*x+1)+7
/48/c^3*d*b^2*ln(c*x-1)^2-1/3/c^3*d*b^2*dilog(1/2+1/2*c*x)+1/6*d*b^2*arctan
h(c*x)*x^3+1/4*c*a^2*d*x^4
```

Maxima [A] time = 2.10101, size = 543, normalized size = 2.3

$$\frac{1}{4}a^2cdx^4 + \frac{1}{3}a^2dx^3 + \frac{1}{12}\left(6x^4\,\text{artanh}\,(cx) + c\left(\frac{2\left(c^2x^3 + 3x\right)}{c^4} - \frac{3\,\log\,(cx+1)}{c^5} + \frac{3\,\log\,(cx-1)}{c^5}\right)\right)abcd + \frac{1}{3}\left(2x^3\,\text{artar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
+ 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c*d + 1/3*(2*x^3
*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*d + 1/3*(log(c*x +
1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*d/c^3 + 1/6*b^2*d*log(c*
x + 1)/c^3 + 1/2*b^2*d*log(c*x - 1)/c^3 + 1/48*(4*b^2*c^2*d*x^2 + 16*b^2*c*
d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x + 1)^2 + (3*b^2*c
^4*d*x^4 + 4*b^2*c^3*d*x^3 - 7*b^2*d)*log(-c*x + 1)^2 + 4*(b^2*c^3*d*x^3 +
2*b^2*c^2*d*x^2 + 3*b^2*c*d*x)*log(c*x + 1) - 2*(2*b^2*c^3*d*x^3 + 4*b^2*c^
2*d*x^2 + 6*b^2*c*d*x + (3*b^2*c^4*d*x^4 + 4*b^2*c^3*d*x^3 + b^2*d)*log(c*x
 + 1))*log(-c*x + 1))/c^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2cdx^3 + a^2dx^2 + \left(b^2cdx^3 + b^2dx^2\right)\text{artanh}\,(cx)^2 + 2\left(abcdx^3 + abdx^2\right)\text{artanh}\,(cx)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^3 + a^2*d*x^2 + (b^2*c*d*x^3 + b^2*d*x^2)*arctanh(c*x)^2
 + 2*(a*b*c*d*x^3 + a*b*d*x^2)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int a^2x^2\,dx + \int a^2cx^3\,dx + \int b^2x^2\,\text{atanh}^2\,(cx)\,dx + \int 2abx^2\,\text{atanh}\,(cx)\,dx + \int b^2cx^3\,\text{atanh}^2\,(cx)\,dx + \int 2abcx^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x**2, x) + Integral(a**2*c*x**3, x) + Integral(b**2*x**2*a
tanh(c*x)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(b**2*c*x**
3*atanh(c*x)**2, x) + Integral(2*a*b*c*x**3*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)(b \operatorname{artanh}(cx) + a)^2 x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x^2, x)
```

3.70 $\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=196

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2$$

```
[Out] (a*b*d*x)/c + (b^2*d*x)/(3*c) - (b^2*d*ArcTanh[c*x])/(3*c^2) + (b^2*d*x*Arc
Tanh[c*x])/c + (b*d*x^2*(a + b*ArcTanh[c*x]))/3 - (d*(a + b*ArcTanh[c*x])^2
)/(6*c^2) + (d*x^2*(a + b*ArcTanh[c*x])^2)/2 + (c*d*x^3*(a + b*ArcTanh[c*x]
)^2)/3 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (b^2*d*Log
[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rubi [A] time = 0.393827, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5940, 5916, 5980, 5910, 260, 5948, 321, 206, 5984, 5918, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{3c^2}+\frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (a*b*d*x)/c + (b^2*d*x)/(3*c) - (b^2*d*ArcTanh[c*x])/(3*c^2) + (b^2*d*x*Arc
Tanh[c*x])/c + (b*d*x^2*(a + b*ArcTanh[c*x]))/3 - (d*(a + b*ArcTanh[c*x])^2
)/(6*c^2) + (d*x^2*(a + b*ArcTanh[c*x])^2)/2 + (c*d*x^3*(a + b*ArcTanh[c*x]
)^2)/3 - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (b^2*d*Log
[1 - c^2*x^2])/(2*c^2) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int \left(dx\left(a+b\tanh^{-1}(cx)\right)^2 + cdx^2\left(a+b\tanh^{-1}(cx)\right)^2\right)dx$$
$$= d\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx + (cd)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$$
$$= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2 - (bcd)\int \frac{x^2\left(a+b\text{ta}\right.}{1-c}$$
$$= \frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}cdx^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}(2bd)\int x\left(a+b\text{t}\right.$$
$$= \frac{abdx}{c} + \frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right) - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{6c^2} + \frac{1}{2}dx^2\left(a+b\tan\right.$$
$$= \frac{abdx}{c} + \frac{b^2dx}{3c} + \frac{b^2dx\tanh^{-1}(cx)}{c} + \frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx)\right) - \frac{d\left(a+b\tanh\right.}{6c^2}$$
$$= \frac{abdx}{c} + \frac{b^2dx}{3c} - \frac{b^2d\tanh^{-1}(cx)}{3c^2} + \frac{b^2dx\tanh^{-1}(cx)}{c} + \frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx\right.$$
$$= \frac{abdx}{c} + \frac{b^2dx}{3c} - \frac{b^2d\tanh^{-1}(cx)}{3c^2} + \frac{b^2dx\tanh^{-1}(cx)}{c} + \frac{1}{3}bdx^2\left(a+b\tanh^{-1}(cx\right.$$

Mathematica [A] time = 0.479758, size = 201, normalized size = 1.03

$$\frac{d\left(2b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 2a^2c^3x^3 + 3a^2c^2x^2 + 2abc^2x^2 + 2ab\log\left(c^2x^2-1\right) + 2b\tanh^{-1}(cx)\left(ac^2x^2(2cx+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(6*a*b*c*x + 2*b^2*c*x + 3*a^2*c^2*x^2 + 2*a*b*c^2*x^2 + 2*a^2*c^3*x^3 +
 b^2*(-5 + 3*c^2*x^2 + 2*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*
x^2*(3 + 2*c*x) + b*(-1 + 3*c*x + c^2*x^2) - 2*b*Log[1 + E^(-2*ArcTanh[c*x]
)]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*b^2*Log[1 - c^2*x^2] + 2*
a*b*Log[-1 + c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(6*c^2)
```

Maple [A] time = 0.048, size = 341, normalized size = 1.7

$$\frac{ca^2dx^3}{3} + \frac{a^2dx^2}{2} + \frac{cdb^2\left(\text{Artanh}(cx)\right)^2x^3}{3} + \frac{db^2\left(\text{Artanh}(cx)\right)^2x^2}{2} + \frac{db^2\text{Artanh}(cx)x^2}{3} + \frac{b^2dx\text{Artanh}(cx)}{c} + \frac{5\,db^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/3*c*a^2*d*x^3+1/2*a^2*d*x^2+1/3*c*d*b^2*arctanh(c*x)^2*x^3+1/2*d*b^2*arct
anh(c*x)^2*x^2+1/3*d*b^2*arctanh(c*x)*x^2+b^2*d*x*arctanh(c*x)/c+5/6/c^2*d*
b^2*arctanh(c*x)*ln(c*x-1)-1/6/c^2*d*b^2*arctanh(c*x)*ln(c*x+1)+5/24/c^2*d*
b^2*ln(c*x-1)^2-1/3/c^2*d*b^2*dilog(1/2+1/2*c*x)-5/12/c^2*d*b^2*ln(c*x-1)*l
n(1/2+1/2*c*x)+1/12/c^2*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/12/c^2*d*b
^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/24/c^2*d*b^2*ln(c*x+1)^2+1/3*b^2*d*x/c+2/3/
c^2*d*b^2*ln(c*x-1)+1/3/c^2*d*b^2*ln(c*x+1)+2/3*c*d*a*b*arctanh(c*x)*x^3+d*
a*b*arctanh(c*x)*x^2+1/3*d*a*b*x^2+a*b*d*x/c+5/6/c^2*d*a*b*ln(c*x-1)-1/6/c^
```

```
2*d*a*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctanh(c*x)^2 + 1/3*(2*x^3*arctanh(c*x) +
c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d - 1/216*(2*c^4*(2*(c^2*x^3 + 3*
x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c*x - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^
2*x^2 - 1)/c^6) - 648*c^3*integrate(1/9*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x
) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c^5 + log(c*x - 1)/c^5) - 324*c*integrate
(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2), x) - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*
c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(c^3*x^3 + 1)*log(c*x + 1))*log(-c*x + 1))/
c^3 - (2*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1) + 2) + 27*(c*x -
1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 54*(c*x - 1)*(log(-c*x + 1
)^2 - 2*log(-c*x + 1) + 2))/c^3 + 18*log(9*c^4*x^2 - 9*c^2)/c^3 - 324*integ
rate(1/9*log(c*x + 1)/(c^4*x^2 - c^2), x))*b^2*c*d + 1/2*a^2*d*x^2 + 1/2*(2
*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*
d + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) -
 (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*l
og(c*x - 1))/c^2)*b^2*d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2cdx^2 + a^2dx + \left(b^2cdx^2 + b^2dx\right)\text{artanh}\left(cx\right)^2 + 2\left(abcdx^2 + abdx\right)\text{artanh}\left(cx\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x^2 + a^2*d*x + (b^2*c*d*x^2 + b^2*d*x)*arctanh(c*x)^2 + 2
*(a*b*c*d*x^2 + a*b*d*x)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int a^2x\,dx + \int a^2cx^2\,dx + \int b^2x\,\text{atanh}^2\left(cx\right)dx + \int 2abx\,\text{atanh}\left(cx\right)dx + \int b^2cx^2\,\text{atanh}^2\left(cx\right)dx + \int 2abcx^2\,\text{atanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2*x, x) + Integral(a**2*c*x**2, x) + Integral(b**2*x*atanh(c
*x)**2, x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(b**2*c*x**2*atanh(c
*x)**2, x) + Integral(2*a*b*c*x**2*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)(b\,\text{artanh}\,(cx) + a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2*x, x)
```

3.71 $\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=112

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+abdx+\frac{b^2d\log\left(1-\right.}{2c}$$

```
[Out] a*b*d*x + b^2*d*x*ArcTanh[c*x] + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x])^2)/(2*
c) - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b^2*d*Log[1 - c^2*x
^2])/(2*c) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rubi [A] time = 0.118684, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5928, 5910, 260, 1586, 5918, 2402, 2315}

$$-\frac{b^2d\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{d(cx+1)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c}-\frac{2bd\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}+abdx+\frac{b^2d\log\left(1-\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] a*b*d*x + b^2*d*x*ArcTanh[c*x] + (d*(1 + c*x)^2*(a + b*ArcTanh[c*x])^2)/(2*
c) - (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b^2*d*Log[1 - c^2*x
^2])/(2*c) - (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{b\int\left(-d^2\left(a+b\tanh^{-1}(cx)\right)+\frac{2\left(d^2+cd^2x\right)\left(a+b\tanh\ldots\right.}{1-c^2x^2}\ldots}{d}$$

$$= \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{(2b)\int\frac{\left(d^2+cd^2x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx}{d} + (bd)\int\left(a+\ldots\right.$$

$$= abdx + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{(2b)\int\frac{a+b\tanh^{-1}(cx)}{\frac{1}{d^2}-\frac{cx}{d^2}}dx}{d} + \left(b^2d\right)\int\tanh\ldots$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd\left(a+b\tanh^{-1}(\ldots\right.}{c}$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd\left(a+b\tanh^{-1}(\ldots\right.}{c}$$

$$= abdx + b^2dx\tanh^{-1}(cx) + \frac{d(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c} - \frac{2bd\left(a+b\tanh^{-1}(\ldots\right.}{c}$$

Mathematica [A] time = 0.31873, size = 156, normalized size = 1.39

$$\frac{d\left(2b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+a^2c^2x^2+2a^2cx+2ab\log\left(1-c^2x^2\right)+2abcx+ab\log(1-cx)-ab\log(cx+1)+2\ldots\right.}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*d*x)*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d*(2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-3 + 2*c*x + c^2*x^2)*ArcTan
h[c*x]^2 + 2*b*ArcTanh[c*x]*(c*x*(2*a + b + a*c*x) - 2*b*Log[1 + E^(-2*ArcT
anh[c*x])]) + a*b*Log[1 - c*x] - a*b*Log[1 + c*x] + 2*a*b*Log[1 - c^2*x^2]
+ b^2*Log[1 - c^2*x^2] + 2*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

Maple [B] time = 0.051, size = 296, normalized size = 2.6

$$\frac{ca^2dx^2}{2}+a^2dx+\frac{cdb^2\left(\text{Artanh}\left(cx\right)\right)^2x^2}{2}+b^2\left(\text{Artanh}\left(cx\right)\right)^2xd+b^2dx\text{Artanh}\left(cx\right)+\frac{3\,b^2\text{Artanh}\left(cx\right)\ln\left(cx-1\right)d}{2\,c}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/2*c*a^2*d*x^2+a^2*d*x+1/2*c*d*b^2*arctanh(c*x)^2*x^2+b^2*arctanh(c*x)^2*x
*d+b^2*d*x*arctanh(c*x)+3/2/c*b^2*arctanh(c*x)*ln(c*x-1)*d+1/2/c*b^2*arctan
h(c*x)*ln(c*x+1)*d-1/4/c*b^2*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*d+1/4/c*b^2*l
n(-1/2*c*x+1/2)*ln(c*x+1)*d-1/c*b^2*dilog(1/2+1/2*c*x)*d-1/8/c*b^2*ln(c*x+1
)^2*d+1/2/c*d*b^2*ln(c*x-1)+1/2/c*d*b^2*ln(c*x+1)+3/8/c*b^2*ln(c*x-1)^2*d-3
/4/c*b^2*ln(1/2+1/2*c*x)*ln(c*x-1)*d+c*d*a*b*arctanh(c*x)*x^2+2*a*b*arctanh
(c*x)*x*d+a*b*d*x+3/2/c*a*b*ln(c*x-1)*d+1/2/c*a*b*ln(c*x+1)*d
```

Maxima [B] time = 1.74104, size = 392, normalized size = 3.5

$$\frac{1}{2}a^2cdx^2+\frac{1}{2}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)abcd+a^2dx+\frac{\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+1\right.\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c*d*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
 log(c*x - 1)/c^3))*a*b*c*d + a^2*d*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a*b*d/c + (log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b
^2*d/c + 1/2*b^2*d*log(c*x + 1)/c + 1/2*b^2*d*log(c*x - 1)/c + 1/8*(4*b^2*c
*d*x*log(c*x + 1) + (b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1)^2 +
(b^2*c^2*d*x^2 + 2*b^2*c*d*x - 3*b^2*d)*log(-c*x + 1)^2 - 2*(2*b^2*c*d*x +
(b^2*c^2*d*x^2 + 2*b^2*c*d*x + b^2*d)*log(c*x + 1))*log(-c*x + 1))/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(a^2cdx+a^2d+\left(b^2cdx+b^2d\right)\operatorname{artanh}(cx)^2+2\left(abcdx+abd\right)\operatorname{artanh}(cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c*
d*x + a*b*d)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int a^2\,dx+\int b^2\operatorname{atanh}^2(cx)\,dx+\int 2ab\operatorname{atanh}(cx)\,dx+\int a^2cx\,dx+\int b^2cx\operatorname{atanh}^2(cx)\,dx+\int 2abcx\operatorname{atanh}(cx)\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2,x)
```

```
[Out] d*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*ata
nh(c*x), x) + Integral(a**2*c*x, x) + Integral(b**2*c*x*atanh(c*x)**2, x) +
```

```
Integral(2*a*b*c*x*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)(b \operatorname{artanh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2, x)
```

3.72 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=191

$$-bd\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-d)\text{PolyLog}\left(2,1\right.$$

```
[Out] d*(a + b*ArcTanh[c*x])^2 + c*d*x*(a + b*ArcTanh[c*x])^2 + 2*d*(a + b*ArcTan
h[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 -
c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 - c*x)] - b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)] + b*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x
)] + (b^2*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```

Rubi [A] time = 0.450618, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5914, 6052, 5948, 6058, 6610}

$$-bd\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-d)\text{PolyLog}\left(2,1\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```


```
[Out] d*(a + b*ArcTanh[c*x])^2 + c*d*x*(a + b*ArcTanh[c*x])^2 + 2*d*(a + b*ArcTan
h[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1 -
c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 - c*x)] - b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - 2/(1 - c*x)] + b*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x
)] + (b^2*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d*PolyLog[3, -1 + 2/(1 -
c*x)])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(cd\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + (cd)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx$$

$$= cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right) - (4bcd)\int$$

$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\Big($$

$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\Big($$

$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\Big($$

$$= d\left(a+b\tanh^{-1}(cx)\right)^2 + cdx\left(a+b\tanh^{-1}(cx)\right)^2 + 2d\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\Big($$

Mathematica [C] time = 0.530007, size = 228, normalized size = 1.19

$$d\Big(ab(\text{PolyLog}(2,cx) - \text{PolyLog}(2,-cx)) + b^2\left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx)\left((cx-1)\tanh^{-1}(cx) - 2\log\Big(e\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] d*(a^2*c*x + a^2*Log[c*x] + a*b*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2]) + b
^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])])
 + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + a*b*(-PolyLog[2, -(c*x)] + PolyLog[2
, c*x]) + b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 +
E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh
[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTa
nh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*
x])]/2))
```

Maple [C] time = 0.403, size = 3644, normalized size = 19.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x)
```

```
[Out] a^2*d*ln(c*x)-1/2*d*b^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-d*b^2*dilog(1+I*
(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+
d*b^2*arctanh(c*x)^2-d*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*d*b^2*
polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*d*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(
1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(
-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1
))*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d*a*b*dilog(c*x)+2*d*b^2
*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+d*a*b*ln(c*x-1)+d*a*b*l
```

n(c*x+1)+d*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+a^2*d*c*x+2*
d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x
)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-d*b^2*arctanh(c*x)*ln((c*x+1)^2/(-c^2*
x^2+1)+1)-d*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d*b^2*arcta
nh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-d*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-
c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x)^2*ln(c*x)-d*b^2*arctanh(c*x)*polylog(2
,-(c*x+1)^2/(-c^2*x^2+1))-d*a*b*dilog(c*x+1)+2*d*a*b*arctanh(c*x)*c*x+1/2*I
*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*d
ilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*I*d*b^2*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*dilog(1+I*(c*x+
1)/(-c^2*x^2+1)^(1/2))-1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c
*x+1)^2/(-c^2*x^2+1)+1))^3*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+2*d*a*b*arcta
nh(c*x)*ln(c*x)-d*a*b*ln(c*x)*ln(c*x+1)-1/2*I*d*b^2*Pi*csgn(I/((c*x+1)^2/(-
c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+
1))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-
c^2*x^2+1)^(1/2))+d*b^2*arctanh(c*x)^2*c*x+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x
^2+1)^(1/2))+1/4*I*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*polylog(2,-(c*x+1)^2/(-
c^2*x^2+1))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+
1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^
2+1)+1))*arctanh(c*x)*ln((c*x+1)^2/(-c^2*x^2+1)+1)-1/2*I*d*b^2*Pi*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)*ln((c*x
+1)^2/(-c^2*x^2+1)+1)-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*dilog(1-I*(c*x
+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((
c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))
-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(
I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/
(-c^2*x^2+1)+1))^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*Pi*csg
n(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^
2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*
dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*d*b^2*Pi*csgn(I/((c
*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2
*x^2+1)+1))^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*I*d*b^2*P
i*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1
/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2
+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1
)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2
+1)+1))*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(
1/2))-1/2*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-
c^2*x^2+1)^(1/2))+1/2*I*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln((c
*x+1)^2/(-c^2*x^2+1)+1)-1/4*I*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*c
sgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1

```
)^2/(-c^2*x^2+1)+1))*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*I*d*b^2*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2
/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln((c*x+1)^2/(-c^2*x^2+1)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\,b^2cdx\log\left(-cx+1\right)^2+a^2cdx+\left(2\,cx\,\mathrm{artanh}\left(cx\right)+\log\left(-c^2x^2+1\right)\right)abd+a^2d\log\left(x\right)-\int-\frac{\left(b^2c^2dx^2-b^2d\right)\log\left(cx+1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*c*d*x*log(-c*x + 1)^2 + a^2*c*d*x + (2*c*x*arctanh(c*x) + log(-c^2*
x^2 + 1))*a*b*d + a^2*d*log(x) - integrate(-1/4*((b^2*c^2*d*x^2 - b^2*d)*lo
g(c*x + 1)^2 + 4*(a*b*c*d*x - a*b*d)*log(c*x + 1) - 2*(b^2*c^2*d*x^2 + 2*a*
b*c*d*x - 2*a*b*d + (b^2*c^2*d*x^2 - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c
*x^2 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2cdx+a^2d+\left(b^2cdx+b^2d\right)\mathrm{artanh}\left(cx\right)^2+2\left(abcdx+abd\right)\mathrm{artanh}\left(cx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int a^2c\,dx+\int\frac{a^2}{x}\,dx+\int b^2c\,\mathrm{atanh}^2\left(cx\right)dx+\int\frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{x}\,dx+\int 2abc\,\mathrm{atanh}\left(cx\right)dx+\int\frac{2ab\,\mathrm{atanh}\left(cx\right)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d*(Integral(a**2*c, x) + Integral(a**2/x, x) + Integral(b**2*c*atanh(c*x)**
2, x) + Integral(b**2*atanh(c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x), x)
 + Integral(2*a*b*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cdx+d)(b\,\mathrm{artanh}\left(cx\right)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x, x)
```

3.73 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=201

$$-bcd\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bcd\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-c)d\text{PolyLog}\left(2\right.$$

```
[Out] c*d*(a + b*ArcTanh[c*x])^2 - (d*(a + b*ArcTanh[c*x])^2)/x + 2*c*d*(a + b*Ar
cTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 2*b*c*d*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] +
b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d*PolyLog[2
, -1 + 2/(1 + c*x)] + (b^2*c*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*c*d*Po
lyLog[3, -1 + 2/(1 - c*x)])/2
```

Rubi [A] time = 0.484035, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610}

$$-bcd\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bcd\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)+b^2(-c)d\text{PolyLog}\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] c*d*(a + b*ArcTanh[c*x])^2 - (d*(a + b*ArcTanh[c*x])^2)/x + 2*c*d*(a + b*Ar
cTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 2*b*c*d*(a + b*ArcTanh[c*x])*Log[2
 - 2/(1 + c*x)] - b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] +
b*c*d*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d*PolyLog[2
, -1 + 2/(1 + c*x)] + (b^2*c*d*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*c*d*Po
lyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx + (cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right) + (2bcd)\int$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}$$

$$= cd\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 2cd\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}$$

Mathematica [C] time = 0.515846, size = 249, normalized size = 1.24

$$-\frac{d\left(abcx(\mathrm{PolyLog}(2,-cx)-\mathrm{PolyLog}(2,cx))+b^2\left(cx\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\left((1-cx)\tanh^{-1}(cx)-2c\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] -((d*(a^2 - a^2*c*x*Log[x] + a*b*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1
 - c^2*x^2])) + b^2*(ArcTanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E
^(-2*ArcTanh[c*x])]) + c*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + a*b*c*x*(Poly
Log[2, -(c*x)] - PolyLog[2, c*x]) - b^2*c*x*((I/24)*Pi^3 - (2*ArcTanh[c*x]^
3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 -
 E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcT
anh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/
2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/x)
```

Maple [C] time = 0.82, size = 3104, normalized size = 15.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] -1/8*I*c*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*dilog((c*x+1)^2/(-c^2*x^2+1)+1)+1
/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+
1/8*I*c*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))
+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))
-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^
```

```
2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*dilog((c*x+1)^2/(-c^2*x^2+1)+1)+1
/8*I*c*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1
/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*dilog((c*x+1)^2/(-c^2*x^2+1))-1/2*I
*c*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*c*d*b^2*Pi*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-
c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/4*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2
+1))+1/8*I*c*d*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*dilog((c*x+1)^2/(-c^2*x^2+1))
+3/4*c*d*b^2*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+1/4*c*d*b^2*dilog((c*x+1)^2/
(-c^2*x^2+1)+1)-1/2*c*d*b^2*polylog(3,(c*x+1)^2/(-c^2*x^2+1))-1/4*c*d*b^2*p
olylog(2,-(c*x+1)^2/(-c^2*x^2+1))-c*d*b^2*arctanh(c*x)^2+1/2*c*d*b^2*polylo
g(3,-(c*x+1)^2/(-c^2*x^2+1))-1/4*c*d*b^2*dilog((c*x+1)^2/(-c^2*x^2+1))+c*a^
2*d*ln(c*x)-d*b^2*arctanh(c*x)^2/x-a^2*d/x+2*c*d*a*b*arctanh(c*x)*ln(c*x)-c
*d*a*b*ln(c*x)*ln(c*x+1)-1/4*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))-1/8*I*c*d
*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*pol
ylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*dilog((c*x+1)^2/(-c^2*x^2+1)+1)-1/8*I
*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3
*polylog(2,(c*x+1)^2/(-c^2*x^2+1))-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2
*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*dilog((c*x+1)^2/(-c^2*x^2+1))+1/2*
I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^
3*arctanh(c*x)^2+1/2*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I
/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(
-c^2*x^2+1)+1))*arctanh(c*x)^2-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1
)/((c*x+1)^2/(-c^2*x^2+1)+1))*dilog((c*x+1)^2/(-c^2*x^2+1))+1/4*I*c*d*b^2*P
i*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))+1/8*I*c
*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)
+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*dilog((c
*x+1)^2/(-c^2*x^2+1)+1)-1/8*I*c*d*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x
+1)^2/(-c^2*x^2+1)+1))*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/4*I*c*d*b^2*Pi*
csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+
1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+1))-1/8*I*c*d
*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1
))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*polylog(2,
(c*x+1)^2/(-c^2*x^2+1))-2*d*a*b*arctanh(c*x)/x-c*d*a*b*dilog(c*x+1)+c*d*b^2
*arctanh(c*x)*polylog(2,(c*x+1)^2/(-c^2*x^2+1))+c*d*b^2*arctanh(c*x)^2*ln(1
-(c*x+1)^2/(-c^2*x^2+1))+3/2*c*d*b^2*arctanh(c*x)*ln(1-(c*x+1)^2/(-c^2*x^2+
1))-c*d*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+c*d*b^2*arctanh(c*x
)^2*ln(c*x)-c*d*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-c*d*a*b
*dilog(c*x)-c*d*a*b*ln(c*x-1)+2*c*d*a*b*ln(c*x)-c*d*a*b*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2cd\log(x) - \left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\operatorname{artanh}(cx)}{x}\right)abd - \frac{b^2d\log(-cx+1)^2}{4x} - \frac{a^2d}{x} - \int -\frac{\left(b^2c^2dx^2 - b^2d\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] a^2*c*d*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d
 - 1/4*b^2*d*log(-c*x + 1)^2/x - a^2*d/x - integrate(-1/4*((b^2*c^2*d*x^2 -
 b^2*d)*log(c*x + 1)^2 + 4*(a*b*c^2*d*x^2 - a*b*c*d*x)*log(c*x + 1) - 2*(2*
a*b*c^2*d*x^2 - (2*a*b*c*d + b^2*c*d)*x + (b^2*c^2*d*x^2 - b^2*d)*log(c*x +
 1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2cdx + a^2d + \left(b^2cdx + b^2d\right)\text{artanh}\left(cx\right)^2 + 2\left(abcdx + abd\right)\text{artanh}\left(cx\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int \frac{a^2}{x^2}\,dx + \int \frac{a^2c}{x}\,dx + \int \frac{b^2\,\text{atanh}^2\left(cx\right)}{x^2}\,dx + \int \frac{2ab\,\text{atanh}\left(cx\right)}{x^2}\,dx + \int \frac{b^2c\,\text{atanh}^2\left(cx\right)}{x}\,dx + \int \frac{2abc\,\text{atanh}\left(cx\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d*(Integral(a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(b**2*atanh(c*x
)**2/x**2, x) + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(b**2*c*atanh(
c*x)**2/x, x) + Integral(2*a*b*c*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)(b\,\text{artanh}\left(cx\right) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^2, x)
```

3.74 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=151

$$-b^2c^2d\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2+2bc^2d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}$$

```
[Out] -((b*c*d*(a + b*ArcTanh[c*x]))/x) + (3*c^2*d*(a + b*ArcTanh[c*x])^2)/2 - (d
*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (c*d*(a + b*ArcTanh[c*x])^2)/x + b^2*c^2
*d*Log[x] - (b^2*c^2*d*Log[1 - c^2*x^2])/2 + 2*b*c^2*d*(a + b*ArcTanh[c*x])
*Log[2 - 2/(1 + c*x)] - b^2*c^2*d*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rubi [A] time = 0.365271, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {5940, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447}

$$-b^2c^2d\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2+2bc^2d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d*(a + b*ArcTanh[c*x]))/x) + (3*c^2*d*(a + b*ArcTanh[c*x])^2)/2 - (d
*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (c*d*(a + b*ArcTanh[c*x])^2)/x + b^2*c^2
*d*Log[x] - (b^2*c^2*d*Log[1 - c^2*x^2])/2 + 2*b*c^2*d*(a + b*ArcTanh[c*x])
*Log[2 - 2/(1 + c*x)] - b^2*c^2*d*PolyLog[2, -1 + 2/(1 + c*x)]
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx + (cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (bcd)\int \frac{a+b\tanh^{-1}(cx)}{x^2\left(1-c^2x^2\right)}\,d$$

$$= c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x} + (b$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} -$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} -$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} -$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{3}{2}c^2d\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} -$$

Mathematica [A] time = 0.281313, size = 206, normalized size = 1.36

$$-\frac{d\left(2b^2c^2x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) + 2a^2cx + a^2 - 4abc^2x^2\log(cx) + abc^2x^2\log(1-cx) - abc^2x^2\log(cx+1) + 2ab\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -(d*(a^2 + 2*a^2*c*x + 2*a*b*c*x + b^2*(1 + 2*c*x - 3*c^2*x^2)*ArcTanh[c*x]
^2 + 2*b*ArcTanh[c*x]*(a + 2*a*c*x + b*c*x - 2*b*c^2*x^2*Log[1 - E^(-2*ArcT
anh[c*x])]) - 4*a*b*c^2*x^2*Log[c*x] + a*b*c^2*x^2*Log[1 - c*x] - a*b*c^2*x
^2*Log[1 + c*x] - 2*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^2*x^
2*Log[1 - c^2*x^2] + 2*b^2*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]))/(2*x^2
)
```

Maple [B] time = 0.071, size = 400, normalized size = 2.7

$$2c^2db^2\text{Artanh}(cx)\ln(cx) - 2\frac{cdab\text{Artanh}(cx)}{x} + c^2db^2\ln(cx) - \frac{3c^2db^2(\ln(cx-1))^2}{8} + c^2db^2\text{dilog}\left(\frac{1}{2}+\frac{cx}{2}\right) + \frac{c^2d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -2*c*d*a*b*arctanh(c*x)/x+c^2*d*b^2*ln(c*x)-3/8*c^2*d*b^2*ln(c*x-1)^2+c^2*d
*b^2*dilog(1/2+1/2*c*x)+1/8*c^2*d*b^2*ln(c*x+1)^2-c^2*d*b^2*dilog(c*x)-1/2*
d*b^2*arctanh(c*x)^2/x^2-1/2*c^2*d*b^2*ln(c*x-1)-1/2*c^2*d*b^2*ln(c*x+1)-c^
2*d*b^2*dilog(c*x+1)-c*a^2*d/x-1/2*a^2*d/x^2+2*c^2*d*a*b*ln(c*x)-1/2*c^2*d*
```

```
a*b*ln(c*x+1)+2*c^2*d*b^2*arctanh(c*x)*ln(c*x)-3/2*c^2*d*b^2*arctanh(c*x)*l
n(c*x-1)-1/2*c^2*d*b^2*arctanh(c*x)*ln(c*x+1)+3/4*c^2*d*b^2*ln(c*x-1)*ln(1/
2+1/2*c*x)-3/2*c^2*d*a*b*ln(c*x-1)-c*d*b^2*arctanh(c*x)^2/x-c*d*b^2*arctanh
(c*x)/x+1/4*c^2*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/4*c^2*d*b^2*ln(-1/
2*c*x+1/2)*ln(c*x+1)-c^2*d*b^2*ln(c*x)*ln(c*x+1)-d*a*b*arctanh(c*x)/x^2-c*d
*a*b/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\operatorname{artanh}(cx)}{x}\right)abcd-\frac{1}{4}\,b^2cd\left(\frac{\log(-cx+1)^2}{x}+\int-\frac{(cx-1)\log(cx+1)^2+2\,(cx-(cx-}{cx^3-x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c*d - 1/4*b^2*c*d
*(log(-c*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x
- 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) + 1/2*((c*log(c*x + 1)
 - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*d + 1/8*((2*(log(c*x -
 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1) +
8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c*arctanh(c*x))*b
^2*d - a^2*c*d/x - 1/2*b^2*d*arctanh(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2cdx+a^2d+\left(b^2cdx+b^2d\right)\operatorname{artanh}(cx)^2+2\,(abcdx+abd)\operatorname{artanh}(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a^2}{x^3}\,dx+\int\frac{a^2c}{x^2}\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{x^3}\,dx+\int\frac{2ab\operatorname{atanh}(cx)}{x^3}\,dx+\int\frac{b^2c\operatorname{atanh}^2(cx)}{x^2}\,dx+\int\frac{2abc\operatorname{atanh}(cx)}{x^2}\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d*(Integral(a**2/x**3, x) + Integral(a**2*c/x**2, x) + Integral(b**2*atanh(
c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(b**2*c*ata
nh(c*x)**2/x**2, x) + Integral(2*a*b*c*atanh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)(b\,\text{artanh}\,(cx)+a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^3, x)
```

3.75 $\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^4} dx$

Optimal. Leaf size=206

$$-\frac{1}{3}b^2c^3d\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{2}{3}bc^3d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\right.$$

```
[Out] -(b^2*c^2*d)/(3*x) + (b^2*c^3*d*ArcTanh[c*x])/3 - (b*c*d*(a + b*ArcTanh[c*x
]))/(3*x^2) - (b*c^2*d*(a + b*ArcTanh[c*x]))/x + (5*c^3*d*(a + b*ArcTanh[c*
x])^2)/6 - (d*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (c*d*(a + b*ArcTanh[c*x])^2
)/(2*x^2) + b^2*c^3*d*Log[x] - (b^2*c^3*d*Log[1 - c^2*x^2])/2 + (2*b*c^3*d*
(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*d*PolyLog[2, -1 + 2
/(1 + c*x)])/3
```

Rubi [A] time = 0.453118, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.65, Rules used = {5940, 5916, 5982, 325, 206, 5988, 5932, 2447, 266, 36, 29, 31, 5948}

$$-\frac{1}{3}b^2c^3d\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)+\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{2}{3}bc^3d\log\left(2-\frac{2}{cx+1}\right)\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```


```
[Out] -(b^2*c^2*d)/(3*x) + (b^2*c^3*d*ArcTanh[c*x])/3 - (b*c*d*(a + b*ArcTanh[c*x
]))/(3*x^2) - (b*c^2*d*(a + b*ArcTanh[c*x]))/x + (5*c^3*d*(a + b*ArcTanh[c*
x])^2)/6 - (d*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (c*d*(a + b*ArcTanh[c*x])^2
)/(2*x^2) + b^2*c^3*d*Log[x] - (b^2*c^3*d*Log[1 - c^2*x^2])/2 + (2*b*c^3*d*
(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*d*PolyLog[2, -1 + 2
/(1 + c*x)])/3
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(d+cdx)\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}+\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\right)dx$$

$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx+(cd)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}(2bcd)\int \frac{a+b\tanh^{-1}(cx)}{x^3\left(1-c^2x^2\right)}\,d$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{cd\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}+\frac{1}{3}(2bcd)\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,d$$

$$= -\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{6}c^3d\left(a+b\tanh^{-1}(cx)\right)^2-$$

$$= -\frac{b^2c^2d}{3x}-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{6}c^3d\left(a+b\tanh^{-1}\right.$$

$$= -\frac{b^2c^2d}{3x}+\frac{1}{3}b^2c^3d\tanh^{-1}(cx)-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}$$

$$= -\frac{b^2c^2d}{3x}+\frac{1}{3}b^2c^3d\tanh^{-1}(cx)-\frac{bcd\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{bc^2d\left(a+b\tanh^{-1}(cx)\right)}{x}$$

Mathematica [A] time = 0.472233, size = 246, normalized size = 1.19

$$-\frac{d\left(2b^2c^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+3a^2cx+2a^2+6abc^2x^2-4abc^3x^3\log(cx)+3abc^3x^3\log(1-cx)-3abc^3x^3\log(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(d*(2*a^2 + 3*a^2*c*x + 2*a*b*c*x + 6*a*b*c^2*x^2 + 2*b^2*c^2*x^2 + b^2*(2
 + 3*c*x - 5*c^3*x^3)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*(2 + 3*c*x) + b*
c*x*(1 + 3*c*x - c^2*x^2) - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) - 4*a
*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*Log[1 - c*x] - 3*a*b*c^3*x^3*Log[1 + c*
x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 - c^2
*x^2] + 2*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]))/(6*x^3)
```

Maple [B] time = 0.07, size = 440, normalized size = 2.1

$$\frac{2\,c^3db^2\text{Artanh}\,(cx)\ln(cx)}{3}-\frac{cdb^2\text{Artanh}\,(cx)}{3\,x^2}+\frac{2\,c^3dab\ln(cx)}{3}+\frac{c^3dab\ln(cx+1)}{6}+\frac{5\,c^3db^2\ln(cx-1)}{12}\ln\left(\frac{1}{2}+\frac{cx}{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -c*d*a*b*arctanh(c*x)/x^2+2/3*c^3*d*a*b*ln(c*x)+1/6*c^3*d*a*b*ln(c*x+1)+2/3
*c^3*d*b^2*arctanh(c*x)*ln(c*x)-5/6*c^3*d*b^2*arctanh(c*x)*ln(c*x-1)+1/6*c^
3*d*b^2*arctanh(c*x)*ln(c*x+1)+5/12*c^3*d*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/1
2*c^3*d*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/12*c^3*d*b^2*ln(-1/2*c*x+1/2
)*ln(c*x+1)-1/3*c^3*d*b^2*ln(c*x)*ln(c*x+1)-c^2*d*b^2*arctanh(c*x)/x-1/2*c*
d*b^2*arctanh(c*x)^2/x^2-1/3*c*d*a*b/x^2-c^2*d*a*b/x-2/3*d*a*b*arctanh(c*x)
/x^3-1/3*c*d*b^2*arctanh(c*x)/x^2-1/3*b^2*c^2*d/x-5/6*c^3*d*a*b*ln(c*x-1)-1
/3*a^2*d/x^3-5/24*c^3*d*b^2*ln(c*x-1)^2-1/3*d*b^2*arctanh(c*x)^2/x^3+c^3*d*
b^2*ln(c*x)+1/3*c^3*d*b^2*dilog(1/2+1/2*c*x)-1/24*c^3*d*b^2*ln(c*x+1)^2-1/3
*c^3*d*b^2*dilog(c*x)-2/3*c^3*d*b^2*ln(c*x-1)-1/2*c*a^2*d/x^2-1/3*c^3*d*b^2
*ln(c*x+1)-1/3*c^3*d*b^2*dilog(c*x+1)
```

Maxima [B] time = 3.06758, size = 563, normalized size = 2.73

$$-\frac{1}{3}\left(\log\left(cx+1\right)\log\left(-\frac{1}{2}\,cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}\,cx+\frac{1}{2}\right)\right)b^2c^3d-\frac{1}{3}\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2\left(-cx+1\right)\right)b^2c^3d+\frac{1}{3}\left(\log\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d -
1/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d + 1/3*(log(c*x + 1
)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d - 1/3*b^2*c^3*d*log(c*x + 1) - 2/3*
b^2*c^3*d*log(c*x - 1) + b^2*c^3*d*log(x) + 1/2*((c*log(c*x + 1) - c*log(c*
x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d - 1/3*((c^2*log(c^2*x^2 - 1)
- c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d - 1/2*a^2*c*d/x^2 - 1
/3*a^2*d/x^3 - 1/24*(8*b^2*c^2*d*x^2 - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2
*d)*log(c*x + 1)^2 - (5*b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(-c*x + 1
)^2 + 4*(3*b^2*c^2*d*x^2 + b^2*c*d*x)*log(c*x + 1) - 2*(6*b^2*c^2*d*x^2 + 2
*b^2*c*d*x - (b^2*c^3*d*x^3 - 3*b^2*c*d*x - 2*b^2*d)*log(c*x + 1))*log(-c*x
 + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2cdx+a^2d+\left(b^2cdx+b^2d\right)\text{artanh}\left(cx\right)^2+2\left(abcdx+abd\right)\text{artanh}\left(cx\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c*d*x + a^2*d + (b^2*c*d*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*c
*d*x + a*b*d)*arctanh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d\left(\int\frac{a^2}{x^4}\,dx+\int\frac{a^2c}{x^3}\,dx+\int\frac{b^2\,\text{atanh}^2\left(cx\right)}{x^4}\,dx+\int\frac{2ab\,\text{atanh}\left(cx\right)}{x^4}\,dx+\int\frac{b^2c\,\text{atanh}^2\left(cx\right)}{x^3}\,dx+\int\frac{2abc\,\text{atanh}\left(cx\right)}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d*(Integral(a**2/x**4, x) + Integral(a**2*c/x**3, x) + Integral(b**2*atanh(
c*x)**2/x**4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(b**2*c*ata
nh(c*x)**2/x**3, x) + Integral(2*a*b*c*atanh(c*x)/x**3, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)(b\operatorname{artanh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)*(b*arctanh(c*x) + a)^2/x^4, x)
```

3.76 $\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=356

$$-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{5abd^2x}{6c^3}-\frac{d^2\left(a+b\tanh\right.}{60c^4}$$

```
[Out] (5*a*b*d^2*x)/(6*c^3) + (3*b^2*d^2*x)/(5*c^3) + (31*b^2*d^2*x^2)/(180*c^2)
+ (b^2*d^2*x^3)/(15*c) + (b^2*d^2*x^4)/60 - (3*b^2*d^2*ArcTanh[c*x])/(5*c^4
) + (5*b^2*d^2*x*ArcTanh[c*x])/(6*c^3) + (2*b*d^2*x^2*(a + b*ArcTanh[c*x]))
/(5*c^2) + (5*b*d^2*x^3*(a + b*ArcTanh[c*x]))/(18*c) + (b*d^2*x^4*(a + b*Ar
cTanh[c*x]))/5 + (b*c*d^2*x^5*(a + b*ArcTanh[c*x]))/15 - (d^2*(a + b*ArcTan
h[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 + (2*c*d^2*x^5*(a
+ b*ArcTanh[c*x])^2)/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x])^2)/6 - (4*b*d^2*
(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (53*b^2*d^2*Log[1 - c^2*x^
2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```

Rubi [A] time = 1.02416, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 266, 43, 5910, 260, 5948, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^4}+\frac{1}{6}c^2d^2x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{5abd^2x}{6c^3}-\frac{d^2\left(a+b\tanh\right.}{60c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (5*a*b*d^2*x)/(6*c^3) + (3*b^2*d^2*x)/(5*c^3) + (31*b^2*d^2*x^2)/(180*c^2)
+ (b^2*d^2*x^3)/(15*c) + (b^2*d^2*x^4)/60 - (3*b^2*d^2*ArcTanh[c*x])/(5*c^4
) + (5*b^2*d^2*x*ArcTanh[c*x])/(6*c^3) + (2*b*d^2*x^2*(a + b*ArcTanh[c*x]))
/(5*c^2) + (5*b*d^2*x^3*(a + b*ArcTanh[c*x]))/(18*c) + (b*d^2*x^4*(a + b*Ar
cTanh[c*x]))/5 + (b*c*d^2*x^5*(a + b*ArcTanh[c*x]))/15 - (d^2*(a + b*ArcTan
h[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 + (2*c*d^2*x^5*(a
+ b*ArcTanh[c*x])^2)/5 + (c^2*d^2*x^6*(a + b*ArcTanh[c*x])^2)/6 - (4*b*d^2*
(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^4) + (53*b^2*d^2*Log[1 - c^2*x^
2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^4)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int\left(d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^5\left(a+b\,\mathrm{t}\right.\right.$$

$$= d^2\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(c^2\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}c^2d^2x^6\left(a+b\,\mathrm{t}\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{5}cd^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{6}c^2d^2x^6\left(a+b\,\mathrm{t}\right.$$

$$= \frac{bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{1}{5}bd^2x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{15}bcd^2x^5\left(a+b\,\mathrm{ta}\right.$$

$$= \frac{abd^2x}{2c^3}+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{5bd^2x^3\left(a+b\tanh^{-1}(cx)\right)}{18c}+\frac{1}{5}bd^2x^4$$

$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{b^2d^2x^3}{15c}+\frac{b^2d^2x\tanh^{-1}(cx)}{2c^3}+\frac{2bd^2x^2\left(a+b\tanh^{-1}(cx\right.}{5c^2}$$

$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{7b^2d^2x^2}{60c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh^{-1}(cx)}{5c^4}+$$

$$= \frac{5abd^2x}{6c^3}+\frac{3b^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{b^2d^2x^3}{15c}+\frac{1}{60}b^2d^2x^4-\frac{3b^2d^2\tanh^{-1}(cx)}{5c^4}+$$

Mathematica [A] time = 1.04655, size = 329, normalized size = 0.92

$$d^2\left(72b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+30a^2c^6x^6+72a^2c^5x^5+45a^2c^4x^4+12abc^5x^5+36abc^4x^4+50abc^3x^3+72abc^2x\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-108*a*b - 34*b^2 + 150*a*b*c*x + 108*b^2*c*x + 72*a*b*c^2*x^2 + 31*b
^2*c^2*x^2 + 50*a*b*c^3*x^3 + 12*b^2*c^3*x^3 + 45*a^2*c^4*x^4 + 36*a*b*c^4*
x^4 + 3*b^2*c^4*x^4 + 72*a^2*c^5*x^5 + 12*a*b*c^5*x^5 + 30*a^2*c^6*x^6 + 3*
b^2*(-49 + 15*c^4*x^4 + 24*c^5*x^5 + 10*c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTa
nh[c*x]*(3*a*c^4*x^4*(15 + 24*c*x + 10*c^2*x^2) + b*(-54 + 75*c*x + 36*c^2*
x^2 + 25*c^3*x^3 + 18*c^4*x^4 + 6*c^5*x^5) - 72*b*Log[1 + E^(-2*ArcTanh[c*x
])]) + 75*a*b*Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 106*b^2*Log[1 - c^2*x^2]
 + 72*a*b*Log[-1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(18
0*c^4)
```

Maple [A] time = 0.054, size = 569, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 1/4*d^2*a^2*x^4+3/5*b^2*d^2*x/c^3+31/180*b^2*d^2*x^2/c^2+1/15*b^2*d^2*x^3/c
+8/9/c^4*d^2*b^2*ln(c*x-1)+13/45/c^4*d^2*b^2*ln(c*x+1)+1/6*c^2*d^2*a^2*x^6+
2/5*c*d^2*a^2*x^5+1/5*d^2*a*b*x^4-2/5/c^4*d^2*b^2*dilog(1/2+1/2*c*x)+1/240/
c^4*d^2*b^2*ln(c*x+1)^2+49/240/c^4*d^2*b^2*ln(c*x-1)^2+1/5*d^2*b^2*arctanh(
c*x)*x^4+1/4*d^2*b^2*arctanh(c*x)^2*x^4+1/15*c*d^2*b^2*arctanh(c*x)*x^5+5/1
8/c*d^2*a*b*x^3+1/6*c^2*d^2*b^2*arctanh(c*x)^2*x^6+2/5*c*d^2*b^2*arctanh(c*
x)^2*x^5+1/2*d^2*a*b*arctanh(c*x)*x^4+5/18/c*d^2*b^2*arctanh(c*x)*x^3+2/5/c
^2*d^2*b^2*arctanh(c*x)*x^2-1/60/c^4*d^2*a*b*ln(c*x+1)+49/60/c^4*d^2*b^2*ar
ctanh(c*x)*ln(c*x-1)-1/60/c^4*d^2*b^2*arctanh(c*x)*ln(c*x+1)-49/120/c^4*d^2
*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+49/60/c^4*d^2*a*b*ln(c*x-1)-1/120/c^4*d^2*b^
2*ln(-1/2*c*x+1/2)*ln(c*x+1)+2/5/c^2*d^2*a*b*x^2+1/15*c*d^2*a*b*x^5+1/60*b^
2*d^2*x^4+5/6*a*b*d^2*x/c^3+5/6*b^2*d^2*x*arctanh(c*x)/c^3+1/120/c^4*d^2*b^
2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/3*c^2*d^2*a*b*arctanh(c*x)*x^6+4/5*c*d
^2*a*b*arctanh(c*x)*x^5
```

Maxima [B] time = 2.24754, size = 1034, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^2*d^2*x^6 + 2/5*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctanh(c*x)^2 +
1/4*a^2*d^2*x^4 + 1/90*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 +
 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^2 + 1/5*
(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a
*b*c*d^2 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x
+ 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*d^2 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4
- 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3
*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*
log(c*x - 1))/c^4)*b^2*d^2 + 2/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(
1/2*c*x + 1/2))*b^2*d^2/c^4 - 2/45*b^2*d^2*log(c*x + 1)/c^4 + 5/9*b^2*d^2*l
og(c*x - 1)/c^4 + 1/360*(6*b^2*c^4*d^2*x^4 + 24*b^2*c^3*d^2*x^3 + 32*b^2*c^
```

```
2*d^2*x^2 + 216*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 + 7
*b^2*d^2)*log(c*x + 1)^2 + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 - 17*b
^2*d^2)*log(-c*x + 1)^2 + 4*(3*b^2*c^5*d^2*x^5 + 9*b^2*c^4*d^2*x^4 + 5*b^2*
c^3*d^2*x^3 + 18*b^2*c^2*d^2*x^2 + 15*b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*
c^5*d^2*x^5 + 18*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 36*b^2*c^2*d^2*x^2
+ 30*b^2*c*d^2*x + 3*(5*b^2*c^6*d^2*x^6 + 12*b^2*c^5*d^2*x^5 + 7*b^2*d^2)*l
og(c*x + 1))*log(-c*x + 1))/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^2d^2x^5 + 2\,a^2cd^2x^4 + a^2d^2x^3 + \left(b^2c^2d^2x^5 + 2\,b^2cd^2x^4 + b^2d^2x^3\right)\text{artanh}\,(cx)^2 + 2\left(abc^2d^2x^5 + 2\,abcd^2x^4 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^5 + 2*a^2*c*d^2*x^4 + a^2*d^2*x^3 + (b^2*c^2*d^2*x^5
 + 2*b^2*c*d^2*x^4 + b^2*d^2*x^3)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^5 + 2*a
*b*c*d^2*x^4 + a*b*d^2*x^3)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2x^3\,dx + \int 2a^2cx^4\,dx + \int a^2c^2x^5\,dx + \int b^2x^3\,\text{atanh}^2\,(cx)\,dx + \int 2abx^3\,\text{atanh}\,(cx)\,dx + \int 2b^2cx^4\,\text{atanh}^2\,(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x**3, x) + Integral(2*a**2*c*x**4, x) + Integral(a**2*c
**2*x**5, x) + Integral(b**2*x**3*atanh(c*x)**2, x) + Integral(2*a*b*x**3*a
tanh(c*x), x) + Integral(2*b**2*c*x**4*atanh(c*x)**2, x) + Integral(b**2*c*
*2*x**5*atanh(c*x)**2, x) + Integral(4*a*b*c*x**4*atanh(c*x), x) + Integral
(2*a*b*c**2*x**5*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)^2(b\,\text{artanh}\,(cx) + a)^2x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2*x^3, x)
```

3.77 $\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=312

$$-\frac{8b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abd^2x}{c^2}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1-cx}\right)\left(a+\right.}{15c^3}$$

```
[Out] (a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + (b^2*d^2*x^2)/(6*c) + (b^2*d^2*
x^3)/30 - (19*b^2*d^2*ArcTanh[c*x])/(30*c^3) + (b^2*d^2*x*ArcTanh[c*x])/c^2
 + (8*b*d^2*x^2*(a + b*ArcTanh[c*x]))/(15*c) + (b*d^2*x^3*(a + b*ArcTanh[c*
x]))/3 + (b*c*d^2*x^4*(a + b*ArcTanh[c*x]))/10 + (d^2*(a + b*ArcTanh[c*x])^
2)/(30*c^3) + (d^2*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d^2*x^4*(a + b*ArcTan
h[c*x])^2)/2 + (c^2*d^2*x^5*(a + b*ArcTanh[c*x])^2)/5 - (16*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)])/(15*c^3) + (2*b^2*d^2*Log[1 - c^2*x^2])/(3*c^
3) - (8*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Rubi [A] time = 0.883728, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 266, 43, 5910, 260, 5948, 302}

$$-\frac{8b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{1}{5}c^2d^2x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{abd^2x}{c^2}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1-cx}\right)\left(a+\right.}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + (b^2*d^2*x^2)/(6*c) + (b^2*d^2*
x^3)/30 - (19*b^2*d^2*ArcTanh[c*x])/(30*c^3) + (b^2*d^2*x*ArcTanh[c*x])/c^2
 + (8*b*d^2*x^2*(a + b*ArcTanh[c*x]))/(15*c) + (b*d^2*x^3*(a + b*ArcTanh[c*
x]))/3 + (b*c*d^2*x^4*(a + b*ArcTanh[c*x]))/10 + (d^2*(a + b*ArcTanh[c*x])^
2)/(30*c^3) + (d^2*x^3*(a + b*ArcTanh[c*x])^2)/3 + (c*d^2*x^4*(a + b*ArcTan
h[c*x])^2)/2 + (c^2*d^2*x^5*(a + b*ArcTanh[c*x])^2)/5 - (16*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)])/(15*c^3) + (2*b^2*d^2*Log[1 - c^2*x^2])/(3*c^
3) - (8*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```

```
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^4\left(a+b\tanh\right.\right.$$

$$=d^2\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(c^2d^2\right)$$

$$=\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}c^2d^2x^5\left(a+b\tan\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}cd^2x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{5}c^2d^2x^5\left(a+b\tan\right.$$

$$=\frac{bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{3}bd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{10}bcd^2x^4\left(a+b\tanh\right.$$

$$=\frac{abd^2x}{c^2}+\frac{b^2d^2x}{3c^2}+\frac{8bd^2x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{1}{3}bd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{10}$$

$$=\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{1}{30}b^2d^2x^3-\frac{b^2d^2\tanh^{-1}(cx)}{3c^3}+\frac{b^2d^2x\tanh^{-1}(cx)}{c^2}+\frac{8bd^2x}{}$$

$$=\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2d^2x\tanh^{-1}(}{c^2}$$

$$=\frac{abd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{b^2d^2x^2}{6c}+\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\tanh^{-1}(cx)}{30c^3}+\frac{b^2d^2x\tanh^{-1}(}{c^2}$$

Mathematica [A] time = 0.999511, size = 297, normalized size = 0.95

$$\frac{d^2\left(16b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+6a^2c^5x^5+15a^2c^4x^4+10a^2c^3x^3+3abc^4x^4+10abc^3x^3+16abc^2x^2+16ab\log\left(c^2x^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-9*a*b - 5*b^2 + 30*a*b*c*x + 19*b^2*c*x + 16*a*b*c^2*x^2 + 5*b^2*c^2
*x^2 + 10*a^2*c^3*x^3 + 10*a*b*c^3*x^3 + b^2*c^3*x^3 + 15*a^2*c^4*x^4 + 3*a
*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-31 + 10*c^3*x^3 + 15*c^4*x^4 + 6*c^5*x^5
)*ArcTanh[c*x]^2 + b*ArcTanh[c*x]*(2*a*c^3*x^3*(10 + 15*c*x + 6*c^2*x^2) +
b*(-19 + 30*c*x + 16*c^2*x^2 + 10*c^3*x^3 + 3*c^4*x^4) - 32*b*Log[1 + E^(-2
*ArcTanh[c*x])]) + 15*a*b*Log[1 - c*x] - 15*a*b*Log[1 + c*x] + 20*b^2*Log[1
 - c^2*x^2] + 16*a*b*Log[-1 + c^2*x^2] + 16*b^2*PolyLog[2, -E^(-2*ArcTanh[c
*x])]))/(30*c^3)
```

Maple [A] time = 0.053, size = 521, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] a*b*d^2*x/c^2+b^2*d^2*x*arctanh(c*x)/c^2+c*d^2*a*b*arctanh(c*x)*x^4+2/5*c^2
*d^2*a*b*arctanh(c*x)*x^5+19/30*b^2*d^2*x/c^2+1/6*b^2*d^2*x^2/c+59/60/c^3*d
^2*b^2*ln(c*x-1)+1/5*c^2*d^2*a^2*x^5+1/2*c*d^2*a^2*x^4-8/15/c^3*d^2*b^2*dil
og(1/2+1/2*c*x)+7/20/c^3*d^2*b^2*ln(c*x+1)-1/120/c^3*d^2*b^2*ln(c*x+1)^2+1/
3*d^2*a^2*x^3+31/120/c^3*d^2*b^2*ln(c*x-1)^2+1/3*d^2*b^2*arctanh(c*x)*x^3+1
/3*d^2*b^2*arctanh(c*x)^2*x^3+1/3*d^2*a*b*x^3+1/30*b^2*d^2*x^3+1/5*c^2*d^2*
b^2*arctanh(c*x)^2*x^5+2/3*d^2*a*b*arctanh(c*x)*x^3+8/15/c*d^2*a*b*x^2+1/10
*c*d^2*a*b*x^4+1/2*c*d^2*b^2*arctanh(c*x)^2*x^4+31/30/c^3*d^2*b^2*arctanh(c
*x)*ln(c*x-1)+1/30/c^3*d^2*b^2*arctanh(c*x)*ln(c*x+1)-31/60/c^3*d^2*b^2*ln(
c*x-1)*ln(1/2+1/2*c*x)+1/60/c^3*d^2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/60/c^3
*d^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+31/30/c^3*d^2*a*b*ln(c*x-1)+1/30/
c^3*d^2*a*b*ln(c*x+1)+8/15/c*d^2*b^2*arctanh(c*x)*x^2+1/10*c*d^2*b^2*arctan
h(c*x)*x^4
```

Maxima [B] time = 2.15045, size = 815, normalized size = 2.61

$$\frac{1}{5}a^2c^2d^2x^5 + \frac{1}{2}a^2cd^2x^4 + \frac{1}{10}\left(4x^5\operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2x^2}{c^4} + \frac{2\log\left(c^2x^2 - 1\right)}{c^6}\right)\right)abc^2d^2 + \frac{1}{3}a^2d^2x^3 + \frac{1}{6}\left(6x^4\operatorname{arta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^2*d^2*x^5 + 1/2*a^2*c*d^2*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((c^
2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2*x^3
 + 1/6*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5
+ 3*log(c*x - 1)/c^5))*a*b*c*d^2 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*d^2 + 8/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + di
log(1/2*c*x + 1/2))*b^2*d^2/c^3 + 7/20*b^2*d^2*log(c*x + 1)/c^3 + 59/60*b^2
*d^2*log(c*x - 1)/c^3 + 1/120*(4*b^2*c^3*d^2*x^3 + 20*b^2*c^2*d^2*x^2 + 76*
b^2*c*d^2*x + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3
+ b^2*d^2)*log(c*x + 1)^2 + (6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^
2*c^3*d^2*x^3 - 31*b^2*d^2)*log(-c*x + 1)^2 + 2*(3*b^2*c^4*d^2*x^4 + 10*b^2
*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x)*log(c*x + 1) - 2*(3*b^2
*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + 16*b^2*c^2*d^2*x^2 + 30*b^2*c*d^2*x + (
6*b^2*c^5*d^2*x^5 + 15*b^2*c^4*d^2*x^4 + 10*b^2*c^3*d^2*x^3 + b^2*d^2)*log(
```

```
c*x + 1))*log(-c*x + 1))/c^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^2d^2x^4 + 2\,a^2cd^2x^3 + a^2d^2x^2 + \left(b^2c^2d^2x^4 + 2\,b^2cd^2x^3 + b^2d^2x^2\right)\text{artanh}\left(cx\right)^2 + 2\left(abc^2d^2x^4 + 2\,abcd^2x^3 + ab\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^4 + 2*a^2*c*d^2*x^3 + a^2*d^2*x^2 + (b^2*c^2*d^2*x^4
 + 2*b^2*c*d^2*x^3 + b^2*d^2*x^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^4 + 2*a
*b*c*d^2*x^3 + a*b*d^2*x^2)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2x^2\,dx + \int 2a^2cx^3\,dx + \int a^2c^2x^4\,dx + \int b^2x^2\,\text{atanh}^2\left(cx\right)dx + \int 2abx^2\,\text{atanh}\left(cx\right)dx + \int 2b^2cx^3\,\text{atanh}^2\left(cx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x**2, x) + Integral(2*a**2*c*x**3, x) + Integral(a**2*c
**2*x**4, x) + Integral(b**2*x**2*atanh(c*x)**2, x) + Integral(2*a*b*x**2*a
tanh(c*x), x) + Integral(2*b**2*c*x**3*atanh(c*x)**2, x) + Integral(b**2*c*
*2*x**4*atanh(c*x)**2, x) + Integral(4*a*b*c*x**3*atanh(c*x), x) + Integral
(2*a*b*c**2*x**4*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)^2(b\,\text{artanh}\left(cx\right) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2*x^2, x)
```

3.78 $\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=280

$$-\frac{2b^2d^2\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{12c^2}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}\right.}{3c^2}$$

```
[Out] (3*a*b*d^2*x)/(2*c) + (2*b^2*d^2*x)/(3*c) + (b^2*d^2*x^2)/12 - (2*b^2*d^2*A
rcTanh[c*x])/(3*c^2) + (3*b^2*d^2*x*ArcTanh[c*x])/(2*c) + (2*b*d^2*x^2*(a +
 b*ArcTanh[c*x]))/3 + (b*c*d^2*x^3*(a + b*ArcTanh[c*x]))/6 - (d^2*(a + b*Ar
cTanh[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTanh[c*x])^2)/2 + (2*c*d^2*x^3
*(a + b*ArcTanh[c*x])^2)/3 + (c^2*d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 - (4*b*
d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (5*b^2*d^2*Log[1 - c^2
*x^2])/(6*c^2) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```

Rubi [A] time = 0.652195, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5940, 5916, 5980, 5910, 260, 5948, 321, 206, 5984, 5918, 2402, 2315, 266, 43}

$$-\frac{2b^2d^2\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^2}+\frac{1}{4}c^2d^2x^4\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{12c^2}-\frac{4bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (3*a*b*d^2*x)/(2*c) + (2*b^2*d^2*x)/(3*c) + (b^2*d^2*x^2)/12 - (2*b^2*d^2*A
rcTanh[c*x])/(3*c^2) + (3*b^2*d^2*x*ArcTanh[c*x])/(2*c) + (2*b*d^2*x^2*(a +
 b*ArcTanh[c*x]))/3 + (b*c*d^2*x^3*(a + b*ArcTanh[c*x]))/6 - (d^2*(a + b*Ar
cTanh[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTanh[c*x])^2)/2 + (2*c*d^2*x^3
*(a + b*ArcTanh[c*x])^2)/3 + (c^2*d^2*x^4*(a + b*ArcTanh[c*x])^2)/4 - (4*b*
d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^2) + (5*b^2*d^2*Log[1 - c^2
*x^2])/(6*c^2) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
```

```
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int\left(d^2x\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+c^2d^2x^3\left(a+b\tan\right.\right.$$

$$= d^2\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(2cd^2\right)\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(c^2d^2\right.$$

$$= \frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}c^2d^2x^4\left(a+b\tan\right.$$

$$= \frac{1}{2}d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{2}{3}cd^2x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{4}c^2d^2x^4\left(a+b\tan\right.$$

$$= \frac{abd^2x}{c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}bcd^2x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2\left(a+\right.}{}$$

$$= \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{b^2d^2x\tanh^{-1}(cx)}{c}+\frac{2}{3}bd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{6}bcd^2$$

$$= \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}(cx)}{2c}+\frac{2}{3}bd^2x^2\left(a+b\right.$$

$$= \frac{3abd^2x}{2c}+\frac{2b^2d^2x}{3c}+\frac{1}{12}b^2d^2x^2-\frac{2b^2d^2\tanh^{-1}(cx)}{3c^2}+\frac{3b^2d^2x\tanh^{-1}(cx)}{2c}+\frac{2}{3}$$

Mathematica [A] time = 0.748979, size = 263, normalized size = 0.94

$$\frac{d^2\left(8b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+3a^2c^4x^4+8a^2c^3x^3+6a^2c^2x^2+2abc^3x^3+8abc^2x^2+8ab\log\left(c^2x^2-1\right)+2b\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(-b^2 + 18*a*b*c*x + 8*b^2*c*x + 6*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + b^2*c
^2*x^2 + 8*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(-17 + 6*c^2*x
^2 + 8*c^3*x^3 + 3*c^4*x^4)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c^2*x^2*(6
 + 8*c*x + 3*c^2*x^2) + b*(-4 + 9*c*x + 4*c^2*x^2 + c^3*x^3) - 8*b*Log[1 +
E^(-2*ArcTanh[c*x])]) + 9*a*b*Log[1 - c*x] - 9*a*b*Log[1 + c*x] + 10*b^2*Lo
g[1 - c^2*x^2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^(-2*ArcTanh[
c*x])]))/(12*c^2)
```

Maple [A] time = 0.049, size = 478, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 2/3*b^2*d^2*x/c+1/48/c^2*d^2*b^2*ln(c*x+1)^2+7/6/c^2*d^2*b^2*ln(c*x-1)+2/3*
d^2*a*b*x^2+1/4*c^2*d^2*a^2*x^4+2/3*c*d^2*a^2*x^3+2/3*d^2*b^2*arctanh(c*x)*
x^2+1/2*d^2*b^2*arctanh(c*x)^2*x^2+1/2/c^2*d^2*b^2*ln(c*x+1)+17/48/c^2*d^2*
b^2*ln(c*x-1)^2-2/3/c^2*d^2*b^2*dilog(1/2+1/2*c*x)+1/6*c*d^2*a*b*x^3+d^2*a*
b*arctanh(c*x)*x^2+2/3*c*d^2*b^2*arctanh(c*x)^2*x^3+1/6*c*d^2*b^2*arctanh(c
*x)*x^3-1/12/c^2*d^2*b^2*arctanh(c*x)*ln(c*x+1)+1/24/c^2*d^2*b^2*ln(-1/2*c*
x+1/2)*ln(1/2+1/2*c*x)+17/12/c^2*d^2*a*b*ln(c*x-1)+1/4*c^2*d^2*b^2*arctanh(
c*x)^2*x^4-1/12/c^2*d^2*a*b*ln(c*x+1)-1/24/c^2*d^2*b^2*ln(-1/2*c*x+1/2)*ln(
c*x+1)+17/12/c^2*d^2*b^2*arctanh(c*x)*ln(c*x-1)+1/2*c^2*d^2*a*b*arctanh(c*x
)*x^4+1/2*d^2*a^2*x^2+1/12*b^2*d^2*x^2-17/24/c^2*d^2*b^2*ln(c*x-1)*ln(1/2+1
/2*c*x)+4/3*c*d^2*a*b*arctanh(c*x)*x^3+3/2*a*b*d^2*x/c+3/2*b^2*d^2*x*arctan
h(c*x)/c
```

Maxima [B] time = 2.16436, size = 824, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^2*d^2*x^4 + 2/3*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctanh(c*x)^2 +
1/12*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 +
3*log(c*x - 1)/c^5))*a*b*c^2*d^2 + 2/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*c*d^2 + 1/2*a^2*d^2*x^2 + 1/2*(2*x^2*arctanh(c*x)
 + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*d^2 + 1/8*(4*c*(2
*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x -
1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^
2)*b^2*d^2 + 2/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*
b^2*d^2/c^2 + 2/3*b^2*d^2*log(c*x - 1)/c^2 + 1/48*(4*b^2*c^2*d^2*x^2 + 32*b
^2*c*d^2*x + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x +
1)^2 + (3*b^2*c^4*d^2*x^4 + 8*b^2*c^3*d^2*x^3 - 11*b^2*d^2)*log(-c*x + 1)^2
 + 4*(b^2*c^3*d^2*x^3 + 4*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x)*log(c*x + 1) - 2
*(2*b^2*c^3*d^2*x^3 + 8*b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (3*b^2*c^4*d^2*x^
4 + 8*b^2*c^3*d^2*x^3 + 5*b^2*d^2)*log(c*x + 1))*log(-c*x + 1))/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^2d^2x^3 + 2\,a^2cd^2x^2 + a^2d^2x + \left(b^2c^2d^2x^3 + 2\,b^2cd^2x^2 + b^2d^2x\right)\operatorname{artanh}(cx)^2 + 2\left(abc^2d^2x^3 + 2\,abcd^2x^2 + abd^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^3 + 2*a^2*c*d^2*x^2 + a^2*d^2*x + (b^2*c^2*d^2*x^3 +
 2*b^2*c*d^2*x^2 + b^2*d^2*x)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^3 + 2*a*b*c
```

```
*d^2*x^2 + a*b*d^2*x)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2x\,dx+\int 2a^2cx^2\,dx+\int a^2c^2x^3\,dx+\int b^2x\operatorname{atanh}^2(cx)\,dx+\int 2abx\operatorname{atanh}(cx)\,dx+\int 2b^2cx^2\operatorname{atanh}^2(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2*x, x) + Integral(2*a**2*c*x**2, x) + Integral(a**2*c**2
*x**3, x) + Integral(b**2*x*atanh(c*x)**2, x) + Integral(2*a*b*x*atanh(c*x)
, x) + Integral(2*b**2*c*x**2*atanh(c*x)**2, x) + Integral(b**2*c**2*x**3*a
tanh(c*x)**2, x) + Integral(4*a*b*c*x**2*atanh(c*x), x) + Integral(2*a*b*c*
*2*x**3*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx+d)^2(b\operatorname{artanh}(cx)+a)^2x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2*x, x)
```

3.79 $\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=175

$$-\frac{4b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c}+\frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c}-\frac{8bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\text{t}\right.}{3c}$$

```
[Out] 2*a*b*d^2*x + (b^2*d^2*x)/3 - (b^2*d^2*ArcTanh[c*x])/(3*c) + 2*b^2*d^2*x*Ar
cTanh[c*x] + (b*c*d^2*x^2*(a + b*ArcTanh[c*x]))/3 + (d^2*(1 + c*x)^3*(a + b
*ArcTanh[c*x])^2)/(3*c) - (8*b*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(
3*c) + (b^2*d^2*Log[1 - c^2*x^2])/c - (4*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)
])/(3*c)
```

Rubi [A] time = 0.164386, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5928, 5910, 260, 5916, 321, 206, 1586, 5918, 2402, 2315}

$$-\frac{4b^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c}+\frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^2(cx+1)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c}-\frac{8bd^2\log\left(\frac{2}{1-cx}\right)\left(a+b\text{t}\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] 2*a*b*d^2*x + (b^2*d^2*x)/3 - (b^2*d^2*ArcTanh[c*x])/(3*c) + 2*b^2*d^2*x*Ar
cTanh[c*x] + (b*c*d^2*x^2*(a + b*ArcTanh[c*x]))/3 + (d^2*(1 + c*x)^3*(a + b
*ArcTanh[c*x])^2)/(3*c) - (8*b*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(
3*c) + (b^2*d^2*Log[1 - c^2*x^2])/c - (4*b^2*d^2*PolyLog[2, 1 - 2/(1 - c*x)
])/(3*c)
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+cdx)^2 \left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(2b)\int \left(-3d^3\left(a+b\tanh^{-1}(cx)\right) - cd^3x\left(a+b\tanh\right.\right.}{3d}$$

$$= \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(8b)\int \frac{\left(d^3+cd^3x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}dx}{3d} + \left(2bd^2\right)\int \left(a\right.$$

$$= 2abd^2x + \frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c} - \frac{(8b)\int}{}$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{d^2(1+cx}{}$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x - \frac{b^2d^2\tanh^{-1}(cx)}{3c} + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a+b\tanh\right.$$

$$= 2abd^2x + \frac{1}{3}b^2d^2x - \frac{b^2d^2\tanh^{-1}(cx)}{3c} + 2b^2d^2x\tanh^{-1}(cx) + \frac{1}{3}bcd^2x^2\left(a+b\tanh\right.$$

Mathematica [A] time = 0.668222, size = 227, normalized size = 1.3

$$\frac{d^2\left(4b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + a^2c^3x^3 + 3a^2c^2x^2 + 3a^2cx + abc^2x^2 + 3ab\log\left(1-c^2x^2\right) + ab\log\left(c^2x^2-1\right) + b\tan\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*d*x)^2*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^2*(3*a^2*c*x + 6*a*b*c*x + b^2*c*x + 3*a^2*c^2*x^2 + a*b*c^2*x^2 + a^2*c
^3*x^3 + b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTanh[
c*x]*(2*a*c*x*(3 + 3*c*x + c^2*x^2) + b*(-1 + 6*c*x + c^2*x^2) - 8*b*Log[1
+ E^(-2*ArcTanh[c*x])]) + 3*a*b*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 3*a*b*L
og[1 - c^2*x^2] + 3*b^2*Log[1 - c^2*x^2] + a*b*Log[-1 + c^2*x^2] + 4*b^2*Po
lyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c)
```

Maple [B] time = 0.048, size = 372, normalized size = 2.1

$$2\,d^2ab\text{Artanh}\,(cx)\,x + \frac{2\,c^2d^2ab\text{Artanh}\,(cx)\,x^3}{3} + 2\,cd^2ab\text{Artanh}\,(cx)\,x^2 + 2\,abd^2x - \frac{d^2b^2}{3\,c} + \frac{d^2a^2}{3\,c} + xa^2d^2 + \frac{b^2d^2x}{3} + \frac{2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 2*c*d^2*a*b*arctanh(c*x)*x^2+2/3*c^2*d^2*a*b*arctanh(c*x)*x^3+2*a*b*d^2*x+2
*b^2*d^2*x*arctanh(c*x)-1/3/c*d^2*b^2+1/3/c*d^2*a^2+x*a^2*d^2+1/3*b^2*d^2*x
+2/3/c*d^2*b^2*ln(c*x-1)^2+7/6/c*d^2*b^2*ln(c*x-1)+5/6/c*d^2*b^2*ln(c*x+1)+
1/3/c*d^2*b^2*arctanh(c*x)^2-4/3/c*d^2*b^2*dilog(1/2+1/2*c*x)+1/3*c^2*x^3*a
^2*d^2+c*x^2*a^2*d^2+d^2*b^2*arctanh(c*x)^2*x+2*d^2*a*b*arctanh(c*x)*x+c*d^
2*b^2*arctanh(c*x)^2*x^2+8/3/c*d^2*a*b*ln(c*x-1)+8/3/c*d^2*b^2*arctanh(c*x)
*ln(c*x-1)-4/3/c*d^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+1/3*c^2*d^2*b^2*arctanh(
c*x)^2*x^3+1/3*c*d^2*b^2*arctanh(c*x)*x^2+2/3/c*d^2*a*b*arctanh(c*x)+1/3*c*
d^2*a*b*x^2
```

Maxima [B] time = 1.77482, size = 626, normalized size = 3.58

$$\frac{1}{3}a^2c^2d^2x^3+\frac{1}{3}\left(2x^3\operatorname{artanh}(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)abc^2d^2+a^2cd^2x^2+\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+}{c^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^2*d^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 -
1)/c^4))*a*b*c^2*d^2 + a^2*c*d^2*x^2 + (2*x^2*arctanh(c*x) + c*(2*x/c^2 - l
og(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b*c*d^2 + a^2*d^2*x + (2*c*x*arctanh
(c*x) + log(-c^2*x^2 + 1))*a*b*d^2/c + 4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2
) + dilog(1/2*c*x + 1/2))*b^2*d^2/c + 5/6*b^2*d^2*log(c*x + 1)/c + 7/6*b^2*
d^2*log(c*x - 1)/c + 1/12*(4*b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2
*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 + (b^2*c^3*d^2*x^3 + 3*b^2*c
^2*d^2*x^2 + 3*b^2*c*d^2*x - 7*b^2*d^2)*log(-c*x + 1)^2 + 2*(b^2*c^2*d^2*x^
2 + 6*b^2*c*d^2*x)*log(c*x + 1) - 2*(b^2*c^2*d^2*x^2 + 6*b^2*c*d^2*x + (b^2
*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1))*l
og(-c*x + 1))/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^2d^2x^2+2a^2cd^2x+a^2d^2+\left(b^2c^2d^2x^2+2b^2cd^2x+b^2d^2\right)\operatorname{artanh}(cx)^2+2\left(abc^2d^2x^2+2abcd^2x+abd^2\right)\text{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*b
^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x +
 a*b*d^2)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2\,dx+\int b^2\operatorname{atanh}^2(cx)\,dx+\int 2ab\operatorname{atanh}(cx)\,dx+\int 2a^2cx\,dx+\int a^2c^2x^2\,dx+\int 2b^2cx\operatorname{atanh}^2(cx)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**2*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*
atanh(c*x), x) + Integral(2*a**2*c*x, x) + Integral(a**2*c**2*x**2, x) + In
tegral(2*b**2*c*x*atanh(c*x)**2, x) + Integral(b**2*c**2*x**2*atanh(c*x)**2
, x) + Integral(4*a*b*c*x*atanh(c*x), x) + Integral(2*a*b*c**2*x**2*atanh(c
*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2, x)
```

3.80 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=278

$$-bd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-2b^2d^2\text{PolyLog}\left(\right.$$

```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTanh[c*x] + (3*d^2*(a + b*ArcTanh[c*x])^2)/2 +
 2*c*d^2*x*(a + b*ArcTanh[c*x])^2 + (c^2*d^2*x^2*(a + b*ArcTanh[c*x])^2)/2
+ 2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 4*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)] + (b^2*d^2*Log[1 - c^2*x^2])/2 - 2*b^2*d^2*Pol
yLog[2, 1 - 2/(1 - c*x)] - b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)] + b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^2
*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rubi [A] time = 0.586404, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 260}

$$-bd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-2b^2d^2\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] a*b*c*d^2*x + b^2*c*d^2*x*ArcTanh[c*x] + (3*d^2*(a + b*ArcTanh[c*x])^2)/2 +
 2*c*d^2*x*(a + b*ArcTanh[c*x])^2 + (c^2*d^2*x^2*(a + b*ArcTanh[c*x])^2)/2
+ 2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 4*b*d^2*(a + b*Ar
cTanh[c*x])*Log[2/(1 - c*x)] + (b^2*d^2*Log[1 - c^2*x^2])/2 - 2*b^2*d^2*Pol
yLog[2, 1 - 2/(1 - c*x)] - b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
 c*x)] + b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^2
*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(2cd^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(\right.\right.$$

$$= d^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx+\left(2cd^2\right)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(c^2d^2\right)\int x$$

$$= 2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}(cx)\right)^2+2d^2\left(a+b\tanh^{-1}\right.$$

$$= 2d^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^2x^2\left(a+b\tanh^{-1}\right.$$

$$= abcd^2x+\frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^2x^2\left(a+\right.$$

$$= abcd^2x+b^2cd^2x\tanh^{-1}(cx)+\frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x\left(a+b\tanh^{-1}\right.$$

$$= abcd^2x+b^2cd^2x\tanh^{-1}(cx)+\frac{3}{2}d^2\left(a+b\tanh^{-1}(cx)\right)^2+2cd^2x\left(a+b\tanh^{-1}\right.$$

Mathematica [C] time = 0.704943, size = 324, normalized size = 1.17

$$\frac{1}{2}d^2\left(2ab(\text{PolyLog}(2,cx)-\text{PolyLog}(2,-cx))+4b^2\left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\left((cx-1)\tanh^{-1}(cx)-\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] (d^2*(4*a^2*c*x + a^2*c^2*x^2 + 2*a^2*Log[c*x] + a*b*(2*c*x + 2*c^2*x^2*Arc
Tanh[c*x] + Log[1 - c*x] - Log[1 + c*x]) + 4*a*b*(2*c*x*ArcTanh[c*x] + Log[
1 - c^2*x^2]) + b^2*(2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + L
og[1 - c^2*x^2]) + 4*b^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 +
 E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + 2*a*b*(-PolyLo
g[2, -(c*x)] + PolyLog[2, c*x]) + 2*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3
 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(
2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[
c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 -
PolyLog[3, E^(2*ArcTanh[c*x])]/2)))/2
```

Maple [C] time = 0.853, size = 1082, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x)
```

```
[Out] a*b*c*d^2*x+b^2*c*d^2*x*arctanh(c*x)+d^2*a^2*ln(c*x)+3/2*d^2*b^2*arctanh(c*
x)^2-2*d^2*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^2*b^2*polylog(3,-(
c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*d^2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-d
^2*b^2*ln((c*x+1)^2/(-c^2*x^2+1)+1)-4*d^2*b^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1
)^(1/2))-4*d^2*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+d^2*b^2*arctanh(c*
x)+2*d^2*a*b*arctanh(c*x)*ln(c*x)-d^2*a*b*ln(c*x)*ln(c*x+1)+d^2*a*b*arctanh
(c*x)*c^2*x^2+4*d^2*a*b*arctanh(c*x)*c*x+1/2*I*d^2*b^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*d^2*a^2*c
^2*x^2-4*d^2*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c*x*a^2*
d^2+d^2*b^2*arctanh(c*x)^2*ln(c*x)-d^2*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^
2*x^2+1)-1)+d^2*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^2*b
^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+d^2*b^2*arctanh(c*x)^
2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1
)/(-c^2*x^2+1)^(1/2))-d^2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1
))-4*d^2*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-d^2*a*b*dilog(
c*x)-d^2*a*b*dilog(c*x+1)+5/2*d^2*a*b*ln(c*x-1)+3/2*d^2*a*b*ln(c*x+1)+1/2*I
*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+
1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctan
h(c*x)^2+1/2*d^2*b^2*arctanh(c*x)^2*c^2*x^2+2*d^2*b^2*arctanh(c*x)^2*c*x-1/
2*I*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*d^2*b^2*Pi*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2
/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,a^2c^2d^2x^2 + 2\,a^2cd^2x + 2\left(2\,cx\,\mathrm{artanh}\left(cx\right) + \log\left(-c^2x^2 + 1\right)\right)abd^2 + a^2d^2\log\left(x\right) + \frac{1}{8}\left(b^2c^2d^2x^2 + 4\,b^2cd^2x\right)\log\left(-cx + 1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + 2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
 + 1))*a*b*d^2 + a^2*d^2*log(x) + 1/8*(b^2*c^2*d^2*x^2 + 4*b^2*c*d^2*x)*log(
-c*x + 1)^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^
2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*d^2*x^2 + a*b*
c*d^2*x - a*b*d^2)*log(c*x + 1) - (4*a*b*c*d^2*x - 4*a*b*d^2 + (4*a*b*c^3*d
^2 + b^2*c^3*d^2)*x^3 - 4*(a*b*c^2*d^2 - b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*
x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1))*log(-c*x + 1))
/(c*x^2 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2c^2d^2x^2 + 2\,a^2cd^2x + a^2d^2 + \left(b^2c^2d^2x^2 + 2\,b^2cd^2x + b^2d^2\right)\mathrm{artanh}\left(cx\right)^2 + 2\left(abc^2d^2x^2 + 2\,abcd^2x + abd^2\right)\mathrm{art}}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int 2a^2c\,dx + \int \frac{a^2}{x}\,dx + \int a^2c^2x\,dx + \int 2b^2c\,\mathrm{atanh}^2(cx)\,dx + \int \frac{b^2\,\mathrm{atanh}^2(cx)}{x}\,dx + \int 4abc\,\mathrm{atanh}(cx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d**2*(Integral(2*a**2*c, x) + Integral(a**2/x, x) + Integral(a**2*c**2*x, x
) + Integral(2*b**2*c*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x, x)
 + Integral(4*a*b*c*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x, x) + Inte
gral(b**2*c**2*x*atanh(c*x)**2, x) + Integral(2*a*b*c**2*x*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^2(b\,\mathrm{artanh}(cx)+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x, x)
```

3.81 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=283

$$-2bcd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-b^2cd^2\text{PolyLog}$$

```
[Out] 2*c*d^2*(a + b*ArcTanh[c*x])^2 - (d^2*(a + b*ArcTanh[c*x])^2)/x + c^2*d^2*x
*(a + b*ArcTanh[c*x])^2 + 4*c*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 -
 c*x)] - 2*b*c*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] + 2*b*c*d^2*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*d^2*PolyLog[2, 1 - 2/(1 - c*x)]
 - 2*b*c*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 2*b*c*d^2*(
a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^2*PolyLog[2, -1
+ 2/(1 + c*x)] + b^2*c*d^2*PolyLog[3, 1 - 2/(1 - c*x)] - b^2*c*d^2*PolyLog[
3, -1 + 2/(1 - c*x)]
```

Rubi [A] time = 0.629137, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610}

$$-2bcd^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+2bcd^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-b^2cd^2\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```


```
[Out] 2*c*d^2*(a + b*ArcTanh[c*x])^2 - (d^2*(a + b*ArcTanh[c*x])^2)/x + c^2*d^2*x
*(a + b*ArcTanh[c*x])^2 + 4*c*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 -
 c*x)] - 2*b*c*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] + 2*b*c*d^2*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c*d^2*PolyLog[2, 1 - 2/(1 - c*x)]
 - 2*b*c*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 2*b*c*d^2*(
a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^2*PolyLog[2, -1
+ 2/(1 + c*x)] + b^2*c*d^2*PolyLog[3, 1 - 2/(1 - c*x)] - b^2*c*d^2*PolyLog[
3, -1 + 2/(1 - c*x)]
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}+\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}\right.$$

$$= d^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\left(2cd^2\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx+\left(c^2d^2\right)\int \left(a+\right.$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2+4cd^2\left(a+b\tanh^{-1}(cx)\right)^2$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2+$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2+$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2+$$

$$= 2cd^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+c^2d^2x\left(a+b\tanh^{-1}(cx)\right)^2+$$

Mathematica [C] time = 0.528593, size = 341, normalized size = 1.2

$$\frac{d^2\left(-24abcx\text{PolyLog}(2,-cx)+24abcx\text{PolyLog}(2,cx)+24b^2cx\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)+12b^2cx\left(2\tanh^{-1}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(-12*a^2 + I*b^2*c*Pi^3*x + 12*a^2*c^2*x^2 - 24*a*b*ArcTanh[c*x] + 24*
a*b*c^2*x^2*ArcTanh[c*x] - 12*b^2*ArcTanh[c*x]^2 + 12*b^2*c^2*x^2*ArcTanh[c
*x]^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 24*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2*A
rcTanh[c*x])] - 24*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b
^2*c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x
]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 24*a*b*c*x*Log[c*x] +
 12*b^2*c*x*(1 + 2*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]*PolyLog[2, E^
(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x)] + 24*a*b*c*x*PolyLog[2, c
*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c*x*PolyLog[3, E
^(2*ArcTanh[c*x])]))/(12*x)
```

Maple [C] time = 0.839, size = 6039, normalized size = 21.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2c^2d^2x - \frac{1}{2}b^2c^2d^2\int \log(cx+1)\log(-cx+1)\,dx + \frac{1}{4}b^2c^2d^2\int \frac{\log(cx+1)^2}{c^2x^2}\,dx + \left(2\,cx\,\mathrm{artanh}(cx) + \log\left(-c^2x^2 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d^2*x - 1/2*b^2*c^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1), x) +
1/4*b^2*c^2*d^2*integrate(log(c*x + 1)^2/(c^2*x^2), x) + (2*c*x*arctanh(c*x
) + log(-c^2*x^2 + 1))*a*b*c*d^2 + 1/2*(c*x - (c*x - 1)*log(-c*x + 1) - 1)*
b^2*c*d^2 + 1/4*b^2*c*d^2*gamma(3, -log(c*x + 1)) + 1/2*b^2*c*d^2*integrate
(log(c*x + 1)^2/x, x) - b^2*c*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x, x
) + 2*a*b*c*d^2*integrate(log(c*x + 1)/x, x) - 2*a*b*c*d^2*integrate(log(-c
*x + 1)/x, x) - 1/2*b^2*c*d^2*integrate(log(-c*x + 1)/x, x) + 2*a^2*c*d^2*l
og(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*d^2 - 1/2*
b^2*d^2*integrate(log(c*x + 1)*log(-c*x + 1)/x^2, x) - a^2*d^2/x + 1/4*(b^2
*c^2*d^2*x^2 - b^2*d^2)*log(-c*x + 1)^2/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2c^2d^2x^2 + 2\,a^2cd^2x + a^2d^2 + \left(b^2c^2d^2x^2 + 2\,b^2cd^2x + b^2d^2\right)\mathrm{artanh}(cx)^2 + 2\left(abc^2d^2x^2 + 2\,abcd^2x + abd^2\right)}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int a^2c^2\,dx+\int\frac{a^2}{x^2}\,dx+\int\frac{2a^2c}{x}\,dx+\int b^2c^2\operatorname{atanh}^2(cx)\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{x^2}\,dx+\int 2abc^2\operatorname{atanh}(cx)\,dx+\int\frac{2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d**2*(Integral(a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(2*a**2*c/x
, x) + Integral(b**2*c**2*atanh(c*x)**2, x) + Integral(b**2*atanh(c*x)**2/x
**2, x) + Integral(2*a*b*c**2*atanh(c*x), x) + Integral(2*a*b*atanh(c*x)/x*
*2, x) + Integral(2*b**2*c*atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x
)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^2, x)
```

3.82 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=313

$$-bc^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^2d^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-2b^2c^2d^2\text{Poly}$$

```
[Out] -((b*c*d^2*(a + b*ArcTanh[c*x]))/x) + (5*c^2*d^2*(a + b*ArcTanh[c*x])^2)/2
- (d^2*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x])^2)/x
 + 2*c^2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + b^2*c^2*d^2*
Log[x] - (b^2*c^2*d^2*Log[1 - c^2*x^2])/2 + 4*b*c^2*d^2*(a + b*ArcTanh[c*x]
)*Log[2 - 2/(1 + c*x)] - b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1
 - c*x)] + b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 2*
b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 + c*x)] + (b^2*c^2*d^2*PolyLog[3, 1 - 2/(1
 - c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rubi [A] time = 0.673329, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 5914, 6052, 6058, 6610}

$$-bc^2d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^2d^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-2b^2c^2d^2\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*d^2*(a + b*ArcTanh[c*x]))/x) + (5*c^2*d^2*(a + b*ArcTanh[c*x])^2)/2
- (d^2*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (2*c*d^2*(a + b*ArcTanh[c*x])^2)/x
 + 2*c^2*d^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + b^2*c^2*d^2*
Log[x] - (b^2*c^2*d^2*Log[1 - c^2*x^2])/2 + 4*b*c^2*d^2*(a + b*ArcTanh[c*x]
)*Log[2 - 2/(1 + c*x)] - b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1
 - c*x)] + b*c^2*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 2*
b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 + c*x)] + (b^2*c^2*d^2*PolyLog[3, 1 - 2/(1
 - c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
```

```
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx = \int\left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}+\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}+\frac{c^2d^2\left(a+b\tanh^{-1}(\right.}{x}\right.$$

$$= d^2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx+\left(2cd^2\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx+\left(c^2d^2\right)\int$$

$$= -\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)^2}{x}+2c^2d^2\left(a+b\tanh^{-1}(cx\right.$$

$$= 2c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{2cd^2\left(a+b\tanh^{-1}(cx)\right)}{x}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx\right)}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx\right)}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx\right)}{2x^2}$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{5}{2}c^2d^2\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^2\left(a+b\tanh^{-1}(cx\right)}{2x^2}$$

Mathematica [C] time = 0.749529, size = 370, normalized size = 1.18

$$-\frac{d^2\left(2abc^2x^2(\text{PolyLog}(2,-cx)-\text{PolyLog}(2,cx))-2b^2c^2x^2\left(\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+\tanh^{-1}(cx)\text{Po}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] -(d^2*(a^2 + 4*a^2*c*x - 2*a^2*c^2*x^2*Log[x] + a*b*(2*ArcTanh[c*x] + c*x*(
2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])) + b^2*(2*c*x*ArcTanh[c*x] + (1 -
c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) + 4*a*b*c
*x*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])) + 4*b^2*c*x*(Arc
Tanh[c*x]*((1 - c*x)*ArcTanh[c*x] - 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + c
*x*PolyLog[2, E^(-2*ArcTanh[c*x])]) + 2*a*b*c^2*x^2*(PolyLog[2, -(c*x)] - P
olyLog[2, c*x]) - 2*b^2*c^2*x^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTa
nh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTan
h[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*Pol
yLog[2, E^(2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[
3, E^(2*ArcTanh[c*x])]/2)))/(2*x^2)
```

Maple [C] time = 1.217, size = 1167, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -1/2*I*c^2*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*c^2*d^2*
b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-2*c*d^2*a^2/x+4*c^2*d^2*b^2*d
ilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-4*c^2*d^2*b^2*dilog((c*x+1)/(-c^2*x^2+1)
^(1/2))+c^2*d^2*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*d^2*b^2*polylog(
3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*d^2*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)
^(1/2))+1/2*c^2*d^2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-c^2*d^2*b^2*arct
anh(c*x)+c^2*d^2*a^2*ln(c*x)-3/2*c^2*d^2*b^2*arctanh(c*x)^2+c^2*d^2*b^2*ln(
(c*x+1)/(-c^2*x^2+1)^(1/2)-1)-1/2*d^2*b^2*arctanh(c*x)^2/x^2-1/2*d^2*a^2/x^
2+1/2*I*c^2*d^2*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2
/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)
+1))*arctanh(c*x)^2-4*c*d^2*a*b*arctanh(c*x)/x+2*c^2*d^2*a*b*arctanh(c*x)*l
n(c*x)-c^2*d^2*a*b*ln(c*x)*ln(c*x+1)+1/2*I*c^2*d^2*b^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-c*d^2*a*b/x-d
^2*a*b*arctanh(c*x)/x^2-c^2*d^2*a*b*dilog(c*x)-c^2*d^2*a*b*dilog(c*x+1)+4*c
^2*d^2*a*b*ln(c*x)-5/2*c^2*d^2*a*b*ln(c*x-1)-3/2*c^2*d^2*a*b*ln(c*x+1)+c^2*
d^2*b^2*arctanh(c*x)^2*ln(c*x)-c^2*d^2*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^
2*x^2+1)-1)+c^2*d^2*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c
^2*d^2*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+c^2*d^2*b^2*a
rctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*d^2*b^2*arctanh(c*x)*p
olylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-c^2*d^2*b^2*arctanh(c*x)*polylog(2,-(
c*x+1)^2/(-c^2*x^2+1))+4*c^2*d^2*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)
^(1/2))-2*c*d^2*b^2*arctanh(c*x)^2/x-c*d^2*b^2*arctanh(c*x)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2c^2d^2\log(x) - 2\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\mathrm{artanh}(cx)}{x}\right)abcd^2 + \frac{1}{2}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c - \frac{2\,\mathrm{art}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^2*d^2*log(x) - 2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c*d^2 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x
)/x^2)*a*b*d^2 - 2*a^2*c*d^2/x - 1/2*a^2*d^2/x^2 - 1/8*(4*b^2*c*d^2*x + b^2
*d^2)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^3*d^2*x^3 + b^2*c^2*d^2*
x^2 - b^2*c*d^2*x - b^2*d^2)*log(c*x + 1)^2 + 4*(a*b*c^3*d^2*x^3 - a*b*c^2*
d^2*x^2)*log(c*x + 1) - (4*a*b*c^3*d^2*x^3 - b^2*c*d^2*x - 4*(a*b*c^2*d^2 +
 b^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d^2*x^3 + b^2*c^2*d^2*x^2 - b^2*c*d^2*x - b^
2*d^2)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^2d^2x^2 + 2\,a^2cd^2x + a^2d^2 + \left(b^2c^2d^2x^2 + 2\,b^2cd^2x + b^2d^2\right)\text{artanh}\,(cx)^2 + 2\left(abc^2d^2x^2 + 2\,abcd^2x + abd^2\right)\,a}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int \frac{a^2}{x^3}\,dx + \int \frac{2a^2c}{x^2}\,dx + \int \frac{a^2c^2}{x}\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x^3}\,dx + \int \frac{2ab\,\text{atanh}\,(cx)}{x^3}\,dx + \int \frac{2b^2c\,\text{atanh}^2\,(cx)}{x^2}\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d**2*(Integral(a**2/x**3, x) + Integral(2*a**2*c/x**2, x) + Integral(a**2*c
**2/x, x) + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*atanh(c*x
)/x**3, x) + Integral(2*b**2*c*atanh(c*x)**2/x**2, x) + Integral(b**2*c**2*
atanh(c*x)**2/x, x) + Integral(4*a*b*c*atanh(c*x)/x**2, x) + Integral(2*a*b
*c**2*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^2(b\,\text{artanh}\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^3, x)
```

3.83 $\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=244

$$-\frac{4}{3}b^2c^3d^2\text{PolyLog}(2,-cx)+\frac{4}{3}b^2c^3d^2\text{PolyLog}(2,cx)+\frac{4}{3}b^2c^3d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{8}{3}abc^3d^2\log(x)-\frac{2bc^2d^2\left(a+\right.}{}$$

```
[Out] -(b^2*c^2*d^2)/(3*x) + (b^2*c^3*d^2*ArcTanh[c*x])/3 - (b*c*d^2*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (2*b*c^2*d^2*(a + b*ArcTanh[c*x]))/x - (d^2*(1 + c*x)^3
*(a + b*ArcTanh[c*x])^2)/(3*x^3) + (8*a*b*c^3*d^2*Log[x])/3 + 2*b^2*c^3*d^2
*Log[x] + (8*b*c^3*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/3 - b^2*c^3*d
^2*Log[1 - c^2*x^2] - (4*b^2*c^3*d^2*PolyLog[2, -(c*x)])/3 + (4*b^2*c^3*d^2
*PolyLog[2, c*x])/3 + (4*b^2*c^3*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/3
```

Rubi [A] time = 0.256134, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {37, 5938, 5916, 325, 206, 266, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-\frac{4}{3}b^2c^3d^2\text{PolyLog}(2,-cx)+\frac{4}{3}b^2c^3d^2\text{PolyLog}(2,cx)+\frac{4}{3}b^2c^3d^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)+\frac{8}{3}abc^3d^2\log(x)-\frac{2bc^2d^2\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^2)/(3*x) + (b^2*c^3*d^2*ArcTanh[c*x])/3 - (b*c*d^2*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (2*b*c^2*d^2*(a + b*ArcTanh[c*x]))/x - (d^2*(1 + c*x)^3
*(a + b*ArcTanh[c*x])^2)/(3*x^3) + (8*a*b*c^3*d^2*Log[x])/3 + 2*b^2*c^3*d^2
*Log[x] + (8*b*c^3*d^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/3 - b^2*c^3*d
^2*Log[1 - c^2*x^2] - (4*b^2*c^3*d^2*PolyLog[2, -(c*x)])/3 + (4*b^2*c^3*d^2
*PolyLog[2, c*x])/3 + (4*b^2*c^3*d^2*PolyLog[2, 1 - 2/(1 - c*x)])/3
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^2\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} - (2bc)\int\left(-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{3x^3} - \frac{cd^2\left(a+b\right.}{}\right.$$

$$= -\frac{d^2(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3} + \frac{1}{3}\left(2bcd^2\right)\int\frac{a+b\tanh^{-1}(cx)}{x^3}\,dx + \left(2bc^2d^2\right)$$

$$= -\frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x} - \frac{d^2(1+cx)^3\left(a+b\tanh\right.}{3x^3}$$

$$= -\frac{b^2c^2d^2}{3x} - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-1}(cx)\right)}{x} - \frac{d^2(1+cx)^3\left(a\right.}{}$$

$$= -\frac{b^2c^2d^2}{3x} + \frac{1}{3}b^2c^3d^2\tanh^{-1}(cx) - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-}\right.}{x}$$

$$= -\frac{b^2c^2d^2}{3x} + \frac{1}{3}b^2c^3d^2\tanh^{-1}(cx) - \frac{bcd^2\left(a+b\tanh^{-1}(cx)\right)}{3x^2} - \frac{2bc^2d^2\left(a+b\tanh^{-}\right.}{x}$$

Mathematica [A] time = 0.639116, size = 270, normalized size = 1.11

$$-\frac{d^2\left(4b^2c^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+3a^2c^2x^2+3a^2cx+a^2+6abc^2x^2-8abc^3x^3\log(cx)+3abc^3x^3\log(1-cx)-3ab\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^2*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(d^2*(a^2 + 3*a^2*c*x + a*b*c*x + 3*a^2*c^2*x^2 + 6*a*b*c^2*x^2 + b^2*c^2*
x^2 + b^2*(1 + 3*c*x + 3*c^2*x^2 - 7*c^3*x^3)*ArcTanh[c*x]^2 + b*ArcTanh[c*
x]*(b*c*x*(1 + 6*c*x - c^2*x^2) + a*(2 + 6*c*x + 6*c^2*x^2) - 8*b*c^3*x^3*L
og[1 - E^(-2*ArcTanh[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + 3*a*b*c^3*x^3*Log[1
 - c*x] - 3*a*b*c^3*x^3*Log[1 + c*x] - 6*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^2
*x^2]] + 4*a*b*c^3*x^3*Log[1 - c^2*x^2] + 4*b^2*c^3*x^3*PolyLog[2, E^(-2*Ar
cTanh[c*x])]))/(3*x^3)
```

Maple [B] time = 0.076, size = 550, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] -2*c*d^2*a*b*arctanh(c*x)/x^2-2*c^2*d^2*a*b*arctanh(c*x)/x-1/3*b^2*c^2*d^2/
x-1/3*d^2*a^2/x^3-1/3*c*d^2*b^2*arctanh(c*x)/x^2-2*c^2*d^2*b^2*arctanh(c*x)
/x-1/3*c*d^2*a*b/x^2-2*c^2*d^2*a*b/x-2/3*d^2*a*b*arctanh(c*x)/x^3-c*d^2*b^2
*arctanh(c*x)^2/x^2-c^2*d^2*b^2*arctanh(c*x)^2/x+8/3*c^3*d^2*b^2*arctanh(c*
x)*ln(c*x)-4/3*c^3*d^2*b^2*ln(c*x)*ln(c*x+1)-7/3*c^3*d^2*b^2*arctanh(c*x)*l
```

```
n(c*x-1)+1/6*c^3*d^2*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/3*c^3*d^2*b^2*a
rctanh(c*x)*ln(c*x+1)+7/6*c^3*d^2*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-7/3*c^3*d^2
*a*b*ln(c*x-1)-1/3*c^3*d^2*a*b*ln(c*x+1)+8/3*c^3*d^2*a*b*ln(c*x)-1/6*c^3*d^
2*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/3*d^2*b^2*arctanh(c*x)^2/x^3-7/12*c^3*d^
2*b^2*ln(c*x-1)^2+4/3*c^3*d^2*b^2*dilog(1/2+1/2*c*x)+1/12*c^3*d^2*b^2*ln(c*
x+1)^2-7/6*c^3*d^2*b^2*ln(c*x-1)-c*d^2*a^2/x^2-c^2*d^2*a^2/x-4/3*c^3*d^2*b^
2*dilog(c*x)-5/6*c^3*d^2*b^2*ln(c*x+1)+2*c^3*d^2*b^2*ln(c*x)-4/3*c^3*d^2*b^
2*dilog(c*x+1)
```

Maxima [B] time = 3.07804, size = 749, normalized size = 3.07

$$-\frac{4}{3}\left(\log\left(cx+1\right)\log\left(-\frac{1}{2}\,cx+\frac{1}{2}\right)+\mathrm{Li}_2\left(\frac{1}{2}\,cx+\frac{1}{2}\right)\right)b^2c^3d^2-\frac{4}{3}\left(\log\left(cx\right)\log\left(-cx+1\right)+\mathrm{Li}_2\left(-cx+1\right)\right)b^2c^3d^2+\frac{4}{3}\left(\log\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -4/3*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^3*d^2
- 4/3*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^3*d^2 + 4/3*(log(c*x
 + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^3*d^2 - 5/6*b^2*c^3*d^2*log(c*x + 1
) - 7/6*b^2*c^3*d^2*log(c*x - 1) + 2*b^2*c^3*d^2*log(x) - (c*(log(c^2*x^2 -
 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b*c^2*d^2 + ((c*log(c*x + 1) - c*log(
c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c*d^2 - 1/3*((c^2*log(c^2*x^2 -
 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*d^2 - a^2*c^2*d^2/x
 - a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/12*(4*b^2*c^2*d^2*x^2 + (b^2*c^3*d^2
*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*d^2)*log(c*x + 1)^2 - (7*b^2
*c^3*d^2*x^3 - 3*b^2*c^2*d^2*x^2 - 3*b^2*c*d^2*x - b^2*d^2)*log(-c*x + 1)^2
 + 2*(6*b^2*c^2*d^2*x^2 + b^2*c*d^2*x)*log(c*x + 1) - 2*(6*b^2*c^2*d^2*x^2
+ b^2*c*d^2*x + (b^2*c^3*d^2*x^3 + 3*b^2*c^2*d^2*x^2 + 3*b^2*c*d^2*x + b^2*
d^2)*log(c*x + 1))*log(-c*x + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2c^2d^2x^2+2\,a^2cd^2x+a^2d^2+\left(b^2c^2d^2x^2+2\,b^2cd^2x+b^2d^2\right)\mathrm{artanh}\left(cx\right)^2+2\left(abc^2d^2x^2+2\,abcd^2x+abd^2\right)}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^2*d^2*x^2 + 2*a^2*c*d^2*x + a^2*d^2 + (b^2*c^2*d^2*x^2 + 2*
b^2*c*d^2*x + b^2*d^2)*arctanh(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + 2*a*b*c*d^2*x
+ a*b*d^2)*arctanh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^2\left(\int\frac{a^2}{x^4}\,dx+\int\frac{2a^2c}{x^3}\,dx+\int\frac{a^2c^2}{x^2}\,dx+\int\frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{x^4}\,dx+\int\frac{2ab\,\mathrm{atanh}\left(cx\right)}{x^4}\,dx+\int\frac{2b^2c\,\mathrm{atanh}^2\left(cx\right)}{x^3}\,dx+\int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**2*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d**2*(Integral(a**2/x**4, x) + Integral(2*a**2*c/x**3, x) + Integral(a**2*c
**2/x**2, x) + Integral(b**2*atanh(c*x)**2/x**4, x) + Integral(2*a*b*atanh(
c*x)/x**4, x) + Integral(2*b**2*c*atanh(c*x)**2/x**3, x) + Integral(b**2*c*
*2*atanh(c*x)**2/x**2, x) + Integral(4*a*b*c*atanh(c*x)/x**3, x) + Integral
(2*a*b*c**2*atanh(c*x)/x**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^2(b\operatorname{artanh}(cx)+a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^2*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^2*(b*arctanh(c*x) + a)^2/x^4, x)
```

3.84 $\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=415

$$-\frac{26b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{35c^4}+\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{21}bc^2d^3x^6\left(a+b\tan\right.$$

```
[Out] (3*a*b*d^3*x)/(2*c^3) + (122*b^2*d^3*x)/(105*c^3) + (7*b^2*d^3*x^2)/(20*c^2
) + (44*b^2*d^3*x^3)/(315*c) + (b^2*d^3*x^4)/20 + (b^2*c*d^3*x^5)/105 - (12
2*b^2*d^3*ArcTanh[c*x])/(105*c^4) + (3*b^2*d^3*x*ArcTanh[c*x])/(2*c^3) + (2
6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/(35*c^2) + (b*d^3*x^3*(a + b*ArcTanh[c*x]
))/(2*c) + (13*b*d^3*x^4*(a + b*ArcTanh[c*x]))/35 + (b*c*d^3*x^5*(a + b*Arc
Tanh[c*x]))/5 + (b*c^2*d^3*x^6*(a + b*ArcTanh[c*x]))/21 - (d^3*(a + b*ArcTa
nh[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c*d^3*x^5*(
a + b*ArcTanh[c*x])^2)/5 + (c^2*d^3*x^6*(a + b*ArcTanh[c*x])^2)/2 + (c^3*d^
3*x^7*(a + b*ArcTanh[c*x])^2)/7 - (52*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 -
 c*x)])/(35*c^4) + (11*b^2*d^3*Log[1 - c^2*x^2])/(10*c^4) - (26*b^2*d^3*Pol
yLog[2, 1 - 2/(1 - c*x)])/(35*c^4)
```

Rubi [A] time = 1.45901, antiderivative size = 415, normalized size of antiderivative = 1., number of steps used = 62, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 266, 43, 5910, 260, 5948, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{26b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{35c^4}+\frac{1}{7}c^3d^3x^7\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{21}bc^2d^3x^6\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (3*a*b*d^3*x)/(2*c^3) + (122*b^2*d^3*x)/(105*c^3) + (7*b^2*d^3*x^2)/(20*c^2
) + (44*b^2*d^3*x^3)/(315*c) + (b^2*d^3*x^4)/20 + (b^2*c*d^3*x^5)/105 - (12
2*b^2*d^3*ArcTanh[c*x])/(105*c^4) + (3*b^2*d^3*x*ArcTanh[c*x])/(2*c^3) + (2
6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/(35*c^2) + (b*d^3*x^3*(a + b*ArcTanh[c*x]
))/(2*c) + (13*b*d^3*x^4*(a + b*ArcTanh[c*x]))/35 + (b*c*d^3*x^5*(a + b*Arc
Tanh[c*x]))/5 + (b*c^2*d^3*x^6*(a + b*ArcTanh[c*x]))/21 - (d^3*(a + b*ArcTa
nh[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c*d^3*x^5*(
a + b*ArcTanh[c*x])^2)/5 + (c^2*d^3*x^6*(a + b*ArcTanh[c*x])^2)/2 + (c^3*d^
3*x^7*(a + b*ArcTanh[c*x])^2)/7 - (52*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 -
 c*x)])/(35*c^4) + (11*b^2*d^3*Log[1 - c^2*x^2])/(10*c^4) - (26*b^2*d^3*Pol
yLog[2, 1 - 2/(1 - c*x)])/(35*c^4)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^3(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+3c^2d^3x^5\left(a+b\right.\right.$$

$$=d^3\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3cd^3\right)\int x^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3c\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a+b\,\mathrm{t}\right.$$

$$=\frac{1}{4}d^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}cd^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{2}c^2d^3x^6\left(a+b\,\mathrm{t}\right.$$

$$=\frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{6c}+\frac{3}{10}bd^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{5}bcd^3x^5\left(a+b\,\mathrm{ta}\right.$$

$$=\frac{abd^3x}{2c^3}+\frac{3bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{5c^2}+\frac{bd^3x^3\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{13}{35}bd^3x^4$$

$$=\frac{3abd^3x}{2c^3}+\frac{199b^2d^3x}{210c^3}+\frac{73b^2d^3x^3}{630c}+\frac{1}{105}b^2cd^3x^5+\frac{b^2d^3x\tanh^{-1}(cx)}{2c^3}+\frac{26bd}{}$$

$$=\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{11b^2d^3x^2}{60c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5-\frac{1}{}$$

$$=\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5-\frac{12}{}$$

$$=\frac{3abd^3x}{2c^3}+\frac{122b^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44b^2d^3x^3}{315c}+\frac{1}{20}b^2d^3x^4+\frac{1}{105}b^2cd^3x^5-\frac{12}{}$$

Mathematica [A] time = 1.63267, size = 385, normalized size = 0.93

$$\frac{d^3\left(936b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+180a^2c^7x^7+630a^2c^6x^6+756a^2c^5x^5+315a^2c^4x^4+60abc^6x^6+252abc^5x^5+468\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-1464*a*b - 504*b^2 + 1890*a*b*c*x + 1464*b^2*c*x + 936*a*b*c^2*x^2 +
 441*b^2*c^2*x^2 + 630*a*b*c^3*x^3 + 176*b^2*c^3*x^3 + 315*a^2*c^4*x^4 + 46
8*a*b*c^4*x^4 + 63*b^2*c^4*x^4 + 756*a^2*c^5*x^5 + 252*a*b*c^5*x^5 + 12*b^2
*c^5*x^5 + 630*a^2*c^6*x^6 + 60*a*b*c^6*x^6 + 180*a^2*c^7*x^7 + 9*b^2*(-209
 + 35*c^4*x^4 + 84*c^5*x^5 + 70*c^6*x^6 + 20*c^7*x^7)*ArcTanh[c*x]^2 + 6*b*
ArcTanh[c*x]*(3*a*c^4*x^4*(35 + 84*c*x + 70*c^2*x^2 + 20*c^3*x^3) + b*(-244
 + 315*c*x + 156*c^2*x^2 + 105*c^3*x^3 + 78*c^4*x^4 + 42*c^5*x^5 + 10*c^6*x
^6) - 312*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 945*a*b*Log[1 - c*x] - 945*a*b*
Log[1 + c*x] + 1386*b^2*Log[1 - c^2*x^2] + 936*a*b*Log[-1 + c^2*x^2] + 936*
b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(1260*c^4)
```

Maple [A] time = 0.057, size = 662, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 122/105*b^2*d^3*x/c^3+7/20*b^2*d^3*x^2/c^2+44/315*b^2*d^3*x^3/c+1/105*b^2*c
*d^3*x^5+353/210/c^4*d^3*b^2*ln(c*x-1)+1/4*d^3*a^2*x^4+1/20*b^2*d^3*x^4+109
/210/c^4*d^3*b^2*ln(c*x+1)+1/560/c^4*d^3*b^2*ln(c*x+1)^2+209/560/c^4*d^3*b^
2*ln(c*x-1)^2-26/35/c^4*d^3*b^2*dilog(1/2+1/2*c*x)+13/35*d^3*a*b*x^4+1/4*d^
3*b^2*arctanh(c*x)^2*x^4+13/35*d^3*b^2*arctanh(c*x)*x^4+1/7*c^3*d^3*a^2*x^7
+1/2*c^2*d^3*a^2*x^6+3/5*c*d^3*a^2*x^5-1/140/c^4*d^3*a*b*ln(c*x+1)+3/2*a*b*
d^3*x/c^3+3/2*b^2*d^3*x*arctanh(c*x)/c^3+1/280/c^4*d^3*b^2*ln(-1/2*c*x+1/2)
*ln(1/2+1/2*c*x)+1/7*c^3*d^3*b^2*arctanh(c*x)^2*x^7+1/5*c*d^3*b^2*arctanh(c
*x)*x^5+26/35/c^2*d^3*b^2*arctanh(c*x)*x^2+1/2*c^2*d^3*b^2*arctanh(c*x)^2*x
^6+3/5*c*d^3*b^2*arctanh(c*x)^2*x^5+209/140/c^4*d^3*a*b*ln(c*x-1)+1/2*d^3*a
*b*arctanh(c*x)*x^4-209/280/c^4*d^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+209/140/c
^4*d^3*b^2*arctanh(c*x)*ln(c*x-1)-1/140/c^4*d^3*b^2*arctanh(c*x)*ln(c*x+1)-
1/280/c^4*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/2/c*d^3*b^2*arctanh(c*x)*x^3
+1/21*c^2*d^3*b^2*arctanh(c*x)*x^6+1/21*c^2*d^3*a*b*x^6+1/5*c*d^3*a*b*x^5+1
/2/c*d^3*a*b*x^3+26/35/c^2*d^3*a*b*x^2+6/5*c*d^3*a*b*arctanh(c*x)*x^5+2/7*c
^3*d^3*a*b*arctanh(c*x)*x^7+c^2*d^3*a*b*arctanh(c*x)*x^6
```

Maxima [B] time = 2.21887, size = 1253, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*a^2*c^3*d^3*x^7 + 1/2*a^2*c^2*d^3*x^6 + 3/5*a^2*c*d^3*x^5 + 1/4*b^2*d^3
*x^4*arctanh(c*x)^2 + 1/42*(12*x^7*arctanh(c*x) + c*((2*c^4*x^6 + 3*c^2*x^4
 + 6*x^2)/c^6 + 6*log(c^2*x^2 - 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*x^4 + 1/
30*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(
c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*b*c^2*d^3 + 3/10*(4*x^5*arctanh(c*x)
 + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c*d^3 + 1/12*(6*
x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*
x - 1)/c^5))*a*b*d^3 + 1/48*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^
5 + 3*log(c*x - 1)/c^5)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*
log(c*x + 1) + 3*log(c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*
b^2*d^3 + 26/35*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b
^2*d^3/c^4 + 13/70*b^2*d^3*log(c*x + 1)/c^4 + 283/210*b^2*d^3*log(c*x - 1)/
c^4 + 1/2520*(24*b^2*c^5*d^3*x^5 + 126*b^2*c^4*d^3*x^4 + 352*b^2*c^3*d^3*x^
3 + 672*b^2*c^2*d^3*x^2 + 2928*b^2*c*d^3*x + 9*(10*b^2*c^7*d^3*x^7 + 35*b^2
*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1)^2 + 9*(10*b^2*
c^7*d^3*x^7 + 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 - 87*b^2*d^3)*log(-c*
x + 1)^2 + 12*(5*b^2*c^6*d^3*x^6 + 21*b^2*c^5*d^3*x^5 + 39*b^2*c^4*d^3*x^4
+ 35*b^2*c^3*d^3*x^3 + 78*b^2*c^2*d^3*x^2 + 105*b^2*c*d^3*x)*log(c*x + 1) -
 6*(10*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 78*b^2*c^4*d^3*x^4 + 70*b^2*c
^3*d^3*x^3 + 156*b^2*c^2*d^3*x^2 + 210*b^2*c*d^3*x + 3*(10*b^2*c^7*d^3*x^7
+ 35*b^2*c^6*d^3*x^6 + 42*b^2*c^5*d^3*x^5 + 17*b^2*d^3)*log(c*x + 1))*log(-
c*x + 1))/c^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^3d^3x^6 + 3\,a^2c^2d^3x^5 + 3\,a^2cd^3x^4 + a^2d^3x^3 + \left(b^2c^3d^3x^6 + 3\,b^2c^2d^3x^5 + 3\,b^2cd^3x^4 + b^2d^3x^3\right)\text{artanh}\left(cx\right)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^6 + 3*a^2*c^2*d^3*x^5 + 3*a^2*c*d^3*x^4 + a^2*d^3*x^
3 + (b^2*c^3*d^3*x^6 + 3*b^2*c^2*d^3*x^5 + 3*b^2*c*d^3*x^4 + b^2*d^3*x^3)*a
rctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^6 + 3*a*b*c^2*d^3*x^5 + 3*a*b*c*d^3*x^4 +
a*b*d^3*x^3)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int a^2x^3\,dx + \int 3a^2cx^4\,dx + \int 3a^2c^2x^5\,dx + \int a^2c^3x^6\,dx + \int b^2x^3\,\text{atanh}^2\left(cx\right)dx + \int 2abx^3\,\text{atanh}\left(cx\right)dx + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x**3, x) + Integral(3*a**2*c*x**4, x) + Integral(3*a**2
*c**2*x**5, x) + Integral(a**2*c**3*x**6, x) + Integral(b**2*x**3*atanh(c*x
)**2, x) + Integral(2*a*b*x**3*atanh(c*x), x) + Integral(3*b**2*c*x**4*atan
h(c*x)**2, x) + Integral(3*b**2*c**2*x**5*atanh(c*x)**2, x) + Integral(b**2
*c**3*x**6*atanh(c*x)**2, x) + Integral(6*a*b*c*x**4*atanh(c*x), x) + Integ
ral(6*a*b*c**2*x**5*atanh(c*x), x) + Integral(2*a*b*c**3*x**6*atanh(c*x), x
))
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx+d)^3 (b \operatorname{artanh}(cx)+a)^2 x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^3, x)
```

3.85 $\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=377

$$-\frac{14b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{15}bc^2d^3x^5\left(a+b\tan\right.$$

```
[Out] (11*a*b*d^3*x)/(6*c^2) + (37*b^2*d^3*x)/(30*c^2) + (61*b^2*d^3*x^2)/(180*c)
 + (b^2*d^3*x^3)/10 + (b^2*c*d^3*x^4)/60 - (37*b^2*d^3*ArcTanh[c*x])/(30*c^
3) + (11*b^2*d^3*x*ArcTanh[c*x])/(6*c^2) + (14*b*d^3*x^2*(a + b*ArcTanh[c*x
]))/(15*c) + (11*b*d^3*x^3*(a + b*ArcTanh[c*x]))/18 + (3*b*c*d^3*x^4*(a + b
*ArcTanh[c*x]))/10 + (b*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/15 + (d^3*(a + b*
ArcTanh[c*x])^2)/(60*c^3) + (d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + (3*c*d^3*x
^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x])^2)/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x])^2)/6 - (28*b*d^3*(a + b*ArcTanh[c*x])*Log[
2/(1 - c*x)])/(15*c^3) + (113*b^2*d^3*Log[1 - c^2*x^2])/(90*c^3) - (14*b^2*
d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```

Rubi [A] time = 1.22981, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5940, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 266, 43, 5910, 260, 5948, 302}

$$-\frac{14b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{15c^3}+\frac{1}{6}c^3d^3x^6\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5\left(a+b\tanh^{-1}(cx)\right)^2+\frac{1}{15}bc^2d^3x^5\left(a+b\tan\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (11*a*b*d^3*x)/(6*c^2) + (37*b^2*d^3*x)/(30*c^2) + (61*b^2*d^3*x^2)/(180*c)
 + (b^2*d^3*x^3)/10 + (b^2*c*d^3*x^4)/60 - (37*b^2*d^3*ArcTanh[c*x])/(30*c^
3) + (11*b^2*d^3*x*ArcTanh[c*x])/(6*c^2) + (14*b*d^3*x^2*(a + b*ArcTanh[c*x
]))/(15*c) + (11*b*d^3*x^3*(a + b*ArcTanh[c*x]))/18 + (3*b*c*d^3*x^4*(a + b
*ArcTanh[c*x]))/10 + (b*c^2*d^3*x^5*(a + b*ArcTanh[c*x]))/15 + (d^3*(a + b*
ArcTanh[c*x])^2)/(60*c^3) + (d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + (3*c*d^3*x
^4*(a + b*ArcTanh[c*x])^2)/4 + (3*c^2*d^3*x^5*(a + b*ArcTanh[c*x])^2)/5 + (
c^3*d^3*x^6*(a + b*ArcTanh[c*x])^2)/6 - (28*b*d^3*(a + b*ArcTanh[c*x])*Log[
2/(1 - c*x)])/(15*c^3) + (113*b^2*d^3*Log[1 - c^2*x^2])/(90*c^3) - (14*b^2*
d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(15*c^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx = \int\left(d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2+3cd^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+3c^2d^3x^4\left(a+b\right.\right.$$

$$= d^3\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3cd^3\right)\int x^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx+\left(3c\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5\left(a+b\,\mathrm{t}\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{4}cd^3x^4\left(a+b\tanh^{-1}(cx)\right)^2+\frac{3}{5}c^2d^3x^5\left(a+b\,\mathrm{t}\right.$$

$$= \frac{bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{3c}+\frac{1}{2}bd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{3}{10}bcd^3x^4\left(a+b\,\mathrm{t}\right.$$

$$= \frac{3abd^3x}{2c^2}+\frac{b^2d^3x}{3c^2}+\frac{14bd^3x^2\left(a+b\tanh^{-1}(cx)\right)}{15c}+\frac{11}{18}bd^3x^3\left(a+b\tanh^{-1}(cx\right.$$

$$= \frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{1}{10}b^2d^3x^3-\frac{b^2d^3\tanh^{-1}(cx)}{3c^3}+\frac{3b^2d^3x\tanh^{-1}(cx)}{2c^2}+$$

$$= \frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{17b^2d^3x^2}{60c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2d^3\tanh^{-1}}{30c^3}$$

$$= \frac{11abd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61b^2d^3x^2}{180c}+\frac{1}{10}b^2d^3x^3+\frac{1}{60}b^2cd^3x^4-\frac{37b^2d^3\tanh^{-1}}{30c^3}$$

Mathematica [A] time = 1.28933, size = 356, normalized size = 0.94

$$\frac{d^3\left(168b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+30a^2c^6x^6+108a^2c^5x^5+135a^2c^4x^4+60a^2c^3x^3+12abc^5x^5+54abc^4x^4+110a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-162*a*b - 64*b^2 + 330*a*b*c*x + 222*b^2*c*x + 168*a*b*c^2*x^2 + 61*
b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3 + 135*a^2*c
^4*x^4 + 54*a*b*c^4*x^4 + 3*b^2*c^4*x^4 + 108*a^2*c^5*x^5 + 12*a*b*c^5*x^5
+ 30*a^2*c^6*x^6 + 3*b^2*(-111 + 20*c^3*x^3 + 45*c^4*x^4 + 36*c^5*x^5 + 10*
c^6*x^6)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*c^3*x^3*(20 + 45*c*x + 36*c
^2*x^2 + 10*c^3*x^3) + b*(-111 + 165*c*x + 84*c^2*x^2 + 55*c^3*x^3 + 27*c^4
*x^4 + 6*c^5*x^5) - 168*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 165*a*b*Log[1 - c
*x] - 165*a*b*Log[1 + c*x] + 226*b^2*Log[1 - c^2*x^2] + 168*a*b*Log[-1 + c^
2*x^2] + 168*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(180*c^3)
```

Maple [A] time = 0.055, size = 618, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 37/30*b^2*d^3*x/c^2+61/180*b^2*d^3*x^2/c+1/60*b^2*c*d^3*x^4-37/40/c^3*d^3*b
^2*ln(c*x-1)*ln(1/2+1/2*c*x)+37/20/c^3*d^3*a*b*ln(c*x-1)+1/60/c^3*d^3*a*b*l
n(c*x+1)+1/6*c^3*d^3*b^2*arctanh(c*x)^2*x^6+3/4*c*d^3*b^2*arctanh(c*x)^2*x^
4+14/15/c*d^3*b^2*arctanh(c*x)*x^2+2/3*d^3*a*b*arctanh(c*x)*x^3+1/15*c^2*d^
3*a*b*x^5+3/10*c*d^3*a*b*x^4+14/15/c*d^3*a*b*x^2+11/18*d^3*a*b*x^3+1/3*d^3*
a^2*x^3+3/4*c*d^3*a^2*x^4+1/6*c^3*d^3*a^2*x^6+3/5*c^2*d^3*a^2*x^5+37/80/c^3
*d^3*b^2*ln(c*x-1)^2+337/180/c^3*d^3*b^2*ln(c*x-1)+23/36/c^3*d^3*b^2*ln(c*x
+1)-1/240/c^3*d^3*b^2*ln(c*x+1)^2-14/15/c^3*d^3*b^2*dilog(1/2+1/2*c*x)+11/1
8*d^3*b^2*arctanh(c*x)*x^3+1/3*d^3*b^2*arctanh(c*x)^2*x^3+1/15*c^2*d^3*b^2*
arctanh(c*x)*x^5+37/20/c^3*d^3*b^2*arctanh(c*x)*ln(c*x-1)+1/60/c^3*d^3*b^2*
arctanh(c*x)*ln(c*x+1)+1/120/c^3*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/120/c
^3*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/10*c*d^3*b^2*arctanh(c*x)*x^4
+1/10*b^2*d^3*x^3+3/5*c^2*d^3*b^2*arctanh(c*x)^2*x^5+11/6*a*b*d^3*x/c^2+11/
6*b^2*d^3*x*arctanh(c*x)/c^2+6/5*c^2*d^3*a*b*arctanh(c*x)*x^5+3/2*c*d^3*a*b
*arctanh(c*x)*x^4+1/3*c^3*d^3*a*b*arctanh(c*x)*x^6
```

Maxima [B] time = 2.15919, size = 1046, normalized size = 2.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*c^3*d^3*x^6 + 3/5*a^2*c^2*d^3*x^5 + 3/4*a^2*c*d^3*x^4 + 1/90*(30*x^
6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/
c^7 + 15*log(c*x - 1)/c^7))*a*b*c^3*d^3 + 3/10*(4*x^5*arctanh(c*x) + c*((c^
2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3
 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5
+ 3*log(c*x - 1)/c^5))*a*b*c*d^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + l
og(c^2*x^2 - 1)/c^4))*a*b*d^3 + 14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + d
ilog(1/2*c*x + 1/2))*b^2*d^3/c^3 + 23/36*b^2*d^3*log(c*x + 1)/c^3 + 337/180
```

```
*b^2*d^3*log(c*x - 1)/c^3 + 1/720*(12*b^2*c^4*d^3*x^4 + 72*b^2*c^3*d^3*x^3
+ 244*b^2*c^2*d^3*x^2 + 888*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^
5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*log(c*x + 1)
^2 + 3*(10*b^2*c^6*d^3*x^6 + 36*b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b
^2*c^3*d^3*x^3 - 111*b^2*d^3)*log(-c*x + 1)^2 + 4*(6*b^2*c^5*d^3*x^5 + 27*b
^2*c^4*d^3*x^4 + 55*b^2*c^3*d^3*x^3 + 84*b^2*c^2*d^3*x^2 + 165*b^2*c*d^3*x)
*log(c*x + 1) - 2*(12*b^2*c^5*d^3*x^5 + 54*b^2*c^4*d^3*x^4 + 110*b^2*c^3*d^
3*x^3 + 168*b^2*c^2*d^3*x^2 + 330*b^2*c*d^3*x + 3*(10*b^2*c^6*d^3*x^6 + 36*
b^2*c^5*d^3*x^5 + 45*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 + b^2*d^3)*log(c*
x + 1))*log(-c*x + 1))/c^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(a^2c^3d^3x^5 + 3\,a^2c^2d^3x^4 + 3\,a^2cd^3x^3 + a^2d^3x^2 + \left(b^2c^3d^3x^5 + 3\,b^2c^2d^3x^4 + 3\,b^2cd^3x^3 + b^2d^3x^2\right)\mathrm{artanh}\,(cx)^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^5 + 3*a^2*c^2*d^3*x^4 + 3*a^2*c*d^3*x^3 + a^2*d^3*x^
2 + (b^2*c^3*d^3*x^5 + 3*b^2*c^2*d^3*x^4 + 3*b^2*c*d^3*x^3 + b^2*d^3*x^2)*a
rctanh(c*x)^2 + 2*(a*b*c^3*d^3*x^5 + 3*a*b*c^2*d^3*x^4 + 3*a*b*c*d^3*x^3 +
a*b*d^3*x^2)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int a^2x^2\,dx + \int 3a^2cx^3\,dx + \int 3a^2c^2x^4\,dx + \int a^2c^3x^5\,dx + \int b^2x^2\,\mathrm{atanh}^2\,(cx)\,dx + \int 2abx^2\,\mathrm{atanh}\,(cx)\,dx + \int\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x**2, x) + Integral(3*a**2*c*x**3, x) + Integral(3*a**2
*c**2*x**4, x) + Integral(a**2*c**3*x**5, x) + Integral(b**2*x**2*atanh(c*x
)**2, x) + Integral(2*a*b*x**2*atanh(c*x), x) + Integral(3*b**2*c*x**3*atan
h(c*x)**2, x) + Integral(3*b**2*c**2*x**4*atanh(c*x)**2, x) + Integral(b**2
*c**3*x**5*atanh(c*x)**2, x) + Integral(6*a*b*c*x**3*atanh(c*x), x) + Integ
ral(6*a*b*c**2*x**4*atanh(c*x), x) + Integral(2*a*b*c**3*x**5*atanh(c*x), x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)^3(b\,\mathrm{artanh}\,(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x^2, x)
```

3.86 $\int x(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=286

$$-\frac{6b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^2}+\frac{1}{10}bc^2d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tan\right.}{4c^2}$$

```
[Out] (5*a*b*d^3*x)/(2*c) + (13*b^2*d^3*x)/(10*c) + (b^2*d^3*x^2)/4 + (b^2*c*d^3*
x^3)/30 - (13*b^2*d^3*ArcTanh[c*x])/(10*c^2) + (5*b^2*d^3*x*ArcTanh[c*x])/(
2*c) + (6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/5 + (b*c*d^3*x^3*(a + b*ArcTanh[c
*x]))/2 + (b*c^2*d^3*x^4*(a + b*ArcTanh[c*x]))/10 - (d^3*(1 + c*x)^4*(a + b
*ArcTanh[c*x])^2)/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x])^2)/(5*c^2
) - (12*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^2) + (3*b^2*d^3*L
og[1 - c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^2)
```

Rubi [A] time = 0.597233, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.7, Rules used = {5940, 5928, 5910, 260, 5916, 321, 206, 266, 43, 1586, 5918, 2402, 2315, 302}

$$-\frac{6b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^2}+\frac{1}{10}bc^2d^3x^4\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{d^3(cx+1)^4\left(a+b\tan\right.}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```


```
[Out] (5*a*b*d^3*x)/(2*c) + (13*b^2*d^3*x)/(10*c) + (b^2*d^3*x^2)/4 + (b^2*c*d^3*
x^3)/30 - (13*b^2*d^3*ArcTanh[c*x])/(10*c^2) + (5*b^2*d^3*x*ArcTanh[c*x])/(
2*c) + (6*b*d^3*x^2*(a + b*ArcTanh[c*x]))/5 + (b*c*d^3*x^3*(a + b*ArcTanh[c
*x]))/2 + (b*c^2*d^3*x^4*(a + b*ArcTanh[c*x]))/10 - (d^3*(1 + c*x)^4*(a + b
*ArcTanh[c*x])^2)/(4*c^2) + (d^3*(1 + c*x)^5*(a + b*ArcTanh[c*x])^2)/(5*c^2
) - (12*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^2) + (3*b^2*d^3*L
og[1 - c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx=\int\left(-\frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{c}+\frac{(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{cd}\right)dx$$

$$=-\frac{\int(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c}+\frac{\int(d+cdx)^4\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{(2b)\int\left(-\right.}{}$$

$$=-\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4c^2}+\frac{d^3(1+cx)^5\left(a+b\tanh^{-1}(cx)\right)^2}{5c^2}-\frac{(32b)\int}{}$$

$$=\frac{5abd^3x}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}bcd^3x^3\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{10}bc^2d^3x$$

$$=\frac{5abd^3x}{2c}+\frac{6b^2d^3x}{5c}+\frac{5b^2d^3x\tanh^{-1}(cx)}{2c}+\frac{6}{5}bd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{1}{2}bcd^3x^3$$

$$=\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{30}b^2cd^3x^3-\frac{6b^2d^3\tanh^{-1}(cx)}{5c^2}+\frac{5b^2d^3x\tanh^{-1}(cx)}{2c}+\frac{6}{5}$$

$$=\frac{5abd^3x}{2c}+\frac{13b^2d^3x}{10c}+\frac{1}{4}b^2d^3x^2+\frac{1}{30}b^2cd^3x^3-\frac{13b^2d^3\tanh^{-1}(cx)}{10c^2}+\frac{5b^2d^3x\tanh}{2c}$$

Mathematica [A] time = 1.18882, size = 325, normalized size = 1.14

$$d^3\left(72b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+12a^2c^5x^5+45a^2c^4x^4+60a^2c^3x^3+30a^2c^2x^2+6abc^4x^4+30abc^3x^3+72abc^2x^2+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-18*a*b - 15*b^2 + 150*a*b*c*x + 78*b^2*c*x + 30*a^2*c^2*x^2 + 72*a*b
*c^2*x^2 + 15*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 30*a*b*c^3*x^3 + 2*b^2*c^3*x^3
 + 45*a^2*c^4*x^4 + 6*a*b*c^4*x^4 + 12*a^2*c^5*x^5 + 3*b^2*(-49 + 10*c^2*x^
2 + 20*c^3*x^3 + 15*c^4*x^4 + 4*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*
(a*c^2*x^2*(10 + 20*c*x + 15*c^2*x^2 + 4*c^3*x^3) + b*(-13 + 25*c*x + 12*c^
2*x^2 + 5*c^3*x^3 + c^4*x^4) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 75*a*b*
Log[1 - c*x] - 75*a*b*Log[1 + c*x] + 90*b^2*Log[1 - c^2*x^2] + 72*a*b*Log[-
1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(60*c^2)
```

Maple [B] time = 0.054, size = 570, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] 13/10*b^2*d^3*x/c+1/30*b^2*c*d^3*x^3+3/4*c^2*d^3*a^2*x^4+c*d^3*a^2*x^3+1/5*
c^3*d^3*a^2*x^5+49/80/c^2*d^3*b^2*ln(c*x-1)^2+1/80/c^2*d^3*b^2*ln(c*x+1)^2-
6/5/c^2*d^3*b^2*dilog(1/2+1/2*c*x)+17/20/c^2*d^3*b^2*ln(c*x+1)+43/20/c^2*d^
3*b^2*ln(c*x-1)+6/5*d^3*b^2*arctanh(c*x)*x^2+1/2*d^3*b^2*arctanh(c*x)^2*x^2
+6/5*d^3*a*b*x^2+1/2*c*d^3*a*b*x^3-1/20/c^2*d^3*a*b*ln(c*x+1)+3/4*c^2*d^3*b
^2*arctanh(c*x)^2*x^4+1/10*c^2*d^3*b^2*arctanh(c*x)*x^4+1/2*c*d^3*b^2*arcta
nh(c*x)*x^3+49/20/c^2*d^3*b^2*arctanh(c*x)*ln(c*x-1)-1/20/c^2*d^3*b^2*arcta
nh(c*x)*ln(c*x+1)-1/40/c^2*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/10*c^2*d^3*
a*b*x^4-49/40/c^2*d^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+c*d^3*b^2*arctanh(c*x)^
2*x^3+49/20/c^2*d^3*a*b*ln(c*x-1)+d^3*a*b*arctanh(c*x)*x^2+1/40/c^2*d^3*b^2
*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/5*c^3*d^3*b^2*arctanh(c*x)^2*x^5+1/2*d^
3*a^2*x^2+1/4*b^2*d^3*x^2+5/2*a*b*d^3*x/c+5/2*b^2*d^3*x*arctanh(c*x)/c+2/5*
c^3*d^3*a*b*arctanh(c*x)*x^5+2*c*d^3*a*b*arctanh(c*x)*x^3+3/2*c^2*d^3*a*b*a
rctanh(c*x)*x^4
```

Maxima [B] time = 2.15455, size = 1053, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*c^3*d^3*x^5 + 3/4*a^2*c^2*d^3*x^4 + 1/10*(4*x^5*arctanh(c*x) + c*((
c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c^6))*a*b*c^3*d^3 + a^2*c*d^3*x^3
 + 1/2*b^2*d^3*x^2*arctanh(c*x)^2 + 1/4*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3
 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^2*d^3 + (2*x^
3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^
3*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x -
 1)/c^3))*a*b*d^3 + 1/8*(4*c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3
)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(
c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2*d^3 + 6/5*(log(c*x + 1)*log(-1/2*c*x
+ 1/2) + dilog(1/2*c*x + 1/2))*b^2*d^3/c^2 + 7/20*b^2*d^3*log(c*x + 1)/c^2
+ 33/20*b^2*d^3*log(c*x - 1)/c^2 + 1/240*(8*b^2*c^3*d^3*x^3 + 60*b^2*c^2*d^
3*x^2 + 312*b^2*c*d^3*x + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 20*b^
2*c^3*d^3*x^3 + 9*b^2*d^3)*log(c*x + 1)^2 + 3*(4*b^2*c^5*d^3*x^5 + 15*b^2*c
^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3 - 39*b^2*d^3)*log(-c*x + 1)^2 + 12*(b^2*c^4
*d^3*x^4 + 5*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 15*b^2*c*d^3*x)*log(c*x
 + 1) - 6*(2*b^2*c^4*d^3*x^4 + 10*b^2*c^3*d^3*x^3 + 24*b^2*c^2*d^3*x^2 + 30
*b^2*c*d^3*x + (4*b^2*c^5*d^3*x^5 + 15*b^2*c^4*d^3*x^4 + 20*b^2*c^3*d^3*x^3
 + 9*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^3d^3x^4 + 3\,a^2c^2d^3x^3 + 3\,a^2cd^3x^2 + a^2d^3x + \left(b^2c^3d^3x^4 + 3\,b^2c^2d^3x^3 + 3\,b^2cd^3x^2 + b^2d^3x\right)\text{artanh}\left(cx\right)^2 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^4 + 3*a^2*c^2*d^3*x^3 + 3*a^2*c*d^3*x^2 + a^2*d^3*x
+ (b^2*c^3*d^3*x^4 + 3*b^2*c^2*d^3*x^3 + 3*b^2*c*d^3*x^2 + b^2*d^3*x)*arcta
nh(c*x)^2 + 2*(a*b*c^3*d^3*x^4 + 3*a*b*c^2*d^3*x^3 + 3*a*b*c*d^3*x^2 + a*b*
d^3*x)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int a^2x\,dx + \int 3a^2cx^2\,dx + \int 3a^2c^2x^3\,dx + \int a^2c^3x^4\,dx + \int b^2x\,\text{atanh}^2\,(cx)\,dx + \int 2abx\,\text{atanh}\,(cx)\,dx + \int 3b^2cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2*x, x) + Integral(3*a**2*c*x**2, x) + Integral(3*a**2*c*
*2*x**3, x) + Integral(a**2*c**3*x**4, x) + Integral(b**2*x*atanh(c*x)**2,
x) + Integral(2*a*b*x*atanh(c*x), x) + Integral(3*b**2*c*x**2*atanh(c*x)**2
, x) + Integral(3*b**2*c**2*x**3*atanh(c*x)**2, x) + Integral(b**2*c**3*x**
4*atanh(c*x)**2, x) + Integral(6*a*b*c*x**2*atanh(c*x), x) + Integral(6*a*b
*c**2*x**3*atanh(c*x), x) + Integral(2*a*b*c**3*x**4*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)^3 (b\,\text{artanh}\,(cx) + a)^2 x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2*x, x)
```

3.87 $\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=206

$$-\frac{2b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right)+bcd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}\right.}{4c}$$

```
[Out] (7*a*b*d^3*x)/2 + b^2*d^3*x + (b^2*c*d^3*x^2)/12 - (b^2*d^3*ArcTanh[c*x])/c
 + (7*b^2*d^3*x*ArcTanh[c*x])/2 + b*c*d^3*x^2*(a + b*ArcTanh[c*x]) + (b*c^2
*d^3*x^3*(a + b*ArcTanh[c*x]))/6 + (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)
/(4*c) - (4*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (11*b^2*d^3*Lo
g[1 - c^2*x^2])/(6*c) - (2*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rubi [A] time = 0.214082, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {5928, 5910, 260, 5916, 321, 206, 266, 43, 1586, 5918, 2402, 2315}

$$-\frac{2b^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{1}{6}bc^2d^3x^3\left(a+b\tanh^{-1}(cx)\right)+bcd^3x^2\left(a+b\tanh^{-1}(cx)\right)+\frac{d^3(cx+1)^4\left(a+b\tanh^{-1}\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (7*a*b*d^3*x)/2 + b^2*d^3*x + (b^2*c*d^3*x^2)/12 - (b^2*d^3*ArcTanh[c*x])/c
 + (7*b^2*d^3*x*ArcTanh[c*x])/2 + b*c*d^3*x^2*(a + b*ArcTanh[c*x]) + (b*c^2
*d^3*x^3*(a + b*ArcTanh[c*x]))/6 + (d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^2)
/(4*c) - (4*b*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (11*b^2*d^3*Lo
g[1 - c^2*x^2])/(6*c) - (2*b^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/c
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int (d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2 dx = \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{b\int \left(-7d^4 \left(a+b\tanh^{-1}(cx)\right) - 4cd^4x \left(a+b\right.\right.}{}$$

$$= \frac{d^3(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{(4b)\int \frac{\left(d^4+cd^4x\right)\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2} dx}{d} + \frac{1}{2}\left(7bd^3\right)$$

$$= \frac{7}{2}abd^3x + bcd^3x^2 \left(a+b\tanh^{-1}(cx)\right) + \frac{1}{6}bc^2d^3x^3 \left(a+b\tanh^{-1}(cx)\right) + \frac{d^3(1+c}{}$$

$$= \frac{7}{2}abd^3x + b^2d^3x + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x^2 \left(a+b\tanh^{-1}(cx)\right) + \frac{1}{6}bc^2d^3x^3$$

$$= \frac{7}{2}abd^3x + b^2d^3x - \frac{b^2d^3\tanh^{-1}(cx)}{c} + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x^2 \left(a+b\tanh\right.$$

$$= \frac{7}{2}abd^3x + b^2d^3x + \frac{1}{12}b^2cd^3x^2 - \frac{b^2d^3\tanh^{-1}(cx)}{c} + \frac{7}{2}b^2d^3x\tanh^{-1}(cx) + bcd^3x$$

Mathematica [A] time = 0.862084, size = 293, normalized size = 1.42

$$\frac{d^3 \left(24b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 3a^2c^4x^4 + 12a^2c^3x^3 + 18a^2c^2x^2 + 12a^2cx + 2abc^3x^3 + 12abc^2x^2 + 12ab\log\left(1-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + c*d*x)^3*(a + b*ArcTanh[c*x])^2,x]
```

```
[Out] (d^3*(-b^2 + 12*a^2*c*x + 42*a*b*c*x + 12*b^2*c*x + 18*a^2*c^2*x^2 + 12*a*b
*c^2*x^2 + b^2*c^2*x^2 + 12*a^2*c^3*x^3 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 3
*b^2*(-15 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 + c^4*x^4)*ArcTanh[c*x]^2 + 2*b*A
rcTanh[c*x]*(3*a*c*x*(4 + 6*c*x + 4*c^2*x^2 + c^3*x^3) + b*(-6 + 21*c*x + 6
*c^2*x^2 + c^3*x^3) - 24*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 21*a*b*Log[1 - c
*x] - 21*a*b*Log[1 + c*x] + 12*a*b*Log[1 - c^2*x^2] + 22*b^2*Log[1 - c^2*x^
2] + 12*a*b*Log[-1 + c^2*x^2] + 24*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(
12*c)
```

Maple [B] time = 0.05, size = 462, normalized size = 2.2

$$b^2d^3x + 3\,cd^3ab\text{Artanh}\,(cx)\,x^2 + 2\,d^3ab\text{Artanh}\,(cx)\,x + \frac{7\,abd^3x}{2} + \frac{b^2cd^3x^2}{12} + \frac{c^2d^3abx^3}{6} + \frac{c^3d^3ab\text{Artanh}\,(cx)\,x^4}{2} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x)
```

```
[Out] b^2*d^3*x+3*c*d^3*a*b*arctanh(c*x)*x^2+2*c^2*d^3*a*b*arctanh(c*x)*x^3+1/2*c
^3*d^3*a*b*arctanh(c*x)*x^4+7/2*a*b*d^3*x+1/12*b^2*c*d^3*x^2+7/2*b^2*d^3*x*
arctanh(c*x)+1/6*c^2*d^3*a*b*x^3-13/12/c*d^3*b^2+1/4/c*d^3*a^2+x*a^2*d^3-2/
c*d^3*b^2*dilog(1/2+1/2*c*x)+7/3/c*d^3*b^2*ln(c*x-1)+1/c*d^3*b^2*ln(c*x-1)^
2+1/4/c*d^3*b^2*arctanh(c*x)^2+d^3*b^2*arctanh(c*x)^2*x+4/3/c*d^3*b^2*ln(c*
x+1)+c^2*x^3*a^2*d^3+3/2*c*x^2*a^2*d^3+1/4*c^3*x^4*a^2*d^3-2/c*d^3*b^2*ln(c
*x-1)*ln(1/2+1/2*c*x)+c*d^3*a*b*x^2+2*d^3*a*b*arctanh(c*x)*x+3/2*c*d^3*b^2*
```

```
arctanh(c*x)^2*x^2+1/2/c*d^3*a*b*arctanh(c*x)+4/c*d^3*a*b*ln(c*x-1)+c^2*d^3
*b^2*arctanh(c*x)^2*x^3+1/4*c^3*d^3*b^2*arctanh(c*x)^2*x^4+c*d^3*b^2*arctan
h(c*x)*x^2+1/6*c^2*d^3*b^2*arctanh(c*x)*x^3+4/c*d^3*b^2*arctanh(c*x)*ln(c*x
-1)
```

Maxima [B] time = 1.76048, size = 846, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*c^3*d^3*x^4 + a^2*c^2*d^3*x^3 + 1/12*(6*x^4*arctanh(c*x) + c*(2*(c^
2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b*c^3*d^3 +
(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a*b*c^2*d^3 + 3/2
*a^2*c*d^3*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 +
log(c*x - 1)/c^3))*a*b*c*d^3 + a^2*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x
^2 + 1))*a*b*d^3/c + 2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x +
1/2))*b^2*d^3/c + 4/3*b^2*d^3*log(c*x + 1)/c + 7/3*b^2*d^3*log(c*x - 1)/c +
 1/48*(4*b^2*c^2*d^3*x^2 + 48*b^2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*
d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x + b^2*d^3)*log(c*x + 1)^2 + 3*(
b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c*d^3*x - 1
5*b^2*d^3)*log(-c*x + 1)^2 + 4*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 21*b^
2*c*d^3*x)*log(c*x + 1) - 2*(2*b^2*c^3*d^3*x^3 + 12*b^2*c^2*d^3*x^2 + 42*b^
2*c*d^3*x + 3*(b^2*c^4*d^3*x^4 + 4*b^2*c^3*d^3*x^3 + 6*b^2*c^2*d^3*x^2 + 4*
b^2*c*d^3*x + b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/c
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\text{artanh}\,(cx)^2 + 2\left(abc^3d^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (b
^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)^
2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arcta
nh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int a^2\,dx + \int b^2\,\text{atanh}^2\,(cx)\,dx + \int 2ab\,\text{atanh}\,(cx)\,dx + \int 3a^2cx\,dx + \int 3a^2c^2x^2\,dx + \int a^2c^3x^3\,dx + \int 3b^2cx\,\text{atan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2,x)
```

```
[Out] d**3*(Integral(a**2, x) + Integral(b**2*atanh(c*x)**2, x) + Integral(2*a*b*
atanh(c*x), x) + Integral(3*a**2*c*x, x) + Integral(3*a**2*c**2*x**2, x) +
```

```
Integral(a**2*c**3*x**3, x) + Integral(3*b**2*c*x*atanh(c*x)**2, x) + Integ
ral(3*b**2*c**2*x**2*atanh(c*x)**2, x) + Integral(b**2*c**3*x**3*atanh(c*x)
**2, x) + Integral(6*a*b*c*x*atanh(c*x), x) + Integral(6*a*b*c**2*x**2*atan
h(c*x), x) + Integral(2*a*b*c**3*x**3*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cdx + d)^3 (b \operatorname{artanh}(cx) + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2, x)
```

3.88 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=355

$$-bd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{10}{3}b^2d^3\text{PolyLog}\left(2,\right.$$

```
[Out] 3*a*b*c*d^3*x + (b^2*c*d^3*x)/3 - (b^2*d^3*ArcTanh[c*x])/3 + 3*b^2*c*d^3*x*
ArcTanh[c*x] + (b*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/3 + (11*d^3*(a + b*ArcT
anh[c*x])^2)/6 + 3*c*d^3*x*(a + b*ArcTanh[c*x])^2 + (3*c^2*d^3*x^2*(a + b*A
rcTanh[c*x])^2)/2 + (c^3*d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + 2*d^3*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - (20*b*d^3*(a + b*ArcTanh[c*x])*Lo
g[2/(1 - c*x)])/3 + (3*b^2*d^3*Log[1 - c^2*x^2])/2 - (10*b^2*d^3*PolyLog[2,
 1 - 2/(1 - c*x)])/3 - b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x
)] + b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^3*Pol
yLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rubi [A] time = 0.812787, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 260, 321, 206}

$$-bd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{10}{3}b^2d^3\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```


```
[Out] 3*a*b*c*d^3*x + (b^2*c*d^3*x)/3 - (b^2*d^3*ArcTanh[c*x])/3 + 3*b^2*c*d^3*x*
ArcTanh[c*x] + (b*c^2*d^3*x^2*(a + b*ArcTanh[c*x]))/3 + (11*d^3*(a + b*ArcT
anh[c*x])^2)/6 + 3*c*d^3*x*(a + b*ArcTanh[c*x])^2 + (3*c^2*d^3*x^2*(a + b*A
rcTanh[c*x])^2)/2 + (c^3*d^3*x^3*(a + b*ArcTanh[c*x])^2)/3 + 2*d^3*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - (20*b*d^3*(a + b*ArcTanh[c*x])*Lo
g[2/(1 - c*x)])/3 + (3*b^2*d^3*Log[1 - c^2*x^2])/2 - (10*b^2*d^3*PolyLog[2,
 1 - 2/(1 - c*x)])/3 - b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x
)] + b*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] + (b^2*d^3*Pol
yLog[3, 1 - 2/(1 - c*x)])/2 - (b^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx = \int \left(3cd^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x\left(a+b\tanh^{-1}(cx\right.\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx + \left(3cd^3\right)\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx + \left(3c^2d^3\right)\int x\left(\right.$$

$$= 3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{3}c^3d^3x^3\left(a+b\tanh\right.$$

$$= 3d^3\left(a+b\tanh^{-1}(cx)\right)^2 + 3cd^3x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2}c^2d^3x^2\left(a+b\tanh^{-1}(cx\right.$$

$$= 3abcd^3x + \frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{11}{6}d^3\left(a+b\tanh^{-1}(cx)\right)^2 + 3cd^3x\left(a+\right.$$

$$= 3abcd^3x + \frac{1}{3}b^2cd^3x + 3b^2cd^3x\tanh^{-1}(cx) + \frac{1}{3}bc^2d^3x^2\left(a+b\tanh^{-1}(cx)\right) + \frac{11}{6}d^3$$

$$= 3abcd^3x + \frac{1}{3}b^2cd^3x - \frac{1}{3}b^2d^3\tanh^{-1}(cx) + 3b^2cd^3x\tanh^{-1}(cx) + \frac{1}{3}bc^2d^3x^2\left(a+b\right.$$

$$= 3abcd^3x + \frac{1}{3}b^2cd^3x - \frac{1}{3}b^2d^3\tanh^{-1}(cx) + 3b^2cd^3x\tanh^{-1}(cx) + \frac{1}{3}bc^2d^3x^2\left(a+b\right.$$

Mathematica [C] time = 0.827737, size = 448, normalized size = 1.26

$$\frac{1}{24}d^3\left(-24ab\text{PolyLog}(2,-cx)+24ab\text{PolyLog}(2,cx)+8b^2\left(3\tanh^{-1}(cx)+10\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+24b^2\,\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x,x]
```

```
[Out] (d^3*(I*b^2*Pi^3 + 72*a^2*c*x + 72*a*b*c*x + 8*b^2*c*x + 36*a^2*c^2*x^2 + 8
*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - 8*b^2*ArcTanh[c*x] + 144*a*b*c*x*ArcTanh[c*x
] + 72*b^2*c*x*ArcTanh[c*x] + 72*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*A
rcTanh[c*x] + 16*a*b*c^3*x^3*ArcTanh[c*x] - 116*b^2*ArcTanh[c*x]^2 + 72*b^2
*c*x*ArcTanh[c*x]^2 + 36*b^2*c^2*x^2*ArcTanh[c*x]^2 + 8*b^2*c^3*x^3*ArcTanh
[c*x]^2 - 16*b^2*ArcTanh[c*x]^3 - 160*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTan
h[c*x])] - 24*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*ArcT
anh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*Log[c*x] + 36*a*b*Log[1 - c
*x] - 36*a*b*Log[1 + c*x] + 72*a*b*Log[1 - c^2*x^2] + 36*b^2*Log[1 - c^2*x^
2] + 8*a*b*Log[-1 + c^2*x^2] + 8*b^2*(10 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-
2*ArcTanh[c*x])] + 24*b^2*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*
a*b*PolyLog[2, -(c*x)] + 24*a*b*PolyLog[2, c*x] + 12*b^2*PolyLog[3, -E^(-2*
ArcTanh[c*x])] - 12*b^2*PolyLog[3, E^(2*ArcTanh[c*x])]))/24
```

Maple [C] time = 1.046, size = 1186, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x)
```

```
[Out] -1/3*d^3*b^2+1/3*b^2*c*d^3*x+8/3*b^2*d^3*arctanh(c*x)+3*a*b*c*d^3*x+3*b^2*c
*d^3*x*arctanh(c*x)+1/2*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-3*d^3*b^
2*ln((c*x+1)^2/(-c^2*x^2+1)+1)-2*d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1
/2))-20/3*d^3*b^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*d^3*b^2*dilog(
1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1
/2))+1/2*I*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/
(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+
1))*arctanh(c*x)^2+3*d^3*b^2*arctanh(c*x)^2*c*x+3/2*d^3*a^2*c^2*x^2-d^3*a*b
*dilog(c*x)-d^3*a*b*dilog(c*x+1)+1/3*d^3*a^2*c^3*x^3+3*c*x*a^2*d^3-d^3*b^2*
arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+d^3*b^2*arctanh(c*x)^2*ln(1
+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^
2*x^2+1)^(1/2))-20/3*d^3*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)-20/3*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+11/6*d^3*a*b
*ln(c*x+1)+29/6*d^3*a*b*ln(c*x-1)-d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2
*x^2+1)-1)+d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*d^3*b^
2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+d^3*b^2*arctanh(c*x)^2
*ln(c*x)+d^3*a^2*ln(c*x)+11/6*d^3*b^2*arctanh(c*x)^2+6*d^3*a*b*arctanh(c*x)
*c*x+3*d^3*a*b*arctanh(c*x)*c^2*x^2+2/3*d^3*a*b*arctanh(c*x)*c^3*x^3+1/2*I*
d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*
arctanh(c*x)^2-1/2*I*d^3*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*
I*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/2*d^3*b^2*arctanh(c*x
)^2*c^2*x^2+1/3*d^3*b^2*arctanh(c*x)^2*c^3*x^3+1/3*d^3*b^2*arctanh(c*x)*c^2
*x^2+1/3*d^3*a*b*c^2*x^2+2*d^3*a*b*arctanh(c*x)*ln(c*x)-d^3*a*b*ln(c*x)*ln(
```

```
c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2c^3d^3x^3 + \frac{3}{2}a^2c^2d^3x^2 + 3a^2cd^3x + 3\left(2cx\operatorname{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)abd^3 + a^2d^3\log(x) + \frac{1}{24}\left(2b^2c^3d^3x^3 + 9b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*c^3*d^3*x^3 + 3/2*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + 3*(2*c*x*arctan
h(c*x) + log(-c^2*x^2 + 1))*a*b*d^3 + a^2*d^3*log(x) + 1/24*(2*b^2*c^3*d^3*
x^3 + 9*b^2*c^2*d^3*x^2 + 18*b^2*c*d^3*x)*log(-c*x + 1)^2 - integrate(-1/12
*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x
 + 1)^2 + 12*(a*b*c^4*d^3*x^4 + 2*a*b*c^3*d^3*x^3 - 3*a*b*c^2*d^3*x^2 + a*b
*c*d^3*x - a*b*d^3)*log(c*x + 1) - (12*a*b*c*d^3*x - 12*a*b*d^3 + 2*(6*a*b*
c^4*d^3 + b^2*c^4*d^3)*x^4 + 3*(8*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 - 18*(2*
a*b*c^2*d^3 - b^2*c^2*d^3)*x^2 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2
*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^2 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^2c^3d^3x^3 + 3a^2c^2d^3x^2 + 3a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3b^2c^2d^3x^2 + 3b^2cd^3x + b^2d^3\right)\operatorname{artanh}(cx)^2 + 2\left(abc^3d^3\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3a^2c\,dx + \int \frac{a^2}{x}\,dx + \int 3a^2c^2x\,dx + \int a^2c^3x^2\,dx + \int 3b^2c\operatorname{atanh}^2(cx)\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x}\,dx + \int 6abc\operatorname{atanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x,x)
```

```
[Out] d**3*(Integral(3*a**2*c, x) + Integral(a**2/x, x) + Integral(3*a**2*c**2*x,
 x) + Integral(a**2*c**3*x**2, x) + Integral(3*b**2*c*atanh(c*x)**2, x) + I
ntegral(b**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x), x) + Integr
al(2*a*b*atanh(c*x)/x, x) + Integral(3*b**2*c**2*x*atanh(c*x)**2, x) + Inte
gral(b**2*c**3*x**2*atanh(c*x)**2, x) + Integral(6*a*b*c**2*x*atanh(c*x), x
) + Integral(2*a*b*c**3*x**2*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3 (b \operatorname{artanh}(cx) + a)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x, x)
```

3.89 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=361

$$-3bcd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bcd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-3b^2cd^3\text{PolyLog}$$

```
[Out] a*b*c^2*d^3*x + b^2*c^2*d^3*x*ArcTanh[c*x] + (7*c*d^3*(a + b*ArcTanh[c*x])^
2)/2 - (d^3*(a + b*ArcTanh[c*x])^2)/x + 3*c^2*d^3*x*(a + b*ArcTanh[c*x])^2
+ (c^3*d^3*x^2*(a + b*ArcTanh[c*x])^2)/2 + 6*c*d^3*(a + b*ArcTanh[c*x])^2*A
rcTanh[1 - 2/(1 - c*x)] - 6*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] +
 (b^2*c*d^3*Log[1 - c^2*x^2])/2 + 2*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2 - 2/
(1 + c*x)] - 3*b^2*c*d^3*PolyLog[2, 1 - 2/(1 - c*x)] - 3*b*c*d^3*(a + b*Arc
Tanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c*d^3*(a + b*ArcTanh[c*x])*Pol
yLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^3*PolyLog[2, -1 + 2/(1 + c*x)] + (3*b^2
*c*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (3*b^2*c*d^3*PolyLog[3, -1 + 2/(1 -
 c*x)])/2
```

Rubi [A] time = 0.776101, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5980, 260}

$$-3bcd^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bcd^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-3b^2cd^3\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```


```
[Out] a*b*c^2*d^3*x + b^2*c^2*d^3*x*ArcTanh[c*x] + (7*c*d^3*(a + b*ArcTanh[c*x])^
2)/2 - (d^3*(a + b*ArcTanh[c*x])^2)/x + 3*c^2*d^3*x*(a + b*ArcTanh[c*x])^2
+ (c^3*d^3*x^2*(a + b*ArcTanh[c*x])^2)/2 + 6*c*d^3*(a + b*ArcTanh[c*x])^2*A
rcTanh[1 - 2/(1 - c*x)] - 6*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] +
 (b^2*c*d^3*Log[1 - c^2*x^2])/2 + 2*b*c*d^3*(a + b*ArcTanh[c*x])*Log[2 - 2/
(1 + c*x)] - 3*b^2*c*d^3*PolyLog[2, 1 - 2/(1 - c*x)] - 3*b*c*d^3*(a + b*Arc
Tanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c*d^3*(a + b*ArcTanh[c*x])*Pol
yLog[2, -1 + 2/(1 - c*x)] - b^2*c*d^3*PolyLog[2, -1 + 2/(1 + c*x)] + (3*b^2
*c*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (3*b^2*c*d^3*PolyLog[3, -1 + 2/(1 -
 c*x)])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
```

```
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx = \int \left(3c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} + \frac{3cd^3\left(a+b\tanh^{-1}\right.}{x}\right.$$

$$= d^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}dx + \left(3cd^3\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx + \left(3c^2d^3\right)\int$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}c^3d^3x^2\left(a+b\tanh\right.$$

$$= 4cd^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x\left(a+b\tanh^{-1}(cx\right.$$

$$= abc^2d^3x + \frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x} + 3c^2d^3x\left(a+b\right.$$

$$= abc^2d^3x + b^2c^2d^3x\tanh^{-1}(cx) + \frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}\right.}{x}$$

$$= abc^2d^3x + b^2c^2d^3x\tanh^{-1}(cx) + \frac{7}{2}cd^3\left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3\left(a+b\tanh^{-1}\right.}{x}$$

Mathematica [C] time = 0.635039, size = 479, normalized size = 1.33

$$\frac{d^3\left(-24abcx\text{PolyLog}(2,-cx) + 24abcx\text{PolyLog}(2,cx) + 24b^2cx\left(\tanh^{-1}(cx)+1\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) - 8b^2c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^2,x]
```

```
[Out] (d^3*(-8*a^2 + I*b^2*c*Pi^3*x + 24*a^2*c^2*x^2 + 8*a*b*c^2*x^2 + 4*a^2*c^3*
x^3 - 16*a*b*ArcTanh[c*x] + 48*a*b*c^2*x^2*ArcTanh[c*x] + 8*b^2*c^2*x^2*Arc
Tanh[c*x] + 8*a*b*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 20*b^2*c*x*
ArcTanh[c*x]^2 + 24*b^2*c^2*x^2*ArcTanh[c*x]^2 + 4*b^2*c^3*x^3*ArcTanh[c*x]
^2 - 16*b^2*c*x*ArcTanh[c*x]^3 + 16*b^2*c*x*ArcTanh[c*x]*Log[1 - E^(-2*ArcT
anh[c*x])] - 48*b^2*c*x*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^2*
c*x*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 24*b^2*c*x*ArcTanh[c*x]^2
*Log[1 - E^(2*ArcTanh[c*x])] + 24*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 4*
a*b*c*x*Log[1 - c*x] - 4*a*b*c*x*Log[1 + c*x] + 16*a*b*c*x*Log[1 - c^2*x^2]
 + 4*b^2*c*x*Log[1 - c^2*x^2] + 24*b^2*c*x*(1 + ArcTanh[c*x])*PolyLog[2, -E
^(-2*ArcTanh[c*x])] - 8*b^2*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*b^2*c*
x*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c*x*PolyLog[2, -(c*x
)] + 24*a*b*c*x*PolyLog[2, c*x] + 12*b^2*c*x*PolyLog[3, -E^(-2*ArcTanh[c*x]
)] - 12*b^2*c*x*PolyLog[3, E^(2*ArcTanh[c*x])]))/(8*x)
```

Maple [C] time = 1.339, size = 1270, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x)
```

```
[Out] 3/2*I*c*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1
)+1))^3*arctanh(c*x)^2+a*b*c^2*d^3*x+b^2*c^2*d^3*x*arctanh(c*x)-3/2*I*c*d^3
*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1
)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/2*I*c*d^3*b^2*Pi*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^
2*x^2+1)+1))^2*arctanh(c*x)^2+3/2*I*c*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-
1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-d^3*a^2/x-6*c*d^3*b^2*dilog(1
+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(
1/2))-6*c*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c*d^3*b^2*dilog(1
+(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c*d^3*b^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-
d^3*b^2*arctanh(c*x)^2/x+1/2*d^3*a^2*c^3*x^2+3*c^2*x*a^2*d^3+3*c*d^3*a^2*ln
(c*x)+3/2*c*d^3*b^2*arctanh(c*x)^2+c*d^3*b^2*arctanh(c*x)+3/2*c*d^3*b^2*pol
ylog(3,-(c*x+1)^2/(-c^2*x^2+1))-c*d^3*b^2*ln((c*x+1)^2/(-c^2*x^2+1)+1)-6*c*
d^3*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*d^3*a*b*arctanh(c*x)/x+5/2
*c*d^3*a*b*ln(c*x-1)+2*c*d^3*a*b*ln(c*x)+3*d^3*b^2*arctanh(c*x)^2*c^2*x-3*c
*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3*c*d^3*b^2*arctan
h(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c*d^3*b^2*arctanh(c*x)*polylog(
2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c*d^3*b^2*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^
2*x^2+1)^(1/2))-6*c*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))
+2*c*d^3*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3*c*d^3*b^2*arct
anh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*c*d^3*b^2*arctanh(c*x)^2*ln(1-(c*
x+1)/(-c^2*x^2+1)^(1/2))+6*c*d^3*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x
^2+1)^(1/2))+3*c*d^3*b^2*arctanh(c*x)^2*ln(c*x)+1/2*d^3*b^2*arctanh(c*x)^2*
c^3*x^2-3*c*d^3*a*b*dilog(c*x)-3*c*d^3*a*b*dilog(c*x+1)+3/2*c*d^3*a*b*ln(c*
x+1)+6*c*d^3*a*b*arctanh(c*x)*ln(c*x)-3*c*d^3*a*b*ln(c*x)*ln(c*x+1)+6*d^3*a
*b*arctanh(c*x)*c^2*x+d^3*a*b*arctanh(c*x)*c^3*x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2c^3d^3x^2 + 3a^2c^2d^3x + 3\left(2cx\,\text{artanh}\,(cx) + \log\left(-c^2x^2+1\right)\right)abcd^3 + 3a^2cd^3\log\left(x\right) - \left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*c^3*d^3*x^2 + 3*a^2*c^2*d^3*x + 3*(2*c*x*arctanh(c*x) + log(-c^2*x^
2 + 1))*a*b*c*d^3 + 3*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^2)) +
 2*arctanh(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/8*(b^2*c^3*d^3*x^3 + 6*b^2*c^2*d
^3*x^2 - 2*b^2*d^3)*log(-c*x + 1)^2/x - integrate(-1/4*((b^2*c^4*d^3*x^4 +
2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 4*(a*b*c^4*d^
3*x^4 - a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 - 3*a*b*c*d^3*x)*log(c*x + 1) -
 (12*a*b*c^2*d^3*x^2 + (4*a*b*c^4*d^3 + b^2*c^4*d^3)*x^4 - 2*(2*a*b*c^3*d^3
 - 3*b^2*c^3*d^3)*x^3 - 2*(6*a*b*c*d^3 + b^2*c*d^3)*x + 2*(b^2*c^4*d^3*x^4
+ 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))
/(c*x^3 - x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3a^2c^2d^3x^2 + 3a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3b^2c^2d^3x^2 + 3b^2cd^3x + b^2d^3\right)\text{artanh}\,(cx)^2 + 2\left(abc^3d^3\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int 3a^2c^2\,dx + \int \frac{a^2}{x^2}\,dx + \int \frac{3a^2c}{x}\,dx + \int a^2c^3x\,dx + \int 3b^2c^2\operatorname{atanh}^2(cx)\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^2}\,dx + \int 6abc^2\operatorname{at}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**2,x)
```

```
[Out] d**3*(Integral(3*a**2*c**2, x) + Integral(a**2/x**2, x) + Integral(3*a**2*c
/x, x) + Integral(a**2*c**3*x, x) + Integral(3*b**2*c**2*atanh(c*x)**2, x)
+ Integral(b**2*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c**2*atanh(c*x), x)
 + Integral(2*a*b*atanh(c*x)/x**2, x) + Integral(3*b**2*c*atanh(c*x)**2/x,
x) + Integral(b**2*c**3*x*atanh(c*x)**2, x) + Integral(6*a*b*c*atanh(c*x)/x
, x) + Integral(2*a*b*c**3*x*atanh(c*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3 (b\operatorname{artanh}(cx) + a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^2, x)
```

3.90 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=385

$$-3bc^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bc^2d^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-b^2c^2d^3\text{PolyL}$$

```
[Out] -((b*c*d^3*(a + b*ArcTanh[c*x]))/x) + (9*c^2*d^3*(a + b*ArcTanh[c*x])^2)/2
- (d^3*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/x
 + c^3*d^3*x*(a + b*ArcTanh[c*x])^2 + 6*c^2*d^3*(a + b*ArcTanh[c*x])^2*ArcT
anh[1 - 2/(1 - c*x)] + b^2*c^2*d^3*Log[x] - 2*b*c^2*d^3*(a + b*ArcTanh[c*x]
)*Log[2/(1 - c*x)] - (b^2*c^2*d^3*Log[1 - c^2*x^2])/2 + 6*b*c^2*d^3*(a + b*
ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)
] - 3*b*c^2*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c^2*
d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 3*b^2*c^2*d^3*PolyL
og[2, -1 + 2/(1 + c*x)] + (3*b^2*c^2*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (
3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rubi [A] time = 0.78765, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 20, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 5914, 6052, 6058, 6610}

$$-3bc^2d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+3bc^2d^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-b^2c^2d^3\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```


```
[Out] -((b*c*d^3*(a + b*ArcTanh[c*x]))/x) + (9*c^2*d^3*(a + b*ArcTanh[c*x])^2)/2
- (d^3*(a + b*ArcTanh[c*x])^2)/(2*x^2) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/x
 + c^3*d^3*x*(a + b*ArcTanh[c*x])^2 + 6*c^2*d^3*(a + b*ArcTanh[c*x])^2*ArcT
anh[1 - 2/(1 - c*x)] + b^2*c^2*d^3*Log[x] - 2*b*c^2*d^3*(a + b*ArcTanh[c*x]
)*Log[2/(1 - c*x)] - (b^2*c^2*d^3*Log[1 - c^2*x^2])/2 + 6*b*c^2*d^3*(a + b*
ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 - c*x)
] - 3*b*c^2*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)] + 3*b*c^2*
d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - 3*b^2*c^2*d^3*PolyL
og[2, -1 + 2/(1 + c*x)] + (3*b^2*c^2*d^3*PolyLog[3, 1 - 2/(1 - c*x)])/2 - (
3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^3} dx = \int \left(c^3d^3 \left(a+b\tanh^{-1}(cx)\right)^2 + \frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x^3} + \frac{3cd^3 \left(a+b\tanh^{-1}(c\right.}{x^2}\right.$$

$$= d^3 \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3} dx + \left(3cd^3\right) \int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2} dx + \left(3c^2d^3\right) \int$$

$$= -\frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3cd^3 \left(a+b\tanh^{-1}(cx)\right)^2}{x} + c^3d^3x \left(a+b\tanh^{-1}(cx\right.$$

$$= 4c^2d^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx)\right)^2}{2x^2} - \frac{3cd^3 \left(a+b\tanh^{-1}(cx)\right)}{x}$$

$$= -\frac{bcd^3 \left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx\right.}{2x^2}$$

$$= -\frac{bcd^3 \left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx\right.}{2x^2}$$

$$= -\frac{bcd^3 \left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx\right.}{2x^2}$$

$$= -\frac{bcd^3 \left(a+b\tanh^{-1}(cx)\right)}{x} + \frac{9}{2}c^2d^3 \left(a+b\tanh^{-1}(cx)\right)^2 - \frac{d^3 \left(a+b\tanh^{-1}(cx\right.}{2x^2}$$

Mathematica [C] time = 1.00393, size = 461, normalized size = 1.2

$$\frac{1}{2}d^3 \left(-6abc^2(\text{PolyLog}(2,-cx) - \text{PolyLog}(2,cx)) + 2b^2c^2 \left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + \tanh^{-1}(cx)\left((cx-1)\tanh^{-1}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^3,x]
```

```
[Out] (d^3*(-(a^2/x^2) - (6*a^2*c)/x + 2*a^2*c^3*x + 6*a^2*c^2*Log[x] - (a*b*(2*A
rcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Log[1 + c*x])))/x^2 + (b^2*(-
2*c*x*ArcTanh[c*x] + (-1 + c^2*x^2)*ArcTanh[c*x]^2 + 2*c^2*x^2*Log[(c*x)/Sq
rt[1 - c^2*x^2]]))/x^2 + 2*a*b*c^2*(2*c*x*ArcTanh[c*x] + Log[1 - c^2*x^2])
- (6*a*b*c*(2*ArcTanh[c*x] + c*x*(-2*Log[c*x] + Log[1 - c^2*x^2])))/x + 2*b
^2*c^2*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x
])]) + PolyLog[2, -E^(-2*ArcTanh[c*x])]) + (6*b^2*c*(ArcTanh[c*x]*((-1 + c*
x)*ArcTanh[c*x] + 2*c*x*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*PolyLog[2, E^(-
2*ArcTanh[c*x])]))/x - 6*a*b*c^2*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]) + 6
*b^2*c^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2
*ArcTanh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]
*PolyLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*
x])] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/
2)))/2
```

Maple [C] time = 1.214, size = 1358, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x)
```

```
[Out] -1/2*d^3*a^2/x^2+2*c^3*d^3*a*b*arctanh(c*x)*x-6*c*d^3*a*b*arctanh(c*x)/x+6*
c^2*d^3*a*b*arctanh(c*x)*ln(c*x)-3*c^2*d^3*a*b*ln(c*x)*ln(c*x+1)-d^3*a*b*ar
ctanh(c*x)/x^2-c*d^3*b^2*arctanh(c*x)/x-3*c*d^3*b^2*arctanh(c*x)^2/x+c^3*d^
3*b^2*arctanh(c*x)^2*x-3*c^2*d^3*a*b*dilog(c*x)-3*c^2*d^3*a*b*dilog(c*x+1)-
3/2*c^2*d^3*a*b*ln(c*x+1)-5/2*c^2*d^3*a*b*ln(c*x-1)+6*c^2*d^3*a*b*ln(c*x)-3
*c^2*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3*c^2*d^3*b^2*
arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c^2*d^3*b^2*arctanh(c*x)*
polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*d^3*b^2*arctanh(c*x)*ln(1+I*(c
*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*d^3*b^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x
^2+1)^(1/2))+6*c^2*d^3*b^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3*
c^2*d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*c^2*d^3*b^2*arcta
nh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c^2*d^3*b^2*arctanh(c*x)*polyl
og(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*d^3*b^2*arctanh(c*x)^2*ln(c*x)-3/2*I
*c^2*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/2*I*c^2*d^3*b^2*Pi
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x
+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-c*d^3*a*b/x-3*c*d^3*a^2/x-1/2*d^3*b
^2*arctanh(c*x)^2/x^2-6*c^2*d^3*b^2*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2
*d^3*a^2*ln(c*x)-3/2*c^2*d^3*b^2*arctanh(c*x)^2+c^2*d^3*b^2*ln(1+(c*x+1)/(-
c^2*x^2+1)^(1/2))+c^2*d^3*b^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)-c^2*d^3*b^2*
arctanh(c*x)+3/2*c^2*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*c^2*d^3*b
^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*d^3*b^2*dilog(1+I*(c*x+1)/(
-c^2*x^2+1)^(1/2))-2*c^2*d^3*b^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c^
2*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c^2*d^3*b^2*dilog(1+(c*x+
1)/(-c^2*x^2+1)^(1/2))+c^3*x*a^2*d^3+3/2*I*c^2*d^3*b^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+3/2*I*c^2*d^3
*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1
))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*
x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2c^3d^3x + \left(2\,cx\,\operatorname{artanh}(cx) + \log\left(-c^2x^2+1\right)\right)abc^2d^3 + 3\,a^2c^2d^3\log(x) - 3\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\operatorname{artanh}(cx)}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*c^3*d^3*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b*c^2*d^3 + 3*a^
2*c^2*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a
*b*c*d^3 + 1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/
x^2)*a*b*d^3 - 3*a^2*c*d^3/x - 1/2*a^2*d^3/x^2 + 1/8*(2*b^2*c^3*d^3*x^3 - 6
*b^2*c*d^3*x - b^2*d^3)*log(-c*x + 1)^2/x^2 - integrate(-1/4*((b^2*c^4*d^3*
x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b
*c^3*d^3*x^3 - a*b*c^2*d^3*x^2)*log(c*x + 1) - (2*b^2*c^4*d^3*x^4 + 12*a*b*
c^3*d^3*x^3 - b^2*c*d^3*x - 6*(2*a*b*c^2*d^3 + b^2*c^2*d^3)*x^2 + 2*(b^2*c^
4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(
-c*x + 1))/(c*x^4 - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\text{artanh}\,(cx)^2 + 2\left(abc\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int a^2c^3\,dx + \int \frac{a^2}{x^3}\,dx + \int \frac{3a^2c}{x^2}\,dx + \int \frac{3a^2c^2}{x}\,dx + \int b^2c^3\,\text{atanh}^2\,(cx)\,dx + \int \frac{b^2\,\text{atanh}^2\,(cx)}{x^3}\,dx + \int 2abc^3\,\text{ata}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**3,x)
```

```
[Out] d**3*(Integral(a**2*c**3, x) + Integral(a**2/x**3, x) + Integral(3*a**2*c/x
**2, x) + Integral(3*a**2*c**2/x, x) + Integral(b**2*c**3*atanh(c*x)**2, x)
 + Integral(b**2*atanh(c*x)**2/x**3, x) + Integral(2*a*b*c**3*atanh(c*x), x
) + Integral(2*a*b*atanh(c*x)/x**3, x) + Integral(3*b**2*c*atanh(c*x)**2/x*
*2, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(
c*x)/x**2, x) + Integral(6*a*b*c**2*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3(b\,\text{artanh}\,(cx) + a)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^3, x)
```

3.91 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=396

$$-bc^3d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^3d^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{10}{3}b^2c^3d^3\text{PolyL}$$

```
[Out] -(b^2*c^2*d^3)/(3*x) + (b^2*c^3*d^3*ArcTanh[c*x])/3 - (b*c*d^3*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/x + (29*c^3*d^3*(a +
 b*ArcTanh[c*x])^2)/6 - (d^3*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (3*c*d^3*(a
+ b*ArcTanh[c*x])^2)/(2*x^2) - (3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/x + 2*c^3
*d^3*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 3*b^2*c^3*d^3*Log[x]
 - (3*b^2*c^3*d^3*Log[1 - c^2*x^2])/2 + (20*b*c^3*d^3*(a + b*ArcTanh[c*x])*
Log[2 - 2/(1 + c*x)])/3 - b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)] + b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - (
10*b^2*c^3*d^3*PolyLog[2, -1 + 2/(1 + c*x)])/3 + (b^2*c^3*d^3*PolyLog[3, 1
- 2/(1 - c*x)])/2 - (b^2*c^3*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rubi [A] time = 0.926942, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5916, 5982, 325, 206, 5988, 5932, 2447, 266, 36, 29, 31, 5948, 5914, 6052, 6058, 6610}

$$-bc^3d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)+bc^3d^3\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)-\frac{10}{3}b^2c^3d^3\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] -(b^2*c^2*d^3)/(3*x) + (b^2*c^3*d^3*ArcTanh[c*x])/3 - (b*c*d^3*(a + b*ArcTa
nh[c*x]))/(3*x^2) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/x + (29*c^3*d^3*(a +
 b*ArcTanh[c*x])^2)/6 - (d^3*(a + b*ArcTanh[c*x])^2)/(3*x^3) - (3*c*d^3*(a
+ b*ArcTanh[c*x])^2)/(2*x^2) - (3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/x + 2*c^3
*d^3*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] + 3*b^2*c^3*d^3*Log[x]
 - (3*b^2*c^3*d^3*Log[1 - c^2*x^2])/2 + (20*b*c^3*d^3*(a + b*ArcTanh[c*x])*
Log[2 - 2/(1 + c*x)])/3 - b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(
1 - c*x)] + b*c^3*d^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)] - (
10*b^2*c^3*d^3*PolyLog[2, -1 + 2/(1 + c*x)])/3 + (b^2*c^3*d^3*PolyLog[3, 1
- 2/(1 - c*x)])/2 - (b^2*c^3*d^3*PolyLog[3, -1 + 2/(1 - c*x)])/2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx = \int\left(\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}+\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}+\frac{3c^2d^3\left(a+b\tanh^{-1}\right.}{x^2}\right.$$

$$= d^3\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}dx+\left(3cd^3\right)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}dx+\left(3c^2d^3\right)\int$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{2x^2}-\frac{3c^2d^3\left(a+b\tanh^{-1}(cx\right.}{x}$$

$$= 3c^3d^3\left(a+b\tanh^{-1}(cx)\right)^2-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{3x^3}-\frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)}{2x^2}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{29}{6}c^3d^3\left(a+b\tanh^{-1}\right.$$

$$= -\frac{b^2c^2d^3}{3x}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x}+\frac{29}{6}c^3d^3\left(a\right.$$

$$= -\frac{b^2c^2d^3}{3x}+\frac{1}{3}b^2c^3d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{3bc^2d^3\left(a+b\tan\right.}{x}$$

$$= -\frac{b^2c^2d^3}{3x}+\frac{1}{3}b^2c^3d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{3x^2}-\frac{3bc^2d^3\left(a+b\tan\right.}{x}$$

Mathematica [C] time = 0.721247, size = 569, normalized size = 1.44

$$\frac{d^3\left(-24abc^3x^3\text{PolyLog}(2,-cx)+24abc^3x^3\text{PolyLog}(2,cx)+24b^2c^3x^3\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-80b^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^4,x]
```

```
[Out] (d^3*(-8*a^2 - 36*a^2*c*x - 8*a*b*c*x - 72*a^2*c^2*x^2 - 72*a*b*c^2*x^2 - 8
*b^2*c^2*x^2 + I*b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTanh[c*x] - 72*a*b*c*x*ArcTan
h[c*x] - 8*b^2*c*x*ArcTanh[c*x] - 144*a*b*c^2*x^2*ArcTanh[c*x] - 72*b^2*c^2
*x^2*ArcTanh[c*x] + 8*b^2*c^3*x^3*ArcTanh[c*x] - 8*b^2*ArcTanh[c*x]^2 - 36*
b^2*c*x*ArcTanh[c*x]^2 - 72*b^2*c^2*x^2*ArcTanh[c*x]^2 + 116*b^2*c^3*x^3*Ar
cTanh[c*x]^2 - 16*b^2*c^3*x^3*ArcTanh[c*x]^3 + 160*b^2*c^3*x^3*ArcTanh[c*x]
*Log[1 - E^(-2*ArcTanh[c*x])] - 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 + E^(-2
*ArcTanh[c*x])] + 24*b^2*c^3*x^3*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])]
 + 24*a^2*c^3*x^3*Log[x] + 160*a*b*c^3*x^3*Log[c*x] - 36*a*b*c^3*x^3*Log[1
- c*x] + 36*a*b*c^3*x^3*Log[1 + c*x] + 72*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 - c^
2*x^2]] - 80*a*b*c^3*x^3*Log[1 - c^2*x^2] + 24*b^2*c^3*x^3*ArcTanh[c*x]*Pol
yLog[2, -E^(-2*ArcTanh[c*x])] - 80*b^2*c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x
])] + 24*b^2*c^3*x^3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 24*a*b*c
^3*x^3*PolyLog[2, -(c*x)] + 24*a*b*c^3*x^3*PolyLog[2, c*x] + 12*b^2*c^3*x^3
*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 12*b^2*c^3*x^3*PolyLog[3, E^(2*ArcTanh[
c*x])]))/(24*x^3)
```

Maple [C] time = 1.52, size = 1337, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x)
```

```
[Out] 1/2*c^3*d^3*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-2*c^3*d^3*b^2*polylog(3,
-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^3*d^3*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(
1/2))+20/3*c^3*d^3*b^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3*c^3*d^3*b^2
*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-1/3*d^3*b^2*arctanh(c*x)^2/x^3+1/2*I*c^3
*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3
*arctanh(c*x)^2-8/3*b^2*c^3*d^3*arctanh(c*x)-1/2*I*c^3*d^3*b^2*Pi*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^
2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*c^3*d^3*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*
x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*
arctanh(c*x)^2-3*c*d^3*a*b*arctanh(c*x)/x^2-6*c^2*d^3*a*b*arctanh(c*x)/x+2*
c^3*d^3*a*b*arctanh(c*x)*ln(c*x)-c^3*d^3*a*b*ln(c*x)*ln(c*x+1)-1/3*d^3*a^2/
x^3+1/2*I*c^3*d^3*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)
^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))*arctanh(c*x)^2-3*c^2*d^3*a^2/x-3/2*c*d^3*a^2/x^2+c^3*d^3*a^2*ln(c*x)
-11/6*c^3*d^3*b^2*arctanh(c*x)^2+3*c^3*d^3*b^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1
/2))+3*c^3*d^3*b^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+2*c^3*d^3*b^2*arctanh(c
*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+20/3*c^3*d^3*b^2*arctanh(c*x)*ln
(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*d^3*b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^
2*x^2+1)-1)+c^3*d^3*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c
^3*d^3*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+c^3*d^3*b^2*a
rctanh(c*x)^2*ln(c*x)+1/3*c^3*d^3*b^2/((-c^2*x^2+1)^(1/2)+c*x+1)*(-c^2*x^2+
1)^(1/2)-1/3*c^3*d^3*b^2/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)-c^3*
d^3*a*b*dilog(c*x)-c^3*d^3*a*b*dilog(c*x+1)-11/6*c^3*d^3*a*b*ln(c*x+1)-3/2*
c*d^3*b^2*arctanh(c*x)^2/x^2-3*c^2*d^3*b^2*arctanh(c*x)/x-3*c^2*d^3*b^2*arc
tanh(c*x)^2/x-1/3*c*d^3*b^2*arctanh(c*x)/x^2-3*c^2*d^3*a*b/x-1/3*c*d^3*a*b/
x^2-2/3*d^3*a*b*arctanh(c*x)/x^3-29/6*c^3*d^3*a*b*ln(c*x-1)+20/3*c^3*d^3*a*
b*ln(c*x)-c^3*d^3*b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+c^3*d
^3*b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2c^3d^3\log(x) - 3\left(c\left(\log\left(c^2x^2-1\right) - \log\left(x^2\right)\right) + \frac{2\,\operatorname{artanh}(cx)}{x}\right)abc^2d^3 + \frac{3}{2}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] a^2*c^3*d^3*log(x) - 3*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)
*a*b*c^2*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c
*x)/x^2)*a*b*c*d^3 - 1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c +
 2*arctanh(c*x)/x^3)*a*b*d^3 - 3*a^2*c^2*d^3/x - 3/2*a^2*c*d^3/x^2 - 1/3*a^
2*d^3/x^3 - 1/24*(18*b^2*c^2*d^3*x^2 + 9*b^2*c*d^3*x + 2*b^2*d^3)*log(-c*x
+ 1)^2/x^3 - integrate(-1/12*(3*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*x^3 - 2*b^
2*c*d^3*x - b^2*d^3)*log(c*x + 1)^2 + 12*(a*b*c^4*d^3*x^4 - a*b*c^3*d^3*x^3
)*log(c*x + 1) - (12*a*b*c^4*d^3*x^4 - 9*b^2*c^2*d^3*x^2 - 2*b^2*c*d^3*x -
6*(2*a*b*c^3*d^3 + 3*b^2*c^3*d^3)*x^3 + 6*(b^2*c^4*d^3*x^4 + 2*b^2*c^3*d^3*
x^3 - 2*b^2*c*d^3*x - b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\operatorname{artanh}(cx)^2 + 2\left(abc\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x^4}\,dx + \int \frac{3a^2c}{x^3}\,dx + \int \frac{3a^2c^2}{x^2}\,dx + \int \frac{a^2c^3}{x}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^4}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^4}\,dx + \int \frac{3b^2c\operatorname{atan}}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**4,x)
```

```
[Out] d**3*(Integral(a**2/x**4, x) + Integral(3*a**2*c/x**3, x) + Integral(3*a**2
*c**2/x**2, x) + Integral(a**2*c**3/x, x) + Integral(b**2*atanh(c*x)**2/x**
4, x) + Integral(2*a*b*atanh(c*x)/x**4, x) + Integral(3*b**2*c*atanh(c*x)**
2/x**3, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**2, x) + Integral(b**2*c*
*3*atanh(c*x)**2/x, x) + Integral(6*a*b*c*atanh(c*x)/x**3, x) + Integral(6*
a*b*c**2*atanh(c*x)/x**2, x) + Integral(2*a*b*c**3*atanh(c*x)/x, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3(b\operatorname{artanh}(cx) + a)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^4, x)
```

3.92 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=271

$$-2b^2c^4d^3\text{PolyLog}(2,-cx)+2b^2c^4d^3\text{PolyLog}(2,cx)+2b^2c^4d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}+4ab$$

```
[Out] -(b^2*c^2*d^3)/(12*x^2) - (b^2*c^3*d^3)/x + b^2*c^4*d^3*ArcTanh[c*x] - (b*c
*d^3*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/x^2 -
 (7*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh
[c*x])^2)/(4*x^4) + 4*a*b*c^4*d^3*Log[x] + (11*b^2*c^4*d^3*Log[x])/3 + 4*b*
c^4*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] - (11*b^2*c^4*d^3*Log[1 - c^2
*x^2])/6 - 2*b^2*c^4*d^3*PolyLog[2, -(c*x)] + 2*b^2*c^4*d^3*PolyLog[2, c*x]
 + 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - c*x)]
```

Rubi [A] time = 0.307835, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {37, 5938, 5916, 266, 44, 325, 206, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-2b^2c^4d^3\text{PolyLog}(2,-cx)+2b^2c^4d^3\text{PolyLog}(2,cx)+2b^2c^4d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2}+4ab$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```


```
[Out] -(b^2*c^2*d^3)/(12*x^2) - (b^2*c^3*d^3)/x + b^2*c^4*d^3*ArcTanh[c*x] - (b*c
*d^3*(a + b*ArcTanh[c*x]))/(6*x^3) - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/x^2 -
 (7*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4*(a + b*ArcTanh
[c*x])^2)/(4*x^4) + 4*a*b*c^4*d^3*Log[x] + (11*b^2*c^4*d^3*Log[x])/3 + 4*b*
c^4*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)] - (11*b^2*c^4*d^3*Log[1 - c^2
*x^2])/6 - 2*b^2*c^4*d^3*PolyLog[2, -(c*x)] + 2*b^2*c^4*d^3*PolyLog[2, c*x]
 + 2*b^2*c^4*d^3*PolyLog[2, 1 - 2/(1 - c*x)]
```


Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

```
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4} - (2bc)\int\left(-\frac{d^3\left(a+b\tanh^{-1}(cx)\right)}{4x^4} - \frac{cd^3\left(a+b\right.}{}\right.$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bcd^3\right)\int\frac{a+b\tanh^{-1}(cx)}{x^4}\,dx + \left(2bc^2d^3\right)$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{7bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{2x}$$

$$= -\frac{b^2c^3d^3}{x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{x^2} - \frac{7bc^3d^3\left(a+b\,\mathrm{ta}\right.}{2x}$$

$$= -\frac{b^2c^3d^3}{x} + b^2c^4d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\tanh^{-1}(c\right.}{x^2}$$

$$= -\frac{b^2c^2d^3}{12x^2} - \frac{b^2c^3d^3}{x} + b^2c^4d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{6x^3} - \frac{bc^2d^3\left(a+b\right.}{x}$$

Mathematica [A] time = 0.771332, size = 343, normalized size = 1.27

$$-\frac{d^3\left(24b^2c^4x^4\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+12a^2c^3x^3+18a^2c^2x^2+12a^2cx+3a^2+42abc^3x^3+12abc^2x^2-48abc^4x^4\log(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^5,x]
```

```
[Out] -(d^3*(3*a^2 + 12*a^2*c*x + 2*a*b*c*x + 18*a^2*c^2*x^2 + 12*a*b*c^2*x^2 + b
^2*c^2*x^2 + 12*a^2*c^3*x^3 + 42*a*b*c^3*x^3 + 12*b^2*c^3*x^3 - b^2*c^4*x^4
 + 3*b^2*(1 + 4*c*x + 6*c^2*x^2 + 4*c^3*x^3 - 15*c^4*x^4)*ArcTanh[c*x]^2 +
2*b*ArcTanh[c*x]*(b*c*x*(1 + 6*c*x + 21*c^2*x^2 - 6*c^3*x^3) + 3*a*(1 + 4*c
*x + 6*c^2*x^2 + 4*c^3*x^3) - 24*b*c^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])]) -
48*a*b*c^4*x^4*Log[c*x] + 21*a*b*c^4*x^4*Log[1 - c*x] - 21*a*b*c^4*x^4*Log[
1 + c*x] - 44*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 24*a*b*c^4*x^4*Log
[1 - c^2*x^2] + 24*b^2*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])]))/(12*x^4)
```

Maple [B] time = 0.075, size = 646, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x)
```

```
[Out] -1/12*b^2*c^2*d^3/x^2-1/4*d^3*a^2/x^4-b^2*c^3*d^3/x-2*c^3*d^3*a*b*arctanh(c
*x)/x-3*c^2*d^3*a*b*arctanh(c*x)/x^2-2*c*d^3*a*b*arctanh(c*x)/x^3-15/4*c^4*
d^3*a*b*ln(c*x-1)-1/4*c^4*d^3*a*b*ln(c*x+1)-3/2*c^2*d^3*b^2*arctanh(c*x)^2/
x^2-7/2*c^3*d^3*b^2*arctanh(c*x)/x-c^3*d^3*b^2*arctanh(c*x)^2/x-c*d^3*b^2*a
rctanh(c*x)^2/x^3-c^2*d^3*b^2*arctanh(c*x)/x^2-1/6*c*d^3*b^2*arctanh(c*x)/x
^3-1/2*d^3*a*b*arctanh(c*x)/x^4-1/6*c*d^3*a*b/x^3-7/2*c^3*d^3*a*b/x-15/4*c^
4*d^3*b^2*arctanh(c*x)*ln(c*x-1)-1/4*c^4*d^3*b^2*arctanh(c*x)*ln(c*x+1)-1/8
*c^4*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/8*c^4*d^3*b^2*ln(-1/2*c*x+1/2)*ln
(1/2+1/2*c*x)-2*c^4*d^3*b^2*ln(c*x)*ln(c*x+1)+4*c^4*d^3*b^2*arctanh(c*x)*ln
(c*x)+15/8*c^4*d^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)+4*c^4*d^3*a*b*ln(c*x)-2*c^
4*d^3*b^2*dilog(c*x+1)+1/16*c^4*d^3*b^2*ln(c*x+1)^2-4/3*c^4*d^3*b^2*ln(c*x+
1)+11/3*c^4*d^3*b^2*ln(c*x)+2*c^4*d^3*b^2*dilog(1/2+1/2*c*x)-7/3*c^4*d^3*b^
2*ln(c*x-1)-15/16*c^4*d^3*b^2*ln(c*x-1)^2-2*c^4*d^3*b^2*dilog(c*x)-c^3*d^3*
a^2/x-3/2*c^2*d^3*a^2/x^2-c*d^3*a^2/x^3-1/4*d^3*b^2*arctanh(c*x)^2/x^4-c^2*
d^3*a*b/x^2
```

Maxima [B] time = 3.10516, size = 1098, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] -2*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^4*d^3 -
2*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^4*d^3 + 2*(log(c*x + 1)*
log(-c*x) + dilog(c*x + 1))*b^2*c^4*d^3 - b^2*c^4*d^3*log(c*x + 1) - 2*b^2*
c^4*d^3*log(c*x - 1) + 3*b^2*c^4*d^3*log(x) - (c*(log(c^2*x^2 - 1) - log(x^
2)) + 2*arctanh(c*x)/x)*a*b*c^3*d^3 + 3/2*((c*log(c*x + 1) - c*log(c*x - 1)
 - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^2*d^3 - ((c^2*log(c^2*x^2 - 1) - c^2*
log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c*d^3 - a^2*c^3*d^3/x + 1/12*
((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*ar
ctanh(c*x)/x^4)*a*b*d^3 + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)^2
+ 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log(c*x
 - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1) - 3*c
^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2*d^3 - 3/2*a^2*
c^2*d^3/x^2 - a^2*c*d^3/x^3 - 1/4*b^2*d^3*arctanh(c*x)^2/x^4 - 1/4*a^2*d^3/
x^4 - 1/8*(8*b^2*c^3*d^3*x^2 + (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2
*c^2*d^3*x + 2*b^2*c*d^3)*log(c*x + 1)^2 - (7*b^2*c^4*d^3*x^3 - 2*b^2*c^3*d
^3*x^2 - 3*b^2*c^2*d^3*x - 2*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(3*b^2*c^3*d^3*
x^2 + b^2*c^2*d^3*x)*log(c*x + 1) - 2*(6*b^2*c^3*d^3*x^2 + 2*b^2*c^2*d^3*x
+ (b^2*c^4*d^3*x^3 + 2*b^2*c^3*d^3*x^2 + 3*b^2*c^2*d^3*x + 2*b^2*c*d^3)*log
(c*x + 1))*log(-c*x + 1))/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\operatorname{artanh}\left(cx\right)^2 + 2\left(abc\right.}{x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x^5}\,dx + \int \frac{3a^2c}{x^4}\,dx + \int \frac{3a^2c^2}{x^3}\,dx + \int \frac{a^2c^3}{x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^5}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^5}\,dx + \int \frac{3b^2c\operatorname{atanh}^2(}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**5,x)
```

```
[Out] d**3*(Integral(a**2/x**5, x) + Integral(3*a**2*c/x**4, x) + Integral(3*a**2
*c**2/x**3, x) + Integral(a**2*c**3/x**2, x) + Integral(b**2*atanh(c*x)**2/
x**5, x) + Integral(2*a*b*atanh(c*x)/x**5, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**4, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**3, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**2, x) + Integral(6*a*b*c*atanh(c*x)/x**4, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**3, x) + Integral(2*a*b*c**3*atanh(c*x)/x**2, x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx+d)^3(b\operatorname{artanh}(cx)+a)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^5, x)
```

3.93 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^6}\,dx$

Optimal. Leaf size=352

$$-\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,-cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{6bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{5x^2}$$

```
[Out] -(b^2*c^2*d^3)/(30*x^3) - (b^2*c^3*d^3)/(4*x^2) - (13*b^2*c^4*d^3)/(10*x) +
 (13*b^2*c^5*d^3*ArcTanh[c*x])/10 - (b*c*d^3*(a + b*ArcTanh[c*x]))/(10*x^4)
 - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/(2*x^3) - (6*b*c^3*d^3*(a + b*ArcTanh[c
*x]))/(5*x^2) - (5*b*c^4*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4
*(a + b*ArcTanh[c*x])^2)/(5*x^5) + (c*d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^
2)/(20*x^4) + (12*a*b*c^5*d^3*Log[x])/5 + 3*b^2*c^5*d^3*Log[x] + (12*b*c^5*
d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/5 - (3*b^2*c^5*d^3*Log[1 - c^2*x
^2])/2 - (6*b^2*c^5*d^3*PolyLog[2, -(c*x)])/5 + (6*b^2*c^5*d^3*PolyLog[2, c
*x])/5 + (6*b^2*c^5*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/5
```

Rubi [A] time = 0.368186, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {45, 37, 5938, 5916, 325, 206, 266, 44, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,-cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}(2,cx)+\frac{6}{5}b^2c^5d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{6bc^3d^3\left(a+b\tanh^{-1}(cx)\right)}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```


```
[Out] -(b^2*c^2*d^3)/(30*x^3) - (b^2*c^3*d^3)/(4*x^2) - (13*b^2*c^4*d^3)/(10*x) +
 (13*b^2*c^5*d^3*ArcTanh[c*x])/10 - (b*c*d^3*(a + b*ArcTanh[c*x]))/(10*x^4)
 - (b*c^2*d^3*(a + b*ArcTanh[c*x]))/(2*x^3) - (6*b*c^3*d^3*(a + b*ArcTanh[c
*x]))/(5*x^2) - (5*b*c^4*d^3*(a + b*ArcTanh[c*x]))/(2*x) - (d^3*(1 + c*x)^4
*(a + b*ArcTanh[c*x])^2)/(5*x^5) + (c*d^3*(1 + c*x)^4*(a + b*ArcTanh[c*x])^
2)/(20*x^4) + (12*a*b*c^5*d^3*Log[x])/5 + 3*b^2*c^5*d^3*Log[x] + (12*b*c^5*
d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/5 - (3*b^2*c^5*d^3*Log[1 - c^2*x
^2])/2 - (6*b^2*c^5*d^3*PolyLog[2, -(c*x)])/5 + (6*b^2*c^5*d^3*PolyLog[2, c
*x])/5 + (6*b^2*c^5*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/5
```


Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^6}\,dx = -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{20x^4}-(2bc)$$

$$= -\frac{d^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5}+\frac{cd^3(1+cx)^4\left(a+b\tanh^{-1}(cx)\right)^2}{20x^4}+\frac{1}{5}\left(2bc\right.$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3}-\frac{6bc^3d^3\left(a+b\tanh^{-1}(\right.}{5x^2}$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{6b^2c^4d^3}{5x}-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-\frac{bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{2x^3}-6$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{13b^2c^4d^3}{10x}+\frac{6}{5}b^2c^5d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4}-bc^2d^3$$

$$= -\frac{b^2c^2d^3}{30x^3}-\frac{b^2c^3d^3}{4x^2}-\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\tanh^{-1}(cx)-\frac{bcd^3\left(a+b\tanh^{-1}(c\right.}{10x^4}$$

Mathematica [A] time = 1.1037, size = 372, normalized size = 1.06

$$-\frac{d^3\left(72b^2c^5x^5\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+30a^2c^3x^3+60a^2c^2x^2+45a^2cx+12a^2+150abc^4x^4+72abc^3x^3+30abc^2x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^6,x]
```

```
[Out] -(d^3*(12*a^2 + 45*a^2*c*x + 6*a*b*c*x + 60*a^2*c^2*x^2 + 30*a*b*c^2*x^2 +
2*b^2*c^2*x^2 + 30*a^2*c^3*x^3 + 72*a*b*c^3*x^3 + 15*b^2*c^3*x^3 + 150*a*b*
c^4*x^4 + 78*b^2*c^4*x^4 - 15*b^2*c^5*x^5 + 3*b^2*(4 + 15*c*x + 20*c^2*x^2
+ 10*c^3*x^3 - 49*c^5*x^5)*ArcTanh[c*x]^2 + 6*b*ArcTanh[c*x]*(a*(4 + 15*c*x
 + 20*c^2*x^2 + 10*c^3*x^3) + b*c*x*(1 + 5*c*x + 12*c^2*x^2 + 25*c^3*x^3 -
13*c^4*x^4) - 24*b*c^5*x^5*Log[1 - E^(-2*ArcTanh[c*x])]) - 144*a*b*c^5*x^5*
Log[c*x] + 75*a*b*c^5*x^5*Log[1 - c*x] - 75*a*b*c^5*x^5*Log[1 + c*x] - 180*
b^2*c^5*x^5*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 72*a*b*c^5*x^5*Log[1 - c^2*x^2]
+ 72*b^2*c^5*x^5*PolyLog[2, E^(-2*ArcTanh[c*x])]))/(60*x^5)
```

Maple [B] time = 0.083, size = 691, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x)
```

```
[Out] -1/10*c*d^3*a*b/x^4-3/4*c*d^3*b^2*arctanh(c*x)^2/x^4-1/2*c^3*d^3*b^2*arctan
h(c*x)^2/x^2-5/2*c^4*d^3*b^2*arctanh(c*x)/x-c^2*d^3*b^2*arctanh(c*x)^2/x^3-
6/5*c^3*d^3*b^2*arctanh(c*x)/x^2-1/10*c*d^3*b^2*arctanh(c*x)/x^4-6/5*c^5*d^
3*b^2*ln(c*x)*ln(c*x+1)+12/5*c^5*d^3*b^2*arctanh(c*x)*ln(c*x)+49/40*c^5*d^3
*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-49/20*c^5*d^3*b^2*arctanh(c*x)*ln(c*x-1)+1/2
0*c^5*d^3*b^2*arctanh(c*x)*ln(c*x+1)+1/40*c^5*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c
*x+1)-1/40*c^5*d^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-49/20*c^5*d^3*a*b*l
n(c*x-1)+12/5*c^5*d^3*a*b*ln(c*x)-1/2*c^2*d^3*a*b/x^3-5/2*c^4*d^3*a*b/x-6/5
*c^3*d^3*a*b/x^2-2/5*d^3*a*b*arctanh(c*x)/x^5-1/30*b^2*c^2*d^3/x^3-1/4*b^2*
c^3*d^3/x^2-13/10*b^2*c^4*d^3/x-1/5*d^3*a^2/x^5-3/4*c*d^3*a^2/x^4-1/2*c^3*d
^3*a^2/x^2-c^2*d^3*a^2/x^3-6/5*c^5*d^3*b^2*dilog(c*x)-6/5*c^5*d^3*b^2*dilog
(c*x+1)+3*c^5*d^3*b^2*ln(c*x)-1/80*c^5*d^3*b^2*ln(c*x+1)^2+6/5*c^5*d^3*b^2*
dilog(1/2+1/2*c*x)-43/20*c^5*d^3*b^2*ln(c*x-1)-49/80*c^5*d^3*b^2*ln(c*x-1)^
2-17/20*c^5*d^3*b^2*ln(c*x+1)-1/5*d^3*b^2*arctanh(c*x)^2/x^5-c^3*d^3*a*b*ar
ctanh(c*x)/x^2-2*c^2*d^3*a*b*arctanh(c*x)/x^3-3/2*c*d^3*a*b*arctanh(c*x)/x^
4+1/20*c^5*d^3*a*b*ln(c*x+1)-1/2*c^2*d^3*b^2*arctanh(c*x)/x^3
```

Maxima [B] time = 3.1543, size = 1057, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="maxima")
```

```
[Out] -6/5*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^5*d^3
- 6/5*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^5*d^3 + 6/5*(log(c*x
 + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^5*d^3 - 17/20*b^2*c^5*d^3*log(c*x +
 1) - 43/20*b^2*c^5*d^3*log(c*x - 1) + 3*b^2*c^5*d^3*log(x) + 1/2*((c*log(c
*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b*c^3*d^3 - ((c^2
*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c^2*d
^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)
*c - 6*arctanh(c*x)/x^4)*a*b*c*d^3 - 1/10*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*
log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*d^3 - 1/2*a^2*c
^3*d^3/x^2 - a^2*c^2*d^3/x^3 - 3/4*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/240*
(312*b^2*c^4*d^3*x^4 + 60*b^2*c^3*d^3*x^3 + 8*b^2*c^2*d^3*x^2 - 3*(b^2*c^5*
```

```
d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*
d^3)*log(c*x + 1)^2 - 3*(49*b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c
^2*d^3*x^2 - 15*b^2*c*d^3*x - 4*b^2*d^3)*log(-c*x + 1)^2 + 12*(25*b^2*c^4*d
^3*x^4 + 12*b^2*c^3*d^3*x^3 + 5*b^2*c^2*d^3*x^2 + b^2*c*d^3*x)*log(c*x + 1)
 - 6*(50*b^2*c^4*d^3*x^4 + 24*b^2*c^3*d^3*x^3 + 10*b^2*c^2*d^3*x^2 + 2*b^2*
c*d^3*x - (b^2*c^5*d^3*x^5 - 10*b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 - 15*b
^2*c*d^3*x - 4*b^2*d^3)*log(c*x + 1))*log(-c*x + 1))/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\text{artanh}\left(cx\right)^2 + 2\,\left(abc\right.}{x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int\frac{a^2}{x^6}\,dx + \int\frac{3a^2c}{x^5}\,dx + \int\frac{3a^2c^2}{x^4}\,dx + \int\frac{a^2c^3}{x^3}\,dx + \int\frac{b^2\,\text{atanh}^2\left(cx\right)}{x^6}\,dx + \int\frac{2ab\,\text{atanh}\left(cx\right)}{x^6}\,dx + \int\frac{3b^2c\,\text{atan}}{x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**6,x)
```

```
[Out] d**3*(Integral(a**2/x**6, x) + Integral(3*a**2*c/x**5, x) + Integral(3*a**2
*c**2/x**4, x) + Integral(a**2*c**3/x**3, x) + Integral(b**2*atanh(c*x)**2/
x**6, x) + Integral(2*a*b*atanh(c*x)/x**6, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**5, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**4, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**3, x) + Integral(6*a*b*c*atanh(c*x)/x**5, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**4, x) + Integral(2*a*b*c**3*atanh(c*x)/x**3, x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(cdx + d)^3(b\,\text{artanh}\left(cx\right) + a)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^6, x)
```

3.94 $\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^7}\,dx$

Optimal. Leaf size=479

$$-\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,-cx)+\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,cx)+\frac{37}{40}b^2c^6d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{1}{120}b^2c^6d^3\text{PolyLog}\left(2,1-\right.$$

```
[Out] -(b^2*c^2*d^3)/(60*x^4) - (b^2*c^3*d^3)/(10*x^3) - (61*b^2*c^4*d^3)/(180*x^
2) - (37*b^2*c^5*d^3)/(30*x) + (37*b^2*c^6*d^3*ArcTanh[c*x])/30 - (b*c*d^3*
(a + b*ArcTanh[c*x]))/(15*x^5) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/(10*x^4
) - (11*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(18*x^3) - (14*b*c^4*d^3*(a + b*Arc
Tanh[c*x]))/(15*x^2) - (11*b*c^5*d^3*(a + b*ArcTanh[c*x]))/(6*x) - (d^3*(a
+ b*ArcTanh[c*x])^2)/(6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/(5*x^5) - (
3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x])^2
)/(3*x^3) + (28*a*b*c^6*d^3*Log[x])/15 + (113*b^2*c^6*d^3*Log[x])/45 + (37*
b*c^6*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/20 + (b*c^6*d^3*(a + b*Arc
Tanh[c*x])*Log[2/(1 + c*x)])/60 - (113*b^2*c^6*d^3*Log[1 - c^2*x^2])/90 - (
14*b^2*c^6*d^3*PolyLog[2, -(c*x)])/15 + (14*b^2*c^6*d^3*PolyLog[2, c*x])/15
 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/40 - (b^2*c^6*d^3*PolyLog[2
, 1 - 2/(1 + c*x)])/120
```

Rubi [A] time = 0.509132, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {43, 5938, 5916, 266, 44, 325, 206, 36, 29, 31, 5912, 5918, 2402, 2315}

$$-\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,-cx)+\frac{14}{15}b^2c^6d^3\text{PolyLog}(2,cx)+\frac{37}{40}b^2c^6d^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)-\frac{1}{120}b^2c^6d^3\text{PolyLog}\left(2,1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```


```
[Out] -(b^2*c^2*d^3)/(60*x^4) - (b^2*c^3*d^3)/(10*x^3) - (61*b^2*c^4*d^3)/(180*x^
2) - (37*b^2*c^5*d^3)/(30*x) + (37*b^2*c^6*d^3*ArcTanh[c*x])/30 - (b*c*d^3*
(a + b*ArcTanh[c*x]))/(15*x^5) - (3*b*c^2*d^3*(a + b*ArcTanh[c*x]))/(10*x^4
) - (11*b*c^3*d^3*(a + b*ArcTanh[c*x]))/(18*x^3) - (14*b*c^4*d^3*(a + b*Arc
Tanh[c*x]))/(15*x^2) - (11*b*c^5*d^3*(a + b*ArcTanh[c*x]))/(6*x) - (d^3*(a
+ b*ArcTanh[c*x])^2)/(6*x^6) - (3*c*d^3*(a + b*ArcTanh[c*x])^2)/(5*x^5) - (
3*c^2*d^3*(a + b*ArcTanh[c*x])^2)/(4*x^4) - (c^3*d^3*(a + b*ArcTanh[c*x])^2
)/(3*x^3) + (28*a*b*c^6*d^3*Log[x])/15 + (113*b^2*c^6*d^3*Log[x])/45 + (37*
b*c^6*d^3*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/20 + (b*c^6*d^3*(a + b*Arc
Tanh[c*x])*Log[2/(1 + c*x)])/60 - (113*b^2*c^6*d^3*Log[1 - c^2*x^2])/90 - (
14*b^2*c^6*d^3*PolyLog[2, -(c*x)])/15 + (14*b^2*c^6*d^3*PolyLog[2, c*x])/15
 + (37*b^2*c^6*d^3*PolyLog[2, 1 - 2/(1 - c*x)])/40 - (b^2*c^6*d^3*PolyLog[2
, 1 - 2/(1 + c*x)])/120
```


Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(d+cdx)^3\left(a+b\tanh^{-1}(cx)\right)^2}{x^7}\,dx = -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} - \frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}$$

$$= -\frac{d^3\left(a+b\tanh^{-1}(cx)\right)^2}{6x^6} - \frac{3cd^3\left(a+b\tanh^{-1}(cx)\right)^2}{5x^5} - \frac{3c^2d^3\left(a+b\tanh^{-1}(cx)\right)^2}{4x^4}$$

$$= -\frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4} - \frac{11bc^3d^3\left(a+b\tanh^{-1}(c\right.}{18x^3}$$

$$= -\frac{b^2c^3d^3}{10x^3} - \frac{14b^2c^5d^3}{15x} - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3\left(a+b\tanh^{-1}(cx)\right)}{10x^4} - 11$$

$$= -\frac{b^2c^3d^3}{10x^3} - \frac{37b^2c^5d^3}{30x} + \frac{14}{15}b^2c^6d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\tanh^{-1}(cx)\right)}{15x^5} - \frac{3bc^2d^3}{}$$

$$= -\frac{b^2c^2d^3}{60x^4} - \frac{b^2c^3d^3}{10x^3} - \frac{61b^2c^4d^3}{180x^2} - \frac{37b^2c^5d^3}{30x} + \frac{37}{30}b^2c^6d^3\tanh^{-1}(cx) - \frac{bcd^3\left(a+b\,t\right.}{15}$$

Mathematica [A] time = 1.37705, size = 402, normalized size = 0.84

$$-\frac{d^3\left(168b^2c^6x^6\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+60a^2c^3x^3+135a^2c^2x^2+108a^2cx+30a^2+330abc^5x^5+168abc^4x^4+110abc^3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + c*d*x)^3*(a + b*ArcTanh[c*x])^2)/x^7,x]
```

```
[Out] -(d^3*(30*a^2 + 108*a^2*c*x + 12*a*b*c*x + 135*a^2*c^2*x^2 + 54*a*b*c^2*x^2
 + 3*b^2*c^2*x^2 + 60*a^2*c^3*x^3 + 110*a*b*c^3*x^3 + 18*b^2*c^3*x^3 + 168*
a*b*c^4*x^4 + 61*b^2*c^4*x^4 + 330*a*b*c^5*x^5 + 222*b^2*c^5*x^5 - 64*b^2*c
^6*x^6 + 3*b^2*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x^3 - 111*c^6*x^6)*ArcTan
```

```
h[c*x]^2 + 2*b*ArcTanh[c*x]*(3*a*(10 + 36*c*x + 45*c^2*x^2 + 20*c^3*x^3) +
b*c*x*(6 + 27*c*x + 55*c^2*x^2 + 84*c^3*x^3 + 165*c^4*x^4 - 111*c^5*x^5) -
168*b*c^6*x^6*Log[1 - E^(-2*ArcTanh[c*x])]) - 336*a*b*c^6*x^6*Log[c*x] + 16
5*a*b*c^6*x^6*Log[1 - c*x] - 165*a*b*c^6*x^6*Log[1 + c*x] - 452*b^2*c^6*x^6
*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 168*a*b*c^6*x^6*Log[1 - c^2*x^2] + 168*b^2*
c^6*x^6*PolyLog[2, E^(-2*ArcTanh[c*x])]))/(180*x^6)
```

Maple [A] time = 0.073, size = 736, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x)
```

```
[Out] -1/60*b^2*c^2*d^3/x^4-1/10*b^2*c^3*d^3/x^3-61/180*b^2*c^4*d^3/x^2-37/30*b^2
*c^5*d^3/x-11/6*c^5*d^3*a*b/x-37/20*c^6*d^3*a*b*ln(c*x-1)-11/18*c^3*d^3*b^2
*arctanh(c*x)/x^3-1/15*c*d^3*a*b/x^5-3/10*c^2*d^3*a*b/x^4-11/18*c^3*d^3*a*b
/x^3-1/6*d^3*a^2/x^6-14/15*c^6*d^3*b^2*dilog(c*x)-3/2*c^2*d^3*a*b*arctanh(c
*x)/x^4-6/5*c*d^3*a*b*arctanh(c*x)/x^5-2/3*c^3*d^3*a*b*arctanh(c*x)/x^3-14/
15*c^4*d^3*a*b/x^2-1/3*c^3*d^3*b^2*arctanh(c*x)^2/x^3-14/15*c^4*d^3*b^2*arc
tanh(c*x)/x^2-3/4*c^2*d^3*b^2*arctanh(c*x)^2/x^4-1/60*c^6*d^3*b^2*arctanh(c
*x)*ln(c*x+1)-14/15*c^6*d^3*b^2*dilog(c*x+1)+113/45*c^6*d^3*b^2*ln(c*x)+1/2
40*c^6*d^3*b^2*ln(c*x+1)^2-23/36*c^6*d^3*b^2*ln(c*x+1)-1/6*d^3*b^2*arctanh(
c*x)^2/x^6-3/4*c^2*d^3*a^2/x^4-3/5*c*d^3*a^2/x^5-1/3*c^3*d^3*a^2/x^3+14/15*
c^6*d^3*b^2*dilog(1/2+1/2*c*x)-337/180*c^6*d^3*b^2*ln(c*x-1)-37/80*c^6*d^3*
b^2*ln(c*x-1)^2-1/120*c^6*d^3*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/120*c^6*d^3*
b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+28/15*c^6*d^3*b^2*arctanh(c*x)*ln(c*x)
+37/40*c^6*d^3*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-37/20*c^6*d^3*b^2*arctanh(c*x)
*ln(c*x-1)-1/15*c*d^3*b^2*arctanh(c*x)/x^5-11/6*c^5*d^3*b^2*arctanh(c*x)/x-
1/3*d^3*a*b*arctanh(c*x)/x^6-3/10*c^2*d^3*b^2*arctanh(c*x)/x^4-3/5*c*d^3*b^
2*arctanh(c*x)^2/x^5+28/15*c^6*d^3*a*b*ln(c*x)-14/15*c^6*d^3*b^2*ln(c*x)*ln
(c*x+1)-1/60*c^6*d^3*a*b*ln(c*x+1)
```

Maxima [B] time = 3.17225, size = 1297, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="maxima")
```

```
[Out] -14/15*(log(c*x + 1)*log(-1/2*c*x + 1/2) + dilog(1/2*c*x + 1/2))*b^2*c^6*d^
3 - 14/15*(log(c*x)*log(-c*x + 1) + dilog(-c*x + 1))*b^2*c^6*d^3 + 14/15*(l
og(c*x + 1)*log(-c*x) + dilog(c*x + 1))*b^2*c^6*d^3 - 23/60*b^2*c^6*d^3*log
(c*x + 1) - 97/60*b^2*c^6*d^3*log(c*x - 1) + 2*b^2*c^6*d^3*log(x) - 1/3*((c
^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)*a*b*c^3
*d^3 + 1/4*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^
3)*c - 6*arctanh(c*x)/x^4)*a*b*c^2*d^3 - 3/10*((2*c^4*log(c^2*x^2 - 1) - 2*
c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c + 4*arctanh(c*x)/x^5)*a*b*c*d^3 + 1/9
0*((15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*c^2*x^2 +
 3)/x^5)*c - 30*arctanh(c*x)/x^6)*a*b*d^3 + 1/360*((184*c^4*log(x) - (15*c^
4*x^4*log(c*x + 1)^2 + 15*c^4*x^4*log(c*x - 1)^2 + 92*c^4*x^4*log(c*x - 1)
+ 32*c^2*x^2 - 2*(15*c^4*x^4*log(c*x - 1) - 46*c^4*x^4)*log(c*x + 1) + 6)/x
```

```
^4)*c^2 + 4*(15*c^5*log(c*x + 1) - 15*c^5*log(c*x - 1) - 2*(15*c^4*x^4 + 5*
c^2*x^2 + 3)/x^5)*c*arctanh(c*x))*b^2*d^3 - 1/3*a^2*c^3*d^3/x^3 - 3/4*a^2*c
^2*d^3/x^4 - 3/5*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctanh(c*x)^2/x^6 - 1/6*a^2*d
^3/x^6 - 1/240*(296*b^2*c^5*d^3*x^4 + 60*b^2*c^4*d^3*x^3 + 24*b^2*c^3*d^3*x
^2 + (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x + 36*b^2*c
*d^3)*log(c*x + 1)^2 - (101*b^2*c^6*d^3*x^5 - 20*b^2*c^3*d^3*x^2 - 45*b^2*c
^2*d^3*x - 36*b^2*c*d^3)*log(-c*x + 1)^2 + 4*(45*b^2*c^5*d^3*x^4 + 28*b^2*c
^4*d^3*x^3 + 15*b^2*c^3*d^3*x^2 + 9*b^2*c^2*d^3*x)*log(c*x + 1) - 2*(90*b^2
*c^5*d^3*x^4 + 56*b^2*c^4*d^3*x^3 + 30*b^2*c^3*d^3*x^2 + 18*b^2*c^2*d^3*x +
 (11*b^2*c^6*d^3*x^5 + 20*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x + 36*b^2*c*d^3
)*log(c*x + 1))*log(-c*x + 1))/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2c^3d^3x^3 + 3\,a^2c^2d^3x^2 + 3\,a^2cd^3x + a^2d^3 + \left(b^2c^3d^3x^3 + 3\,b^2c^2d^3x^2 + 3\,b^2cd^3x + b^2d^3\right)\operatorname{artanh}\left(cx\right)^2 + 2\left(abc^3d^3\right.}{x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^3*d^3*x^3 + 3*a^2*c^2*d^3*x^2 + 3*a^2*c*d^3*x + a^2*d^3 + (
b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 + 3*b^2*c*d^3*x + b^2*d^3)*arctanh(c*x)
^2 + 2*(a*b*c^3*d^3*x^3 + 3*a*b*c^2*d^3*x^2 + 3*a*b*c*d^3*x + a*b*d^3)*arct
anh(c*x))/x^7, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$d^3\left(\int \frac{a^2}{x^7}\,dx + \int \frac{3a^2c}{x^6}\,dx + \int \frac{3a^2c^2}{x^5}\,dx + \int \frac{a^2c^3}{x^4}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{x^7}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{x^7}\,dx + \int \frac{3b^2c\operatorname{atanh}^2(}{x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)**3*(a+b*atanh(c*x))**2/x**7,x)
```

```
[Out] d**3*(Integral(a**2/x**7, x) + Integral(3*a**2*c/x**6, x) + Integral(3*a**2
*c**2/x**5, x) + Integral(a**2*c**3/x**4, x) + Integral(b**2*atanh(c*x)**2/
x**7, x) + Integral(2*a*b*atanh(c*x)/x**7, x) + Integral(3*b**2*c*atanh(c*x
)**2/x**6, x) + Integral(3*b**2*c**2*atanh(c*x)**2/x**5, x) + Integral(b**2
*c**3*atanh(c*x)**2/x**4, x) + Integral(6*a*b*c*atanh(c*x)/x**6, x) + Integ
ral(6*a*b*c**2*atanh(c*x)/x**5, x) + Integral(2*a*b*c**3*atanh(c*x)/x**4, x
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdx + d)^3 (b\operatorname{artanh}(cx) + a)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*x+d)^3*(a+b*arctanh(c*x))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*d*x + d)^3*(b*arctanh(c*x) + a)^2/x^7, x)
```

3.95 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=329

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^4d}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{x^2\left(a+b\tanh\right.}{2c^2d}$$

```
[Out] -((a*b*x)/(c^3*d)) + (b^2*x)/(3*c^3*d) - (b^2*ArcTanh[c*x])/(3*c^4*d) - (b^
2*x*ArcTanh[c*x])/(c^3*d) + (b*x^2*(a + b*ArcTanh[c*x]))/(3*c^2*d) + (11*(a
 + b*ArcTanh[c*x])^2)/(6*c^4*d) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d) - (x^2
*(a + b*ArcTanh[c*x])^2)/(2*c^2*d) + (x^3*(a + b*ArcTanh[c*x])^2)/(3*c*d) -
 (8*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^4*d) + ((a + b*ArcTanh[c*
x])^2*Log[2/(1 + c*x)])/(c^4*d) - (b^2*Log[1 - c^2*x^2])/(2*c^4*d) - (4*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^4*d) - (b*(a + b*ArcTanh[c*x])*PolyLog[2
, 1 - 2/(1 + c*x)])/(c^4*d) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d)
```

Rubi [A] time = 0.84035, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5930, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 5910, 260, 5948, 6056, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d}-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^4d}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d}-\frac{x^2\left(a+b\tanh\right.}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```


```
[Out] -((a*b*x)/(c^3*d)) + (b^2*x)/(3*c^3*d) - (b^2*ArcTanh[c*x])/(3*c^4*d) - (b^
2*x*ArcTanh[c*x])/(c^3*d) + (b*x^2*(a + b*ArcTanh[c*x]))/(3*c^2*d) + (11*(a
 + b*ArcTanh[c*x])^2)/(6*c^4*d) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d) - (x^2
*(a + b*ArcTanh[c*x])^2)/(2*c^2*d) + (x^3*(a + b*ArcTanh[c*x])^2)/(3*c*d) -
 (8*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^4*d) + ((a + b*ArcTanh[c*
x])^2*Log[2/(1 + c*x)])/(c^4*d) - (b^2*Log[1 - c^2*x^2])/(2*c^4*d) - (4*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^4*d) - (b*(a + b*ArcTanh[c*x])*PolyLog[2
, 1 - 2/(1 + c*x)])/(c^4*d) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d)
```


Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx &= -\frac{\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c}+\frac{\int x^2\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}\\
&= \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd}+\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^2}-\frac{(2b)\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{3d}-\frac{\int x\left(a+\cdots\right.}{}\\
&= -\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d}+\frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3cd}-\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^3}+\frac{\int \left(a+b\,\mathrm{t}\cdots\right.}{}\\
&= \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c^4d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d}-\frac{x^2\left(a+b\,\mathrm{ta}\cdots\right.}{2c^2\cdots}\\
&= -\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a+b\tanh^{-1}(cx)\right)^2}{6c^4d}+\frac{x\left(a+b\tanh^{-1}(c\cdots\right.}{c^3d}\\
&= -\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a+b\cdots\right.}{}\\
&= -\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a+b\cdots\right.}{}\\
&= -\frac{abx}{c^3d}+\frac{b^2x}{3c^3d}-\frac{b^2\tanh^{-1}(cx)}{3c^4d}-\frac{b^2x\tanh^{-1}(cx)}{c^3d}+\frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^2d}+\frac{11\left(a+b\cdots\right.}{}
\end{aligned}
$$

Mathematica [A] time = 0.863035, size = 347, normalized size = 1.05

$$
\frac{ab\left(-3\,\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-8\log\left(\frac{1}{\sqrt{1-c^2x^2}}\right)+\left(1-c^2x^2\right)\left(-2cx\tanh^{-1}(cx)+3\tanh^{-1}(cx)-1\right)-3cx+8cx\,\mathrm{ta}\cdots\right.}{3c^4d}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a^2*x)/(c^3*d) - (a^2*x^2)/(2*c^2*d) + (a^2*x^3)/(3*c*d) - (a^2*Log[1 + c*
x])/(c^4*d) + (a*b*(-3*c*x + 8*c*x*ArcTanh[c*x] + (1 - c^2*x^2)*(-1 + 3*Arc
Tanh[c*x] - 2*c*x*ArcTanh[c*x]) + 6*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x]
```

```
)] - 8*Log[1/Sqrt[1 - c^2*x^2]] - 3*PolyLog[2, -E^(-2*ArcTanh[c*x])]))/(3*c
^4*d) + (b^2*(2*c*x - 6*c*x*ArcTanh[c*x] - 2*(1 - c^2*x^2)*ArcTanh[c*x] - 8
*ArcTanh[c*x]^2 + 8*c*x*ArcTanh[c*x]^2 + 3*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 2
*c*x*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 16*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*Log[1/Sqrt[1 - c^
2*x^2]] + (8 - 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 3*PolyLog
[3, -E^(-2*ArcTanh[c*x])]))/(6*c^4*d)
```

Maple [C] time = 0.964, size = 1298, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] 1/3*b^2*x/c^3/d-4/3*b^2*arctanh(c*x)/d/c^4+1/c^3*b^2/d*arctanh(c*x)^2*x+1/3
/c^2*b^2/d*arctanh(c*x)*x^2+1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)^3*arctanh(c*x)^2+1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)
^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2/c^2*a^2/d*x^2+1/c^3*a^2/d*x+1/3/c*
a^2/d*x^3+11/6/c^4*b^2/d*arctanh(c*x)^2-8/3/c^4*b^2/d*dilog(1+I*(c*x+1)/(-c
^2*x^2+1)^(1/2))-8/3/c^4*b^2/d*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2/c^
4*b^2/d*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/c^4*a^2/d*ln(c*x+1)-2/3/c^4*b^
2/d*arctanh(c*x)^3+1/c^4*b^2/d*ln((c*x+1)^2/(-c^2*x^2+1)+1)-a*b*x/c^3/d-b^2
*x*arctanh(c*x)/c^3/d-1/3/c^4*b^2/d+1/3/c^2*a*b/d*x^2+1/3/c*b^2/d*x^3*arcta
nh(c*x)^2+1/c^4*b^2/d*arctanh(c*x)^2*ln(2)+1/c^4*a*b/d*dilog(1/2+1/2*c*x)+1
/2/c^4*a*b/d*ln(c*x+1)^2+5/6/c^4*a*b/d*ln(c*x-1)+11/6/c^4*a*b/d*ln(c*x+1)+2
/c^4*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-8/3/c^4*b^2/d*arct
anh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/c^4*b^2/d*arctanh(c*x)*polylo
g(2,-(c*x+1)^2/(-c^2*x^2+1))-1/c^4*b^2/d*arctanh(c*x)^2*ln(c*x+1)-8/3/c^4*b
^2/d*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2/c^2*b^2/d*arctanh(
c*x)^2*x^2+I/c^4*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2*arctanh(c*x)^2+1/2*I/c^4*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2
*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arct
anh(c*x)^2+1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(
c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-1/2*I/c^4*b^2/d*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arcta
nh(c*x)^2-4/3/c^4*a*b/d+2/3/c*a*b/d*x^3*arctanh(c*x)-1/c^2*a*b/d*arctanh(c*
x)*x^2+2/c^3*a*b/d*arctanh(c*x)*x-2/c^4*a*b/d*arctanh(c*x)*ln(c*x+1)-1/c^4*
a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/c^4*a*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*
x)-1/2*I/c^4*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctan
h(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}a^2\left(\frac{2c^2x^3 - 3cx^2 + 6x}{c^3d} - \frac{6\log(cx+1)}{c^4d}\right) + \frac{\left(2b^2c^3x^3 - 3b^2c^2x^2 + 6b^2cx - 6b^2\log(cx+1)\right)\log(-cx+1)^2}{24c^4d} - \int -\frac{3\left(b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*a^2*((2*c^2*x^3 - 3*c*x^2 + 6*x)/(c^3*d) - 6*log(c*x + 1)/(c^4*d)) + 1/
24*(2*b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 6*b^2*c*x - 6*b^2*log(c*x + 1))*log(-c*
```

```
x + 1)^2/(c^4*d) - integrate(-1/12*(3*(b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x +
 1)^2 + 12*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (3*b^2*c^2*x^2 + 2*(6
*a*b*c^4 + b^2*c^4)*x^4 + 6*b^2*c*x - (12*a*b*c^3 + b^2*c^3)*x^3 + 6*(b^2*c
^4*x^4 - b^2*c^3*x^3 - b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d*x
^2 - c^3*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^3\,\mathrm{artanh}\,(cx)^2 + 2\,abx^3\,\mathrm{artanh}\,(cx) + a^2x^3}{cdx+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c*d*x
 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^3}{cx+1}\,dx + \int \frac{b^2x^3\,\mathrm{atanh}^2\,(cx)}{cx+1}\,dx + \int \frac{2abx^3\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x**3/(c*x + 1), x) + Integral(b**2*x**3*atanh(c*x)**2/(c*x +
 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2x^3}{cdx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d), x)
```

3.96 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=247

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d}+\frac{abx}{c^2d}-\frac{x\left(a+b\tanh\right.}{c^2d}$$

```
[Out] (a*b*x)/(c^2*d) + (b^2*x*ArcTanh[c*x])/(c^2*d) - (3*(a + b*ArcTanh[c*x])^2)
/(2*c^3*d) - (x*(a + b*ArcTanh[c*x])^2)/(c^2*d) + (x^2*(a + b*ArcTanh[c*x])
^2)/(2*c*d) + (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d) - ((a + b
*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d) + (b^2*Log[1 - c^2*x^2])/(2*c^3*
d) + (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^3*d) + (b*(a + b*ArcTanh[c*x])*Po
lyLog[2, 1 - 2/(1 + c*x)])/(c^3*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c
^3*d)
```

Rubi [A] time = 0.533158, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5930, 5916, 5980, 5910, 260, 5948, 5984, 5918, 2402, 2315, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d}+\frac{abx}{c^2d}-\frac{x\left(a+b\tanh\right.}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a*b*x)/(c^2*d) + (b^2*x*ArcTanh[c*x])/(c^2*d) - (3*(a + b*ArcTanh[c*x])^2)
/(2*c^3*d) - (x*(a + b*ArcTanh[c*x])^2)/(c^2*d) + (x^2*(a + b*ArcTanh[c*x])
^2)/(2*c*d) + (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d) - ((a + b
*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d) + (b^2*Log[1 - c^2*x^2])/(2*c^3*
d) + (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^3*d) + (b*(a + b*ArcTanh[c*x])*Po
lyLog[2, 1 - 2/(1 + c*x)])/(c^3*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c
^3*d)
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```

```
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2 \left(a+b \tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c} + \frac{\int x\left(a+b\tanh^{-1}(cx)\right)^2 dx}{cd}$$

$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c^2} - \frac{b\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d} - \frac{\int \left(a+b\tanh \cdots\right.}{c^2 d}$$

$$= -\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^3 d} + \frac{b\int \cdots}{}$$

$$= \frac{abx}{c^2 d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2cd} - \frac{\left(a+b \cdots\right.}{}$$

$$= \frac{abx}{c^2 d} + \frac{b^2 x\tanh^{-1}(cx)}{c^2 d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2\left(a+b\tanh \cdots\right.}{2cd}$$

$$= \frac{abx}{c^2 d} + \frac{b^2 x\tanh^{-1}(cx)}{c^2 d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2\left(a+b\tanh \cdots\right.}{2cd}$$

$$= \frac{abx}{c^2 d} + \frac{b^2 x\tanh^{-1}(cx)}{c^2 d} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3 d} - \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2 d} + \frac{x^2\left(a+b\tanh \cdots\right.}{2cd}$$

Mathematica [A] time = 0.503607, size = 260, normalized size = 1.05

$$2b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)-b\right)+b^2\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+a^2c^2x^2-2a^2cx+2a^2\log(cx+1)\cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (-2*a^2*c*x + 2*a*b*c*x + a^2*c^2*x^2 - 2*a*b*ArcTanh[c*x] - 4*a*b*c*x*ArcT
anh[c*x] + 2*b^2*c*x*ArcTanh[c*x] + 2*a*b*c^2*x^2*ArcTanh[c*x] + b^2*ArcTan
h[c*x]^2 - 2*b^2*c*x*ArcTanh[c*x]^2 + b^2*c^2*x^2*ArcTanh[c*x]^2 - 4*a*b*Ar
cTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 4*b^2*ArcTanh[c*x]*Log[1 + E^(-2*
ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*
Log[1 + c*x] - 2*a*b*Log[1 - c^2*x^2] + b^2*Log[1 - c^2*x^2] + 2*b*(a - b +
 b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*A
rcTanh[c*x])])/(2*c^3*d)
```

Maple [C] time = 0.623, size = 1192, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] 1/2*I/c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-I/c^3*b^2/d*Pi*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-1/2*I
```

```
/c^3*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^
2-1))*arctanh(c*x)^2-1/2*I/c^3*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-2
/c^3*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/c^3*b^2/d*arctan
h(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/c^3*b^2/d*arctanh(c*x)*polylog(
2,-(c*x+1)^2/(-c^2*x^2+1))+1/c^3*b^2/d*arctanh(c*x)^2*ln(c*x+1)+2/c^3*b^2/d
*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/c^3*b^2/d*arctanh(c*x)^2
*ln(2)-1/c^3*a*b/d*dilog(1/2+1/2*c*x)+1/2/c*b^2/d*arctanh(c*x)^2*x^2-1/c^2*
b^2/d*arctanh(c*x)^2*x-1/2*I/c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I/c^3*b^2/d*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/2*I/c^3*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*
x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x
+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2+a*b*x/c^2/d+b^2*x*arctanh(c*x)/c^2/d+
1/c*a*b/d*arctanh(c*x)*x^2-2/c^2*a*b/d*arctanh(c*x)*x+2/c^3*a*b/d*arctanh(c
*x)*ln(c*x+1)+1/c^3*a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c^3*a*b/d*ln(-1/2*c*
x+1/2)*ln(1/2+1/2*c*x)+1/c^3*a*b/d+1/2/c*a^2/d*x^2-1/c^2*a^2/d*x+1/c^3*a^2/
d*ln(c*x+1)+2/3/c^3*b^2/d*arctanh(c*x)^3+1/c^3*b^2/d*arctanh(c*x)-1/c^3*b^2
/d*ln((c*x+1)^2/(-c^2*x^2+1)+1)-3/2/c^3*b^2/d*arctanh(c*x)^2+2/c^3*b^2/d*di
log(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+2/c^3*b^2/d*dilog(1-I*(c*x+1)/(-c^2*x^2
+1)^(1/2))+1/2/c^3*b^2/d*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/2/c^3*a*b/d*l
n(c*x+1)^2-1/2/c^3*a*b/d*ln(c*x-1)-3/2/c^3*a*b/d*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{cx^2-2x}{c^2d}+\frac{2\log(cx+1)}{c^3d}\right)+\frac{\left(b^2c^2x^2-2b^2cx+2b^2\log(cx+1)\right)\log(-cx+1)^2}{8c^3d}-\int-\frac{\left(b^2c^3x^3-b^2c^2x^2\right)\log(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((c*x^2 - 2*x)/(c^2*d) + 2*log(c*x + 1)/(c^3*d)) + 1/8*(b^2*c^2*x^2
 - 2*b^2*c*x + 2*b^2*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d) - integrate(-1/4
*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2
)*log(c*x + 1) + (2*b^2*c*x - (4*a*b*c^3 + b^2*c^3)*x^3 + (4*a*b*c^2 + b^2*
c^2)*x^2 - 2*(b^2*c^3*x^3 - b^2*c^2*x^2 + b^2*c*x + b^2)*log(c*x + 1))*log(
-c*x + 1))/(c^4*d*x^2 - c^2*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\,\text{artanh}(cx)^2+2\,abx^2\,\text{artanh}(cx)+a^2x^2}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c*d*x
 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^2}{cx+1}\,dx + \int \frac{b^2x^2\,\mathrm{atanh}^2\,(cx)}{cx+1}\,dx + \int \frac{2abx^2\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x**2/(c*x + 1), x) + Integral(b**2*x**2*atanh(c*x)**2/(c*x +
 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2 x^2}{cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d), x)
```

3.97 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=172

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{\left(a+b\tanh^{-1}(\right.}{c^2d}$$

```
[Out] (a + b*ArcTanh[c*x])^2/(c^2*d) + (x*(a + b*ArcTanh[c*x])^2)/(c*d) - (2*b*(a
 + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^2*d) + ((a + b*ArcTanh[c*x])^2*Log[
2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^2*d) - (b*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[3, 1 -
 2/(1 + c*x)])/(2*c^2*d)
```

Rubi [A] time = 0.30629, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {5930, 5910, 5984, 5918, 2402, 2315, 5948, 6056, 6610}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^2d}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d}+\frac{\left(a+b\tanh^{-1}(\right.}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (a + b*ArcTanh[c*x])^2/(c^2*d) + (x*(a + b*ArcTanh[c*x])^2)/(c*d) - (2*b*(a
 + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^2*d) + ((a + b*ArcTanh[c*x])^2*Log[
2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^2*d) - (b*(a +
 b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^2*d) - (b^2*PolyLog[3, 1 -
 2/(1 + c*x)])/(2*c^2*d)
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx}{c}+\frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{cd}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d}-\frac{(2b)\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d}-\frac{(2b)}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d}-\frac{b\left(a+b\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}+\frac{\left(a+b\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}+\frac{\left(a+b\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{cd}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{c^2d}+\frac{\left(a+b\right.}{}$$

Mathematica [A] time = 0.482965, size = 140, normalized size = 0.81

$$\frac{-2b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)-b\right)-b^2\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+2a\left(acx-a\log(cx+1)+b\text{l}\right.}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x),x]
```

```
[Out] (2*b^2*ArcTanh[c*x]^2*(-1 + c*x + Log[1 + E^(-2*ArcTanh[c*x])]) + 4*b*ArcTa
nh[c*x]*(a*c*x + (a - b)*Log[1 + E^(-2*ArcTanh[c*x])]) + 2*a*(a*c*x - a*Log
[1 + c*x] + b*Log[1 - c^2*x^2]) - 2*b*(a - b + b*ArcTanh[c*x])*PolyLog[2, -
E^(-2*ArcTanh[c*x])] - b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2*c^2*d)
```

Maple [C] time = 0.507, size = 5361, normalized size = 31.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{x}{cd}-\frac{\log(cx+1)}{c^2d}\right)+\frac{\left(b^2cx-b^2\log(cx+1)\right)\log(-cx+1)^2}{4\,c^2d}-\int-\frac{\left(b^2c^2x^2-b^2cx\right)\log(cx+1)^2+4\left(abc^2x^2-abcx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(x/(c*d) - log(c*x + 1)/(c^2*d)) + 1/4*(b^2*c*x - b^2*log(c*x + 1))*log
(-c*x + 1)^2/(c^2*d) - integrate(-1/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)
^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(c*x + 1) - 2*((2*a*b*c^2 + b^2*c^2)*x^2
- (2*a*b*c - b^2*c)*x + (b^2*c^2*x^2 - 2*b^2*c*x - b^2)*log(c*x + 1))*log(-
c*x + 1))/(c^3*d*x^2 - c*d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x\,\text{artanh}(cx)^2+2\,abx\,\text{artanh}(cx)+a^2x}{cdx+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c*d*x + d),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x}{cx+1}\,dx + \int \frac{b^2x\,\mathrm{atanh}^2\,(cx)}{cx+1}\,dx + \int \frac{2abx\,\mathrm{atanh}\,(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2*x/(c*x + 1), x) + Integral(b**2*x*atanh(c*x)**2/(c*x + 1), x
) + Integral(2*a*b*x*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2 x}{cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d), x)
```

3.98 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx$

Optimal. Leaf size=84

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{cd}$$

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c*d)) + (b*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 + c*x)])/(c*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2
*c*d)
```

Rubi [A] time = 0.1457, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5918, 5948, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{cd}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2cd}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c*d)) + (b*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 + c*x)])/(c*d) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2
*c*d)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+cdx}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{(2b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}dx}{d}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{cd} - \frac{b^2\int \frac{\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{1-c^2x^2}dx}{d}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{cd} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\mathrm{Li}_2\left(1-\frac{2}{1+cx}\right)}{cd} + \frac{b^2\mathrm{Li}_3\left(1-\frac{2}{1+cx}\right)}{2cd}$$

Mathematica [A] time = 0.203521, size = 102, normalized size = 1.21

$$\frac{2b\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)+b^2\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+2a^2\log(cx+1)-4ab\tanh^{-1}(cx)\log}{2cd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x),x]
```

```
[Out] (-4*a*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 2*b^2*ArcTanh[c*x]^2*Lo
g[1 + E^(-2*ArcTanh[c*x])] + 2*a^2*Log[1 + c*x] + 2*b*(a + b*ArcTanh[c*x])*
PolyLog[2, -E^(-2*ArcTanh[c*x])] + b^2*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(2
*c*d)
```

Maple [C] time = 0.266, size = 822, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d),x)
```

```
[Out] 1/c*a^2/d*ln(c*x+1)+1/c*b^2/d*arctanh(c*x)^2*ln(c*x+1)-2/c*b^2/d*arctanh(c*
x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+2/3/c*b^2/d*arctanh(c*x)^3+1/2*I/c*b^2/
d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/
(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I/c*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+1/2*I/c*b^2/d*Pi*
csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I/c*b^2/d
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-1/2*I/c*b^2/d*Pi*csgn(I*
(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I/c*
b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((
c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-I/c*b^2/d*Pi*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-1/c*b^2/d*ar
ctanh(c*x)^2*ln(2)-1/c*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1)
)+1/2/c*b^2/d*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+2/c*a*b/d*arctanh(c*x)*ln(
c*x+1)+1/c*a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c*a*b/d*ln(-1/2*c*x+1/2)*ln(1
/2+1/2*c*x)-1/c*a*b/d*dilog(1/2+1/2*c*x)-1/2/c*a*b/d*ln(c*x+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2\log(cx+1)\log(-cx+1)^2}{4\,cd} + \frac{a^2\log(cdx+d)}{cd} - \int -\frac{\left(b^2cx-b^2\right)\log(cx+1)^2+4\,(abcx-ab)\log(cx+1)-4\left(b^2cx\log\right.}{4\left(c^2dx^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/(c*d) + a^2*log(c*d*x + d)/(c*d) - int
egrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1
) - 4*(b^2*c*x*log(c*x + 1) + a*b*c*x - a*b)*log(-c*x + 1))/(c^2*d*x^2 - d)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab\operatorname{artanh}(cx) + a^2}{cdx + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{cx+1}\,dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{cx+1}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx+1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c*x + 1), x) +
Integral(2*a*b*atanh(c*x)/(c*x + 1), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2}{cdx + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(c*d*x + d), x)
```

3.99 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx$

Optimal. Leaf size=77

$$-\frac{b\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}$$

```
[Out] ((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (b*(a + b*ArcTanh[c*x])*P
olyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*PolyLog[3, -1 + 2/(1 + c*x)])/(2*d)
```

Rubi [A] time = 0.165733, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5932, 5948, 6056, 6610}

$$-\frac{b\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{\log\left(2-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

```
[Out] ((a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (b*(a + b*ArcTanh[c*x])*P
olyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*PolyLog[3, -1 + 2/(1 + c*x)])/(2*d)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx = \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{(2bc)\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{1-c^2x^2}dx}{d}$$
$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{d} + \frac{\left(b^2c\right)\int \frac{\text{Li}_2(}{}}{}$$
$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} - \frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(-1+\frac{2}{1+cx}\right)}{d} - \frac{b^2\text{Li}_3\left(-1+\right.}{2d}$$

Mathematica [C] time = 0.374449, size = 132, normalized size = 1.71

$$\frac{ab\left(2\tanh^{-1}(cx)\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)\right)+b^2\left(\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right)-\frac{1}{2}\text{Po}\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)),x]
```

```
[Out] (a^2*Log[c*x] - a^2*Log[1 + c*x] + a*b*(2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTan
h[c*x])] - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^2*((I/24)*Pi^3 - (2*ArcTanh
[c*x]^3)/3 + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*Poly
Log[2, E^(2*ArcTanh[c*x])] - PolyLog[3, E^(2*ArcTanh[c*x])]/2))/d
```

Maple [C] time = 0.306, size = 1389, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d),x)
```

```
[Out] 2*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*
ln(2)-b^2/d*arctanh(c*x)^2*ln(c*x+1)+a*b/d*dilog(1/2+1/2*c*x)+1/2*a*b/d*ln(
c*x+1)^2+a^2/d*ln(c*x)-2/3*b^2/d*arctanh(c*x)^3+b^2/d*arctanh(c*x)^2*ln(c*x
)+b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x
)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/
(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/
2))-b^2/d*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-a*b/d*dilog(c*x)-a*b/
d*dilog(c*x+1)-2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*polylo
g(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-a^2/d*ln(c*x+1)+1/2*I*b^2/d*Pi*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^2/d*Pi
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2+1/2*I*b^2/d*
Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^
2+1/2*I*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/2*I*b^2/d
*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh
(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*b^2/d*P
i*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+I*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+1/2*I*b^2/d*Pi*c
```

```
sgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(
-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)
^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-1/2*I*b^2/d*Pi*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*arctanh(c*x)^2-2*a*b/d*arctanh(c*x)*ln(c*x+1)-a*b/d*ln(-1/2*c*x+1/
2)*ln(c*x+1)+a*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-a*b/d*ln(c*x)*ln(c*x+1)
+2*a*b/d*arctanh(c*x)*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2\log(cx+1)\log(-cx+1)^2}{4d}-a^2\left(\frac{\log(cx+1)}{d}-\frac{\log(x)}{d}\right)+\int\frac{\left(b^2cx-b^2\right)\log(cx+1)^2+4\,(abcx-ab)\log(cx+1)-2}{4\left(c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*log(c*x + 1)*log(-c*x + 1)^2/d - a^2*(log(c*x + 1)/d - log(x)/d) +
 integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x
+ 1) - 2*(2*a*b*c*x - 2*a*b - (b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x +
1))/(c^2*d*x^3 - d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\,(cx)^2+2\,ab\,\mathrm{artanh}\,(cx)+a^2}{cdx^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^2 + d*x), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{cx^2+x}\,dx+\int\frac{b^2\,\mathrm{atanh}^2\,(cx)}{cx^2+x}\,dx+\int\frac{2ab\,\mathrm{atanh}\,(cx)}{cx^2+x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2/(c*x**2 + x),
 x) + Integral(2*a*b*atanh(c*x)/(c*x**2 + x), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\mathrm{artanh}\,(cx)+a)^2}{(cdx+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x), x)
```

3.100 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx$

Optimal. Leaf size=162

$$\frac{bc\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2c\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}+\frac{b^2c\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{c\left(a+b\tanh^{-1}(\right.}{d}$$

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) + (2*b*c*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (c*(a + b*ArcTanh[c*x])^2*Log[2 -
2/(1 + c*x)])/d - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b*c*(a + b*ArcT
anh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c*PolyLog[3, -1 + 2/(1 + c
*x)])/(2*d)
```

Rubi [A] time = 0.412051, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5934, 5916, 5988, 5932, 2447, 5948, 6056, 6610}

$$\frac{bc\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{b^2c\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}+\frac{b^2c\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{c\left(a+b\tanh^{-1}(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) + (2*b*c*(a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d - (c*(a + b*ArcTanh[c*x])^2*Log[2 -
2/(1 + c*x)])/d - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b*c*(a + b*ArcT
anh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c*PolyLog[3, -1 + 2/(1 + c
*x)])/(2*d)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d}+\frac{(2bc)\int \frac{a+b\tanh^{-1}(cx)}{x\left(1-c^2x^2\right)}\,dx}{d}+$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}-\frac{c\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d}+$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}-$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx}+\frac{2bc\left(a+b\tanh^{-1}(cx)\right)\log\left(2-\frac{2}{1+cx}\right)}{d}-$$

Mathematica [C] time = 0.639727, size = 225, normalized size = 1.39

$$\frac{ab\left(cx\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-2\tanh^{-1}(cx)\left(cx\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+1\right)\right)}{x}+b^2c\left(-\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{2\tanh^{-1}(cx)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)),x]
```

```
[Out] (-(a^2/x) - a^2*c*Log[x] + a^2*c*Log[1 + c*x] + (a*b*(-2*ArcTanh[c*x]*(1 +
c*x*Log[1 - E^(-2*ArcTanh[c*x])]) + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]] + c*
x*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x + b^2*c*((-I/24)*Pi^3 + ArcTanh[c*x]^
2 - ArcTanh[c*x]^2/(c*x) + (2*ArcTanh[c*x]^3)/3 + 2*ArcTanh[c*x]*Log[1 - E^
(-2*ArcTanh[c*x])] - ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - PolyLog[2
, E^(-2*ArcTanh[c*x])] - ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + Poly
Log[3, E^(2*ArcTanh[c*x])]/2))/d
```

Maple [C] time = 0.685, size = 7232, normalized size = 44.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{c\log(cx+1)}{d} - \frac{c\log(x)}{d} - \frac{1}{dx}\right) + \frac{\left(b^2cx\log(cx+1) - b^2\right)\log(-cx+1)^2}{4\,dx} - \int -\frac{\left(b^2cx - b^2\right)\log(cx+1)^2 + 4\,(abcx - a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(c*log(c*x + 1)/d - c*log(x)/d - 1/(d*x)) + 1/4*(b^2*c*x*log(c*x + 1) -
 b^2)*log(-c*x + 1)^2/(d*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1)^
2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(b^2*c^2*x^2 + 2*a*b - (2*a*b*c - b^
2*c)*x - (b^2*c^3*x^3 + b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x
 + 1))/(c^2*d*x^4 - d*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}(cx)^2 + 2\,ab\,\text{artanh}(cx) + a^2}{cdx^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^3 + d*x^2),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{cx^3+x^2}\,dx + \int \frac{b^2\operatorname{atanh}^2(cx)}{cx^3+x^2}\,dx + \int \frac{2ab\operatorname{atanh}(cx)}{cx^3+x^2}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)**2/(c*x**3 +
x**2), x) + Integral(2*a*b*atanh(c*x)/(c*x**3 + x**2), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2}{(cdx + d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^2), x)
```

3.101 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx$

Optimal. Leaf size=250

$$-\frac{bc^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{b^2c^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}-\frac{c^2\left(a+b\tan\right.}{2d}$$

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/(d*x)) - (c^2*(a + b*ArcTanh[c*x])^2)/(2*d) -
(a + b*ArcTanh[c*x])^2/(2*d*x^2) + (c*(a + b*ArcTanh[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 - c^2*x^2])/(2*d) - (2*b*c^2*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)])/d + (c^2*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 +
c*x)])/d + (b^2*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b*c^2*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*c^2*PolyLog[3, -1 + 2/(1 + c*
x)])/(2*d)
```

Rubi [A] time = 0.634345, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5934, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 6056, 6610}

$$-\frac{bc^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{b^2c^2\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{d}-\frac{b^2c^2\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}-\frac{c^2\left(a+b\tan\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

```
[Out] -((b*c*(a + b*ArcTanh[c*x]))/(d*x)) - (c^2*(a + b*ArcTanh[c*x])^2)/(2*d) -
(a + b*ArcTanh[c*x])^2/(2*d*x^2) + (c*(a + b*ArcTanh[c*x])^2)/(d*x) + (b^2*
c^2*Log[x])/d - (b^2*c^2*Log[1 - c^2*x^2])/(2*d) - (2*b*c^2*(a + b*ArcTanh[
c*x])*Log[2 - 2/(1 + c*x)])/d + (c^2*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 +
c*x)])/d + (b^2*c^2*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b*c^2*(a + b*ArcTanh
[c*x])*PolyLog[2, -1 + 2/(1 + c*x)])/d - (b^2*c^2*PolyLog[3, -1 + 2/(1 + c*
x)])/(2*d)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

```
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
```

```
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx\right) + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} + c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d} + \frac{(bc)\int \frac{a+b\,\mathrm{ta}}{x^2(}}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{dx} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(2-\frac{2}{1+cx}\right)}{d} + \frac{(b}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(c\right.}{dx}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(c\right.}{dx}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(c\right.}{dx}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{dx} - \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2} + \frac{c\left(a+b\tanh^{-1}(c\right.}{dx}$$

Mathematica [C] time = 1.0157, size = 317, normalized size = 1.27

$$\frac{\frac{2ab\left(-c^2x^2\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-cx\left(2cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+1\right)+\tanh^{-1}(cx)\left(c^2x^2+2c^2x^2\log\left(1-e^{-2\tanh^{-1}(cx)}\right)+2cx-1\right)\right)}{x^2} + 2b^2c^2\left(\tanh^{-1}(cx)\mathrm{PolyLo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)),x]
```

```
[Out] (-(a^2/x^2) + (2*a^2*c)/x + 2*a^2*c^2*Log[x] - 2*a^2*c^2*Log[1 + c*x] + (2*
a*b*(ArcTanh[c*x]*(-1 + 2*c*x + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-2*ArcTanh[c
*x])]) - c*x*(1 + 2*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - c^2*x^2*PolyLog[2,
E^(-2*ArcTanh[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTanh[c*x]/(c*x) -
ArcTanh[c*x]^2/2 - ArcTanh[c*x]^2/(2*c^2*x^2) + ArcTanh[c*x]^2/(c*x) - (2*A
rcTanh[c*x]^3)/3 - 2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + ArcTanh[c*
x]^2*Log[1 - E^(2*ArcTanh[c*x])] + Log[(c*x)/Sqrt[1 - c^2*x^2]] + PolyLog[2
, E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - Poly
Log[3, E^(2*ArcTanh[c*x])]/2))/(2*d)
```

Maple [C] time = 1.205, size = 1841, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x)
```

```
[Out] I*c^2*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2
-1))^2*arctanh(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))
^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-1/2*I*c^2*b^2/d*Pi*csgn(I*(
c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+
1))^2*arctanh(c*x)^2-1/2*I*c^2*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*
csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x
)^2-1/2*I*c^2*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*c^2*b^2
/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x
+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+c^2*a*b/d*dilog(1/2+1/2*c*x)+1/2*c^
2*a*b/d*ln(c*x+1)^2+1/2*c^2*a*b/d*ln(c*x-1)+3/2*c^2*a*b/d*ln(c*x+1)+c^2*b^2
/d*arctanh(c*x)^2*ln(c*x)-c*b^2/d*arctanh(c*x)/x+c*b^2/d*arctanh(c*x)^2/x-a
*b/d*arctanh(c*x)/x^2-2*c^2*b^2/d*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1
/2))+c^2*b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*b^2/d*
arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+c^2*b^2/d*arctanh(c*x)^
2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*b^2/d*arctanh(c*x)*polylog(2,(c*x+
1)/(-c^2*x^2+1)^(1/2))+1/2*I*c^2*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)
/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*I*c^2*b^2/d*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*I*c^2*b
^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-1/2*a^2/d/x^2+1/2*I*
c^2*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2
+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arcta
nh(c*x)^2-1/2*I*c^2*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))
*arctanh(c*x)^2+2*c*a*b/d*arctanh(c*x)/x-c^2*a*b/d*ln(c*x)*ln(c*x+1)+2*c^2*
a*b/d*arctanh(c*x)*ln(c*x)-2*c^2*a*b/d*arctanh(c*x)*ln(c*x+1)-c^2*a*b/d*ln(
-1/2*c*x+1/2)*ln(c*x+1)+c^2*a*b/d*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-c^2*b^2/
d*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+c*a^2/d/x-1/2*b^2/d*arctanh(c
*x)^2/x^2+c^2*b^2/d*ln((c*x+1)/(-c^2*x^2+1)^(1/2)-1)+c^2*b^2/d*ln(1+(c*x+1)
/(-c^2*x^2+1)^(1/2))-2*c^2*b^2/d*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^2*
b^2/d*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^2*b^2/d*polylog(3,(c*x+1)/(-c^2
*x^2+1)^(1/2))-2*c^2*b^2/d*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-c^2*a^2/d
*ln(c*x+1)-2/3*c^2*b^2/d*arctanh(c*x)^3-c^2*b^2/d*arctanh(c*x)+3/2*c^2*b^2/
d*arctanh(c*x)^2+c^2*a^2/d*ln(c*x)+2*c^2*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-
c^2*x^2+1)^(1/2))-c^2*b^2/d*arctanh(c*x)^2*ln(c*x+1)-c^2*a*b/d*dilog(c*x)-c
^2*a*b/d*dilog(c*x+1)-2*c^2*a*b/d*ln(c*x)-c*a*b/d/x+c^2*b^2/d*arctanh(c*x)^
2*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(\frac{2\,c^2\log\left(cx+1\right)}{d}-\frac{2\,c^2\log\left(x\right)}{d}-\frac{2\,cx-1}{dx^2}\right)a^2-\frac{\left(2\,b^2c^2x^2\log\left(cx+1\right)-2\,b^2cx+b^2\right)\log\left(-cx+1\right)^2}{8\,dx^2}+\int\frac{\left(b^2cx-b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*log(c*x + 1)/d - 2*c^2*log(x)/d - (2*c*x - 1)/(d*x^2))*a^2 - 1/
8*(2*b^2*c^2*x^2*log(c*x + 1) - 2*b^2*c*x + b^2)*log(-c*x + 1)^2/(d*x^2) +
integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x +
 1) - (2*b^2*c^3*x^3 + b^2*c^2*x^2 - 4*a*b + (4*a*b*c - b^2*c)*x - 2*(b^2*c
^4*x^4 + b^2*c^3*x^3 - b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x
```

```
^5 - d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\,(cx)^2+2\,ab\,\text{artanh}\,(cx)+a^2}{cdx^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^4 + d*x^3),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{cx^4+x^3}\,dx+\int \frac{b^2\,\text{atanh}^2\,(cx)}{cx^4+x^3}\,dx+\int \frac{2ab\,\text{atanh}\,(cx)}{cx^4+x^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)**2/(c*x**4 +
x**3), x) + Integral(2*a*b*atanh(c*x)/(c*x**4 + x**3), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{(cdx+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^3), x)
```

3.102 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4(d+cdx)}\,dx$

Optimal. Leaf size=334

$$\frac{bc^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{4b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{3d}+\frac{b^2c^3\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{5c^3\left(a+\right.}{}$$

```
[Out] -(b^2*c^2)/(3*d*x) + (b^2*c^3*ArcTanh[c*x])/(3*d) - (b*c*(a + b*ArcTanh[c*x
]))/(3*d*x^2) + (b*c^2*(a + b*ArcTanh[c*x]))/(d*x) + (5*c^3*(a + b*ArcTanh[
c*x])^2)/(6*d) - (a + b*ArcTanh[c*x])^2/(3*d*x^3) + (c*(a + b*ArcTanh[c*x])
^2)/(2*d*x^2) - (c^2*(a + b*ArcTanh[c*x])^2)/(d*x) - (b^2*c^3*Log[x])/d + (
b^2*c^3*Log[1 - c^2*x^2])/(2*d) + (8*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(
1 + c*x)])/(3*d) - (c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (4
*b^2*c^3*PolyLog[2, -1 + 2/(1 + c*x)])/(3*d) + (b*c^3*(a + b*ArcTanh[c*x])*
PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c^3*PolyLog[3, -1 + 2/(1 + c*x)])/(2
*d)
```

Rubi [A] time = 0.97563, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5934, 5916, 5982, 325, 206, 5988, 5932, 2447, 266, 36, 29, 31, 5948, 6056, 6610}

$$\frac{bc^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d}-\frac{4b^2c^3\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{3d}+\frac{b^2c^3\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{2d}+\frac{5c^3\left(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

```
[Out] -(b^2*c^2)/(3*d*x) + (b^2*c^3*ArcTanh[c*x])/(3*d) - (b*c*(a + b*ArcTanh[c*x
]))/(3*d*x^2) + (b*c^2*(a + b*ArcTanh[c*x]))/(d*x) + (5*c^3*(a + b*ArcTanh[
c*x])^2)/(6*d) - (a + b*ArcTanh[c*x])^2/(3*d*x^3) + (c*(a + b*ArcTanh[c*x])
^2)/(2*d*x^2) - (c^2*(a + b*ArcTanh[c*x])^2)/(d*x) - (b^2*c^3*Log[x])/d + (
b^2*c^3*Log[1 - c^2*x^2])/(2*d) + (8*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(
1 + c*x)])/(3*d) - (c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)])/d - (4
*b^2*c^3*PolyLog[2, -1 + 2/(1 + c*x)])/(3*d) + (b*c^3*(a + b*ArcTanh[c*x])*
PolyLog[2, -1 + 2/(1 + c*x)])/d + (b^2*c^3*PolyLog[3, -1 + 2/(1 + c*x)])/(2
*d)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4(d+cdx)}\,dx = -\left(c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)}\,dx\right)+\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^4}\,dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)}\,dx-\frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d}+\frac{(2bc)\int}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+\frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2dx^2}-c^3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)}\,dx+\frac{(2bc)\int}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{c^3\left(a+b\tanh^{-1}(cx)\right)^2}{3d}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3dx^3}+\frac{c\left(a+b\tanh\right.}{2dx^2}$$

$$= -\frac{b^2c^2}{3dx}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\left(a+b\tanh^{-1}(cx)\right)^2}{6d}-\frac{(a}{}$$

$$= -\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\left(a+b\right.}{}$$

$$= -\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\left(a+b\right.}{}$$

$$= -\frac{b^2c^2}{3dx}+\frac{b^2c^3\tanh^{-1}(cx)}{3d}-\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{3dx^2}+\frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{dx}+\frac{5c^3\left(a+b\right.}{}$$

Mathematica [C] time = 1.42146, size = 388, normalized size = 1.16

$$\frac{-\frac{8ab\left(-3c^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-cx\left(c^2x^2+8c^2x^2\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+3cx-1\right)+\tanh^{-1}(cx)\left(3c^3x^3+6c^2x^2+6c^3x^3\log\left(1-e^{-2\tanh^{-1}(cx)}\right)-3cx+2\right)\right)}{x^3}+b^2c^3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^4*(d + c*d*x)),x]
```

```
[Out] ((-8*a^2)/x^3 + (12*a^2*c)/x^2 - (24*a^2*c^2)/x - 24*a^2*c^3*Log[x] + 24*a^
2*c^3*Log[1 + c*x] - (8*a*b*(ArcTanh[c*x]*(2 - 3*c*x + 6*c^2*x^2 + 3*c^3*x^
3 + 6*c^3*x^3*Log[1 - E^(-2*ArcTanh[c*x])]) - c*x*(-1 + 3*c*x + c^2*x^2 + 8
*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x^2]]) - 3*c^3*x^3*PolyLog[2, E^(-2*ArcTanh
[c*x])]))/x^3 + b^2*c^3*((-I)*Pi^3 - 8/(c*x) + 8*ArcTanh[c*x] - (8*ArcTanh[
c*x])/(c^2*x^2) + (24*ArcTanh[c*x])/(c*x) + 20*ArcTanh[c*x]^2 - (8*ArcTanh[
c*x]^2)/(c^3*x^3) + (12*ArcTanh[c*x]^2)/(c^2*x^2) - (24*ArcTanh[c*x]^2)/(c*
x) + 16*ArcTanh[c*x]^3 + 64*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*
ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*Log[(c*x)/Sqrt[1 - c^2*x^2]
] - 32*PolyLog[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*Ar
cTanh[c*x])] + 12*PolyLog[3, E^(2*ArcTanh[c*x])]))/(24*d)
```

Maple [C] time = 1.181, size = 2010, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x)
```

```
[Out] -1/2*I*c^3*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*arctanh(c*x)^2-1/2*I*c^3*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1
)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*
x)^2+1/2*I*c^3*b^2/d*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2+1/2*I*c^3*b^
2/d*arctanh(c*x)^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2+1/2*I*c^3*b^2/d*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)^2*arctanh(c*x)^2-I*c^3*b^2/d*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+4/3*b^2*c^3*arctanh(c*x)/d-1/3*a^2
/d/x^3-2*c^2*a*b/d*arctanh(c*x)/x+c*a*b/d*arctanh(c*x)/x^2+c^3*a*b/d*ln(c*x
)*ln(c*x+1)-2*c^3*a*b/d*arctanh(c*x)*ln(c*x)+2*c^3*a*b/d*arctanh(c*x)*ln(c*
x+1)+c^3*a*b/d*ln(-1/2*c*x+1/2)*ln(c*x+1)-c^3*a*b/d*ln(-1/2*c*x+1/2)*ln(1/2
+1/2*c*x)+1/2*c*a^2/d/x^2-c^2*a^2/d/x-1/3*b^2/d*arctanh(c*x)^2/x^3+8/3*c^3*
b^2/d*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-8/3*c^3*b^2/d*dilog((c*x+1)/(-c^2
*x^2+1)^(1/2))+2*c^3*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^3*b^2/
d*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+c^3*a^2/d*ln(c*x+1)+2/3*c^3*b^2/d*
arctanh(c*x)^3-11/6*c^3*b^2/d*arctanh(c*x)^2-c^3*a^2/d*ln(c*x)-c^3*b^2/d*ln
((c*x+1)/(-c^2*x^2+1)^(1/2)-1)-c^3*b^2/d*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-1
/2*I*c^3*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)
+1))^3*arctanh(c*x)^2-1/2*I*c^3*b^2/d*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x
+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I*c^3*b^2/d*Pi*csgn(I*(c*x+1)^2
/(c^2*x^2-1))^3*arctanh(c*x)^2+c^3*a*b/d*dilog(c*x)+c^3*a*b/d*dilog(c*x+1)+
8/3*c^3*a*b/d*ln(c*x)-c^3*b^2/d*arctanh(c*x)^2*ln(2)-c^3*a*b/d*dilog(1/2+1/
2*c*x)-1/2*c^3*a*b/d*ln(c*x+1)^2-5/6*c^3*a*b/d*ln(c*x-1)-11/6*c^3*a*b/d*ln(
c*x+1)-c^3*b^2/d*arctanh(c*x)^2*ln(c*x)+8/3*c^3*b^2/d*arctanh(c*x)*ln(1+(c*
x+1)/(-c^2*x^2+1)^(1/2))-c^3*b^2/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)
^(1/2))-2*c^3*b^2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-c^3
*b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*c^3*b^2/d*arctanh(
c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+c^3*b^2/d*arctanh(c*x)^2*ln((c*x
+1)^2/(-c^2*x^2+1)-1)-2*c^3*b^2/d*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1
/2))+c^3*b^2/d*arctanh(c*x)^2*ln(c*x+1)+1/3*c^3*b^2/d/((-c^2*x^2+1)^(1/2)+c
*x+1)*(-c^2*x^2+1)^(1/2)-1/3*c^3*b^2/d/(c*x+1-(-c^2*x^2+1)^(1/2))*(-c^2*x^2
```

```
+1)^(1/2)-1/3*c*a*b/d/x^2+c^2*a*b/d/x-2/3*a*b/d*arctanh(c*x)/x^3-c^2*b^2/d*
arctanh(c*x)^2/x+1/2*c*b^2/d*arctanh(c*x)^2/x^2+c^2*b^2/d*arctanh(c*x)/x-1/
3*c*b^2/d*arctanh(c*x)/x^2+1/2*I*c^3*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1
)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2
/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*c^3*b^2/d*arctanh(c*x)^2*Pi*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+
1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\left(\frac{6\,c^3\log\left(cx+1\right)}{d}-\frac{6\,c^3\log\left(x\right)}{d}-\frac{6\,c^2x^2-3\,cx+2}{dx^3}\right)a^2+\frac{\left(6\,b^2c^3x^3\log\left(cx+1\right)-6\,b^2c^2x^2+3\,b^2cx-2\,b^2\right)\log\left(-cx\right.}{24\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^3*log(c*x + 1)/d - 6*c^3*log(x)/d - (6*c^2*x^2 - 3*c*x + 2)/(d*x^3
))*a^2 + 1/24*(6*b^2*c^3*x^3*log(c*x + 1) - 6*b^2*c^2*x^2 + 3*b^2*c*x - 2*b
^2)*log(-c*x + 1)^2/(d*x^3) - integrate(-1/12*(3*(b^2*c*x - b^2)*log(c*x +
1)^2 + 12*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b
^2*c^2*x^2 + 12*a*b - 2*(6*a*b*c - b^2*c)*x - 6*(b^2*c^5*x^5 + b^2*c^4*x^4
+ b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*d*x^6 - d*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}\left(cx\right)^2+2\,ab\,\text{artanh}\left(cx\right)+a^2}{cdx^5+dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c*d*x^5 + d*x^4),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{cx^5+x^4}\,dx+\int \frac{b^2\,\text{atanh}^2\left(cx\right)}{cx^5+x^4}\,dx+\int \frac{2ab\,\text{atanh}\left(cx\right)}{cx^5+x^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**4/(c*d*x+d),x)
```

```
[Out] (Integral(a**2/(c*x**5 + x**4), x) + Integral(b**2*atanh(c*x)**2/(c*x**5 +
x**4), x) + Integral(2*a*b*atanh(c*x)/(c*x**5 + x**4), x))/d
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4/(c*d*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)*x^4), x)
```

3.103 $\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=394

$$-\frac{4b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2}-\frac{10b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^5d^2}-\frac{2b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^2}+\frac{x^3\left(a+b\,\mathrm{t}\right.}{3c}$$

```
[Out] (-2*a*b*x)/(c^4*d^2) + (b^2*x)/(3*c^4*d^2) - b^2/(2*c^5*d^2*(1 + c*x)) + (b
^2*ArcTanh[c*x])/(6*c^5*d^2) - (2*b^2*x*ArcTanh[c*x])/(c^4*d^2) + (b*x^2*(a
 + b*ArcTanh[c*x]))/(3*c^3*d^2) - (b*(a + b*ArcTanh[c*x]))/(c^5*d^2*(1 + c*
x)) + (29*(a + b*ArcTanh[c*x])^2)/(6*c^5*d^2) + (3*x*(a + b*ArcTanh[c*x])^2
)/(c^4*d^2) - (x^2*(a + b*ArcTanh[c*x])^2)/(c^3*d^2) + (x^3*(a + b*ArcTanh[
c*x])^2)/(3*c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^5*d^2*(1 + c*x)) - (20*b*(
a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^5*d^2) + (4*(a + b*ArcTanh[c*x])
^2*Log[2/(1 + c*x)])/(c^5*d^2) - (b^2*Log[1 - c^2*x^2])/(c^5*d^2) - (10*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^5*d^2) - (4*b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/(c^5*d^2) - (2*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^
5*d^2)
```

Rubi [A] time = 0.841405, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 19, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.864, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 321, 206, 5928, 5926, 627, 44, 207, 6056, 6610}

$$-\frac{4b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2}-\frac{10b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^5d^2}-\frac{2b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^2}+\frac{x^3\left(a+b\,\mathrm{t}\right.}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```



```
[Out] (-2*a*b*x)/(c^4*d^2) + (b^2*x)/(3*c^4*d^2) - b^2/(2*c^5*d^2*(1 + c*x)) + (b
^2*ArcTanh[c*x])/(6*c^5*d^2) - (2*b^2*x*ArcTanh[c*x])/(c^4*d^2) + (b*x^2*(a
 + b*ArcTanh[c*x]))/(3*c^3*d^2) - (b*(a + b*ArcTanh[c*x]))/(c^5*d^2*(1 + c*
x)) + (29*(a + b*ArcTanh[c*x])^2)/(6*c^5*d^2) + (3*x*(a + b*ArcTanh[c*x])^2
)/(c^4*d^2) - (x^2*(a + b*ArcTanh[c*x])^2)/(c^3*d^2) + (x^3*(a + b*ArcTanh[
c*x])^2)/(3*c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^5*d^2*(1 + c*x)) - (20*b*(
a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(3*c^5*d^2) + (4*(a + b*ArcTanh[c*x])
^2*Log[2/(1 + c*x)])/(c^5*d^2) - (b^2*Log[1 - c^2*x^2])/(c^5*d^2) - (10*b^2
*PolyLog[2, 1 - 2/(1 - c*x)])/(3*c^5*d^2) - (4*b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/(c^5*d^2) - (2*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^
5*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int\left(\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} + \frac{(a+b\,\mathrm{ta}}{c^4d^2(}\right.$$

$$= \frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{c^4d^2} + \frac{3\int\left(a+b\tanh^{-1}(cx)\right)^2 dx}{c^4d^2} - \frac{4\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}dx}{c^4d^2} - \frac{2\int x\left(a+b\right.}{}$$

$$= \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{3c^2d^2} - \frac{(a+b\tanh}{c^5d^2(1-}$$

$$= \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^2} + \frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^3\left(a+b\tan\right.}{3c^2}$$

$$= -\frac{2abx}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2(1+cx)} + \frac{29\left(a+b\tanh^{-1}(cx)\right)^2}{6c^5d^2} + \frac{3x(}{}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^5d^2(1+cx)} + \frac{2}{}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2\tanh^{-1}(cx)}{3c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}(cx)\right)}{3c^3d^2} - \frac{b(a+b}{c^5d^2}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2}{2c^5d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{3c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}\right.}{3c^3d^2}$$

$$= -\frac{2abx}{c^4d^2} + \frac{b^2x}{3c^4d^2} - \frac{b^2}{2c^5d^2(1+cx)} + \frac{b^2\tanh^{-1}(cx)}{6c^5d^2} - \frac{2b^2x\tanh^{-1}(cx)}{c^4d^2} + \frac{bx^2\left(a+b\tanh^{-1}\right.}{3c^3d^2}$$

Mathematica [A] time = 1.68042, size = 425, normalized size = 1.08

$$\frac{2ab\left(-24\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2c^2x^2+20\log\left(1-c^2x^2\right)+2\tanh^{-1}(cx)\left(2c^3x^3-6c^2x^2+18cx+24\log\left(e^{-2\tanh}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (36*a^2*c*x - 12*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(1 + c*x) - 48*a^2*
Log[1 + c*x] + b^2*(4*c*x - 4*ArcTanh[c*x] - 24*c*x*ArcTanh[c*x] + 4*c^2*x^
2*ArcTanh[c*x] - 28*ArcTanh[c*x]^2 + 36*c*x*ArcTanh[c*x]^2 - 12*c^2*x^2*Arc
Tanh[c*x]^2 + 4*c^3*x^3*ArcTanh[c*x]^2 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh
[c*x]*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 80*Arc
Tanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 48*ArcTanh[c*x]^2*Log[1 + E^(-2*Ar
cTanh[c*x])] - 12*Log[1 - c^2*x^2] - 8*(-5 + 6*ArcTanh[c*x])*PolyLog[2, -E^
(-2*ArcTanh[c*x])] - 24*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh
[c*x]] + 6*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcT
anh[c*x]]) + 2*a*b*(-2 - 12*c*x + 2*c^2*x^2 - 3*Cosh[2*ArcTanh[c*x]] + 20*L
og[1 - c^2*x^2] - 24*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*
x]] + 2*ArcTanh[c*x]*(6 + 18*c*x - 6*c^2*x^2 + 2*c^3*x^3 - 3*Cosh[2*ArcTanh
[c*x]] + 24*Log[1 + E^(-2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]])))/(12*c^
5*d^2)
```

Maple [C] time = 0.806, size = 1467, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] -2*I/c^5*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+2*I/c^5*b^2/d^2*arctanh(c*
x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c
*x+1)^2/(-c^2*x^2+1)+1))^2+4*I/c^5*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(
1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+2*I/c^5*b^2/d^2*Pi*csg
n(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x
)^2+1/2/c^4*b^2/d^2*arctanh(c*x)/(c*x+1)*x+2/3/c^2*a*b/d^2*arctanh(c*x)*x^3
-2/c^3*a*b/d^2*arctanh(c*x)*x^2+6/c^4*a*b/d^2*arctanh(c*x)*x-2/c^5*a*b/d^2*
arctanh(c*x)/(c*x+1)-8/c^5*a*b/d^2*arctanh(c*x)*ln(c*x+1)-4/c^5*a*b/d^2*ln(
-1/2*c*x+1/2)*ln(c*x+1)+4/c^5*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/3*
b^2*x/d^2/c^4-1/4*b^2/c^5/d^2/(c*x+1)-7/3*b^2*arctanh(c*x)/c^5/d^2-1/3/c^5*
b^2/d^2-2*a*b*x/d^2/c^4-2*b^2*x*arctanh(c*x)/d^2/c^4+2*I/c^5*b^2/d^2*Pi*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+2*I/
c^5*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-2*I/c^5*b^2/d
^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-7/3/c^5
*a*b/d^2-1/c^5*a^2/d^2/(c*x+1)+1/3/c^2*a^2/d^2*x^3-1/c^3*a^2/d^2*x^2+3/c^4*
a^2/d^2*x-4/c^5*a^2/d^2*ln(c*x+1)-20/3/c^5*b^2/d^2*dilog(1+I*(c*x+1)/(-c^2*
x^2+1)^(1/2))+2/c^5*b^2/d^2*ln((c*x+1)^2/(-c^2*x^2+1)+1)-8/3/c^5*b^2/d^2*ar
ctanh(c*x)^3-2/c^5*b^2/d^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-20/3/c^5*b^2/
d^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+29/6/c^5*b^2/d^2*arctanh(c*x)^2-1
/c^5*a*b/d^2/(c*x+1)+1/4/c^4*b^2/d^2/(c*x+1)*x+1/3/c^3*a*b/d^2*x^2+4/c^5*b^
2/d^2*arctanh(c*x)^2*ln(2)+4/c^5*a*b/d^2*dilog(1/2+1/2*c*x)+2/c^5*a*b/d^2*l
n(c*x+1)^2+11/6/c^5*a*b/d^2*ln(c*x-1)+29/6/c^5*a*b/d^2*ln(c*x+1)+1/3/c^2*b^
2/d^2*arctanh(c*x)^2*x^3-1/c^3*b^2/d^2*arctanh(c*x)^2*x^2+3/c^4*b^2/d^2*arc
tanh(c*x)^2*x+1/3/c^3*b^2/d^2*arctanh(c*x)*x^2-20/3/c^5*b^2/d^2*arctanh(c*x
)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-20/3/c^5*b^2/d^2*arctanh(c*x)*ln(1-I*(
c*x+1)/(-c^2*x^2+1)^(1/2))-4/c^5*b^2/d^2*arctanh(c*x)^2*ln(c*x+1)-1/c^5*b^2
/d^2*arctanh(c*x)^2/(c*x+1)+8/c^5*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x
^2+1)^(1/2))-1/2/c^5*b^2/d^2*arctanh(c*x)/(c*x+1)+4/c^5*b^2/d^2*arctanh(c*x
)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}a^2\left(\frac{3}{c^6d^2x+c^5d^2}-\frac{c^2x^3-3cx^2+9x}{c^4d^2}+\frac{12\log(cx+1)}{c^5d^2}\right)+\frac{\left(b^2c^4x^4-2b^2c^3x^3+6b^2c^2x^2+9b^2cx-3b^2-12\left(b^2cx\right.\right.}{12\left(c^6d^2x+c^5d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/3*a^2*(3/(c^6*d^2*x + c^5*d^2) - (c^2*x^3 - 3*c*x^2 + 9*x)/(c^4*d^2) + 1
2*log(c*x + 1)/(c^5*d^2)) + 1/12*(b^2*c^4*x^4 - 2*b^2*c^3*x^3 + 6*b^2*c^2*x
^2 + 9*b^2*c*x - 3*b^2 - 12*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(
c^6*d^2*x + c^5*d^2) - integrate(-1/12*(3*(b^2*c^5*x^5 - b^2*c^4*x^4)*log(c
*x + 1)^2 + 12*(a*b*c^5*x^5 - a*b*c^4*x^4)*log(c*x + 1) - 2*(4*b^2*c^3*x^3
+ 15*b^2*c^2*x^2 + (6*a*b*c^5 + b^2*c^5)*x^5 - (6*a*b*c^4 + b^2*c^4)*x^4 +
6*b^2*c*x - 3*b^2 + 3*(b^2*c^5*x^5 - b^2*c^4*x^4 - 4*b^2*c^2*x^2 - 8*b^2*c*
```

```
x - 4*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^7*d^2*x^3 + c^6*d^2*x^2 - c^5*d^
2*x - c^4*d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4\,\text{artanh}\,(cx)^2 + 2\,abx^4\,\text{artanh}\,(cx) + a^2x^4}{c^2d^2x^2 + 2\,cd^2x + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^4}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^4\,\text{atanh}^2\,(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^4\,\text{atanh}\,(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**4/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**4*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**4*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^2x^4}{(cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=331

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{2b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^2}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{x^2\left(a+b\tan\right.}{2c^2d}$$

```
[Out] (a*b*x)/(c^3*d^2) + b^2/(2*c^4*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^4*d
^2) + (b^2*x*ArcTanh[c*x])/(c^3*d^2) + (b*(a + b*ArcTanh[c*x]))/(c^4*d^2*(1
 + c*x)) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^2) - (2*x*(a + b*ArcTanh[c*x])
^2)/(c^3*d^2) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^2*d^2) + (a + b*ArcTanh[c
*x])^2/(c^4*d^2*(1 + c*x)) + (4*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c
^4*d^2) - (3*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^2) + (b^2*Log[
1 - c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^4*d^2) +
 (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^4*d^2) + (3*b^2*
PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```

Rubi [A] time = 0.62533, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 5928, 5926, 627, 44, 207, 6056, 6610}

$$\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2}+\frac{2b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^2}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^2}+\frac{x^2\left(a+b\tan\right.}{2c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```


```
[Out] (a*b*x)/(c^3*d^2) + b^2/(2*c^4*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^4*d
^2) + (b^2*x*ArcTanh[c*x])/(c^3*d^2) + (b*(a + b*ArcTanh[c*x]))/(c^4*d^2*(1
 + c*x)) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^2) - (2*x*(a + b*ArcTanh[c*x])
^2)/(c^3*d^2) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^2*d^2) + (a + b*ArcTanh[c
*x])^2/(c^4*d^2*(1 + c*x)) + (4*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c
^4*d^2) - (3*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^2) + (b^2*Log[
1 - c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^4*d^2) +
 (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^4*d^2) + (3*b^2*
PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int \left(-\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)^2} + \frac{3\left(a+b\tan\right.}{c^3d^2(1}\right.$$

$$= -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{c^3d^2} - \frac{2\int \left(a+b\tanh^{-1}(cx)\right)^2 dx}{c^3d^2} + \frac{3\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}dx}{c^3d^2} + \frac{\int x\left(a+b\right.}{}$$

$$= -\frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2(1+cx)} - \frac{3\left(a+b\tanh\right.}{}$$

$$= -\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{\left(a+b\tanh\right.}{c^4d^2(1}$$

$$= \frac{abx}{c^3d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2} + \frac{x^2\left(a+\right.}{}$$

$$= \frac{abx}{c^3d^2} + \frac{b^2x\tanh^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh\right.}{c^3d^2}$$

$$= \frac{abx}{c^3d^2} + \frac{b^2x\tanh^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2} - \frac{2x\left(a+b\tanh\right.}{c^3d^2}$$

$$= \frac{abx}{c^3d^2} + \frac{b^2}{2c^4d^2(1+cx)} + \frac{b^2x\tanh^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)} - \frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^2}$$

$$= \frac{abx}{c^3d^2} + \frac{b^2}{2c^4d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{2c^4d^2} + \frac{b^2x\tanh^{-1}(cx)}{c^3d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^4d^2(1+cx)} - \frac{3\left(a+\right.}{}$$

Mathematica [A] time = 1.25602, size = 354, normalized size = 1.07

$$\frac{2ab\left(6\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-4\log\left(1-c^2x^2\right)+2\tanh^{-1}(cx)\left(c^2x^2-4cx-6\log\left(e^{-2\tanh^{-1}(cx)}+1\right)-\sinh\left(2\tanh\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (-8*a^2*c*x + 2*a^2*c^2*x^2 + (4*a^2)/(1 + c*x) + 12*a^2*Log[1 + c*x] + 2*a
*b*(2*c*x + Cosh[2*ArcTanh[c*x]] - 4*Log[1 - c^2*x^2] + 6*PolyLog[2, -E^(-2
*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(-1 - 4*c*x + c^2*x^2 + Cosh[2*ArcTanh[c*x
]] - 6*Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTan
h[c*x]]) + b^2*(4*c*x*ArcTanh[c*x] + 6*ArcTanh[c*x]^2 - 8*c*x*ArcTanh[c*x]^
2 + 2*c^2*x^2*ArcTanh[c*x]^2 + Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2
*ArcTanh[c*x]] + 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 16*ArcTanh[c*x]*Lo
g[1 + E^(-2*ArcTanh[c*x])] - 12*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])]
 + 2*Log[1 - c^2*x^2] + 4*(-2 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c
*x])] + 6*PolyLog[3, -E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTa
nh[c*x]*Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c
^4*d^2)
```

Maple [C] time = 0.666, size = 1354, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] 1/4*b^2/c^4/d^2/(c*x+1)+b^2*arctanh(c*x)/d^2/c^4+1/2/c^4*b^2/d^2*arctanh(c*
x)/(c*x+1)-3/c^4*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/
2*I/c^4*b^2/d^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c
*x)^2+a*b*x/c^3/d^2+b^2*x*arctanh(c*x)/c^3/d^2-3*I/c^4*b^2/d^2*Pi*csgn(I*(c
*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+3/
2*I/c^4*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-
1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/2*I/c^4*b^2/d^2*arctanh(c
*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((
c*x+1)^2/(-c^2*x^2+1)+1))^2-3/2*I/c^4*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2+2/c^4*a*b/d^2*arcta
nh(c*x)/(c*x+1)+6/c^4*a*b/d^2*arctanh(c*x)*ln(c*x+1)+3/c^4*a*b/d^2*ln(-1/2*
c*x+1/2)*ln(c*x+1)-3/c^4*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/2/c^3*b
^2/d^2*arctanh(c*x)/(c*x+1)*x+1/c^2*a*b/d^2*arctanh(c*x)*x^2-4/c^3*a*b/d^2*
arctanh(c*x)*x+1/c^4*a*b/d^2+1/c^4*a^2/d^2/(c*x+1)+1/2/c^2*a^2/d^2*x^2-2/c^
3*a^2/d^2*x+2/c^4*b^2/d^2*arctanh(c*x)^3+3/2/c^4*b^2/d^2*polylog(3,-(c*x+1)
^2/(-c^2*x^2+1))+4/c^4*b^2/d^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))-3/c^4*
b^2/d^2*arctanh(c*x)^2+3/c^4*a^2/d^2*ln(c*x+1)+4/c^4*b^2/d^2*dilog(1+I*(c*x
+1)/(-c^2*x^2+1)^(1/2))-1/c^4*b^2/d^2*ln((c*x+1)^2/(-c^2*x^2+1)+1)-3/2*I/c^
4*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/2*I/c^4*b^2/d
^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*
x)^2+1/c^4*a*b/d^2/(c*x+1)-1/4/c^3*b^2/d^2/(c*x+1)*x+1/2/c^2*b^2/d^2*arctan
h(c*x)^2*x^2-2/c^3*b^2/d^2*arctanh(c*x)^2*x-3/c^4*b^2/d^2*arctanh(c*x)^2*ln
(2)-3/c^4*a*b/d^2*dilog(1/2+1/2*c*x)-3/2/c^4*a*b/d^2*ln(c*x+1)^2-1/c^4*a*b/
d^2*ln(c*x-1)-3/c^4*a*b/d^2*ln(c*x+1)+4/c^4*b^2/d^2*arctanh(c*x)*ln(1+I*(c*
x+1)/(-c^2*x^2+1)^(1/2))+4/c^4*b^2/d^2*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^
2+1)^(1/2))+3/c^4*b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+1/c^4*b^2/d^2*arctanh(c*
x)^2/(c*x+1)-6/c^4*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{2}{c^5d^2x+c^4d^2}+\frac{cx^2-4x}{c^3d^2}+\frac{6\log(cx+1)}{c^4d^2}\right)+\frac{\left(b^2c^3x^3-3b^2c^2x^2-4b^2cx+2b^2+6\left(b^2cx+b^2\right)\log(cx+1)\right)\log}{8\left(c^5d^2x+c^4d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2/(c^5*d^2*x + c^4*d^2) + (c*x^2 - 4*x)/(c^3*d^2) + 6*log(c*x + 1)
/(c^4*d^2)) + 1/8*(b^2*c^3*x^3 - 3*b^2*c^2*x^2 - 4*b^2*c*x + 2*b^2 + 6*(b^2
*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^5*d^2*x + c^4*d^2) - integrate
(-1/4*((b^2*c^4*x^4 - b^2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^
3*x^3)*log(c*x + 1) + (7*b^2*c^2*x^2 - (4*a*b*c^4 + b^2*c^4)*x^4 + 2*b^2*c*
x + 2*(2*a*b*c^3 + b^2*c^3)*x^3 - 2*b^2 - 2*(b^2*c^4*x^4 - b^2*c^3*x^3 + 3*
b^2*c^2*x^2 + 6*b^2*c*x + 3*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^6*d^2*x^3
+ c^5*d^2*x^2 - c^4*d^2*x - c^3*d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^3\,\text{artanh}(cx)^2+2\,abx^3\,\text{artanh}(cx)+a^2x^3}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^3}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**3/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**3*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**3*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx)+a)^2x^3}{(cdx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^2, x)
```

3.105 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=260

$$-\frac{2b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d^2}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b\left(a+b\tanh^{-1}\right.}{c^3d^2(cx+}$$

```
[Out] -b^2/(2*c^3*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c^3*d^2) - (b*(a + b*Arc
Tanh[c*x]))/(c^3*d^2*(1 + c*x)) + (3*(a + b*ArcTanh[c*x])^2)/(2*c^3*d^2) +
(x*(a + b*ArcTanh[c*x])^2)/(c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^3*d^2*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d^2) + (2*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d^2) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^3*d^2) - (2*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
3*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rubi [A] time = 0.475892, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5928, 5926, 627, 44, 207, 5948, 6056, 6610}

$$-\frac{2b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3d^2}-\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^3d^2}-\frac{b\left(a+b\tanh^{-1}\right.}{c^3d^2(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] -b^2/(2*c^3*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c^3*d^2) - (b*(a + b*Arc
Tanh[c*x]))/(c^3*d^2*(1 + c*x)) + (3*(a + b*ArcTanh[c*x])^2)/(2*c^3*d^2) +
(x*(a + b*ArcTanh[c*x])^2)/(c^2*d^2) - (a + b*ArcTanh[c*x])^2/(c^3*d^2*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^3*d^2) + (2*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^3*d^2) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^3*d^2) - (2*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
3*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^3*d^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx=\int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)^2}-\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)}\right)dx$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^2d^2}+\frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^2d^2}-\frac{2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{c^2d^2}$$

$$=\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^3d^2}+\frac{(2\cdots}{\cdots}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^2(1+cx)}+\frac{2\left(a+b\tanh^{-}\cdots\right.}{c\cdots}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh\cdots\right.}{c^3d^2(1\cdots}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh\cdots\right.}{c^3d^2(1\cdots}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}-\frac{\left(a+b\tanh\cdots\right.}{c^3d^2(1\cdots}$$

$$=-\frac{b^2}{2c^3d^2(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x\left(a+b\tanh^{-1}(cx\cdots\right.}{c^2d^2}$$

$$=-\frac{b^2}{2c^3d^2(1+cx)}+\frac{b^2\tanh^{-1}(cx)}{2c^3d^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^3d^2(1+cx)}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^2}+\frac{x}{\cdots}$$

Mathematica [A] time = 0.918927, size = 295, normalized size = 1.13

$$\frac{2ab\left(-4\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+2\log\left(1-c^2x^2\right)+\sinh\left(2\tanh^{-1}(cx)\right)-\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(2\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] (4*a^2*c*x - (4*a^2)/(1 + c*x) - 8*a^2*Log[1 + c*x] + b^2*(-4*ArcTanh[c*x]^
2 + 4*c*x*ArcTanh[c*x]^2 - Cosh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Cosh[2*Arc
Tanh[c*x]] - 2*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 8*ArcTanh[c*x]*Log[1 +
 E^(-2*ArcTanh[c*x])] + 8*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (4
- 8*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 4*PolyLog[3, -E^(-2*Ar
cTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] +
 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 2*a*b*(-Cosh[2*ArcTanh[c*x]] + 2*
Log[1 - c^2*x^2] - 4*PolyLog[2, -E^(-2*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]
] + 2*ArcTanh[c*x]*(2*c*x - Cosh[2*ArcTanh[c*x]] + 4*Log[1 + E^(-2*ArcTanh[
c*x])] + Sinh[2*ArcTanh[c*x]])))/(4*c^3*d^2)
```

Maple [C] time = 0.507, size = 5542, normalized size = 21.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^2\left(\frac{1}{c^4d^2x+c^3d^2}-\frac{x}{c^2d^2}+\frac{2\log(cx+1)}{c^3d^2}\right)+\frac{\left(b^2c^2x^2+b^2cx-b^2-2\left(b^2cx+b^2\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(c^4d^2x+c^3d^2\right)}-\int-\frac{\left(b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*(1/(c^4*d^2*x + c^3*d^2) - x/(c^2*d^2) + 2*log(c*x + 1)/(c^3*d^2)) + 1
/4*(b^2*c^2*x^2 + b^2*c*x - b^2 - 2*(b^2*c*x + b^2)*log(c*x + 1))*log(-c*x
+ 1)^2/(c^4*d^2*x + c^3*d^2) - integrate(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*
log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*c^2*x^2)*log(c*x + 1) - 2*((2*a*b*c^3
 + b^2*c^3)*x^3 - 2*(a*b*c^2 - b^2*c^2)*x^2 - b^2 + (b^2*c^3*x^3 - 3*b^2*c^
2*x^2 - 4*b^2*c*x - 2*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^5*d^2*x^3 + c^4*
d^2*x^2 - c^3*d^2*x - c^2*d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\operatorname{artanh}(cx)^2+2\,abx^2\operatorname{artanh}(cx)+a^2x^2}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^2*d
^2*x^2 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^2}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x**2*atanh(
c*x)**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x**2*atanh(c*x)/(c**2*
x**2 + 2*c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)^2x^2}{(cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^2, x)
```

3.106 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=188

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(cx+1)}-$$

```
[Out] b^2/(2*c^2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^2*d^2) + (b*(a + b*ArcT
anh[c*x]))/(c^2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*c^2*d^2) + (a +
b*ArcTanh[c*x])^2/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(1 +
c*x)])/(c^2*d^2) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```

Rubi [A] time = 0.342551, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5940, 5928, 5926, 627, 44, 207, 5948, 5918, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^2d^2}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(cx+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```


```
[Out] b^2/(2*c^2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*c^2*d^2) + (b*(a + b*ArcT
anh[c*x]))/(c^2*d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*c^2*d^2) + (a +
b*ArcTanh[c*x])^2/(c^2*d^2*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(1 +
c*x)])/(c^2*d^2) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^2*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = \int \left(-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{cd^2} + \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}dx}{cd^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2} - \frac{(2b)\int \left(\frac{a+b\tanh^{-1}(cx)}{2(1+cx)^2} - \frac{a+b\tanh^{-1}(c}{2\left(-1+c^2x^2\right)}\right.}{cd^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^2d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1+c}\right.}{c^2d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2}$$

$$= \frac{b^2}{2c^2d^2(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^2(1+cx)} - \frac{(a}{}$$

$$= \frac{b^2}{2c^2d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{2c^2d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{c^2d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2d^2} + \frac{(a+b\tan}{c^2d^2(1}$$

Mathematica [A] time = 0.625016, size = 233, normalized size = 1.24

$$\frac{2ab\left(2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) - \sinh\left(2\tanh^{-1}(cx)\right) + \cosh\left(2\tanh^{-1}(cx)\right) + 2\tanh^{-1}(cx)\left(-2\log\left(e^{-2\tanh^{-1}(cx)}+1\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^2,x]
```

```
[Out] ((4*a^2)/(1 + c*x) + 4*a^2*Log[1 + c*x] + 2*a*b*(Cosh[2*ArcTanh[c*x]] + 2*P
olyLog[2, -E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[2*ArcTanh[c*x]] - 2*
Log[1 + E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) - Sinh[2*ArcTanh[c*x]]
) + b^2*(Cosh[2*ArcTanh[c*x]] + 2*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 2*Arc
Tanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c
*x])] + 4*ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*PolyLog[3, -E^(
-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]] - 2*ArcTanh[c*x]*Sinh[2*ArcTanh[c*
x]] - 2*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]))/(4*c^2*d^2)
```

Maple [C] time = 0.277, size = 1030, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] 1/4*b^2/c^2/d^2/(c*x+1)-1/2*I/c^2*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1
/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-I/c^2*b^2/d^2*arctanh(c
*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi
-1/2*I/c^2*b^2/d^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I/c^2*b^2/d^2*a
rctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*Pi+2/c^2*a*b/d^2*arctanh(c*x)*ln(c*x+1)-1/2/c*b
^2/d^2*arctanh(c*x)/(c*x+1)*x+1/c^2*a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/c^
2*a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+2/c^2*a*b/d^2*arctanh(c*x)/(c*x+
1)+1/2*I/c^2*b^2/d^2*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*Pi+1/c^2*a^2/d^2/(c*x+1)+1/c^2*a^2/d^2*ln(c*x+1)+2/3/c^2*b^2/d^2*ar
ctanh(c*x)^3+1/2/c^2*b^2/d^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/2/c^2*b^2
/d^2*arctanh(c*x)^2+1/c^2*a*b/d^2/(c*x+1)-1/4/c*b^2/d^2/(c*x+1)*x-1/c^2*b^2
/d^2*arctanh(c*x)^2*ln(2)-1/c^2*a*b/d^2*dilog(1/2+1/2*c*x)-1/2/c^2*a*b/d^2*
ln(c*x+1)^2+1/2/c^2*a*b/d^2*ln(c*x-1)-1/2/c^2*a*b/d^2*ln(c*x+1)+1/c^2*b^2/d
^2*arctanh(c*x)^2*ln(c*x+1)+1/c^2*b^2/d^2*arctanh(c*x)^2/(c*x+1)-2/c^2*b^2/
d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+1/2/c^2*b^2/d^2*arctanh(c
*x)/(c*x+1)-1/c^2*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-1
/2*I/c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)^3*arctanh(c*x)^2-1/2*I/c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arc
tanh(c*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{1}{c^3d^2x+c^2d^2}+\frac{\log(cx+1)}{c^2d^2}\right)+\frac{\left(b^2+\left(b^2cx+b^2\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(c^3d^2x+c^2d^2\right)}-\int-\frac{\left(b^2c^2x^2-b^2cx\right)\log(cx+1)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^2*(1/(c^3*d^2*x + c^2*d^2) + log(c*x + 1)/(c^2*d^2)) + 1/4*(b^2 + (b^2*c*
x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c^3*d^2*x + c^2*d^2) - integrate(-1
/4*((b^2*c^2*x^2 - b^2*c*x)*log(c*x + 1)^2 + 4*(a*b*c^2*x^2 - a*b*c*x)*log(
c*x + 1) - 2*(2*a*b*c^2*x^2 + b^2 - (2*a*b*c - b^2*c)*x + (2*b^2*c^2*x^2 +
b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d^2*x^3 + c^3*d^2*x^2 - c^
2*d^2*x - c*d^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x\,\text{artanh}(cx)^2+2\,abx\,\text{artanh}(cx)+a^2x}{c^2d^2x^2+2\,cd^2x+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(c^2*d^2*x^2
 + 2*c*d^2*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x}{c^2x^2+2cx+1}\,dx + \int \frac{b^2x\,\mathrm{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx + \int \frac{2abx\,\mathrm{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2*x/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*x*atanh(c*x)**
2/(c**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*x*atanh(c*x)/(c**2*x**2 + 2*
c*x + 1), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}(cx) + a)^2 x}{(cdx + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d)^2, x)
```

3.107 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{b^2}{2cd^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}$$

```
[Out] -b^2/(2*c*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c*d^2) - (b*(a + b*ArcTanh
[c*x]))/(c*d^2*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(2*c*d^2) - (a + b*ArcTa
nh[c*x])^2/(c*d^2*(1 + c*x))
```

Rubi [A] time = 0.124376, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(cx+1)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2}-\frac{b^2}{2cd^2(cx+1)}+\frac{b^2\tanh^{-1}(cx)}{2cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

```
[Out] -b^2/(2*c*d^2*(1 + c*x)) + (b^2*ArcTanh[c*x])/(2*c*d^2) - (b*(a + b*ArcTanh
[c*x]))/(c*d^2*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(2*c*d^2) - (a + b*ArcTa
nh[c*x])^2/(c*d^2*(1 + c*x))
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
```

```
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{(2b)\int \left(\frac{a+b\tanh^{-1}(cx)}{2d(1+cx)^2} - \frac{a+b\tanh^{-1}(cx)}{2d\left(-1+c^2x^2\right)}\right)dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b\int \frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx}{d^2} - \frac{b\int \frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{d^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b^2\int \frac{1}{(1+cx)\left(1-c^2x^2\right)}\,dx}{d^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b^2\int \frac{1}{(1-cx)(1+cx)^2}\,dx}{d^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} + \frac{b^2\int \left(\frac{1}{2(1+cx)^2} - \frac{}{2(-1\cdots}\right.}{d^2}$$

$$= -\frac{b^2}{2cd^2(1+cx)} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{cd^2(1+cx)} - \frac{b^2\int\cdots}{}$$

$$= -\frac{b^2}{2cd^2(1+cx)} + \frac{b^2\tanh^{-1}(cx)}{2cd^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)}{cd^2(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^2} - \frac{\left(a+b\tan\cdots\right.}{cd^2(1\cdots}$$

Mathematica [A] time = 0.131612, size = 124, normalized size = 1.16

$$\frac{-4a^2+2ab\log(cx+1)+2abcx\log(cx+1)-b(2a+b)(cx+1)\log(1-cx)-4b(2a+b)\tanh^{-1}(cx)-4ab+b^2\log(cx+1\cdots}{4cd^2(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^2,x]
```

```
[Out] (-4*a^2 - 4*a*b - 2*b^2 - 4*b*(2*a + b)*ArcTanh[c*x] + 2*b^2*(-1 + c*x)*Arc
Tanh[c*x]^2 - b*(2*a + b)*(1 + c*x)*Log[1 - c*x] + 2*a*b*Log[1 + c*x] + b^2
*Log[1 + c*x] + 2*a*b*c*x*Log[1 + c*x] + b^2*c*x*Log[1 + c*x])/(4*c*d^2*(1
+ c*x))
```

Maple [B] time = 0.056, size = 341, normalized size = 3.2

$$-\frac{a^2}{cd^2(cx+1)} - \frac{b^2(\mathrm{Artanh}(cx))^2}{cd^2(cx+1)} - \frac{b^2\mathrm{Artanh}(cx)\ln(cx-1)}{2cd^2} - \frac{b^2\mathrm{Artanh}(cx)}{cd^2(cx+1)} + \frac{b^2\mathrm{Artanh}(cx)\ln(cx+1)}{2cd^2} - \frac{b^2(\ln(c\cdots}{8c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x)
```

```
[Out] -1/c*a^2/d^2/(c*x+1)-1/c*b^2/d^2*arctanh(c*x)^2/(c*x+1)-1/2/c*b^2/d^2*arcta
nh(c*x)*ln(c*x-1)-1/c*b^2/d^2*arctanh(c*x)/(c*x+1)+1/2/c*b^2/d^2*arctanh(c*
x)*ln(c*x+1)-1/8/c*b^2/d^2*ln(c*x-1)^2+1/4/c*b^2/d^2*ln(c*x-1)*ln(1/2+1/2*c
*x)-1/4/c*b^2/d^2*ln(c*x-1)-1/2*b^2/c/d^2/(c*x+1)+1/4/c*b^2/d^2*ln(c*x+1)-1
/4/c*b^2/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/4/c*b^2/d^2*ln(-1/2*c*x+1/2
)*ln(c*x+1)-1/8/c*b^2/d^2*ln(c*x+1)^2-2/c*a*b/d^2*arctanh(c*x)/(c*x+1)-1/2/
c*a*b/d^2*ln(c*x-1)-1/c*a*b/d^2/(c*x+1)+1/2/c*a*b/d^2*ln(c*x+1)
```

Maxima [B] time = 0.990188, size = 374, normalized size = 3.5

$$-\frac{1}{2}\left(c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\log(cx-1)}{c^2d^2}\right)+\frac{4\,\text{artanh}(cx)}{c^2d^2x+cd^2}\right)ab-\frac{1}{8}\left(4\,c\left(\frac{2}{c^3d^2x+c^2d^2}-\frac{\log(cx+1)}{c^2d^2}+\frac{\log(c\cdots}{c^2\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(2/(c^3*d^2*x + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c
^2*d^2)) + 4*arctanh(c*x)/(c^2*d^2*x + c*d^2))*a*b - 1/8*(4*c*(2/(c^3*d^2*x
 + c^2*d^2) - log(c*x + 1)/(c^2*d^2) + log(c*x - 1)/(c^2*d^2))*arctanh(c*x)
 + ((c*x + 1)*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1
)*log(c*x - 1) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*d
^2*x + c^3*d^2))*b^2 - b^2*arctanh(c*x)^2/(c^2*d^2*x + c*d^2) - a^2/(c^2*d^
2*x + c*d^2)
```

Fricas [A] time = 2.05308, size = 215, normalized size = 2.01

$$\frac{\left(b^2cx-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2-8\,a^2-8\,ab-4\,b^2+2\left(\left(2\,ab+b^2\right)cx-2\,ab-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{8\left(c^2d^2x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b^2*c*x - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 8*a^2 - 8*a*b - 4*b^2 +
2*((2*a*b + b^2)*c*x - 2*a*b - b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^2*d^2*x +
 c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^2x^2+2cx+1}\,dx+\int\frac{b^2\,\text{atanh}^2(cx)}{c^2x^2+2cx+1}\,dx+\int\frac{2ab\,\text{atanh}(cx)}{c^2x^2+2cx+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**2 + 2*c*x + 1), x) + Integral(b**2*atanh(c*x)**2/(c
**2*x**2 + 2*c*x + 1), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**2 + 2*c*x +
1), x))/d**2
```

Giac [A] time = 1.1786, size = 203, normalized size = 1.9

$$\frac{1}{8}\left(\frac{b^2}{cd^2}-\frac{2\,b^2}{(cdx+d)cd}\right)\log\left(\frac{1}{\frac{2\,d}{cdx+d}-1}\right)^2-\frac{\left(2\,ab+b^2\right)\log\left(-\frac{2\,d}{cdx+d}+1\right)}{4\,cd^2}-\frac{\left(2\,ab+b^2\right)\log\left(\frac{1}{\frac{2\,d}{cdx+d}-1}\right)}{2\,(cdx+d)cd}-\frac{2\,a^2+2\,ab+b^2}{2\,(cdx+d)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2/(c*d^2) - 2*b^2/((c*d*x + d)*c*d))*log(1/(2*d/(c*d*x + d) - 1))^2
- 1/4*(2*a*b + b^2)*log(-2*d/(c*d*x + d) + 1)/(c*d^2) - 1/2*(2*a*b + b^2)*l
og(1/(2*d/(c*d*x + d) - 1))/((c*d*x + d)*c*d) - 1/2*(2*a^2 + 2*a*b + b^2)/(
(c*d*x + d)*c*d)
```

3.108 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^2}\,dx$

Optimal. Leaf size=295

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(}{d^2}$$

```
[Out] b^2/(2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*d^2) + (b*(a + b*ArcTanh[c*x]
))/(d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*d^2) + (a + b*ArcTanh[c*x])^
2/(d^2*(1 + c*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2
 + ((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)])/d^2 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
2/(1 - c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^
2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) - (b^2*PolyLog[3, -1 + 2/(1 -
 c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rubi [A] time = 0.640484, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```


```
[Out] b^2/(2*d^2*(1 + c*x)) - (b^2*ArcTanh[c*x])/(2*d^2) + (b*(a + b*ArcTanh[c*x]
))/(d^2*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/(2*d^2) + (a + b*ArcTanh[c*x])^
2/(d^2*(1 + c*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2
 + ((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)])/d^2 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 +
2/(1 - c*x)])/d^2 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^
2 + (b^2*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) - (b^2*PolyLog[3, -1 + 2/(1 -
 c*x)])/(2*d^2) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)^2} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^2} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}\,dx}{d^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2 \text{lo}}{d^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2 \text{lo}}{d^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx\right.}{}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx\right.}{}$$

$$= \frac{b^2}{2d^2(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{2\left(a\right.}{}$$

$$= \frac{b^2}{2d^2(1+cx)} - \frac{b^2\tanh^{-1}(cx)}{2d^2} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} + \frac{\left(a+b\tan\right.}{d^2(1}$$

Mathematica [C] time = 0.847599, size = 254, normalized size = 0.86

$$\frac{12ab\left(-2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-\sinh\left(2\tanh^{-1}(cx)\right)+\cosh\left(2\tanh^{-1}(cx)\right)+2\tanh^{-1}(cx)\left(2\log\left(1-e^{-2\tanh^{-1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^2),x]
```

```
[Out] ((24*a^2)/(1 + c*x) + 24*a^2*Log[c*x] - 24*a^2*Log[1 + c*x] + 12*a*b*(Cosh[
2*ArcTanh[c*x]] - 2*PolyLog[2, E^(-2*ArcTanh[c*x])] + 2*ArcTanh[c*x]*(Cosh[
2*ArcTanh[c*x]] + 2*Log[1 - E^(-2*ArcTanh[c*x])] - Sinh[2*ArcTanh[c*x]]) -
Sinh[2*ArcTanh[c*x]]) + b^2*(I*Pi^3 - 16*ArcTanh[c*x]^3 + 6*Cosh[2*ArcTanh[
c*x]] + 12*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] + 12*ArcTanh[c*x]^2*Cosh[2*Arc
Tanh[c*x]] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])] - 6*Si
nh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 12*ArcTanh[c*x]
^2*Sinh[2*ArcTanh[c*x]]))/(24*d^2)
```

Maple [C] time = 0.339, size = 1566, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x)
```

```
[Out] -a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+a*b/d^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x
)+b^2/d^2*arctanh(c*x)^2*ln(2)-b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+b^2/d^2*arc
tanh(c*x)^2/(c*x+1)+2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))
+1/2*b^2/d^2*arctanh(c*x)/(c*x+1)+a*b/d^2/(c*x+1)+a*b/d^2*dilog(1/2+1/2*c*x
)+1/2*a*b/d^2*ln(c*x+1)^2+1/2*a*b/d^2*ln(c*x-1)-1/2*a*b/d^2*ln(c*x+1)+1/4*b
^2/d^2/(c*x+1)-2*b^2/d^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-a^2/d^2*ln(c
*x+1)+a^2/d^2/(c*x+1)+1/2*I*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*c
sgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1
)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^
2*arctanh(c*x)^2+I*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*
x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-2*b^2/d^2*polylog(3,-(c*x+1)/(-c^2*x^2
+1)^(1/2))+a^2/d^2*ln(c*x)-2/3*b^2/d^2*arctanh(c*x)^3-1/2*b^2/d^2*arctanh(c
*x)^2-1/2*I*b^2/d^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/
(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arcta
nh(c*x)^2+2*b^2/d^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2
/d^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d^2*arctanh(c*x)
*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d^2*arctanh(c*x)^2*ln(c*x)-b^2/d
^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-a*b/d^2*dilog(c*x+1)-a*b/d^2
*dilog(c*x)+b^2/d^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*a*b/d
^2*arctanh(c*x)*ln(c*x)-a*b/d^2*ln(c*x)*ln(c*x+1)-1/4*b^2/d^2/(c*x+1)*c*x+2
*a*b/d^2*arctanh(c*x)/(c*x+1)-2*a*b/d^2*arctanh(c*x)*ln(c*x+1)+1/2*I*b^2/d^
2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+
1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-1/2*I*b^2/d^
2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/(
(c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*
arctanh(c*x)^2+1/2*I*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x
)^2-1/2*b^2/d^2*arctanh(c*x)/(c*x+1)*c*x+1/2*I*b^2/d^2*Pi*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*I*b^2/d^2
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)
^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{1}{cd^2x+d^2} - \frac{\log(cx+1)}{d^2} + \frac{\log(x)}{d^2}\right) + \frac{\left(b^2 - \left(b^2cx + b^2\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(cd^2x+d^2\right)} + \int \frac{\left(b^2cx - b^2\right)\log(cx+1)^2 + 4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] a^2*(1/(c*d^2*x + d^2) - log(c*x + 1)/d^2 + log(x)/d^2) + 1/4*(b^2 - (b^2*c
*x + b^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x + d^2) + integrate(1/4*((b
^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - 2*(b^2*c^2*
x^2 - 2*a*b + (2*a*b*c + b^2*c)*x - (b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2)*log
(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^4 + c^2*d^2*x^3 - c*d^2*x^2 - d^2*x),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{c^2d^2x^3 + 2\,cd^2x^2 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^3 + 2*c
*d^2*x^2 + d^2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^3+2cx^2+x}\,dx + \int \frac{b^2 \operatorname{atanh}^2(cx)}{c^2x^3+2cx^2+x}\,dx + \int \frac{2ab \operatorname{atanh}(cx)}{c^2x^3+2cx^2+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(b**2*atanh(c*x)**2
/(c**2*x**3 + 2*c*x**2 + x), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**3 + 2*
c*x**2 + x), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(cdx + d)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x), x)
```

3.109 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^2}\,dx$

Optimal. Leaf size=371

$$\frac{2bc\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2bc\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{2bc\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right.}{d^2}$$

```
[Out] -(b^2*c)/(2*d^2*(1 + c*x)) + (b^2*c*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*Arc
Tanh[c*x]))/(d^2*(1 + c*x)) + (3*c*(a + b*ArcTanh[c*x])^2)/(2*d^2) - (a + b
*ArcTanh[c*x])^2/(d^2*x) - (c*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c*x)) - (4*
c*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (2*c*(a + b*ArcTan
h[c*x])^2*Log[2/(1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
+ c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2
 - (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (2*b*c*(
a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1
 + 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/d^2 + (b^2*c*Pol
yLog[3, -1 + 2/(1 - c*x)])/d^2 + (b^2*c*PolyLog[3, 1 - 2/(1 + c*x)])/d^2
```

Rubi [A] time = 0.797292, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$\frac{2bc\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{2bc\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{2bc\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

```
[Out] -(b^2*c)/(2*d^2*(1 + c*x)) + (b^2*c*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*Arc
Tanh[c*x]))/(d^2*(1 + c*x)) + (3*c*(a + b*ArcTanh[c*x])^2)/(2*d^2) - (a + b
*ArcTanh[c*x])^2/(d^2*x) - (c*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c*x)) - (4*
c*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (2*c*(a + b*ArcTan
h[c*x])^2*Log[2/(1 + c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
+ c*x)])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2
 - (2*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (2*b*c*(
a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1
 + 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/d^2 + (b^2*c*Pol
yLog[3, -1 + 2/(1 - c*x)])/d^2 + (b^2*c*PolyLog[3, 1 - 2/(1 + c*x)])/d^2
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)^2} + \frac{2c^2\left(a+b\right.}{d^2(}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} - \frac{(2c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^2} + \frac{\left(2c^2\right)\int \frac{(a+b\,\mathrm{t}}{}}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} - \frac{4c\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right.}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} - \frac{4c\left(a+b\tanh^{-1}\right.}{}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh\right.}{d^2(1+c}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh\right.}{d^2(1+c}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c\left(a+b\tanh\right.}{d^2(1+c}$$

$$= -\frac{b^2c}{2d^2(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} -$$

$$= -\frac{b^2c}{2d^2(1+cx)} + \frac{b^2c\tanh^{-1}(cx)}{2d^2} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} + \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2} - \frac{(a+}{}$$

Mathematica [C] time = 1.71242, size = 347, normalized size = 0.94

$$\frac{6abc\left(4\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+4\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+\sinh\left(2\tanh^{-1}(cx)\right)-\cosh\left(2\tanh^{-1}(cx)\right)+\tanh^{-1}(cx)\left(-\frac{4}{cx}-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^2),x]
```

```
[Out] ((-12*a^2)/x - (12*a^2*c)/(1 + c*x) - 24*a^2*c*Log[x] + 24*a^2*c*Log[1 + c*
x] + b^2*c*((-I)*Pi^3 + 12*ArcTanh[c*x]^2 - (12*ArcTanh[c*x]^2)/(c*x) + 16*
ArcTanh[c*x]^3 - 3*Cosh[2*ArcTanh[c*x]] - 6*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x
]] - 6*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + 24*ArcTanh[c*x]*Log[1 - E^(-2*
ArcTanh[c*x])] - 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 12*PolyLog
[2, E^(-2*ArcTanh[c*x])] - 24*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] +
 12*PolyLog[3, E^(2*ArcTanh[c*x])] + 3*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x
]*Sinh[2*ArcTanh[c*x]] + 6*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]]) + 6*a*b*c*(
-Cosh[2*ArcTanh[c*x]] + 4*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 4*PolyLog[2, E^(-2
*ArcTanh[c*x])] + Sinh[2*ArcTanh[c*x]] + ArcTanh[c*x]*(-4/(c*x) - 2*Cosh[2*
ArcTanh[c*x]] - 8*Log[1 - E^(-2*ArcTanh[c*x])] + 2*Sinh[2*ArcTanh[c*x]])))/
(12*d^2)
```

Maple [C] time = 0.682, size = 7397, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^2\left(\frac{2\,cx+1}{cd^2x^2+d^2x}-\frac{2\,c\log(cx+1)}{d^2}+\frac{2\,c\log(x)}{d^2}\right)-\frac{\left(2\,b^2cx+b^2-2\left(b^2c^2x^2+b^2cx\right)\log(cx+1)\right)\log(-cx+1)^2}{4\left(cd^2x^2+d^2x\right)}-\int -\frac{(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -a^2*((2*c*x + 1)/(c*d^2*x^2 + d^2*x) - 2*c*log(c*x + 1)/d^2 + 2*c*log(x)/d
^2) - 1/4*(2*b^2*c*x + b^2 - 2*(b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c
*x + 1)^2/(c*d^2*x^2 + d^2*x) - integrate(-1/4*((b^2*c*x - b^2)*log(c*x + 1
)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + 2*(2*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 2
*a*b - (2*a*b*c - b^2*c)*x - (2*b^2*c^4*x^4 + 4*b^2*c^3*x^3 + 2*b^2*c^2*x^2
 + b^2*c*x - b^2)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^5 + c^2*d^2*x^4 -
 c*d^2*x^3 - d^2*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}(cx)^2+2\,ab\,\text{artanh}(cx)+a^2}{c^2d^2x^4+2\,cd^2x^3+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^4 + 2*c
*d^2*x^3 + d^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^2x^4+2cx^3+x^2}\,dx+\int \frac{b^2\,\text{atanh}^2(cx)}{c^2x^4+2cx^3+x^2}\,dx+\int \frac{2ab\,\text{atanh}(cx)}{c^2x^4+2cx^3+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(b**2*atanh(c*x)
**2/(c**2*x**4 + 2*c*x**3 + x**2), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**
4 + 2*c*x**3 + x**2), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{artanh}\,(cx) + a)^2}{(cdx + d)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^2), x)
```

3.110 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)^2}\,dx$

Optimal. Leaf size=480

$$-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{3bc^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{3bc^2\text{PolyLog}\left(2,1-\right.}{}$$

```
[Out] (b^2*c^2)/(2*d^2*(1 + c*x)) - (b^2*c^2*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*
ArcTanh[c*x]))/(d^2*x) + (b*c^2*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*
c^2*(a + b*ArcTanh[c*x])^2)/d^2 - (a + b*ArcTanh[c*x])^2/(2*d^2*x^2) + (2*c
*(a + b*ArcTanh[c*x])^2)/(d^2*x) + (c^2*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c
*x)) + (6*c^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 + (b^2*c
^2*Log[x])/d^2 + (3*c^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b^2
*c^2*Log[1 - c^2*x^2])/(2*d^2) - (4*b*c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
 + c*x)])/d^2 - (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/
d^2 + (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 - (3*
b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 + (2*b^2*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/d^2 + (3*b^2*c^2*PolyLog[3, 1 - 2/(1 - c*x)])/(
2*d^2) - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^2) - (3*b^2*c^2*Poly
Log[3, 1 - 2/(1 + c*x)])/(2*d^2)
```

Rubi [A] time = 0.953069, antiderivative size = 480, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 22, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {5940, 5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 2447, 5914, 6052, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$-\frac{3bc^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{3bc^2\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{3bc^2\text{PolyLog}\left(2,1-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```


```
[Out] (b^2*c^2)/(2*d^2*(1 + c*x)) - (b^2*c^2*ArcTanh[c*x])/(2*d^2) - (b*c*(a + b*
ArcTanh[c*x]))/(d^2*x) + (b*c^2*(a + b*ArcTanh[c*x]))/(d^2*(1 + c*x)) - (2*
c^2*(a + b*ArcTanh[c*x])^2)/d^2 - (a + b*ArcTanh[c*x])^2/(2*d^2*x^2) + (2*c
*(a + b*ArcTanh[c*x])^2)/(d^2*x) + (c^2*(a + b*ArcTanh[c*x])^2)/(d^2*(1 + c
*x)) + (6*c^2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 + (b^2*c
^2*Log[x])/d^2 + (3*c^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/d^2 - (b^2
*c^2*Log[1 - c^2*x^2])/(2*d^2) - (4*b*c^2*(a + b*ArcTanh[c*x])*Log[2 - 2/(1
 + c*x)])/d^2 - (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/
d^2 + (3*b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 - (3*
b*c^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 + (2*b^2*c^2*Po
lyLog[2, -1 + 2/(1 + c*x)])/d^2 + (3*b^2*c^2*PolyLog[3, 1 - 2/(1 - c*x)])/(
2*d^2) - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^2) - (3*b^2*c^2*Poly
Log[3, 1 - 2/(1 + c*x)])/(2*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3(d+cdx)^2}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^3} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x^2} + \frac{3c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} - \frac{c^3\left(a+b\right.}{d^2(}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^3}\,dx}{d^2} - \frac{(2c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d^2} + \frac{\left(3c^2\right)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} - \frac{c^3\int \frac{(a+b\,\mathrm{t}}{(}}{}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(1+cx)} + \frac{6c^2\left(a+b\,\mathrm{ta}\right.}{}$$

$$= -\frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^2x^2} + \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{c^2\left(a+b\,\mathrm{ta}\right.}{d^2(1}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\,\mathrm{tan}\right.}{2d}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\,\mathrm{tan}\right.}{2d}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2} - \frac{\left(a+b\,\mathrm{tan}\right.}{2d}$$

$$= \frac{b^2c^2}{2d^2(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2}$$

$$= \frac{b^2c^2}{2d^2(1+cx)} - \frac{b^2c^2\tanh^{-1}(cx)}{2d^2} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{bc^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(1+cx)} - \frac{2c^2\left(\right.}{}$$

Mathematica [C] time = 2.15167, size = 452, normalized size = 0.94

$$\frac{4ab\left(-6c^2x^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+cx\left(-8cx\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)-cx\sinh\left(2\tanh^{-1}(cx)\right)+cx\cosh\left(2\tanh^{-1}(cx)\right)-2\right)+2\tanh^{-1}(cx)\left(c^2x^2+6c^2x^2\log\left(1-e^{-2\tanh^{-1}(\cdots}\right.\right.\right.}{x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^3*(d + c*d*x)^2),x]
```

```
[Out] ((-4*a^2)/x^2 + (16*a^2*c)/x + (8*a^2*c^2)/(1 + c*x) + 24*a^2*c^2*Log[x] -
24*a^2*c^2*Log[1 + c*x] + b^2*c^2*(I*Pi^3 - (8*ArcTanh[c*x])/(c*x) - 12*Arc
Tanh[c*x]^2 - (4*ArcTanh[c*x]^2)/(c^2*x^2) + (16*ArcTanh[c*x]^2)/(c*x) - 16
*ArcTanh[c*x]^3 + 2*Cosh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*Cosh[2*ArcTanh[c*
x]] + 4*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] - 32*ArcTanh[c*x]*Log[1 - E^(-2
*ArcTanh[c*x])] + 24*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 8*Log[(c*
x)/Sqrt[1 - c^2*x^2]] + 16*PolyLog[2, E^(-2*ArcTanh[c*x])] + 24*ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - 12*PolyLog[3, E^(2*ArcTanh[c*x])] - 2*Si
nh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Sinh[2*ArcTanh[c*x]] - 4*ArcTanh[c*x]^2
*Sinh[2*ArcTanh[c*x]]) + (4*a*b*(-6*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])]
 + c*x*(-2 + c*x*Cosh[2*ArcTanh[c*x]] - 8*c*x*Log[(c*x)/Sqrt[1 - c^2*x^2]]
- c*x*Sinh[2*ArcTanh[c*x]]) + 2*ArcTanh[c*x]*(-1 + 4*c*x + c^2*x^2 + c^2*x^
2*Cosh[2*ArcTanh[c*x]] + 6*c^2*x^2*Log[1 - E^(-2*ArcTanh[c*x])] - c^2*x^2*S
inh[2*ArcTanh[c*x]])))/x^2)/(8*d^2)
```

Maple [C] time = 1., size = 2009, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x)
```

```
[Out] c^2*a^2/d^2/(c*x+1)+2*c*a^2/d^2/x-6*c^2*b^2/d^2*polylog(3,(c*x+1)/(-c^2*x^2
+1)^(1/2))-3*c^2*a^2/d^2*ln(c*x+1)+4*c^2*b^2/d^2*dilog((c*x+1)/(-c^2*x^2+1)
^(1/2))-4*c^2*b^2/d^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+c^2*b^2/d^2*ln((c
*x+1)/(-c^2*x^2+1)^(1/2)-1)+c^2*b^2/d^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-6*
c^2*b^2/d^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*a^2/d^2*ln(c*x)-2*
c^2*b^2/d^2*arctanh(c*x)^3+2*c^2*b^2/d^2*arctanh(c*x)^2-1/2*b^2/d^2*arctanh
(c*x)^2/x^2+1/4*b^2*c^2/d^2/(c*x+1)-b^2*c^2*arctanh(c*x)/d^2+3/2*I*c^2*b^2/
d^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-3/2*I*c^
2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c
*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-3/2*I
*c^2*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/2*I*c^2*b^2/d^2*Pi
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x
+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*a^2/d^2/x^2-1/2*c^3*b^2/d^2*arc
tanh(c*x)/(c*x+1)*x+4*c*a*b/d^2*arctanh(c*x)/x+2*c^2*a*b/d^2*ln(c*x+1)-4*c^
2*b^2/d^2*arctanh(c*x)*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*b^2/d^2*arcta
nh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c^2*b^2/d^2*arctanh(c*x)*polyl
og(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c^2*b^2/d^2*arctanh(c*x)^2*ln(1-(c*x+1)
/(-c^2*x^2+1)^(1/2))+6*c^2*b^2/d^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2
+1)^(1/2))+3*c^2*b^2/d^2*arctanh(c*x)^2*ln(c*x)-3*c^2*b^2/d^2*arctanh(c*x)^
```

```
2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-3*c^2*b^2/d^2*arctanh(c*x)^2*ln(c*x+1)+c^2*b
^2/d^2*arctanh(c*x)^2/(c*x+1)+6*c^2*b^2/d^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2
*x^2+1)^(1/2))-4*c^2*a*b/d^2*ln(c*x)-3*c^2*a*b/d^2*dilog(c*x+1)-3*c^2*a*b/d
^2*dilog(c*x)-c*b^2/d^2/x*arctanh(c*x)+2*c*b^2/d^2*arctanh(c*x)^2/x+1/2*c^2
*b^2/d^2*arctanh(c*x)/(c*x+1)+3*c^2*b^2/d^2*arctanh(c*x)^2*ln(2)+3*c^2*a*b/
d^2*dilog(1/2+1/2*c*x)-a*b/d^2*arctanh(c*x)/x^2-c*a*b/d^2/x+3/2*I*c^2*b^2/d
^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))
*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)
^2-3/2*I*c^2*b^2/d^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arct
anh(c*x)^2+6*c^2*a*b/d^2*arctanh(c*x)*ln(c*x)-3*c^2*a*b/d^2*ln(c*x)*ln(c*x+
1)+2*c^2*a*b/d^2*arctanh(c*x)/(c*x+1)-6*c^2*a*b/d^2*arctanh(c*x)*ln(c*x+1)-
3*c^2*a*b/d^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3*c^2*a*b/d^2*ln(-1/2*c*x+1/2)*ln(
1/2+1/2*c*x)+3/2*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c
*x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^
2+1)+1))^3*arctanh(c*x)^2+3/2*I*c^2*b^2/d^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+c^2*a*b/d^2/(c*x+1)-1/4*
c^3*b^2/d^2/(c*x+1)*x+3/2*c^2*a*b/d^2*ln(c*x+1)^2+2*c^2*a*b/d^2*ln(c*x-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a^2\left(\frac{6c^2\log(cx+1)}{d^2}-\frac{6c^2\log(x)}{d^2}-\frac{6c^2x^2+3cx-1}{cd^2x^3+d^2x^2}\right)+\frac{\left(6b^2c^2x^2+3b^2cx-b^2-6\left(b^2c^3x^3+b^2c^2x^2\right)\log(cx+1)\right)}{8\left(cd^2x^3+d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*(6*c^2*log(c*x + 1)/d^2 - 6*c^2*log(x)/d^2 - (6*c^2*x^2 + 3*c*x -
1)/(c*d^2*x^3 + d^2*x^2)) + 1/8*(6*b^2*c^2*x^2 + 3*b^2*c*x - b^2 - 6*(b^2*c
^3*x^3 + b^2*c^2*x^2)*log(c*x + 1))*log(-c*x + 1)^2/(c*d^2*x^3 + d^2*x^2) +
 integrate(1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x
+ 1) - (6*b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 2*b^2*c^2*x^2 - 4*a*b + (4*a*b*c -
b^2*c)*x - 2*(3*b^2*c^5*x^5 + 6*b^2*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x + b^2
)*log(c*x + 1))*log(-c*x + 1))/(c^3*d^2*x^6 + c^2*d^2*x^5 - c*d^2*x^4 - d^2
*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}(cx)^2+2\,ab\,\text{artanh}(cx)+a^2}{c^2d^2x^5+2\,cd^2x^4+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^2*d^2*x^5 + 2*c
*d^2*x^4 + d^2*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^2x^5+2cx^4+x^3}\,dx+\int\frac{b^2\,\text{atanh}^2(cx)}{c^2x^5+2cx^4+x^3}\,dx+\int\frac{2ab\,\text{atanh}(cx)}{c^2x^5+2cx^4+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**3/(c*d*x+d)**2,x)
```

```
[Out] (Integral(a**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(b**2*atanh(c*x)
**2/(c**2*x**5 + 2*c*x**4 + x**3), x) + Integral(2*a*b*atanh(c*x)/(c**2*x**
5 + 2*c*x**4 + x**3), x))/d**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)^2}{(cdx+d)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^3/(c*d*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^2*x^3), x)
```

3.111 $\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=408

$$\frac{6b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}+\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^5d^3}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{x^2\left(a+b\tan\right.}{2c^3d}$$

```
[Out] (a*b*x)/(c^4*d^3) - b^2/(16*c^5*d^3*(1 + c*x)^2) + (29*b^2)/(16*c^5*d^3*(1
+ c*x)) - (29*b^2*ArcTanh[c*x])/(16*c^5*d^3) + (b^2*x*ArcTanh[c*x])/(c^4*d^
3) - (b*(a + b*ArcTanh[c*x]))/(4*c^5*d^3*(1 + c*x)^2) + (15*b*(a + b*ArcTan
h[c*x]))/(4*c^5*d^3*(1 + c*x)) - (43*(a + b*ArcTanh[c*x])^2)/(8*c^5*d^3) -
(3*x*(a + b*ArcTanh[c*x])^2)/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^
3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh
[c*x])^2)/(c^5*d^3*(1 + c*x)) + (6*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])
/(c^5*d^3) - (6*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^5*d^3) + (b^2*L
og[1 - c^2*x^2])/(2*c^5*d^3) + (3*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^5*d^3
) + (6*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3) + (3*b
^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^5*d^3)
```

Rubi [A] time = 0.810864, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 5928, 5926, 627, 44, 207, 6056, 6610}

$$\frac{6b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^5d^3}+\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^5d^3}+\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{c^5d^3}+\frac{x^2\left(a+b\tan\right.}{2c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```


```
[Out] (a*b*x)/(c^4*d^3) - b^2/(16*c^5*d^3*(1 + c*x)^2) + (29*b^2)/(16*c^5*d^3*(1
+ c*x)) - (29*b^2*ArcTanh[c*x])/(16*c^5*d^3) + (b^2*x*ArcTanh[c*x])/(c^4*d^
3) - (b*(a + b*ArcTanh[c*x]))/(4*c^5*d^3*(1 + c*x)^2) + (15*b*(a + b*ArcTan
h[c*x]))/(4*c^5*d^3*(1 + c*x)) - (43*(a + b*ArcTanh[c*x])^2)/(8*c^5*d^3) -
(3*x*(a + b*ArcTanh[c*x])^2)/(c^4*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*c^
3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^5*d^3*(1 + c*x)^2) + (4*(a + b*ArcTanh
[c*x])^2)/(c^5*d^3*(1 + c*x)) + (6*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])
/(c^5*d^3) - (6*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^5*d^3) + (b^2*L
og[1 - c^2*x^2])/(2*c^5*d^3) + (3*b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c^5*d^3
) + (6*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^5*d^3) + (3*b
^2*PolyLog[3, 1 - 2/(1 + c*x)])/(c^5*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
```

```
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
```

```
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \int\left(-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1+cx)^3}-\frac{4\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1\cdots}\right.$$

$$=\frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}dx}{c^4d^3}-\frac{3\int\left(a+b\tanh^{-1}(cx)\right)^2dx}{c^4d^3}-\frac{4\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{c^4d^3}+\frac{6\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+\cdots}}{c^4\cdots}$$

$$=-\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5d^3(1+cx)^2}+\frac{4\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3(1+\cdots}$$

$$=-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^5d^3}-\frac{3x\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^5d^3(1\cdots}$$

$$=\frac{abx}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)}-\frac{43\left(a+b\tanh^{-1}(cx)\right)^2}{8c^5d^3}-\frac{3x\left(a\cdots\right.}{}$$

$$=\frac{abx}{c^4d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)}-\frac{43\left(a+b\tanh^{-1}\cdots\right.}{8c^5d^3}$$

$$=\frac{abx}{c^4d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)}-\frac{43\left(a+b\tanh^{-1}\cdots\right.}{8c^5d^3}$$

$$=\frac{abx}{c^4d^3}-\frac{b^2}{16c^5d^3(1+cx)^2}+\frac{29b^2}{16c^5d^3(1+cx)}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^5d^3(1+cx)^2}+\frac{15b\cdots}{\cdots}$$

$$=\frac{abx}{c^4d^3}-\frac{b^2}{16c^5d^3(1+cx)^2}+\frac{29b^2}{16c^5d^3(1+cx)}-\frac{29b^2\tanh^{-1}(cx)}{16c^5d^3}+\frac{b^2x\tanh^{-1}(cx)}{c^4d^3}-\frac{b\left(a+b\cdots\right.}{4c^5\cdots}$$

Mathematica [A] time = 2.01257, size = 420, normalized size = 1.03

$$ab\left(96\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-48\log\left(1-c^2x^2\right)+4\tanh^{-1}(cx)\left(4c^2x^2-24cx-48\log\left(e^{-2\tanh^{-1}(cx)}+1\right)-14\sinh\cdots\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (-48*a^2*c*x + 8*a^2*c^2*x^2 - (8*a^2)/(1 + c*x)^2 + (64*a^2)/(1 + c*x) + 9
6*a^2*Log[1 + c*x] + a*b*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh
[c*x]] - 48*Log[1 - c^2*x^2] + 96*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 28*Sin
h[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(-4 - 24*c*x + 4*
c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2
*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])) + 16*b^2
*((-3 + 6*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + (56*Cosh[2*ArcTa
nh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[1 - c^2*x^2] + 192*PolyLog[3, -E^(
-2*ArcTanh[c*x])] - 56*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]] + 4*ArcT
anh[c*x]*(16*c*x + 28*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 96*Log[
1 + E^(-2*ArcTanh[c*x])] - 28*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]])
+ 8*ArcTanh[c*x]^2*(20 - 24*c*x + 4*c^2*x^2 + 14*Cosh[2*ArcTanh[c*x]] - Cos
h[4*ArcTanh[c*x]] - 48*Log[1 + E^(-2*ArcTanh[c*x])] - 14*Sinh[2*ArcTanh[c*x
]] + Sinh[4*ArcTanh[c*x]]))/64))/(16*c^5*d^3)
```

Maple [C] time = 0.816, size = 1565, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] -7/4/c^4*b^2/d^3*arctanh(c*x)/(c*x+1)*x-1/16/c^3*b^2/d^3*arctanh(c*x)/(c*x+
1)^2*x^2+1/8/c^4*b^2/d^3*arctanh(c*x)/(c*x+1)^2*x+1/c^3*a*b/d^3*arctanh(c*x
)*x^2-6/c^4*a*b/d^3*arctanh(c*x)*x-6/c^5*a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/
2*c*x)-1/c^5*a*b/d^3*arctanh(c*x)/(c*x+1)^2+8/c^5*a*b/d^3*arctanh(c*x)/(c*x
+1)+12/c^5*a*b/d^3*arctanh(c*x)*ln(c*x+1)+6/c^5*a*b/d^3*ln(-1/2*c*x+1/2)*ln
(c*x+1)-3*I/c^5*b^2/d^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-3*I/c^5*b^2/d^3*arctanh(c*x)^2*Pi*csgn(I/
((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x
^2+1)+1))^2-6*I/c^5*b^2/d^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c
*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2+3*I/c^5*b^2/d^3*Pi*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arct
anh(c*x)^2+3*I/c^5*b^2/d^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)*arctanh(c*x)^2-1/64*b^2/c^5/d^3/(c*x+1)^2+7/8*b^2/c^5/d^3/(c*x+1)+b^2*arc
tanh(c*x)/c^5/d^3+a*b*x/c^4/d^3+b^2*x*arctanh(c*x)/c^4/d^3-3*I/c^5*b^2/d^3*
Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^
2-3*I/c^5*b^2/d^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2+1/c^5*a
*b/d^3-1/2/c^5*a^2/d^3/(c*x+1)^2+4/c^5*a^2/d^3/(c*x+1)+1/2/c^3*a^2/d^3*x^2-
3/c^4*a^2/d^3*x-43/8/c^5*b^2/d^3*arctanh(c*x)^2-1/c^5*b^2/d^3*ln((c*x+1)^2/
(-c^2*x^2+1)+1)+6/c^5*b^2/d^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+4/c^5*b
^2/d^3*arctanh(c*x)^3+6/c^5*b^2/d^3*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+3
/c^5*b^2/d^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+6/c^5*a^2/d^3*ln(c*x+1)-1/2
/c^5*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+4/c^5*b^2/d^3*arctanh(c*x)^2/(c*x+1)-
6/c^5*b^2/d^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+7/4/c^5*b^2/d
^3*arctanh(c*x)/(c*x+1)-1/16/c^5*b^2/d^3*arctanh(c*x)/(c*x+1)^2+1/2/c^3*b^2
/d^3*arctanh(c*x)^2*x^2-3/c^4*b^2/d^3*arctanh(c*x)^2*x-6/c^5*a*b/d^3*dilog(
1/2+1/2*c*x)-3/c^5*a*b/d^3*ln(c*x+1)^2-5/8/c^5*a*b/d^3*ln(c*x-1)-43/8/c^5*a
*b/d^3*ln(c*x+1)-6/c^5*b^2/d^3*arctanh(c*x)^2*ln(2)+6/c^5*b^2/d^3*arctanh(c
*x)*ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-1/4/c^5*a*b/d^3/(c*x+1)^2+15/4/c^5*a
*b/d^3/(c*x+1)-1/64/c^3*b^2/d^3/(c*x+1)^2*x^2+1/32/c^4*b^2/d^3/(c*x+1)^2*x-
7/8/c^4*b^2/d^3/(c*x+1)*x+6/c^5*b^2/d^3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x
^2+1)^(1/2))-12/c^5*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))+6
/c^5*b^2/d^3*arctanh(c*x)^2*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{8\,cx+7}{c^7d^3x^2+2\,c^6d^3x+c^5d^3}+\frac{cx^2-6\,x}{c^4d^3}+\frac{12\,\log\left(cx+1\right)}{c^5d^3}\right)+\frac{\left(b^2c^4x^4-4\,b^2c^3x^3-11\,b^2c^2x^2+2\,b^2cx+7\,b^2+12\,\left(b^2\right.\right.}{8\left(c^7d^3x^2+2\,c^6d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((8*c*x + 7)/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) + (c*x^2 - 6*x)/
(c^4*d^3) + 12*log(c*x + 1)/(c^5*d^3)) + 1/8*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 -
 11*b^2*c^2*x^2 + 2*b^2*c*x + 7*b^2 + 12*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*lo
g(c*x + 1))*log(-c*x + 1)^2/(c^7*d^3*x^2 + 2*c^6*d^3*x + c^5*d^3) - integra
te(-1/4*((b^2*c^5*x^5 - b^2*c^4*x^4)*log(c*x + 1)^2 + 4*(a*b*c^5*x^5 - a*b*
c^4*x^4)*log(c*x + 1) + (15*b^2*c^3*x^3 + 9*b^2*c^2*x^2 - (4*a*b*c^5 + b^2*
c^5)*x^5 + (4*a*b*c^4 + 3*b^2*c^4)*x^4 - 9*b^2*c*x - 7*b^2 - 2*(b^2*c^5*x^5
 - b^2*c^4*x^4 + 6*b^2*c^3*x^3 + 18*b^2*c^2*x^2 + 18*b^2*c*x + 6*b^2)*log(c
*x + 1))*log(-c*x + 1))/(c^8*d^3*x^4 + 2*c^7*d^3*x^3 - 2*c^5*d^3*x - c^4*d^
```

```
3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^4\,\text{artanh}\,(cx)^2+2\,abx^4\,\text{artanh}\,(cx)+a^2x^4}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2x^4}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int\frac{b^2x^4\,\text{atanh}^2\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx+\int\frac{2abx^4\,\text{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**4/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**4*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**4*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\text{artanh}\,(cx)+a)^2x^4}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^4/(c*d*x + d)^3, x)
```

3.112 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=337

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^3}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(cx}$$

```
[Out] b^2/(16*c^4*d^3*(1 + c*x)^2) - (21*b^2)/(16*c^4*d^3*(1 + c*x)) + (21*b^2*Ar
cTanh[c*x])/(16*c^4*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)^2)
 - (11*b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)) + (19*(a + b*ArcTanh[c
*x])^2)/(8*c^4*d^3) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d^3) + (a + b*ArcTanh
[c*x])^2/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^3*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^4*d^3) + (3*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^3) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^4*d^3) - (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
4*d^3) - (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```

Rubi [A] time = 0.661555, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5928, 5926, 627, 44, 207, 5948, 6056, 6610}

$$-\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^4d^3}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4d^3}-\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^4d^3}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(cx}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```


```
[Out] b^2/(16*c^4*d^3*(1 + c*x)^2) - (21*b^2)/(16*c^4*d^3*(1 + c*x)) + (21*b^2*Ar
cTanh[c*x])/(16*c^4*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)^2)
 - (11*b*(a + b*ArcTanh[c*x]))/(4*c^4*d^3*(1 + c*x)) + (19*(a + b*ArcTanh[c
*x])^2)/(8*c^4*d^3) + (x*(a + b*ArcTanh[c*x])^2)/(c^3*d^3) + (a + b*ArcTanh
[c*x])^2/(2*c^4*d^3*(1 + c*x)^2) - (3*(a + b*ArcTanh[c*x])^2)/(c^4*d^3*(1 +
 c*x)) - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(c^4*d^3) + (3*(a + b*
ArcTanh[c*x])^2*Log[2/(1 + c*x)])/(c^4*d^3) - (b^2*PolyLog[2, 1 - 2/(1 - c*
x)])/(c^4*d^3) - (3*b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/(c^
4*d^3) - (3*b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^4*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx=\int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)^3}+\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)^2}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1 \ldots}\right.$$

$$=\frac{\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{c^3d^3}-\frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{c^3d^3}+\frac{3\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{c^3d^3}-\frac{3\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{\ldots}}{c \ldots}$$

$$=\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^4d^3(1+cx)^2}-\frac{3\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3(1+cx)}+\frac{3\left(a+b\tanh \ldots\right.}{\ldots}$$

$$=\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^4d^3}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^4d^3(1+cx)^2}-\frac{3\left(a+b\tanh^{-}\ldots\right.}{c^4d^3(1+c\ldots}$$

$$=\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)}+\frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3}+\frac{x\left(a+b\,\mathrm{ta}\ldots\right.}{c^3\ldots}$$

$$=\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)}+\frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3}+\frac{x\left(a+b\,\mathrm{ta}\ldots\right.}{c^3\ldots}$$

$$=\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)}+\frac{19\left(a+b\tanh^{-1}(cx)\right)^2}{8c^4d^3}+\frac{x\left(a+b\,\mathrm{ta}\ldots\right.}{c^3\ldots}$$

$$=\frac{b^2}{16c^4d^3(1+cx)^2}-\frac{21b^2}{16c^4d^3(1+cx)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2}-\frac{11b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)}+\ldots$$

$$=\frac{b^2}{16c^4d^3(1+cx)^2}-\frac{21b^2}{16c^4d^3(1+cx)}+\frac{21b^2\tanh^{-1}(cx)}{16c^4d^3}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^4d^3(1+cx)^2}-\frac{11b\left(a\ldots\right.}{4\ldots}$$

Mathematica [A] time = 1.30706, size = 418, normalized size = 1.24

$$\frac{4ab\left(-48\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+16\log\left(1-c^2x^2\right)+20\sinh\left(2\tanh^{-1}(cx)\right)-\sinh\left(4\tanh^{-1}(cx)\right)-20\cosh\left(2\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (64*a^2*c*x + (32*a^2)/(1 + c*x)^2 - (192*a^2)/(1 + c*x) - 192*a^2*Log[1 +
c*x] + 4*a*b*(-20*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 16*Log[1 -
c^2*x^2] - 48*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 20*Sinh[2*ArcTanh[c*x]] +
4*ArcTanh[c*x]*(8*c*x - 10*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 24
*Log[1 + E^(-2*ArcTanh[c*x])] + 10*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*
x]]) - Sinh[4*ArcTanh[c*x]]) + b^2*(-64*ArcTanh[c*x]^2 + 64*c*x*ArcTanh[c*x
]^2 - 40*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*A
rcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*
Cosh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Cosh[4*ArcTanh[c*x]] - 128*ArcTanh[
c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 192*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTan
h[c*x])] - 64*(-1 + 3*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 96*P
olyLog[3, -E^(-2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]
*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - Sinh[4*Arc
Tanh[c*x]] - 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 8*ArcTanh[c*x]^2*Sinh[4*
ArcTanh[c*x]]))/(64*c^4*d^3)
```

Maple [C] time = 0.556, size = 5750, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a^2\left(\frac{6\,cx+5}{c^6d^3x^2+2\,c^5d^3x+c^4d^3}-\frac{2\,x}{c^3d^3}+\frac{6\,\log\left(cx+1\right)}{c^4d^3}\right)+\frac{\left(2\,b^2c^3x^3+4\,b^2c^2x^2-4\,b^2cx-5\,b^2-6\left(b^2c^2x^2+2\,b^2cx+b^2\right)\right.}{8\left(c^6d^3x^2+2\,c^5d^3x+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*c*x + 5)/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - 2*x/(c^3*d^3)
 + 6*log(c*x + 1)/(c^4*d^3)) + 1/8*(2*b^2*c^3*x^3 + 4*b^2*c^2*x^2 - 4*b^2*c
*x - 5*b^2 - 6*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*log(c*x + 1))*log(-c*x + 1)^
2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3) - integrate(-1/4*((b^2*c^4*x^4 - b^
2*c^3*x^3)*log(c*x + 1)^2 + 4*(a*b*c^4*x^4 - a*b*c^3*x^3)*log(c*x + 1) - (2
*(2*a*b*c^4 + b^2*c^4)*x^4 - 9*b^2*c*x - 2*(2*a*b*c^3 - 3*b^2*c^3)*x^3 - 5*
b^2 + 2*(b^2*c^4*x^4 - 4*b^2*c^3*x^3 - 9*b^2*c^2*x^2 - 9*b^2*c*x - 3*b^2)*l
og(c*x + 1))*log(-c*x + 1))/(c^7*d^3*x^4 + 2*c^6*d^3*x^3 - 2*c^4*d^3*x - c^
3*d^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^3\,\mathrm{artanh}\left(cx\right)^2+2\,abx^3\,\mathrm{artanh}\left(cx\right)+a^2x^3}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^3*arctanh(c*x)^2 + 2*a*b*x^3*arctanh(c*x) + a^2*x^3)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^3}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^3\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^3\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**3/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**3*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**3*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx)+a)^2x^3}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^3/(c*d*x + d)^3, x)
```

3.113 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=265

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)^2}+$$

```
[Out] -b^2/(16*c^3*d^3*(1 + c*x)^2) + (13*b^2)/(16*c^3*d^3*(1 + c*x)) - (13*b^2*A
rcTanh[c*x])/(16*c^3*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)^2
) + (7*b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)) - (7*(a + b*ArcTanh[c*
x])^2)/(8*c^3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^3*d^3*(1 + c*x)^2) + (2*(a
 + b*ArcTanh[c*x])^2)/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(
1 + c*x)])/(c^3*d^3) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])
/(c^3*d^3) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rubi [A] time = 0.543568, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5940, 5928, 5926, 627, 44, 207, 5948, 5918, 6056, 6610}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^3d^3}+\frac{b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)}{2c^3d^3}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(cx+1)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] -b^2/(16*c^3*d^3*(1 + c*x)^2) + (13*b^2)/(16*c^3*d^3*(1 + c*x)) - (13*b^2*A
rcTanh[c*x])/(16*c^3*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)^2
) + (7*b*(a + b*ArcTanh[c*x]))/(4*c^3*d^3*(1 + c*x)) - (7*(a + b*ArcTanh[c*
x])^2)/(8*c^3*d^3) - (a + b*ArcTanh[c*x])^2/(2*c^3*d^3*(1 + c*x)^2) + (2*(a
 + b*ArcTanh[c*x])^2)/(c^3*d^3*(1 + c*x)) - ((a + b*ArcTanh[c*x])^2*Log[2/(
1 + c*x)])/(c^3*d^3) + (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])
/(c^3*d^3) + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*c^3*d^3)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
```

```
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}dx=\int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)^3}-\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{c^2d^3(1+cx)}\right)dx$$

$$=\frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}dx}{c^2d^3}+\frac{\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1+cx}dx}{c^2d^3}-\frac{2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}dx}{c^2d^3}$$

$$=-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b\int\Big(a$$

$$=-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^3d^3(1+cx)^2}+\frac{2\left(a+b\tanh^{-1}(cx)\right)^2}{c^3d^3(1+cx)}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{c^3d^3}+\frac{b\big(a+}{}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)}-\frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3}-\frac{\left(a+b\tanh^{-1}\right.}{2c^3d^3(1+c}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)}-\frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3}-\frac{\left(a+b\tanh^{-1}\right.}{2c^3d^3(1+c}$$

$$=-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)}-\frac{7\left(a+b\tanh^{-1}(cx)\right)^2}{8c^3d^3}-\frac{\left(a+b\tanh^{-1}\right.}{2c^3d^3(1+c}$$

$$=-\frac{b^2}{16c^3d^3(1+cx)^2}+\frac{13b^2}{16c^3d^3(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}+\frac{7b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)}-7\big($$

$$=-\frac{b^2}{16c^3d^3(1+cx)^2}+\frac{13b^2}{16c^3d^3(1+cx)}-\frac{13b^2\tanh^{-1}(cx)}{16c^3d^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^3d^3(1+cx)^2}+\frac{7b\big(a+}{4c^3}$$

Mathematica [A] time = 1.39504, size = 310, normalized size = 1.17

$$ab\left(16\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)-12\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)+12\cosh\left(2\tanh^{-1}(cx)\right)-\cosh\left(4\,\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] ((-8*a^2)/(1 + c*x)^2 + (32*a^2)/(1 + c*x) + 16*a^2*Log[1 + c*x] + 16*b^2*(
ArcTanh[c*x]*PolyLog[2, -E^(-2*ArcTanh[c*x])] + PolyLog[3, -E^(-2*ArcTanh[c
*x])]/2 + ((-Cosh[2*ArcTanh[c*x]] + Sinh[2*ArcTanh[c*x]])*(-24 + Cosh[2*Arc
Tanh[c*x]] + 4*ArcTanh[c*x]*(-12 + Cosh[2*ArcTanh[c*x]] - Sinh[2*ArcTanh[c*
x]]) - Sinh[2*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*(-6 + Cosh[2*ArcTanh[c*x]]*(
1 + 8*Log[1 + E^(-2*ArcTanh[c*x])]) + (-1 + 8*Log[1 + E^(-2*ArcTanh[c*x])])
*Sinh[2*ArcTanh[c*x]])))/64) + a*b*(12*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTan
h[c*x]] + 16*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + S
inh[4*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6*Cosh[2*ArcTanh[c*x]] - Cosh[4*ArcTa
nh[c*x]] - 8*Log[1 + E^(-2*ArcTanh[c*x])] - 6*Sinh[2*ArcTanh[c*x]] + Sinh[4
*ArcTanh[c*x]])))/(16*c^3*d^3)
```

Maple [C] time = 0.345, size = 1241, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] 1/2*I/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2
+1)+1))-1/64*b^2/c^3/d^3/(c*x+1)^2+3/8*b^2/c^3/d^3/(c*x+1)-1/2/c^3*a^2/d^3/
(c*x+1)^2+2/c^3*a^2/d^3/(c*x+1)+1/2/c^3*b^2/d^3*polylog(3,-(c*x+1)^2/(-c^2*
x^2+1))+1/c^3*a^2/d^3*ln(c*x+1)-7/8/c^3*b^2/d^3*arctanh(c*x)^2+2/3/c^3*b^2/
d^3*arctanh(c*x)^3-1/2*I/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-
c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2-I
/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))+1/2*I/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2-1/
2*I/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x
+1)/(-c^2*x^2+1)^(1/2))^2-3/8/c^2*b^2/d^3/(c*x+1)*x-1/2/c^3*a*b/d^3*ln(c*x+
1)^2+7/8/c^3*a*b/d^3*ln(c*x-1)-7/8/c^3*a*b/d^3*ln(c*x+1)-1/c^3*b^2/d^3*arct
anh(c*x)^2*ln(2)-2/c^3*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)
)+1/c^3*b^2/d^3*arctanh(c*x)^2*ln(c*x+1)-1/2/c^3*b^2/d^3*arctanh(c*x)^2/(c*
x+1)^2+2/c^3*b^2/d^3*arctanh(c*x)^2/(c*x+1)-1/c^3*b^2/d^3*arctanh(c*x)*poly
log(2,-(c*x+1)^2/(-c^2*x^2+1))+3/4/c^3*b^2/d^3*arctanh(c*x)/(c*x+1)-1/16/c^
3*b^2/d^3*arctanh(c*x)/(c*x+1)^2-1/c^3*a*b/d^3*dilog(1/2+1/2*c*x)-1/4/c^3*a
*b/d^3/(c*x+1)^2+7/4/c^3*a*b/d^3/(c*x+1)-1/64/c*b^2/d^3/(c*x+1)^2*x^2+1/32/
c^2*b^2/d^3/(c*x+1)^2*x-1/2*I/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^
2/(c^2*x^2-1))^3-1/2*I/c^3*b^2/d^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3-1/c^3*a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+
1/2*c*x)-3/4/c^2*b^2/d^3*arctanh(c*x)/(c*x+1)*x-1/16/c*b^2/d^3*arctanh(c*x)
/(c*x+1)^2*x^2+1/8/c^2*b^2/d^3*arctanh(c*x)/(c*x+1)^2*x-1/c^3*a*b/d^3*arcta
nh(c*x)/(c*x+1)^2+4/c^3*a*b/d^3*arctanh(c*x)/(c*x+1)+2/c^3*a*b/d^3*arctanh(
c*x)*ln(c*x+1)+1/c^3*a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{4\,cx+3}{c^5d^3x^2+2\,c^4d^3x+c^3d^3}+\frac{2\,\log\left(cx+1\right)}{c^3d^3}\right)+\frac{\left(4\,b^2cx+3\,b^2+2\left(b^2c^2x^2+2\,b^2cx+b^2\right)\log\left(cx+1\right)\right)\log\left(-cx+1\right)}{8\left(c^5d^3x^2+2\,c^4d^3x+c^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((4*c*x + 3)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) + 2*log(c*x + 1)
/(c^3*d^3)) + 1/8*(4*b^2*c*x + 3*b^2 + 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)*lo
g(c*x + 1))*log(-c*x + 1)^2/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - integra
te(-1/4*((b^2*c^3*x^3 - b^2*c^2*x^2)*log(c*x + 1)^2 + 4*(a*b*c^3*x^3 - a*b*
c^2*x^2)*log(c*x + 1) - (4*a*b*c^3*x^3 + 7*b^2*c*x - 4*(a*b*c^2 - b^2*c^2)*
x^2 + 3*b^2 + 2*(2*b^2*c^3*x^3 + 2*b^2*c^2*x^2 + 3*b^2*c*x + b^2)*log(c*x +
 1))*log(-c*x + 1))/(c^6*d^3*x^4 + 2*c^5*d^3*x^3 - 2*c^3*d^3*x - c^2*d^3),
x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2x^2\,\mathrm{artanh}\left(cx\right)^2+2\,abx^2\,\mathrm{artanh}\left(cx\right)+a^2x^2}{c^3d^3x^3+3\,c^2d^3x^2+3\,cd^3x+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(c^3*d
^3*x^3 + 3*c^2*d^3*x^2 + 3*c*d^3*x + d^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x^2\operatorname{atanh}^2(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx^2\operatorname{atanh}(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b*
*2*x**2*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(
2*a*b*x**2*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx)+a)^2x^2}{(cdx+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(c*d*x + d)^3, x)
```

3.114 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(cx+1)^2}-\frac{5b^2}{16c^2d^3(cx+1)}+\frac{}{1}$$

```
[Out] b^2/(16*c^2*d^3*(1 + c*x)^2) - (5*b^2)/(16*c^2*d^3*(1 + c*x)) + (5*b^2*ArcT
anh[c*x])/(16*c^2*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)^2) -
 (3*b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*d^3*(1 + c*x)^2)
```

Rubi [A] time = 0.21377, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {37, 5938, 5926, 627, 44, 207, 5948}

$$-\frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)}+\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(cx+1)^2}-\frac{5b^2}{16c^2d^3(cx+1)}+\frac{}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] b^2/(16*c^2*d^3*(1 + c*x)^2) - (5*b^2)/(16*c^2*d^3*(1 + c*x)) + (5*b^2*ArcT
anh[c*x])/(16*c^2*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)^2) -
 (3*b*(a + b*ArcTanh[c*x]))/(4*c^2*d^3*(1 + c*x)) - (a + b*ArcTanh[c*x])^2/
(8*c^2*d^3) + (x^2*(a + b*ArcTanh[c*x])^2)/(2*d^3*(1 + c*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 5938

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Di
st[(a + b*ArcTanh[c*x])^p, u, x] - Dist[b*c*p, Int[ExpandIntegrand[(a + b*A
rcTanh[c*x])^(p - 1), u/(1 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 - e^2, 0] && IntegersQ[m, q] && NeQ
[m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - (2bc)\int\left(\frac{a+b\tanh^{-1}(cx)}{4c^2d^3(1+cx)^3} - \frac{3\left(a+b\tanh^{-1}(cx)\right)}{8c^2d^3(1+cx)^2} - \frac{a+b\tanh}{8c^2d^3\left(-1+\right.}\right.$$

$$= \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx}{4cd^3} - \frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}\,dx}{2cd^3} + \frac{(3b)\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}}{4cd^3}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+b\tanh^{-1}(c\right.}{2d^3(1+cx)^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+b\tanh^{-1}(c\right.}{2d^3(1+cx)^2}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8c^2d^3} + \frac{x^2\left(a+b\tanh^{-1}(c\right.}{2d^3(1+cx)^2}$$

$$= \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)} - \frac{(a+}{}$$

$$= \frac{b^2}{16c^2d^3(1+cx)^2} - \frac{5b^2}{16c^2d^3(1+cx)} + \frac{5b^2\tanh^{-1}(cx)}{16c^2d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4c^2d^3(1+cx)^2} - \frac{3b\left(a+b\,\mathrm{ta}\right.}{4c^2d^3(1}$$

Mathematica [A] time = 0.308307, size = 150, normalized size = 0.96

$$\frac{-2\left(16a^2+12ab+5b^2\right)(cx+1)+2\left(8a^2+4ab+b^2\right)-b(12a+5b)(cx+1)^2\log(1-cx)+b(12a+5b)(cx+1)^2\log(cx+1}{32c^2d^3(cx+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + c*d*x)^3,x]
```

```
[Out] (2*(8*a^2 + 4*a*b + b^2) - 2*(16*a^2 + 12*a*b + 5*b^2)*(1 + c*x) - 8*b*(b*(
2 + 3*c*x) + a*(4 + 8*c*x))*ArcTanh[c*x] + 4*b^2*(-1 - 2*c*x + 3*c^2*x^2)*A
```

```
rcTanh[c*x]^2 - b*(12*a + 5*b)*(1 + c*x)^2*Log[1 - c*x] + b*(12*a + 5*b)*(1
 + c*x)^2*Log[1 + c*x])/(32*c^2*d^3*(1 + c*x)^2)
```

Maple [B] time = 0.066, size = 460, normalized size = 2.9

$$\frac{a^2}{2c^2d^3(cx+1)^2}-\frac{a^2}{c^2d^3(cx+1)}+\frac{b^2(\operatorname{Artanh}(cx))^2}{2c^2d^3(cx+1)^2}-\frac{b^2(\operatorname{Artanh}(cx))^2}{c^2d^3(cx+1)}-\frac{3b^2\operatorname{Artanh}(cx)\ln(cx-1)}{8c^2d^3}+\frac{b^2\operatorname{Artanh}}{4c^2d^3(cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] 1/2/c^2*a^2/d^3/(c*x+1)^2-1/c^2*a^2/d^3/(c*x+1)+1/2/c^2*b^2/d^3*arctanh(c*x
)^2/(c*x+1)^2-1/c^2*b^2/d^3*arctanh(c*x)^2/(c*x+1)-3/8/c^2*b^2/d^3*arctanh(
c*x)*ln(c*x-1)+1/4/c^2*b^2/d^3*arctanh(c*x)/(c*x+1)^2-3/4/c^2*b^2/d^3*arcta
nh(c*x)/(c*x+1)+3/8/c^2*b^2/d^3*arctanh(c*x)*ln(c*x+1)-3/32/c^2*b^2/d^3*ln(
c*x-1)^2+3/16/c^2*b^2/d^3*ln(c*x-1)*ln(1/2+1/2*c*x)-3/16/c^2*b^2/d^3*ln(-1/
2*c*x+1/2)*ln(1/2+1/2*c*x)+3/16/c^2*b^2/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-3/32
/c^2*b^2/d^3*ln(c*x+1)^2-5/32/c^2*b^2/d^3*ln(c*x-1)+1/16*b^2/c^2/d^3/(c*x+1
)^2-5/16*b^2/c^2/d^3/(c*x+1)+5/32/c^2*b^2/d^3*ln(c*x+1)+1/c^2*a*b/d^3*arcta
nh(c*x)/(c*x+1)^2-2/c^2*a*b/d^3*arctanh(c*x)/(c*x+1)-3/8/c^2*a*b/d^3*ln(c*x
-1)+1/4/c^2*a*b/d^3/(c*x+1)^2-3/4/c^2*a*b/d^3/(c*x+1)+3/8/c^2*a*b/d^3*ln(c*
x+1)
```

Maxima [B] time = 1.02855, size = 579, normalized size = 3.69

$$-\frac{(2cx+1)b^2\operatorname{artanh}(cx)^2}{2\left(c^4d^3x^2+2c^3d^3x+c^2d^3\right)}-\frac{1}{8}\left(c\left(\frac{2(3cx+2)}{c^5d^3x^2+2c^4d^3x+c^3d^3}-\frac{3\log(cx+1)}{c^3d^3}+\frac{3\log(cx-1)}{c^3d^3}\right)+\frac{8(2cx+1)\operatorname{artanh}}{c^4d^3x^2+2c^3d^3x+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x + 1)*b^2*arctanh(c*x)^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) -
 1/8*(c*(2*(3*c*x + 2)/(c^5*d^3*x^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x +
1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d^3)) + 8*(2*c*x + 1)*arctanh(c*x)/(c^4*
d^3*x^2 + 2*c^3*d^3*x + c^2*d^3))*a*b - 1/32*(4*c*(2*(3*c*x + 2)/(c^5*d^3*x
^2 + 2*c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^
3*d^3))*arctanh(c*x) + (3*(c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + 3*(c^2*x^2
 + 2*c*x + 1)*log(c*x - 1)^2 + 10*c*x - (5*c^2*x^2 + 10*c*x + 6*(c^2*x^2 +
2*c*x + 1)*log(c*x - 1) + 5)*log(c*x + 1) + 5*(c^2*x^2 + 2*c*x + 1)*log(c*x
 - 1) + 8)*c^2/(c^6*d^3*x^2 + 2*c^5*d^3*x + c^4*d^3))*b^2 - 1/2*(2*c*x + 1)
*a^2/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Fricas [A] time = 1.97724, size = 354, normalized size = 2.25

$$-\frac{2\left(16a^2+12ab+5b^2\right)cx-\left(3b^2c^2x^2-2b^2cx-b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2+16a^2+16ab+8b^2-\left(\left(12ab+5b^2\right)c^2x^2-2\left(4\right.\right.}{32\left(c^4d^3x^2+2c^3d^3x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(16*a^2 + 12*a*b + 5*b^2)*c*x - (3*b^2*c^2*x^2 - 2*b^2*c*x - b^2)*
log(-(c*x + 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((12*a*b + 5*b^2)*c
^2*x^2 - 2*(4*a*b + b^2)*c*x - 4*a*b - 3*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c
^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2x}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2x\,\mathrm{atanh}^2\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2abx\,\mathrm{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2*x/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*
x*atanh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*
x*atanh(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2 x}{(cdx + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(c*d*x + d)^3, x)
```

3.115 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{3b^2}{16cd^3(cx+1)}-\frac{}{16cd^3(}$$

```
[Out] -b^2/(16*c*d^3*(1 + c*x)^2) - (3*b^2)/(16*c*d^3*(1 + c*x)) + (3*b^2*ArcTanh
[c*x])/(16*c*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)^2) - (b*(a
+ b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(8*c*d^3) -
 (a + b*ArcTanh[c*x])^2/(2*c*d^3*(1 + c*x)^2)
```

Rubi [A] time = 0.177749, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(cx+1)^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(cx+1)^2}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{3b^2}{16cd^3(cx+1)}-\frac{}{16cd^3(}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```


```
[Out] -b^2/(16*c*d^3*(1 + c*x)^2) - (3*b^2)/(16*c*d^3*(1 + c*x)) + (3*b^2*ArcTanh
[c*x])/(16*c*d^3) - (b*(a + b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)^2) - (b*(a
+ b*ArcTanh[c*x]))/(4*c*d^3*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(8*c*d^3) -
 (a + b*ArcTanh[c*x])^2/(2*c*d^3*(1 + c*x)^2)
```


Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(d+cdx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}+\frac{b\int\left(\frac{a+b\tanh^{-1}(cx)}{2d^2(1+cx)^3}+\frac{a+b\tanh^{-1}(cx)}{4d^2(1+cx)^2}-\frac{a+b\tanh^{-1}(cx)}{4d^2\left(-1+c^2x^2\right)}\right)dx}{d}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}+\frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}dx}{4d^3}-\frac{b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}dx}{4d^3}+\frac{b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^3}dx}{2d^3}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{8cd^3}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2cd^3(1+cx)^2}$$

$$= -\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)}+\frac{\left(a+b\text{ta}\right.}{8}$$

$$= -\frac{b^2}{16cd^3(1+cx)^2}-\frac{3b^2}{16cd^3(1+cx)}+\frac{3b^2\tanh^{-1}(cx)}{16cd^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{4cd^3(1+cx)^2}-\frac{b\left(a+b\tanh\right.}{4cd^3(1+c}$$

Mathematica [A] time = 0.136996, size = 183, normalized size = 1.17

$$\frac{-8a^2-4ab-b^2}{16cd^3(cx+1)^2}+\frac{\left(-4ab-3b^2\right)\log(1-cx)}{32cd^3}+\frac{\left(4ab+3b^2\right)\log(cx+1)}{32cd^3}-\frac{b(4a+3b)}{16cd^3(cx+1)}-\frac{b\tanh^{-1}(cx)(4a+bcx+2b)}{4cd^3(cx+1)^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + c*d*x)^3,x]
```

```
[Out] (-8*a^2 - 4*a*b - b^2)/(16*c*d^3*(1 + c*x)^2) - (b*(4*a + 3*b))/(16*c*d^3*(
1 + c*x)) - (b*(4*a + 2*b + b*c*x)*ArcTanh[c*x])/(4*c*d^3*(1 + c*x)^2) + (b
^2*(-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^2)/(8*c*d^3*(1 + c*x)^2) + ((-4*a*b
- 3*b^2)*Log[1 - c*x])/(32*c*d^3) + ((4*a*b + 3*b^2)*Log[1 + c*x])/(32*c*d^
3)
```

Maple [B] time = 0.062, size = 398, normalized size = 2.5

$$-\frac{a^2}{2\,cd^3\,(cx+1)^2}-\frac{b^2\,(\mathrm{Artanh}\,(cx))^2}{2\,cd^3\,(cx+1)^2}-\frac{b^2\mathrm{Artanh}\,(cx)\ln\,(cx-1)}{8\,cd^3}-\frac{b^2\mathrm{Artanh}\,(cx)}{4\,cd^3\,(cx+1)^2}-\frac{b^2\mathrm{Artanh}\,(cx)}{4\,cd^3\,(cx+1)}+\frac{b^2\mathrm{Artanh}\,(cx}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x)
```

```
[Out] -1/2/c*a^2/d^3/(c*x+1)^2-1/2/c*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2-1/8/c*b^2/d
^3*arctanh(c*x)*ln(c*x-1)-1/4/c*b^2/d^3*arctanh(c*x)/(c*x+1)^2-1/4/c*b^2/d^
3*arctanh(c*x)/(c*x+1)+1/8/c*b^2/d^3*arctanh(c*x)*ln(c*x+1)-1/32/c*b^2/d^3*
ln(c*x-1)^2+1/16/c*b^2/d^3*ln(c*x-1)*ln(1/2+1/2*c*x)-1/16/c*b^2/d^3*ln(-1/2
*c*x+1/2)*ln(1/2+1/2*c*x)+1/16/c*b^2/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/32/c*
b^2/d^3*ln(c*x+1)^2-3/32/c*b^2/d^3*ln(c*x-1)-1/16*b^2/c/d^3/(c*x+1)^2-3/16*
b^2/c/d^3/(c*x+1)+3/32/c*b^2/d^3*ln(c*x+1)-1/c*a*b/d^3*arctanh(c*x)/(c*x+1)
^2-1/8/c*a*b/d^3*ln(c*x-1)-1/4/c*a*b/d^3/(c*x+1)^2-1/4/c*a*b/d^3/(c*x+1)+1/
8/c*a*b/d^3*ln(c*x+1)
```

Maxima [B] time = 1.03299, size = 539, normalized size = 3.43

$$-\frac{1}{8}\left(c\left(\frac{2\,(cx+2)}{c^4d^3x^2+2\,c^3d^3x+c^2d^3}-\frac{\log\,(cx+1)}{c^2d^3}+\frac{\log\,(cx-1)}{c^2d^3}\right)+\frac{8\,\mathrm{artanh}\,(cx)}{c^3d^3x^2+2\,c^2d^3x+cd^3}\right)ab-\frac{1}{32}\left(4\,c\left(\frac{2\,(cx+2}{c^4d^3x^2+2\,c^3d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2*d^3) - log(c*x + 1)/(
c^2*d^3) + log(c*x - 1)/(c^2*d^3)) + 8*arctanh(c*x)/(c^3*d^3*x^2 + 2*c^2*d^
3*x + c*d^3))*a*b - 1/32*(4*c*(2*(c*x + 2)/(c^4*d^3*x^2 + 2*c^3*d^3*x + c^2
*d^3) - log(c*x + 1)/(c^2*d^3) + log(c*x - 1)/(c^2*d^3))*arctanh(c*x) + ((c
^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 +
 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log
(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*d^3*x^2 + 2*
c^4*d^3*x + c^3*d^3))*b^2 - 1/2*b^2*arctanh(c*x)^2/(c^3*d^3*x^2 + 2*c^2*d^3
*x + c*d^3) - 1/2*a^2/(c^3*d^3*x^2 + 2*c^2*d^3*x + c*d^3)
```

Fricas [A] time = 1.98649, size = 338, normalized size = 2.15

$$-\frac{2\left(4\,ab+3\,b^2\right)cx-\left(b^2c^2x^2+2\,b^2cx-3\,b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2+16\,a^2+16\,ab+8\,b^2-\left(\left(4\,ab+3\,b^2\right)c^2x^2+2\left(4\,ab+b^2\right)cx\right.}{32\left(c^3d^3x^2+2\,c^2d^3x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(2*(4*a*b + 3*b^2)*c*x - (b^2*c^2*x^2 + 2*b^2*c*x - 3*b^2)*log(-(c*x
+ 1)/(c*x - 1))^2 + 16*a^2 + 16*a*b + 8*b^2 - ((4*a*b + 3*b^2)*c^2*x^2 + 2*
(4*a*b + b^2)*c*x - 12*a*b - 5*b^2)*log(-(c*x + 1)/(c*x - 1)))/(c^3*d^3*x^2
 + 2*c^2*d^3*x + c*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a^2}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{b^2\,\mathrm{atanh}^2\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx + \int \frac{2ab\,\mathrm{atanh}\,(cx)}{c^3x^3+3c^2x^2+3cx+1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(b**2*at
anh(c*x)**2/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x) + Integral(2*a*b*atan
h(c*x)/(c**3*x**3 + 3*c**2*x**2 + 3*c*x + 1), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2}{(cdx + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(c*d*x + d)^3, x)
```

3.116 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^3}\,dx$

Optimal. Leaf size=362

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(}{d^3}$$

```
[Out] b^2/(16*d^3*(1 + c*x)^2) + (11*b^2)/(16*d^3*(1 + c*x)) - (11*b^2*ArcTanh[c*
x])/(16*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) + (5*b*(a + b*A
rcTanh[c*x]))/(4*d^3*(1 + c*x)) - (5*(a + b*ArcTanh[c*x])^2)/(8*d^3) + (a +
 b*ArcTanh[c*x])^2/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])^2/(d^3*(1 + c
*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 + ((a + b*Ar
cTanh[c*x])^2*Log[2/(1 + c*x)])/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1
- 2/(1 - c*x)])/d^3 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])
/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 + (b^2*Poly
Log[3, 1 - 2/(1 - c*x)])/(2*d^3) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^
3) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rubi [A] time = 0.804001, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)(}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

```
[Out] b^2/(16*d^3*(1 + c*x)^2) + (11*b^2)/(16*d^3*(1 + c*x)) - (11*b^2*ArcTanh[c*
x])/(16*d^3) + (b*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) + (5*b*(a + b*A
rcTanh[c*x]))/(4*d^3*(1 + c*x)) - (5*(a + b*ArcTanh[c*x])^2)/(8*d^3) + (a +
 b*ArcTanh[c*x])^2/(2*d^3*(1 + c*x)^2) + (a + b*ArcTanh[c*x])^2/(d^3*(1 + c
*x)) + (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 + ((a + b*Ar
cTanh[c*x])^2*Log[2/(1 + c*x)])/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1
- 2/(1 - c*x)])/d^3 + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])
/d^3 - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 + (b^2*Poly
Log[3, 1 - 2/(1 - c*x)])/(2*d^3) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d^
3) - (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^3)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
```

reeQ[{a, b, c}, x] && IGtQ[p, 1]

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+cdx)^3}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^3} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^2} - \frac{c\left(a+b\tanh\right.}{d^3(1+}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx}{d^3} - \frac{c\int \frac{\left(a+b\tanh^{-1}(cx)\right.}{1+cx}}{d^3}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^3} +$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} + \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^3} +$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{\left(a+b\tanh^{-1}(\right.}{2d^3(1+cx)}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{\left(a+b\tanh^{-1}(\right.}{2d^3(1+cx)}$$

$$= \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} + \frac{\left(a+b\tanh^{-1}(\right.}{2d^3(1+cx)}$$

$$= \frac{b^2}{16d^3(1+cx)^2} + \frac{11b^2}{16d^3(1+cx)} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} - \frac{5\left(a+b\right.}{}$$

$$= \frac{b^2}{16d^3(1+cx)^2} + \frac{11b^2}{16d^3(1+cx)} - \frac{11b^2\tanh^{-1}(cx)}{16d^3} + \frac{b\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} + \frac{5b\left(a+b\tan\right.}{4d^3(1+}$$

Mathematica [C] time = 1.40131, size = 376, normalized size = 1.04

$$12ab\left(-16\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)-12\sinh\left(2\tanh^{-1}(cx)\right)-\sinh\left(4\tanh^{-1}(cx)\right)+12\cosh\left(2\tanh^{-1}(cx)\right)+\cosh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + c*d*x)^3),x]
```

```
[Out] ((96*a^2)/(1 + c*x)^2 + (192*a^2)/(1 + c*x) + 192*a^2*Log[c*x] - 192*a^2*Lo
g[1 + c*x] + 12*a*b*(12*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] - 16*Po
lyLog[2, E^(-2*ArcTanh[c*x])] - 12*Sinh[2*ArcTanh[c*x]] + 4*ArcTanh[c*x]*(6
*Cosh[2*ArcTanh[c*x]] + Cosh[4*ArcTanh[c*x]] + 8*Log[1 - E^(-2*ArcTanh[c*x]
)] - 6*Sinh[2*ArcTanh[c*x]] - Sinh[4*ArcTanh[c*x]]) - Sinh[4*ArcTanh[c*x]])
 + b^2*((8*I)*Pi^3 - 128*ArcTanh[c*x]^3 + 72*Cosh[2*ArcTanh[c*x]] + 144*Arc
Tanh[c*x]*Cosh[2*ArcTanh[c*x]] + 144*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]] +
3*Cosh[4*ArcTanh[c*x]] + 12*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] + 24*ArcTanh[
c*x]^2*Cosh[4*ArcTanh[c*x]] + 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])
] + 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] - 96*PolyLog[3, E^(2*Ar
cTanh[c*x])] - 72*Sinh[2*ArcTanh[c*x]] - 144*ArcTanh[c*x]*Sinh[2*ArcTanh[c*
x]] - 144*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] - 3*Sinh[4*ArcTanh[c*x]] - 12
*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] - 24*ArcTanh[c*x]^2*Sinh[4*ArcTanh[c*x]]
))/(192*d^3)
```

Maple [C] time = 0.432, size = 1752, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x)
```

```
[Out] 1/2*a^2/d^3/(c*x+1)^2+a^2/d^3/(c*x+1)-a^2/d^3*ln(c*x+1)+a^2/d^3*ln(c*x)-5/8
*b^2/d^3*arctanh(c*x)^2-2/3*b^2/d^3*arctanh(c*x)^3+1/2*I*b^2/d^3*Pi*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^2/
d^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csg
n(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2+1/64*b
^2/d^3/(c*x+1)^2+3/8*b^2/d^3/(c*x+1)+1/2*I*b^2/d^3*Pi*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-1/2*I*b^2/d^3*
Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-
c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*b^2/d^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*ar
ctanh(c*x)^2-1/2*I*b^2/d^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*
b^2/d^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/4*b^2/d^3*arcta
nh(c*x)/(c*x+1)*c*x+1/16*b^2/d^3*arctanh(c*x)/(c*x+1)^2*c^2*x^2-1/8*b^2/d^3
*arctanh(c*x)/(c*x+1)^2*c*x+1/2*I*b^2/d^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
(c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*I*b^2/d^3*Pi*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+1/2*I*b^2/
d^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+I*b^2/d^3*Pi*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-2*b^2/d^3*poly
log(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d^3*polylog(3,(c*x+1)/(-c^2*x^2+1)
^(1/2))+1/64*b^2/d^3/(c*x+1)^2*c^2*x^2-1/32*b^2/d^3/(c*x+1)^2*c*x-3/8*b^2/d
^3/(c*x+1)*c*x+a*b/d^3*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+a*b/d^3*arctanh(c*x
)/(c*x+1)^2+2*a*b/d^3*arctanh(c*x)/(c*x+1)-2*a*b/d^3*arctanh(c*x)*ln(c*x+1)
-a*b/d^3*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/4*a*b/d^3/(c*x+1)^2+5/4*a*b/d^3/(c*x+
1)+a*b/d^3*dilog(1/2+1/2*c*x)+1/2*a*b/d^3*ln(c*x+1)^2+5/8*a*b/d^3*ln(c*x-1)
-5/8*a*b/d^3*ln(c*x+1)+2*b^2/d^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/
2))-b^2/d^3*arctanh(c*x)^2*ln(c*x+1)+1/2*b^2/d^3*arctanh(c*x)^2/(c*x+1)^2+b
^2/d^3*arctanh(c*x)^2/(c*x+1)+b^2/d^3*arctanh(c*x)^2*ln(2)+3/4*b^2/d^3*arct
anh(c*x)/(c*x+1)+1/16*b^2/d^3*arctanh(c*x)/(c*x+1)^2+2*b^2/d^3*arctanh(c*x)
*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-b^2/d^3*arctanh(c*x)^2*ln((c*x+1)^2
```

```
/(-c^2*x^2+1)-1)+b^2/d^3*arctanh(c*x)^2*ln(c*x)-a*b/d^3*dilog(c*x)-a*b/d^3*
dilog(c*x+1)+b^2/d^3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/
d^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d^3*arctanh(c*x)
^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*a*b/d^3*arctanh(c*x)*ln(c*x)-a*b/d^3*
ln(c*x)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{2\,cx+3}{c^2d^3x^2+2\,cd^3x+d^3}-\frac{2\,\log\left(cx+1\right)}{d^3}+\frac{2\,\log\left(x\right)}{d^3}\right)+\frac{\left(2\,b^2cx+3\,b^2-2\left(b^2c^2x^2+2\,b^2cx+b^2\right)\log\left(cx+1\right)\right)\log\left(\right.}{8\left(c^2d^3x^2+2\,cd^3x+d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((2*c*x + 3)/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) - 2*log(c*x + 1)/d^3 +
 2*log(x)/d^3) + 1/8*(2*b^2*c*x + 3*b^2 - 2*(b^2*c^2*x^2 + 2*b^2*c*x + b^2)
*log(c*x + 1))*log(-c*x + 1)^2/(c^2*d^3*x^2 + 2*c*d^3*x + d^3) + integrate(
1/4*((b^2*c*x - b^2)*log(c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) - (2*b
^2*c^3*x^3 + 5*b^2*c^2*x^2 - 4*a*b + (4*a*b*c + 3*b^2*c)*x - 2*(b^2*c^4*x^4
 + 3*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2)*log(c*x + 1))*log(-c*x + 1))/(c^4*d
^3*x^5 + 2*c^3*d^3*x^4 - 2*c*d^3*x^2 - d^3*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\,\mathrm{artanh}\left(cx\right)^2+2\,ab\,\mathrm{artanh}\left(cx\right)+a^2}{c^3d^3x^4+3\,c^2d^3x^3+3\,cd^3x^2+d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^4 + 3*c
^2*d^3*x^3 + 3*c*d^3*x^2 + d^3*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^3x^4+3c^2x^3+3cx^2+x}\,dx+\int\frac{b^2\,\mathrm{atanh}^2\left(cx\right)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx+\int\frac{2ab\,\mathrm{atanh}\left(cx\right)}{c^3x^4+3c^2x^3+3cx^2+x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(b**2
*atanh(c*x)**2/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x) + Integral(2*a*
b*atanh(c*x)/(c**3*x**4 + 3*c**2*x**3 + 3*c*x**2 + x), x))/d**3
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2}{(cdx + d)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x), x)
```

3.117 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^3}\,dx$

Optimal. Leaf size=448

$$\frac{3bc\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{3bc\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{3bc\text{PolyLog}\left(2,1-\frac{}{c}\right.}{}$$

```
[Out] -(b^2*c)/(16*d^3*(1 + c*x)^2) - (19*b^2*c)/(16*d^3*(1 + c*x)) + (19*b^2*c*A
rcTanh[c*x])/(16*d^3) - (b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) - (9
*b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)) + (17*c*(a + b*ArcTanh[c*x])^2
)/(8*d^3) - (a + b*ArcTanh[c*x])^2/(d^3*x) - (c*(a + b*ArcTanh[c*x])^2)/(2*
d^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x])^2)/(d^3*(1 + c*x)) - (6*c*(a +
 b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 - (3*c*(a + b*ArcTanh[c*x]
)^2*Log[2/(1 + c*x)])/d^3 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)
])/d^3 + (3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^3 - (3*
b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^3 + (3*b*c*(a + b*
ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 - (b^2*c*PolyLog[2, -1 + 2/(
1 + c*x)])/d^3 - (3*b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*P
olyLog[3, -1 + 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*PolyLog[3, 1 - 2/(1 + c*x)]
)/(2*d^3)
```

Rubi [A] time = 0.982703, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5928, 5926, 627, 44, 207, 5918, 6056}

$$\frac{3bc\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}-\frac{3bc\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^3}+\frac{3bc\text{PolyLog}\left(2,1-\frac{}{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```


```
[Out] -(b^2*c)/(16*d^3*(1 + c*x)^2) - (19*b^2*c)/(16*d^3*(1 + c*x)) + (19*b^2*c*A
rcTanh[c*x])/(16*d^3) - (b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)^2) - (9
*b*c*(a + b*ArcTanh[c*x]))/(4*d^3*(1 + c*x)) + (17*c*(a + b*ArcTanh[c*x])^2
)/(8*d^3) - (a + b*ArcTanh[c*x])^2/(d^3*x) - (c*(a + b*ArcTanh[c*x])^2)/(2*
d^3*(1 + c*x)^2) - (2*c*(a + b*ArcTanh[c*x])^2)/(d^3*(1 + c*x)) - (6*c*(a +
 b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^3 - (3*c*(a + b*ArcTanh[c*x]
)^2*Log[2/(1 + c*x)])/d^3 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)
])/d^3 + (3*b*c*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^3 - (3*
b*c*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^3 + (3*b*c*(a + b*
ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d^3 - (b^2*c*PolyLog[2, -1 + 2/(
1 + c*x)])/d^3 - (3*b^2*c*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*P
olyLog[3, -1 + 2/(1 - c*x)])/(2*d^3) + (3*b^2*c*PolyLog[3, 1 - 2/(1 + c*x)]
)/(2*d^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+cdx)^3}\,dx = \int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x^2} - \frac{3c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} + \frac{c^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)^3} + \frac{2c^2\left(a+b\tan\right.}{d^3(1+}\right.$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}dx}{d^3} - \frac{(3c)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}dx}{d^3} + \frac{c^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^3}dx}{d^3} + \frac{\left(2c^2\right)\int \frac{\left(a+b\tanh\right.}{(1+c}}{d^3}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3(1+cx)} - \frac{6c\left(a+b\tanh^{-1}\right.}{}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d^3x} - \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{2d^3(1+cx)^2} - \frac{2c\left(a+b\tanh^{-1}(cx\right.}{d^3(1+cx)}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tanh\right.}{d^3x}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tanh\right.}{d^3x}$$

$$= -\frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a+b\tanh^{-1}(cx)\right)^2}{8d^3} - \frac{\left(a+b\tanh\right.}{d^3x}$$

$$= -\frac{b^2c}{16d^3(1+cx)^2} - \frac{19b^2c}{16d^3(1+cx)} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)} + \frac{17c\left(a\right.}{}$$

$$= -\frac{b^2c}{16d^3(1+cx)^2} - \frac{19b^2c}{16d^3(1+cx)} + \frac{19b^2c\tanh^{-1}(cx)}{16d^3} - \frac{bc\left(a+b\tanh^{-1}(cx)\right)}{4d^3(1+cx)^2} - \frac{9bc\left(a+b\tan\right.}{4d^3(1+}$$

Mathematica [C] time = 2.26037, size = 479, normalized size = 1.07

$$\frac{\frac{4ab\left(48cx\,\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+cx\left(32\log\left(\frac{cx}{\sqrt{1-c^2x^2}}\right)+20\sinh\left(2\tanh^{-1}(cx)\right)+\sinh\left(4\tanh^{-1}(cx)\right)-20\cosh\left(2\tanh^{-1}(cx)\right)-\cosh\left(4\tanh^{-1}(cx)\right)\right)-4\tanh^{-1}\right.}{x}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + c*d*x)^3),x]
```

```
[Out] ((-64*a^2)/x - (32*a^2*c)/(1 + c*x)^2 - (128*a^2*c)/(1 + c*x) - 192*a^2*c*L
og[x] + 192*a^2*c*Log[1 + c*x] + b^2*c*((-8*I)*Pi^3 + 64*ArcTanh[c*x]^2 - (
64*ArcTanh[c*x]^2)/(c*x) + 128*ArcTanh[c*x]^3 - 40*Cosh[2*ArcTanh[c*x]] - 8
0*ArcTanh[c*x]*Cosh[2*ArcTanh[c*x]] - 80*ArcTanh[c*x]^2*Cosh[2*ArcTanh[c*x]
] - Cosh[4*ArcTanh[c*x]] - 4*ArcTanh[c*x]*Cosh[4*ArcTanh[c*x]] - 8*ArcTanh[
c*x]^2*Cosh[4*ArcTanh[c*x]] + 128*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])]
 - 192*ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 64*PolyLog[2, E^(-2*Arc
Tanh[c*x])] - 192*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 96*PolyLog[
3, E^(2*ArcTanh[c*x])] + 40*Sinh[2*ArcTanh[c*x]] + 80*ArcTanh[c*x]*Sinh[2*A
rcTanh[c*x]] + 80*ArcTanh[c*x]^2*Sinh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]
] + 4*ArcTanh[c*x]*Sinh[4*ArcTanh[c*x]] + 8*ArcTanh[c*x]^2*Sinh[4*ArcTanh[c
*x]]) + (4*a*b*(48*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*x*(-20*Cosh[2*Ar
cTanh[c*x]] - Cosh[4*ArcTanh[c*x]] + 32*Log[(c*x)/Sqrt[1 - c^2*x^2]] + 20*S
inh[2*ArcTanh[c*x]] + Sinh[4*ArcTanh[c*x]]) - 4*ArcTanh[c*x]*(8 + 10*c*x*Co
sh[2*ArcTanh[c*x]] + c*x*Cosh[4*ArcTanh[c*x]] + 24*c*x*Log[1 - E^(-2*ArcTan
h[c*x])] - 10*c*x*Sinh[2*ArcTanh[c*x]] - c*x*Sinh[4*ArcTanh[c*x]])))/x)/(64
*d^3)
```

Maple [C] time = 0.723, size = 7593, normalized size = 17.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a^2\left(\frac{6c^2x^2+9cx+2}{c^2d^3x^3+2cd^3x^2+d^3x}-\frac{6c\log(cx+1)}{d^3}+\frac{6c\log(x)}{d^3}\right)-\frac{\left(6b^2c^2x^2+9b^2cx+2b^2-6\left(b^2c^3x^3+2b^2c^2x^2+b^2c\right.\right.}{8\left(c^2d^3x^3+2cd^3x^2+d^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a^2*((6*c^2*x^2 + 9*c*x + 2)/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - 6*c
*log(c*x + 1)/d^3 + 6*c*log(x)/d^3) - 1/8*(6*b^2*c^2*x^2 + 9*b^2*c*x + 2*b^
2 - 6*(b^2*c^3*x^3 + 2*b^2*c^2*x^2 + b^2*c*x)*log(c*x + 1))*log(-c*x + 1)^2
/(c^2*d^3*x^3 + 2*c*d^3*x^2 + d^3*x) - integrate(-1/4*((b^2*c*x - b^2)*log(
c*x + 1)^2 + 4*(a*b*c*x - a*b)*log(c*x + 1) + (6*b^2*c^4*x^4 + 15*b^2*c^3*x
^3 + 11*b^2*c^2*x^2 + 4*a*b - 2*(2*a*b*c - b^2*c)*x - 2*(3*b^2*c^5*x^5 + 9*
b^2*c^4*x^4 + 9*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + b^2*c*x - b^2)*log(c*x + 1))*
log(-c*x + 1))/(c^4*d^3*x^6 + 2*c^3*d^3*x^5 - 2*c*d^3*x^3 - d^3*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\operatorname{artanh}(cx)^2+2ab\operatorname{artanh}(cx)+a^2}{c^3d^3x^5+3c^2d^3x^4+3cd^3x^3+d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(c^3*d^3*x^5 + 3*c
^2*d^3*x^4 + 3*c*d^3*x^3 + d^3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{a^2}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx+\int\frac{b^2\operatorname{atanh}^2(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx+\int\frac{2ab\operatorname{atanh}(cx)}{c^3x^5+3c^2x^4+3cx^3+x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(c*d*x+d)**3,x)
```

```
[Out] (Integral(a**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integral(b
**2*atanh(c*x)**2/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x) + Integra
l(2*a*b*atanh(c*x)/(c**3*x**5 + 3*c**2*x**4 + 3*c*x**3 + x**2), x))/d**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)^2}{(cdx+d)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(c*d*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((c*d*x + d)^3*x^2), x)
```

3.118 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^4}\,dx$

Optimal. Leaf size=176

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(cx+1)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(cx+1)^3}-$$

```
[Out] -b^2/(54*c*(1 + c*x)^3) - (5*b^2)/(144*c*(1 + c*x)^2) - (11*b^2)/(144*c*(1
+ c*x)) + (11*b^2*ArcTanh[c*x])/(144*c) - (b*(a + b*ArcTanh[c*x]))/(9*c*(1
+ c*x)^3) - (b*(a + b*ArcTanh[c*x]))/(12*c*(1 + c*x)^2) - (b*(a + b*ArcTanh
[c*x]))/(12*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(24*c) - (a + b*ArcTanh[c
*x])^2/(3*c*(1 + c*x)^3)
```

Rubi [A] time = 0.217757, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(cx+1)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(cx+1)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(cx+1)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```


```
[Out] -b^2/(54*c*(1 + c*x)^3) - (5*b^2)/(144*c*(1 + c*x)^2) - (11*b^2)/(144*c*(1
+ c*x)) + (11*b^2*ArcTanh[c*x])/(144*c) - (b*(a + b*ArcTanh[c*x]))/(9*c*(1
+ c*x)^3) - (b*(a + b*ArcTanh[c*x]))/(12*c*(1 + c*x)^2) - (b*(a + b*ArcTanh
[c*x]))/(12*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^2/(24*c) - (a + b*ArcTanh[c
*x])^2/(3*c*(1 + c*x)^3)
```


Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(1+cx)^3}+\frac{1}{3}(2b)\int\left(\frac{a+b\tanh^{-1}(cx)}{2(1+cx)^4}+\frac{a+b\tanh^{-1}(cx)}{4(1+cx)^3}+\frac{a+b\tanh^{-1}(cx)}{8(1+cx)^2}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{3c(1+cx)^3}+\frac{1}{12}b\int\frac{a+b\tanh^{-1}(cx)}{(1+cx)^2}\,dx-\frac{1}{12}b\int\frac{a+b\tanh^{-1}(cx)}{-1+c^2x^2}\,dx+\frac{1}{6}b\int$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)^2}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{12c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{24c}$$

$$= -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-\frac{b\left(a+b\tanh^{-1}(cx\right.}{12c(1+cx)^2}$$

$$= -\frac{b^2}{54c(1+cx)^3}-\frac{5b^2}{144c(1+cx)^2}-\frac{11b^2}{144c(1+cx)}+\frac{11b^2\tanh^{-1}(cx)}{144c}-\frac{b\left(a+b\tanh^{-1}(cx)\right)}{9c(1+cx)^3}-$$

Mathematica [A] time = 0.173419, size = 168, normalized size = 0.95

$$-\frac{16\left(18a^2+6ab+b^2\right)+24b\tanh^{-1}(cx)\left(24a+b\left(3c^2x^2+9cx+10\right)\right)+6b(12a+11b)(cx+1)^2+6b(12a+5b)(cx+1)+}{864c(cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(1 + c*x)^4,x]
```

```
[Out] -(16*(18*a^2 + 6*a*b + b^2) + 6*b*(12*a + 5*b)*(1 + c*x) + 6*b*(12*a + 11*b
)*(1 + c*x)^2 + 24*b*(24*a + b*(10 + 9*c*x + 3*c^2*x^2))*ArcTanh[c*x] - 36*
b^2*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[c*x]^2 + 3*b*(12*a + 11*b)*(
1 + c*x)^3*Log[1 - c*x] - 3*b*(12*a + 11*b)*(1 + c*x)^3*Log[1 + c*x])/(864*
c*(1 + c*x)^3)
```

Maple [B] time = 0.062, size = 386, normalized size = 2.2

$$-\frac{a^2}{3c(cx+1)^3}-\frac{b^2(\operatorname{Artanh}(cx))^2}{3c(cx+1)^3}-\frac{b^2\operatorname{Artanh}(cx)\ln(cx-1)}{24c}-\frac{b^2\operatorname{Artanh}(cx)}{9c(cx+1)^3}-\frac{b^2\operatorname{Artanh}(cx)}{12c(cx+1)^2}-\frac{b^2\operatorname{Artanh}(cx)}{12c(cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(c*x+1)^4,x)
```

```
[Out] -1/3/c*a^2/(c*x+1)^3-1/3/c*b^2/(c*x+1)^3*arctanh(c*x)^2-1/24/c*b^2*arctanh(
c*x)*ln(c*x-1)-1/9/c*b^2*arctanh(c*x)/(c*x+1)^3-1/12/c*b^2*arctanh(c*x)/(c*
x+1)^2-1/12/c*b^2*arctanh(c*x)/(c*x+1)+1/24/c*b^2*arctanh(c*x)*ln(c*x+1)-1/
96/c*b^2*ln(c*x-1)^2+1/48/c*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/48/c*b^2*ln(-1/
2*c*x+1/2)*ln(1/2+1/2*c*x)+1/48/c*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)-1/96/c*b^2
*ln(c*x+1)^2-11/288/c*b^2*ln(c*x-1)-1/54*b^2/c/(c*x+1)^3-5/144*b^2/c/(c*x+1
)^2-11/144*b^2/c/(c*x+1)+11/288/c*b^2*ln(c*x+1)-2/3/c*a*b*arctanh(c*x)/(c*x
+1)^3-1/24/c*a*b*ln(c*x-1)-1/9/c*a*b/(c*x+1)^3-1/12/c*a*b/(c*x+1)^2-1/12/c*
a*b/(c*x+1)+1/24/c*a*b*ln(c*x+1)
```

Maxima [B] time = 1.04532, size = 601, normalized size = 3.41

$$-\frac{1}{72}\left(c\left(\frac{2\left(3c^2x^2+9cx+10\right)}{c^5x^3+3c^4x^2+3c^3x+c^2}-\frac{3\log(cx+1)}{c^2}+\frac{3\log(cx-1)}{c^2}\right)+\frac{48\operatorname{artanh}(cx)}{c^4x^3+3c^3x^2+3c^2x+c}\right)ab-\frac{1}{864}\left(12c\left(\frac{2(}{c^5x^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/72*(c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2)
- 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c
^3*x^2 + 3*c^2*x + c))*a*b - 1/864*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x
^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*
arctanh(c*x) + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x +
1)^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11
*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*
x - 1) + 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x -
1) + 112)*c^2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*b^2 - 1/3*b^2*arctanh(
c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^2/(c^4*x^3 + 3*c^3*x^2 +
 3*c^2*x + c)
```

Fricas [A] time = 1.96812, size = 458, normalized size = 2.6

$$-\frac{6\left(12ab+11b^2\right)c^2x^2+54\left(4ab+3b^2\right)cx-9\left(b^2c^3x^3+3b^2c^2x^2+3b^2cx-7b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)^2+288a^2+240ab+1}{864\left(c^4x^3+3c^3x^2+3c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/864*(6*(12*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b + 3*b^2)*c*x - 9*(b^2*c^3*x
^3 + 3*b^2*c^2*x^2 + 3*b^2*c*x - 7*b^2)*log(-(c*x + 1)/(c*x - 1))^2 + 288*a
^2 + 240*a*b + 112*b^2 - 3*((12*a*b + 11*b^2)*c^3*x^3 + 3*(12*a*b + 7*b^2)*
```

```
c^2*x^2 + 3*(12*a*b - b^2)*c*x - 84*a*b - 29*b^2)*log(-(c*x + 1)/(c*x - 1))
)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))^2}{(cx + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(c*x+1)**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(c*x + 1)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)^2}{(cx + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(c*x + 1)^4, x)
```

3.119 $\int \frac{\tanh^{-1}(ax)^2}{cx-acx^2}\,dx$

Optimal. Leaf size=67

$$-\frac{\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2c}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{c}$$

```
[Out] (ArcTanh[a*x]^2*Log[2 - 2/(1 - a*x)])/c + (ArcTanh[a*x]*PolyLog[2, -1 + 2/(
1 - a*x)])/c - PolyLog[3, -1 + 2/(1 - a*x)]/(2*c)
```

Rubi [A] time = 0.138105, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1593, 5932, 5948, 6058, 6610}

$$-\frac{\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{2c}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}+\frac{\log\left(2-\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*Log[2 - 2/(1 - a*x)])/c + (ArcTanh[a*x]*PolyLog[2, -1 + 2/(
1 - a*x)])/c - PolyLog[3, -1 + 2/(1 - a*x)]/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{cx-acx^2}\,dx=\int \frac{\tanh^{-1}(ax)^2}{x(c-acx)}\,dx$$
$$=\frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}-\frac{(2a)\int \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$=\frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}+\frac{\tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c}-\frac{a\int \frac{\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$
$$=\frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1-ax}\right)}{c}+\frac{\tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c}-\frac{\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{2c}$$

Mathematica [A] time = 0.150091, size = 59, normalized size = 0.88

$$\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)}{c}-\frac{\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)}{2c}+\frac{\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])])/c + (ArcTanh[a*x]*PolyLog[2, E
^(2*ArcTanh[a*x])])/c - PolyLog[3, E^(2*ArcTanh[a*x])]/(2*c)
```

Maple [C] time = 0.32, size = 717, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a*c*x^2+c*x),x)
```

```
[Out] -1/c*arctanh(a*x)^2*ln(a*x-1)+1/c*arctanh(a*x)^2*ln(a*x)-1/c*arctanh(a*x)^2
*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))+2/c*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2/c*polylog(3
,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(
1/2))+2/c*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2/c*polylog(3
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/c*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-I/c*arctanh(a*x)^2*Pi*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))^2+I/c*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))^3+I/c*arctanh(a*x)^2*Pi-1/2*I/c*arctanh(a*x)^2*Pi*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2-1/2*I/c*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I/c*arctanh
(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+1/c*arc
tanh(a*x)^2*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(-ax+1\right)^{3}}{12\,c}+\frac{1}{4}\int -\frac{\log\left(ax+1\right)^{2}-2\,\log\left(ax+1\right)\log\left(-ax+1\right)}{acx^{2}-cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(-a*x + 1)^3/c + 1/4*integrate(-(log(a*x + 1)^2 - 2*log(a*x + 1)*l
og(-a*x + 1))/(a*c*x^2 - c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\operatorname{artanh}\left(ax\right)^{2}}{acx^{2}-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a*c*x^2 - c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{atanh}^{2}\left(ax\right)}{ax^{2}-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a*c*x**2+c*x),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a*x**2 - x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}\left(ax\right)^{2}}{acx^{2}-cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/(a*c*x^2 - c*x), x)
```

3.120 $\int (1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=306

$$-\frac{6b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{11b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}+\frac{1}{4}b^2cx^2\left(a+\right.$$

```
[Out] 3*a*b^2*x + (b^3*x)/4 - (b^3*ArcTanh[c*x])/(4*c) + 3*b^3*x*ArcTanh[c*x] + (
b^2*c*x^2*(a + b*ArcTanh[c*x]))/4 + (4*b*(a + b*ArcTanh[c*x])^2)/c + (21*b*
x*(a + b*ArcTanh[c*x])^2)/4 + (3*b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + (b*c^2
*x^3*(a + b*ArcTanh[c*x])^2)/4 + ((1 + c*x)^4*(a + b*ArcTanh[c*x])^3)/(4*c)
 - (11*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c - (6*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c + (3*b^3*Log[1 - c^2*x^2])/(2*c) - (11*b^3*PolyL
og[2, 1 - 2/(1 - c*x)])/(2*c) - (6*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 -
2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rubi [A] time = 0.658268, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 321, 206, 1586, 6058, 6610}

$$-\frac{6b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{11b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}+\frac{1}{4}b^2cx^2\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] 3*a*b^2*x + (b^3*x)/4 - (b^3*ArcTanh[c*x])/(4*c) + 3*b^3*x*ArcTanh[c*x] + (
b^2*c*x^2*(a + b*ArcTanh[c*x]))/4 + (4*b*(a + b*ArcTanh[c*x])^2)/c + (21*b*
x*(a + b*ArcTanh[c*x])^2)/4 + (3*b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + (b*c^2
*x^3*(a + b*ArcTanh[c*x])^2)/4 + ((1 + c*x)^4*(a + b*ArcTanh[c*x])^3)/(4*c)
 - (11*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c - (6*b*(a + b*ArcTanh[c
*x])^2*Log[2/(1 - c*x)])/c + (3*b^3*Log[1 - c^2*x^2])/(2*c) - (11*b^3*PolyL
og[2, 1 - 2/(1 - c*x)])/(2*c) - (6*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 -
2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
```

```
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)^3 \left(a+b\tanh^{-1}(cx)\right)^3 dx = \frac{(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^3}{4c} - \frac{1}{4}(3b)\int \left(-7\left(a+b\tanh^{-1}(cx)\right)^2 - 4cx\left(a+b\tanh\right.\right.$$

$$= \frac{(1+cx)^4 \left(a+b\tanh^{-1}(cx)\right)^3}{4c} + \frac{1}{4}(21b)\int \left(a+b\tanh^{-1}(cx)\right)^2 dx - (6b)\int \frac{(1+cx}{}$$

$$= \frac{21}{4}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{4}bc^2x^3\left(a+b\tanh^{-1}(cx)\right.$$

$$= \frac{21b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} + \frac{21}{4}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{3}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2$$

$$= 3ab^2x + \frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right) + \frac{4b\left(a+b\tanh^{-1}(cx)\right)^2}{c} + \frac{21}{4}bx\left(a+b\tanh^{-1}\right.$$

$$= 3ab^2x + \frac{b^3x}{4} + 3b^3x\tanh^{-1}(cx) + \frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right) + \frac{4b\left(a+b\tanh^{-1}(c\right.}{c}$$

$$= 3ab^2x + \frac{b^3x}{4} - \frac{b^3\tanh^{-1}(cx)}{4c} + 3b^3x\tanh^{-1}(cx) + \frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right) + \frac{4b}{}$$

$$= 3ab^2x + \frac{b^3x}{4} - \frac{b^3\tanh^{-1}(cx)}{4c} + 3b^3x\tanh^{-1}(cx) + \frac{1}{4}b^2cx^2\left(a+b\tanh^{-1}(cx)\right) + \frac{4b}{}$$

Mathematica [B] time = 1.44633, size = 644, normalized size = 2.1

$$\frac{4b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(12a+12b\tanh^{-1}(cx)+11b\right)+24b^3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+2a^2bc^3x^3+12a^2bc^2x^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c*x)^3*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (-2*a*b^2 + 8*a^3*c*x + 42*a^2*b*c*x + 24*a*b^2*c*x + 2*b^3*c*x + 12*a^3*c^
2*x^2 + 12*a^2*b*c^2*x^2 + 2*a*b^2*c^2*x^2 + 8*a^3*c^3*x^3 + 2*a^2*b*c^3*x^
3 + 2*a^3*c^4*x^4 - 24*a*b^2*ArcTanh[c*x] - 2*b^3*ArcTanh[c*x] + 24*a^2*b*c
*x*ArcTanh[c*x] + 84*a*b^2*c*x*ArcTanh[c*x] + 24*b^3*c*x*ArcTanh[c*x] + 36*
a^2*b*c^2*x^2*ArcTanh[c*x] + 24*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^2*x^2*
ArcTanh[c*x] + 24*a^2*b*c^3*x^3*ArcTanh[c*x] + 4*a*b^2*c^3*x^3*ArcTanh[c*x]
 + 6*a^2*b*c^4*x^4*ArcTanh[c*x] - 90*a*b^2*ArcTanh[c*x]^2 - 56*b^3*ArcTanh[
c*x]^2 + 24*a*b^2*c*x*ArcTanh[c*x]^2 + 42*b^3*c*x*ArcTanh[c*x]^2 + 36*a*b^2
*c^2*x^2*ArcTanh[c*x]^2 + 12*b^3*c^2*x^2*ArcTanh[c*x]^2 + 24*a*b^2*c^3*x^3*
ArcTanh[c*x]^2 + 2*b^3*c^3*x^3*ArcTanh[c*x]^2 + 6*a*b^2*c^4*x^4*ArcTanh[c*x
]^2 - 30*b^3*ArcTanh[c*x]^3 + 8*b^3*c*x*ArcTanh[c*x]^3 + 12*b^3*c^2*x^2*Arc
Tanh[c*x]^3 + 8*b^3*c^3*x^3*ArcTanh[c*x]^3 + 2*b^3*c^4*x^4*ArcTanh[c*x]^3 -
 96*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 88*b^3*ArcTanh[c*x]*L
og[1 + E^(-2*ArcTanh[c*x])] - 48*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c
*x])] + 45*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 44*a*b^2*Log[1 - c^2
*x^2] + 12*b^3*Log[1 - c^2*x^2] + 4*b^2*(12*a + 11*b + 12*b*ArcTanh[c*x])*P
olyLog[2, -E^(-2*ArcTanh[c*x])] + 24*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])])/
(8*c)
```

Maple [C] time = 0.316, size = 963, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)^3*(a+b*arctanh(c*x))^3,x)
```

```
[Out] -6*I/c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+6*I/c*b^3
*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/4*b^3*x+9/2*c*a^2
*b*arctanh(c*x)*x^2+12/c*a*b^2*arctanh(c*x)*ln(c*x-1)-6/c*a*b^2*ln(c*x-1)*l
n(1/2+1/2*c*x)+3/4*c^3*a*b^2*arctanh(c*x)^2*x^4+3*c^2*a*b^2*arctanh(c*x)^2*
x^3+9/2*c*a*b^2*arctanh(c*x)^2*x^2-6*I/c*b^3*Pi*arctanh(c*x)^2+11/4*b^3*arc
tanh(c*x)/c+3*b^3*x*arctanh(c*x)+3/2*c*a^2*b*x^2+b^3*x*arctanh(c*x)^3+1/4/c
*b^3*arctanh(c*x)^3+3/c*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-13/4/c*a*b^2
+21/4*x*a^2*b+1/4*c^3*x^4*a^3+c^2*x^3*a^3+3/2*c*x^2*a^3+4/c*b^3*arctanh(c*x
)^2-11/c*b^3*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-11/c*b^3*dilog(1-I*(c*x+
1)/(-c^2*x^2+1)^(1/2))-3/c*b^3*ln((c*x+1)^2/(-c^2*x^2+1)+1)+21/4*b^3*arctan
h(c*x)^2*x+3*a*b^2*x-6/c*b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1)
)+3*x*a*b^2*arctanh(c*x)^2+3/4/c*a*b^2*arctanh(c*x)^2+3*x*a^2*b*arctanh(c*x
)+3/c*a*b^2*ln(c*x-1)^2-6/c*a*b^2*dilog(1/2+1/2*c*x)-11/c*b^3*arctanh(c*x)*
ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))+6/c*b^3*arctanh(c*x)^2*ln(c*x-1)-11/c*b^
3*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2))+1/4*c^2*b^3*arctanh(c*x)^
2*x^3+3/2*c*b^3*arctanh(c*x)^2*x^2+1/4*c*b^3*arctanh(c*x)*x^2+3/2*c*b^3*arc
tanh(c*x)^3*x^2+c^2*b^3*arctanh(c*x)^3*x^3+1/4*c^3*b^3*arctanh(c*x)^3*x^4+2
1/2*a*b^2*arctanh(c*x)*x+1/4*c^2*a^2*b*x^3+1/4*c*x^2*a*b^2-6/c*b^3*arctanh(
c*x)^2*ln(2)+3/4/c*a^2*b*arctanh(c*x)+6/c*a^2*b*ln(c*x-1)+7/c*a*b^2*ln(c*x-
1)+4/c*a*b^2*ln(c*x+1)+x*a^3+1/2*c^2*a*b^2*arctanh(c*x)*x^3+3*c*a*b^2*arcta
nh(c*x)*x^2+3/4*c^3*a^2*b*arctanh(c*x)*x^4+3*c^2*a^2*b*arctanh(c*x)*x^3-1/4
*b^3/c+1/4/c*a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*c^3*x^4 + a^3*c^2*x^3 + 1/8*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3
*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b*c^3 + 3/2*(2*x^3*
arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*a^2*b*c^2 + 3/2*a^3*c*x^
2 + 9/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/
c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/
c - 1/32*((b^3*c^4*x^4 + 4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x - 15*b^3
)*log(-c*x + 1)^3 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^3 + 12*
(3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(b^3*c^4*x^4 +
4*b^3*c^3*x^3 + 6*b^3*c^2*x^2 + 4*b^3*c*x + b^3)*log(c*x + 1))*log(-c*x + 1
)^2)/c - integrate(-1/16*(2*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*c*x - b^3)
*log(c*x + 1)^3 + 12*(a*b^2*c^4*x^4 + 2*a*b^2*c^3*x^3 - 2*a*b^2*c*x - a*b^2
)*log(c*x + 1)^2 - (6*a*b^2*c^4*x^4 + 2*(12*a*b^2*c^3 + b^3*c^3)*x^3 + 12*(
3*a*b^2*c^2 + b^3*c^2)*x^2 + 6*(b^3*c^4*x^4 + 2*b^3*c^3*x^3 - 2*b^3*c*x - b
^3)*log(c*x + 1)^2 + 6*(4*a*b^2*c + 7*b^3*c)*x + 3*(6*b^3*c^2*x^2 + (8*a*b^
2*c^4 + b^3*c^4)*x^4 + 4*(4*a*b^2*c^3 + b^3*c^3)*x^3 - 8*a*b^2 + b^3 - 4*(4
*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3c^3x^3 + 3\,a^3c^2x^2 + 3\,a^3cx + \left(b^3c^3x^3 + 3\,b^3c^2x^2 + 3\,b^3cx + b^3\right)\text{artanh}\left(cx\right)^3 + a^3 + 3\left(ab^2c^3x^3 + 3\,ab^2c^2x^2 + 3\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c^3*x^3 + 3*a^3*c^2*x^2 + 3*a^3*c*x + (b^3*c^3*x^3 + 3*b^3*c^2
*x^2 + 3*b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c^3*x^3 + 3*a*b^2*c
^2*x^2 + 3*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(a^2*b*c^3*x^3 + 3*a^2*b*c
^2*x^2 + 3*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b\,\text{atanh}\left(cx\right)\right)^3\left(cx + 1\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cx + 1)^3(b\,\text{artanh}\left(cx\right) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)^3*(b*arctanh(c*x) + a)^3, x)
```

3.121 $\int (1+cx)^2 \left(a+b\tanh^{-1}(cx)\right)^3 dx$

Optimal. Leaf size=240

$$-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{2b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}-\frac{6b^2\log\left(\frac{2}{1-\cdots}\right.}{\cdots}$$

```
[Out] a*b^2*x + b^3*x*ArcTanh[c*x] + (5*b*(a + b*ArcTanh[c*x])^2)/(2*c) + 3*b*x*(
a + b*ArcTanh[c*x])^2 + (b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + ((1 + c*x)^3*(
a + b*ArcTanh[c*x])^3)/(3*c) - (6*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]
)/c - (4*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c + (b^3*Log[1 - c^2*x^
2])/(2*c) - (3*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c - (4*b^2*(a + b*ArcTanh[c
*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (2*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rubi [A] time = 0.442961, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 1586, 6058, 6610}

$$-\frac{4b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{2b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{c}-\frac{6b^2\log\left(\frac{2}{1-\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] a*b^2*x + b^3*x*ArcTanh[c*x] + (5*b*(a + b*ArcTanh[c*x])^2)/(2*c) + 3*b*x*(
a + b*ArcTanh[c*x])^2 + (b*c*x^2*(a + b*ArcTanh[c*x])^2)/2 + ((1 + c*x)^3*(
a + b*ArcTanh[c*x])^3)/(3*c) - (6*b^2*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]
)/c - (4*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c + (b^3*Log[1 - c^2*x^
2])/(2*c) - (3*b^3*PolyLog[2, 1 - 2/(1 - c*x)])/c - (4*b^2*(a + b*ArcTanh[c
*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (2*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/c
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3\,dx = \frac{(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^3}{3c} - b\int\left(-3\left(a+b\tanh^{-1}(cx)\right)^2 - cx\left(a+b\tanh^{-1}\right.\right.$$

$$= \frac{(1+cx)^3\left(a+b\tanh^{-1}(cx)\right)^3}{3c} + (3b)\int\left(a+b\tanh^{-1}(cx)\right)^2 dx - (4b)\int \frac{(1+cx}{}$$

$$= 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^3\left(a+b\tanh^{-1}\right.}{3c}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-1}(cx)\right)^2$$

$$= ab^2x + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}bcx^2\left(a+b\tanh^{-}\right.$$

$$= ab^2x + b^3x\tanh^{-1}(cx) + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}$$

$$= ab^2x + b^3x\tanh^{-1}(cx) + \frac{5b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + 3bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{1}{2}$$

Mathematica [B] time = 0.988259, size = 488, normalized size = 2.03

$$\frac{6b^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(4a+4b\tanh^{-1}(cx)+3b\right)+12b^3\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+3a^2bc^2x^2+6a^2bc^3x^3\,\text{t}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c*x)^2*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (6*a^3*c*x + 18*a^2*b*c*x + 6*a*b^2*c*x + 6*a^3*c^2*x^2 + 3*a^2*b*c^2*x^2 +
 2*a^3*c^3*x^3 - 6*a*b^2*ArcTanh[c*x] + 18*a^2*b*c*x*ArcTanh[c*x] + 36*a*b^
2*c*x*ArcTanh[c*x] + 6*b^3*c*x*ArcTanh[c*x] + 18*a^2*b*c^2*x^2*ArcTanh[c*x]
 + 6*a*b^2*c^2*x^2*ArcTanh[c*x] + 6*a^2*b*c^3*x^3*ArcTanh[c*x] - 42*a*b^2*A
rcTanh[c*x]^2 - 21*b^3*ArcTanh[c*x]^2 + 18*a*b^2*c*x*ArcTanh[c*x]^2 + 18*b^
3*c*x*ArcTanh[c*x]^2 + 18*a*b^2*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^2*x^2*ArcT
anh[c*x]^2 + 6*a*b^2*c^3*x^3*ArcTanh[c*x]^2 - 14*b^3*ArcTanh[c*x]^3 + 6*b^3
*c*x*ArcTanh[c*x]^3 + 6*b^3*c^2*x^2*ArcTanh[c*x]^3 + 2*b^3*c^3*x^3*ArcTanh[
c*x]^3 - 48*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 36*b^3*ArcTan
h[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 24*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*A
rcTanh[c*x])] + 21*a^2*b*Log[1 - c*x] + 3*a^2*b*Log[1 + c*x] + 18*a*b^2*Log
[1 - c^2*x^2] + 3*b^3*Log[1 - c^2*x^2] + 6*b^2*(4*a + 3*b + 4*b*ArcTanh[c*x
])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 12*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x]
)])/(6*c)
```

Maple [C] time = 0.278, size = 811, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)^2*(a+b*arctanh(c*x))^3,x)
```

```
[Out] 3*c*a^2*b*arctanh(c*x)*x^2+8/c*a*b^2*arctanh(c*x)*ln(c*x-1)-4/c*a*b^2*ln(c*
x-1)*ln(1/2+1/2*c*x)+c^2*a*b^2*arctanh(c*x)^2*x^3+3*c*a*b^2*arctanh(c*x)^2*
x^2+b^3*arctanh(c*x)/c+b^3*x*arctanh(c*x)+1/2*c*a^2*b*x^2+b^3*x*arctanh(c*x
)^3+1/3/c*b^3*arctanh(c*x)^3+2/c*b^3*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/c
*a*b^2+3*x*a^2*b+1/3*c^2*x^3*a^3+c*x^2*a^3+5/2/c*b^3*arctanh(c*x)^2-6/c*b^3
*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-6/c*b^3*dilog(1-I*(c*x+1)/(-c^2*x^2+
1)^(1/2))-1/c*b^3*ln((c*x+1)^2/(-c^2*x^2+1)+1)+3*b^3*arctanh(c*x)^2*x+a*b^2
*x-4/c*b^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3*x*a*b^2*arctan
h(c*x)^2+1/c*a*b^2*arctanh(c*x)^2+3*x*a^2*b*arctanh(c*x)+2/c*a*b^2*ln(c*x-1
)^2-4/c*a*b^2*dilog(1/2+1/2*c*x)-6/c*b^3*arctanh(c*x)*ln(1+I*(c*x+1)/(-c^2*
x^2+1)^(1/2))+4/c*b^3*arctanh(c*x)^2*ln(c*x-1)-6/c*b^3*arctanh(c*x)*ln(1-I*
(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*c*b^3*arctanh(c*x)^2*x^2+c*b^3*arctanh(c*x)
^3*x^2+1/3*c^2*b^3*arctanh(c*x)^3*x^3+6*a*b^2*arctanh(c*x)*x-4/c*b^3*arctan
h(c*x)^2*ln(2)+1/c*a^2*b*arctanh(c*x)+4/c*a^2*b*ln(c*x-1)+7/2/c*a*b^2*ln(c*
x-1)+5/2/c*a*b^2*ln(c*x+1)-4*I/c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^
3*arctanh(c*x)^2+4*I/c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(
c*x)^2+x*a^3+c*a*b^2*arctanh(c*x)*x^2+c^2*a^2*b*arctanh(c*x)*x^3-4*I/c*b^3*
Pi*arctanh(c*x)^2+1/3/c*a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^3c^2x^3 + \frac{1}{2}\left(2x^3\operatorname{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)a^2bc^2 + a^3cx^2 + \frac{3}{2}\left(2x^2\operatorname{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*c^2*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c
^4))*a^2*b*c^2 + a^3*c*x^2 + 3/2*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x
 + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) +
log(-c^2*x^2 + 1))*a^2*b/c - 1/24*((b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x
 - 7*b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 + (6*a*b^2*c^2 + b^3*c^2)*x^
2 + 6*(a*b^2*c + b^3*c)*x + (b^3*c^3*x^3 + 3*b^3*c^2*x^2 + 3*b^3*c*x + b^3)
*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^3*x^3 + b^3*c^2*
x^2 - b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^3 + a*b^2*c^2*x^2 - a*
b^2*c*x - a*b^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*(6*a*b^2*c^2 + b^3*c
^2)*x^2 + 3*(b^3*c^3*x^3 + b^3*c^2*x^2 - b^3*c*x - b^3)*log(c*x + 1)^2 + 12
*(a*b^2*c + b^3*c)*x + 2*((6*a*b^2*c^3 + b^3*c^3)*x^3 - 6*a*b^2 + b^3 + 3*(
2*a*b^2*c^2 + b^3*c^2)*x^2 - 3*(2*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*
x + 1))/(c*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(a^3c^2x^2 + 2a^3cx + \left(b^3c^2x^2 + 2b^3cx + b^3\right)\operatorname{artanh}(cx)^3 + a^3 + 3\left(ab^2c^2x^2 + 2ab^2cx + ab^2\right)\operatorname{artanh}(cx)^2 + 3\left(a^2b\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c^2*x^2 + 2*a^3*c*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^3)*arctanh(
c*x)^3 + a^3 + 3*(a*b^2*c^2*x^2 + 2*a*b^2*c*x + a*b^2)*arctanh(c*x)^2 + 3*(
```

```
a^2*b*c^2*x^2 + 2*a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\,\mathrm{atanh}\,(cx))^3\,(cx + 1)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)**2*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cx + 1)^2(b\,\mathrm{artanh}\,(cx) + a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)^2*(b*arctanh(c*x) + a)^3, x)
```

3.122 $\int (1+cx)\left(a+b\tanh^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=191

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)}{}$$

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c) + (3*b*x*(a + b*ArcTanh[c*x])^2)/2 + ((1
 + c*x)^2*(a + b*ArcTanh[c*x])^3)/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - (3*b^3*P
olyLog[2, 1 - 2/(1 - c*x)])/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rubi [A] time = 0.301005, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5928, 5910, 5984, 5918, 2402, 2315, 1586, 5948, 6058, 6610}

$$-\frac{3b^2\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c}-\frac{3b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}+\frac{3b^3\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}-\frac{3b^2\log\left(\frac{2}{1-cx}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c) + (3*b*x*(a + b*ArcTanh[c*x])^2)/2 + ((1
 + c*x)^2*(a + b*ArcTanh[c*x])^3)/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - (3*b^3*P
olyLog[2, 1 - 2/(1 - c*x)])/(2*c) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2,
1 - 2/(1 - c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(2*c)
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int (1+cx)\left(a+b\tanh^{-1}(cx)\right)^3\,dx = \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{1}{2}(3b)\int \left(-\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{2(1+cx)\left(a+b\,\mathrm{t}\right.}{1-c^2}\right.$$

$$= \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} + \frac{1}{2}(3b)\int \left(a+b\tanh^{-1}(cx)\right)^2 dx - (3b)\int \frac{(1+cx)\left(\right.}{}$$

$$= \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - (3b)\int \frac{\left(a+b\tanh^{-1}(cx\right.}{1-cx}$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} -$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} -$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} -$$

$$= \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c} + \frac{3}{2}bx\left(a+b\tanh^{-1}(cx)\right)^2 + \frac{(1+cx)^2\left(a+b\tanh^{-1}(cx)\right)^3}{2c} -$$

Mathematica [A] time = 0.501247, size = 334, normalized size = 1.75

$$\frac{6b^2\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(2a+2b\tanh^{-1}(cx)+b\right)+6b^3\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)+6a^2bc^2x^2\tanh^{-1}(cx)+6a^2b}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + c*x)*(a + b*ArcTanh[c*x])^3,x]
```

```
[Out] (4*a^3*c*x + 6*a^2*b*c*x + 2*a^3*c^2*x^2 + 12*a^2*b*c*x*ArcTanh[c*x] + 12*a
*b^2*c*x*ArcTanh[c*x] + 6*a^2*b*c^2*x^2*ArcTanh[c*x] - 18*a*b^2*ArcTanh[c*x
]^2 - 6*b^3*ArcTanh[c*x]^2 + 12*a*b^2*c*x*ArcTanh[c*x]^2 + 6*b^3*c*x*ArcTan
h[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTanh[c*x]^2 - 6*b^3*ArcTanh[c*x]^3 + 4*b^3*c*
x*ArcTanh[c*x]^3 + 2*b^3*c^2*x^2*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log
[1 + E^(-2*ArcTanh[c*x])] - 12*b^3*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])
] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 9*a^2*b*Log[1 - c*
x] + 3*a^2*b*Log[1 + c*x] + 6*a*b^2*Log[1 - c^2*x^2] + 6*b^2*(2*a + b + 2*b
*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*A
rcTanh[c*x])])/(4*c)
```

Maple [C] time = 0.58, size = 6440, normalized size = 33.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x+1)*(a+b*arctanh(c*x))^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^3cx^2+\frac{3}{4}\left(2x^2\operatorname{artanh}(cx)+c\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)\right)a^2bc+a^3x+\frac{3\left(2cx\operatorname{artanh}(cx)+\log\left(-c^2x^2+\right.\right.}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*c*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + l
og(c*x - 1)/c^3))*a^2*b*c + a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*a^2*b/c - 1/16*((b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-c*x + 1)^3 - 3
*(2*a*b^2*c^2*x^2 + 2*(2*a*b^2*c + b^3*c)*x + (b^3*c^2*x^2 + 2*b^3*c*x + b^
3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c^2*x^2 - b^3)*l
og(c*x + 1)^3 + 6*(a*b^2*c^2*x^2 - a*b^2)*log(c*x + 1)^2 - 3*(2*a*b^2*c^2*x
^2 + (b^3*c^2*x^2 - b^3)*log(c*x + 1)^2 + 2*(2*a*b^2*c + b^3*c)*x + (2*b^3*
c*x - 4*a*b^2 + b^3 + (4*a*b^2*c^2 + b^3*c^2)*x^2)*log(c*x + 1))*log(-c*x +
 1))/(c*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(a^3cx+\left(b^3cx+b^3\right)\operatorname{artanh}(cx)^3+a^3+3\left(ab^2cx+ab^2\right)\operatorname{artanh}(cx)^2+3\left(a^2bcx+a^2b\right)\operatorname{artanh}(cx),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(a^3*c*x + (b^3*c*x + b^3)*arctanh(c*x)^3 + a^3 + 3*(a*b^2*c*x + a*
b^2)*arctanh(c*x)^2 + 3*(a^2*b*c*x + a^2*b)*arctanh(c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\operatorname{atanh}(cx))^3(cx+1)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3*(c*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (cx+1)(b\operatorname{artanh}(cx)+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x+1)*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((c*x + 1)*(b*arctanh(c*x) + a)^3, x)
```

3.123 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{1+cx}\,dx$

Optimal. Leaf size=111

$$\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{3b^3\text{PolyLog}\left(4,1-\frac{2}{cx+1}\right.}{4c}$$

```
[Out] -(((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/c) + (3*b*(a + b*ArcTanh[c*x])^
2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[
3, 1 - 2/(1 + c*x)])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 + c*x)])/(4*c)
```

Rubi [A] time = 0.221338, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5918, 5948, 6056, 6060, 6610}

$$\frac{3b^2\text{PolyLog}\left(3,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{2c}+\frac{3b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)^2}{2c}+\frac{3b^3\text{PolyLog}\left(4,1-\frac{2}{cx+1}\right.}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^3*Log[2/(1 + c*x)])/c) + (3*b*(a + b*ArcTanh[c*x])^
2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c) + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[
3, 1 - 2/(1 + c*x)])/(2*c) + (3*b^3*PolyLog[4, 1 - 2/(1 + c*x)])/(4*c)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
```

```
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{1+cx}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c} + (3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}\,dx$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c} - \left(3b^2\right)\int \frac{(a+\ldots}{\ldots}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c} + \frac{3b^2\left(a+b\,\text{ta}\ldots\right.}{\ldots}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3\log\left(\frac{2}{1+cx}\right)}{c} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2c} + \frac{3b^2\left(a+b\,\text{ta}\ldots\right.}{\ldots}$$

Mathematica [A] time = 0.287449, size = 152, normalized size = 1.37

$$\frac{6b^2\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)+6b\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)^2+3b^3\text{PolyLo}\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x),x]
```

```
[Out] (-12*a^2*b*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - 12*a*b^2*ArcTanh[c*x
]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 4*b^3*ArcTanh[c*x]^3*Log[1 + E^(-2*ArcTa
nh[c*x])] + 4*a^3*Log[1 + c*x] + 6*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, -E^(
-2*ArcTanh[c*x])] + 6*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -E^(-2*ArcTanh[c*
x])] + 3*b^3*PolyLog[4, -E^(-2*ArcTanh[c*x])])/(4*c)
```

Maple [C] time = 0.313, size = 1491, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1),x)
```

```
[Out] -3*I/c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*Pi+3/2*I/c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-3/2*I/
c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x
^2+1)^(1/2))^2*Pi+1/2*I/c*b^3*arctanh(c*x)^3*csgn(I/((c*x+1)^2/(-c^2*x^2+1)
+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/
(-c^2*x^2+1)+1))*Pi-3/2*I/c*a*b^2*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^
2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi+a^3/
c*ln(c*x+1)-3/4/c*a^2*b*ln(c*x+1)^2+2/c*a*b^2*arctanh(c*x)^3+3/2/c*a*b^2*po
```

```
lylog(3,-(c*x+1)^2/(-c^2*x^2+1))+1/c*b^3*ln(c*x+1)*arctanh(c*x)^3-2/c*b^3*a
rctanh(c*x)^3*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3/2/c*b^3*arctanh(c*x)^2*polyl
og(2,-(c*x+1)^2/(-c^2*x^2+1))+3/2/c*b^3*arctanh(c*x)*polylog(3,-(c*x+1)^2/(
-c^2*x^2+1))-1/c*b^3*arctanh(c*x)^3*ln(2)-3/2/c*a^2*b*dilog(1/2+1/2*c*x)-3/
4/c*b^3*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))+1/2/c*b^3*arctanh(c*x)^4+3/2*I/c
*a*b^2*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi-3/2
*I/c*a*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi-3/2*I/c*a*b^2*
arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*P
i-1/2*I/c*b^3*arctanh(c*x)^3*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-I/c*b^3*arctanh(c*x)^3*c
sgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi+1/2*I/
c*b^3*arctanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-1/2*I/c*b^3*arctanh(c*x)^3*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi-1/2*I/c*b^3*arc
tanh(c*x)^3*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi-1/2*I/c*b^3*arctanh(c*x)^3*c
sgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi-6/c*a*b^2*arct
anh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3/c*a*b^2*arctanh(c*x)*polylog(2,
-(c*x+1)^2/(-c^2*x^2+1))+3/c*a*b^2*ln(c*x+1)*arctanh(c*x)^2-3/2/c*a^2*b*ln(
-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/c*a^2*b*ln(c*x+1)*arctanh(c*x)-3/c*a*b^2*ar
ctanh(c*x)^2*ln(2)+3/2/c*a^2*b*ln(-1/2*c*x+1/2)*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3\log(cx+1)\log(-cx+1)^3}{8c}+\frac{a^3\log(cx+1)}{c}+\int\frac{\left(b^3cx-b^3\right)\log(cx+1)^3+6\left(ab^2cx-ab^2\right)\log(cx+1)^2+6\left(b^3cx\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*log(c*x + 1)*log(-c*x + 1)^3/c + a^3*log(c*x + 1)/c + integrate(1/
8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 +
6*(b^3*c*x*log(c*x + 1) + a*b^2*c*x - a*b^2)*log(-c*x + 1)^2 + 12*(a^2*b*c*
x - a^2*b)*log(c*x + 1) - 3*(4*a^2*b*c*x - 4*a^2*b + (b^3*c*x - b^3)*log(c*
x + 1)^2 + 4*(a*b^2*c*x - a*b^2)*log(c*x + 1))*log(-c*x + 1))/(c^2*x^2 - 1)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\,\text{artanh}(cx)^3+3\,ab^2\,\text{artanh}(cx)^2+3\,a^2b\,\text{artanh}(cx)+a^3}{cx+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3)/(c*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\text{atanh}(cx))^3}{cx+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1),x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)^3}{cx + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(c*x + 1), x)
```

3.124 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(cx+1)}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(cx+1)}$$

```
[Out] (-3*b^3)/(4*c*(1 + c*x)) + (3*b^3*ArcTanh[c*x])/(4*c) - (3*b^2*(a + b*ArcTa
nh[c*x]))/(2*c*(1 + c*x)) + (3*b*(a + b*ArcTanh[c*x])^2)/(4*c) - (3*b*(a +
b*ArcTanh[c*x])^2)/(2*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(2*c) - (a + b*
ArcTanh[c*x])^3/(c*(1 + c*x))
```

Rubi [A] time = 0.193227, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(cx+1)}+\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(cx+1)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c}-\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

```
[Out] (-3*b^3)/(4*c*(1 + c*x)) + (3*b^3*ArcTanh[c*x])/(4*c) - (3*b^2*(a + b*ArcTa
nh[c*x]))/(2*c*(1 + c*x)) + (3*b*(a + b*ArcTanh[c*x])^2)/(4*c) - (3*b*(a +
b*ArcTanh[c*x])^2)/(2*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(2*c) - (a + b*
ArcTanh[c*x])^3/(c*(1 + c*x))
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + (3b)\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^2} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2\left(-1+c^2x^2\right)}\right)dx$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \frac{1}{2}(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx - \frac{1}{2}(3b)\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{-1+c^2x^2}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \left(3b^2\right)\int \Big(\frac{a+b}{2}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c} - \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{c(1+cx)} + \frac{1}{2}\left(3b^2\right)\int \frac{a+}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{(a+b\,\mathrm{ta}}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{(a+b\,\mathrm{ta}}{}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{2c(1+cx)} + \frac{(a+b\,\mathrm{ta}}{}$$

$$= -\frac{3b^3}{4c(1+cx)} - \frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - \frac{3b\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)}$$

$$= -\frac{3b^3}{4c(1+cx)} + \frac{3b^3\tanh^{-1}(cx)}{4c} - \frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{2c(1+cx)} + \frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{4c} - 3b\,($$

Mathematica [A] time = 0.148799, size = 198, normalized size = 1.42

$$\frac{-3b\left(2a^2+2ab+b^2\right)(cx+1)\log(1-cx) - 12b\left(2a^2+2ab+b^2\right)\tanh^{-1}(cx) + 6a^2b\log(cx+1) + 6a^2bcx\log(cx+1) -}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^2,x]
```

```
[Out] (-8*a^3 - 12*a^2*b - 12*a*b^2 - 6*b^3 - 12*b*(2*a^2 + 2*a*b + b^2)*ArcTanh[
c*x] + 6*b^2*(2*a + b)*(-1 + c*x)*ArcTanh[c*x]^2 + 4*b^3*(-1 + c*x)*ArcTanh
[c*x]^3 - 3*b*(2*a^2 + 2*a*b + b^2)*(1 + c*x)*Log[1 - c*x] + 6*a^2*b*Log[1
+ c*x] + 6*a*b^2*Log[1 + c*x] + 3*b^3*Log[1 + c*x] + 6*a^2*b*c*x*Log[1 + c*
```

```
x] + 6*a*b^2*c*x*Log[1 + c*x] + 3*b^3*c*x*Log[1 + c*x])/(8*c*(1 + c*x))
```

Maple [C] time = 0.377, size = 1895, normalized size = 13.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^2,x)
```

```
[Out] -3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2
+1)^(1/2))^2*arctanh(c*x)^2*Pi-3/4*I/c*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^
2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*Pi+3/8*I/c*b^3/(c
*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2*Pi-3/4*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c
^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)^2*Pi*x-3/8*I*b
^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(
I/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2*Pi*x+3/8*I*b^3/(c*x+1)*csgn(I*
(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*arctanh(c*x)^2*Pi*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*arctanh(c*x)^2*Pi*x-3/8*I/c*b^3/(c*x+1
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I/((c*x+1
)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2*Pi-1/c*a^3/(c*x+1)+3/8*b^3/(c*x+1)*x+3/
4*I*b^3/(c*x+1)*arctanh(c*x)^2*Pi*x+3/4*I/c*b^3/(c*x+1)*arctanh(c*x)^2*Pi-3
/2/c*a*b^2/(c*x+1)-3/2/c*a^2*b/(c*x+1)-3/4/c*b^3/(c*x+1)*arctanh(c*x)-3/4/c
*b^3*arctanh(c*x)^2/(c*x+1)-1/2/c*b^3/(c*x+1)*arctanh(c*x)^3+1/2*b^3/(c*x+1
)*arctanh(c*x)^3*x+3/4*b^3/(c*x+1)*arctanh(c*x)*x+3/4*b^3/(c*x+1)*arctanh(c
*x)^2*x-3/2/c*a*b^2*arctanh(c*x)*ln(c*x-1)+3/4/c*a*b^2*ln(c*x-1)*ln(1/2+1/2
*c*x)-3/8*b^3/c/(c*x+1)+3/2/c*a*b^2*arctanh(c*x)*ln(c*x+1)+3/4/c*a*b^2*ln(-
1/2*c*x+1/2)*ln(c*x+1)-3/4/c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/8/c*a
*b^2*ln(c*x-1)^2-3/4/c*b^3*arctanh(c*x)^2*ln(c*x-1)-3/4/c*a^2*b*ln(c*x-1)-3
/4/c*a*b^2*ln(c*x-1)+3/4/c*a*b^2*ln(c*x+1)+3/8*I/c*b^3/(c*x+1)*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2*Pi+3/8*I*b^3/(c*x+1)*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2*Pi*x-3/8*I/c*b^3/
(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2*Pi-3/8*I/c*b^3/(c*x+
1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^
2*Pi-3/4*I/c*b^3/(c*x+1)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^
2*Pi+3/4/c*b^3*arctanh(c*x)^2*ln(c*x+1)-3/2/c*b^3*arctanh(c*x)^2*ln((c*x+1)
/(-c^2*x^2+1)^(1/2))-3/8/c*a*b^2*ln(c*x+1)^2+3/4/c*a^2*b*ln(c*x+1)-3/c*a*b^
2/(c*x+1)*arctanh(c*x)^2-3/c*a^2*b/(c*x+1)*arctanh(c*x)-3/c*a*b^2/(c*x+1)*a
rctanh(c*x)+3/4*I*b^3/(c*x+1)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(
c*x)^2*Pi*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^
2*Pi*x-3/8*I*b^3/(c*x+1)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^3*arctanh(c*x)^2*Pi*x-3/4*I*b^3/(c*x+1)*csgn(I/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*arctanh(c*x)^2*Pi*x+3/4*I/c*b^3/(c*x+1)*csgn(I/((c*x+1)^2/(-c^2*x^
2+1)+1))^3*arctanh(c*x)^2*Pi
```

Maxima [B] time = 1.06134, size = 714, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="maxima")
```

```
[Out] -b^3*arctanh(c*x)^3/(c^2*x + c) - 3/4*(c*(2/(c^3*x + c^2) - log(c*x + 1)/c^
2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*a^2*b - 3/8*(4*c*(2/(c^
3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c*x + 1)
*log(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x - 1
) + 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c^2/(c^4*x + c^3))*a*b^
2 - 1/16*(12*c*(2/(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arct
anh(c*x)^2 - (((c*x + 1)*log(c*x + 1)^3 - (c*x + 1)*log(c*x - 1)^3 - 3*(c*x
 + (c*x + 1)*log(c*x - 1) + 1)*log(c*x + 1)^2 - 3*(c*x + 1)*log(c*x - 1)^2
+ 3*((c*x + 1)*log(c*x - 1)^2 + 2*c*x + 2*(c*x + 1)*log(c*x - 1) + 2)*log(c
*x + 1) - 6*(c*x + 1)*log(c*x - 1) - 12)*c^2/(c^5*x + c^4) - 6*((c*x + 1)*l
og(c*x + 1)^2 + (c*x + 1)*log(c*x - 1)^2 - 2*(c*x + (c*x + 1)*log(c*x - 1)
+ 1)*log(c*x + 1) + 2*(c*x + 1)*log(c*x - 1) + 4)*c*arctanh(c*x)/(c^4*x + c
^3))*c)*b^3 - 3*a*b^2*arctanh(c*x)^2/(c^2*x + c) - a^3/(c^2*x + c)
```

Fricas [A] time = 2.02394, size = 354, normalized size = 2.55

$$\frac{\left(b^3cx - b^3\right)\log\left(-\frac{cx+1}{cx-1}\right)^3 - 16\,a^3 - 24\,a^2b - 24\,ab^2 - 12\,b^3 - 3\left(2\,ab^2 + b^3 - \left(2\,ab^2 + b^3\right)cx\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 6\left(2\,a^2b + 2\right.}{16\left(c^2x + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*((b^3*c*x - b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 16*a^3 - 24*a^2*b - 24*
a*b^2 - 12*b^3 - 3*(2*a*b^2 + b^3 - (2*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(c*
x - 1))^2 - 6*(2*a^2*b + 2*a*b^2 + b^3 - (2*a^2*b + 2*a*b^2 + b^3)*c*x)*log
(-(c*x + 1)/(c*x - 1)))/(c^2*x + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{atanh}\,(cx))^3}{(cx + 1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**2, x)
```

Giac [A] time = 1.19108, size = 279, normalized size = 2.01

$$\frac{1}{16}\left(\frac{b^3}{c} - \frac{2\,b^3}{(cx+1)c}\right)\log\left(\frac{1}{\frac{2}{cx+1} - 1}\right)^3 + \frac{3}{16}\left(\frac{2\,ab^2 + b^3}{c} - \frac{2\left(2\,ab^2 + b^3\right)}{(cx+1)c}\right)\log\left(\frac{1}{\frac{2}{cx+1} - 1}\right)^2 - \frac{3\left(2\,a^2b + 2\,ab^2 + b^3\right)\log\left(-\right.}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*(b^3/c - 2*b^3/((c*x + 1)*c))*log(1/(2/(c*x + 1) - 1))^3 + 3/16*((2*a*
b^2 + b^3)/c - 2*(2*a*b^2 + b^3)/((c*x + 1)*c))*log(1/(2/(c*x + 1) - 1))^2
- 3/8*(2*a^2*b + 2*a*b^2 + b^3)*log(-2/(c*x + 1) + 1)/c - 3/4*(2*a^2*b + 2*
a*b^2 + b^3)*log(1/(2/(c*x + 1) - 1))/((c*x + 1)*c) - 1/4*(4*a^3 + 6*a^2*b
+ 6*a*b^2 + 3*b^3)/((c*x + 1)*c)
```

3.125 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)^2}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)}-\frac{3b\left(a+b\tan\right.}{8c(cx+}$$

```
[Out] (-3*b^3)/(64*c*(1 + c*x)^2) - (21*b^3)/(64*c*(1 + c*x)) + (21*b^3*ArcTanh[c
*x])/(64*c) - (3*b^2*(a + b*ArcTanh[c*x]))/(16*c*(1 + c*x)^2) - (9*b^2*(a +
 b*ArcTanh[c*x]))/(16*c*(1 + c*x)) + (9*b*(a + b*ArcTanh[c*x])^2)/(32*c) -
(3*b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)^2) - (3*b*(a + b*ArcTanh[c*x])^
2)/(8*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(8*c) - (a + b*ArcTanh[c*x])^3/
(2*c*(1 + c*x)^2)
```

Rubi [A] time = 0.371169, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(cx+1)^2}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(cx+1)}-\frac{3b\left(a+b\tan\right.}{8c(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```


```
[Out] (-3*b^3)/(64*c*(1 + c*x)^2) - (21*b^3)/(64*c*(1 + c*x)) + (21*b^3*ArcTanh[c
*x])/(64*c) - (3*b^2*(a + b*ArcTanh[c*x]))/(16*c*(1 + c*x)^2) - (9*b^2*(a +
 b*ArcTanh[c*x]))/(16*c*(1 + c*x)) + (9*b*(a + b*ArcTanh[c*x])^2)/(32*c) -
(3*b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)^2) - (3*b*(a + b*ArcTanh[c*x])^
2)/(8*c*(1 + c*x)) + (a + b*ArcTanh[c*x])^3/(8*c) - (a + b*ArcTanh[c*x])^3/
(2*c*(1 + c*x)^2)
```


Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^3}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(1+cx)^2}+\frac{1}{2}(3b)\int\left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^3}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4(1+cx)^2}-\frac{\left(a+b\tan\right.}{4\left(-1+\right.}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{2c(1+cx)^2}+\frac{1}{8}(3b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx-\frac{1}{8}(3b)\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{-1+c^2x^2}\,dx$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{\left(a+b\tanh^{-1}(c\right.}{2c(1+cx)^2}$$

$$= -\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2}-\frac{3b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)}+\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{8c}-\frac{\left(a+b\tanh^{-1}(c\right.}{2c(1+cx)^2}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tan\right.}{8c(1+}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tan\right.}{8c(1+}$$

$$= -\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\tanh^{-1}(cx)\right)^2}{32c}-\frac{3b\left(a+b\tan\right.}{8c(1+}$$

$$= -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)}+\frac{9b\left(a+b\right.}{}$$

$$= -\frac{3b^3}{64c(1+cx)^2}-\frac{21b^3}{64c(1+cx)}+\frac{21b^3\tanh^{-1}(cx)}{64c}-\frac{3b^2\left(a+b\tanh^{-1}(cx)\right)}{16c(1+cx)^2}-\frac{9b^2\left(a+b\tanh\right.}{16c(1+c}$$

Mathematica [A] time = 0.212471, size = 215, normalized size = 1.03

$$\frac{-6b\left(8a^2+12ab+7b^2\right)(cx+1)-3b\left(8a^2+12ab+7b^2\right)(cx+1)^2\log(1-cx)+3b\left(8a^2+12ab+7b^2\right)(cx+1)^2\log(cx+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^3,x]
```

```
[Out] (-2*(32*a^3 + 24*a^2*b + 12*a*b^2 + 3*b^3) - 6*b*(8*a^2 + 12*a*b + 7*b^2)*(
1 + c*x) - 24*b*(8*a^2 + 4*a*b*(2 + c*x) + b^2*(4 + 3*c*x))*ArcTanh[c*x] +
12*b^2*(-1 + c*x)*(4*a*(3 + c*x) + b*(5 + 3*c*x))*ArcTanh[c*x]^2 + 16*b^3*(
-3 + 2*c*x + c^2*x^2)*ArcTanh[c*x]^3 - 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*
x)^2*Log[1 - c*x] + 3*b*(8*a^2 + 12*a*b + 7*b^2)*(1 + c*x)^2*Log[1 + c*x])/
(128*c*(1 + c*x)^2)
```

Maple [C] time = 0.432, size = 2752, normalized size = 13.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^3,x)
```

```
[Out] -1/2/c*a^3/(c*x+1)^2+1/8*c*b^3/(c*x+1)^2*arctanh(c*x)^3*x^2-3/4/c*a*b^2/(c*
x+1)^2*arctanh(c*x)-3/2/c*a*b^2/(c*x+1)^2*arctanh(c*x)^2-3/2/c*a^2*b/(c*x+1
)^2*arctanh(c*x)+9/32*c*b^3/(c*x+1)^2*arctanh(c*x)^2*x^2+21/64*c*b^3/(c*x+1
)^2*arctanh(c*x)*x^2+3/128*b^3/(c*x+1)^2*x+3/32*I*c*b^3/(c*x+1)^2*arctanh(c
*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1
)^2/(-c^2*x^2+1)+1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*x^2-9/16/c*a*b^2/(c
*x+1)-3/8/c*a^2*b/(c*x+1)-3/8/c*b^3*arctanh(c*x)^2/(c*x+1)-3/32*I*c*b^3/(c*
x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*x^2+3/16*I*b^3/(c*x+1)^2*arctan
h(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*x-3/16*I*c*b^3/(
c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(
-c^2*x^2+1)^(1/2))*x^2+3/32*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x
+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))
^2*x^2-3/32*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2+3/32*I/c*b^3/(c*x+1)^2*Pi*arcta
nh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))-3/8/c*a*b^2*arctan
h(c*x)*ln(c*x-1)+3/16/c*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-51/256*b^3/c/(c*x+1
)^2+3/8/c*a*b^2*arctanh(c*x)*ln(c*x+1)+3/16/c*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x
+1)-3/16/c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-3/32/c*a*b^2*ln(c*x-1)^2-
3/16/c*b^3*arctanh(c*x)^2*ln(c*x-1)-3/16/c*a^2*b*ln(c*x-1)-9/32/c*a*b^2*ln(
c*x-1)+9/32/c*a*b^2*ln(c*x+1)+3/16*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*x^2-
3/16*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))
^2-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x
-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+
1)^2/(-c^2*x^2+1)+1))^3*x+3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I/((c*
x+1)^2/(-c^2*x^2+1)+1))^3*x-3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I/((
c*x+1)^2/(-c^2*x^2+1)+1))^2*x-3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn
(I*(c*x+1)^2/(c^2*x^2-1))^3+3/16*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I
/((c*x+1)^2/(-c^2*x^2+1)+1))^3-3/8*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x+3/16*I*b^3/(
c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*x-3/16*I*b^3/(c*x+1)^2*arctanh(c*x
)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x
-3/16*I*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+
1)^2/(-c^2*x^2+1)+1))^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*x-3/32*I*c*b^3/(
c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^
2+1)+1))^3*x^2+3/16*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(
-c^2*x^2+1)+1))^3*x^2-3/16*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(I/((c*x
+1)^2/(-c^2*x^2+1)+1))^2*x^2-3/32*I*c*b^3/(c*x+1)^2*arctanh(c*x)^2*Pi*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^3*x^2+3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csg
```

```
n(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^
2+1)+1))^2-3/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-3/32*I/c*b^3/(c*x+1)^2*Pi*arcta
nh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
(c*x+1)^2/(-c^2*x^2+1)+1))^2-3/16*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(
I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-3/16/c*a*b^2/
(c*x+1)^2-3/8/c*a^2*b/(c*x+1)^2+45/256*c*b^3/(c*x+1)^2*x^2+1/4*b^3/(c*x+1)^
2*arctanh(c*x)^3*x+9/16*b^3/(c*x+1)^2*arctanh(c*x)^2*x+3/32*b^3/(c*x+1)^2*a
rctanh(c*x)*x-27/64/c*b^3/(c*x+1)^2*arctanh(c*x)-3/32/c*b^3*arctanh(c*x)^2/
(c*x+1)^2-3/8/c*b^3/(c*x+1)^2*arctanh(c*x)^3+3/16/c*b^3*arctanh(c*x)^2*ln(c
*x+1)-3/8/c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3/32/c*a*b^2*
ln(c*x+1)^2+3/16/c*a^2*b*ln(c*x+1)-3/4/c*a*b^2/(c*x+1)*arctanh(c*x)+3/8*I*b
^3/(c*x+1)^2*arctanh(c*x)^2*Pi*x+3/16*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2-3
/32*I/c*b^3/(c*x+1)^2*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+
1)^2/(-c^2*x^2+1)+1))^3
```

Maxima [B] time = 1.09875, size = 1075, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*arctanh(c*x)^3/(c^3*x^2 + 2*c^2*x + c) - 3/16*(c*(2*(c*x + 2)/(c^4
*x^2 + 2*c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + 8*arctanh(c*
x)/(c^3*x^2 + 2*c^2*x + c))*a^2*b - 3/32*(4*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3
*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2)*arctanh(c*x) + ((c^2*x^2 +
 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^2 + 6*c*x -
 (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1
) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c^2/(c^5*x^2 + 2*c^4*x + c^3)
)*a*b^2 - 1/128*(24*c*(2*(c*x + 2)/(c^4*x^2 + 2*c^3*x + c^2) - log(c*x + 1)
/c^2 + log(c*x - 1)/c^2)*arctanh(c*x)^2 - ((2*(c^2*x^2 + 2*c*x + 1)*log(c*x
 + 1)^3 - 2*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1)^3 - 3*(3*c^2*x^2 + 6*c*x + 2
*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 3)*log(c*x + 1)^2 - 9*(c^2*x^2 + 2*c*
x + 1)*log(c*x - 1)^2 - 42*c*x + 3*(7*c^2*x^2 + 2*(c^2*x^2 + 2*c*x + 1)*log
(c*x - 1)^2 + 14*c*x + 6*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 7)*log(c*x +
1) - 21*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) - 48)*c^2/(c^6*x^2 + 2*c^5*x + c
^4) - 12*((c^2*x^2 + 2*c*x + 1)*log(c*x + 1)^2 + (c^2*x^2 + 2*c*x + 1)*log(
c*x - 1)^2 + 6*c*x - (3*c^2*x^2 + 6*c*x + 2*(c^2*x^2 + 2*c*x + 1)*log(c*x -
 1) + 3)*log(c*x + 1) + 3*(c^2*x^2 + 2*c*x + 1)*log(c*x - 1) + 8)*c*arctanh
(c*x)/(c^5*x^2 + 2*c^4*x + c^3))*c)*b^3 - 3/2*a*b^2*arctanh(c*x)^2/(c^3*x^2
 + 2*c^2*x + c) - 1/2*a^3/(c^3*x^2 + 2*c^2*x + c)
```

Fricas [A] time = 1.90281, size = 554, normalized size = 2.66

$$2\left(b^3c^2x^2+2\,b^3cx-3\,b^3\right)\log\left(-\frac{cx+1}{cx-1}\right)^3-64\,a^3-96\,a^2b-96\,ab^2-48\,b^3-6\left(8\,a^2b+12\,ab^2+7\,b^3\right)cx+3\left(\left(4\,ab^2+3\,b^3\right)c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(2*(b^3*c^2*x^2 + 2*b^3*c*x - 3*b^3)*log(-(c*x + 1)/(c*x - 1))^3 - 64
*a^3 - 96*a^2*b - 96*a*b^2 - 48*b^3 - 6*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x +
3*((4*a*b^2 + 3*b^3)*c^2*x^2 - 12*a*b^2 - 5*b^3 + 2*(4*a*b^2 + b^3)*c*x)*lo
g(-(c*x + 1)/(c*x - 1))^2 + 3*((8*a^2*b + 12*a*b^2 + 7*b^3)*c^2*x^2 - 24*a^
2*b - 20*a*b^2 - 9*b^3 + 2*(8*a^2*b + 4*a*b^2 + b^3)*c*x)*log(-(c*x + 1)/(c
*x - 1)))/(c^3*x^2 + 2*c^2*x + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{(cx + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{(cx + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(c*x + 1)^3, x)
```

3.126 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^4}\,dx$

Optimal. Leaf size=275

$$-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)^2}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(cx+1)^3}+\frac{11b\left(a+b\tanh^{-1}(cx)\right)^2}{96c}-\frac{b\left(a+b\tanh^{-1}\right.}{8c(cx+1)}$$

```
[Out] -b^3/(108*c*(1 + c*x)^3) - (19*b^3)/(576*c*(1 + c*x)^2) - (85*b^3)/(576*c*(
1 + c*x)) + (85*b^3*ArcTanh[c*x])/(576*c) - (b^2*(a + b*ArcTanh[c*x]))/(18*
c*(1 + c*x)^3) - (5*b^2*(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)^2) - (11*b^2*
(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)) + (11*b*(a + b*ArcTanh[c*x])^2)/(96*
c) - (b*(a + b*ArcTanh[c*x])^2)/(6*c*(1 + c*x)^3) - (b*(a + b*ArcTanh[c*x])
^2)/(8*c*(1 + c*x)^2) - (b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)) + (a + b
*ArcTanh[c*x])^3/(24*c) - (a + b*ArcTanh[c*x])^3/(3*c*(1 + c*x)^3)
```

Rubi [A] time = 0.613865, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5928, 5926, 627, 44, 207, 5948}

$$-\frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)}-\frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(cx+1)^2}-\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(cx+1)^3}+\frac{11b\left(a+b\tanh^{-1}(cx)\right)^2}{96c}-\frac{b\left(a+b\tanh^{-1}\right.}{8c(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

```
[Out] -b^3/(108*c*(1 + c*x)^3) - (19*b^3)/(576*c*(1 + c*x)^2) - (85*b^3)/(576*c*(
1 + c*x)) + (85*b^3*ArcTanh[c*x])/(576*c) - (b^2*(a + b*ArcTanh[c*x]))/(18*
c*(1 + c*x)^3) - (5*b^2*(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)^2) - (11*b^2*
(a + b*ArcTanh[c*x]))/(48*c*(1 + c*x)) + (11*b*(a + b*ArcTanh[c*x])^2)/(96*
c) - (b*(a + b*ArcTanh[c*x])^2)/(6*c*(1 + c*x)^3) - (b*(a + b*ArcTanh[c*x])
^2)/(8*c*(1 + c*x)^2) - (b*(a + b*ArcTanh[c*x])^2)/(8*c*(1 + c*x)) + (a + b
*ArcTanh[c*x])^3/(24*c) - (a + b*ArcTanh[c*x])^3/(3*c*(1 + c*x)^3)
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^3}{(1+cx)^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(1+cx)^3} + b\int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2(1+cx)^4} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{4(1+cx)^3} + \frac{\left(a+b\tanh\right.}{8(1+c}\right.$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^3}{3c(1+cx)^3} + \frac{1}{8}b\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{(1+cx)^2}\,dx - \frac{1}{8}b\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{-1+c^2x^2}\,dx +$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(1+cx)^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)} + \frac{\left(a+b\tanh^{-}\right.}{24c}$$

$$= -\frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{6c(1+cx)^3} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)^2} - \frac{b\left(a+b\tanh^{-1}(cx)\right)^2}{8c(1+cx)} + \frac{\left(a+b\tanh^{-}\right.}{24c}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2} - \frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)} + \frac{11b\left(a+b\right.}{}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2} - \frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)} + \frac{11b\left(a+b\right.}{}$$

$$= -\frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)^2} - \frac{11b^2\left(a+b\tanh^{-1}(cx)\right)}{48c(1+cx)} + \frac{11b\left(a+b\right.}{}$$

$$= -\frac{b^3}{108c(1+cx)^3} - \frac{19b^3}{576c(1+cx)^2} - \frac{85b^3}{576c(1+cx)} - \frac{b^2\left(a+b\tanh^{-1}(cx)\right)}{18c(1+cx)^3} - \frac{5b^2\left(a+b\tan\right.}{48c(1+}$$

$$= -\frac{b^3}{108c(1+cx)^3} - \frac{19b^3}{576c(1+cx)^2} - \frac{85b^3}{576c(1+cx)} + \frac{85b^3\tanh^{-1}(cx)}{576c} - \frac{b^2\left(a+b\tanh^{-1}(c\right.}{18c(1+cx)^3}$$

Mathematica [A] time = 0.217635, size = 279, normalized size = 1.01

$$-\frac{24b\tanh^{-1}(cx)\left(144a^2+12ab\left(3c^2x^2+9cx+10\right)+b^2\left(33c^2x^2+81cx+56\right)\right)+6b\left(72a^2+132ab+85b^2\right)(cx+1)^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^3/(1 + c*x)^4,x]
```

```
[Out] -(32*(36*a^3 + 18*a^2*b + 6*a*b^2 + b^3) + 6*b*(72*a^2 + 60*a*b + 19*b^2)*(
1 + c*x) + 6*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^2 + 24*b*(144*a^2 + 12
*a*b*(10 + 9*c*x + 3*c^2*x^2) + b^2*(56 + 81*c*x + 33*c^2*x^2))*ArcTanh[c*x
] - 36*b^2*(-1 + c*x)*(12*a*(7 + 4*c*x + c^2*x^2) + b*(29 + 32*c*x + 11*c^2
*x^2))*ArcTanh[c*x]^2 - 144*b^3*(-7 + 3*c*x + 3*c^2*x^2 + c^3*x^3)*ArcTanh[
c*x]^3 + 3*b*(72*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 - c*x] - 3*b*(72
*a^2 + 132*a*b + 85*b^2)*(1 + c*x)^3*Log[1 + c*x])/(3456*c*(1 + c*x)^3)
```

Maple [C] time = 0.454, size = 3637, normalized size = 13.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^3/(c*x+1)^4,x)
```

```
[Out] -1/3/c*a^3/(c*x+1)^3-1/4/c*a*b^2/(c*x+1)^2*arctanh(c*x)-3/32*I*c*b^3/(c*x+1
)^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^
2+1)^(1/2))^2*x^2-11/48/c*a*b^2/(c*x+1)-1/8/c*a^2*b/(c*x+1)-1/8/c*b^3*arcta
nh(c*x)^2/(c*x+1)-1/8/c*a*b^2*arctanh(c*x)*ln(c*x-1)+1/16/c*a*b^2*ln(c*x-1)
*ln(1/2+1/2*c*x)-1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-
c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2+3/32*I*b^3/(c*x+1)^3*arct
anh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((
c*x+1)^2/(-c^2*x^2+1)+1))^2*x-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x-3/32*I*b^3/(
c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*x-3/16*I*b^3/(c*x+1)^3*arctan
h(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/
2))*x+1/32*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2-1/32*I/c*b^3/(
c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x
+1)^2/(c^2*x^2-1))-3/32*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*x^2-1/32*I*c^2*b^3/(c*x+1)^3*ar
ctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^3-1/32*I*c^2*b^3/(c*x+1)^
3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)^3*x^3+3/16*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^
2+1)+1))^3*x^2-1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2
/(-c^2*x^2+1)+1))^2*x^3-3/32*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1))^3*x^2+1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I
/((c*x+1)^2/(-c^2*x^2+1)+1))^3*x^3-3/16*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*x^2-737/6912*b^3/c/(c*x+1)^3-1/32*I/c
*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/32*I/c*b
^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^
2*x^2+1)+1))^3+1/8/c*a*b^2*arctanh(c*x)*ln(c*x+1)+1/16/c*a*b^2*ln(-1/2*c*x+
1/2)*ln(c*x+1)-1/16/c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/32*I/c*b^3/(
c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1
)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2-1/32/c*a*b^2*ln(c*x-1)^2-1/16
/c*b^3*arctanh(c*x)^2*ln(c*x-1)-1/16/c*a^2*b*ln(c*x-1)-11/96/c*a*b^2*ln(c*x
-1)+11/96/c*a*b^2*ln(c*x+1)+1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2+3/16*I
*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*x-181/2304*b^3/(c*x+1)^3*x-1/16*I*c^2*b^3/
(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/
(-c^2*x^2+1)^(1/2))*x^3+1/32*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I/(
(c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*x^3+3/32*I*c*b^3/(c*x+1)^3*arctanh(c
*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*c
```

```
sgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*x^2-1/18/c*a*b^2/(c
*x+1)^3-1/6/c*a^2*b/(c*x+1)^3+575/6912*c^2*b^3/(c*x+1)^3*x^3+235/2304*c*b^3
/(c*x+1)^3*x^2-5/96/c*b^3*arctanh(c*x)^2/(c*x+1)^3-139/576/c*b^3/(c*x+1)^3*
arctanh(c*x)-7/24/c*b^3/(c*x+1)^3*arctanh(c*x)^3+11/32*b^3/(c*x+1)^3*arctan
h(c*x)^2*x-23/192*b^3/(c*x+1)^3*arctanh(c*x)*x+1/8*b^3/(c*x+1)^3*arctanh(c*
x)^3*x-5/48/c*a*b^2/(c*x+1)^2-1/8/c*a^2*b/(c*x+1)^2-1/8/c*b^3*arctanh(c*x)^
2/(c*x+1)^2+1/16/c*b^3*arctanh(c*x)^2*ln(c*x+1)-1/8/c*b^3*arctanh(c*x)^2*ln
((c*x+1)/(-c^2*x^2+1)^(1/2))-1/32/c*a*b^2*ln(c*x+1)^2+1/16/c*a^2*b*ln(c*x+1
)-1/16*I/c*b^3/(c*x+1)^3*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1
))^2+1/16*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*x^3+3/16*I*c*b^3/(c*x+1)^3*
arctanh(c*x)^2*Pi*x^2-1/4/c*a*b^2/(c*x+1)*arctanh(c*x)+1/24*c^2*b^3/(c*x+1)
^3*arctanh(c*x)^3*x^3-1/c*a^2*b/(c*x+1)^3*arctanh(c*x)-1/c*a*b^2/(c*x+1)^3*
arctanh(c*x)^2-1/3/c*a*b^2/(c*x+1)^3*arctanh(c*x)+85/576*c^2*b^3/(c*x+1)^3*
arctanh(c*x)*x^3+41/192*c*b^3/(c*x+1)^3*arctanh(c*x)*x^2+1/8*c*b^3/(c*x+1)^
3*arctanh(c*x)^3*x^2+11/96*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*x^3+11/32*c*b^3
/(c*x+1)^3*arctanh(c*x)^2*x^2-3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I
/((c*x+1)^2/(-c^2*x^2+1)+1))^2*x+3/16*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csg
n(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*x-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x-3/32*I*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*x+1/16*I/c*b^3/(c*
x+1)^3*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3-1/32*I*c^2*b^
3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*
x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*x^3+1/32*I/c*b^3/(c*x+1)^3
*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))+3/32*I*b^
3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)
)*x-3/16*I*c*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^2+3/32*I*c*b^3/(c*x+1)^3*arctanh(c*x
)^2*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^
2/(-c^2*x^2+1)+1))^2*x^2+1/32*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I*
(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)
+1))^2*x^3-1/32*I*c^2*b^3/(c*x+1)^3*arctanh(c*x)^2*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^3-3/32*I*c*b^3/(c*x+1)^3*ar
ctanh(c*x)^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^
2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*x^2
```

Maxima [B] time = 1.18677, size = 1465, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="maxima")
```

```
[Out] -1/3*b^3*arctanh(c*x)^3/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/48*(c*(2*(3
*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x +
1)/c^2 + 3*log(c*x - 1)/c^2) + 48*arctanh(c*x)/(c^4*x^3 + 3*c^3*x^2 + 3*c^2
*x + c))*a^2*b - 1/288*(12*c*(2*(3*c^2*x^2 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x
^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 + 3*log(c*x - 1)/c^2)*arctanh(c*x)
 + (66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 + 9*(c^
3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*x^3 + 3
3*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 11)
*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 112)*c^
2/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*a*b^2 - 1/3456*(72*c*(2*(3*c^2*x^2
 + 9*c*x + 10)/(c^5*x^3 + 3*c^4*x^2 + 3*c^3*x + c^2) - 3*log(c*x + 1)/c^2 +
 3*log(c*x - 1)/c^2)*arctanh(c*x)^2 + ((510*c^2*x^2 - 18*(c^3*x^3 + 3*c^2*x
```

```
^2 + 3*c*x + 1)*log(c*x + 1)^3 + 18*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c
*x - 1)^3 + 9*(11*c^3*x^3 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 +
3*c*x + 1)*log(c*x - 1) + 11)*log(c*x + 1)^2 + 99*(c^3*x^3 + 3*c^2*x^2 + 3*
c*x + 1)*log(c*x - 1)^2 + 1134*c*x - 3*(85*c^3*x^3 + 255*c^2*x^2 + 18*(c^3*
x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 255*c*x + 66*(c^3*x^3 + 3*c^2
*x^2 + 3*c*x + 1)*log(c*x - 1) + 85)*log(c*x + 1) + 255*(c^3*x^3 + 3*c^2*x^
2 + 3*c*x + 1)*log(c*x - 1) + 656)*c^2/(c^7*x^3 + 3*c^6*x^2 + 3*c^5*x + c^4
) + 12*(66*c^2*x^2 + 9*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x + 1)^2 + 9
*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1)^2 + 162*c*x - 3*(11*c^3*x^3
 + 33*c^2*x^2 + 33*c*x + 6*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) +
 11)*log(c*x + 1) + 33*(c^3*x^3 + 3*c^2*x^2 + 3*c*x + 1)*log(c*x - 1) + 112
)*c*arctanh(c*x)/(c^6*x^3 + 3*c^5*x^2 + 3*c^4*x + c^3))*c)*b^3 - a*b^2*arct
anh(c*x)^2/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c) - 1/3*a^3/(c^4*x^3 + 3*c^3*x
^2 + 3*c^2*x + c)
```

Fricas [A] time = 1.99753, size = 788, normalized size = 2.87

$$-\frac{6\left(72\,a^2b+132\,ab^2+85\,b^3\right)c^2x^2-18\left(b^3c^3x^3+3\,b^3c^2x^2+3\,b^3cx-7\,b^3\right)\log\left(-\frac{cx+1}{cx-1}\right)^3+1152\,a^3+1440\,a^2b+1344\,ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="fricas")
```

```
[Out] -1/3456*(6*(72*a^2*b + 132*a*b^2 + 85*b^3)*c^2*x^2 - 18*(b^3*c^3*x^3 + 3*b^
3*c^2*x^2 + 3*b^3*c*x - 7*b^3)*log(-(c*x + 1)/(c*x - 1))^3 + 1152*a^3 + 144
0*a^2*b + 1344*a*b^2 + 656*b^3 + 162*(8*a^2*b + 12*a*b^2 + 7*b^3)*c*x - 9*(
(12*a*b^2 + 11*b^3)*c^3*x^3 + 3*(12*a*b^2 + 7*b^3)*c^2*x^2 - 84*a*b^2 - 29*
b^3 + 3*(12*a*b^2 - b^3)*c*x)*log(-(c*x + 1)/(c*x - 1))^2 - 3*((72*a^2*b +
132*a*b^2 + 85*b^3)*c^3*x^3 + 3*(72*a^2*b + 84*a*b^2 + 41*b^3)*c^2*x^2 - 50
4*a^2*b - 348*a*b^2 - 139*b^3 + 3*(72*a^2*b - 12*a*b^2 - 23*b^3)*c*x)*log(-
(c*x + 1)/(c*x - 1)))/(c^4*x^3 + 3*c^3*x^2 + 3*c^2*x + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\,\mathrm{atanh}\,(cx)\right)^3}{\left(cx+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/(c*x+1)**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/(c*x + 1)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\,\mathrm{artanh}\,(cx)+a\right)^3}{\left(cx+1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/(c*x+1)^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/(c*x + 1)^4, x)
```

3.127 $\int \frac{x^2 \tanh^{-1}(ax)^3}{c+acx}\,dx$

Optimal. Leaf size=309

$$-\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^3c}-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a^3c}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a^3c}+$$

```
[Out] (3*ArcTanh[a*x]^2)/(2*a^3*c) + (3*x*ArcTanh[a*x]^2)/(2*a^2*c) - (3*ArcTanh[
a*x]^3)/(2*a^3*c) - (x*ArcTanh[a*x]^3)/(a^2*c) + (x^2*ArcTanh[a*x]^3)/(2*a*
c) - (3*ArcTanh[a*x]*Log[2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]^2*Log[2/(1
 - a*x)])/(a^3*c) - (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^3*c) - (3*PolyLog[
2, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)
])/(a^3*c) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/(2*a^3*c) - (3*
PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(
1 + a*x)])/(2*a^3*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a^3*c)
```

Rubi [A] time = 0.635857, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {5930, 5916, 5980, 5910, 5984, 5918, 2402, 2315, 5948, 6058, 6610, 6056, 6060}

$$-\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^3c}-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a^3c}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a^3c}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (3*ArcTanh[a*x]^2)/(2*a^3*c) + (3*x*ArcTanh[a*x]^2)/(2*a^2*c) - (3*ArcTanh[
a*x]^3)/(2*a^3*c) - (x*ArcTanh[a*x]^3)/(a^2*c) + (x^2*ArcTanh[a*x]^3)/(2*a*
c) - (3*ArcTanh[a*x]*Log[2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]^2*Log[2/(1
 - a*x)])/(a^3*c) - (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^3*c) - (3*PolyLog[
2, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)
])/(a^3*c) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(1 + a*x)])/(2*a^3*c) - (3*
PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(
1 + a*x)])/(2*a^3*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a^3*c)
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx}{a}+\frac{\int x\tanh^{-1}(ax)^3\,dx}{ac}$$

$$= \frac{x^2\tanh^{-1}(ax)^3}{2ac}+\frac{\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx}{a^2}-\frac{3\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{2c}-\frac{\int \tanh^{-1}(ax)^3\,dx}{a^2c}$$

$$= -\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}-\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^3c}+\frac{3\int \tanh^{-1}(ax)^2\,dx}{2a^2c}-\frac{3\int \frac{\tanh^{-1}}{1-a^2}}{2a^2}$$

$$= \frac{3x\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}-\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^3c}$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c}+\frac{3x\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}+\frac{3\tanh}{}$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c}+\frac{3x\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}-\frac{3\tanh}{}$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c}+\frac{3x\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}-\frac{3\tanh}{}$$

$$= \frac{3\tanh^{-1}(ax)^2}{2a^3c}+\frac{3x\tanh^{-1}(ax)^2}{2a^2c}-\frac{3\tanh^{-1}(ax)^3}{2a^3c}-\frac{x\tanh^{-1}(ax)^3}{a^2c}+\frac{x^2\tanh^{-1}(ax)^3}{2ac}-\frac{3\tanh}{}$$

Mathematica [A] time = 0.348015, size = 172, normalized size = 0.56

$$\frac{6\left(\tanh^{-1}(ax)-1\right)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+6\left(\tanh^{-1}(ax)-1\right)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\text{PolyLog}\left(4,-e^{-2t}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]^3 - 4*a*x*ArcTan
h[a*x]^3 + 2*a^2*x^2*ArcTanh[a*x]^3 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh
[a*x])] + 12*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3
*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])^2*PolyLog[2, -E^(-2*A
rcTanh[a*x])] + 6*(-1 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*
PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a^3*c)
```

Maple [A] time = 0.825, size = 400, normalized size = 1.3

$$\frac{x^2 (\mathrm{Artanh}(ax))^3}{2ac} - \frac{x (\mathrm{Artanh}(ax))^3}{a^2c} + \frac{3x (\mathrm{Artanh}(ax))^2}{2a^2c} - \frac{3 (\mathrm{Artanh}(ax))^3}{2a^3c} + \frac{3 (\mathrm{Artanh}(ax))^2}{2a^3c} + \frac{(\mathrm{Artanh}(ax}{2a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(a*c*x+c),x)
```

```
[Out] 1/2*x^2*arctanh(a*x)^3/a/c-x*arctanh(a*x)^3/a^2/c+3/2*x*arctanh(a*x)^2/a^2/
c-3/2*arctanh(a*x)^3/a^3/c+3/2*arctanh(a*x)^2/a^3/c+1/2*arctanh(a*x)^4/a^3/
c-1/a^3/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)-3/2/a^3/c*arctanh(a*x
)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2/a^3/c*arctanh(a*x)*polylog(3,-(a
*x+1)^2/(-a^2*x^2+1))-3/4/a^3/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/a^3/c*
arctanh(a*x)*ln((a*x+1)^2/(-a^2*x^2+1)+1)-3/2/a^3/c*polylog(2,-(a*x+1)^2/(-
a^2*x^2+1))+3/a^3/c*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/a^3/c*arc
tanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2/a^3/c*polylog(3,-(a*x+1)^2
/(-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(a^2x^2 - 2ax + 2\log(ax+1)\right)\log(-ax+1)^3}{16a^3c} + \frac{1}{8}\int \frac{2\left(a^3x^3 - a^2x^2\right)\log(ax+1)^3 - 6\left(a^3x^3 - a^2x^2\right)\log(ax+1)^2 \log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(a^2*x^2 - 2*a*x + 2*log(a*x + 1))*log(-a*x + 1)^3/(a^3*c) + 1/8*inte
grate(1/2*(2*(a^3*x^3 - a^2*x^2)*log(a*x + 1)^3 - 6*(a^3*x^3 - a^2*x^2)*log
(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^3 - a^2*x^2 - 2*a*x + 2*(a^3*x^3 - a^2
*x^2 + a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*c*x^2 - a^2*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2 \operatorname{artanh}(ax)^3}{acx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x^2 \operatorname{atanh}^3(ax)}{ax+1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/(a*x + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{acx + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(a*c*x + c), x)
```

3.128 $\int \frac{x \tanh^{-1}(ax)^3}{c+acx} \, dx$

Optimal. Leaf size=205

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2c}-\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a^2c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a^2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(\right.}{a^2c}$$

```
[Out] ArcTanh[a*x]^3/(a^2*c) + (x*ArcTanh[a*x]^3)/(a*c) - (3*ArcTanh[a*x]^2*Log[2
/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^2*c) - (3*ArcTa
nh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[2,
 1 - 2/(1 + a*x)])/(2*a^2*c) + (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^2*c) -
(3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*a^2*c) - (3*PolyLog[4, 1 -
2/(1 + a*x)])/(4*a^2*c)
```

Rubi [A] time = 0.374202, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5930, 5910, 5984, 5918, 5948, 6058, 6610, 6056, 6060}

$$\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2c}-\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4a^2c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2a^2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(\right.}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```


```
[Out] ArcTanh[a*x]^3/(a^2*c) + (x*ArcTanh[a*x]^3)/(a*c) - (3*ArcTanh[a*x]^2*Log[2
/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a^2*c) - (3*ArcTa
nh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[2,
 1 - 2/(1 + a*x)])/(2*a^2*c) + (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^2*c) -
(3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*a^2*c) - (3*PolyLog[4, 1 -
2/(1 + a*x)])/(4*a^2*c)
```


Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx}{a} + \frac{\int \tanh^{-1}(ax)^3\,dx}{ac}$$
$$= \frac{x\tanh^{-1}(ax)^3}{ac} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - \frac{3\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{c} - \frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{ac}$$
$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - \frac{3\tanh^{-1}(ax)^2\mathrm{Li}_2\left(1-\frac{2}{1+ax}\right)}{2a^2c} - 3\int$$
$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} - \frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - 3\tanh$$
$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} - \frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - 3\tanh$$
$$= \frac{\tanh^{-1}(ax)^3}{a^2c} + \frac{x\tanh^{-1}(ax)^3}{ac} - \frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{a^2c} - 3\tanh$$

Mathematica [A] time = 0.274517, size = 126, normalized size = 0.61

$$-\frac{3}{2}\left(\tanh^{-1}(ax)-2\right)\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) - \frac{3}{2}\left(\tanh^{-1}(ax)-1\right)\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) - \frac{3}{4}\mathrm{Pol}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(c + a*c*x),x]
```

```
[Out] (-ArcTanh[a*x]^3 + a*x*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcT
anh[a*x])] + ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - (3*(-2 + ArcTanh
[a*x])*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])])/2 - (3*(-1 + ArcTanh[
a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])])/2 - (3*PolyLog[4, -E^(-2*ArcTanh[a*
x])])/4)/(a^2*c)
```

Maple [C] time = 0.449, size = 833, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(a*c*x+c),x)
```

```
[Out] x*arctanh(a*x)^3/a/c-1/a^2/c*arctanh(a*x)^3*ln(a*x+1)+2/a^2/c*arctanh(a*x)^
3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2/a^2/c*arctanh(a*x)^2*polylog(2,-(a*x+1
)^2/(-a^2*x^2+1))-3/2/a^2/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))
+3/4/a^2/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-1/2*arctanh(a*x)^4/a^2/c+arct
anh(a*x)^3/a^2/c+1/2*I/a^2/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^3*arctanh(a*x)^3-3/a^2/c*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^
2+1)+1)-3/a^2/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2/a^2/c*p
olylog(3,-(a*x+1)^2/(-a^2*x^2+1))-1/2*I/a^2/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3+1/2*I/a^2/c*Pi*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arcta
nh(a*x)^3+1/2*I/a^2/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+1/2
```

```
*I/a^2/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*arctanh(a*x)^3-1/2*I/a^2/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+I/a^2/c*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a
*x)^3+1/a^2/c*ln(2)*arctanh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(ax-\log(ax+1))\log(-ax+1)^3}{8\,a^2c}+\frac{1}{8}\int\frac{\left(a^2x^2-ax\right)\log(ax+1)^3-3\left(a^2x^2-ax\right)\log(ax+1)^2\log(-ax+1)+3\left(a^2x^2\right.}{a^3cx^2-ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(a*x - log(a*x + 1))*log(-a*x + 1)^3/(a^2*c) + 1/8*integrate(((a^2*x^2
 - a*x)*log(a*x + 1)^3 - 3*(a^2*x^2 - a*x)*log(a*x + 1)^2*log(-a*x + 1) + 3
*(a^2*x^2 + a*x + (a^2*x^2 - 2*a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^3
*c*x^2 - a*c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x\,\text{artanh}(ax)^3}{acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(x*arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x\,\text{atanh}^3(ax)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(x*atanh(a*x)**3/(a*x + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,\text{artanh}(ax)^3}{acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")

[Out] integrate(x*arctanh(a*x)^3/(a*c*x + c), x)
```

3.129 $\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx$

Optimal. Leaf size=104

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4ac}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2ac}-\frac{\log\left(\frac{2}{ax+1}\right)\tanh}{ac}$$

```
[Out] -((ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a*c)) + (3*ArcTanh[a*x]^2*PolyLog[2, 1
 - 2/(1 + a*x)])/(2*a*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*
a*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a*c)
```

Rubi [A] time = 0.164142, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5918, 5948, 6056, 6060, 6610}

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{ax+1}\right)}{4ac}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{ax+1}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{ax+1}\right)}{2ac}-\frac{\log\left(\frac{2}{ax+1}\right)\tanh}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(c + a*c*x),x]
```

```
[Out] -((ArcTanh[a*x]^3*Log[2/(1 + a*x)])/(a*c)) + (3*ArcTanh[a*x]^2*PolyLog[2, 1
 - 2/(1 + a*x)])/(2*a*c) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 + a*x)])/(2*
a*c) + (3*PolyLog[4, 1 - 2/(1 + a*x)])/(4*a*c)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
```

```
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{c+acx}\,dx = -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\int\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1+ax}\right)}{1-a^2x^2}dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac}-\frac{3\int\frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{1-a^2x^2}dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{Li}_3\left(1-\frac{2}{1+ax}\right)}{2ac}-3\int$$

$$= -\frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1+ax}\right)}{ac}+\frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\tanh^{-1}(ax)\text{Li}_3\left(1-\frac{2}{1+ax}\right)}{2ac}+\frac{3\text{Li}_4}{}$$

Mathematica [A] time = 0.0836102, size = 82, normalized size = 0.79

$$\frac{6\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)}{4ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(c + a*c*x),x]
```

```
[Out] (-4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]^2*PolyLog[
2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])]
+ 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a*c)
```

Maple [C] time = 0.191, size = 703, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(a*c*x+c),x)
```

```
[Out] 1/a/c*arctanh(a*x)^3*ln(a*x+1)-2/a/c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)
^(1/2))+1/2/a/c*arctanh(a*x)^4+1/2*I/a/c*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/2*I/a/c*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
^2-1/2*I/a/c*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-I/a/c*arctan
h(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/2*I/a/c*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I/a/c*arctanh(a*x)^3*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2-1/2*I/a/c
```

```
*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
^3-1/a/c*arctanh(a*x)^3*ln(2)-3/2/a/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(
-a^2*x^2+1))+3/2/a/c*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-3/4/a/
c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(ax+1)\log(-ax+1)^3}{8\,ac}+\frac{1}{8}\int\frac{6\,ax\log(ax+1)\log(-ax+1)^2+(ax-1)\log(ax+1)^3-3\,(ax-1)\log(ax+1)^2\log}{a^2cx^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*log(a*x + 1)*log(-a*x + 1)^3/(a*c) + 1/8*integrate((6*a*x*log(a*x + 1)
*log(-a*x + 1)^2 + (a*x - 1)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*lo
g(-a*x + 1))/(a^2*c*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{artanh}(ax)^3}{acx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{atanh}^3(ax)}{ax+1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x + 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{artanh}(ax)^3}{acx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a*c*x + c), x)
```

3.130 $\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx$

Optimal. Leaf size=93

$$-\frac{3\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{\log\left(2-\frac{2}{ax\right.}$$

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rubi [A] time = 0.171676, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5932, 5948, 6056, 6060, 6610}

$$-\frac{3\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{\log\left(2-\frac{2}{ax\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
```

```
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx = \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{(3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(-1+\frac{2}{1+ax}\right)}{2c} + \frac{(3a}{}$$

$$= \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(-1+\frac{2}{1+ax}\right)}{2c} - \frac{3\text{Li}}{}$$

Mathematica [A] time = 0.147915, size = 86, normalized size = 0.92

$$\frac{96\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 48\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right) - 32\,\text{ta}}{64c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(c + a*c*x)),x]
```

```
[Out] (Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog
[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

Maple [C] time = 0.284, size = 1217, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(a*c*x+c),x)
```

```
[Out] 1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh
(a*x)^3-1/2*I/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a
*x)^3+I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^2*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/2*I/c*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^2*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I*((a*x+1)^
```

```
2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-6/
c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*arctanh(a*x)^3*ln
(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*arc
tanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)^2*polylog(2,-
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)
+2/c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*ln(2)*arctanh(a*x)^3
+1/c*arctanh(a*x)^3*ln(a*x)-1/c*arctanh(a*x)^3*ln(a*x+1)+1/2*I/c*Pi*csgn(I*
(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*
x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3-1/2/c*arct
anh(a*x)^4+6/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,-(a*x+1)
/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log\left(ax+1\right)\log\left(-ax+1\right)^{3}}{8\,c}-\frac{1}{8}\int-\frac{\left(ax-1\right)\log\left(ax+1\right)^{3}-3\left(ax-1\right)\log\left(ax+1\right)^{2}\log\left(-ax+1\right)-3\left(a^{2}x^{2}+1\right)\log\left(a\right.}{a^{2}cx^{3}-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1)
^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x + 1
)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{artanh}\left(ax\right)^{3}}{acx^{2}+cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{atanh}^{3}\left(ax\right)}{ax^{2}+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x), x)
```

3.131 $\int \frac{\tanh^{-1}(ax)^3}{cx+acx^2}\,dx$

Optimal. Leaf size=93

$$-\frac{3\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{\log\left(2-\frac{2}{ax}\right.}{}$$

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rubi [A] time = 0.175708, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1593, 5932, 5948, 6056, 6060, 6610}

$$-\frac{3\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{\log\left(2-\frac{2}{ax}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/(2*c) - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c)
- (3*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{cx + acx^2}\,dx = \int \frac{\tanh^{-1}(ax)^3}{x(c + acx)}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3 \log\left(2 - \frac{2}{1+ax}\right)}{c} - \frac{(3a)\int \frac{\tanh^{-1}(ax)^2 \log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^3 \log\left(2 - \frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1 + \frac{2}{1+ax}\right)}{2c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)\text{Li}_2\left(-1+\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^3 \log\left(2 - \frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1 + \frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(-1 + \frac{2}{1+ax}\right)}{2c} + \frac{(3a}{}$$

$$= \frac{\tanh^{-1}(ax)^3 \log\left(2 - \frac{2}{1+ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(-1 + \frac{2}{1+ax}\right)}{2c} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(-1 + \frac{2}{1+ax}\right)}{2c} - \frac{3\text{Li}}{}$$

Mathematica [A] time = 0.0726874, size = 86, normalized size = 0.92

$$\frac{96\tanh^{-1}(ax)^2\text{PolyLog}\left(2, e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\text{PolyLog}\left(3, e^{2\tanh^{-1}(ax)}\right) + 48\text{PolyLog}\left(4, e^{2\tanh^{-1}(ax)}\right) - 32\,\text{ta}}{64c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(c*x + a*c*x^2),x]
```

```
[Out] (Pi^4 - 32*ArcTanh[a*x]^4 + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*PolyLog
[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])])/(64*c)
```

Maple [C] time = 0.207, size = 1217, normalized size = 13.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(a*c*x^2+c*x),x)
```

```
[Out] 1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh
(a*x)^3-1/2*I/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a
```

```
*x)^3+I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^2*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/2*I/c*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^2*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I*((a*x+1)^
2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*arctanh(a*x)^3-1/2*I/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-6/
c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*arctanh(a*x)^3*ln
(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x
^2+1)^(1/2))-6/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*arc
tanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/c*arctanh(a*x)^2*polylog(2,-
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)
+2/c*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*ln(2)*arctanh(a*x)^3
+1/c*arctanh(a*x)^3*ln(a*x)-1/c*arctanh(a*x)^3*ln(a*x+1)+1/2*I/c*Pi*csgn(I*
(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*((a*x+1)^2/(-a^2*
x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3+1/2*I/c*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3-1/2/c*arct
anh(a*x)^4+6/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+6/c*polylog(4,-(a*x+1)
/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\log(ax+1)\log(-ax+1)^3}{8c} - \frac{1}{8}\int -\frac{(ax-1)\log(ax+1)^3 - 3(ax-1)\log(ax+1)^2\log(-ax+1) - 3\left(a^2x^2+1\right)\log(a}{a^2cx^3 - cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] 1/8*log(a*x + 1)*log(-a*x + 1)^3/c - 1/8*integrate(-((a*x - 1)*log(a*x + 1)
^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + 1)*log(a*x + 1
)*log(-a*x + 1)^2)/(a^2*c*x^3 - c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(ax)^3}{acx^2+cx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{atanh}^3(ax)}{ax^2+x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(a*c*x**2+c*x),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**2 + x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{acx^2 + cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a*c*x^2 + c*x), x)
```

3.132 $\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx$

Optimal. Leaf size=191

$$-\frac{3a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{3a\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}+\frac{3a\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3a\tanh^{-1}(ax)\text{Poly}}{c}$$

```
[Out] (a*ArcTanh[a*x]^3)/c - ArcTanh[a*x]^3/(c*x) + (3*a*ArcTanh[a*x]^2*Log[2 - 2
/(1 + a*x)])/c - (a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a*ArcTanh[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]^2*PolyLog[2, -1 + 2
/(1 + a*x)])/(2*c) - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*ArcTan
h[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*PolyLog[4, -1 + 2/(1 + a*
x)])/(4*c)
```

Rubi [A] time = 0.463386, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5934, 5916, 5988, 5932, 5948, 6056, 6610, 6060}

$$-\frac{3a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}+\frac{3a\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}+\frac{3a\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}-\frac{3a\tanh^{-1}(ax)\text{Poly}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```


```
[Out] (a*ArcTanh[a*x]^3)/c - ArcTanh[a*x]^3/(c*x) + (3*a*ArcTanh[a*x]^2*Log[2 - 2
/(1 + a*x)])/c - (a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a*ArcTanh[a
*x]*PolyLog[2, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]^2*PolyLog[2, -1 + 2
/(1 + a*x)])/(2*c) - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*ArcTan
h[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) + (3*a*PolyLog[4, -1 + 2/(1 + a*
x)])/(4*c)
```


Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx = -\left(a\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx\right) + \frac{\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{cx} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx}{c} + \frac{\left(3a^2\right)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}}{c}$$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{3a\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)}{2c} +$$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + $$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - $$

$$= \frac{a\tanh^{-1}(ax)^3}{c} - \frac{\tanh^{-1}(ax)^3}{cx} + \frac{3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{c} - \frac{a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} - $$

Mathematica [C] time = 0.289606, size = 154, normalized size = 0.81

$$a\left(-\frac{3}{2}\left(\tanh^{-1}(ax)-2\right)\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\left(\tanh^{-1}(ax)-1\right)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-\frac{3}{4}\text{PolyL}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(c + a*c*x)),x]
```

```
[Out] (a*((I/8)*Pi^3 - Pi^4/64 - ArcTanh[a*x]^3 - ArcTanh[a*x]^3/(a*x) + ArcTanh[
a*x]^4/2 + 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^3*Lo
g[1 - E^(2*ArcTanh[a*x])] - (3*(-2 + ArcTanh[a*x])*ArcTanh[a*x]*PolyLog[2,
E^(2*ArcTanh[a*x])])/2 + (3*(-1 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x
])])/2 - (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4))/c
```

Maple [C] time = 0.536, size = 1451, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(a*c*x+c),x)
```

```
[Out] -1/2*I*a/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-arctanh(a*x)^3/c/x+1/2*I*a/
c*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/2*I*a/
c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*
csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^
3-a*arctanh(a*x)^3/c+1/2*a*arctanh(a*x)^4/c-6*a/c*polylog(4,(a*x+1)/(-a^2*x
^2+1)^(1/2))-6*a/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a/c*polylog(3,(
a*x+1)/(-a^2*x^2+1)^(1/2))-6*a/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2
*I*a/c*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctan
h(a*x)^3-1/2*I*a/c*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-1/2*I*
a/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arct
anh(a*x)^3+1/2*I*a/c*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^
2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+1/2*I*a/c*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^2*arctanh(a*x)^3-a/c*arctanh(a*x)^3*ln(a*x)+a/c*arctanh(a*x)^3
*ln(a*x+1)+6*a/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a/c*arc
tanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a/c*arctanh(a*x)*polylog(2,(
a*x+1)/(-a^2*x^2+1)^(1/2))-3*a/c*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))+3*a/c*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a/c*arc
tanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a/c*arctanh(a*x)^2*ln(1-
(a*x+1)/(-a^2*x^2+1)^(1/2))-a/c*ln(2)*arctanh(a*x)^3-2*a/c*arctanh(a*x)^3*l
n((a*x+1)/(-a^2*x^2+1)^(1/2))+a/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-
1)+6*a/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a/c*arctanh(a*
x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a/c*arctanh(a*x)^2*polylog(2,(a*x+1
)/(-a^2*x^2+1)^(1/2))-I*a/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a
*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3-1/2*I*a/c*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3+1/2*I*a/c*Pi*csgn
(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(ax\log(ax+1)-1)\log(-ax+1)^3}{8\,cx}+\frac{1}{8}\int\frac{(ax-1)\log(ax+1)^3-3\,(ax-1)\log(ax+1)^2\log(-ax+1)-3\left(a^2x^2+ax\right.}{a^2cx^4-cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(a*x*log(a*x + 1) - 1)*log(-a*x + 1)^3/(c*x) + 1/8*integrate(((a*x - 1
)*log(a*x + 1)^3 - 3*(a*x - 1)*log(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 +
a*x - (a^3*x^3 + a^2*x^2 + a*x - 1)*log(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x
^4 - c*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)^3}{acx^3+cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^3 + c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\operatorname{atanh}^3(ax)}{ax^3+x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**3 + x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{artanh}(ax)^3}{(acx+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x^2), x)
```

3.133 $\int \frac{\tanh^{-1}(ax)^3}{x^3(c+acx)}\,dx$

Optimal. Leaf size=305

$$-\frac{3a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}+\frac{3a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}-\frac{3a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\right.}{2c}$$

```
[Out] (3*a^2*ArcTanh[a*x]^2)/(2*c) - (3*a*ArcTanh[a*x]^2)/(2*c*x) - (a^2*ArcTanh[
a*x]^3)/(2*c) - ArcTanh[a*x]^3/(2*c*x^2) + (a*ArcTanh[a*x]^3)/(c*x) + (3*a^
2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - (3*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1
 + a*x)])/c + (a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a^2*PolyLog[
2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*
x)])/c - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2
*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 +
2/(1 + a*x)])/(2*c) - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```

Rubi [A] time = 0.748083, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5934, 5916, 5982, 5988, 5932, 2447, 5948, 6056, 6610, 6060}

$$-\frac{3a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)}{2c}+\frac{3a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{2c}-\frac{3a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{4c}-\frac{3a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\right.}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```


```
[Out] (3*a^2*ArcTanh[a*x]^2)/(2*c) - (3*a*ArcTanh[a*x]^2)/(2*c*x) - (a^2*ArcTanh[
a*x]^3)/(2*c) - ArcTanh[a*x]^3/(2*c*x^2) + (a*ArcTanh[a*x]^3)/(c*x) + (3*a^
2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/c - (3*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1
 + a*x)])/c + (a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c - (3*a^2*PolyLog[
2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*
x)])/c - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/(2*c) + (3*a^2
*PolyLog[3, -1 + 2/(1 + a*x)])/(2*c) - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 +
2/(1 + a*x)])/(2*c) - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/(4*c)
```


Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

```
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3(c+acx)}\,dx = -\left(a\int \frac{\tanh^{-1}(ax)^3}{x^2(c+acx)}\,dx\right) + \frac{\int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{2cx^2} + a^2\int \frac{\tanh^{-1}(ax)^3}{x(c+acx)}\,dx - \frac{a\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx}{2c}$$

$$= -\frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} + \frac{a^2\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} + \frac{(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx}{2c} - \frac{(3a^2)}{}$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} + \frac{a^2\tanh^{-1}(ax)^3\log\left(2-\right.}{c}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c} - \frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} - \frac{3a^2\,\mathrm{ta}}{}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c} - \frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\,\mathrm{ta}}{}$$

$$= \frac{3a^2\tanh^{-1}(ax)^2}{2c} - \frac{3a\tanh^{-1}(ax)^2}{2cx} - \frac{a^2\tanh^{-1}(ax)^3}{2c} - \frac{\tanh^{-1}(ax)^3}{2cx^2} + \frac{a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\,\mathrm{ta}}{}$$

Mathematica [C] time = 0.629177, size = 222, normalized size = 0.73

$$\frac{a^2\left(96\left(\tanh^{-1}(ax)-2\right)\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) - 96\mathrm{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(c + a*c*x)),x]
```

```
[Out] (a^2*((-8*I)*Pi^3 + Pi^4 + 96*ArcTanh[a*x]^2 - (96*ArcTanh[a*x]^2)/(a*x) +
96*ArcTanh[a*x]^3 - (32*ArcTanh[a*x]^3)/(a^2*x^2) + (64*ArcTanh[a*x]^3)/(a*
x) - 32*ArcTanh[a*x]^4 + 192*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 19
2*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^
(2*ArcTanh[a*x])] - 96*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*(-2 + ArcTanh[a
*x])*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 96*PolyLog[3, E^(2*ArcTa
nh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4,
E^(2*ArcTanh[a*x])]))/(64*c)
```

Maple [B] time = 0.984, size = 664, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(a*c*x+c),x)
```

```
[Out] a*arctanh(a*x)^3/c/x-3/2*a*arctanh(a*x)^2/c/x-1/2*arctanh(a*x)^3/c/x^2-1/2*
a^2*arctanh(a*x)^4/c+6*a^2/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2/c
*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2/c+3*a^2/c*pol
ylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*polylog(2,(a*x+1)/(-a^2*x^2+1)^
(1/2))+3/2*a^2*arctanh(a*x)^3/c+6*a^2/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
```

```
2))+6*a^2/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)^2*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(3,(a*x+1)/
(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3
*a^2/c*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2/c*arctanh(a*x)^2
*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(2,-(a*x+1)/(
-a^2*x^2+1)^(1/2))-3*a^2/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-
6*a^2/c*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2/c*arctanh(a*
x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2/c*arctanh(a*x)^2*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))-6*a^2/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+
1)^(1/2))+a^2/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,a^2x^2\log\left(ax+1\right)-2\,ax+1\right)\log\left(-ax+1\right)^3}{16\,cx^2}-\frac{1}{8}\int-\frac{2\left(ax-1\right)\log\left(ax+1\right)^3-6\left(ax-1\right)\log\left(ax+1\right)^2\log\left(-ax+1\right)+3}{2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(2*a^2*x^2*log(a*x + 1) - 2*a*x + 1)*log(-a*x + 1)^3/(c*x^2) - 1/8*int
egrate(-1/2*(2*(a*x - 1)*log(a*x + 1)^3 - 6*(a*x - 1)*log(a*x + 1)^2*log(-a
*x + 1) + 3*(2*a^3*x^3 + a^2*x^2 - a*x - 2*(a^4*x^4 + a^3*x^3 - a*x + 1)*lo
g(a*x + 1))*log(-a*x + 1)^2)/(a^2*c*x^5 - c*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\mathrm{artanh}\left(ax\right)^3}{acx^4+cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a*c*x^4 + c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\mathrm{atanh}^3\left(ax\right)}{ax^4+x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(a*c*x+c),x)
```

```
[Out] Integral(atanh(a*x)**3/(a*x**4 + x**3), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(acx+c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a*c*x + c)*x^3), x)
```

3.134 $\int \frac{x^2 \tanh^{-1}(ax)^4}{c-acx}\,dx$

Optimal. Leaf size=384

$$-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^3c}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^3c}-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c}+$$

```
[Out] (-2*ArcTanh[a*x]^3)/(a^3*c) - (2*x*ArcTanh[a*x]^3)/(a^2*c) - ArcTanh[a*x]^4
/(2*a^3*c) - (x*ArcTanh[a*x]^4)/(a^2*c) - (x^2*ArcTanh[a*x]^4)/(2*a*c) + (6
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(a^3*c) + (4*ArcTanh[a*x]^3*Log[2/(1 - a*
x)])/(a^3*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]*
PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(a^3*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c)
 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(a^3*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 -
 2/(1 - a*x)])/(a^3*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^
3*c) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]*PolyLog[4,
 1 - 2/(1 - a*x)])/(a^3*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^3*c)
```

Rubi [A] time = 0.862002, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5930, 5916, 5980, 5910, 5984, 5918, 5948, 6058, 6610, 6062}

$$-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a^3c}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^3c}-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^3c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3c}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```



```
[Out] (-2*ArcTanh[a*x]^3)/(a^3*c) - (2*x*ArcTanh[a*x]^3)/(a^2*c) - ArcTanh[a*x]^4
/(2*a^3*c) - (x*ArcTanh[a*x]^4)/(a^2*c) - (x^2*ArcTanh[a*x]^4)/(2*a*c) + (6
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(a^3*c) + (4*ArcTanh[a*x]^3*Log[2/(1 - a*
x)])/(a^3*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]*
PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c) + (6*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(a^3*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 - 2/(1 - a*x)])/(a^3*c)
 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(a^3*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 -
 2/(1 - a*x)])/(a^3*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^
3*c) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(a^3*c) + (3*ArcTanh[a*x]*PolyLog[4,
 1 - 2/(1 - a*x)])/(a^3*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^3*c)
```


Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^4}{c - acx} dx = \frac{\int \frac{x \tanh^{-1}(ax)^4}{c-acx} dx}{a} - \frac{\int x \tanh^{-1}(ax)^4 dx}{ac}$$
$$= -\frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\int \frac{\tanh^{-1}(ax)^4}{c-acx} dx}{a^2} + \frac{2 \int \frac{x^2 \tanh^{-1}(ax)^3}{1-a^2x^2} dx}{c} - \frac{\int \tanh^{-1}(ax)^4 dx}{a^2 c}$$
$$= -\frac{x \tanh^{-1}(ax)^4}{a^2 c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\tanh^{-1}(ax)^4 \log\left(\frac{2}{1-ax}\right)}{a^3 c} - \frac{2 \int \tanh^{-1}(ax)^3 dx}{a^2 c} + \frac{2 \int \frac{\tanh^{-1}}{1-a^2}}{a^2 c}$$
$$= -\frac{2x \tanh^{-1}(ax)^3}{a^2 c} - \frac{\tanh^{-1}(ax)^4}{2a^3 c} - \frac{x \tanh^{-1}(ax)^4}{a^2 c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{\tanh^{-1}(ax)^4 \log\left(\frac{2}{1-ax}\right)}{a^3 c} -$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3 c} - \frac{2x \tanh^{-1}(ax)^3}{a^2 c} - \frac{\tanh^{-1}(ax)^4}{2a^3 c} - \frac{x \tanh^{-1}(ax)^4}{a^2 c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{4 \tanh}{}$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3 c} - \frac{2x \tanh^{-1}(ax)^3}{a^2 c} - \frac{\tanh^{-1}(ax)^4}{2a^3 c} - \frac{x \tanh^{-1}(ax)^4}{a^2 c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{6 \tanh}{}$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3 c} - \frac{2x \tanh^{-1}(ax)^3}{a^2 c} - \frac{\tanh^{-1}(ax)^4}{2a^3 c} - \frac{x \tanh^{-1}(ax)^4}{a^2 c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{6 \tanh}{}$$
$$= -\frac{2 \tanh^{-1}(ax)^3}{a^3 c} - \frac{2x \tanh^{-1}(ax)^3}{a^2 c} - \frac{\tanh^{-1}(ax)^4}{2a^3 c} - \frac{x \tanh^{-1}(ax)^4}{a^2 c} - \frac{x^2 \tanh^{-1}(ax)^4}{2ac} + \frac{6 \tanh}{}$$

Mathematica [A] time = 0.406843, size = 233, normalized size = 0.61

$$-\frac{2\left(\tanh^{-1}(ax)^2 + 3\tanh^{-1}(ax) + 3\right)\tanh^{-1}(ax)\text{PolyLog}\left(2, -e^{-2\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)\text{PolyLog}\left(4, -e^{-2\tanh^{-1}(ax)}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -((-2*ArcTanh[a*x]^3 + 2*a*x*ArcTanh[a*x]^3 - ArcTanh[a*x]^4 + a*x*ArcTanh[
a*x]^4 - ((1 - a^2*x^2)*ArcTanh[a*x]^4)/2 - (2*ArcTanh[a*x]^5)/5 - 6*ArcTan
h[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcT
anh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*ArcTanh[a*x]*(
3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*(
1 + ArcTanh[a*x])^2*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*
ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])] + (3*PolyL
og[5, -E^(-2*ArcTanh[a*x])])/2)/(a^3*c))
```

Maple [A] time = 0.571, size = 496, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^4/(-a*c*x+c),x)
```

```
[Out] -1/2*x^2*arctanh(a*x)^4/a/c-x*arctanh(a*x)^4/a^2/c-2*x*arctanh(a*x)^3/a^2/c
-1/2*arctanh(a*x)^4/a^3/c-2*arctanh(a*x)^3/a^3/c+1/a^3/c*arctanh(a*x)^4*ln(
(a*x+1)^2/(-a^2*x^2+1)+1)+2/a^3/c*arctanh(a*x)^3*polylog(2,-(a*x+1)^2/(-a^2
```

```
*x^2+1))-3/a^3/c*arctanh(a*x)^2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/a^3/c*
arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/2/a^3/c*polylog(5,-(a*x+1
)^2/(-a^2*x^2+1))+6/a^3/c*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+6/a^3
/c*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/a^3/c*polylog(3,-(a*x+
1)^2/(-a^2*x^2+1))+4/a^3/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)+6/a^
3/c*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-6/a^3/c*arctanh(a*x)*
polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/a^3/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\,\log\left(-ax+1\right)^{5}+5\left(2\,\log\left(-ax+1\right)^{4}-4\,\log\left(-ax+1\right)^{3}+6\,\log\left(-ax+1\right)^{2}-6\,\log\left(-ax+1\right)+3\right)(ax-1)^{2}+40\,(\mathrm{lo}}{320\,a^{3}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/320*(4*log(-a*x + 1)^5 + 5*(2*log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 6*lo
g(-a*x + 1)^2 - 6*log(-a*x + 1) + 3)*(a*x - 1)^2 + 40*(log(-a*x + 1)^4 - 4*
log(-a*x + 1)^3 + 12*log(-a*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a
^3*c) + 1/16*integrate(-(x^2*log(a*x + 1)^4 - 4*x^2*log(a*x + 1)^3*log(-a*x
 + 1) + 6*x^2*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*x^2*log(a*x + 1)*log(-a*x
+ 1)^3)/(a*c*x - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x^{2}\operatorname{artanh}\left(ax\right)^{4}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{x^{2}\operatorname{atanh}^{4}\left(ax\right)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**4/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^{2}\operatorname{artanh}\left(ax\right)^{4}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^4/(a*c*x - c), x)
```

3.135 $\int \frac{x \tanh^{-1}(ax)^4}{c-acx} dx$

Optimal. Leaf size=261

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^2c}-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2c}+\frac{6\tanh^{-1}(ax)^2\text{PolyLog}}{a^2c}$$

```
[Out] -(ArcTanh[a*x]^4/(a^2*c)) - (x*ArcTanh[a*x]^4)/(a*c) + (4*ArcTanh[a*x]^3*Lo
g[2/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^2*c) + (6*Ar
cTanh[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) + (2*ArcTanh[a*x]^3*PolyL
og[2, 1 - 2/(1 - a*x)])/(a^2*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x
)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^2*c) + (3*P
olyLog[4, 1 - 2/(1 - a*x)])/(a^2*c) + (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 -
 a*x)])/(a^2*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^2*c)
```

Rubi [A] time = 0.498128, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {5930, 5910, 5984, 5918, 5948, 6058, 6062, 6610}

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{a^2c}-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2a^2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2c}+\frac{6\tanh^{-1}(ax)^2\text{PolyLog}}{a^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -(ArcTanh[a*x]^4/(a^2*c)) - (x*ArcTanh[a*x]^4)/(a*c) + (4*ArcTanh[a*x]^3*Lo
g[2/(1 - a*x)])/(a^2*c) + (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a^2*c) + (6*Ar
cTanh[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(a^2*c) + (2*ArcTanh[a*x]^3*PolyL
og[2, 1 - 2/(1 - a*x)])/(a^2*c) - (6*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x
)])/(a^2*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a^2*c) + (3*P
olyLog[4, 1 - 2/(1 - a*x)])/(a^2*c) + (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 -
 a*x)])/(a^2*c) - (3*PolyLog[5, 1 - 2/(1 - a*x)])/(2*a^2*c)
```

Rule 5930

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) +
(e_.)*(x_)), x_Symbol] :> Dist[f/e, Int[(f*x)^(m - 1)*(a + b*ArcTanh[c*x])^
p, x], x] - Dist[(d*f)/e, Int[((f*x)^(m - 1)*(a + b*ArcTanh[c*x])^p)/(d + e
*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 -
e^2, 0] && GtQ[m, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx}{a} - \frac{\int \tanh^{-1}(ax)^4\,dx}{ac}$$

$$= -\frac{x\tanh^{-1}(ax)^4}{ac} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{4\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{c} - \frac{4\int \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{ac}$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{2\tanh^{-1}(ax)^3\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2c} + \ldots$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \ldots$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \ldots$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \ldots$$

$$= -\frac{\tanh^{-1}(ax)^4}{a^2c} - \frac{x\tanh^{-1}(ax)^4}{ac} + \frac{4\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2c} + \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{a^2c} + \ldots$$

Mathematica [A] time = 0.269772, size = 172, normalized size = 0.66

$$-\frac{2\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+3\left(\tanh^{-1}(ax)+2\right)\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^4)/(c - a*c*x),x]
```

```
[Out] -((-ArcTanh[a*x]^4 + a*x*ArcTanh[a*x]^4 - (2*ArcTanh[a*x]^5)/5 - 4*ArcTanh[
a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[
a*x])] + 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, -E^(-2*ArcTanh[a*x]
)] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3
*PolyLog[4, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTan
h[a*x])] + (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a^2*c))
```

Maple [C] time = 0.223, size = 454, normalized size = 1.7

$$-\frac{x\left(\text{Artanh}\left(ax\right)\right)^4}{ac}-\frac{\left(\text{Artanh}\left(ax\right)\right)^4\ln\left(ax-1\right)}{a^2c}+2\,\frac{\left(\text{Artanh}\left(ax\right)\right)^3}{a^2c}\text{polylog}\left(2,-\frac{\left(ax+1\right)^2}{-a^2x^2+1}\right)-3\,\frac{\left(\text{Artanh}\left(ax\right)\right)^2}{a^2c}\text{pol}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^4/(-a*c*x+c),x)
```

```
[Out] -x*arctanh(a*x)^4/a/c-1/a^2/c*arctanh(a*x)^4*ln(a*x-1)+2/a^2/c*arctanh(a*x)
^3*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/a^2/c*arctanh(a*x)^2*polylog(3,-(a*
x+1)^2/(-a^2*x^2+1))+3/a^2/c*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1)
)-3/2/a^2/c*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))-I/a^2/c*Pi*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*arctanh(a*x)^4+I/a^2/c*Pi*arctanh(a*x)^4-arctanh(a*x)^4
/a^2/c+4/a^2/c*arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)+1)+6/a^2/c*arctanh(
a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-6/a^2/c*arctanh(a*x)*polylog(3,-(
a*x+1)^2/(-a^2*x^2+1))+3/a^2/c*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+1/a^2/c*l
n(2)*arctanh(a*x)^4+I/a^2/c*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh
(a*x)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(-ax+1\right)^5+5\left(\log\left(-ax+1\right)^4-4\,\log\left(-ax+1\right)^3+12\,\log\left(-ax+1\right)^2-24\,\log\left(-ax+1\right)+24\right)\left(ax-1\right)}{80\,a^2c}+\frac{1}{16}\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*(log(-a*x + 1)^5 + 5*(log(-a*x + 1)^4 - 4*log(-a*x + 1)^3 + 12*log(-a
*x + 1)^2 - 24*log(-a*x + 1) + 24)*(a*x - 1))/(a^2*c) + 1/16*integrate(-(x*
log(a*x + 1)^4 - 4*x*log(a*x + 1)^3*log(-a*x + 1) + 6*x*log(a*x + 1)^2*log(
-a*x + 1)^2 - 4*x*log(a*x + 1)*log(-a*x + 1)^3)/(a*c*x - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x \operatorname{artanh}\left(ax\right)^{4}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{x \operatorname{atanh}^{4}\left(ax\right)}{ax-1}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(x*atanh(a*x)**4/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \operatorname{artanh}\left(ax\right)^{4}}{acx-c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^4/(a*c*x - c), x)
```

3.136 $\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx$

Optimal. Leaf size=131

$$-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2ac}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{ac}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{ac}+\frac{3\tanh^{-1}(}{}$$

```
[Out] (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 -
2/(1 - a*x)])/(a*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a*c)
+ (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 - a*x)])/(a*c) - (3*PolyLog[5, 1 - 2/
(1 - a*x)])/(2*a*c)
```

Rubi [A] time = 0.214482, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5918, 5948, 6058, 6062, 6610}

$$-\frac{3\text{PolyLog}\left(5,1-\frac{2}{1-ax}\right)}{2ac}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{ac}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{ac}+\frac{3\tanh^{-1}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(c - a*c*x),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2/(1 - a*x)])/(a*c) + (2*ArcTanh[a*x]^3*PolyLog[2, 1 -
2/(1 - a*x)])/(a*c) - (3*ArcTanh[a*x]^2*PolyLog[3, 1 - 2/(1 - a*x)])/(a*c)
+ (3*ArcTanh[a*x]*PolyLog[4, 1 - 2/(1 - a*x)])/(a*c) - (3*PolyLog[5, 1 - 2/
(1 - a*x)])/(2*a*c)
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
```

```
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{c-acx}\,dx = \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} - \frac{4\int \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{6\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} + \frac{6\int \frac{\tanh^{-1}}{}}{}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} + \frac{3\tanh^{-1}(}{}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(\frac{2}{1-ax}\right)}{ac} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{ac} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{ac} + \frac{3\tanh^{-1}(}{}$$

Mathematica [A] time = 0.108939, size = 112, normalized size = 0.85

$$-\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)\text{PolyLog}\left(4,-e^{-}\right.}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(c - a*c*x),x]
```

```
[Out] -(((-2*ArcTanh[a*x]^5)/5 - ArcTanh[a*x]^4*Log[1 + E^(-2*ArcTanh[a*x])] + 2*
ArcTanh[a*x]^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]^2*PolyLog[
3, -E^(-2*ArcTanh[a*x])] + 3*ArcTanh[a*x]*PolyLog[4, -E^(-2*ArcTanh[a*x])]
+ (3*PolyLog[5, -E^(-2*ArcTanh[a*x])])/2)/(a*c))
```

Maple [C] time = 0.201, size = 285, normalized size = 2.2

$$-\frac{(\text{Artanh}(ax))^4\ln(ax-1)}{ac} + \frac{i(\text{Artanh}(ax))^4\pi}{ac}\left(\text{csgn}\left(i\left(\frac{(ax+1)^2}{-a^2x^2+1}+1\right)^{-1}\right)\right)^3 - \frac{i(\text{Artanh}(ax))^4\pi}{ac}\left(\text{csgn}\left(i\left(\frac{(ax+1)}{-a^2x^2+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/(-a*c*x+c),x)
```

```
[Out] -1/a/c*arctanh(a*x)^4*ln(a*x-1)+I/a/c*arctanh(a*x)^4*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))^3*Pi-I/a/c*arctanh(a*x)^4*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2
*Pi+I/a/c*arctanh(a*x)^4*Pi+1/a/c*arctanh(a*x)^4*ln(2)+2/a/c*arctanh(a*x)^3
```

```
*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/a/c*arctanh(a*x)^2*polylog(3,-(a*x+1)
^2/(-a^2*x^2+1))+3/a/c*arctanh(a*x)*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/2/
a/c*polylog(5,-(a*x+1)^2/(-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(-ax+1\right)^{5}}{80\,ac}+\frac{1}{16}\int-\frac{\log\left(ax+1\right)^{4}-4\,\log\left(ax+1\right)^{3}\log\left(-ax+1\right)+6\,\log\left(ax+1\right)^{2}\log\left(-ax+1\right)^{2}-4\,\log\left(ax+\right.}{acx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/(a*c) + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x +
 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log
(-a*x + 1)^3)/(a*c*x - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}\left(ax\right)^{4}}{acx-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\text{atanh}^{4}\left(ax\right)}{ax-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x - 1), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\text{artanh}\left(ax\right)^{4}}{acx-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/(a*c*x - c), x)
```

3.137 $\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx$

Optimal. Leaf size=118

$$-\frac{3\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{c}+\frac{3\tanh^{-1}(ax)}{}$$

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rubi [A] time = 0.220625, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5932, 5948, 6058, 6062, 6610}

$$-\frac{3\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{c}+\frac{3\tanh^{-1}(ax)}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
```

```
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx = \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(4a)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{(6a)\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c} +$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c} +$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c} +$$

Mathematica [A] time = 0.168999, size = 102, normalized size = 0.86

$$\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{3\tanh^{-1}(ax)\text{PolyLog}\left(4,e^{2\tanh}\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(x*(c - a*c*x)),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog[
2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x])
])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5, E^
(2*ArcTanh[a*x])])/(2*c)
```

Maple [C] time = 0.277, size = 843, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x/(-a*c*x+c),x)
```

```
[Out] -1/c*arctanh(a*x)^4*ln(a*x-1)+1/c*arctanh(a*x)^4*ln(a*x)-1/c*arctanh(a*x)^4
*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))+4/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctan
h(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,
(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*
```

```
arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/
c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/c*arctanh(a*x)^4*Pi*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+
1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/2*I/c*arctanh(a*x)^4*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2-I/c*arctanh(a*x)^4*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^2+I/c*arctanh(a*x)^4*Pi-1/2*I/c*arctanh(a*x)^4*Pi*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2+I/c*arctanh(a*x)^4*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/2*I/c*arc
tanh(a*x)^4*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3+1/c*arctanh(a*x)^4*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(-ax+1)^5}{80\,c}+\frac{1}{16}\int-\frac{\log(ax+1)^4-4\,\log(ax+1)^3\log(-ax+1)+6\,\log(ax+1)^2\log(-ax+1)^2-4\,\log(ax+1)\,\log(-ax+1)^3}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^
3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-a*
x + 1)^3)/(a*c*x^2 - c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^4}{acx^2-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\operatorname{atanh}^4(ax)}{ax^2-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x), x)
```

3.138 $\int \frac{\tanh^{-1}(ax)^4}{cx-acx^2}\,dx$

Optimal. Leaf size=118

$$-\frac{3\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{c}+\frac{3\tanh^{-1}(ax)}{}$$

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rubi [A] time = 0.225395, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1593, 5932, 5948, 6058, 6062, 6610}

$$-\frac{3\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)}{c}+\frac{3\tanh^{-1}(ax)}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (2*ArcTanh[a*x]^3*PolyLog[2, -1 +
 2/(1 - a*x)])/c - (3*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c + (3*A
rcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*PolyLog[5, -1 + 2/(1 - a*x
)])/(2*c)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

```
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{cx-acx^2}\,dx = \int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} - \frac{(4a)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{(6a)\int \frac{\tanh^{-1}(ax)^2\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{c}$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c} +$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c} +$$

$$= \frac{\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{Li}_3\left(-1+\frac{2}{1-ax}\right)}{c} +$$

Mathematica [A] time = 0.0776738, size = 102, normalized size = 0.86

$$\frac{2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)}{c} - \frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)}{c} + \frac{3\tanh^{-1}(ax)\text{PolyLog}\left(4,e^{2\tanh}\right.}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(c*x - a*c*x^2),x]
```

```
[Out] (ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])])/c + (2*ArcTanh[a*x]^3*PolyLog[
2, E^(2*ArcTanh[a*x])])/c - (3*ArcTanh[a*x]^2*PolyLog[3, E^(2*ArcTanh[a*x])
])/c + (3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])])/c - (3*PolyLog[5, E^
(2*ArcTanh[a*x])])/(2*c)
```

Maple [C] time = 0.214, size = 843, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/(-a*c*x^2+c*x),x)
```

```
[Out] -1/c*arctanh(a*x)^4*ln(a*x-1)+1/c*arctanh(a*x)^4*ln(a*x)-1/c*arctanh(a*x)^4
*ln((a*x+1)^2/(-a^2*x^2+1)-1)+1/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))+4/c*arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctan
h(a*x)^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,
(a*x+1)/(-a^2*x^2+1)^(1/2))-24/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/c*
arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4/c*arctanh(a*x)^3*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12/c*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))+24/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24/
c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/c*arctanh(a*x)^4*Pi*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+
1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/2*I/c*arctanh(a*x)^4*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2-I/c*arctanh(a*x)^4*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^2+I/c*arctanh(a*x)^4*Pi-1/2*I/c*arctanh(a*x)^4*Pi*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2+I/c*arctanh(a*x)^4*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/2*I/c*arc
tanh(a*x)^4*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3+1/c*arctanh(a*x)^4*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log(-ax+1)^5}{80\,c}+\frac{1}{16}\int -\frac{\log(ax+1)^4-4\,\log(ax+1)^3\log(-ax+1)+6\,\log(ax+1)^2\log(-ax+1)^2-4\,\log(ax+1)\,\log(-ax+1)^3}{acx^2-cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="maxima")
```

```
[Out] -1/80*log(-a*x + 1)^5/c + 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^
3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*log(a*x + 1)*log(-a*
x + 1)^3)/(a*c*x^2 - c*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}(ax)^4}{acx^2-cx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^2-x}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/(-a*c*x**2+c*x),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**2 - x), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^4}{acx^2 - cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/(-a*c*x^2+c*x),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/(a*c*x^2 - c*x), x)
```

3.139 $\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx$

Optimal. Leaf size=239

$$-\frac{3a\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{c}-\frac{3a\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2a\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{6a\tanh^{-1}(ax)^2\text{PolyLo}}{c}$$

```
[Out] (a*ArcTanh[a*x]^4)/c - ArcTanh[a*x]^4/(c*x) + (a*ArcTanh[a*x]^4*Log[2 - 2/(
1 - a*x)])/c + (4*a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a*ArcTanh[a
*x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/c - (3*a*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (6
*a*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]*PolyLog
[4, -1 + 2/(1 - a*x)])/c - (3*a*PolyLog[4, -1 + 2/(1 + a*x)])/c - (3*a*Poly
Log[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rubi [A] time = 0.548763, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5934, 5916, 5988, 5932, 5948, 6056, 6060, 6610, 6058, 6062}

$$-\frac{3a\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{c}-\frac{3a\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2a\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\frac{2}{1-ax}-1\right)}{c}-\frac{6a\tanh^{-1}(ax)^2\text{PolyLo}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

```
[Out] (a*ArcTanh[a*x]^4)/c - ArcTanh[a*x]^4/(c*x) + (a*ArcTanh[a*x]^4*Log[2 - 2/(
1 - a*x)])/c + (4*a*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a*ArcTanh[a
*x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a*ArcTanh[a*x]^2*PolyLog[2, -1 +
 2/(1 + a*x)])/c - (3*a*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (6
*a*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c + (3*a*ArcTanh[a*x]*PolyLog
[4, -1 + 2/(1 - a*x)])/c - (3*a*PolyLog[4, -1 + 2/(1 + a*x)])/c - (3*a*Poly
Log[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}dx = a\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}dx + \frac{\int \frac{\tanh^{-1}(ax)^4}{x^2}dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{(4a)\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}dx}{c} - \frac{(4a^2)\int \frac{\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1-ax}\right.}{1-a^2x^2}}{c}$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{2a\tanh^{-1}(ax)^3\text{Li}_2\left(-1+\frac{2}{1-ax}\right)}{c} +$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} +$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} +$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} +$$

$$= \frac{a\tanh^{-1}(ax)^4}{c} - \frac{\tanh^{-1}(ax)^4}{cx} + \frac{a\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{4a\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right)}{c} +$$

Mathematica [C] time = 0.501071, size = 172, normalized size = 0.72

$$-\frac{a\left(-2\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+3\left(\tanh^{-1}(ax)+2\right)\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(x^2*(c - a*c*x)),x]
```

```
[Out] -((a*(-Pi^4/16 + (I/160)*Pi^5 + ArcTanh[a*x]^4 + ArcTanh[a*x]^4/(a*x) - 4*A
rcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*Arc
Tanh[a*x])] - 2*ArcTanh[a*x]^2*(3 + ArcTanh[a*x])*PolyLog[2, E^(2*ArcTanh[a
*x])] + 3*ArcTanh[a*x]*(2 + ArcTanh[a*x])*PolyLog[3, E^(2*ArcTanh[a*x])] -
3*PolyLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a
*x])] + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

Maple [B] time = 0.309, size = 583, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x^2/(-a*c*x+c),x)
```

```
[Out] -a*arctanh(a*x)^4/c-arctanh(a*x)^4/c/x+a/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))+4*a/c*arctanh(a*x)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
-12*a/c*arctanh(a*x)^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a/c*arctan
h(a*x)*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24*a/c*polylog(5,-(a*x+1)/(-a
^2*x^2+1)^(1/2))+a/c*arctanh(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a/c*
arctanh(a*x)^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a/c*arctanh(a*x)^2*
polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a/c*arctanh(a*x)*polylog(4,(a*x+1)
```

```
/(-a^2*x^2+1)^(1/2))-24*a/c*polylog(5,(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a/c*arc
tanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a/c*arctanh(a*x)^2*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24*a/c*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))+24*a/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a/c*arctanh(
a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a/c*arctanh(a*x)^2*polylog(2,(a*
x+1)/(-a^2*x^2+1)^(1/2))-24*a/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)
^(1/2))+24*a/c*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ax\log(-ax+1)^5+5\log(-ax+1)^4}{80\,cx}+\frac{1}{16}\int-\frac{\log(ax+1)^4-4\log(ax+1)^3\log(-ax+1)+6\log(ax+1)^2\log(-}{acx^3-cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/80*(a*x*log(-a*x + 1)^5 + 5*log(-a*x + 1)^4)/(c*x) + 1/16*integrate(-(lo
g(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(a*x + 1)^2*log(-a*x +
 1)^2 - 4*(a*x + log(a*x + 1))*log(-a*x + 1)^3)/(a*c*x^3 - c*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^4}{acx^3-cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^3 - c*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{\operatorname{atanh}^4(ax)}{ax^3-x^2}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x**2/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**3 - x**2), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^2/(-a*c*x+c),x, algorithm="giac")

[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x^2), x)
```

3.140 $\int \frac{\tanh^{-1}(ax)^4}{x^3(c-acx)}\,dx$

Optimal. Leaf size=380

$$-\frac{3a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2a^2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\right.}{c}$$

```
[Out] (2*a^2*ArcTanh[a*x]^3)/c - (2*a*ArcTanh[a*x]^3)/(c*x) + (3*a^2*ArcTanh[a*x]
^4)/(2*c) - ArcTanh[a*x]^4/(2*c*x^2) - (a*ArcTanh[a*x]^4)/(c*x) + (a^2*ArcT
anh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (6*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a
*x)])/c + (4*a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a^2*ArcTanh[a*
x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[2, -1 +
2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/c - (
3*a^2*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[3, -1
 + 2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c +
(3*a^2*ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[4, -1
+ 2/(1 + a*x)])/c - (3*a^2*PolyLog[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rubi [A] time = 0.955561, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5934, 5916, 5982, 5988, 5932, 5948, 6056, 6610, 6060, 6058, 6062}

$$-\frac{3a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)}{c}-\frac{3a^2\text{PolyLog}\left(5,\frac{2}{1-ax}-1\right)}{2c}+\frac{2a^2\tanh^{-1}(ax)^3\text{PolyLog}\left(2,\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

```
[Out] (2*a^2*ArcTanh[a*x]^3)/c - (2*a*ArcTanh[a*x]^3)/(c*x) + (3*a^2*ArcTanh[a*x]
^4)/(2*c) - ArcTanh[a*x]^4/(2*c*x^2) - (a*ArcTanh[a*x]^4)/(c*x) + (a^2*ArcT
anh[a*x]^4*Log[2 - 2/(1 - a*x)])/c + (6*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a
*x)])/c + (4*a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)])/c + (2*a^2*ArcTanh[a*
x]^3*PolyLog[2, -1 + 2/(1 - a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[2, -1 +
2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/c - (
3*a^2*ArcTanh[a*x]^2*PolyLog[3, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[3, -1
 + 2/(1 + a*x)])/c - (6*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/c +
(3*a^2*ArcTanh[a*x]*PolyLog[4, -1 + 2/(1 - a*x)])/c - (3*a^2*PolyLog[4, -1
+ 2/(1 + a*x)])/c - (3*a^2*PolyLog[5, -1 + 2/(1 - a*x)])/(2*c)
```

Rule 5934

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x],
x] - Dist[e/(d*f), Int[((f*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d + e*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
 && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^4}{x^3(c-acx)}\,dx = a\int \frac{\tanh^{-1}(ax)^4}{x^2(c-acx)}\,dx + \frac{\int \frac{\tanh^{-1}(ax)^4}{x^3}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{2cx^2} + a^2\int \frac{\tanh^{-1}(ax)^4}{x(c-acx)}\,dx + \frac{a\int \frac{\tanh^{-1}(ax)^4}{x^2}\,dx}{c} + \frac{(2a)\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)}\,dx}{c}$$

$$= -\frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\tanh^{-1}(ax)^4\log\left(2-\frac{2}{1-ax}\right)}{c} + \frac{(2a)\int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx}{c} + \frac{(4a^2)\ldots}{\ldots}$$

$$= -\frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\tanh^{-1}(ax)^4\log\left(2-\ldots\right.}{c}$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\,\mathrm{ta}\ldots}{\ldots}$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\,\mathrm{ta}\ldots}{\ldots}$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\,\mathrm{ta}\ldots}{\ldots}$$

$$= \frac{2a^2\tanh^{-1}(ax)^3}{c} - \frac{2a\tanh^{-1}(ax)^3}{cx} + \frac{3a^2\tanh^{-1}(ax)^4}{2c} - \frac{\tanh^{-1}(ax)^4}{2cx^2} - \frac{a\tanh^{-1}(ax)^4}{cx} + \frac{a^2\,\mathrm{ta}\ldots}{\ldots}$$

Mathematica [C] time = 0.868193, size = 250, normalized size = 0.66

$$-\frac{a^2\left(-2\left(\tanh^{-1}(ax)^2+3\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-3\tanh^{-1}(ax)\mathrm{PolyLog}\left(4,e^{2\tanh^{-1}(\ldots}\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^4/(x^3*(c - a*c*x)),x]
```

```
[Out] -((a^2*((-I/4)*Pi^3 - Pi^4/16 + (I/160)*Pi^5 + 2*ArcTanh[a*x]^3 + (2*ArcTan
h[a*x]^3)/(a*x) + ArcTanh[a*x]^4/2 + ArcTanh[a*x]^4/(2*a^2*x^2) + ArcTanh[a
*x]^4/(a*x) - 6*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 4*ArcTanh[a*x]
^3*Log[1 - E^(2*ArcTanh[a*x])] - ArcTanh[a*x]^4*Log[1 - E^(2*ArcTanh[a*x])]
 - 2*ArcTanh[a*x]*(3 + 3*ArcTanh[a*x] + ArcTanh[a*x]^2)*PolyLog[2, E^(2*Arc
Tanh[a*x])] + 3*(1 + ArcTanh[a*x])^2*PolyLog[3, E^(2*ArcTanh[a*x])] - 3*Pol
yLog[4, E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[4, E^(2*ArcTanh[a*x])]
 + (3*PolyLog[5, E^(2*ArcTanh[a*x])])/2))/c)
```

Maple [B] time = 0.524, size = 858, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^4/x^3/(-a*c*x+c),x)
```

```
[Out] -a*arctanh(a*x)^4/c/x-2*a*arctanh(a*x)^3/c/x-1/2*a^2*arctanh(a*x)^4/c-1/2*a
rctanh(a*x)^4/c/x^2-2*a^2*arctanh(a*x)^3/c-24*a^2/c*polylog(5,(a*x+1)/(-a^2
*x^2+1)^(1/2))-24*a^2/c*polylog(5,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2/c*arctan
h(a*x)^4*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2/c*arctanh(a*x)^3*polylog(2,
(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2/c*arctanh(a*x)^2*polylog(3,(a*x+1)/(-a^2
*x^2+1)^(1/2))+24*a^2/c*arctanh(a*x)*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+
a^2/c*arctanh(a*x)^4*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2/c*arctanh(a*x)^
3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2/c*arctanh(a*x)^2*polylog(3,
-(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a^2/c*arctanh(a*x)*polylog(4,-(a*x+1)/(-a^2
*x^2+1)^(1/2))+12*a^2/c*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2)
)-24*a^2/c*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2/c*arcta
nh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2/c*arctanh(a*x)*polylog(2,
-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2/c*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+
1)^(1/2))+12*a^2/c*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2
/c*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2/c*arctanh(a*x)^2*
polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-24*a^2/c*arctanh(a*x)*polylog(3,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+4*a^2/c*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+24*a^2/c*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+24*a^2/c*polylog(4,(a
*x+1)/(-a^2*x^2+1)^(1/2))-12*a^2/c*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1
2*a^2/c*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,a^2x^2\log\left(-ax+1\right)^5+5\,(2\,ax+1)\log\left(-ax+1\right)^4}{160\,cx^2}+\frac{1}{16}\int -\frac{\log\left(ax+1\right)^4-4\,\log\left(ax+1\right)^3\log\left(-ax+1\right)+6\,\log\left(ax+\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="maxima")
```

```
[Out] -1/160*(2*a^2*x^2*log(-a*x + 1)^5 + 5*(2*a*x + 1)*log(-a*x + 1)^4)/(c*x^2)
+ 1/16*integrate(-(log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(-a*x + 1) + 6*log(
a*x + 1)^2*log(-a*x + 1)^2 - 2*(2*a^2*x^2 + a*x + 2*log(a*x + 1))*log(-a*x
+ 1)^3)/(a*c*x^4 - c*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}\left(ax\right)^4}{acx^4-cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^4/(a*c*x^4 - c*x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\operatorname{atanh}^4(ax)}{ax^4-x^3}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**4/x**3/(-a*c*x+c),x)
```

```
[Out] -Integral(atanh(a*x)**4/(a*x**4 - x**3), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^4}{(acx-c)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^4/x^3/(-a*c*x+c),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^4/((a*c*x - c)*x^3), x)
```

3.141 $\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{x}{(acx+c)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0405899, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx=\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.30026, size = 0, normalized size = 0.

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x/((c + a*c*x)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.274, size = 0, normalized size = 0.

$$\int \frac{x}{(acx+c)\,\text{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c*x+c)/arctanh(a*x),x)
```

```
[Out] int(x/(a*c*x+c)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(acx+c)\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x}{(acx+c)\,\mathrm{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/((a*c*x + c)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{x}{ax\,\mathrm{atanh}\,(ax)+\mathrm{atanh}\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/atanh(a*x),x)
```

```
[Out] Integral(x/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(acx+c)\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)), x)
```

3.142 $\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=17

$$\text{Unintegrable}\left(\frac{1}{(acx+c)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0247666, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx = \int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.111588, size = 0, normalized size = 0.

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/((c + a*c*x)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.253, size = 0, normalized size = 0.

$$\int \frac{1}{(acx+c)\,\text{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x+c)/arctanh(a*x),x)
```

```
[Out] int(1/(a*c*x+c)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(acx + c)\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(acx + c)\,\text{artanh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a*c*x + c)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ax\,\text{atanh}\,(ax)+\text{atanh}\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/atanh(a*x),x)
```

```
[Out] Integral(1/(a*x*atanh(a*x) + atanh(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(acx + c)\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)), x)
```

3.143 $\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x(acx+c)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0584234, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx = \int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.132888, size = 0, normalized size = 0.

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.298, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(acx+c)\,\mathrm{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c*x+c)/arctanh(a*x),x)
```

```
[Out] int(1/x/(a*c*x+c)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(acx+c)x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(acx^2+cx\right)\mathrm{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a*c*x^2 + c*x)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ax^2\,\mathrm{atanh}\,(ax)+x\,\mathrm{atanh}\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/atanh(a*x),x)
```

```
[Out] Integral(1/(a*x**2*atanh(a*x) + x*atanh(a*x)), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(acx+c)x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)), x)
```

3.144 $\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{x}{(acx+c)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.0390449, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][x/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx=\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.93592, size = 0, normalized size = 0.

$$\int \frac{x}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.252, size = 0, normalized size = 0.

$$\int \frac{x}{(acx+c)\left(\text{Artanh}\left(ax\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(ax^2-x\right)}{ac\log\left(ax+1\right)-ac\log\left(-ax+1\right)}+\int-\frac{2\left(2\,ax-1\right)}{ac\log\left(ax+1\right)-ac\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x^2 - x)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + integrate(-2*(2*a*x
- 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{(acx+c)\,\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{x}{ax\,\text{atanh}^2\left(ax\right)+\text{atanh}^2\left(ax\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(x/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{(acx+c)\,\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a*c*x + c)*arctanh(a*x)^2), x)
```

3.145 $\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=17

$$\text{Unintegrable}\left(\frac{1}{(acx+c)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.0209782, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx = \int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.04628, size = 0, normalized size = 0.

$$\int \frac{1}{(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/((c + a*c*x)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{1}{(acx+c)\left(\text{Artanh}\,(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(1/(a*c*x+c)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(ax-1)}{ac\log\left(ax+1\right)-ac\log\left(-ax+1\right)}+2\int -\frac{1}{c\log\left(ax+1\right)-c\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x - 1)/(a*c*log(a*x + 1) - a*c*log(-a*x + 1)) + 2*integrate(-1/(c*log(
a*x + 1) - c*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(acx+c)\,\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ax\,\text{atanh}^2\,(ax)+\text{atanh}^2\,(ax)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(a*x*atanh(a*x)**2 + atanh(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(acx+c)\,\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*c*x + c)*arctanh(a*x)^2), x)
```

3.146 $\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{x(acx+c)\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.0551411, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx = \int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.2164, size = 0, normalized size = 0.

$$\int \frac{1}{x(c+acx)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(c + a*c*x)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.113, size = 0, normalized size = 0.

$$\int \frac{1}{x\,(acx+c)\,(\text{Artanh}\,(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(a*c*x+c)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(ax-1)}{acx\log\left(ax+1\right)-acx\log\left(-ax+1\right)}+2\int-\frac{1}{acx^2\log\left(ax+1\right)-acx^2\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a*x - 1)/(a*c*x*log(a*x + 1) - a*c*x*log(-a*x + 1)) + 2*integrate(-1/(a*
c*x^2*log(a*x + 1) - a*c*x^2*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(acx^2+cx\right)\mathrm{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a*c*x^2 + c*x)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{1}{ax^2\,\mathrm{atanh}^2\left(ax\right)+x\,\mathrm{atanh}^2\left(ax\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(a*x**2*atanh(a*x)**2 + x*atanh(a*x)**2), x)/c
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(acx+c)x\,\mathrm{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c*x+c)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*c*x + c)*x*arctanh(a*x)^2), x)
```

3.147 $\int \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx$

Optimal. Leaf size=275

$$-\frac{bd^3\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^4}+\frac{bd^3\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^4}+\frac{d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tanh^{-1}(cx\right)}{e^4}$$

```
[Out] (a*d^2*x)/e^3 - (b*d*x)/(2*c*e^2) + (b*x^2)/(6*c*e) + (b*d*ArcTanh[c*x])/(2
*c^2*e^2) + (b*d^2*x*ArcTanh[c*x])/e^3 - (d*x^2*(a + b*ArcTanh[c*x]))/(2*e^
2) + (x^3*(a + b*ArcTanh[c*x]))/(3*e) + (d^3*(a + b*ArcTanh[c*x])*Log[2/(1
+ c*x)])/e^4 - (d^3*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1
+ c*x))])/e^4 + (b*d^2*Log[1 - c^2*x^2])/(2*c*e^3) + (b*Log[1 - c^2*x^2])/(
6*c^3*e) - (b*d^3*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^4) + (b*d^3*PolyLog[2,
1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^4)
```

Rubi [A] time = 0.255716, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {5940, 5910, 260, 5916, 321, 206, 266, 43, 5920, 2402, 2315, 2447}

$$-\frac{bd^3\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^4}+\frac{bd^3\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^4}+\frac{d^3\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^4}-\frac{d^3\left(a+b\tanh^{-1}(cx\right)}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (a*d^2*x)/e^3 - (b*d*x)/(2*c*e^2) + (b*x^2)/(6*c*e) + (b*d*ArcTanh[c*x])/(2
*c^2*e^2) + (b*d^2*x*ArcTanh[c*x])/e^3 - (d*x^2*(a + b*ArcTanh[c*x]))/(2*e^
2) + (x^3*(a + b*ArcTanh[c*x]))/(3*e) + (d^3*(a + b*ArcTanh[c*x])*Log[2/(1
+ c*x)])/e^4 - (d^3*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1
+ c*x))])/e^4 + (b*d^2*Log[1 - c^2*x^2])/(2*c*e^3) + (b*Log[1 - c^2*x^2])/(
6*c^3*e) - (b*d^3*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^4) + (b*d^3*PolyLog[2,
1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^4)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^3} - \frac{dx\left(a+b\tanh^{-1}(cx)\right)}{e^2} + \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{e} - \frac{d^3\left(a+b\,\mathrm{ta}\right.}{e^3(d+}\right.$$

$$= \frac{d^2\int\left(a+b\tanh^{-1}(cx)\right)dx}{e^3} - \frac{d^3\int\frac{a+b\tanh^{-1}(cx)}{d+ex}dx}{e^3} - \frac{d\int x\left(a+b\tanh^{-1}(cx)\right)dx}{e^2} + \frac{\int x^2\left(\right.}{}$$

$$= \frac{ad^2x}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e} + \frac{d^3\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right.}{e^4}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\tanh^{-1}(cx)\right)}{3e} + \frac{d^3\left(\right.}{}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bd\tanh^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a+b\,\mathrm{ta}\right.}{3e}$$

$$= \frac{ad^2x}{e^3} - \frac{bdx}{2ce^2} + \frac{bx^2}{6ce} + \frac{bd\tanh^{-1}(cx)}{2c^2e^2} + \frac{bd^2x\tanh^{-1}(cx)}{e^3} - \frac{dx^2\left(a+b\tanh^{-1}(cx)\right)}{2e^2} + \frac{x^3\left(a\right.}{}$$

Mathematica [C] time = 6.67452, size = 474, normalized size = 1.72

$$\frac{3bd^3\mathrm{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) - 3bd^3\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right) + 6ad^2ex - 6ad^3\log(d+ex) - 3ade^2x^2 + 2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (-((b*e^3)/c^3) + 6*a*d^2*e*x - (3*b*d*e^2*x)/c - 3*a*d*e^2*x^2 + (b*e^3*x^
2)/c + 2*a*e^3*x^3 + (3*b*d*e^2*ArcTanh[c*x])/c^2 - (3*I)*b*d^3*Pi*ArcTanh[
c*x] + 6*b*d^2*e*x*ArcTanh[c*x] - 3*b*d*e^2*x^2*ArcTanh[c*x] + 2*b*e^3*x^3*
ArcTanh[c*x] - 6*b*d^3*ArcTanh[(c*d)/e]*ArcTanh[c*x] + 3*b*d^3*ArcTanh[c*x]
^2 - (3*b*d^2*e*ArcTanh[c*x]^2)/c + (3*b*d^2*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcT
anh[c*x]^2)/(c*E^ArcTanh[(c*d)/e]) + 6*b*d^3*ArcTanh[c*x]*Log[1 + E^(-2*Arc
Tanh[c*x])] + (3*I)*b*d^3*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 6*b*d^3*ArcTanh[
(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*b*d^3*ArcTan
h[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 6*a*d^3*Log[d +
e*x] + (3*b*d^2*e*Log[1 - c^2*x^2])/c + (b*e^3*Log[1 - c^2*x^2])/c^3 + ((3*
I)/2)*b*d^3*Pi*Log[1 - c^2*x^2] + 6*b*d^3*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTa
nh[(c*d)/e] + ArcTanh[c*x]]] - 3*b*d^3*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 3
*b*d^3*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(6*e^4)
```

Maple [A] time = 0.125, size = 381, normalized size = 1.4

$$\frac{x^3a}{3e} - \frac{dax^2}{2e^2} + \frac{axd^2}{e^3} - \frac{ad^3\ln(cxe+cd)}{e^4} + \frac{b\mathrm{Artanh}(cx)x^3}{3e} - \frac{b\mathrm{Artanh}(cx)x^2d}{2e^2} + \frac{bd^2x\mathrm{Artanh}(cx)}{e^3} - \frac{b\mathrm{Artanh}(cx)d^3\mathrm{l}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] 1/3*a*x^3/e-1/2*a/e^2*x^2*d+a*d^2*x/e^3-a*d^3/e^4*ln(c*e*x+c*d)+1/3*b*arcta
nh(c*x)*x^3/e-1/2*b*arctanh(c*x)/e^2*x^2*d+b*d^2*x*arctanh(c*x)/e^3-b*arcta
nh(c*x)*d^3/e^4*ln(c*e*x+c*d)+1/2*b/e^4*d^3*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*
d+e))+1/2*b/e^4*d^3*dilog((c*e*x+e)/(-c*d+e))-1/2*b/e^4*d^3*ln(c*e*x+c*d)*l
n((c*e*x-e)/(-c*d-e))-1/2*b/e^4*d^3*dilog((c*e*x-e)/(-c*d-e))-1/2*b*d*x/c/e
^2-2/3/c*b*d^2/e^3+1/6*b*x^2/c/e+1/2/c*b/e^3*ln(c*e*x+e)*d^2+1/4/c^2*b/e^2*
ln(c*e*x+e)*d+1/6/c^3*b/e*ln(c*e*x+e)+1/2/c*b/e^3*ln(c*e*x-e)*d^2-1/4/c^2*b
/e^2*ln(c*e*x-e)*d+1/6/c^3*b/e*ln(c*e*x-e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}a\left(\frac{6\,d^3\log\left(ex+d\right)}{e^4}-\frac{2\,e^2x^3-3\,dex^2+6\,d^2x}{e^3}\right)+\frac{1}{2}b\int\frac{x^3(\log\left(cx+1\right)-\log\left(-cx+1\right))}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) + 1
/2*b*integrate(x^3*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^3\,\mathrm{artanh}\left(cx\right)+ax^3}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arctanh(c*x) + a*x^3)/(e*x + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\mathrm{artanh}\left(cx\right)+a)x^3}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^3/(e*x + d), x)
```

3.148 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx$

Optimal. Leaf size=214

$$\frac{bd^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^3}-\frac{bd^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^3}-\frac{d^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^3}$$

```
[Out] -((a*d*x)/e^2) + (b*x)/(2*c*e) - (b*ArcTanh[c*x])/(2*c^2*e) - (b*d*x*ArcTan
h[c*x])/e^2 + (x^2*(a + b*ArcTanh[c*x]))/(2*e) - (d^2*(a + b*ArcTanh[c*x])*
Log[2/(1 + c*x)])/e^3 + (d^2*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d
 + e)*(1 + c*x))])/e^3 - (b*d*Log[1 - c^2*x^2])/(2*c*e^2) + (b*d^2*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*e^3) - (b*d^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d
+ e)*(1 + c*x))])/(2*e^3)
```

Rubi [A] time = 0.199672, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5940, 5910, 260, 5916, 321, 206, 5920, 2402, 2315, 2447}

$$\frac{bd^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^3}-\frac{bd^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^3}-\frac{d^2\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```


```
[Out] -((a*d*x)/e^2) + (b*x)/(2*c*e) - (b*ArcTanh[c*x])/(2*c^2*e) - (b*d*x*ArcTan
h[c*x])/e^2 + (x^2*(a + b*ArcTanh[c*x]))/(2*e) - (d^2*(a + b*ArcTanh[c*x])*
Log[2/(1 + c*x)])/e^3 + (d^2*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d
 + e)*(1 + c*x))])/e^3 - (b*d*Log[1 - c^2*x^2])/(2*c*e^2) + (b*d^2*PolyLog[
2, 1 - 2/(1 + c*x)])/(2*e^3) - (b*d^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d
+ e)*(1 + c*x))])/(2*e^3)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
```

```
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)}{d + ex} dx = \int \left(-\frac{d \left(a + b \tanh^{-1}(cx)\right)}{e^2} + \frac{x \left(a + b \tanh^{-1}(cx)\right)}{e} + \frac{d^2 \left(a + b \tanh^{-1}(cx)\right)}{e^2(d + ex)} \right) dx$$

$$= -\frac{d \int \left(a + b \tanh^{-1}(cx)\right) dx}{e^2} + \frac{d^2 \int \frac{a+b \tanh^{-1}(cx)}{d+ex} dx}{e^2} + \frac{\int x \left(a + b \tanh^{-1}(cx)\right) dx}{e}$$

$$= -\frac{adx}{e^2} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)}{2e} - \frac{d^2 \left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2}{1+cx}\right)}{e^3} + \frac{d^2 \left(a + b \tanh^{-1}(cx)\right)}{e^3}$$

$$= -\frac{adx}{e^2} + \frac{bx}{2ce} - \frac{bdx \tanh^{-1}(cx)}{e^2} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)}{2e} - \frac{d^2 \left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2}{1+cx}\right)}{e^3}$$

$$= -\frac{adx}{e^2} + \frac{bx}{2ce} - \frac{b \tanh^{-1}(cx)}{2c^2 e} - \frac{bdx \tanh^{-1}(cx)}{e^2} + \frac{x^2 \left(a + b \tanh^{-1}(cx)\right)}{2e} - \frac{d^2 \left(a + b \tanh^{-1}\right.}{e}$$

Mathematica [C] time = 3.08483, size = 394, normalized size = 1.84

$$\frac{-bd^2 \text{PolyLog}\left(2, e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) + bd^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + 2ad^2 \log(d + ex) - 2adex + ae^2x^2 - \frac{bde\sqrt{1}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (-2*a*d*e*x + (b*e^2*x)/c + a*e^2*x^2 - (b*e^2*ArcTanh[c*x])/c^2 + I*b*d^2*
Pi*ArcTanh[c*x] - 2*b*d*e*x*ArcTanh[c*x] + b*e^2*x^2*ArcTanh[c*x] + 2*b*d^2
*ArcTanh[(c*d)/e]*ArcTanh[c*x] - b*d^2*ArcTanh[c*x]^2 + (b*d*e*ArcTanh[c*x]
^2)/c - (b*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c*d)/e
]) - 2*b*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - I*b*d^2*Pi*Log[1 +
 E^(2*ArcTanh[c*x])] + 2*b*d^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] + 2*b*d^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/
e] + ArcTanh[c*x]))] + 2*a*d^2*Log[d + e*x] - (b*d*e*Log[1 - c^2*x^2])/c -
(I/2)*b*d^2*Pi*Log[1 - c^2*x^2] - 2*b*d^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTa
nh[(c*d)/e] + ArcTanh[c*x]]] + b*d^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] - b*d
^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))])/(2*e^3)
```

Maple [A] time = 0.115, size = 298, normalized size = 1.4

$$\frac{ax^2}{2e} - \frac{adx}{e^2} + \frac{ad^2 \ln(cxe + cd)}{e^3} + \frac{b\text{Artanh}(cx) x^2}{2e} - \frac{bdx\text{Artanh}(cx)}{e^2} + \frac{b\text{Artanh}(cx) d^2 \ln(cxe + cd)}{e^3} + \frac{bx}{2ce} + \frac{bd}{2ce^2} - \frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] 1/2*a/e*x^2-a*d*x/e^2+a*d^2/e^3*ln(c*e*x+c*d)+1/2*b*arctanh(c*x)/e*x^2-b*d*
x*arctanh(c*x)/e^2+b*arctanh(c*x)*d^2/e^3*ln(c*e*x+c*d)+1/2*b*x/c/e+1/2/c*b
*d/e^2-1/2/c*b/e^2*ln(c*e*x+e)*d-1/4/c^2*b/e*ln(c*e*x+e)-1/2/c*b/e^2*ln(c*e
*x-e)*d+1/4/c^2*b/e*ln(c*e*x-e)-1/2*b/e^3*d^2*ln(c*e*x+c*d)*ln((c*e*x+e)/(-
c*d+e))-1/2*b/e^3*d^2*dilog((c*e*x+e)/(-c*d+e))+1/2*b/e^3*d^2*ln(c*e*x+c*d)
*ln((c*e*x-e)/(-c*d-e))+1/2*b/e^3*d^2*dilog((c*e*x-e)/(-c*d-e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right)+\frac{1}{2}b\int\frac{x^2(\log(cx+1)-\log(-cx+1))}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/2*b*integrate(x^2*
(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2\operatorname{artanh}(cx)+ax^2}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arctanh(c*x) + a*x^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2\,(a+b\operatorname{atanh}(cx))}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\operatorname{artanh}(cx)+a)x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x^2/(e*x + d), x)
```

3.149 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx$

Optimal. Leaf size=156

$$-\frac{bd\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^2}+\frac{bd\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^2}+\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\text{lo}}{e^2}$$

```
[Out] (a*x)/e + (b*x*ArcTanh[c*x])/e + (d*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/
e^2 - (d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e
^2 + (b*Log[1 - c^2*x^2])/(2*c*e) - (b*d*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^
2) + (b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^2)
```

Rubi [A] time = 0.153831, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5940, 5910, 260, 5920, 2402, 2315, 2447}

$$-\frac{bd\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e^2}+\frac{bd\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e^2}+\frac{d\log\left(\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)\text{lo}}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```


```
[Out] (a*x)/e + (b*x*ArcTanh[c*x])/e + (d*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/
e^2 - (d*(a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e
^2 + (b*Log[1 - c^2*x^2])/(2*c*e) - (b*d*PolyLog[2, 1 - 2/(1 + c*x)])/(2*e^
2) + (b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)}{d+ex}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{e} - \frac{d\left(a+b\tanh^{-1}(cx)\right)}{e(d+ex)}\right) dx$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2} - \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e^2} - (bcd)\int$$

$$= \frac{ax}{e} + \frac{bx\tanh^{-1}(cx)}{e} + \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2} - \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(}{(cd+e}\right.}{e^2}$$

$$= \frac{ax}{e} + \frac{bx\tanh^{-1}(cx)}{e} + \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e^2} - \frac{d\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(}{(cd+e}\right.}{e^2}$$

Mathematica [C] time = 2.44204, size = 315, normalized size = 2.02

$$\frac{b\left(cd\,\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)-cd\,\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)+e\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}+\frac{1}{2}i\pi cd\log\left(1-c^2x^2\right)+e\log\left(1-c^2x^2\right)-2c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x]))/(d + e*x),x]
```

```
[Out] (2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*e*x*ArcT
anh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*Arc
Tanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e]
 + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*
ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTa
nh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*Arc
Tanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2,
 -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))]))/c)/(2*e^2)
```

Maple [A] time = 0.112, size = 217, normalized size = 1.4

$$\frac{ax}{e} - \frac{ad\ln(cxe+cd)}{e^2} + \frac{bx\text{Artanh}(cx)}{e} - \frac{b\text{Artanh}(cx)\,d\ln(cxe+cd)}{e^2} + \frac{bd\ln(cxe+cd)}{2e^2}\ln\left(\frac{cxe+e}{-cd+e}\right) + \frac{bd}{2e^2}\text{dilog}\left(\frac{cxe}{-cd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] a/e*x-a*d/e^2*ln(c*e*x+c*d)+b*x*arctanh(c*x)/e-b*arctanh(c*x)*d/e^2*ln(c*e*
x+c*d)+1/2*b/e^2*d*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))+1/2*b/e^2*d*dilog((
c*e*x+e)/(-c*d+e))-1/2*b/e^2*d*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))-1/2*b/e
^2*d*dilog((c*e*x-e)/(-c*d-e))+1/2/c*b/e*ln(c^2*d^2-2*(c*e*x+c*d)*c*d+(c*e*
x+c*d)^2-e^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) + \frac{1}{2}b\int\frac{x(\log(cx+1) - \log(-cx+1))}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(x/e - d*log(e*x + d)/e^2) + 1/2*b*integrate(x*(log(c*x + 1) - log(-c*x +
 1))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx\,\text{artanh}(cx) + ax}{ex+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arctanh(c*x) + a*x)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x(a+b\,\text{atanh}(cx))}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)x}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*x/(e*x + d), x)
```

3.150 $\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=114

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tan\right.}{e}$$

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])*Log[(2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(
2*e) - (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rubi [A] time = 0.0708463, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5920, 2402, 2315, 2447}

$$-\frac{b\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2e}+\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}-\frac{\log\left(\frac{2}{cx+1}\right)\left(a+b\tan\right.}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])*Log[(2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*PolyLog[2, 1 - 2/(1 + c*x)])/(
2*e) - (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{d+ex}dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{(bc)\int \frac{\log\left(\frac{2}{1+cx}\right)}{1-c^2x^2}dx}{e}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} - \frac{b\text{Li}_2\left(1-\frac{2c(d+ex)}{(cd+e)(1+c}\right.}{2e}$$
$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{e} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e} + \frac{b\text{Li}_2\left(1-\frac{2}{1+cx}\right)}{2e} -$$

Mathematica [A] time = 0.195548, size = 104, normalized size = 0.91

$$\frac{-b\text{PolyLog}\left(2,\frac{c(d+ex)}{cd-e}\right)+b\text{PolyLog}\left(2,\frac{c(d+ex)}{cd+e}\right)+\log(d+ex)\left(2a-b\log\left(-\frac{e(cx+1)}{cd-e}\right)+b\log\left(\frac{e-cex}{cd+e}\right)+2b\tanh^{-1}(cx)\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(d + e*x),x]
```

```
[Out] (Log[d + e*x]*(2*a + 2*b*ArcTanh[c*x] - b*Log[-((e*(1 + c*x))/(c*d - e))] +
 b*Log[(e - c*e*x)/(c*d + e)]) - b*PolyLog[2, (c*(d + e*x))/(c*d - e)] + b*
PolyLog[2, (c*(d + e*x))/(c*d + e)])/(2*e)
```

Maple [A] time = 0.103, size = 148, normalized size = 1.3

$$\frac{a\ln(cxe+cd)}{e}+\frac{b\ln(cxe+cd)\,\text{Artanh}(cx)}{e}-\frac{b\ln(cxe+cd)}{2e}\ln\left(\frac{cxe+e}{-cd+e}\right)-\frac{b}{2e}\text{dilog}\left(\frac{cxe+e}{-cd+e}\right)+\frac{b\ln(cxe+cd)}{2e}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(e*x+d),x)
```

```
[Out] a*ln(c*e*x+c*d)/e+b*ln(c*e*x+c*d)/e*arctanh(c*x)-1/2*b/e*ln(c*e*x+c*d)*ln((
c*e*x+e)/(-c*d+e))-1/2*b/e*dilog((c*e*x+e)/(-c*d+e))+1/2*b/e*ln(c*e*x+c*d)*
ln((c*e*x-e)/(-c*d-e))+1/2*b/e*dilog((c*e*x-e)/(-c*d-e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b\int \frac{\log(cx+1)-\log(-cx+1)}{ex+d}dx+\frac{a\log(ex+d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x + d), x) + a*log(e*x +
d)/e
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}\,(cx)+a}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a+b\,\text{atanh}\,(cx)}{d+ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\,\text{artanh}\,(cx)+a}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/(e*x + d), x)
```

3.151 $\int \frac{a+b\tanh^{-1}(cx)}{x(d+ex)}\,dx$

Optimal. Leaf size=148

$$\frac{b\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d}-\frac{b\text{PolyLog}(2,-cx)}{2d}+\frac{b\text{PolyLog}(2,cx)}{2d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d}-\frac{\left(a+b\tanh^{-1}(cx\right.}{}$$

```
[Out] (a*Log[x])/d + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[
c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d - (b*PolyLog[2, -(c*x)]
)/(2*d) + (b*PolyLog[2, c*x])/(2*d) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d)
 + (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d)
```

Rubi [A] time = 0.163711, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5940, 5912, 5920, 2402, 2315, 2447}

$$\frac{b\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d}-\frac{b\text{PolyLog}(2,-cx)}{2d}+\frac{b\text{PolyLog}(2,cx)}{2d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d}-\frac{\left(a+b\tanh^{-1}(cx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```



```
[Out] (a*Log[x])/d + ((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[
c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d - (b*PolyLog[2, -(c*x)]
)/(2*d) + (b*PolyLog[2, c*x])/(2*d) - (b*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d)
 + (b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d}$$

$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{b\text{Li}_2(-cx)}{2d} +$$

$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{b\text{Li}_2(-cx)}{2d} +$$

$$= \frac{a\log(x)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{b\text{Li}_2(-cx)}{2d} +$$

Mathematica [C] time = 1.63877, size = 294, normalized size = 1.99

$$\frac{b\left(cd\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right)-cd\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right)+e\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}+\frac{1}{2}i\pi cd\log\left(1-c^2x^2\right)-2cd\tanh^{-1}(cx)\tanh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x*(d + e*x)),x]
```

```
[Out] (2*a*d*Log[x] - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTanh[c*x] - 2*c*d*A
rcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2 + (Sqr
t[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*ArcTanh[c
*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c*x])] - 2
*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 2
*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + (I/2)
*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/
e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2,
E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c)/(2*d^2)
```

Maple [A] time = 0.119, size = 210, normalized size = 1.4

$$-\frac{a\ln(cxe+cd)}{d} + \frac{a\ln(cx)}{d} - \frac{b\text{Artanh}(cx)\ln(cxe+cd)}{d} + \frac{b\text{Artanh}(cx)\ln(cx)}{d} + \frac{b\ln(cxe+cd)}{2d}\ln\left(\frac{cxe+e}{-cd+e}\right) + \frac{b}{2d}\text{dil}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x/(e*x+d),x)
```

```
[Out] -a/d*ln(c*e*x+c*d)+a/d*ln(c*x)-b*arctanh(c*x)/d*ln(c*e*x+c*d)+b*arctanh(c*x
)/d*ln(c*x)+1/2*b/d*ln((c*e*x+e)/(-c*d+e))*ln(c*e*x+c*d)+1/2*b/d*dilog((c*e
*x+e)/(-c*d+e))-1/2*b/d*ln((c*e*x-e)/(-c*d-e))*ln(c*e*x+c*d)-1/2*b/d*dilog(
(c*e*x-e)/(-c*d-e))-1/2*b/d*dilog(c*x)-1/2*b/d*dilog(c*x+1)-1/2*b/d*ln(c*x)
*ln(c*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)+\frac{1}{2}\,b\int\frac{\log(cx+1)-\log(-cx+1)}{ex^2+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a*(log(e*x + d)/d - log(x)/d) + 1/2*b*integrate((log(c*x + 1) - log(-c*x +
 1))/(e*x^2 + d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b\,\mathrm{artanh}(cx)+a}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x^2 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\,\mathrm{atanh}(cx)}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))/(x*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\mathrm{artanh}(cx)+a}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x), x)
```

3.152 $\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+ex)}\,dx$

Optimal. Leaf size=200

$$\frac{be\text{PolyLog}(2,-cx)}{2d^2}-\frac{be\text{PolyLog}(2,cx)}{2d^2}+\frac{be\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}-\frac{be\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d^2}-\frac{e\log\left(\frac{2}{cx+1}\right)(}{}$$

```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
 b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])*Log[(2*c*(
d + e*x))/((c*d + e)*(1 + c*x))])/d^2 - (b*c*Log[1 - c^2*x^2])/(2*d) + (b*e
*PolyLog[2, -(c*x)])/(2*d^2) - (b*e*PolyLog[2, c*x])/(2*d^2) + (b*e*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*d^2) - (b*e*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/(2*d^2)
```

Rubi [A] time = 0.20372, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5940, 5916, 266, 36, 29, 31, 5912, 5920, 2402, 2315, 2447}

$$\frac{be\text{PolyLog}(2,-cx)}{2d^2}-\frac{be\text{PolyLog}(2,cx)}{2d^2}+\frac{be\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^2}-\frac{be\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d^2}-\frac{e\log\left(\frac{2}{cx+1}\right)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```


```
[Out] -((a + b*ArcTanh[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
 b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])*Log[(2*c*(
d + e*x))/((c*d + e)*(1 + c*x))])/d^2 - (b*c*Log[1 - c^2*x^2])/(2*d) + (b*e
*PolyLog[2, -(c*x)])/(2*d^2) - (b*e*PolyLog[2, c*x])/(2*d^2) + (b*e*PolyLog
[2, 1 - 2/(1 + c*x)])/(2*d^2) - (b*e*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/(2*d^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^2(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{d^2(d+ex)}\right)dx$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)\log}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)\log}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)\log}{d^2}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{dx} + \frac{bc\log(x)}{d} - \frac{ae\log(x)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^2} + \frac{e\left(a+b\tan\right.}{}$$

Mathematica [C] time = 3.22214, size = 360, normalized size = 1.8

$$-\frac{-bde\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) + bde\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) + \frac{2ad^2}{x} + 2ade\log(x) - 2ade\log(d+ex)\,\cdot}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^2*(d + e*x)),x]
```

```
[Out] -((2*a*d^2)/x - I*b*d*e*Pi*ArcTanh[c*x] + (2*b*d^2*ArcTanh[c*x])/x - 2*b*d*
e*ArcTanh[(c*d)/e]*ArcTanh[c*x] + b*d*e*ArcTanh[c*x]^2 - (b*e^2*ArcTanh[c*x
]^2)/c + (b*Sqrt[1 - (c^2*d^2)/e^2]*e^2*ArcTanh[c*x]^2)/(c*E^ArcTanh[(c*d)/
e]) + 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*b*d*e*Pi*Log[1
+ E^(2*ArcTanh[c*x])] - 2*b*d*e*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*
d)/e] + ArcTanh[c*x]))] - 2*b*d*e*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)
/e] + ArcTanh[c*x]))] + 2*a*d*e*Log[x] - 2*a*d*e*Log[d + e*x] - 2*b*c*d^2*L
og[(c*x)/Sqrt[1 - c^2*x^2]] + (I/2)*b*d*e*Pi*Log[1 - c^2*x^2] + 2*b*d*e*Arc
Tanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - b*d*e*PolyLog[
2, E^(-2*ArcTanh[c*x])] + b*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTan
h[c*x]))])/(2*d^3)
```

Maple [A] time = 0.134, size = 279, normalized size = 1.4

$$\frac{ae\ln(cxe+cd)}{d^2} - \frac{a}{dx} - \frac{ae\ln(cx)}{d^2} + \frac{b\text{Artanh}(cx)\,e\ln(cxe+cd)}{d^2} - \frac{b\text{Artanh}(cx)}{dx} - \frac{b\text{Artanh}(cx)\,e\ln(cx)}{d^2} - \frac{bc\ln(cx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^2/(e*x+d),x)
```

```
[Out] a/d^2*e*ln(c*e*x+c*d)-a/d/x-a/d^2*e*ln(c*x)+b*arctanh(c*x)/d^2*e*ln(c*e*x+c
*d)-b*arctanh(c*x)/d/x-b*arctanh(c*x)/d^2*e*ln(c*x)-1/2*c*b/d*ln(c*x-1)+c*b
/d*ln(c*x)-1/2*c*b/d*ln(c*x+1)+1/2*b/d^2*e*dilog(c*x)+1/2*b/d^2*e*dilog(c*x
+1)+1/2*b/d^2*e*ln(c*x)*ln(c*x+1)-1/2*b/d^2*ln((c*e*x+e)/(-c*d+e))*ln(c*e*x
```

```
+c*d)*e-1/2*b/d^2*dilog((c*e*x+e)/(-c*d+e))*e+1/2*b/d^2*ln((c*e*x-e)/(-c*d-
e))*ln(c*e*x+c*d)*e+1/2*b/d^2*dilog((c*e*x-e)/(-c*d-e))*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a\left(\frac{e\log\left(ex+d\right)}{d^2}-\frac{e\log\left(x\right)}{d^2}-\frac{1}{dx}\right)+\frac{1}{2}b\int\frac{\log\left(cx+1\right)-\log\left(-cx+1\right)}{ex^3+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 1/2*b*integrate((log(c*x
+ 1) - log(-c*x + 1))/(e*x^3 + d*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}\left(cx\right)+a}{ex^3+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x^3 + d*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{artanh}\left(cx\right)+a}{(ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x^2), x)
```

3.153 $\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+ex)}\,dx$

Optimal. Leaf size=261

$$-\frac{be^2\text{PolyLog}(2,-cx)}{2d^3}+\frac{be^2\text{PolyLog}(2,cx)}{2d^3}-\frac{be^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{be^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d^3}+\frac{e^2\log\left(\right.}{}$$

```
[Out] -(b*c)/(2*d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x^2
) + (e*(a + b*ArcTanh[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])/
d^3 + (e^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (e^2*(a + b*ArcTanh
[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d^3 + (b*c*e*Log[1 - c^2
*x^2])/(2*d^2) - (b*e^2*PolyLog[2, -(c*x)])/(2*d^3) + (b*e^2*PolyLog[2, c*x
])/(2*d^3) - (b*e^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3) + (b*e^2*PolyLog[2
, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^3)
```

Rubi [A] time = 0.245221, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {5940, 5916, 325, 206, 266, 36, 29, 31, 5912, 5920, 2402, 2315, 2447}

$$-\frac{be^2\text{PolyLog}(2,-cx)}{2d^3}+\frac{be^2\text{PolyLog}(2,cx)}{2d^3}-\frac{be^2\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2d^3}+\frac{be^2\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d^3}+\frac{e^2\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -(b*c)/(2*d*x) + (b*c^2*ArcTanh[c*x])/(2*d) - (a + b*ArcTanh[c*x])/(2*d*x^2
) + (e*(a + b*ArcTanh[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])/
d^3 + (e^2*(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/d^3 - (e^2*(a + b*ArcTanh
[c*x])*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d^3 + (b*c*e*Log[1 - c^2
*x^2])/(2*d^2) - (b*e^2*PolyLog[2, -(c*x)])/(2*d^3) + (b*e^2*PolyLog[2, c*x
])/(2*d^3) - (b*e^2*PolyLog[2, 1 - 2/(1 + c*x)])/(2*d^3) + (b*e^2*PolyLog[2
, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^3)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{a+b\tanh^{-1}(cx)}{x^3(d+ex)}\,dx = \int \left(\frac{a+b\tanh^{-1}(cx)}{dx^3} - \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x^2} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)}{d^3x} - \frac{e^3\left(a+b\tanh^{-1}(cx\right.}{d^3(d+ex)}\right.$$

$$= \frac{\int \frac{a+b\tanh^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\tanh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx}{d^3} - \frac{e^3\int \frac{a+b\tanh^{-1}(cx)}{d+ex}\,dx}{d^3}$$

$$= -\frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1+cx}\right)}{d^3}$$

$$= -\frac{bc}{2dx} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)\log\left(\right.}{d^3}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} + \frac{ae^2\log(x)}{d^3} + \frac{e^2\left(a+b\right.}{}$$

$$= -\frac{bc}{2dx} + \frac{bc^2\tanh^{-1}(cx)}{2d} - \frac{a+b\tanh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)}{d^2x} - \frac{bce\log(x)}{d^2} + \frac{ae^2\log(}{d^3}$$

Mathematica [C] time = 6.12119, size = 435, normalized size = 1.67

$$-\frac{b\left(cde^2\text{PolyLog}\left(2,e^{-2\tanh^{-1}(cx)}\right) - cde^2\text{PolyLog}\left(2,e^{-2\left(\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)\right)}\right) - e^3\sqrt{1-\frac{c^2d^2}{e^2}}\tanh^{-1}(cx)^2e^{-\tanh^{-1}\left(\frac{cd}{e}\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])/(x^3*(d + e*x)),x]
```

```
[Out] -a/(2*d*x^2) + (a*e)/(d^2*x) + (a*e^2*Log[x])/d^3 - (a*e^2*Log[d + e*x])/d^
3 - (b*((c^2*d^3)/x + I*c*d*e^2*Pi*ArcTanh[c*x] - (2*c*d^2*e*ArcTanh[c*x])/
x + (c*d^3*(1 - c^2*x^2)*ArcTanh[c*x])/x^2 + 2*c*d*e^2*ArcTanh[(c*d)/e]*Arc
Tanh[c*x] - c*d*e^2*ArcTanh[c*x]^2 + e^3*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*d^
2)/e^2]*e^3*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e^2*ArcTanh[c*x]*Log
[1 - E^(-2*ArcTanh[c*x])] - I*c*d*e^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c*
d*e^2*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] +
2*c*d*e^2*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] +
I*c*d*e^2*Pi*Log[1/Sqrt[1 - c^2*x^2]] + 2*c^2*d^2*e*Log[(c*x)/Sqrt[1 - c^2*
x^2]] - 2*c*d*e^2*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*
x]]] + c*d*e^2*PolyLog[2, E^(-2*ArcTanh[c*x])] - c*d*e^2*PolyLog[2, E^(-2*(
ArcTanh[(c*d)/e] + ArcTanh[c*x]))])))/(2*c*d^4)
```

Maple [A] time = 0.125, size = 367, normalized size = 1.4

$$-\frac{ae^2\ln(cxe+cd)}{d^3} - \frac{a}{2\,dx^2} + \frac{ae^2\ln(cx)}{d^3} + \frac{ae}{d^2x} - \frac{b\text{Artanh}(cx)\,e^2\ln(cxe+cd)}{d^3} - \frac{b\text{Artanh}(cx)}{2\,dx^2} + \frac{b\text{Artanh}(cx)\,e^2\ln}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^3/(e*x+d),x)
```

```
[Out] -a/d^3*e^2*ln(c*e*x+c*d)-1/2*a/d/x^2+a/d^3*e^2*ln(c*x)+a/d^2*e/x-b*arctanh(
c*x)/d^3*e^2*ln(c*e*x+c*d)-1/2*b*arctanh(c*x)/d/x^2+b*arctanh(c*x)/d^3*e^2*
ln(c*x)+b*arctanh(c*x)/d^2*e/x-1/2*b/d^3*e^2*dilog(c*x)-1/2*b/d^3*e^2*dilog
(c*x+1)-1/2*b/d^3*e^2*ln(c*x)*ln(c*x+1)+1/2*b/d^3*e^2*ln((c*e*x+e)/(-c*d+e)
)*ln(c*e*x+c*d)+1/2*b/d^3*e^2*dilog((c*e*x+e)/(-c*d+e))-1/2*b/d^3*e^2*ln((c
*e*x-e)/(-c*d-e))*ln(c*e*x+c*d)-1/2*b/d^3*e^2*dilog((c*e*x-e)/(-c*d-e))-1/4
*c^2*b/d*ln(c*x-1)+1/2*c*b/d^2*ln(c*x-1)*e-1/2*b*c/d/x-c*b/d^2*e*ln(c*x)+1/
4*c^2*b/d*ln(c*x+1)+1/2*c*b/d^2*ln(c*x+1)*e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{2e^2\log(ex+d)}{d^3}-\frac{2e^2\log(x)}{d^3}-\frac{2ex-d}{d^2x^2}\right)+\frac{1}{2}b\int\frac{\log(cx+1)-\log(-cx+1)}{ex^4+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
+ 1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/(e*x^4 + d*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b\,\text{artanh}(cx)+a}{ex^4+dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c*x) + a)/(e*x^4 + d*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\,\text{artanh}(cx)+a}{(ex+d)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)/((e*x + d)*x^3), x)
```

3.154 $\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=385

$$\frac{bd^2\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}-\frac{bd^2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}+\frac{b^2d^2\mathrm{PolyLog}(}{2e^3}$$

```
[Out] (a*b*x)/(c*e) + (b^2*x*ArcTanh[c*x])/(c*e) - (d*(a + b*ArcTanh[c*x])^2)/(c*
e^2) - (a + b*ArcTanh[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTanh[c*x])^2)/e^2
+ (x^2*(a + b*ArcTanh[c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1
 - c*x)])/(c*e^2) - (d^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e^3 + (d^
2*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^3 +
(b^2*Log[1 - c^2*x^2])/(2*c^2*e) + (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e
^2) + (b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^3 - (b*d^2
*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]
)/e^3 + (b^2*d^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e^3) - (b^2*d^2*PolyLog[3,
 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^3)
```

Rubi [A] time = 0.427147, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5916, 5980, 260, 5948, 5922}

$$\frac{bd^2\mathrm{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^3}-\frac{bd^2\left(a+b\tanh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^3}+\frac{b^2d^2\mathrm{PolyLog}(}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (a*b*x)/(c*e) + (b^2*x*ArcTanh[c*x])/(c*e) - (d*(a + b*ArcTanh[c*x])^2)/(c*
e^2) - (a + b*ArcTanh[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTanh[c*x])^2)/e^2
+ (x^2*(a + b*ArcTanh[c*x])^2)/(2*e) + (2*b*d*(a + b*ArcTanh[c*x])*Log[2/(1
 - c*x)])/(c*e^2) - (d^2*(a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e^3 + (d^
2*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^3 +
(b^2*Log[1 - c^2*x^2])/(2*c^2*e) + (b^2*d*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e
^2) + (b*d^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^3 - (b*d^2
*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))]
)/e^3 + (b^2*d^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e^3) - (b^2*d^2*PolyLog[3,
 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e^3)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
```

```
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx = \int \left(-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2}{e^2(d+ex)}\right)dx$$

$$= -\frac{d\int\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{e^2}+\frac{d^2\int\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{e^2}+\frac{\int x\left(a+b\tanh^{-1}(cx)\right)^2\,dx}{e}$$

$$= -\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}-\frac{d^2\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right.}{e^3}$$

$$= -\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a+b\tanh^{-1}(cx)\right)^2}{2e}-\frac{d^2\left(a+\right.}{}$$

$$= \frac{abx}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tanh^{-1}(cx)\right)^2}{e^2}+\frac{x^2\left(a\right.}{}$$

$$= \frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tan\right.}{e^2}$$

$$= \frac{abx}{ce}+\frac{b^2x\tanh^{-1}(cx)}{ce}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{ce^2}-\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{2c^2e}-\frac{dx\left(a+b\tan\right.}{e^2}$$

Mathematica [C] time = 16.0594, size = 1297, normalized size = 3.37

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] -((a^2*d*x)/e^2) + (a^2*x^2)/(2*e) + (a^2*d^2*Log[d + e*x])/e^3 + (a*b*(c*e
^2*x + I*c^2*d^2*Pi*ArcTanh[c*x] - 2*c^2*d*e*x*ArcTanh[c*x] - e^2*(1 - c^2*
x^2)*ArcTanh[c*x] + 2*c^2*d^2*ArcTanh[(c*d)/e]*ArcTanh[c*x] - c^2*d^2*ArcTa
nh[c*x]^2 + c*d*e*ArcTanh[c*x]^2 - (c*d*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c
*x]^2)/E^ArcTanh[(c*d)/e] - 2*c^2*d^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*
x])] - I*c^2*d^2*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c^2*d^2*ArcTanh[(c*d)/e
]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*ArcTanh[c*x
]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c*d*e*Log[1/Sqrt[1
- c^2*x^2]] + I*c^2*d^2*Pi*Log[1/Sqrt[1 - c^2*x^2]] - 2*c^2*d^2*ArcTanh[(c*
d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + c^2*d^2*PolyLog[2, -E^
(-2*ArcTanh[c*x])] - c^2*d^2*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))]))/(c^2*e^3) + (b^2*((6*c*e^2*x*ArcTanh[c*x] + 6*c*d*e*ArcTanh[c*x]^2
 - 6*c^2*d*e*x*ArcTanh[c*x]^2 - 3*e^2*(1 - c^2*x^2)*ArcTanh[c*x]^2 - 2*c^2*
d^2*ArcTanh[c*x]^3 + 2*c*d*e*ArcTanh[c*x]^3 + 12*c*d*e*ArcTanh[c*x]*Log[1 +
 E^(-2*ArcTanh[c*x])] - 6*c^2*d^2*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x]
)] - 6*e^2*Log[1/Sqrt[1 - c^2*x^2]] + 6*c*d*(-e + c*d*ArcTanh[c*x])*PolyLog
[2, -E^(-2*ArcTanh[c*x])] + 3*c^2*d^2*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(6*
e^3) - (c*d*(-(c*d) + e)*(c*d + e)*(-6*c*d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x
]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*
I)*c*d*Pi*ArcTanh[c*x]*Log[(1 + E^(2*ArcTanh[c*x]))/(2*E^ArcTanh[c*x])] - 6
*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] + 6*c
*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcT
```

```
anh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x
]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 12*c*d*ArcTanh[(c*d)
/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*
(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^
(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] + (6*
I)*c*d*Pi*ArcTanh[c*x]*Log[1/Sqrt[1 - c^2*x^2]] - 6*c*d*ArcTanh[c*x]^2*Log[
(c*d)/Sqrt[1 - c^2*x^2] + (c*e*x)/Sqrt[1 - c^2*x^2]] - 12*c*d*ArcTanh[(c*d)
/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcTa
nh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] + 12*c*d*Ar
cTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTan
h[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]
*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -((
(c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[(
c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[
c*x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(e^3*(
6*c^2*d^2 - 6*e^2))))/c^2
```

Maple [C] time = 1.298, size = 1656, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] c*b^2*d^3/e^3/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-
c*d+e))+c*b^2*d^3/e^3/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^
2*x^2+1)/(-c*d+e))+1/2*I*b^2/e^3*d^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*
e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x
)^2-1/2*I*b^2/e^3*d^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^
2/(-c^2*x^2+1)+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c
^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*b^2/e^3*d^
2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*
e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x
)^2+a*b*x/c/e+b^2*x*arctanh(c*x)/c/e+1/c^2*b^2*arctanh(c*x)/e-1/2/c^2*b^2/e
*arctanh(c*x)^2-1/c^2*b^2/e*ln((c*x+1)^2/(-c^2*x^2+1)+1)+1/2*b^2*arctanh(c*
x)^2/e*x^2+a^2*d^2/e^3*ln(c*e*x+c*d)+1/2*b^2*d^2/e^3*polylog(3,-(c*x+1)^2/(
-c^2*x^2+1))-a^2*d/e^2*x+1/c*a*b*d/e^2+1/2*I*b^2/e^3*d^2*Pi*csgn(I/((c*x+1)
^2/(-c^2*x^2+1)+1))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c
^2*x^2+1)+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^
2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2+1/2*a^2/e*x^2+a*b/e^3*d
^2*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+b^2*d^2/e^2/(c*d+e)*arctanh(c*x)^2*
ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b^2*d^2/e^2/(c*d+e)*arctanh(c
*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-2*a*b*arctanh(c*x)*d
/e^2*x+2*a*b*arctanh(c*x)*d^2/e^3*ln(c*e*x+c*d)-a*b/e^3*d^2*ln(c*e*x+c*d)*l
n((c*e*x+e)/(-c*d+e))-1/c*a*b/e^2*ln(c*e*x-e)*d-1/2*c*b^2*d^3/e^3/(c*d+e)*p
olylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+2/c*b^2/e^2*ln(1+I*(c*x+1
)/(-c^2*x^2+1)^(1/2))*arctanh(c*x)*d+2/c*b^2/e^2*ln(1-I*(c*x+1)/(-c^2*x^2+1
)^(1/2))*arctanh(c*x)*d-1/c*a*b/e^2*ln(c*e*x+e)*d-1/2/c^2*a*b/e*ln(c*e*x+e)
+1/2/c^2*a*b/e*ln(c*e*x-e)+2/c*b^2/e^2*dilog(1-I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)*d-1/c*b^2/e^2*arctanh(c*x)^2*d+2/c*b^2/e^2*dilog(1+I*(c*x+1)/(-c^2*x^2+1)
^(1/2))*d-a*b/e^3*d^2*dilog((c*e*x+e)/(-c*d+e))+a*b/e^3*d^2*dilog((c*e*x-e)
/(-c*d-e))-1/2*b^2*d^2/e^2/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)
/(-c*d+e))+a*b*arctanh(c*x)/e*x^2-b^2*arctanh(c*x)^2*d/e^2*x-b^2*d^2/e^3*ar
ctanh(c*x)^2*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)
)-b^2*d^2/e^3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+b^2*arctanh(c
*x)^2*d^2/e^3*ln(c*e*x+c*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^2\left(\frac{2d^2\log(ex+d)}{e^3}+\frac{ex^2-2dx}{e^2}\right)+\frac{(b^2ex^2-2b^2dx)\log(-cx+1)^2}{8e^2}-\int-\frac{(b^2ce^2x^3-b^2e^2x^2)\log(cx+1)^2+4(abc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/8*(b^2*e*x^2 - 2
*b^2*d*x)*log(-c*x + 1)^2/e^2 - integrate(-1/4*((b^2*c*e^2*x^3 - b^2*e^2*x^
2)*log(c*x + 1)^2 + 4*(a*b*c*e^2*x^3 - a*b*e^2*x^2)*log(c*x + 1) + (2*b^2*c
*d^2*x - (4*a*b*c*e^2 + b^2*c*e^2)*x^3 + (b^2*c*d*e + 4*a*b*e^2)*x^2 - 2*(b
^2*c*e^2*x^3 - b^2*e^2*x^2)*log(c*x + 1))*log(-c*x + 1))/(c*e^3*x^2 - d*e^2
 + (c*d*e^2 - e^3)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^2\,\text{artanh}(cx)^2+2\,abx^2\,\text{artanh}(cx)+a^2x^2}{ex+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2)/(e*x +
 d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\text{artanh}(cx)+a)^2x^2}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x^2/(e*x + d), x)
```

3.155 $\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=279

$$-\frac{bd\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{bd\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}-\frac{b^2d\text{PolyLog}\left(3,1\right.}{2e^2}$$

```
[Out] (a + b*ArcTanh[c*x])^2/(c*e) + (x*(a + b*ArcTanh[c*x])^2)/e - (2*b*(a + b*A
rcTanh[c*x])*Log[2/(1 - c*x)])/(c*e) + (d*(a + b*ArcTanh[c*x])^2*Log[2/(1 +
 c*x)])/e^2 - (d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 +
 c*x))])/e^2 - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e) - (b*d*(a + b*ArcTan
h[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^2 + (b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^2 - (b^2*d*PolyLog[3, 1
- 2/(1 + c*x)])/(2*e^2) + (b^2*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*
(1 + c*x))])/(2*e^2)
```

Rubi [A] time = 0.257585, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5940, 5910, 5984, 5918, 2402, 2315, 5922}

$$-\frac{bd\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e^2}+\frac{bd\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e^2}-\frac{b^2d\text{PolyLog}\left(3,1\right.}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```


```
[Out] (a + b*ArcTanh[c*x])^2/(c*e) + (x*(a + b*ArcTanh[c*x])^2)/e - (2*b*(a + b*A
rcTanh[c*x])*Log[2/(1 - c*x)])/(c*e) + (d*(a + b*ArcTanh[c*x])^2*Log[2/(1 +
 c*x)])/e^2 - (d*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 +
 c*x))])/e^2 - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(c*e) - (b*d*(a + b*ArcTan
h[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e^2 + (b*d*(a + b*ArcTanh[c*x])*PolyLo
g[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e^2 - (b^2*d*PolyLog[3, 1
- 2/(1 + c*x)])/(2*e^2) + (b^2*d*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*
(1 + c*x))])/(2*e^2)
```


Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{e(d+ex)}\right)dx$$

$$= \frac{\int \left(a+b\tanh^{-1}(cx)\right)^2\,dx}{e}-\frac{d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{e}$$

$$= \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\right.}{e^2}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}+\frac{d\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e^2}-\frac{d\left(a\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}+\frac{d\left(a\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}+\frac{d\left(a\right.}{}$$

$$= \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{ce}+\frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{e}-\frac{2b\left(a+b\tanh^{-1}(cx)\right)\log\left(\frac{2}{1-cx}\right)}{ce}+\frac{d\left(a\right.}{}$$

Mathematica [C] time = 15.0144, size = 1036, normalized size = 3.71

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcTanh[c*x])^2)/(d + e*x),x]
```

```
[Out] (6*a^2*e*x - 6*a^2*d*Log[d + e*x] + (6*a*b*((-I)*c*d*Pi*ArcTanh[c*x] + 2*c*
e*x*ArcTanh[c*x] - 2*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2
 - e*ArcTanh[c*x]^2 + (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[
(c*d)/e] + 2*c*d*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1
 + E^(2*ArcTanh[c*x])] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d
)/e] + ArcTanh[c*x]))] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e]
 + ArcTanh[c*x]))] + e*Log[1 - c^2*x^2] + (I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2
*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - c*d*Po
lyLog[2, -E^(-2*ArcTanh[c*x])] + c*d*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))]))/c + (b^2*(-6*e*ArcTanh[c*x]^2 + 6*c*e*x*ArcTanh[c*x]^2 + 8
*c*d*ArcTanh[c*x]^3 - 4*e*ArcTanh[c*x]^3 + (4*Sqrt[1 - (c^2*d^2)/e^2]*e*Arc
Tanh[c*x]^3)/E^ArcTanh[(c*d)/e] - 12*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + (6*I)*c*d*Pi*Ar
cTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] + 6*c*d*ArcTanh[c*x]
^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] - 6*c*d*ArcTanh[c*x]^2
*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[1
+ E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*
(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*
Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])))] - 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x]))
 + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2
*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] + (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c
^2*x^2] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e]
+ ArcTanh[c*x]]] + 6*(e - c*d*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])
] + 6*c*d*ArcTanh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e
))] - 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])]
- 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 6*c
*d*ArcTanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - 3*c*d
*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 3*c*d*PolyLog[3, -(((c*d + e)*E^(2*ArcT
anh[c*x]))/(c*d - e))] + 12*c*d*PolyLog[3, -E^(ArcTanh[(c*d)/e] + ArcTanh[c
*x])] + 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 3*c*d*Poly
Log[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/c)/(6*e^2)
```

Maple [C] time = 1.167, size = 13923, normalized size = 49.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2x\log(-cx+1)^2}{4e} + a^2\left(\frac{x}{e} - \frac{d\log(ex+d)}{e^2}\right) - \int -\frac{\left(b^2cex^2 - b^2ex\right)\log(cx+1)^2 + 4\left(abcex^2 - abex\right)\log(cx+1) - 2\left((2a\right.}{4\left(ce^2x^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x*log(-c*x + 1)^2/e + a^2*(x/e - d*log(e*x + d)/e^2) - integrate(-1
/4*((b^2*c*e*x^2 - b^2*e*x)*log(c*x + 1)^2 + 4*(a*b*c*e*x^2 - a*b*e*x)*log(
c*x + 1) - 2*((2*a*b*c*e + b^2*c*e)*x^2 + (b^2*c*d - 2*a*b*e)*x + (b^2*c*e*
x^2 - b^2*e*x)*log(c*x + 1))*log(-c*x + 1))/(c*e^2*x^2 - d*e + (c*d*e - e^2
)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2 x \operatorname{artanh}(cx)^2 + 2\,abx \operatorname{artanh}(cx) + a^2 x}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*x*arctanh(c*x)^2 + 2*a*b*x*arctanh(c*x) + a^2*x)/(e*x + d), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x (a + b \operatorname{atanh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral(x*(a + b*atanh(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2 x}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*x/(e*x + d), x)
```

3.156 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx$

Optimal. Leaf size=188

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e}-\frac{b^2\text{PolyLog}\left(3,1-\right.}{2e}$$

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/e - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d
+ e*x))/((c*d + e)*(1 + c*x))])/e + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e)
 - (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rubi [A] time = 0.0466511, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5922}

$$-\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{e}+\frac{b\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a+b\tanh^{-1}(cx)\right)}{e}-\frac{b^2\text{PolyLog}\left(3,1-\right.}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] -(((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e) + ((a + b*ArcTanh[c*x])^2*Lo
g[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - 2/(1 + c*x)])/e - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d
+ e*x))/((c*d + e)*(1 + c*x))])/e + (b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e)
 - (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e)
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx=-\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{e}+\frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{e}+\frac{b\left(a+b\tanh^{-1}(\right.}{}$$

Mathematica [C] time = 12.6864, size = 938, normalized size = 4.99

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(d + e*x),x]
```

```
[Out] (6*a^2*Log[d + e*x] + 6*a*b*ArcTanh[c*x]*(Log[1 - c^2*x^2] + 2*Log[I*Sinh[A
rcTanh[(c*d)/e] + ArcTanh[c*x]]]) - (6*I)*a*b*((-I/4)*(Pi - (2*I)*ArcTanh[c
*x])^2 + I*(ArcTanh[(c*d)/e] + ArcTanh[c*x])^2 + (Pi - (2*I)*ArcTanh[c*x])*
Log[1 + E^(2*ArcTanh[c*x])] + (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[1
 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] - (Pi - (2*I)*ArcTanh[c*x])*Lo
g[2/Sqrt[1 - c^2*x^2]] - (2*I)*(ArcTanh[(c*d)/e] + ArcTanh[c*x])*Log[(2*I)*
Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - I*PolyLog[2, -E^(2*ArcTanh[c*x])]
- I*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]) + (b^2*(-8*c*d*Ar
cTanh[c*x]^3 + 4*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*
x]^3)/E^ArcTanh[(c*d)/e] - 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])
] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]) + E^ArcTanh[c*x])/2] -
 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e)] + 6
*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*Ar
cTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c
*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 12*c*d*ArcTanh[(c*
d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(
2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 +
E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))/(2*E^ArcTanh[c*x])] - 6
*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*x^2]] - (3*I)*c*d*Pi*Arc
Tanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sin
h[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + 6*c*d*ArcTanh[c*x]*PolyLog[2, -E^(-2*
ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*
x]))/(c*d - e))] + 12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + Ar
cTanh[c*x])] + 12*c*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh
[c*x])] + 6*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*
x]))] + 3*c*d*PolyLog[3, -E^(-2*ArcTanh[c*x])] + 3*c*d*PolyLog[3, -(((c*d +
 e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[(c*d)/e
] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])]
 - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(c*d))/(6*e)
```

Maple [C] time = 0.272, size = 1170, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/(e*x+d),x)
```

```
[Out] a^2*ln(c*e*x+c*d)/e+b^2*ln(c*e*x+c*d)/e*arctanh(c*x)^2-b^2/e*arctanh(c*x)^2
*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))-1/2*I*b^2/
e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c
^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+
1))+1/2*I*b^2/e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*
((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(((c*x+1)^2/
(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)))*csgn(I/((c*x+1)^2/(-c^2*
x^2+1)+1))+1/2*I*b^2/e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)
*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^3-1/2*I*b^2/
e*arctanh(c*x)^2*Pi*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c
^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)
-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)))-b^2/e*arctanh(c*x)*polylog(2,-(c*x+1
)^2/(-c^2*x^2+1))+1/2*b^2/e*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^2/(c*d+e)*
arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+b^2/(c*d+e)*ar
ctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2*b^2/(c*d+
e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2/e*d/(c*d+e)*arc
tanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+c*b^2/e*d/(c*d+e)
```

```
*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/2*c*b^2/
e*d/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+2*a*b*ln(c*e
*x+c*d)/e*arctanh(c*x)-a*b/e*ln(c*e*x+c*d)*ln((c*e*x+e)/(-c*d+e))-a*b/e*dil
og((c*e*x+e)/(-c*d+e))+a*b/e*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+a*b/e*dil
og((c*e*x-e)/(-c*d-e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2 \log (ex + d)}{e} + \int \frac{b^2(\log (cx+1) - \log (-cx+1))^2}{4\,(ex+d)} + \frac{ab(\log (cx+1) - \log (-cx+1))}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*log(e*x + d)/e + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/(e*
x + d) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2\,ab \operatorname{artanh}(cx) + a^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/(e*x + d), x)
```

3.157 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+ex)}\,dx$

Optimal. Leaf size=319

$$\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}\right.}{}$$

```
[Out] (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d + ((a + b*ArcTanh[c*x
])^2*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + e)*(1 + c*x))])/d - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]
)/d + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d - (b*(a + b*A
rcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + (b^2*PolyLog[3, 1 - 2
/(1 - c*x)])/(2*d) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d) - (b^2*PolyLo
g[3, 1 - 2/(1 + c*x)])/(2*d) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))])/(2*d)
```

Rubi [A] time = 0.430597, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5922}

$$\frac{b\left(a+b\tanh^{-1}(cx)\right)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}-\frac{b\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d}+\frac{b\text{PolyLog}\left(2,\frac{2}{1-cx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d + ((a + b*ArcTanh[c*x
])^2*Log[2/(1 + c*x)])/d - ((a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + e)*(1 + c*x))])/d - (b*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)]
)/d + (b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d - (b*(a + b*A
rcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/d + (b*(a + b*ArcTanh[c*x])*PolyL
og[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + (b^2*PolyLog[3, 1 - 2
/(1 - c*x)])/(2*d) - (b^2*PolyLog[3, -1 + 2/(1 - c*x)])/(2*d) - (b^2*PolyLo
g[3, 1 - 2/(1 + c*x)])/(2*d) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d +
e)*(1 + c*x))])/(2*d)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x(d+ex)}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d} - \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{d}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(\right.}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(\right.}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(\right.}{}$$

$$= \frac{2\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\left(a+b\tanh^{-1}(\right.}{}$$

Mathematica [C] time = 12.1933, size = 1034, normalized size = 3.24

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x*(d + e*x)),x]
```

```
[Out] (a^2*Log[x])/d - (a^2*Log[d + e*x])/d + (a*b*((-I)*c*d*Pi*ArcTanh[c*x] - 2*
c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x] + c*d*ArcTanh[c*x]^2 - e*ArcTanh[c*x]^2 +
 (Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] + 2*c*d*ArcT
anh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] + I*c*d*Pi*Log[1 + E^(2*ArcTanh[c*x])
] - 2*c*d*ArcTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))
] - 2*c*d*ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] +
(I/2)*c*d*Pi*Log[1 - c^2*x^2] + 2*c*d*ArcTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(
c*d)/e] + ArcTanh[c*x]]] - c*d*PolyLog[2, E^(-2*ArcTanh[c*x])] + c*d*PolyLo
g[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(c*d^2) + (b^2*(I*c*d*Pi^3
 - 8*c*d*ArcTanh[c*x]^3 - 8*e*ArcTanh[c*x]^3 + 24*c*d*ArcTanh[c*x]^2*Log[1
- E^(2*ArcTanh[c*x])] + 24*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])]
- 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] - (24*(c*d - e)*(c*d + e)*(-6*c*d*A
rcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTanh[c
*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c*x]
) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*ArcTa
nh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e] + A
rcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTanh[
c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]
))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e] -
 ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*ArcT
anh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x])))
/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c^2*
x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d)/e
]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcTanh
[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] + 12*c*d*ArcT
anh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTanh[
c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]*P
olyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -(((c
*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[(c*
d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh[c*
x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(6*c^2*d
^2 - 6*e^2)))/(24*c*d^2)
```

Maple [C] time = 0.44, size = 1799, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x/(e*x+d),x)
```

```
[Out] 1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(((
c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2
*x^2+1)+1))^2+1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)
-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e
+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^2-a^2/d*ln(c*e
*x+c*d)-b^2/d*e/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/
(-c*d+e))-b^2/d*e/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^
```

```
2+1)/(-c*d+e))-1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1
)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^3+a*b/d*
dilog((c*e*x+e)/(-c*d+e))-a*b/d*dilog((c*e*x-e)/(-c*d-e))-b^2*arctanh(c*x)^
2/d*ln(c*e*x+c*d)+b^2*arctanh(c*x)^2/d*ln(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*
((c*x+1)^2/(-c^2*x^2+1)+1))+1/2*b^2*c/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(
-c^2*x^2+1)/(-c*d+e))+a^2/d*ln(c*x)+b^2/d*arctanh(c*x)^2*ln(c*x)+b^2/d*arct
anh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,
-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)
^(1/2))+2*b^2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-b^2/d*ar
ctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)-a*b/d*dilog(c*x)-a*b/d*dilog(c*x+
1)-2*b^2/d*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2/d*polylog(3,-(c*x+1)
/(-c^2*x^2+1)^(1/2))+1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2
/(-c^2*x^2+1)+1))*arctanh(c*x)^2+1/2*I*b^2/d*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I*b^2/d*Pi*csgn(I*(
(c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c
^2*x^2+1)+1))^2*arctanh(c*x)^2-1/2*I*b^2/d*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1
)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arcta
nh(c*x)^2-1/2*I*b^2/d*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*
csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)))*csgn(
I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/
(-c^2*x^2+1)+1))-2*a*b*arctanh(c*x)/d*ln(c*e*x+c*d)+a*b/d*ln(c*e*x+c*d)*ln(
(c*e*x+e)/(-c*d+e))-a*b/d*ln(c*e*x+c*d)*ln((c*e*x-e)/(-c*d-e))+1/2*b^2/d*e/
(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-b^2*c/(c*d+e)*ar
ctanh(c*x)*polylog(2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-b^2*c/(c*d+e)
*arctanh(c*x)^2*ln(1-(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-a*b/d*ln(c*x)
*ln(c*x+1)+2*a*b/d*arctanh(c*x)*ln(c*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-a^2\left(\frac{\log(ex+d)}{d}-\frac{\log(x)}{d}\right)+\int\frac{b^2(\log(cx+1)-\log(-cx+1))^2}{4\left(ex^2+dx\right)}+\frac{ab(\log(cx+1)-\log(-cx+1))}{ex^2+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] -a^2*(log(e*x + d)/d - log(x)/d) + integrate(1/4*b^2*(log(c*x + 1) - log(-c
*x + 1))^2/(e*x^2 + d*x) + a*b*(log(c*x + 1) - log(-c*x + 1))/(e*x^2 + d*x)
, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}(cx)^2+2\,ab\,\text{artanh}(cx)+a^2}{ex^2+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x^2 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\,\mathrm{atanh}\,(cx))^2}{x\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x/(e*x+d),x)
```

```
[Out] Integral((a + b*atanh(c*x))**2/(x*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx)+a)^2}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x), x)
```

3.158 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx$

Optimal. Leaf size=412

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{be\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a\right.}{d^2}$$

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) - (2*e*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (e*(a + b*ArcTanh[c*x])^2*Lo
g[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d
+ (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2 - (b*e*(a + b*
ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (b*e*(a + b*ArcTanh[c*x])
*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d
- (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 +
c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[
3, -1 + 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
 - (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^2)
```

Rubi [A] time = 0.603766, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5940, 5916, 5988, 5932, 2447, 5914, 6052, 5948, 6058, 6610, 5922}

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}-\frac{be\text{PolyLog}\left(2,\frac{2}{1-cx}-1\right)\left(a+b\tanh^{-1}(cx)\right)}{d^2}+\frac{be\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)\left(a\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] (c*(a + b*ArcTanh[c*x])^2)/d - (a + b*ArcTanh[c*x])^2/(d*x) - (2*e*(a + b*A
rcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)])/d^2 - (e*(a + b*ArcTanh[c*x])^2*Lo
g[2/(1 + c*x)])/d^2 + (e*(a + b*ArcTanh[c*x])^2*Log[(2*c*(d + e*x))/((c*d +
 e)*(1 + c*x))])/d^2 + (2*b*c*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/d
+ (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/d^2 - (b*e*(a + b*
ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/d^2 + (b*e*(a + b*ArcTanh[c*x])
*PolyLog[2, 1 - 2/(1 + c*x)])/d^2 - (b^2*c*PolyLog[2, -1 + 2/(1 + c*x)])/d
- (b*e*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 +
c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[
3, -1 + 2/(1 - c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + c*x)])/(2*d^2)
 - (b^2*e*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*d^2)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2(d+ex)}\,dx = \int \left(\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{d^2x} + \frac{e^2\left(a+b\tanh^{-1}(cx)\right)^2}{d^2(d+ex)}\right) dx$$

$$= \frac{\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x^2}\,dx}{d} - \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{x}\,dx}{d^2} + \frac{e^2\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{d+ex}\,dx}{d^2}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^2 \mathrm{lo}}{d^2}$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} -$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} -$$

$$= \frac{c\left(a+b\tanh^{-1}(cx)\right)^2}{d} - \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{dx} - \frac{2e\left(a+b\tanh^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d^2} -$$

Mathematica [C] time = 13.593, size = 1188, normalized size = 2.88

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/(x^2*(d + e*x)),x]
```

```
[Out] -(a^2/(d*x)) - (a^2*e*Log[x])/d^2 + (a^2*e*Log[d + e*x])/d^2 + (a*b*(I*c*d*
e*Pi*ArcTanh[c*x] - (2*c*d^2*ArcTanh[c*x])/x + 2*c*d*e*ArcTanh[(c*d)/e]*Arc
Tanh[c*x] - c*d*e*ArcTanh[c*x]^2 + e^2*ArcTanh[c*x]^2 - (Sqrt[1 - (c^2*d^2)
/e^2]*e^2*ArcTanh[c*x]^2)/E^ArcTanh[(c*d)/e] - 2*c*d*e*ArcTanh[c*x]*Log[1 -
 E^(-2*ArcTanh[c*x])] - I*c*d*e*Pi*Log[1 + E^(2*ArcTanh[c*x])] + 2*c*d*e*Ar
cTanh[(c*d)/e]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c*d*e*
ArcTanh[c*x]*Log[1 - E^(-2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 2*c^2*d^2*
Log[(c*x)/Sqrt[1 - c^2*x^2]] - (I/2)*c*d*e*Pi*Log[1 - c^2*x^2] - 2*c*d*e*Ar
cTanh[(c*d)/e]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] + c*d*e*PolyLog
[2, E^(-2*ArcTanh[c*x])] - c*d*e*PolyLog[2, E^(-2*(ArcTanh[(c*d)/e] + ArcTa
nh[c*x]))]))/(c*d^3) + (b^2*((-I)*c*d*e*Pi^3 + 24*c^2*d^2*ArcTanh[c*x]^2 -
(24*c*d^2*ArcTanh[c*x]^2)/x + 8*c*d*e*ArcTanh[c*x]^3 + 8*e^2*ArcTanh[c*x]^3
 + 48*c^2*d^2*ArcTanh[c*x]*Log[1 - E^(-2*ArcTanh[c*x])] - 24*c*d*e*ArcTanh[
c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] - 24*c^2*d^2*PolyLog[2, E^(-2*ArcTanh[c*
x])] - 24*c*d*e*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + 12*c*d*e*Poly
Log[3, E^(2*ArcTanh[c*x])]))/(24*c*d^3) + (b^2*(c*d - e)*e*(c*d + e)*(-6*c*
d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*e*ArcTan
```

```
h[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-ArcTanh[c
*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)*E^(2*Ar
cTanh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(c*d)/e]
+ ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e] + ArcTa
nh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c
*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh[(c*d)/e
] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] + 6*c*d*A
rcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcTanh[c*x]
)))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sqrt[1 - c
^2*x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcTanh[(c*d
)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*c*d*ArcT
anh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] + 12*c*d*A
rcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c*d*ArcTa
nh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x
]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyLog[3, -(
((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(ArcTanh[
(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] + ArcTanh
[c*x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]))/(d^3*
(6*c^3*d^2 - 6*c*e^2))
```

Maple [C] time = 2.493, size = 26776, normalized size = 65.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x^2/(e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\frac{e\log(ex+d)}{d^2} - \frac{e\log(x)}{d^2} - \frac{1}{dx}\right) - \frac{b^2\log(-cx+1)^2}{4\,dx} - \int -\frac{\left(b^2cdx - b^2d\right)\log(cx+1)^2 + 4\left(abcdx - abd\right)\log(cx+1\ldots}{4\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/4*b^2*log(-c*x + 1)^2
/(d*x) - integrate(-1/4*((b^2*c*d*x - b^2*d)*log(c*x + 1)^2 + 4*(a*b*c*d*x
- a*b*d)*log(c*x + 1) + 2*(b^2*c*e*x^2 + 2*a*b*d - (2*a*b*c*d - b^2*c*d)*x
- (b^2*c*d*x - b^2*d)*log(c*x + 1))*log(-c*x + 1))/(c*d*e*x^4 - d^2*x^2 + (
c*d^2 - d*e)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\,\text{artanh}(cx)^2 + 2\,ab\,\text{artanh}(cx) + a^2}{ex^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/(e*x^3 + d*x^2), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**2/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2/((e*x + d)*x^2), x)
```

3.159 $\int \frac{\tanh^{-1}(cx)^2}{x(d+ex)}\,dx$

Optimal. Leaf size=275

$$\frac{\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d}+\frac{\tanh^{-1}(cx)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{\text{PolyLog}\left(3,\frac{2}{1-cx}\right.}{2d}$$

```
[Out] (2*ArcTanh[c*x]^2*ArcTanh[1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]^2*Log[2/(1 +
c*x)])/d - (ArcTanh[c*x]^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d -
(ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]*PolyLog[2, -1
+ 2/(1 - c*x)])/d - (ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/d + (ArcTanh
[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + PolyLog[3,
 1 - 2/(1 - c*x)]/(2*d) - PolyLog[3, -1 + 2/(1 - c*x)]/(2*d) - PolyLog[3, 1
 - 2/(1 + c*x)]/(2*d) + PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x)
)]/(2*d)
```

Rubi [A] time = 0.335681, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5940, 5914, 6052, 5948, 6058, 6610, 5922}

$$\frac{\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{2d}+\frac{\tanh^{-1}(cx)\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cx+1)(cd+e)}\right)}{d}+\frac{\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2d}-\frac{\text{PolyLog}\left(3,\frac{2}{1-cx}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

```
[Out] (2*ArcTanh[c*x]^2*ArcTanh[1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]^2*Log[2/(1 +
c*x)])/d - (ArcTanh[c*x]^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d -
(ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 - c*x)])/d + (ArcTanh[c*x]*PolyLog[2, -1
+ 2/(1 - c*x)])/d - (ArcTanh[c*x]*PolyLog[2, 1 - 2/(1 + c*x)])/d + (ArcTanh
[c*x]*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/d + PolyLog[3,
 1 - 2/(1 - c*x)]/(2*d) - PolyLog[3, -1 + 2/(1 - c*x)]/(2*d) - PolyLog[3, 1
 - 2/(1 + c*x)]/(2*d) + PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x)
)]/(2*d)
```

Rule 5940

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^p, (
f*x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0]
 && IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5922

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
 -Simp[((a + b*ArcTanh[c*x])^2*Log[2/(1 + c*x)])/e, x] + (Simp[((a + b*ArcT
anh[c*x])^2*Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Simp[(b*(a
+ b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 + c*x)])/e, x] - Simp[(b*(a + b*ArcTa
nh[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x] + Sim
p[(b^2*PolyLog[3, 1 - 2/(1 + c*x)])/(2*e), x] - Simp[(b^2*PolyLog[3, 1 - (2
*c*(d + e*x))/((c*d + e)*(1 + c*x))])/(2*e), x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(cx)^2}{x(d+ex)}\,dx = \int \left(\frac{\tanh^{-1}(cx)^2}{dx} - \frac{e\tanh^{-1}(cx)^2}{d(d+ex)}\right)dx$$

$$= \frac{\int \frac{\tanh^{-1}(cx)^2}{x}\,dx}{d} - \frac{e\int \frac{\tanh^{-1}(cx)^2}{d+ex}\,dx}{d}$$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{\tanh}{}$$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{\tanh}{}$$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{\tanh}{}$$

$$= \frac{2\tanh^{-1}(cx)^2\tanh^{-1}\left(1-\frac{2}{1-cx}\right)}{d} + \frac{\tanh^{-1}(cx)^2\log\left(\frac{2}{1+cx}\right)}{d} - \frac{\tanh^{-1}(cx)^2\log\left(\frac{2c(d+ex)}{(cd+e)(1+cx)}\right)}{d} - \frac{\tanh}{}$$

Mathematica [C] time = 9.24865, size = 733, normalized size = 2.67

$$-\frac{24(cd-e)(cd+e)\left(-6cd\tanh^{-1}(cx)\text{PolyLog}\left(2,-\frac{(cd+e)e^{2\tanh^{-1}(cx)}}{cd-e}\right)+12cd\tanh^{-1}(cx)\text{PolyLog}\left(2,-e^{\tanh^{-1}\left(\frac{cd}{e}\right)+\tanh^{-1}(cx)}\right)+12cd\tanh^{-1}(cx)\text{PolyLog}\left(2,e^{\tanh}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[c*x]^2/(x*(d + e*x)),x]
```

```
[Out] (I*c*d*Pi^3 - 8*c*d*ArcTanh[c*x]^3 - 8*e*ArcTanh[c*x]^3 + 24*c*d*ArcTanh[c*
x]^2*Log[1 - E^(2*ArcTanh[c*x])] + 24*c*d*ArcTanh[c*x]*PolyLog[2, E^(2*ArcT
anh[c*x])] - 12*c*d*PolyLog[3, E^(2*ArcTanh[c*x])] - (24*(c*d - e)*(c*d + e
)*(-6*c*d*ArcTanh[c*x]^3 + 2*e*ArcTanh[c*x]^3 - (4*Sqrt[1 - (c^2*d^2)/e^2]*
e*ArcTanh[c*x]^3)/E^ArcTanh[(c*d)/e] - (6*I)*c*d*Pi*ArcTanh[c*x]*Log[(E^(-A
rcTanh[c*x]) + E^ArcTanh[c*x])/2] - 6*c*d*ArcTanh[c*x]^2*Log[1 + ((c*d + e)
*E^(2*ArcTanh[c*x]))/(c*d - e)] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(ArcTanh[(
c*d)/e] + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 + E^(ArcTanh[(c*d)/e]
 + ArcTanh[c*x])] + 6*c*d*ArcTanh[c*x]^2*Log[1 - E^(2*(ArcTanh[(c*d)/e] + A
rcTanh[c*x]))] + 12*c*d*ArcTanh[(c*d)/e]*ArcTanh[c*x]*Log[(I/2)*E^(-ArcTanh
[(c*d)/e] - ArcTanh[c*x])*(-1 + E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x])))] +
 6*c*d*ArcTanh[c*x]^2*Log[(e*(-1 + E^(2*ArcTanh[c*x])) + c*d*(1 + E^(2*ArcT
anh[c*x])))/(2*E^ArcTanh[c*x])] - 6*c*d*ArcTanh[c*x]^2*Log[(c*(d + e*x))/Sq
rt[1 - c^2*x^2]] - (3*I)*c*d*Pi*ArcTanh[c*x]*Log[1 - c^2*x^2] - 12*c*d*ArcT
anh[(c*d)/e]*ArcTanh[c*x]*Log[I*Sinh[ArcTanh[(c*d)/e] + ArcTanh[c*x]]] - 6*
c*d*ArcTanh[c*x]*PolyLog[2, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] +
12*c*d*ArcTanh[c*x]*PolyLog[2, -E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 12*c
*d*ArcTanh[c*x]*PolyLog[2, E^(ArcTanh[(c*d)/e] + ArcTanh[c*x])] + 6*c*d*Arc
Tanh[c*x]*PolyLog[2, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))] + 3*c*d*PolyL
og[3, -(((c*d + e)*E^(2*ArcTanh[c*x]))/(c*d - e))] - 12*c*d*PolyLog[3, -E^(
ArcTanh[(c*d)/e] + ArcTanh[c*x])] - 12*c*d*PolyLog[3, E^(ArcTanh[(c*d)/e] +
 ArcTanh[c*x])] - 3*c*d*PolyLog[3, E^(2*(ArcTanh[(c*d)/e] + ArcTanh[c*x]))]
))/(6*c^2*d^2 - 6*e^2))/(24*c*d^2)
```

Maple [C] time = 0.247, size = 1507, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c*x)^2/x/(e*x+d),x)
```

```
[Out] -arctanh(c*x)^2/d*ln(c*e*x+c*d)+arctanh(c*x)^2/d*ln(c*x)+arctanh(c*x)^2/d*l
n(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))-arctanh(c*x)
^2/d*ln((c*x+1)^2/(-c^2*x^2+1)-1)+1/d*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2
+1)^(1/2))+2/d*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2/d*polyl
og(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+1/d*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+
1)^(1/2))+2/d*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2/d*polyl
og(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))-1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-
c^2*x^2+1)+1))^2-1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-
1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^3+1/2*I/d*
Pi*arctanh(c*x)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)
+1))^3-1/2*I/d*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*
(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)))*csgn(I*(((c*
x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x
^2+1)+1))+1/2*I/d*Pi*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn
(I*(((c*x+1)^2/(-c^2*x^2+1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2
/(-c^2*x^2+1)+1))^2+1/2*I/d*Pi*arctanh(c*x)^2*csgn(I*(((c*x+1)^2/(-c^2*x^2+
1)-1)*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1)))*csgn(I*(((c*x+1)^2/(-c^2*x^2+1)-1)
*e+c*d*((c*x+1)^2/(-c^2*x^2+1)+1))/((c*x+1)^2/(-c^2*x^2+1)+1))^2-1/2*I/d*Pi
```

```
*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2-1/d*e/(c*d+e)*arctanh(c*x)^2*ln(1-(
c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-1/d*e/(c*d+e)*arctanh(c*x)*polylog(
2,(c*d+e)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/2/d*e/(c*d+e)*polylog(3,(c*d+e
)*(c*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-c/(c*d+e)*arctanh(c*x)^2*ln(1-(c*d+e)*(c
*x+1)^2/(-c^2*x^2+1)/(-c*d+e))-c/(c*d+e)*arctanh(c*x)*polylog(2,(c*d+e)*(c*
x+1)^2/(-c^2*x^2+1)/(-c*d+e))+1/2*c/(c*d+e)*polylog(3,(c*d+e)*(c*x+1)^2/(-c
^2*x^2+1)/(-c*d+e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(cx)^2}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}(cx)^2}{ex^2+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(arctanh(c*x)^2/(e*x^2 + d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(cx)}{x(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c*x)**2/x/(e*x+d),x)
```

```
[Out] Integral(atanh(c*x)**2/(x*(d + e*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(cx)^2}{(ex+d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)^2/x/(e*x+d),x, algorithm="giac")

[Out] integrate(arctanh(c*x)^2/((e*x + d)*x), x)
```

3.160 $\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi [A] time = 0.0306718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.510134, size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)\left(a+b\tan^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

Maple [A] time = 0.684, size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)\,(a+b\arctan(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

```
[Out] int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{aex+ad+(bex+bd)\arctan(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e*x + a*d + (b*e*x + b*d)*arctan(c*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

3.161 $\int x^4 \left(1-a^2x^2\right) \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=72

$$\frac{x^2}{35a^3}+\frac{\log\left(1-a^2x^2\right)}{35a^5}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{ax^6}{42}+\frac{x^4}{70a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Rubi [A] time = 0.107275, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 266, 43}

$$\frac{x^2}{35a^3}+\frac{\log\left(1-a^2x^2\right)}{35a^5}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{ax^6}{42}+\frac{x^4}{70a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx=-\left(a^2\int x^6\tanh^{-1}(ax)\,dx\right)+\int x^4\tanh^{-1}(ax)\,dx$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{5}a\int\frac{x^5}{1-a^2x^2}\,dx+\frac{1}{7}a^3\int\frac{x^7}{1-a^2x^2}\,dx$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{10}a\,\mathrm{Subst}\left(\int\frac{x^2}{1-a^2x}\,dx,x,x^2\right)+\frac{1}{14}a^3\,\mathrm{Sub}$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{1}{10}a\,\mathrm{Subst}\left(\int\left(-\frac{1}{a^4}-\frac{x}{a^2}-\frac{1}{a^4\left(-1+a^2x\right)}\right)\right.$$

$$=\frac{x^2}{35a^3}+\frac{x^4}{70a}-\frac{ax^6}{42}+\frac{1}{5}x^5\tanh^{-1}(ax)-\frac{1}{7}a^2x^7\tanh^{-1}(ax)+\frac{\log\left(1-a^2x^2\right)}{35a^5}$$

Mathematica [A] time = 0.0186719, size = 72, normalized size = 1.

$$\frac{x^2}{35a^3}+\frac{\log\left(1-a^2x^2\right)}{35a^5}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)-\frac{ax^6}{42}+\frac{x^4}{70a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(35*a^3) + x^4/(70*a) - (a*x^6)/42 + (x^5*ArcTanh[a*x])/5 - (a^2*x^7*Ar
cTanh[a*x])/7 + Log[1 - a^2*x^2]/(35*a^5)
```

Maple [A] time = 0.03, size = 67, normalized size = 0.9

$$-\frac{a^2x^7\mathrm{Artanh}(ax)}{7}+\frac{x^5\mathrm{Artanh}(ax)}{5}-\frac{x^6a}{42}+\frac{x^4}{70\,a}+\frac{x^2}{35\,a^3}+\frac{\ln(ax-1)}{35\,a^5}+\frac{\ln(ax+1)}{35\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/7*a^2*x^7*arctanh(a*x)+1/5*x^5*arctanh(a*x)-1/42*x^6*a+1/70*x^4/a+1/35*x
^2/a^3+1/35/a^5*ln(a*x-1)+1/35/a^5*ln(a*x+1)
```

Maxima [A] time = 0.952599, size = 99, normalized size = 1.38

$$-\frac{1}{210}\,a\left(\frac{5\,a^4x^6-3\,a^2x^4-6\,x^2}{a^4}-\frac{6\,\log(ax+1)}{a^6}-\frac{6\,\log(ax-1)}{a^6}\right)-\frac{1}{35}\left(5\,a^2x^7-7\,x^5\right)\mathrm{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*log(
a*x - 1)/a^6) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)
```

Fricas [A] time = 2.31199, size = 166, normalized size = 2.31

$$-\frac{5\,a^{6}x^{6}-3\,a^{4}x^{4}-6\,a^{2}x^{2}+3\left(5\,a^{7}x^{7}-7\,a^{5}x^{5}\right)\log\left(-\frac{ax+1}{ax-1}\right)-6\,\log\left(a^{2}x^{2}-1\right)}{210\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*x^6 - 3*a^4*x^4 - 6*a^2*x^2 + 3*(5*a^7*x^7 - 7*a^5*x^5)*log(-
(a*x + 1)/(a*x - 1)) - 6*log(a^2*x^2 - 1))/a^5
```

Sympy [A] time = 3.30478, size = 71, normalized size = 0.99

$$\begin{cases} -\frac{a^{2}x^{7}\operatorname{atanh}(ax)}{7}-\frac{ax^{6}}{42}+\frac{x^{5}\operatorname{atanh}(ax)}{5}+\frac{x^{4}}{70a}+\frac{x^{2}}{35a^{3}}+\frac{2\log\left(x-\frac{1}{a}\right)}{35a^{5}}+\frac{2\operatorname{atanh}(ax)}{35a^{5}} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**7*atanh(a*x)/7 - a*x**6/42 + x**5*atanh(a*x)/5 + x**4/(
70*a) + x**2/(35*a**3) + 2*log(x - 1/a)/(35*a**5) + 2*atanh(a*x)/(35*a**5),
 Ne(a, 0)), (0, True))
```

Giac [A] time = 1.17489, size = 105, normalized size = 1.46

$$-\frac{1}{70}\left(5\,a^{2}x^{7}-7\,x^{5}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{\log\left(\left|a^{2}x^{2}-1\right|\right)}{35\,a^{5}}-\frac{5\,a^{7}x^{6}-3\,a^{5}x^{4}-6\,a^{3}x^{2}}{210\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/70*(5*a^2*x^7 - 7*x^5)*log(-(a*x + 1)/(a*x - 1)) + 1/35*log(abs(a^2*x^2
- 1))/a^5 - 1/210*(5*a^7*x^6 - 3*a^5*x^4 - 6*a^3*x^2)/a^6
```

3.162 $\int x^3 \left(1-a^2x^2\right) \tanh^{-1}(ax) \, dx$

Optimal. Leaf size=63

$$-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{x}{12a^3}-\frac{\tanh^{-1}(ax)}{12a^4}-\frac{ax^5}{30}+\frac{x^3}{36a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 - ArcTanh[a*x]/(12*a^4) + (x^4*ArcTanh
[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/6
```

Rubi [A] time = 0.0807167, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 302, 206}

$$-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{x}{12a^3}-\frac{\tanh^{-1}(ax)}{12a^4}-\frac{ax^5}{30}+\frac{x^3}{36a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 - ArcTanh[a*x]/(12*a^4) + (x^4*ArcTanh
[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/6
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx = -\left(a^2\int x^5\tanh^{-1}(ax)\,dx\right)+\int x^3\tanh^{-1}(ax)\,dx$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{1}{4}a\int\frac{x^4}{1-a^2x^2}\,dx+\frac{1}{6}a^3\int\frac{x^6}{1-a^2x^2}\,dx$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)-\frac{1}{4}a\int\left(-\frac{1}{a^4}-\frac{x^2}{a^2}+\frac{1}{a^4\left(1-a^2x^2\right)}\right)dx+\frac{1}{6}a^3\int\Big($$
$$= \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{\int\frac{1}{1-a^2x^2}\,dx}{6a^3}-\frac{\int\frac{1}{1-a^2x^2}\,dx}{4a^3}$$
$$= \frac{x}{12a^3}+\frac{x^3}{36a}-\frac{ax^5}{30}-\frac{\tanh^{-1}(ax)}{12a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)-\frac{1}{6}a^2x^6\tanh^{-1}(ax)$$

Mathematica [A] time = 0.0174197, size = 79, normalized size = 1.25

$$-\frac{1}{6}a^2x^6\tanh^{-1}(ax)+\frac{x}{12a^3}+\frac{\log(1-ax)}{24a^4}-\frac{\log(ax+1)}{24a^4}-\frac{ax^5}{30}+\frac{x^3}{36a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(12*a^3) + x^3/(36*a) - (a*x^5)/30 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcT
anh[a*x])/6 + Log[1 - a*x]/(24*a^4) - Log[1 + a*x]/(24*a^4)
```

Maple [A] time = 0.027, size = 65, normalized size = 1.

$$-\frac{a^2x^6\mathrm{Artanh}\,(ax)}{6}+\frac{x^4\mathrm{Artanh}\,(ax)}{4}-\frac{ax^5}{30}+\frac{x^3}{36\,a}+\frac{x}{12\,a^3}+\frac{\ln(ax-1)}{24\,a^4}-\frac{\ln(ax+1)}{24\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/6*a^2*x^6*arctanh(a*x)+1/4*x^4*arctanh(a*x)-1/30*a*x^5+1/36*x^3/a+1/12*x
/a^3+1/24/a^4*ln(a*x-1)-1/24/a^4*ln(a*x+1)
```

Maxima [A] time = 0.949963, size = 97, normalized size = 1.54

$$-\frac{1}{360}\,a\left(\frac{2\left(6\,a^4x^5-5\,a^2x^3-15\,x\right)}{a^4}+\frac{15\,\log(ax+1)}{a^5}-\frac{15\,\log(ax-1)}{a^5}\right)-\frac{1}{12}\left(2\,a^2x^6-3\,x^4\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/360*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*l
og(a*x - 1)/a^5) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)
```

Fricas [A] time = 2.25821, size = 143, normalized size = 2.27

$$-\frac{12\,a^5x^5-10\,a^3x^3-30\,ax+15\left(2\,a^6x^6-3\,a^4x^4+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{360\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/360*(12*a^5*x^5 - 10*a^3*x^3 - 30*a*x + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*l
og(-(a*x + 1)/(a*x - 1)))/a^4
```

Sympy [A] time = 2.2913, size = 54, normalized size = 0.86

$$\begin{cases}-\frac{a^2x^6\operatorname{atanh}(ax)}{6}-\frac{ax^5}{30}+\frac{x^4\operatorname{atanh}(ax)}{4}+\frac{x^3}{36a}+\frac{x}{12a^3}-\frac{\operatorname{atanh}(ax)}{12a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**6*atanh(a*x)/6 - a*x**5/30 + x**4*atanh(a*x)/4 + x**3/(
36*a) + x/(12*a**3) - atanh(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.17286, size = 113, normalized size = 1.79

$$-\frac{1}{24}\left(2\,a^2x^6-3\,x^4\right)\log\left(-\frac{ax+1}{ax-1}\right)-\frac{\log\left(|ax+1|\right)}{24\,a^4}+\frac{\log\left(|ax-1|\right)}{24\,a^4}-\frac{6\,a^{11}x^5-5\,a^9x^3-15\,a^7x}{180\,a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/24*(2*a^2*x^6 - 3*x^4)*log(-(a*x + 1)/(a*x - 1)) - 1/24*log(abs(a*x + 1)
)/a^4 + 1/24*log(abs(a*x - 1))/a^4 - 1/180*(6*a^11*x^5 - 5*a^9*x^3 - 15*a^7
*x)/a^10
```

3.163 $\int x^2 \left(1-a^2x^2\right) \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=62

$$\frac{\log\left(1-a^2x^2\right)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{ax^4}{20}+\frac{x^2}{15a}+\frac{1}{3}x^3\tanh^{-1}(ax)$$

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

Rubi [A] time = 0.0874176, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 266, 43}

$$\frac{\log\left(1-a^2x^2\right)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)-\frac{ax^4}{20}+\frac{x^2}{15a}+\frac{1}{3}x^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(1-a^2x^2\right) \tanh^{-1}(ax)\,dx = -\left(a^2\int x^4 \tanh^{-1}(ax)\,dx\right) + \int x^2 \tanh^{-1}(ax)\,dx$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{1}{5}a^2x^5 \tanh^{-1}(ax) - \frac{1}{3}a\int \frac{x^3}{1-a^2x^2}\,dx + \frac{1}{5}a^3\int \frac{x^5}{1-a^2x^2}\,dx$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{1}{5}a^2x^5 \tanh^{-1}(ax) - \frac{1}{6}a\,\text{Subst}\left(\int \frac{x}{1-a^2x}\,dx, x, x^2\right) + \frac{1}{10}a^3\,\text{Subs}$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{1}{5}a^2x^5 \tanh^{-1}(ax) - \frac{1}{6}a\,\text{Subst}\left(\int \left(-\frac{1}{a^2} - \frac{1}{a^2\left(-1+a^2x\right)}\right) dx, x, x$$
$$= \frac{x^2}{15a} - \frac{ax^4}{20} + \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{1}{5}a^2x^5 \tanh^{-1}(ax) + \frac{\log\left(1-a^2x^2\right)}{15a^3}$$

Mathematica [A] time = 0.014658, size = 62, normalized size = 1.

$$\frac{\log\left(1-a^2x^2\right)}{15a^3} - \frac{1}{5}a^2x^5 \tanh^{-1}(ax) - \frac{ax^4}{20} + \frac{x^2}{15a} + \frac{1}{3}x^3 \tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x^2/(15*a) - (a*x^4)/20 + (x^3*ArcTanh[a*x])/3 - (a^2*x^5*ArcTanh[a*x])/5 +
 Log[1 - a^2*x^2]/(15*a^3)
```

Maple [A] time = 0.03, size = 59, normalized size = 1.

$$-\frac{a^2x^5 \text{Artanh}\,(ax)}{5} + \frac{x^3 \text{Artanh}\,(ax)}{3} - \frac{x^4 a}{20} + \frac{x^2}{15\,a} + \frac{\ln\,(ax-1)}{15\,a^3} + \frac{\ln\,(ax+1)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/5*a^2*x^5*arctanh(a*x)+1/3*x^3*arctanh(a*x)-1/20*x^4*a+1/15*x^2/a+1/15/a
^3*ln(a*x-1)+1/15/a^3*ln(a*x+1)
```

Maxima [A] time = 0.951199, size = 88, normalized size = 1.42

$$-\frac{1}{60}\,a\left(\frac{3\,a^2x^4 - 4\,x^2}{a^2} - \frac{4\,\log\,(ax+1)}{a^4} - \frac{4\,\log\,(ax-1)}{a^4}\right) - \frac{1}{15}\left(3\,a^2x^5 - 5\,x^3\right)\text{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/60*a*((3*a^2*x^4 - 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)
 - 1/15*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)
```

Fricas [A] time = 2.18957, size = 149, normalized size = 2.4

$$-\frac{3\,a^{4}x^{4}-4\,a^{2}x^{2}+2\left(3\,a^{5}x^{5}-5\,a^{3}x^{3}\right)\log\left(-\frac{ax+1}{ax-1}\right)-4\,\log\left(a^{2}x^{2}-1\right)}{60\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/60*(3*a^4*x^4 - 4*a^2*x^2 + 2*(3*a^5*x^5 - 5*a^3*x^3)*log(-(a*x + 1)/(a*
x - 1)) - 4*log(a^2*x^2 - 1))/a^3
```

Sympy [A] time = 1.83875, size = 63, normalized size = 1.02

$$\begin{cases} -\frac{a^{2}x^{5}\operatorname{atanh}(ax)}{5}-\frac{ax^{4}}{20}+\frac{x^{3}\operatorname{atanh}(ax)}{3}+\frac{x^{2}}{15a}+\frac{2\log\left(x-\frac{1}{a}\right)}{15a^{3}}+\frac{2\operatorname{atanh}(ax)}{15a^{3}} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**5*atanh(a*x)/5 - a*x**4/20 + x**3*atanh(a*x)/3 + x**2/(
15*a) + 2*log(x - 1/a)/(15*a**3) + 2*atanh(a*x)/(15*a**3), Ne(a, 0)), (0, T
rue))
```

Giac [A] time = 1.16168, size = 95, normalized size = 1.53

$$-\frac{1}{30}\left(3\,a^{2}x^{5}-5\,x^{3}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{\log\left(\left|a^{2}x^{2}-1\right|\right)}{15\,a^{3}}-\frac{3\,a^{5}x^{4}-4\,a^{3}x^{2}}{60\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/30*(3*a^2*x^5 - 5*x^3)*log(-(a*x + 1)/(a*x - 1)) + 1/15*log(abs(a^2*x^2
- 1))/a^3 - 1/60*(3*a^5*x^4 - 4*a^3*x^2)/a^4
```

3.164 $\int x \left(1-a^2x^2\right) \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=40

$$-\frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{4a^2}-\frac{ax^3}{12}+\frac{x}{4a}$$

```
[Out] x/(4*a) - (a*x^3)/12 - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*a^2)
```

Rubi [A] time = 0.0215924, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5994}

$$-\frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{4a^2}-\frac{ax^3}{12}+\frac{x}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(4*a) - (a*x^3)/12 - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx &= -\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{4a^2}+\frac{\int \left(1-a^2x^2\right)\,dx}{4a}\\ &= \frac{x}{4a}-\frac{ax^3}{12}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.0140596, size = 69, normalized size = 1.72

$$-\frac{1}{4}a^2x^4\tanh^{-1}(ax)+\frac{\log(1-ax)}{8a^2}-\frac{\log(ax+1)}{8a^2}-\frac{ax^3}{12}+\frac{1}{2}x^2\tanh^{-1}(ax)+\frac{x}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] x/(4*a) - (a*x^3)/12 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTanh[a*x])/4 + Lo
g[1 - a*x]/(8*a^2) - Log[1 + a*x]/(8*a^2)
```

Maple [A] time = 0.027, size = 57, normalized size = 1.4

$$-\frac{a^{2}\operatorname{Artanh}\left(ax\right)x^{4}}{4}+\frac{\operatorname{Artanh}\left(ax\right)x^{2}}{2}-\frac{x^{3}a}{12}+\frac{x}{4a}+\frac{\ln\left(ax-1\right)}{8a^{2}}-\frac{\ln\left(ax+1\right)}{8a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/4*a^2*arctanh(a*x)*x^4+1/2*arctanh(a*x)*x^2-1/12*x^3*a+1/4*x/a+1/8/a^2*l
n(a*x-1)-1/8/a^2*ln(a*x+1)
```

Maxima [A] time = 0.965126, size = 50, normalized size = 1.25

$$-\frac{\left(a^{2}x^{2}-1\right)^{2}\operatorname{artanh}\left(ax\right)}{4a^{2}}-\frac{a^{2}x^{3}-3x}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*x^2 - 1)^2*arctanh(a*x)/a^2 - 1/12*(a^2*x^3 - 3*x)/a
```

Fricas [A] time = 2.19443, size = 117, normalized size = 2.92

$$-\frac{2a^{3}x^{3}-6ax+3\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{24a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/24*(2*a^3*x^3 - 6*a*x + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1)))/a^2
```

Sympy [A] time = 1.36611, size = 46, normalized size = 1.15

$$\begin{cases}-\frac{a^{2}x^{4}\operatorname{atanh}\left(ax\right)}{4}-\frac{ax^{3}}{12}+\frac{x^{2}\operatorname{atanh}\left(ax\right)}{2}+\frac{x}{4a}-\frac{\operatorname{atanh}\left(ax\right)}{4a^{2}} & \text{for}\: a\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**4*atanh(a*x)/4 - a*x**3/12 + x**2*atanh(a*x)/2 + x/(4*a
) - atanh(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

Giac [B] time = 1.16326, size = 100, normalized size = 2.5

$$-\frac{1}{8}\left(a^{2}x^{4}-2\,x^{2}\right)\log\left(-\frac{ax+1}{ax-1}\right)-\frac{\log\left(|ax+1|\right)}{8\,a^{2}}+\frac{\log\left(|ax-1|\right)}{8\,a^{2}}-\frac{a^{7}x^{3}-3\,a^{5}x}{12\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/8*(a^2*x^4 - 2*x^2)*log(-(a*x + 1)/(a*x - 1)) - 1/8*log(abs(a*x + 1))/a^
2 + 1/8*log(abs(a*x - 1))/a^2 - 1/12*(a^7*x^3 - 3*a^5*x)/a^6
```

3.165 $\int (1-a^2x^2)\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=64

$$\frac{1-a^2x^2}{6a}+\frac{\log(1-a^2x^2)}{3a}+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{2}{3}x\tanh^{-1}(ax)$$

```
[Out] (1 - a^2*x^2)/(6*a) + (2*x*ArcTanh[a*x])/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x])
/3 + Log[1 - a^2*x^2]/(3*a)
```

Rubi [A] time = 0.0212375, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5942, 5910, 260}

$$\frac{1-a^2x^2}{6a}+\frac{\log(1-a^2x^2)}{3a}+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{2}{3}x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] (1 - a^2*x^2)/(6*a) + (2*x*ArcTanh[a*x])/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x])
/3 + Log[1 - a^2*x^2]/(3*a)
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int (1-a^2x^2)\tanh^{-1}(ax)\,dx &= \frac{1-a^2x^2}{6a}+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{2}{3}\int \tanh^{-1}(ax)\,dx\\
&= \frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)-\frac{1}{3}(2a)\int \frac{x}{1-a^2x^2}\,dx\\
&= \frac{1-a^2x^2}{6a}+\frac{2}{3}x\tanh^{-1}(ax)+\frac{1}{3}x(1-a^2x^2)\tanh^{-1}(ax)+\frac{\log(1-a^2x^2)}{3a}
\end{aligned}$$

Mathematica [A] time = 0.0089424, size = 47, normalized size = 0.73

$$\frac{\log\left(1-a^2x^2\right)}{3a}-\frac{1}{3}a^2x^3\tanh^{-1}(ax)-\frac{ax^2}{6}+x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x],x]
```

```
[Out] -(a*x^2)/6 + x*ArcTanh[a*x] - (a^2*x^3*ArcTanh[a*x])/3 + Log[1 - a^2*x^2]/(
3*a)
```

Maple [A] time = 0.026, size = 48, normalized size = 0.8

$$-\frac{a^2\mathrm{Artanh}\,(ax)\,x^3}{3}+x\mathrm{Artanh}\,(ax)-\frac{ax^2}{6}+\frac{\ln\,(ax-1)}{3\,a}+\frac{\ln\,(ax+1)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x),x)
```

```
[Out] -1/3*a^2*arctanh(a*x)*x^3+x*arctanh(a*x)-1/6*a*x^2+1/3/a*ln(a*x-1)+1/3/a*ln
(a*x+1)
```

Maxima [A] time = 0.954495, size = 63, normalized size = 0.98

$$-\frac{1}{6}\left(x^2-\frac{2\,\log\,(ax+1)}{a^2}-\frac{2\,\log\,(ax-1)}{a^2}\right)a-\frac{1}{3}\left(a^2x^3-3\,x\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/6*(x^2 - 2*log(a*x + 1)/a^2 - 2*log(a*x - 1)/a^2)*a - 1/3*(a^2*x^3 - 3*x
)*arctanh(a*x)
```

Fricas [A] time = 2.21157, size = 115, normalized size = 1.8

$$-\frac{a^2x^2+\left(a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-2\,\log\left(a^2x^2-1\right)}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/6*(a^2*x^2 + (a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 2*log(a^2*x^2
 - 1))/a
```

Sympy [A] time = 0.995619, size = 49, normalized size = 0.77

$$\begin{cases} -\frac{a^2x^3\operatorname{atanh}(ax)}{3} - \frac{ax^2}{6} + x\operatorname{atanh}(ax) + \frac{2\log\left(x-\frac{1}{a}\right)}{3a} + \frac{2\operatorname{atanh}(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x),x)
```

```
[Out] Piecewise((-a**2*x**3*atanh(a*x)/3 - a*x**2/6 + x*atanh(a*x) + 2*log(x - 1/
a)/(3*a) + 2*atanh(a*x)/(3*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.16982, size = 69, normalized size = 1.08

$$-\frac{1}{6}ax^2 - \frac{1}{6}\left(a^2x^3 - 3x\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{\log\left(\left|a^2x^2-1\right|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/6*a*x^2 - 1/6*(a^2*x^3 - 3*x)*log(-(a*x + 1)/(a*x - 1)) + 1/3*log(abs(a^
2*x^2 - 1))/a
```

3.166 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)$$

```
[Out] -(a*x)/2 + ArcTanh[a*x]/2 - (a^2*x^2*ArcTanh[a*x])/2 - PolyLog[2, -(a*x)]/2
 + PolyLog[2, a*x]/2
```

Rubi [A] time = 0.0473091, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6014, 5912, 5916, 321, 206}

$$-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

```
[Out] -(a*x)/2 + ArcTanh[a*x]/2 - (a^2*x^2*ArcTanh[a*x])/2 - PolyLog[2, -(a*x)]/2
 + PolyLog[2, a*x]/2
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{x}\,dx &= -\left(a^2\int x\tanh^{-1}(ax)\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x}\,dx\\ &= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\mathrm{Li}_2(-ax)}{2}+\frac{\mathrm{Li}_2(ax)}{2}+\frac{1}{2}a^3\int \frac{x^2}{1-a^2x^2}\,dx\\ &= -\frac{ax}{2}-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\mathrm{Li}_2(-ax)}{2}+\frac{\mathrm{Li}_2(ax)}{2}+\frac{1}{2}a\int \frac{1}{1-a^2x^2}\,dx\\ &= -\frac{ax}{2}+\frac{1}{2}\tanh^{-1}(ax)-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{\mathrm{Li}_2(-ax)}{2}+\frac{\mathrm{Li}_2(ax)}{2}\end{aligned}$$

Mathematica [A] time = 0.0177563, size = 60, normalized size = 1.25

$$\frac{1}{2}(\mathrm{PolyLog}(2,ax)-\mathrm{PolyLog}(2,-ax))-\frac{1}{2}a^2x^2\tanh^{-1}(ax)-\frac{ax}{2}-\frac{1}{4}\log(1-ax)+\frac{1}{4}\log(ax+1)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x,x]
```

```
[Out] -(a*x)/2 - (a^2*x^2*ArcTanh[a*x])/2 - Log[1 - a*x]/4 + Log[1 + a*x]/4 + (-P
olyLog[2, -(a*x)] + PolyLog[2, a*x])/2
```

Maple [A] time = 0.046, size = 69, normalized size = 1.4

$$-\frac{a^2x^2\mathrm{Artanh}\,(ax)}{2}+\mathrm{Artanh}\,(ax)\ln(ax)-\frac{ax}{2}-\frac{\ln(ax-1)}{4}+\frac{\ln(ax+1)}{4}-\frac{\mathrm{dilog}\,(ax)}{2}-\frac{\mathrm{dilog}\,(ax+1)}{2}-\frac{\ln(ax)\ln(ax}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x,x)
```

```
[Out] -1/2*a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*a*x-1/4*ln(a*x-1)+1/4*ln
(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Maxima [B] time = 0.952142, size = 120, normalized size = 2.5

$$-\frac{1}{4}a\left(2x+\frac{2(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax))}{a}-\frac{2(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax))}{a}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)-\frac{1}{2}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] -1/4*a*(2*x + 2*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 2*(log(-a*x + 1)*lo
g(x) + dilog(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) - 1/2*(a^2*x^2 - lo
g(x^2))*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\text{atanh}\left(ax\right)}{x}\,dx-\int a^2x\,\text{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x,x)
```

```
[Out] -Integral(-atanh(a*x)/x, x) - Integral(a**2*x*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)/x, x)
```

3.167 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=38

$$-a\log\left(1-a^2x^2\right)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]
```

Rubi [A] time = 0.0511531, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6014, 5916, 266, 36, 29, 31, 5910, 260}

$$-a\log\left(1-a^2x^2\right)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^2}\,dx &= -\left(a^2\int \tanh^{-1}(ax)\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\int \frac{1}{x(1-a^2x^2)}\,dx+a^3\int \frac{x}{1-a^2x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)-\frac{1}{2}a\log(1-a^2x^2)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x(1-a^2x)}\,dx,x,
\right.\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)-\frac{1}{2}a\log(1-a^2x^2)+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^3\\
&= -\frac{\tanh^{-1}(ax)}{x}-a^2x\tanh^{-1}(ax)+a\log(x)-a\log(1-a^2x^2)
\end{aligned}$$

Mathematica [A] time = 0.0097317, size = 38, normalized size = 1.

$$-a\log(1-a^2x^2)+a^2(-x)\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(ArcTanh[a*x]/x) - a^2*x*ArcTanh[a*x] + a*Log[x] - a*Log[1 - a^2*x^2]
```

Maple [A] time = 0.033, size = 45, normalized size = 1.2

$$-a^2x\text{Artanh}(ax)-\frac{\text{Artanh}(ax)}{x}-a\ln(ax-1)+a\ln(ax)-a\ln(ax+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^2,x)
```

```
[Out] -a^2*x*arctanh(a*x)-arctanh(a*x)/x-a*ln(a*x-1)+a*ln(a*x)-a*ln(a*x+1)
```

Maxima [A] time = 0.94342, size = 49, normalized size = 1.29

$$-a(\log(ax+1)+\log(ax-1)-\log(x))-\left(a^2x+\frac{1}{x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] -a*(log(a*x + 1) + log(a*x - 1) - log(x)) - (a^2*x + 1/x)*arctanh(a*x)
```

Fricas [A] time = 2.23007, size = 122, normalized size = 3.21

$$-\frac{2\,ax\log\left(a^2x^2-1\right)-2\,ax\log(x)+\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*log(a^2*x^2 - 1) - 2*a*x*log(x) + (a^2*x^2 + 1)*log(-(a*x + 1)/
(a*x - 1)))/x
```

Sympy [A] time = 1.23114, size = 41, normalized size = 1.08

$$\begin{cases} -a^2x\operatorname{atanh}(ax)+a\log(x)-2a\log\left(x-\frac{1}{a}\right)-2a\operatorname{atanh}(ax)-\frac{\operatorname{atanh}(ax)}{x} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**2,x)
```

```
[Out] Piecewise((-a**2*x*atanh(a*x) + a*log(x) - 2*a*log(x - 1/a) - 2*a*atanh(a*x
) - atanh(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.15541, size = 65, normalized size = 1.71

$$\frac{1}{2}\,a\log\left(x^2\right)-\frac{1}{2}\left(a^2x+\frac{1}{x}\right)\log\left(-\frac{ax+1}{ax-1}\right)-a\log\left(\left|a^2x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*a*log(x^2) - 1/2*(a^2*x + 1/x)*log(-(a*x + 1)/(a*x - 1)) - a*log(abs(a^
2*x^2 - 1))
```

3.168 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=56

$$\frac{1}{2}a^2\text{PolyLog}(2,-ax)-\frac{1}{2}a^2\text{PolyLog}(2,ax)+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

```
[Out] -a/(2*x) + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*PolyLog[2, -(
a*x)])/2 - (a^2*PolyLog[2, a*x])/2
```

Rubi [A] time = 0.0509202, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6014, 5916, 325, 206, 5912}

$$\frac{1}{2}a^2\text{PolyLog}(2,-ax)-\frac{1}{2}a^2\text{PolyLog}(2,ax)+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -a/(2*x) + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*PolyLog[2, -(
a*x)])/2 - (a^2*PolyLog[2, a*x])/2
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{x^3}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{x}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\text{Li}_2(-ax)-\frac{1}{2}a^2\text{Li}_2(ax)+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx\\ &= -\frac{a}{2x}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\text{Li}_2(-ax)-\frac{1}{2}a^2\text{Li}_2(ax)+\frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\,dx\\ &= -\frac{a}{2x}+\frac{1}{2}a^2\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^2\text{Li}_2(-ax)-\frac{1}{2}a^2\text{Li}_2(ax)\end{aligned}$$

Mathematica [A] time = 0.0313162, size = 68, normalized size = 1.21

$$-\frac{1}{2}a^2(\text{PolyLog}(2,ax)-\text{PolyLog}(2,-ax))-\frac{1}{4}a^2\log(1-ax)+\frac{1}{4}a^2\log(ax+1)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -a/(2*x) - ArcTanh[a*x]/(2*x^2) - (a^2*Log[1 - a*x])/4 + (a^2*Log[1 + a*x])
/4 - (a^2*(-PolyLog[2, -(a*x)] + PolyLog[2, a*x]))/2
```

Maple [A] time = 0.044, size = 87, normalized size = 1.6

$$-a^2\text{Artanh}\,(ax)\ln(ax)-\frac{\text{Artanh}\,(ax)}{2\,x^2}-\frac{a}{2\,x}-\frac{a^2\ln(ax-1)}{4}+\frac{a^2\ln(ax+1)}{4}+\frac{a^2\text{dilog}\,(ax)}{2}+\frac{a^2\text{dilog}\,(ax+1)}{2}+\frac{a^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^3,x)
```

```
[Out] -a^2*arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/x^2-1/2*a/x-1/4*a^2*ln(a*x-1)+1/
4*a^2*ln(a*x+1)+1/2*a^2*dilog(a*x)+1/2*a^2*dilog(a*x+1)+1/2*a^2*ln(a*x)*ln(
a*x+1)
```

Maxima [A] time = 0.956987, size = 109, normalized size = 1.95

$$\frac{1}{4}\left(2\left(\log(ax+1)\log(x)+\text{Li}_2(-ax)\right)a-2\left(\log(-ax+1)\log(x)+\text{Li}_2(ax)\right)a+a\log(ax+1)-a\log(ax-1)-\frac{2}{x}\right)a-\frac{1}{2}\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + di
log(a*x))*a + a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*a - 1/2*(a^2*log(x^2)
+ 1/x^2)*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\text{atanh}\left(ax\right)}{x^3}\,dx - \int \frac{a^2\,\text{atanh}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**3,x)
```

```
[Out] -Integral(-atanh(a*x)/x**3, x) - Integral(a**2*atanh(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)/x^3, x)
```

3.169 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=58

$$\frac{1}{3}a^3\log\left(1-a^2x^2\right)-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

Rubi [A] time = 0.0763112, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6014, 5916, 266, 44, 36, 29, 31}

$$\frac{1}{3}a^3\log\left(1-a^2x^2\right)-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^4}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^4}\,dx \\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{3}a\int \frac{1}{x^3(1-a^2x^2)}\,dx-a^3\int \frac{1}{x(1-a^2x^2)}\,dx \\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{6}a\,\text{Subst}\left(\int \frac{1}{x^2(1-a^2x)}\,dx,x,x^2\right)-\frac{1}{2}a^3\,\text{Subst}\left(\int \right. \\
&= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}+\frac{1}{6}a\,\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{a^2}{x}-\frac{a^4}{-1+a^2x}\right)dx,x,x^2\right)-\frac{1}{2}a \\
&= -\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{2}{3}a^3\log(x)+\frac{1}{3}a^3\log(1-a^2x^2)
\end{aligned}$$

Mathematica [A] time = 0.0154878, size = 58, normalized size = 1.

$$\frac{1}{3}a^3\log(1-a^2x^2)-\frac{2}{3}a^3\log(x)+\frac{a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (a^2*ArcTanh[a*x])/x - (2*a^3*Log[x])/3
 + (a^3*Log[1 - a^2*x^2])/3
```

Maple [A] time = 0.038, size = 59, normalized size = 1.

$$\frac{a^2\text{Artanh}(ax)}{x}-\frac{\text{Artanh}(ax)}{3x^3}+\frac{a^3\ln(ax-1)}{3}-\frac{a}{6x^2}-\frac{2a^3\ln(ax)}{3}+\frac{a^3\ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^4,x)
```

```
[Out] a^2*arctanh(a*x)/x-1/3*arctanh(a*x)/x^3+1/3*a^3*ln(a*x-1)-1/6*a/x^2-2/3*a^3
*ln(a*x)+1/3*a^3*ln(a*x+1)
```

Maxima [A] time = 0.945411, size = 72, normalized size = 1.24

$$\frac{1}{6}\left(2\,a^2\log\left(a^2x^2-1\right)-2\,a^2\log\left(x^2\right)-\frac{1}{x^2}\right)a+\frac{\left(3\,a^2x^2-1\right)\operatorname{artanh}\left(ax\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x^2) - 1/x^2)*a + 1/3*(3*a^2*x^2 -
1)*arctanh(a*x)/x^3
```

Fricas [A] time = 2.24186, size = 144, normalized size = 2.48

$$\frac{2\,a^3x^3\log\left(a^2x^2-1\right)-4\,a^3x^3\log\left(x\right)-ax+\left(3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*x^3*log(a^2*x^2 - 1) - 4*a^3*x^3*log(x) - a*x + (3*a^2*x^2 - 1)*
log(-(a*x + 1)/(a*x - 1)))/x^3
```

Sympy [A] time = 1.7318, size = 63, normalized size = 1.09

$$\begin{cases} -\frac{2a^3\log\left(x\right)}{3}+\frac{2a^3\log\left(x-\frac{1}{a}\right)}{3}+\frac{2a^3\operatorname{atanh}\left(ax\right)}{3}+\frac{a^2\operatorname{atanh}\left(ax\right)}{x}-\frac{a}{6x^2}-\frac{\operatorname{atanh}\left(ax\right)}{3x^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**4,x)
```

```
[Out] Piecewise((-2*a**3*log(x)/3 + 2*a**3*log(x - 1/a)/3 + 2*a**3*atanh(a*x)/3 +
 a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3), Ne(a, 0)), (0, True)
)
```

Giac [A] time = 1.17024, size = 99, normalized size = 1.71

$$-\frac{1}{3}\,a^3\log\left(x^2\right)+\frac{1}{3}\,a^3\log\left(\left|a^2x^2-1\right|\right)+\frac{2\,a^3x^2-a}{6\,x^2}+\frac{\left(3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -1/3*a^3*log(x^2) + 1/3*a^3*log(abs(a^2*x^2 - 1)) + 1/6*(2*a^3*x^2 - a)/x^2
 + 1/6*(3*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))/x^3
```

3.170 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=42

$$-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}+\frac{a^3}{4x}-\frac{a}{12x^3}$$

```
[Out] -a/(12*x^3) + a^3/(4*x) - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*x^4)
```

Rubi [A] time = 0.0302711, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6008, 14}

$$-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}+\frac{a^3}{4x}-\frac{a}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -a/(12*x^3) + a^3/(4*x) - ((1 - a^2*x^2)^2*ArcTanh[a*x])/(4*x^4)
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^5}\,dx &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \frac{1-a^2x^2}{x^4}\,dx\\ &= -\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}+\frac{1}{4}a\int \left(\frac{1}{x^4}-\frac{a^2}{x^2}\right)dx\\ &= -\frac{a}{12x^3}+\frac{a^3}{4x}-\frac{(1-a^2x^2)^2\tanh^{-1}(ax)}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.0158646, size = 71, normalized size = 1.69

$$\frac{a^2\tanh^{-1}(ax)}{2x^2}+\frac{a^3}{4x}+\frac{1}{8}a^4\log(1-ax)-\frac{1}{8}a^4\log(ax+1)-\frac{a}{12x^3}-\frac{\tanh^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -a/(12*x^3) + a^3/(4*x) - ArcTanh[a*x]/(4*x^4) + (a^2*ArcTanh[a*x])/(2*x^2)
 + (a^4*Log[1 - a*x])/8 - (a^4*Log[1 + a*x])/8
```

Maple [A] time = 0.036, size = 59, normalized size = 1.4

$$-\frac{\operatorname{Artanh}(ax)}{4x^4}+\frac{a^2\operatorname{Artanh}(ax)}{2x^2}+\frac{a^4\ln(ax-1)}{8}+\frac{a^3}{4x}-\frac{a}{12x^3}-\frac{a^4\ln(ax+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^5,x)
```

```
[Out] -1/4*arctanh(a*x)/x^4+1/2*a^2*arctanh(a*x)/x^2+1/8*a^4*ln(a*x-1)+1/4*a^3/x-
1/12*a/x^3-1/8*a^4*ln(a*x+1)
```

Maxima [A] time = 0.956346, size = 82, normalized size = 1.95

$$-\frac{1}{24}\left(3a^3\log(ax+1)-3a^3\log(ax-1)-\frac{2\left(3a^2x^2-1\right)}{x^3}\right)a+\frac{\left(2a^2x^2-1\right)\operatorname{artanh}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a^2*x^2 - 1)/x^3)*a +
 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/x^4
```

Fricas [A] time = 2.24379, size = 116, normalized size = 2.76

$$\frac{6a^3x^3-2ax-3\left(a^4x^4-2a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^3*x^3 - 2*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1)))/x^4
```

Sympy [A] time = 1.23657, size = 46, normalized size = 1.1

$$-\frac{a^4\operatorname{atanh}(ax)}{4}+\frac{a^3}{4x}+\frac{a^2\operatorname{atanh}(ax)}{2x^2}-\frac{a}{12x^3}-\frac{\operatorname{atanh}(ax)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**5,x)
```

```
[Out] -a**4*atanh(a*x)/4 + a**3/(4*x) + a**2*atanh(a*x)/(2*x**2) - a/(12*x**3) -
atanh(a*x)/(4*x**4)
```

Giac [B] time = 1.23577, size = 97, normalized size = 2.31

$$-\frac{1}{8}\,a^4\log\left(|ax+1|\right)+\frac{1}{8}\,a^4\log\left(|ax-1|\right)+\frac{3\,a^3x^2-a}{12\,x^3}+\frac{\left(2\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*a^4*log(abs(a*x + 1)) + 1/8*a^4*log(abs(a*x - 1)) + 1/12*(3*a^3*x^2 -
a)/x^3 + 1/8*(2*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))/x^4
```

3.171 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{15x^2}+\frac{1}{15}a^5\log\left(1-a^2x^2\right)+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)-\frac{a}{20x^4}-\frac{\tanh^{-1}(ax)}{5x^5}$$

```
[Out] -a/(20*x^4) + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Rubi [A] time = 0.0876415, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6014, 5916, 266, 44}

$$\frac{a^3}{15x^2}+\frac{1}{15}a^5\log\left(1-a^2x^2\right)+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)-\frac{a}{20x^4}-\frac{\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] -a/(20*x^4) + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)}{x^6}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{5}a\int \frac{1}{x^5(1-a^2x^2)}\,dx-\frac{1}{3}a^3\int \frac{1}{x^3(1-a^2x^2)}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3(1-a^2x)}\,dx,x,x^2\right)-\frac{1}{6}a^3\,\text{Subst}\Big(\dots$$

$$= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}+\frac{1}{10}a\,\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{a^2}{x^2}+\frac{a^4}{x}-\frac{a^6}{-1+a^2x}\right)dx,x,x^2\dots$$

$$= -\frac{a}{20x^4}+\frac{a^3}{15x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)+\frac{1}{15}a^5\log(1-a^2x^2)$$

Mathematica [A] time = 0.0160814, size = 71, normalized size = 1.

$$\frac{a^3}{15x^2}+\frac{1}{15}a^5\log(1-a^2x^2)+\frac{a^2\tanh^{-1}(ax)}{3x^3}-\frac{2}{15}a^5\log(x)-\frac{a}{20x^4}-\frac{\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] -a/(20*x^4) + a^3/(15*x^2) - ArcTanh[a*x]/(5*x^5) + (a^2*ArcTanh[a*x])/(3*x
^3) - (2*a^5*Log[x])/15 + (a^5*Log[1 - a^2*x^2])/15
```

Maple [A] time = 0.039, size = 68, normalized size = 1.

$$-\frac{\text{Artanh}(ax)}{5x^5}+\frac{a^2\text{Artanh}(ax)}{3x^3}+\frac{a^5\ln(ax-1)}{15}-\frac{a}{20x^4}+\frac{a^3}{15x^2}-\frac{2a^5\ln(ax)}{15}+\frac{a^5\ln(ax+1)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)/x^6,x)
```

```
[Out] -1/5*arctanh(a*x)/x^5+1/3*a^2*arctanh(a*x)/x^3+1/15*a^5*ln(a*x-1)-1/20*a/x^
4+1/15*a^3/x^2-2/15*a^5*ln(a*x)+1/15*a^5*ln(a*x+1)
```

Maxima [A] time = 0.956459, size = 84, normalized size = 1.18

$$\frac{1}{60}\left(4a^4\log(a^2x^2-1)-4a^4\log(x^2)+\frac{4a^2x^2-3}{x^4}\right)a+\frac{(5a^2x^2-3)\,\text{artanh}(ax)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 - 3)/x^4)*a + 1/
15*(5*a^2*x^2 - 3)*arctanh(a*x)/x^5
```

Fricas [A] time = 2.24535, size = 167, normalized size = 2.35

$$\frac{4\,a^5x^5\log\left(a^2x^2-1\right)-8\,a^5x^5\log\left(x\right)+4\,a^3x^3-3\,ax+2\left(5\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(4*a^5*x^5*log(a^2*x^2 - 1) - 8*a^5*x^5*log(x) + 4*a^3*x^3 - 3*a*x + 2
*(5*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

Sympy [A] time = 2.90935, size = 75, normalized size = 1.06

$$\begin{cases} -\frac{2a^5\log\left(x\right)}{15}+\frac{2a^5\log\left(x-\frac{1}{a}\right)}{15}+\frac{2a^5\operatorname{atanh}\left(ax\right)}{15}+\frac{a^3}{15x^2}+\frac{a^2\operatorname{atanh}\left(ax\right)}{3x^3}-\frac{a}{20x^4}-\frac{\operatorname{atanh}\left(ax\right)}{5x^5} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)/x**6,x)
```

```
[Out] Piecewise((-2*a**5*log(x)/15 + 2*a**5*log(x - 1/a)/15 + 2*a**5*atanh(a*x)/1
5 + a**3/(15*x**2) + a**2*atanh(a*x)/(3*x**3) - a/(20*x**4) - atanh(a*x)/(5
*x**5), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18245, size = 109, normalized size = 1.54

$$-\frac{1}{15}\,a^5\log\left(x^2\right)+\frac{1}{15}\,a^5\log\left(\left|a^2x^2-1\right|\right)+\frac{6\,a^5x^4+4\,a^3x^2-3\,a}{60\,x^4}+\frac{\left(5\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*a^5*log(x^2) + 1/15*a^5*log(abs(a^2*x^2 - 1)) + 1/60*(6*a^5*x^4 + 4*a
^3*x^2 - 3*a)/x^4 + 1/30*(5*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1))/x^5
```

3.172 $\int x^4 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=162

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a^5}-\frac{2x^3}{315a^2}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{4x}{105a^4}+\frac{2\tanh^{-1}(ax)^2}{35a^5}-\frac{4\tanh^{-1}(ax)}{105a^5}$$

```
[Out] (4*x)/(105*a^4) - (2*x^3)/(315*a^2) - x^5/105 - (4*ArcTanh[a*x])/(105*a^5)
+ (2*x^2*ArcTanh[a*x])/(35*a^3) + (x^4*ArcTanh[a*x])/(35*a) - (a*x^6*ArcTan
h[a*x])/21 + (2*ArcTanh[a*x]^2)/(35*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (a^2*x^
7*ArcTanh[a*x]^2)/7 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(35*a^5) - (2*PolyL
og[2, 1 - 2/(1 - a*x)])/(35*a^5)
```

Rubi [A] time = 0.579493, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6014, 5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a^5}-\frac{2x^3}{315a^2}-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{4x}{105a^4}+\frac{2\tanh^{-1}(ax)^2}{35a^5}-\frac{4\tanh^{-1}(ax)}{105a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (4*x)/(105*a^4) - (2*x^3)/(315*a^2) - x^5/105 - (4*ArcTanh[a*x])/(105*a^5)
+ (2*x^2*ArcTanh[a*x])/(35*a^3) + (x^4*ArcTanh[a*x])/(35*a) - (a*x^6*ArcTan
h[a*x])/21 + (2*ArcTanh[a*x]^2)/(35*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (a^2*x^
7*ArcTanh[a*x]^2)/7 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(35*a^5) - (2*PolyL
og[2, 1 - 2/(1 - a*x)])/(35*a^5)
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\left(a^2\int x^6\tanh^{-1}(ax)^2\,dx\right)+\int x^4\tanh^{-1}(ax)^2\,dx$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2-\frac{1}{5}(2a)\int\frac{x^5\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{7}\left(2a^3\right)\int x\ldots$$

$$=\frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{2\int x^3\tanh^{-1}(ax)\,dx}{5a}-\frac{2\int\frac{x^3\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{5a}\ldots$$

$$=\frac{x^4\tanh^{-1}(ax)}{10a}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\frac{1}{5}x^5\tanh^{-1}(ax)^2-\frac{1}{7}a^2x^7\tanh^{-1}(ax)^2-\frac{1}{10}\ldots$$

$$=\frac{x^2\tanh^{-1}(ax)}{5a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{5a^5}+\frac{1}{5}x^5\tanh^{-1}\ldots$$

$$=\frac{53x}{210a^4}+\frac{11x^3}{630a^2}-\frac{x^5}{105}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\tanh^{-1}(ax)+\ldots$$

$$=\frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{53\tanh^{-1}(ax)}{210a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax\ldots$$

$$=\frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{4\tanh^{-1}(ax)}{105a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\ldots$$

$$=\frac{4x}{105a^4}-\frac{2x^3}{315a^2}-\frac{x^5}{105}-\frac{4\tanh^{-1}(ax)}{105a^5}+\frac{2x^2\tanh^{-1}(ax)}{35a^3}+\frac{x^4\tanh^{-1}(ax)}{35a}-\frac{1}{21}ax^6\ldots$$

Mathematica [A] time = 0.902051, size = 113, normalized size = 0.7

$$-\frac{-18\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+3a^5x^5+2a^3x^3+9\left(5a^7x^7-7a^5x^5+2\right)\tanh^{-1}(ax)^2+3\tanh^{-1}(ax)\left(5a^6x^6-3a^4x\ldots\right.}{315a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -(-12*a*x + 2*a^3*x^3 + 3*a^5*x^5 + 9*(2 - 7*a^5*x^5 + 5*a^7*x^7)*ArcTanh[a
*x]^2 + 3*ArcTanh[a*x]*(4 - 6*a^2*x^2 - 3*a^4*x^4 + 5*a^6*x^6 + 12*Log[1 +
E^(-2*ArcTanh[a*x])]) - 18*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(315*a^5)
```

Maple [A] time = 0.049, size = 225, normalized size = 1.4

$$-\frac{a^2x^7\left(\text{Artanh}\left(ax\right)\right)^2}{7}+\frac{x^5\left(\text{Artanh}\left(ax\right)\right)^2}{5}-\frac{ax^6\text{Artanh}\left(ax\right)}{21}+\frac{x^4\text{Artanh}\left(ax\right)}{35\,a}+\frac{2\,x^2\text{Artanh}\left(ax\right)}{35\,a^3}+\frac{2\,\text{Artanh}\left(ax\ldots\right.}{35\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/7*a^2*x^7*arctanh(a*x)^2+1/5*x^5*arctanh(a*x)^2-1/21*a*x^6*arctanh(a*x)+
1/35*x^4*arctanh(a*x)/a+2/35*x^2*arctanh(a*x)/a^3+2/35/a^5*arctanh(a*x)*ln(
a*x-1)+2/35/a^5*arctanh(a*x)*ln(a*x+1)+1/70/a^5*ln(a*x-1)^2-2/35/a^5*dilog(
1/2+1/2*a*x)-1/35/a^5*ln(a*x-1)*ln(1/2+1/2*a*x)-1/35/a^5*ln(-1/2*a*x+1/2)*l
n(1/2+1/2*a*x)+1/35/a^5*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/70/a^5*ln(a*x+1)^2-1/1
```

```
05*x^5-2/315*x^3/a^2+4/105*x/a^4+2/105/a^5*ln(a*x-1)-2/105/a^5*ln(a*x+1)
```

Maxima [A] time = 0.983634, size = 257, normalized size = 1.59

$$-\frac{1}{630}a^2\left(\frac{6a^5x^5+4a^3x^3-24ax+9\log(ax+1)^2-18\log(ax+1)\log(ax-1)-9\log(ax-1)^2-12\log(ax-1)}{a^7}+\frac{36}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/630*a^2*((6*a^5*x^5 + 4*a^3*x^3 - 24*a*x + 9*log(a*x + 1)^2 - 18*log(a*x
 + 1)*log(a*x - 1) - 9*log(a*x - 1)^2 - 12*log(a*x - 1))/a^7 + 36*(log(a*x
- 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^7 + 12*log(a*x + 1)/a^7)
 - 1/105*a*((5*a^4*x^6 - 3*a^2*x^4 - 6*x^2)/a^4 - 6*log(a*x + 1)/a^6 - 6*lo
g(a*x - 1)/a^6)*arctanh(a*x) - 1/35*(5*a^2*x^7 - 7*x^5)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2x^6-x^4\right)\mathrm{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^6 - x^4)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -x^4\,\mathrm{atanh}^2\left(ax\right)dx-\int a^2x^6\,\mathrm{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(-x**4*atanh(a*x)**2, x) - Integral(a**2*x**6*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2x^2-1\right)x^4\,\mathrm{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x^4*arctanh(a*x)^2, x)
```

3.173 $\int x^3 (1-a^2x^2) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=116

$$-\frac{x^2}{180a^2}+\frac{7\log(1-a^2x^2)}{90a^4}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2+\frac{x\tanh^{-1}(ax)}{6a^3}-\frac{\tanh^{-1}(ax)^2}{12a^4}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(a$$

```
[Out] -x^2/(180*a^2) - x^4/60 + (x*ArcTanh[a*x])/(6*a^3) + (x^3*ArcTanh[a*x])/(18
*a) - (a*x^5*ArcTanh[a*x])/15 - ArcTanh[a*x]^2/(12*a^4) + (x^4*ArcTanh[a*x]
^2)/4 - (a^2*x^6*ArcTanh[a*x]^2)/6 + (7*Log[1 - a^2*x^2])/(90*a^4)
```

Rubi [A] time = 0.439878, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6014, 5916, 5980, 266, 43, 5910, 260, 5948}

$$-\frac{x^2}{180a^2}+\frac{7\log(1-a^2x^2)}{90a^4}-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2+\frac{x\tanh^{-1}(ax)}{6a^3}-\frac{\tanh^{-1}(ax)^2}{12a^4}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -x^2/(180*a^2) - x^4/60 + (x*ArcTanh[a*x])/(6*a^3) + (x^3*ArcTanh[a*x])/(18
*a) - (a*x^5*ArcTanh[a*x])/15 - ArcTanh[a*x]^2/(12*a^4) + (x^4*ArcTanh[a*x]
^2)/4 - (a^2*x^6*ArcTanh[a*x]^2)/6 + (7*Log[1 - a^2*x^2])/(90*a^4)
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\left(a^2\int x^5\tanh^{-1}(ax)^2\,dx\right)+\int x^3\tanh^{-1}(ax)^2\,dx$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\frac{1}{2}a\int\frac{x^4\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{3}a^3\int\frac{x^6\tanh^{-1}(}{1-a^2x^2}$$
$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2+\frac{\int x^2\tanh^{-1}(ax)\,dx}{2a}-\frac{\int\frac{x^2\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{2a}-\frac{1}{3}a$$
$$= \frac{x^3\tanh^{-1}(ax)}{6a}-\frac{1}{15}ax^5\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{6}a^2x^6\tanh^{-1}(ax)^2-\frac{1}{6}\int\frac{}{1}$$
$$= \frac{x\tanh^{-1}(ax)}{2a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{4a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)$$
$$= \frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{1}{4}x^4\tanh^{-1}(ax)$$
$$= \frac{x^2}{20a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{12a^4}+\frac{1}{4}x$$
$$= -\frac{x^2}{180a^2}-\frac{x^4}{60}+\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{15}ax^5\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{12a^4}+$$

Mathematica [A] time = 0.0446643, size = 88, normalized size = 0.76

$$-\frac{3a^4x^4+a^2x^2-14\log\left(1-a^2x^2\right)+2ax\left(6a^4x^4-5a^2x^2-15\right)\tanh^{-1}(ax)+15\left(2a^6x^6-3a^4x^4+1\right)\tanh^{-1}(ax)^2}{180a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -(a^2*x^2 + 3*a^4*x^4 + 2*a*x*(-15 - 5*a^2*x^2 + 6*a^4*x^4)*ArcTanh[a*x] +
15*(1 - 3*a^4*x^4 + 2*a^6*x^6)*ArcTanh[a*x]^2 - 14*Log[1 - a^2*x^2])/(180*a
```

```
^4)
```

Maple [B] time = 0.045, size = 205, normalized size = 1.8

$$-\frac{a^{2}x^{6}\left(\mathrm{Artanh}\left(ax\right)\right)^{2}}{6}+\frac{x^{4}\left(\mathrm{Artanh}\left(ax\right)\right)^{2}}{4}-\frac{a x^{5}\mathrm{Artanh}\left(ax\right)}{15}+\frac{x^{3}\mathrm{Artanh}\left(ax\right)}{18\,a}+\frac{x\mathrm{Artanh}\left(ax\right)}{6\,a^{3}}+\frac{\mathrm{Artanh}\left(ax\right)\ln}{12\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/6*a^2*x^6*arctanh(a*x)^2+1/4*x^4*arctanh(a*x)^2-1/15*a*x^5*arctanh(a*x)+
1/18*x^3*arctanh(a*x)/a+1/6*x*arctanh(a*x)/a^3+1/12/a^4*arctanh(a*x)*ln(a*x
-1)-1/12/a^4*arctanh(a*x)*ln(a*x+1)+1/48/a^4*ln(a*x-1)^2-1/24/a^4*ln(a*x-1)
*ln(1/2+1/2*a*x)+1/24/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/24/a^4*ln(-1/2
*a*x+1/2)*ln(a*x+1)+1/48/a^4*ln(a*x+1)^2-1/60*x^4-1/180*x^2/a^2+7/90/a^4*ln
(a*x-1)+7/90/a^4*ln(a*x+1)
```

Maxima [A] time = 1.01882, size = 197, normalized size = 1.7

$$-\frac{1}{180}\,a\left(\frac{2\left(6\,a^{4}x^{5}-5\,a^{2}x^{3}-15\,x\right)}{a^{4}}+\frac{15\,\log\left(ax+1\right)}{a^{5}}-\frac{15\,\log\left(ax-1\right)}{a^{5}}\right)\mathrm{artanh}\left(ax\right)-\frac{1}{12}\left(2\,a^{2}x^{6}-3\,x^{4}\right)\mathrm{artanh}\left(ax\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/180*a*(2*(6*a^4*x^5 - 5*a^2*x^3 - 15*x)/a^4 + 15*log(a*x + 1)/a^5 - 15*l
og(a*x - 1)/a^5)*arctanh(a*x) - 1/12*(2*a^2*x^6 - 3*x^4)*arctanh(a*x)^2 - 1
/720*(12*a^4*x^4 + 4*a^2*x^2 + 2*(15*log(a*x - 1) - 28)*log(a*x + 1) - 15*l
og(a*x + 1)^2 - 15*log(a*x - 1)^2 - 56*log(a*x - 1))/a^4
```

Fricas [A] time = 2.12239, size = 247, normalized size = 2.13

$$-\frac{12\,a^{4}x^{4}+4\,a^{2}x^{2}+15\left(2\,a^{6}x^{6}-3\,a^{4}x^{4}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}+4\left(6\,a^{5}x^{5}-5\,a^{3}x^{3}-15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-56\,\log\left(a^{2}x^{2}-1\right.}{720\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/720*(12*a^4*x^4 + 4*a^2*x^2 + 15*(2*a^6*x^6 - 3*a^4*x^4 + 1)*log(-(a*x +
 1)/(a*x - 1))^2 + 4*(6*a^5*x^5 - 5*a^3*x^3 - 15*a*x)*log(-(a*x + 1)/(a*x -
 1)) - 56*log(a^2*x^2 - 1))/a^4
```

Sympy [A] time = 3.20107, size = 114, normalized size = 0.98

$$\begin{cases}-\frac{a^{2}x^{6}\operatorname{atanh}^{2}(ax)}{6}-\frac{ax^{5}\operatorname{atanh}(ax)}{15}+\frac{x^{4}\operatorname{atanh}^{2}(ax)}{4}-\frac{x^{4}}{60}+\frac{x^{3}\operatorname{atanh}(ax)}{18a}-\frac{x^{2}}{180a^{2}}+\frac{x\operatorname{atanh}(ax)}{6a^{3}}+\frac{7\log\left(x-\frac{1}{a}\right)}{45a^{4}}-\frac{\operatorname{atanh}^{2}(ax)}{12a^{4}}+\frac{7\operatorname{atanh}}{45a} \\ 0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*x**6*atanh(a*x)**2/6 - a*x**5*atanh(a*x)/15 + x**4*atanh(a
*x)**2/4 - x**4/60 + x**3*atanh(a*x)/(18*a) - x**2/(180*a**2) + x*atanh(a*x
)/(6*a**3) + 7*log(x - 1/a)/(45*a**4) - atanh(a*x)**2/(12*a**4) + 7*atanh(a
*x)/(45*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18136, size = 139, normalized size = 1.2

$$-\frac{1}{60}\,x^4-\frac{1}{48}\left(2\,a^2x^6-3\,x^4+\frac{1}{a^4}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-\frac{1}{180}\left(6\,ax^5-\frac{5\,x^3}{a}-\frac{15\,x}{a^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)-\frac{x^2}{180\,a^2}+\frac{7\,\log\left(a^2x^2-1\right)}{90\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/60*x^4 - 1/48*(2*a^2*x^6 - 3*x^4 + 1/a^4)*log(-(a*x + 1)/(a*x - 1))^2 -
1/180*(6*a*x^5 - 5*x^3/a - 15*x/a^3)*log(-(a*x + 1)/(a*x - 1)) - 1/180*x^2/
a^2 + 7/90*log(a^2*x^2 - 1)/a^4
```

3.174 $\int x^2 \left(1-a^2x^2\right) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=138

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{x}{30a^2}+\frac{2\tanh^{-1}(ax)^2}{15a^3}-\frac{\tanh^{-1}(ax)}{30a^3}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{15a^3}-\cdots$$

```
[Out] x/(30*a^2) - x^3/30 - ArcTanh[a*x]/(30*a^3) + (2*x^2*ArcTanh[a*x])/(15*a) -
 (a*x^4*ArcTanh[a*x])/10 + (2*ArcTanh[a*x]^2)/(15*a^3) + (x^3*ArcTanh[a*x]^
2)/3 - (a^2*x^5*ArcTanh[a*x]^2)/5 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a
^3) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(15*a^3)
```

Rubi [A] time = 0.413488, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6014, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 302}

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a^3}-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{x}{30a^2}+\frac{2\tanh^{-1}(ax)^2}{15a^3}-\frac{\tanh^{-1}(ax)}{30a^3}-\frac{4\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{15a^3}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] x/(30*a^2) - x^3/30 - ArcTanh[a*x]/(30*a^3) + (2*x^2*ArcTanh[a*x])/(15*a) -
 (a*x^4*ArcTanh[a*x])/10 + (2*ArcTanh[a*x]^2)/(15*a^3) + (x^3*ArcTanh[a*x]^
2)/3 - (a^2*x^5*ArcTanh[a*x]^2)/5 - (4*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a
^3) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(15*a^3)
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\left(a^2\int x^4\tanh^{-1}(ax)^2\,dx\right)+\int x^2\tanh^{-1}(ax)^2\,dx$$

$$= \frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2-\frac{1}{3}(2a)\int\frac{x^3\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\frac{1}{5}\left(2a^3\right)\int \frac{x}{}$$

$$= \frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{1}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{2\int x\tanh^{-1}(ax)\,dx}{3a}-\frac{2\int\frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{3a}$$

$$= \frac{x^2\tanh^{-1}(ax)}{3a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{1}{5}a^2x^5\tan$$

$$= \frac{x}{3a^2}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{15a^3}+\frac{1}{3}x^3\tanh^{-1}(ax)^2-$$

$$= \frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{15a^3}$$

$$= \frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{30a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{15a^3}$$

$$= \frac{x}{30a^2}-\frac{x^3}{30}-\frac{\tanh^{-1}(ax)}{30a^3}+\frac{2x^2\tanh^{-1}(ax)}{15a}-\frac{1}{10}ax^4\tanh^{-1}(ax)+\frac{2\tanh^{-1}(ax)^2}{15a^3}$$

Mathematica [A] time = 0.241826, size = 95, normalized size = 0.69

$$-\frac{-4\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+a^3x^3+2\left(3a^5x^5-5a^3x^3+2\right)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\left(3a^4x^4-4a^2x^2+8\log\left(e^{-}\right.\right.}{30a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -(-(a*x) + a^3*x^3 + 2*(2 - 5*a^3*x^3 + 3*a^5*x^5)*ArcTanh[a*x]^2 + ArcTanh
[a*x]*(1 - 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^(-2*ArcTanh[a*x])]) - 4*Poly
Log[2, -E^(-2*ArcTanh[a*x])])/(30*a^3)
```

Maple [A] time = 0.048, size = 205, normalized size = 1.5

$$-\frac{a^2x^5\left(\text{Artanh}\,(ax)\right)^2}{5}+\frac{x^3\left(\text{Artanh}\,(ax)\right)^2}{3}-\frac{ax^4\text{Artanh}\,(ax)}{10}+\frac{2\,x^2\text{Artanh}\,(ax)}{15\,a}+\frac{2\,\text{Artanh}\,(ax)\ln(ax-1)}{15\,a^3}+\frac{2\,\text{A}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/5*a^2*x^5*arctanh(a*x)^2+1/3*x^3*arctanh(a*x)^2-1/10*a*x^4*arctanh(a*x)+
2/15*x^2*arctanh(a*x)/a+2/15/a^3*arctanh(a*x)*ln(a*x-1)+2/15/a^3*arctanh(a*
x)*ln(a*x+1)+1/30/a^3*ln(a*x-1)^2-2/15/a^3*dilog(1/2+1/2*a*x)-1/15/a^3*ln(a
*x-1)*ln(1/2+1/2*a*x)-1/15/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/15/a^3*ln
(-1/2*a*x+1/2)*ln(a*x+1)-1/30/a^3*ln(a*x+1)^2-1/30*x^3+1/30*x/a^2+1/60/a^3*
ln(a*x-1)-1/60/a^3*ln(a*x+1)
```

Maxima [A] time = 1.01269, size = 234, normalized size = 1.7

$$-\frac{1}{60}a^2\left(\frac{2\,a^3x^3-2\,ax+2\,\log\left(ax+1\right)^2-4\,\log\left(ax+1\right)\log\left(ax-1\right)-2\,\log\left(ax-1\right)^2-\log\left(ax-1\right)}{a^5}+\frac{8\left(\log\left(ax-1\right)\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/60*a^2*((2*a^3*x^3 - 2*a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x -
 1) - 2*log(a*x - 1)^2 - log(a*x - 1))/a^5 + 8*(log(a*x - 1)*log(1/2*a*x +
1/2) + dilog(-1/2*a*x + 1/2))/a^5 + log(a*x + 1)/a^5) - 1/30*a*((3*a^2*x^4
- 4*x^2)/a^2 - 4*log(a*x + 1)/a^4 - 4*log(a*x - 1)/a^4)*arctanh(a*x) - 1/15
*(3*a^2*x^5 - 5*x^3)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2x^4-x^2\right)\text{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^4 - x^2)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -x^2\,\text{atanh}^2\left(ax\right)dx-\int a^2x^4\,\text{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(-x**2*atanh(a*x)**2, x) - Integral(a**2*x**4*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2x^2-1\right)x^2\,\text{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x^2*arctanh(a*x)^2, x)
```

3.175 $\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=95

$$\frac{1-a^2x^2}{12a^2}+\frac{\log\left(1-a^2x^2\right)}{6a^2}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2}+\frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a}+\frac{x\tanh^{-1}(ax)}{3a}$$

```
[Out] (1 - a^2*x^2)/(12*a^2) + (x*ArcTanh[a*x])/(3*a) + (x*(1 - a^2*x^2)*ArcTanh[
a*x])/(6*a) - ((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/(4*a^2) + Log[1 - a^2*x^2]/(
6*a^2)
```

Rubi [A] time = 0.0486081, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5994, 5942, 5910, 260}

$$\frac{1-a^2x^2}{12a^2}+\frac{\log\left(1-a^2x^2\right)}{6a^2}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2}+\frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a}+\frac{x\tanh^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (1 - a^2*x^2)/(12*a^2) + (x*ArcTanh[a*x])/(3*a) + (x*(1 - a^2*x^2)*ArcTanh[
a*x])/(6*a) - ((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/(4*a^2) + Log[1 - a^2*x^2]/(
6*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = -\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\int\left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx}{2a}$$
$$= \frac{1-a^2x^2}{12a^2} + \frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a} - \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\int\tanh^{-1}(ax)\,dx}{3a}$$
$$= \frac{1-a^2x^2}{12a^2} + \frac{x\tanh^{-1}(ax)}{3a} + \frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a} - \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2} - \frac{1}{3}\int$$
$$= \frac{1-a^2x^2}{12a^2} + \frac{x\tanh^{-1}(ax)}{3a} + \frac{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{6a} - \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4a^2} + \frac{\log(}{}$$

Mathematica [A] time = 0.0295572, size = 66, normalized size = 0.69

$$\frac{-a^2x^2+2\log\left(1-a^2x^2\right)-3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2+\left(6ax-2a^3x^3\right)\tanh^{-1}(ax)}{12a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] (-(a^2*x^2) + (6*a*x - 2*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTanh
[a*x]^2 + 2*Log[1 - a^2*x^2])/(12*a^2)
```

Maple [B] time = 0.045, size = 185, normalized size = 2.

$$-\frac{a^2\left(\mathrm{Artanh}\left(ax\right)\right)^2x^4}{4}+\frac{\left(\mathrm{Artanh}\left(ax\right)\right)^2x^2}{2}-\frac{a\mathrm{Artanh}\left(ax\right)x^3}{6}+\frac{x\mathrm{Artanh}\left(ax\right)}{2a}+\frac{\mathrm{Artanh}\left(ax\right)\ln\left(ax-1\right)}{4a^2}-\frac{\mathrm{Artanh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/4*a^2*arctanh(a*x)^2*x^4+1/2*arctanh(a*x)^2*x^2-1/6*a*arctanh(a*x)*x^3+1
/2*x*arctanh(a*x)/a+1/4/a^2*arctanh(a*x)*ln(a*x-1)-1/4/a^2*arctanh(a*x)*ln(
a*x+1)+1/16/a^2*ln(a*x-1)^2-1/8/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8/a^2*ln(-1
/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/16/a^2*ln(
a*x+1)^2-1/12*x^2+1/6/a^2*ln(a*x-1)+1/6/a^2*ln(a*x+1)
```

Maxima [A] time = 0.965173, size = 100, normalized size = 1.05

$$-\frac{\left(a^2x^2-1\right)^2\operatorname{artanh}\left(ax\right)^2}{4\,a^2}-\frac{\left(x^2-\frac{2\,\log\left(ax+1\right)}{a^2}-\frac{2\,\log\left(ax-1\right)}{a^2}\right)a+2\left(a^2x^3-3\,x\right)\operatorname{artanh}\left(ax\right)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*x^2 - 1)^2*arctanh(a*x)^2/a^2 - 1/12*((x^2 - 2*log(a*x + 1)/a^2 -
 2*log(a*x - 1)/a^2)*a + 2*(a^2*x^3 - 3*x)*arctanh(a*x))/a
```

Fricas [A] time = 2.19842, size = 203, normalized size = 2.14

$$-\frac{4\,a^2x^2+3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-8\,\log\left(a^2x^2-1\right)}{48\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(4*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
+ 4*(a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 8*log(a^2*x^2 - 1))/a^2
```

Sympy [A] time = 1.82278, size = 88, normalized size = 0.93

$$\begin{cases} -\frac{a^2x^4\operatorname{atanh}^2(ax)}{4}-\frac{ax^3\operatorname{atanh}(ax)}{6}+\frac{x^2\operatorname{atanh}^2(ax)}{2}-\frac{x^2}{12}+\frac{x\operatorname{atanh}(ax)}{2a}+\frac{\log\left(x-\frac{1}{a}\right)}{3a^2}-\frac{\operatorname{atanh}^2(ax)}{4a^2}+\frac{\operatorname{atanh}(ax)}{3a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*x**4*atanh(a*x)**2/4 - a*x**3*atanh(a*x)/6 + x**2*atanh(a*
x)**2/2 - x**2/12 + x*atanh(a*x)/(2*a) + log(x - 1/a)/(3*a**2) - atanh(a*x)
**2/(4*a**2) + atanh(a*x)/(3*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.16943, size = 115, normalized size = 1.21

$$-\frac{1}{16}\left(a^2x^4-2\,x^2+\frac{1}{a^2}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-\frac{1}{12}\,x^2-\frac{1}{12}\left(ax^3-\frac{3\,x}{a}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{\log\left(a^2x^2-1\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*(a^2*x^4 - 2*x^2 + 1/a^2)*log(-(a*x + 1)/(a*x - 1))^2 - 1/12*x^2 - 1/
12*(a*x^3 - 3*x/a)*log(-(a*x + 1)/(a*x - 1)) + 1/6*log(a^2*x^2 - 1)/a^2
```

3.176 $\int (1 - a^2x^2) \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=115

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{2\tanh^{-1}(ax)^2}{3a}-\frac{4}{}$$

```
[Out] -x/3 + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*a) + (2*ArcTanh[a*x]^2)/(3*a) + (2*x
*ArcTanh[a*x]^2)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/3 - (4*ArcTanh[a*x]*L
og[2/(1 - a*x)])/(3*a) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(3*a)
```

Rubi [A] time = 0.10326, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5944, 5910, 5984, 5918, 2402, 2315, 8}

$$-\frac{2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{2\tanh^{-1}(ax)^2}{3a}-\frac{4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -x/3 + ((1 - a^2*x^2)*ArcTanh[a*x])/(3*a) + (2*ArcTanh[a*x]^2)/(3*a) + (2*x
*ArcTanh[a*x]^2)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/3 - (4*ArcTanh[a*x]*L
og[2/(1 - a*x)])/(3*a) - (2*PolyLog[2, 1 - 2/(1 - a*x)])/(3*a)
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

```
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(1-a^2x^2\right)\tanh^{-1}(ax)^2\,dx = \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2-\frac{\int 1\,dx}{3}+\frac{2}{3}\int \tanh^{-1}(ax)^2\,dx$$

$$= -\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2-\frac{1}{3}(4a)$$

$$= -\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x^2\right)\tan$$

$$= -\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x^2\right)\tan$$

$$= -\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x^2\right)\tan$$

$$= -\frac{x}{3}+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{3a}+\frac{2\tanh^{-1}(ax)^2}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^2+\frac{1}{3}x\left(1-a^2x^2\right)\tan$$

Mathematica [A] time = 0.0498392, size = 71, normalized size = 0.62

$$-\frac{-2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(a^2x^2+4\log\left(e^{-2\tanh^{-1}(ax)}+1\right)-1\right)+ax+(ax-1)^2(ax+2)\tanh^{-1}(a}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^2,x]
```

```
[Out] -(a*x + (-1 + a*x)^2*(2 + a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-1 + a^2*x^2
+ 4*Log[1 + E^(-2*ArcTanh[a*x])]) - 2*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(3*
a)
```

Maple [A] time = 0.047, size = 182, normalized size = 1.6

$$-\frac{a^2\left(\text{Artanh}\,(ax)\right)^2x^3}{3}+x\left(\text{Artanh}\,(ax)\right)^2-\frac{a\text{Artanh}\,(ax)\,x^2}{3}+\frac{2\,\text{Artanh}\,(ax)\ln\,(ax-1)}{3\,a}+\frac{2\,\text{Artanh}\,(ax)\ln\,(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2,x)
```

```
[Out] -1/3*a^2*arctanh(a*x)^2*x^3+x*arctanh(a*x)^2-1/3*a*arctanh(a*x)*x^2+2/3/a*a
rctanh(a*x)*ln(a*x-1)+2/3/a*arctanh(a*x)*ln(a*x+1)-1/3*x-1/6/a*ln(a*x-1)+1/
6/a*ln(a*x+1)+1/6/a*ln(a*x-1)^2-2/3/a*dilog(1/2+1/2*a*x)-1/3/a*ln(a*x-1)*ln
(1/2+1/2*a*x)-1/3/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/3/a*ln(-1/2*a*x+1/2)
*ln(a*x+1)-1/6/a*ln(a*x+1)^2
```

Maxima [A] time = 0.98019, size = 194, normalized size = 1.69

$$-\frac{1}{6}a^2\left(\frac{2\,ax+\log\left(ax+1\right)^2-2\,\log\left(ax+1\right)\log\left(ax-1\right)-\log\left(ax-1\right)^2+\log\left(ax-1\right)}{a^3}+\frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{I}\right.}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/6*a^2*((2*a*x + log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x -
 1)^2 + log(a*x - 1))/a^3 + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2
*a*x + 1/2))/a^3 - log(a*x + 1)/a^3) - 1/3*(x^2 - 2*log(a*x + 1)/a^2 - 2*lo
g(a*x - 1)/a^2)*a*arctanh(a*x) - 1/3*(a^2*x^3 - 3*x)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int a^2x^2\,\mathrm{atanh}^2\left(ax\right)dx-\int-\mathrm{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2,x)
```

```
[Out] -Integral(a**2*x**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\left(a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2,x, algorithm="giac")

[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

3.177 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=146

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\tanh^{-1}(ax)\text{PolyLog}\left(2,\right.$$

```
[Out] -(a*x*ArcTanh[a*x]) + ArcTanh[a*x]^2/2 - (a^2*x^2*ArcTanh[a*x]^2)/2 + 2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 - ArcTanh[a*x]*Po
lyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + Pol
yLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rubi [A] time = 0.31188, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6014, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 5910, 260}

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\tanh^{-1}(ax)\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

```
[Out] -(a*x*ArcTanh[a*x]) + ArcTanh[a*x]^2/2 - (a^2*x^2*ArcTanh[a*x]^2)/2 + 2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 - ArcTanh[a*x]*Po
lyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + Pol
yLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x}\,dx = -\left(a^2\int x\tanh^{-1}(ax)^2\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x}\,dx$$

$$= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-(4a)\int \frac{\tanh^{-1}(ax)\tanh}{1-a^2}$$

$$= -\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right)-a\int \tanh^{-1}(ax)\,dx+a$$

$$= -ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1-$$

$$= -ax\tanh^{-1}(ax)+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{2}a^2x^2\tanh^{-1}(ax)^2+2\tanh^{-1}(ax)^2\tanh^{-1}\Big(1-$$

Mathematica [A] time = 0.0437522, size = 145, normalized size = 0.99

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{-ax-1}{ax-1}\right)+\frac{1}{2}\text{PolyLog}\left(3,\frac{ax+1}{ax-1}\right)+\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{-ax-1}{ax-1}\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{ax+1}{ax-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x,x]
```

```
[Out] -(a*x*ArcTanh[a*x]) - ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/2 + 2*ArcTanh[a*x]^2*
ArcTanh[1 - 2/(1 - a*x)] - Log[1 - a^2*x^2]/2 + ArcTanh[a*x]*PolyLog[2, (-1
 - a*x)/(-1 + a*x)] - ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a*x)] - PolyL
og[3, (-1 - a*x)/(-1 + a*x)]/2 + PolyLog[3, (1 + a*x)/(-1 + a*x)]/2
```

Maple [C] time = 0.836, size = 663, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x,x)
```

```
[Out] -1/2*a^2*x^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1
)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arcta
nh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2
+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*polylog(3,-(a*x+1)^
2/(-a^2*x^2+1))+1/2*arctanh(a*x)^2+ln((a*x+1)^2/(-a^2*x^2+1)+1)-1/2*I*Pi*cs
gn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-(a*x+1)*arctanh(a*x)-1/2*I*Pi*cs
gn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1
)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}a^2x^2\log(-ax+1)^2+\frac{1}{4}\int-\frac{\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)^2-\left(a^3x^3+2\left(a^3x^3-a^2x^2-ax+1\right)\log(ax+1)\right)\log(-ax+1)}{ax^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*x^2*log(-a*x + 1)^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1
)*log(a*x + 1)^2 - (a^3*x^3 + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))
*log(-a*x + 1))/(a*x^2 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\operatorname{atanh}^2\left(ax\right)}{x}\,dx - \int a^2x\operatorname{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x, x) - Integral(a**2*x*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x, x)
```

3.178 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=93

$$a\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-a^2x\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)+$$

```
[Out] -(ArcTanh[a*x]^2/x) - a^2*x*ArcTanh[a*x]^2 + 2*a*ArcTanh[a*x]*Log[2/(1 - a*
x)] + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + a*PolyLog[2, 1 - 2/(1 - a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.217911, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6014, 5916, 5988, 5932, 2447, 5910, 5984, 5918, 2402, 2315}

$$a\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-a^2x\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)+$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -(ArcTanh[a*x]^2/x) - a^2*x*ArcTanh[a*x]^2 + 2*a*ArcTanh[a*x]*Log[2/(1 - a*
x)] + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + a*PolyLog[2, 1 - 2/(1 - a*x)]
 - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
```

```
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^2}\,dx = -\left(a^2\int \tanh^{-1}(ax)^2\,dx\right) + \int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + (2a)\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx + (2a^3)\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + (2a)\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx + (2a^2)\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + 2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\tanh^{-1}(ax)\log\left(2 -\right.$$
$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + 2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\tanh^{-1}(ax)\log\left(2 -\right.$$
$$= -\frac{\tanh^{-1}(ax)^2}{x} - a^2x\tanh^{-1}(ax)^2 + 2a\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right) + 2a\tanh^{-1}(ax)\log\left(2 -\right.$$

Mathematica [A] time = 0.132146, size = 102, normalized size = 1.1

$$-a\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + a\left(\tanh^{-1}(ax)\left(-\frac{\tanh^{-1}(ax)}{ax} + \tanh^{-1}(ax) + 2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right) - \text{PolyLog}\left(2,e\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -(a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcTan
h[a*x])])) - a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + a*(ArcTanh[a*x]*(ArcTanh[
a*x] - ArcTanh[a*x]/(a*x) + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2, E^
(-2*ArcTanh[a*x])])
```

Maple [A] time = 0.055, size = 170, normalized size = 1.8

$$-a^2x\left(\text{Artanh}\left(ax\right)\right)^2 - \frac{\left(\text{Artanh}\left(ax\right)\right)^2}{x} - 2\,a\text{Artanh}\left(ax\right)\ln\left(ax-1\right) + 2\,a\text{Artanh}\left(ax\right)\ln\left(ax\right) - 2\,a\text{Artanh}\left(ax\right)\ln\left(ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x)
```

```
[Out] -a^2*x*arctanh(a*x)^2-arctanh(a*x)^2/x-2*a*arctanh(a*x)*ln(a*x-1)+2*a*arcta
nh(a*x)*ln(a*x)-2*a*arctanh(a*x)*ln(a*x+1)-a*dilog(a*x)-a*dilog(a*x+1)-a*ln
(a*x)*ln(a*x+1)-1/2*a*ln(a*x-1)^2+2*a*dilog(1/2+1/2*a*x)+a*ln(a*x-1)*ln(1/2
+1/2*a*x)-a*ln(-1/2*a*x+1/2)*ln(a*x+1)+a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1
/2*a*ln(a*x+1)^2
```

Maxima [A] time = 0.985686, size = 205, normalized size = 2.2

$$\frac{1}{2}a^2\left(\frac{\log\left(ax+1\right)^2 - 2\,\log\left(ax+1\right)\log\left(ax-1\right) - \log\left(ax-1\right)^2}{a} + \frac{4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right) + \text{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a} - \frac{2\,(\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a
+ 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 2*(log(a*
x + 1)*log(x) + dilog(-a*x))/a + 2*(log(-a*x + 1)*log(x) + dilog(a*x))/a) -
 2*a*(log(a*x + 1) + log(a*x - 1) - log(x))*arctanh(a*x) - (a^2*x + 1/x)*ar
ctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int a^2\,\text{atanh}^2\left(ax\right)dx-\int-\frac{\text{atanh}^2\left(ax\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**2,x)
```

```
[Out] -Integral(a**2*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(a^2x^2-1\right)\text{artanh}\left(ax\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^2, x)
```

3.179 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=172

$$-\frac{1}{2}a^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)+\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)+a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a^2\tanh^{-1}(ax)\text{P}$$

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) - 2
*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^
2*x^2])/2 + a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - a^2*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 - a*x)] - (a^2*PolyLog[3, 1 - 2/(1 - a*x)])/2 + (a^2*
PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rubi [A] time = 0.334493, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {6014, 5916, 5982, 266, 36, 29, 31, 5948, 5914, 6052, 6058, 6610}

$$-\frac{1}{2}a^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)+\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)+a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a^2\tanh^{-1}(ax)\text{P}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) - 2
*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^
2*x^2])/2 + a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - a^2*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 - a*x)] - (a^2*PolyLog[3, 1 - 2/(1 - a*x)])/2 + (a^2*
PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^3}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\right) + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx + (4a^3)\int$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a\int \frac{\tanh^{-1}(ax)}{x^2}\,dx + a^3\int \frac{\tan}{1}$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} - 2a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right.$$

Mathematica [A] time = 0.0735436, size = 174, normalized size = 1.01

$$\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{-ax-1}{ax-1}\right) - \frac{1}{2}a^2\text{PolyLog}\left(3,\frac{ax+1}{ax-1}\right) - a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{-ax-1}{ax-1}\right) + a^2\tanh^{-1}(ax)\text{PolyLog}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) - 2*a^2*Arc
Tanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/
2 - a^2*ArcTanh[a*x]*PolyLog[2, (-1 - a*x)/(-1 + a*x)] + a^2*ArcTanh[a*x]*P
olyLog[2, (1 + a*x)/(-1 + a*x)] + (a^2*PolyLog[3, (-1 - a*x)/(-1 + a*x)])/2
 - (a^2*PolyLog[3, (1 + a*x)/(-1 + a*x)])/2
```

Maple [C] time = 1.208, size = 741, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x)
```

```
[Out] -a^2*arctanh(a*x)^2*ln(a*x)-1/2*arctanh(a*x)^2/x^2+1/2*a^2*arctanh(a*x)^2+a
^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^
2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2+a^2*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)-1)-a^2*arctanh(a*x)-a*arctanh(a*x)/x-1/2*I*a^2*Pi*csgn(I*((a*x+1)^2/(-
a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+1/2*I*a^2*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+1/2*I*a^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^
2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*ar
ctanh(a*x)^2+a^2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-a^2*arctanh(a*
```

```
x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*arctanh(a*x)*polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))+2*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh
(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*arctanh(a*x)*polylog(2,-(a*x
+1)/(-a^2*x^2+1)^(1/2))+2*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ar
ctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2*a^2*polylog(3,-(a*x+1)^2/
(-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(-ax+1\right)^{2}}{8\,x^{2}}+\frac{1}{4}\int-\frac{\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\log\left(ax+1\right)^{2}-\left(ax+2\left(a^{3}x^{3}-a^{2}x^{2}-ax+1\right)\log\left(ax+1\right)\right)\log\left(-ax\right.}{ax^{4}-x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*log(-a*x + 1)^2/x^2 + 1/4*integrate(-((a^3*x^3 - a^2*x^2 - a*x + 1)*lo
g(a*x + 1)^2 - (a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*log(a*x + 1))*log(-a*
x + 1))/(a*x^4 - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^{2}x^{2}-1\right)\operatorname{artanh}\left(ax\right)^{2}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int-\frac{\operatorname{atanh}^{2}\left(ax\right)}{x^{3}}\,dx-\int\frac{a^{2}\operatorname{atanh}^{2}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**3,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**3, x) - Integral(a**2*atanh(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(a^{2}x^{2}-1\right)\operatorname{artanh}\left(ax\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^3, x)
```

3.180 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=116

$$\frac{2}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{4}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\text{ta}$$

```
[Out] -a^2/(3*x) + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (a^2*ArcTanh[a*x]^2)/x - (4*a^3*Arc
Tanh[a*x]*Log[2 - 2/(1 + a*x)])/3 + (2*a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rubi [A] time = 0.307588, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6014, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$\frac{2}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-1}(ax)+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{4}{3}a^3\log\left(2-\frac{2}{ax+1}\right)\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -a^2/(3*x) + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (a^2*ArcTanh[a*x]^2)/x - (4*a^3*Arc
Tanh[a*x]*Log[2 - 2/(1 + a*x)])/3 + (2*a^3*PolyLog[2, -1 + 2/(1 + a*x)])/3
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^4}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\\ &= -\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}+\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx-\left(2a^3\right)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\ &= -a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}+\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3}\,dx+\frac{1}{3}\left(2a^3\right)\\ &= -\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}-2a^3\tanh^{-1}(ax)\,\mathrm{lo}\\ &= -\frac{a^2}{3x}-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(ax)^2}{x}-\frac{4}{3}a^3\tanh^{-1}\\ &= -\frac{a^2}{3x}+\frac{1}{3}a^3\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{3x^2}-\frac{2}{3}a^3\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{3x^3}+\frac{a^2\tanh^{-1}(}{x}\end{aligned}$$

Mathematica [A] time = 0.261702, size = 93, normalized size = 0.8

$$\frac{2a^3x^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-a^2x^2+\tanh^{-1}(ax)\left(a^3x^3-4a^3x^3\log\left(1-e^{-2\tanh^{-1}(ax)}\right)-ax\right)-(ax-1)^2(2ax+1)\tan}{3x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] (-(a^2*x^2) - (-1 + a*x)^2*(1 + 2*a*x)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(-(a*x
) + a^3*x^3 - 4*a^3*x^3*Log[1 - E^(-2*ArcTanh[a*x])]) + 2*a^3*x^3*PolyLog[2
, E^(-2*ArcTanh[a*x])])/(3*x^3)
```

Maple [B] time = 0.06, size = 237, normalized size = 2.

$$\frac{a^2 (\mathrm{Artanh}(ax))^2}{x} - \frac{(\mathrm{Artanh}(ax))^2}{3x^3} + \frac{2a^3 \mathrm{Artanh}(ax)\ln(ax-1)}{3} - \frac{a\mathrm{Artanh}(ax)}{3x^2} - \frac{4a^3 \mathrm{Artanh}(ax)\ln(ax)}{3} + \frac{2a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x)
```

```
[Out] a^2*arctanh(a*x)^2/x-1/3*arctanh(a*x)^2/x^3+2/3*a^3*arctanh(a*x)*ln(a*x-1)-
1/3*a*arctanh(a*x)/x^2-4/3*a^3*arctanh(a*x)*ln(a*x)+2/3*a^3*arctanh(a*x)*ln
(a*x+1)-1/3*a^2/x-1/6*a^3*ln(a*x-1)+1/6*a^3*ln(a*x+1)+2/3*a^3*dilog(a*x)+2/
3*a^3*dilog(a*x+1)+2/3*a^3*ln(a*x)*ln(a*x+1)+1/6*a^3*ln(a*x-1)^2-2/3*a^3*di
log(1/2+1/2*a*x)-1/3*a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-1/3*a^3*ln(-1/2*a*x+1/2)
*ln(1/2+1/2*a*x)+1/3*a^3*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/6*a^3*ln(a*x+1)^2
```

Maxima [A] time = 0.985423, size = 254, normalized size = 2.19

$$-\frac{1}{6}\left(4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a - 4(\log(ax+1)\log(x) + \mathrm{Li}_2(-ax))a + 4(\log(-ax+1)\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(lo
g(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*
a - a*log(a*x + 1) + a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*x +
 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 + 2)/x)*a^2 + 1/3*(2*a^2*log(a^2*x^2
- 1) - 2*a^2*log(x^2) - 1/x^2)*a*arctanh(a*x) + 1/3*(3*a^2*x^2 - 1)*arctanh
(a*x)^2/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{(a^2x^2-1)\,\mathrm{artanh}(ax)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\operatorname{atanh}^2(ax)}{x^4}\,dx - \int \frac{a^2\operatorname{atanh}^2(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**4,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**4, x) - Integral(a**2*atanh(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^4, x)
```

3.181 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^5}\,dx$

Optimal. Leaf size=89

$$-\frac{a^2}{12x^2}+\frac{1}{6}a^4\log\left(1-a^2x^2\right)-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{a\tanh^{-1}(ax)}{6x^3}$$

```
[Out] -a^2/(12*x^2) - (a*ArcTanh[a*x])/(6*x^3) + (a^3*ArcTanh[a*x])/(2*x) - ((1 -
 a^2*x^2)^2*ArcTanh[a*x]^2)/(4*x^4) - (a^4*Log[x])/3 + (a^4*Log[1 - a^2*x^2
])/6
```

Rubi [A] time = 0.107831, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6008, 6014, 5916, 266, 44, 36, 29, 31}

$$-\frac{a^2}{12x^2}+\frac{1}{6}a^4\log\left(1-a^2x^2\right)-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{a\tanh^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```


```
[Out] -a^2/(12*x^2) - (a*ArcTanh[a*x])/(6*x^3) + (a^3*ArcTanh[a*x])/(2*x) - ((1 -
 a^2*x^2)^2*ArcTanh[a*x]^2)/(4*x^4) - (a^4*Log[x])/3 + (a^4*Log[1 - a^2*x^2
])/6
```


Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^5}\,dx &= -\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)}{x^4}\,dx\\
&= -\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{2}a\int\frac{\tanh^{-1}(ax)}{x^4}\,dx-\frac{1}{2}a^3\int\frac{\tanh^{-1}(ax)}{x^2}\,dx\\
&= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{6}a^2\int\frac{1}{x^3\left(1-a^2x^2\right)}\,dx-\\
&= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\,\text{Subst}\left(\int\frac{1}{x^2\left(1-a^2\right.}\right.\\
&= -\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}+\frac{1}{12}a^2\,\text{Subst}\left(\int\left(\frac{1}{x^2}+\frac{a^2}{x}\right.\right.\\
&= -\frac{a^2}{12x^2}-\frac{a\tanh^{-1}(ax)}{6x^3}+\frac{a^3\tanh^{-1}(ax)}{2x}-\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{4x^4}-\frac{1}{3}a^4\log(x)+\frac{1}{6}a^4\,\mathrm{l}
\end{aligned}$$

Mathematica [A] time = 0.0326074, size = 82, normalized size = 0.92

$$\frac{-a^2x^2-4a^4x^4\log(x)+2a^4x^4\log\left(1-a^2x^2\right)-3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2+\left(6a^3x^3-2ax\right)\tanh^{-1}(ax)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] (-(a^2*x^2) + (-2*a*x + 6*a^3*x^3)*ArcTanh[a*x] - 3*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^2 - 4*a^4*x^4*Log[x] + 2*a^4*x^4*Log[1 - a^2*x^2])/(12*x^4)
```

Maple [B] time = 0.06, size = 199, normalized size = 2.2

$$-\frac{(\mathrm{Artanh}(ax))^2}{4x^4}+\frac{a^2(\mathrm{Artanh}(ax))^2}{2x^2}+\frac{a^4\mathrm{Artanh}(ax)\ln(ax-1)}{4}+\frac{a^3\mathrm{Artanh}(ax)}{2x}-\frac{a\mathrm{Artanh}(ax)}{6x^3}-\frac{a^4\mathrm{Artanh}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x)
```

```
[Out] -1/4*arctanh(a*x)^2/x^4+1/2*a^2*arctanh(a*x)^2/x^2+1/4*a^4*arctanh(a*x)*ln(
a*x-1)+1/2*a^3*arctanh(a*x)/x-1/6*a*arctanh(a*x)/x^3-1/4*a^4*arctanh(a*x)*l
n(a*x+1)+1/16*a^4*ln(a*x-1)^2-1/8*a^4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8*a^4*ln(
-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8*a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/16*a^4*l
n(a*x+1)^2+1/6*a^4*ln(a*x-1)-1/12*a^2/x^2-1/3*a^4*ln(a*x)+1/6*a^4*ln(a*x+1)
```

Maxima [B] time = 0.967811, size = 221, normalized size = 2.48

$$-\frac{1}{48}\left(16a^2\log(x)-\frac{3a^2x^2\log(ax+1)^2+3a^2x^2\log(ax-1)^2+8a^2x^2\log(ax-1)-2\left(3a^2x^2\log(ax-1)-4a^2x^2\right)\mathrm{l}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/48*(16*a^2*log(x) - (3*a^2*x^2*log(a*x + 1)^2 + 3*a^2*x^2*log(a*x - 1)^2
 + 8*a^2*x^2*log(a*x - 1) - 2*(3*a^2*x^2*log(a*x - 1) - 4*a^2*x^2)*log(a*x
+ 1) - 4)/x^2)*a^2 - 1/12*(3*a^3*log(a*x + 1) - 3*a^3*log(a*x - 1) - 2*(3*a
^2*x^2 - 1)/x^3)*a*arctanh(a*x) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/x^4
```

Fricas [A] time = 2.51928, size = 239, normalized size = 2.69

$$\frac{8a^4x^4\log\left(a^2x^2-1\right)-16a^4x^4\log(x)-4a^2x^2-3\left(a^4x^4-2a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(3a^3x^3-ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/48*(8*a^4*x^4*log(a^2*x^2 - 1) - 16*a^4*x^4*log(x) - 4*a^2*x^2 - 3*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^3*x^3 - a*x)*log(-
(a*x + 1)/(a*x - 1)))/x^4
```

Sympy [A] time = 2.46072, size = 102, normalized size = 1.15

$$\begin{cases}-\frac{a^4\log(x)}{3}+\frac{a^4\log\left(x-\frac{1}{a}\right)}{3}-\frac{a^4\operatorname{atanh}^2(ax)}{4}+\frac{a^4\operatorname{atanh}(ax)}{3}+\frac{a^3\operatorname{atanh}(ax)}{2x}+\frac{a^2\operatorname{atanh}^2(ax)}{2x^2}-\frac{a^2}{12x^2}-\frac{a\operatorname{atanh}(ax)}{6x^3}-\frac{\operatorname{atanh}^2(ax)}{4x^4} & \text{for} \\ 0 & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**5,x)
```

```
[Out] Piecewise((-a**4*log(x)/3 + a**4*log(x - 1/a)/3 - a**4*atanh(a*x)**2/4 + a*
*4*atanh(a*x)/3 + a**3*atanh(a*x)/(2*x) + a**2*atanh(a*x)**2/(2*x**2) - a**
2/(12*x**2) - a*atanh(a*x)/(6*x**3) - atanh(a*x)**2/(4*x**4), Ne(a, 0)), (0
, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^5, x)
```

3.182 $\int \frac{(1-a^2x^2)\tanh^{-1}(ax)^2}{x^6}\,dx$

Optimal. Leaf size=143

$$\frac{2}{15}a^5\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{30x^3}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{a^4}{30x}-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{1}{30}a^5\tanh^{-1}$$

```
[Out] -a^2/(30*x^3) + a^4/(30*x) - (a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x])/(10*x
^4) + (2*a^3*ArcTanh[a*x])/(15*x^2) - (2*a^5*ArcTanh[a*x]^2)/15 - ArcTanh[a
*x]^2/(5*x^5) + (a^2*ArcTanh[a*x]^2)/(3*x^3) - (4*a^5*ArcTanh[a*x]*Log[2 -
2/(1 + a*x)])/15 + (2*a^5*PolyLog[2, -1 + 2/(1 + a*x)])/15
```

Rubi [A] time = 0.445357, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6014, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$\frac{2}{15}a^5\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{30x^3}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{a^4}{30x}-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{1}{30}a^5\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```


```
[Out] -a^2/(30*x^3) + a^4/(30*x) - (a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x])/(10*x
^4) + (2*a^3*ArcTanh[a*x])/(15*x^2) - (2*a^5*ArcTanh[a*x]^2)/15 - ArcTanh[a
*x]^2/(5*x^5) + (a^2*ArcTanh[a*x]^2)/(3*x^3) - (4*a^5*ArcTanh[a*x]*Log[2 -
2/(1 + a*x)])/15 + (2*a^5*PolyLog[2, -1 + 2/(1 + a*x)])/15
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{x^6}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5\left(1-a^2x^2\right)}\,dx-\frac{1}{3}\left(2a^3\right)\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5}\,dx+\frac{1}{5}\left(2a^3\right)\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,d$$

$$= -\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{a^3\tanh^{-1}(ax)}{3x^2}-\frac{1}{3}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2\tanh^{-1}(ax)^2}{3x^3}+$$

$$= -\frac{a^2}{30x^3}+\frac{a^4}{3x}-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{5x^5}+\frac{a^2}{}$$

$$= -\frac{a^2}{30x^3}+\frac{a^4}{30x}-\frac{1}{3}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh^{-1}(ax)^2$$

$$= -\frac{a^2}{30x^3}+\frac{a^4}{30x}-\frac{1}{30}a^5\tanh^{-1}(ax)-\frac{a\tanh^{-1}(ax)}{10x^4}+\frac{2a^3\tanh^{-1}(ax)}{15x^2}-\frac{2}{15}a^5\tanh^{-1}(ax)^2$$

Mathematica [A] time = 0.430777, size = 114, normalized size = 0.8

$$\frac{4a^5x^5\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)+a^2x^2\left(a^2x^2-1\right)-2\left(2a^5x^5-5a^2x^2+3\right)\tanh^{-1}(ax)^2-ax\tanh^{-1}(ax)\left(a^4x^4-4a^2x^2+\right.}{30x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] (a^2*x^2*(-1 + a^2*x^2) - 2*(3 - 5*a^2*x^2 + 2*a^5*x^5)*ArcTanh[a*x]^2 - a*
x*ArcTanh[a*x]*(3 - 4*a^2*x^2 + a^4*x^4 + 8*a^4*x^4*Log[1 - E^(-2*ArcTanh[a
*x])]) + 4*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*x^5)
```

Maple [B] time = 0.062, size = 258, normalized size = 1.8

$$-\frac{(\text{Artanh}(ax))^2}{5x^5}+\frac{a^2(\text{Artanh}(ax))^2}{3x^3}+\frac{2a^5\text{Artanh}(ax)\ln(ax-1)}{15}-\frac{a\text{Artanh}(ax)}{10x^4}+\frac{2a^3\text{Artanh}(ax)}{15x^2}-\frac{4a^5\text{Arta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x)
```

```
[Out] -1/5*arctanh(a*x)^2/x^5+1/3*a^2*arctanh(a*x)^2/x^3+2/15*a^5*arctanh(a*x)*ln
(a*x-1)-1/10*a*arctanh(a*x)/x^4+2/15*a^3*arctanh(a*x)/x^2-4/15*a^5*arctanh(
a*x)*ln(a*x)+2/15*a^5*arctanh(a*x)*ln(a*x+1)+1/60*a^5*ln(a*x-1)-1/30*a^2/x^
3+1/30*a^4/x-1/60*a^5*ln(a*x+1)+2/15*a^5*dilog(a*x)+2/15*a^5*dilog(a*x+1)+2
/15*a^5*ln(a*x)*ln(a*x+1)+1/30*a^5*ln(a*x-1)^2-2/15*a^5*dilog(1/2+1/2*a*x)-
1/15*a^5*ln(a*x-1)*ln(1/2+1/2*a*x)+1/15*a^5*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/15
*a^5*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/30*a^5*ln(a*x+1)^2
```

Maxima [A] time = 0.99753, size = 308, normalized size = 2.15

$$-\frac{1}{60}\left(8\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^3-8\left(\log(ax+1)\log(x)+\text{Li}_2(-ax)\right)a^3+8\left(\log(-ax+1)\text{l}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 8*
(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 8*(log(-a*x + 1)*log(x) + dilog(a
*x))*a^3 + a^3*log(a*x + 1) - a^3*log(a*x - 1) + 2*(a^3*x^3*log(a*x + 1)^2
- 2*a^3*x^3*log(a*x + 1)*log(a*x - 1) - a^3*x^3*log(a*x - 1)^2 - a^2*x^2 +
1)/x^3)*a^2 + 1/30*(4*a^4*log(a^2*x^2 - 1) - 4*a^4*log(x^2) + (4*a^2*x^2 -
3)/x^4)*a*arctanh(a*x) + 1/15*(5*a^2*x^2 - 3)*arctanh(a*x)^2/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\text{artanh}(ax)^2}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\operatorname{atanh}^2(ax)}{x^6}\,dx - \int \frac{a^2\operatorname{atanh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**2/x**6,x)
```

```
[Out] -Integral(-atanh(a*x)**2/x**6, x) - Integral(a**2*atanh(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(a^2x^2-1)\operatorname{artanh}(ax)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^2/x^6, x)
```

3.183 $\int \left(1-a^2x^2\right) \tanh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=157

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a}-\frac{2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3+\frac{\left(1-a^2\right.}{}$$

```
[Out] -(x*ArcTanh[a*x]) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(2*a) + (2*ArcTanh[a*x]^
3)/(3*a) + (2*x*ArcTanh[a*x]^3)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^3)/3 - (2
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - Log[1 - a^2*x^2]/(2*a) - (2*ArcTanh[a
*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/a
```

Rubi [A] time = 0.192105, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {5944, 5910, 5984, 5918, 5948, 6058, 6610, 260}

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{a}-\frac{2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3+\frac{\left(1-a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

```
[Out] -(x*ArcTanh[a*x]) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(2*a) + (2*ArcTanh[a*x]^
3)/(3*a) + (2*x*ArcTanh[a*x]^3)/3 + (x*(1 - a^2*x^2)*ArcTanh[a*x]^3)/3 - (2
*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a - Log[1 - a^2*x^2]/(2*a) - (2*ArcTanh[a
*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a + PolyLog[3, 1 - 2/(1 - a*x)]/a
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
```

```
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \left(1-a^2x^2\right)\tanh^{-1}(ax)^3\,dx &= \frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3+\frac{2}{3}\int \tanh^{-1}(ax)^3\,dx-\int \tanh^{-1}\\
&= -x\tanh^{-1}(ax)+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{1}{3}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3\\
&= -x\tanh^{-1}(ax)+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{2\tanh^{-1}(ax)^3}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{1}{3}x\left(1-a\right.\\
&= -x\tanh^{-1}(ax)+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{2\tanh^{-1}(ax)^3}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{1}{3}x\left(1-a\right.\\
&= -x\tanh^{-1}(ax)+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{2\tanh^{-1}(ax)^3}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{1}{3}x\left(1-a\right.\\
&= -x\tanh^{-1}(ax)+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a}+\frac{2\tanh^{-1}(ax)^3}{3a}+\frac{2}{3}x\tanh^{-1}(ax)^3+\frac{1}{3}x\left(1-a\right.
\end{aligned}$$

Mathematica [A] time = 0.293761, size = 134, normalized size = 0.85

$$-\frac{-12\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\log\left(1-a^2x^2\right)+2a^3x^3\tanh^{-1}(ax)^3+3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)*ArcTanh[a*x]^3,x]
```

```
[Out] -(6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 + 4*ArcT
anh[a*x]^3 - 6*a*x*ArcTanh[a*x]^3 + 2*a^3*x^3*ArcTanh[a*x]^3 + 12*ArcTanh[a
```

```
*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + 3*Log[1 - a^2*x^2] - 12*ArcTanh[a*x]*P
olyLog[2, -E^(-2*ArcTanh[a*x])] - 6*PolyLog[3, -E^(-2*ArcTanh[a*x])])/(6*a)
```

Maple [C] time = 0.551, size = 829, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)*arctanh(a*x)^3,x)
```

```
[Out] -1/3*a^2*arctanh(a*x)^3*x^3+x*arctanh(a*x)^3-1/2*a*arctanh(a*x)^2*x^2+1/a*a
rctanh(a*x)^2*ln(a*x-1)+1/a*arctanh(a*x)^2*ln(a*x+1)-2/a*arctanh(a*x)^2*ln(
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^
2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi-1/2*I/a*arctanh(a*x)^2*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi-1/2*I/a*arctanh(a*x
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))*Pi-I/a*arctanh(a*x)^2*Pi+1/2*I/a*arctanh(a*x)^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*Pi-I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*Pi+1/2*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+
1)^2/(-a^2*x^2+1)+1))*Pi-1/2*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1
))^3*Pi+I/a*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi-I/a*arct
anh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi+2/3*arctanh(a*x)^3/a-2/a
*arctanh(a*x)^2*ln(2)-x*arctanh(a*x)+1/2*arctanh(a*x)^2/a-arctanh(a*x)/a+1/
a*ln((a*x+1)^2/(-a^2*x^2+1)+1)-2/a*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*
x^2+1))+1/a*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2\,a^3x^3-3\,a^2x^2-12\,ax-6\left(a^3x^3-3\,ax-2\right)\log\left(ax+1\right)\right)\log\left(-ax+1\right)^2}{48\,a}-\frac{\left(\log\left(-ax+1\right)^3-3\,\log\left(-ax+1\right)^2+6\,\log\right.}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(2*a^3*x^3 - 3*a^2*x^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*x + 1)
)*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x
 + 1) - 6)*(a*x - 1)/a + 1/864*(4*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2 +
6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^
2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 - 3*log(-a*x +
1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/3*(3*(a^3*x^3 -
 a^2*x^2 - a*x + 1)*log(a*x + 1)^3 + (2*a^3*x^3 - 3*a^2*x^2 - 9*(a^3*x^3 -
a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 12*a*x - 6*(a^3*x^3 - 3*a*x - 2)*log(a*
x + 1))*log(-a*x + 1))/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2x^2-1\right)\mathrm{artanh}\left(ax\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int a^2x^2 \operatorname{atanh}^3(ax)\,dx - \int -\operatorname{atanh}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)*atanh(a*x)**3,x)
```

```
[Out] -Integral(a**2*x**2*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2x^2 - 1\right)\operatorname{artanh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

3.184 $\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx$

Optimal. Leaf size=193

$$-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)+\frac{1}{4}\text{PolyLog}\left(2,1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)+\log$$

```
[Out] ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Log
[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/2 - (ArcTanh[x/Sqrt[2]]*Log[(
4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/2 - PolyLog[2, 1 - (2*Sqrt[2])/(
Sqrt[2] + x)]/2 + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]
/4 + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/4
```

Rubi [A] time = 0.226503, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5992, 5920, 2402, 2315, 2447}

$$-\frac{1}{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(x+\sqrt{2}\right)}+1\right)+\frac{1}{4}\text{PolyLog}\left(2,1-\frac{4(x+1)}{\left(2+\sqrt{2}\right)\left(x+\sqrt{2}\right)}\right)+\log$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```


```
[Out] ArcTanh[x/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + x)] - (ArcTanh[x/Sqrt[2]]*Log
[(-4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))])/2 - (ArcTanh[x/Sqrt[2]]*Log[(
4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))])/2 - PolyLog[2, 1 - (2*Sqrt[2])/(
Sqrt[2] + x)]/2 + PolyLog[2, 1 + (4*(1 - x))/((2 - Sqrt[2])*(Sqrt[2] + x))]
/4 + PolyLog[2, 1 - (4*(1 + x))/((2 + Sqrt[2])*(Sqrt[2] + x))]/4
```


Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{x\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1-x^2}\,dx = \int \left(-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(-1+x)} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2(1+x)}\right)dx$$

$$= -\left(\frac{1}{2}\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{-1+x}\,dx\right) - \frac{1}{2}\int \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{1+x}\,dx$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\Big($$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\Big($$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(\frac{2\sqrt{2}}{\sqrt{2}+x}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\left(-\frac{4(1-x)}{\left(2-\sqrt{2}\right)\left(\sqrt{2}+x\right)}\right) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)\log\Big($$

Mathematica [A] time = 0.247399, size = 232, normalized size = 1.2

$$\frac{1}{4}\left(-2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)+\text{PolyLog}\left(2,\left(3-2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)+\text{PolyLog}\left(2,\left(3+2\sqrt{2}\right)e^{-2\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}\right)+4\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[x/Sqrt[2]])/(1 - x^2),x]
```

```
[Out] (-4*ArcSinh[1]*ArcTanh[x] + 4*ArcTanh[x/Sqrt[2]]*Log[1 + E^(-2*ArcTanh[x/Sq
rt[2]])] + 2*ArcSinh[1]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])]
- 2*ArcTanh[x/Sqrt[2]]*Log[1 + (-3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] -
 2*ArcSinh[1]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*ArcTanh
[x/Sqrt[2]]*Log[1 - (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])] - 2*PolyLog[2
, -E^(-2*ArcTanh[x/Sqrt[2]])] + PolyLog[2, (3 - 2*Sqrt[2])/E^(2*ArcTanh[x/S
qrt[2]])] + PolyLog[2, (3 + 2*Sqrt[2])/E^(2*ArcTanh[x/Sqrt[2]])])/4
```

Maple [A] time = 0.049, size = 251, normalized size = 1.3

$$-\frac{\ln\left(x^2-1\right)}{2}\text{Artanh}\left(\frac{x\sqrt{2}}{2}\right)-\frac{\ln\left(x^2-1\right)}{4}\ln\left(\frac{x\sqrt{2}}{2}-1\right)+\frac{1}{4}\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}-x\sqrt{2}}{-2+\sqrt{2}}\right)+\frac{1}{4}\ln\left(\frac{x\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}+x\sqrt{}}{2+\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x)
```

```
[Out] -1/2*ln(x^2-1)*arctanh(1/2*x*2^(1/2))-1/4*ln(1/2*x*2^(1/2)-1)*ln(x^2-1)+1/4
*ln(1/2*x*2^(1/2)-1)*ln((2^(1/2)-x*2^(1/2))/(-2+2^(1/2)))+1/4*ln(1/2*x*2^(1
/2)-1)*ln((2^(1/2)+x*2^(1/2))/(2+2^(1/2)))+1/4*dilog((2^(1/2)-x*2^(1/2))/(-
2+2^(1/2)))+1/4*dilog((2^(1/2)+x*2^(1/2))/(2+2^(1/2)))+1/4*ln(1/2*x*2^(1/2)
+1)*ln(x^2-1)-1/4*ln(1/2*x*2^(1/2)+1)*ln((2^(1/2)-x*2^(1/2))/(2+2^(1/2)))-1
/4*ln(1/2*x*2^(1/2)+1)*ln((2^(1/2)+x*2^(1/2))/(-2+2^(1/2)))-1/4*dilog((2^(1
/2)-x*2^(1/2))/(2+2^(1/2)))-1/4*dilog((2^(1/2)+x*2^(1/2))/(-2+2^(1/2)))
```

Maxima [A] time = 1.46133, size = 374, normalized size = 1.94

$$-\frac{1}{2}\,\operatorname{artanh}\left(\frac{1}{2}\,\sqrt{2}x\right)\log\left(x^2-1\right)-\frac{1}{4}\,\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\frac{1}{8}\,\sqrt{2}\left(\sqrt{2}\log\left(x^2-1\right)\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+\sqrt{2}\left(\left(\log\left(2x\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*arctanh(1/2*sqrt(2)*x)*log(x^2 - 1) - 1/4*log(x^2 - 1)*log((x - sqrt(2
))/(x + sqrt(2))) + 1/8*sqrt(2)*(sqrt(2)*log(x^2 - 1)*log((x - sqrt(2))/(x
+ sqrt(2))) + sqrt(2)*((log(2*x + 2*sqrt(2)) - log(2*x - 2*sqrt(2)))*log(x^
2 - 1) - log(x + sqrt(2))*log(-(x + sqrt(2))/(sqrt(2) + 1) + 1) + log(x - s
qrt(2))*log((x - sqrt(2))/(sqrt(2) + 1) + 1) - log(x + sqrt(2))*log(-(x + s
qrt(2))/(sqrt(2) - 1) + 1) + log(x - sqrt(2))*log((x - sqrt(2))/(sqrt(2) -
1) + 1) - dilog((x + sqrt(2))/(sqrt(2) + 1)) + dilog(-(x - sqrt(2))/(sqrt(2
) + 1)) - dilog((x + sqrt(2))/(sqrt(2) - 1)) + dilog(-(x - sqrt(2))/(sqrt(2
) - 1))))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x\,\operatorname{artanh}\left(\frac{1}{2}\,\sqrt{2}x\right)}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\,\operatorname{atanh}\left(\frac{\sqrt{2}x}{2}\right)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(1/2*x*2**(1/2))/(-x**2+1),x)
```

```
[Out] -Integral(x*atanh(sqrt(2)*x/2)/(x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \operatorname{artanh}\left(\frac{1}{2} \sqrt{2}x\right)}{x^2 - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(1/2*x*2^(1/2))/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(1/2*sqrt(2)*x)/(x^2 - 1), x)
```

3.185 $\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Rubi [A] time = 0.0250282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)} \, dx = \int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.74538, size = 0, normalized size = 0.

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x],x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x], x]
```

Maple [A] time = 0.252, size = 0, normalized size = 0.

$$\int \frac{x\left(-a^2x^2+1\right)}{\text{Artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] int(x*(-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)*x/arctanh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^2x^3-x}{\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^3 - x)/arctanh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x}{\operatorname{atanh}(ax)}\,dx-\int \frac{a^2x^3}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] -Integral(-x/atanh(a*x), x) - Integral(a**2*x**3/atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(a^2x^2-1)x}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x/arctanh(a*x), x)
```

3.186 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Rubi [A] time = 0.0144755, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.317716, size = 0, normalized size = 0.

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x],x]
```

```
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x], x]
```

Maple [A] time = 0.233, size = 0, normalized size = 0.

$$\int \frac{-a^2x^2+1}{\text{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/arctanh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{a^2x^2-1}{\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2}{\operatorname{atanh}(ax)}\,dx-\int -\frac{1}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x), x) - Integral(-1/atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x), x)
```

3.187 $\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1-a^2x^2}{x\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0365073, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx=\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.09101, size = 0, normalized size = 0.

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]),x]
```

```
[Out] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]), x]
```

Maple [A] time = 0.26, size = 0, normalized size = 0.

$$\int \frac{-a^2x^2+1}{x\text{Artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/x/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)/x/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2x^2-1}{x\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{1}{x\,\text{atanh}\,(ax)}\,dx-\int \frac{a^2x}{\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/x/atanh(a*x),x)
```

```
[Out] -Integral(-1/(x*atanh(a*x)), x) - Integral(a**2*x/atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{a^2x^2-1}{x\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)), x)
```

3.188 $\int \frac{x(1-a^2x^2)}{\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=20

$$\text{Unintegrable}\left(\frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Rubi [A] time = 0.0243652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} dx = \int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} dx$$

Mathematica [A] time = 0.966105, size = 0, normalized size = 0.

$$\int \frac{x\left(1-a^2x^2\right)}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2))/ArcTanh[a*x]^2, x]
```

Maple [A] time = 0.25, size = 0, normalized size = 0.

$$\int \frac{x\left(-a^2x^2+1\right)}{\left(\text{Artanh}\left(ax\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(x*(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^4x^5-2\,a^2x^3+x\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}-\int-\frac{2\left(5\,a^4x^4-6\,a^2x^2+1\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^5 - 2*a^2*x^3 + x)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrate
(-2*(5*a^4*x^4 - 6*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^2x^3-x}{\operatorname{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^3 - x)/arctanh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int-\frac{x}{\operatorname{atanh}^2\left(ax\right)}\,dx-\int\frac{a^2x^3}{\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(-x/atanh(a*x)**2, x) - Integral(a**2*x**3/atanh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\left(a^2x^2-1\right)x}{\operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)*x/arctanh(a*x)^2, x)
```

3.189 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Rubi [A] time = 0.013154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx=\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.31541, size = 0, normalized size = 0.

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^2, x]
```

Maple [A] time = 0.233, size = 0, normalized size = 0.

$$\int \frac{-a^2x^2+1}{(\text{Artanh}\,(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^4x^4-2\,a^2x^2+1\right)}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}-\int-\frac{8\left(a^3x^3-ax\right)}{\log\left(ax+1\right)-\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*log(a*x + 1) - a*log(-a*x + 1)) - integrate
(-8*(a^3*x^3 - a*x)/(log(a*x + 1) - log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^2x^2-1}{\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^2x^2}{\text{atanh}^2\left(ax\right)}\,dx-\int-\frac{1}{\text{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x)**2, x) - Integral(-1/atanh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{a^2x^2-1}{\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x)^2, x)
```

3.190 $\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{1-a^2x^2}{x\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.0338011, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx=\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 1.14553, size = 0, normalized size = 0.

$$\int \frac{1-a^2x^2}{x\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[(1 - a^2*x^2)/(x*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.263, size = 0, normalized size = 0.

$$\int \frac{-a^2x^2+1}{x\left(\text{Artanh}\,(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)/x/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)}{ax\log\left(ax+1\right)-ax\log\left(-ax+1\right)}-\int-\frac{2\left(3\,a^{4}x^{4}-2\,a^{2}x^{2}-1\right)}{ax^{2}\log\left(ax+1\right)-ax^{2}\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 + 1)/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) - integ
rate(-2*(3*a^4*x^4 - 2*a^2*x^2 - 1)/(a*x^2*log(a*x + 1) - a*x^2*log(-a*x +
1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^{2}x^{2}-1}{x\,\text{artanh}\left(ax\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int-\frac{1}{x\,\text{atanh}^{2}\left(ax\right)}\,dx-\int\frac{a^{2}x}{\text{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/x/atanh(a*x)**2,x)
```

```
[Out] -Integral(-1/(x*atanh(a*x)**2), x) - Integral(a**2*x/atanh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{a^{2}x^{2}-1}{x\,\text{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/x/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/(x*arctanh(a*x)^2), x)
```

3.191 $\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=19

$$\text{Unintegrable}\left(\frac{1-a^2x^2}{\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Rubi [A] time = 0.0132488, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Rubi steps

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} dx = \int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} dx$$

Mathematica [A] time = 1.14183, size = 0, normalized size = 0.

$$\int \frac{1-a^2x^2}{\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3,x]
```

```
[Out] Integrate[(1 - a^2*x^2)/ArcTanh[a*x]^3, x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{-a^2x^2+1}{(\text{Artanh}(ax))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(a^4x^4-2a^2x^2-2\left(a^5x^5-2a^3x^3+ax\right)\log\left(ax+1\right)+2\left(a^5x^5-2a^3x^3+ax\right)\log\left(-ax+1\right)+1\right)}{a\log\left(ax+1\right)^2-2a\log\left(ax+1\right)\log\left(-ax+1\right)+a\log\left(-ax+1\right)^2}+\int-\frac{4\left(5a^4x^4-6\right.}{\log\left(ax+1\right)-\mathrm{l}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a^4*x^4 - 2*a^2*x^2 - 2*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) + 2*(a
^5*x^5 - 2*a^3*x^3 + a*x)*log(-a*x + 1) + 1)/(a*log(a*x + 1)^2 - 2*a*log(a*
x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2) + integrate(-4*(5*a^4*x^4 - 6*a^2
*x^2 + 1)/(log(a*x + 1) - log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{a^2x^2-1}{\mathrm{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^2x^2}{\mathrm{atanh}^3\left(ax\right)}\,dx-\int-\frac{1}{\mathrm{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(a**2*x**2/atanh(a*x)**3, x) - Integral(-1/atanh(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{a^2x^2-1}{\mathrm{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)/arctanh(a*x)^3, x)
```

3.192 $\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=96

$$\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}+\frac{4\log\left(1-a^2x^2\right)}{315a^5}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)-\frac{11ax^6}{378}+\frac{2x^4}{315a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

Rubi [A] time = 0.188829, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6012, 5916, 266, 43}

$$\frac{a^3x^8}{72}+\frac{4x^2}{315a^3}+\frac{4\log\left(1-a^2x^2\right)}{315a^5}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)-\frac{11ax^6}{378}+\frac{2x^4}{315a}+\frac{1}{5}x^5\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \int \left(x^4 \tanh^{-1}(ax) - 2a^2x^6 \tanh^{-1}(ax) + a^4x^8 \tanh^{-1}(ax)\right) dx$$
$$= -\left(\left(2a^2\right) \int x^6 \tanh^{-1}(ax)\,dx\right) + a^4 \int x^8 \tanh^{-1}(ax)\,dx + \int x^4 \tanh^{-1}(ax)\,dx$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{1}{5}a \int \frac{x^5}{1-a^2x^2}\,dx + \frac{1}{7}\left(2a\right.$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{1}{10}a\,\text{Subst}\left(\int \frac{x^2}{1-a^2x}\,dx\right.$$
$$= \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{1}{10}a\,\text{Subst}\left(\int \left(-\frac{1}{a^4} - \frac{x}{a^2}\right.\right.$$
$$= \frac{4x^2}{315a^3} + \frac{2x^4}{315a} - \frac{11ax^6}{378} + \frac{a^3x^8}{72} + \frac{1}{5}x^5 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) + \frac{1}{9}a^4x^9 \tanh^{-1}$$

Mathematica [A] time = 0.0266366, size = 96, normalized size = 1.

$$\frac{a^3x^8}{72} + \frac{4x^2}{315a^3} + \frac{4\log\left(1-a^2x^2\right)}{315a^5} + \frac{1}{9}a^4x^9 \tanh^{-1}(ax) - \frac{2}{7}a^2x^7 \tanh^{-1}(ax) - \frac{11ax^6}{378} + \frac{2x^4}{315a} + \frac{1}{5}x^5 \tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(315*a^3) + (2*x^4)/(315*a) - (11*a*x^6)/378 + (a^3*x^8)/72 + (x^5*
ArcTanh[a*x])/5 - (2*a^2*x^7*ArcTanh[a*x])/7 + (a^4*x^9*ArcTanh[a*x])/9 + (
4*Log[1 - a^2*x^2])/(315*a^5)
```

Maple [A] time = 0.028, size = 87, normalized size = 0.9

$$\frac{a^4x^9 \operatorname{Artanh}(ax)}{9} - \frac{2a^2x^7 \operatorname{Artanh}(ax)}{7} + \frac{x^5 \operatorname{Artanh}(ax)}{5} + \frac{a^3x^8}{72} - \frac{11x^6a}{378} + \frac{2x^4}{315a} + \frac{4x^2}{315a^3} + \frac{4\ln(ax-1)}{315a^5} + \frac{4\ln(ax}{315a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/9*a^4*x^9*arctanh(a*x)-2/7*a^2*x^7*arctanh(a*x)+1/5*x^5*arctanh(a*x)+1/72
*a^3*x^8-11/378*x^6*a+2/315*x^4/a+4/315*x^2/a^3+4/315/a^5*ln(a*x-1)+4/315/a
^5*ln(a*x+1)
```

Maxima [A] time = 0.960876, size = 120, normalized size = 1.25

$$\frac{1}{7560}a\left(\frac{105a^6x^8 - 220a^4x^6 + 48a^2x^4 + 96x^2}{a^4} + \frac{96\log(ax+1)}{a^6} + \frac{96\log(ax-1)}{a^6}\right) + \frac{1}{315}\left(35a^4x^9 - 90a^2x^7 + 63x^5\right) \text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x^4 + 96*x^2)/a^4 + 96*log(a*
x + 1)/a^6 + 96*log(a*x - 1)/a^6) + 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5
```

```
)*arctanh(a*x)
```

Fricas [A] time = 1.98492, size = 213, normalized size = 2.22

$$\frac{105\,a^{8}x^{8}-220\,a^{6}x^{6}+48\,a^{4}x^{4}+96\,a^{2}x^{2}+12\left(35\,a^{9}x^{9}-90\,a^{7}x^{7}+63\,a^{5}x^{5}\right)\log\left(-\frac{ax+1}{ax-1}\right)+96\,\log\left(a^{2}x^{2}-1\right)}{7560\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*x^8 - 220*a^6*x^6 + 48*a^4*x^4 + 96*a^2*x^2 + 12*(35*a^9*x^
9 - 90*a^7*x^7 + 63*a^5*x^5)*log(-(a*x + 1)/(a*x - 1)) + 96*log(a^2*x^2 - 1
))/a^5
```

Sympy [A] time = 4.934, size = 100, normalized size = 1.04

$$\begin{cases}\frac{a^{4}x^{9}\operatorname{atanh}(ax)}{9}+\frac{a^{3}x^{8}}{72}-\frac{2a^{2}x^{7}\operatorname{atanh}(ax)}{7}-\frac{11ax^{6}}{378}+\frac{x^{5}\operatorname{atanh}(ax)}{5}+\frac{2x^{4}}{315a}+\frac{4x^{2}}{315a^{3}}+\frac{8\log\left(x-\frac{1}{a}\right)}{315a^{5}}+\frac{8\operatorname{atanh}(ax)}{315a^{5}} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**9*atanh(a*x)/9 + a**3*x**8/72 - 2*a**2*x**7*atanh(a*x)/7
 - 11*a*x**6/378 + x**5*atanh(a*x)/5 + 2*x**4/(315*a) + 4*x**2/(315*a**3) +
 8*log(x - 1/a)/(315*a**5) + 8*atanh(a*x)/(315*a**5), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.15844, size = 127, normalized size = 1.32

$$\frac{1}{630}\left(35\,a^{4}x^{9}-90\,a^{2}x^{7}+63\,x^{5}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{4\,\log\left(\left|a^{2}x^{2}-1\right|\right)}{315\,a^{5}}+\frac{105\,a^{11}x^{8}-220\,a^{9}x^{6}+48\,a^{7}x^{4}+96\,a^{5}x^{2}}{7560\,a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/630*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5)*log(-(a*x + 1)/(a*x - 1)) + 4/315*
log(abs(a^2*x^2 - 1))/a^5 + 1/7560*(105*a^11*x^8 - 220*a^9*x^6 + 48*a^7*x^4
 + 96*a^5*x^2)/a^8
```

3.193 $\int x^3 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=87

$$\frac{a^3x^7}{56}+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{x}{24a^3}-\frac{\tanh^{-1}(ax)}{24a^4}-\frac{ax^5}{24}+\frac{x^3}{72a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

```
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 - ArcTanh[a*x]/(24*a^4)
 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])
/8
```

Rubi [A] time = 0.137494, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6012, 5916, 302, 206}

$$\frac{a^3x^7}{56}+\frac{1}{8}a^4x^8\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)+\frac{x}{24a^3}-\frac{\tanh^{-1}(ax)}{24a^4}-\frac{ax^5}{24}+\frac{x^3}{72a}+\frac{1}{4}x^4\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```



```
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 - ArcTanh[a*x]/(24*a^4)
 + (x^4*ArcTanh[a*x])/4 - (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])
/8
```


Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \int \left(x^3 \tanh^{-1}(ax) - 2a^2x^5 \tanh^{-1}(ax) + a^4x^7 \tanh^{-1}(ax)\right) dx$$
$$= -\left(\left(2a^2\right) \int x^5 \tanh^{-1}(ax)\,dx\right) + a^4 \int x^7 \tanh^{-1}(ax)\,dx + \int x^3 \tanh^{-1}(ax)\,dx$$
$$= \frac{1}{4}x^4 \tanh^{-1}(ax) - \frac{1}{3}a^2x^6 \tanh^{-1}(ax) + \frac{1}{8}a^4x^8 \tanh^{-1}(ax) - \frac{1}{4}a \int \frac{x^4}{1-a^2x^2}\,dx + \frac{1}{3}$$
$$= \frac{1}{4}x^4 \tanh^{-1}(ax) - \frac{1}{3}a^2x^6 \tanh^{-1}(ax) + \frac{1}{8}a^4x^8 \tanh^{-1}(ax) - \frac{1}{4}a \int \left(-\frac{1}{a^4} - \frac{x^2}{a^2} + \frac{}{a^4}\right.$$
$$= \frac{x}{24a^3} + \frac{x^3}{72a} - \frac{ax^5}{24} + \frac{a^3x^7}{56} + \frac{1}{4}x^4 \tanh^{-1}(ax) - \frac{1}{3}a^2x^6 \tanh^{-1}(ax) + \frac{1}{8}a^4x^8 \tanh^{-1}$$
$$= \frac{x}{24a^3} + \frac{x^3}{72a} - \frac{ax^5}{24} + \frac{a^3x^7}{56} - \frac{\tanh^{-1}(ax)}{24a^4} + \frac{1}{4}x^4 \tanh^{-1}(ax) - \frac{1}{3}a^2x^6 \tanh^{-1}(ax) +$$

Mathematica [A] time = 0.0283403, size = 103, normalized size = 1.18

$$\frac{a^3x^7}{56} + \frac{1}{8}a^4x^8 \tanh^{-1}(ax) - \frac{1}{3}a^2x^6 \tanh^{-1}(ax) + \frac{x}{24a^3} + \frac{\log(1-ax)}{48a^4} - \frac{\log(ax+1)}{48a^4} - \frac{ax^5}{24} + \frac{x^3}{72a} + \frac{1}{4}x^4 \tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(24*a^3) + x^3/(72*a) - (a*x^5)/24 + (a^3*x^7)/56 + (x^4*ArcTanh[a*x])/4
- (a^2*x^6*ArcTanh[a*x])/3 + (a^4*x^8*ArcTanh[a*x])/8 + Log[1 - a*x]/(48*a^
4) - Log[1 + a*x]/(48*a^4)
```

Maple [A] time = 0.027, size = 85, normalized size = 1.

$$\frac{a^4x^8 \operatorname{Artanh}(ax)}{8} - \frac{a^2x^6 \operatorname{Artanh}(ax)}{3} + \frac{x^4 \operatorname{Artanh}(ax)}{4} + \frac{a^3x^7}{56} - \frac{ax^5}{24} + \frac{x^3}{72\,a} + \frac{x}{24\,a^3} + \frac{\ln(ax-1)}{48\,a^4} - \frac{\ln(ax+1)}{48\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/8*a^4*x^8*arctanh(a*x)-1/3*a^2*x^6*arctanh(a*x)+1/4*x^4*arctanh(a*x)+1/56
*a^3*x^7-1/24*a*x^5+1/72*x^3/a+1/24*x/a^3+1/48/a^4*ln(a*x-1)-1/48/a^4*ln(a*
x+1)
```

Maxima [A] time = 0.943079, size = 119, normalized size = 1.37

$$\frac{1}{1008}\,a\left(\frac{2\left(9\,a^6x^7 - 21\,a^4x^5 + 7\,a^2x^3 + 21\,x\right)}{a^4} - \frac{21\,\log(ax+1)}{a^5} + \frac{21\,\log(ax-1)}{a^5}\right) + \frac{1}{24}\left(3\,a^4x^8 - 8\,a^2x^6 + 6\,x^4\right) \operatorname{artan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/1008*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x +
1)/a^5 + 21*log(a*x - 1)/a^5) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6*x^4)*arctan
```

```
h(a*x)
```

Fricas [A] time = 1.95539, size = 177, normalized size = 2.03

$$\frac{18\,a^7x^7 - 42\,a^5x^5 + 14\,a^3x^3 + 42\,ax + 21\left(3\,a^8x^8 - 8\,a^6x^6 + 6\,a^4x^4 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{1008\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/1008*(18*a^7*x^7 - 42*a^5*x^5 + 14*a^3*x^3 + 42*a*x + 21*(3*a^8*x^8 - 8*a
^6*x^6 + 6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^4
```

Sympy [A] time = 4.00134, size = 76, normalized size = 0.87

$$\begin{cases} \frac{a^4x^8\operatorname{atanh}(ax)}{8} + \frac{a^3x^7}{56} - \frac{a^2x^6\operatorname{atanh}(ax)}{3} - \frac{ax^5}{24} + \frac{x^4\operatorname{atanh}(ax)}{4} + \frac{x^3}{72a} + \frac{x}{24a^3} - \frac{\operatorname{atanh}(ax)}{24a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**8*atanh(a*x)/8 + a**3*x**7/56 - a**2*x**6*atanh(a*x)/3 -
 a*x**5/24 + x**4*atanh(a*x)/4 + x**3/(72*a) + x/(24*a**3) - atanh(a*x)/(24
*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18255, size = 135, normalized size = 1.55

$$\frac{1}{48}\left(3\,a^4x^8 - 8\,a^2x^6 + 6\,x^4\right)\log\left(-\frac{ax+1}{ax-1}\right) - \frac{\log\left(|ax+1|\right)}{48\,a^4} + \frac{\log\left(|ax-1|\right)}{48\,a^4} + \frac{9\,a^{17}x^7 - 21\,a^{15}x^5 + 7\,a^{13}x^3 + 21\,a^{11}x}{504\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/48*(3*a^4*x^8 - 8*a^2*x^6 + 6*x^4)*log(-(a*x + 1)/(a*x - 1)) - 1/48*log(a
bs(a*x + 1))/a^4 + 1/48*log(abs(a*x - 1))/a^4 + 1/504*(9*a^17*x^7 - 21*a^15
*x^5 + 7*a^13*x^3 + 21*a^11*x)/a^14
```

3.194 $\int x^2 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=86

$$\frac{a^3x^6}{42}+\frac{4\log\left(1-a^2x^2\right)}{105a^3}+\frac{1}{7}a^4x^7\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)-\frac{9ax^4}{140}+\frac{4x^2}{105a}+\frac{1}{3}x^3\tanh^{-1}(ax)$$

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

Rubi [A] time = 0.161448, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6012, 5916, 266, 43}

$$\frac{a^3x^6}{42}+\frac{4\log\left(1-a^2x^2\right)}{105a^3}+\frac{1}{7}a^4x^7\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)-\frac{9ax^4}{140}+\frac{4x^2}{105a}+\frac{1}{3}x^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \int \left(x^2 \tanh^{-1}(ax) - 2a^2x^4 \tanh^{-1}(ax) + a^4x^6 \tanh^{-1}(ax)\right) dx$$
$$= -\left(\left(2a^2\right)\int x^4 \tanh^{-1}(ax)\,dx\right) + a^4 \int x^6 \tanh^{-1}(ax)\,dx + \int x^2 \tanh^{-1}(ax)\,dx$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{1}{3}a\int \frac{x^3}{1-a^2x^2}\,dx + \frac{1}{5}\left(2a\right.$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{x}{1-a^2x}\,dx,\right.$$
$$= \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{1}{6}a\,\mathrm{Subst}\left(\int\left(-\frac{1}{a^2} - \frac{}{a^2(}\right.\right.$$
$$= \frac{4x^2}{105a} - \frac{9ax^4}{140} + \frac{a^3x^6}{42} + \frac{1}{3}x^3 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) + \frac{4}{}$$

Mathematica [A] time = 0.0213349, size = 86, normalized size = 1.

$$\frac{a^3x^6}{42} + \frac{4\log\left(1-a^2x^2\right)}{105a^3} + \frac{1}{7}a^4x^7 \tanh^{-1}(ax) - \frac{2}{5}a^2x^5 \tanh^{-1}(ax) - \frac{9ax^4}{140} + \frac{4x^2}{105a} + \frac{1}{3}x^3 \tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (4*x^2)/(105*a) - (9*a*x^4)/140 + (a^3*x^6)/42 + (x^3*ArcTanh[a*x])/3 - (2*
a^2*x^5*ArcTanh[a*x])/5 + (a^4*x^7*ArcTanh[a*x])/7 + (4*Log[1 - a^2*x^2])/(
105*a^3)
```

Maple [A] time = 0.029, size = 79, normalized size = 0.9

$$\frac{a^4x^7 \mathrm{Artanh}\,(ax)}{7} - \frac{2\,a^2x^5 \mathrm{Artanh}\,(ax)}{5} + \frac{x^3 \mathrm{Artanh}\,(ax)}{3} + \frac{x^6a^3}{42} - \frac{9\,x^4a}{140} + \frac{4\,x^2}{105\,a} + \frac{4\,\ln\,(ax-1)}{105\,a^3} + \frac{4\,\ln\,(ax+1)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/7*a^4*x^7*arctanh(a*x)-2/5*a^2*x^5*arctanh(a*x)+1/3*x^3*arctanh(a*x)+1/42
*x^6*a^3-9/140*x^4*a+4/105*x^2/a+4/105/a^3*ln(a*x-1)+4/105/a^3*ln(a*x+1)
```

Maxima [A] time = 0.950059, size = 109, normalized size = 1.27

$$\frac{1}{420}\,a\left(\frac{10\,a^4x^6 - 27\,a^2x^4 + 16\,x^2}{a^2} + \frac{16\,\log\,(ax+1)}{a^4} + \frac{16\,\log\,(ax-1)}{a^4}\right) + \frac{1}{105}\left(15\,a^4x^7 - 42\,a^2x^5 + 35\,x^3\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 + 16*
log(a*x - 1)/a^4) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^3)*arctanh(a*x)
```

Fricas [A] time = 2.07963, size = 190, normalized size = 2.21

$$\frac{10\,a^{6}x^{6}-27\,a^{4}x^{4}+16\,a^{2}x^{2}+2\left(15\,a^{7}x^{7}-42\,a^{5}x^{5}+35\,a^{3}x^{3}\right)\log\left(-\frac{ax+1}{ax-1}\right)+16\,\log\left(a^{2}x^{2}-1\right)}{420\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*x^6 - 27*a^4*x^4 + 16*a^2*x^2 + 2*(15*a^7*x^7 - 42*a^5*x^5 +
35*a^3*x^3)*log(-(a*x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a^3
```

Sympy [A] time = 3.11666, size = 90, normalized size = 1.05

$$\begin{cases} \frac{a^{4}x^{7}\operatorname{atanh}(ax)}{7}+\frac{a^{3}x^{6}}{42}-\frac{2a^{2}x^{5}\operatorname{atanh}(ax)}{5}-\frac{9ax^{4}}{140}+\frac{x^{3}\operatorname{atanh}(ax)}{3}+\frac{4x^{2}}{105a}+\frac{8\log\left(x-\frac{1}{a}\right)}{105a^{3}}+\frac{8\operatorname{atanh}(ax)}{105a^{3}} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**7*atanh(a*x)/7 + a**3*x**6/42 - 2*a**2*x**5*atanh(a*x)/5
 - 9*a*x**4/140 + x**3*atanh(a*x)/3 + 4*x**2/(105*a) + 8*log(x - 1/a)/(105*
a**3) + 8*atanh(a*x)/(105*a**3), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.15999, size = 116, normalized size = 1.35

$$\frac{1}{210}\left(15\,a^{4}x^{7}-42\,a^{2}x^{5}+35\,x^{3}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{4\,\log\left(\left|a^{2}x^{2}-1\right|\right)}{105\,a^{3}}+\frac{10\,a^{9}x^{6}-27\,a^{7}x^{4}+16\,a^{5}x^{2}}{420\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/210*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^3)*log(-(a*x + 1)/(a*x - 1)) + 4/105*
log(abs(a^2*x^2 - 1))/a^3 + 1/420*(10*a^9*x^6 - 27*a^7*x^4 + 16*a^5*x^2)/a^
6
```

3.195 $\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=50

$$\frac{a^3x^5}{30}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}-\frac{ax^3}{9}+\frac{x}{6a}$$

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 - ((1 - a^2*x^2)^3*ArcTanh[a*x])/(6*a^2)
```

Rubi [A] time = 0.0370326, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5994, 194}

$$\frac{a^3x^5}{30}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}-\frac{ax^3}{9}+\frac{x}{6a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 - ((1 - a^2*x^2)^3*ArcTanh[a*x])/(6*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx&=-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}+\frac{\int\left(1-a^2x^2\right)^2\,dx}{6a}\\&=-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}+\frac{\int\left(1-2a^2x^2+a^4x^4\right)\,dx}{6a}\\&=\frac{x}{6a}-\frac{ax^3}{9}+\frac{a^3x^5}{30}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}{6a^2}\end{aligned}$$

Mathematica [A] time = 0.0230581, size = 93, normalized size = 1.86

$$\frac{a^3x^5}{30}+\frac{1}{6}a^4x^6\tanh^{-1}(ax)-\frac{1}{2}a^2x^4\tanh^{-1}(ax)+\frac{\log(1-ax)}{12a^2}-\frac{\log(ax+1)}{12a^2}-\frac{ax^3}{9}+\frac{1}{2}x^2\tanh^{-1}(ax)+\frac{x}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] x/(6*a) - (a*x^3)/9 + (a^3*x^5)/30 + (x^2*ArcTanh[a*x])/2 - (a^2*x^4*ArcTan
h[a*x])/2 + (a^4*x^6*ArcTanh[a*x])/6 + Log[1 - a*x]/(12*a^2) - Log[1 + a*x]
/(12*a^2)
```

Maple [A] time = 0.028, size = 77, normalized size = 1.5

$$\frac{a^4 \operatorname{Artanh}(ax)\, x^6}{6} - \frac{a^2 \operatorname{Artanh}(ax)\, x^4}{2} + \frac{\operatorname{Artanh}(ax)\, x^2}{2} + \frac{x^5 a^3}{30} - \frac{x^3 a}{9} + \frac{x}{6a} + \frac{\ln(ax-1)}{12a^2} - \frac{\ln(ax+1)}{12a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/6*a^4*arctanh(a*x)*x^6-1/2*a^2*arctanh(a*x)*x^4+1/2*arctanh(a*x)*x^2+1/30
*x^5*a^3-1/9*x^3*a+1/6*x/a+1/12/a^2*ln(a*x-1)-1/12/a^2*ln(a*x+1)
```

Maxima [A] time = 0.95139, size = 62, normalized size = 1.24

$$\frac{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)}{6a^2} + \frac{3a^4x^5 - 10a^2x^3 + 15x}{90a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 1)^3*arctanh(a*x)/a^2 + 1/90*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)
/a
```

Fricas [A] time = 1.90612, size = 154, normalized size = 3.08

$$\frac{6a^5x^5 - 20a^3x^3 + 30ax + 15\left(a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{180a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/180*(6*a^5*x^5 - 20*a^3*x^3 + 30*a*x + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^
2 - 1)*log(-(a*x + 1)/(a*x - 1)))/a^2
```

Sympy [A] time = 2.18522, size = 68, normalized size = 1.36

$$\begin{cases} \frac{a^4x^6 \operatorname{atanh}(ax)}{6} + \frac{a^3x^5}{30} - \frac{a^2x^4 \operatorname{atanh}(ax)}{2} - \frac{ax^3}{9} + \frac{x^2 \operatorname{atanh}(ax)}{2} + \frac{x}{6a} - \frac{\operatorname{atanh}(ax)}{6a^2} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**6*atanh(a*x)/6 + a**3*x**5/30 - a**2*x**4*atanh(a*x)/2 -
 a*x**3/9 + x**2*atanh(a*x)/2 + x/(6*a) - atanh(a*x)/(6*a**2), Ne(a, 0)), (
0, True))
```

Giac [A] time = 1.1609, size = 77, normalized size = 1.54

$$\frac{\left(a^2x^2-1\right)^3\log\left(-\frac{ax+1}{ax-1}\right)}{12\,a^2}+\frac{3\,a^4x^5-10\,a^2x^3+15\,x}{90\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/12*(a^2*x^2 - 1)^3*log(-(a*x + 1)/(a*x - 1))/a^2 + 1/90*(3*a^4*x^5 - 10*a
^2*x^3 + 15*x)/a
```

3.196 $\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=104

$$\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{2\left(1-a^2x^2\right)}{15a}+\frac{4\log\left(1-a^2x^2\right)}{15a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{8}{15}x\tanh^{-1}(ax)$$

```
[Out] (2*(1 - a^2*x^2))/(15*a) + (1 - a^2*x^2)^2/(20*a) + (8*x*ArcTanh[a*x])/15 +
 (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 +
 (4*Log[1 - a^2*x^2])/(15*a)
```

Rubi [A] time = 0.0437777, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5942, 5910, 260}

$$\frac{\left(1-a^2x^2\right)^2}{20a}+\frac{2\left(1-a^2x^2\right)}{15a}+\frac{4\log\left(1-a^2x^2\right)}{15a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)+\frac{8}{15}x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (2*(1 - a^2*x^2))/(15*a) + (1 - a^2*x^2)^2/(20*a) + (8*x*ArcTanh[a*x])/15 +
 (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x])/5 +
 (4*Log[1 - a^2*x^2])/(15*a)
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{1}{5}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax) + \frac{4}{5}\int \left(1-a^2x^2\right)\tanh^{-1}(ax)\,dx$$
$$= \frac{2\left(1-a^2x^2\right)}{15a} + \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax) + $$
$$= \frac{2\left(1-a^2x^2\right)}{15a} + \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{8}{15}x\tanh^{-1}(ax) + \frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{1}{5}x\left(1-a^2x\right.$$
$$= \frac{2\left(1-a^2x^2\right)}{15a} + \frac{\left(1-a^2x^2\right)^2}{20a} + \frac{8}{15}x\tanh^{-1}(ax) + \frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{1}{5}x\left(1-a^2x\right.$$

Mathematica [A] time = 0.0166325, size = 71, normalized size = 0.68

$$\frac{a^3x^4}{20} + \frac{4\log\left(1-a^2x^2\right)}{15a} + \frac{1}{5}a^4x^5\tanh^{-1}(ax) - \frac{2}{3}a^2x^3\tanh^{-1}(ax) - \frac{7ax^2}{30} + x\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (-7*a*x^2)/30 + (a^3*x^4)/20 + x*ArcTanh[a*x] - (2*a^2*x^3*ArcTanh[a*x])/3
+ (a^4*x^5*ArcTanh[a*x])/5 + (4*Log[1 - a^2*x^2])/(15*a)
```

Maple [A] time = 0.03, size = 68, normalized size = 0.7

$$\frac{a^4\mathrm{Artanh}\,(ax)\,x^5}{5} - \frac{2\,a^2\mathrm{Artanh}\,(ax)\,x^3}{3} + x\mathrm{Artanh}\,(ax) + \frac{x^4a^3}{20} - \frac{7\,ax^2}{30} + \frac{4\,\ln\,(ax-1)}{15\,a} + \frac{4\,\ln\,(ax+1)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x),x)
```

```
[Out] 1/5*a^4*arctanh(a*x)*x^5-2/3*a^2*arctanh(a*x)*x^3+x*arctanh(a*x)+1/20*x^4*a
^3-7/30*a*x^2+4/15/a*ln(a*x-1)+4/15/a*ln(a*x+1)
```

Maxima [A] time = 0.975285, size = 89, normalized size = 0.86

$$\frac{1}{60}\left(3\,a^2x^4 - 14\,x^2 + \frac{16\,\log\,(ax+1)}{a^2} + \frac{16\,\log\,(ax-1)}{a^2}\right)a + \frac{1}{15}\left(3\,a^4x^5 - 10\,a^2x^3 + 15\,x\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*(3*a^2*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 1
/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)
```

Fricas [A] time = 1.95209, size = 161, normalized size = 1.55

$$\frac{3\,a^4x^4 - 14\,a^2x^2 + 2\left(3\,a^5x^5 - 10\,a^3x^3 + 15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) + 16\,\log\left(a^2x^2-1\right)}{60\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*x^4 - 14*a^2*x^2 + 2*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*
x + 1)/(a*x - 1)) + 16*log(a^2*x^2 - 1))/a
```

Sympy [A] time = 1.77116, size = 75, normalized size = 0.72

$$\begin{cases} \frac{a^4x^5 \operatorname{atanh}(ax)}{5} + \frac{a^3x^4}{20} - \frac{2a^2x^3 \operatorname{atanh}(ax)}{3} - \frac{7ax^2}{30} + x \operatorname{atanh}(ax) + \frac{8\log\left(x-\frac{1}{a}\right)}{15a} + \frac{8\operatorname{atanh}(ax)}{15a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x),x)
```

```
[Out] Piecewise((a**4*x**5*atanh(a*x)/5 + a**3*x**4/20 - 2*a**2*x**3*atanh(a*x)/3
 - 7*a*x**2/30 + x*atanh(a*x) + 8*log(x - 1/a)/(15*a) + 8*atanh(a*x)/(15*a)
, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.16773, size = 103, normalized size = 0.99

$$\frac{1}{30}\left(3\,a^4x^5 - 10\,a^2x^3 + 15\,x\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{4\,\log\left(\left|a^2x^2-1\right|\right)}{15\,a} + \frac{3\,a^7x^4 - 14\,a^5x^2}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/30*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*log(-(a*x + 1)/(a*x - 1)) + 4/15*log(a
bs(a^2*x^2 - 1))/a + 1/60*(3*a^7*x^4 - 14*a^5*x^2)/a^4
```

3.197 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x} dx$

Optimal. Leaf size=70

$$-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)+\frac{a^3x^3}{12}+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-a^2x^2\tanh^{-1}(ax)-\frac{3ax}{4}+\frac{3}{4}\tanh^{-1}(ax)$$

```
[Out] (-3*a*x)/4 + (a^3*x^3)/12 + (3*ArcTanh[a*x])/4 - a^2*x^2*ArcTanh[a*x] + (a^
4*x^4*ArcTanh[a*x])/4 - PolyLog[2, -(a*x)]/2 + PolyLog[2, a*x]/2
```

Rubi [A] time = 0.0974804, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6012, 5912, 5916, 321, 206, 302}

$$-\frac{1}{2}\text{PolyLog}(2,-ax)+\frac{1}{2}\text{PolyLog}(2,ax)+\frac{a^3x^3}{12}+\frac{1}{4}a^4x^4\tanh^{-1}(ax)-a^2x^2\tanh^{-1}(ax)-\frac{3ax}{4}+\frac{3}{4}\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

```
[Out] (-3*a*x)/4 + (a^3*x^3)/12 + (3*ArcTanh[a*x])/4 - a^2*x^2*ArcTanh[a*x] + (a^
4*x^4*ArcTanh[a*x])/4 - PolyLog[2, -(a*x)]/2 + PolyLog[2, a*x]/2
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x}\,dx &= \int \left(\frac{\tanh^{-1}(ax)}{x} - 2a^2x\tanh^{-1}(ax) + a^4x^3\tanh^{-1}(ax)\right)dx\\ &= -\left((2a^2)\int x\tanh^{-1}(ax)\,dx\right) + a^4\int x^3\tanh^{-1}(ax)\,dx + \int \frac{\tanh^{-1}(ax)}{x}\,dx\\ &= -a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax)}{2} + \frac{\text{Li}_2(ax)}{2} + a^3\int \frac{x^2}{1-a^2x^2}\,dx - \frac{1}{4}\\ &= -ax - a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax)}{2} + \frac{\text{Li}_2(ax)}{2} + a\int \frac{1}{1-a^2x^2}\,dx\\ &= -\frac{3ax}{4} + \frac{a^3x^3}{12} + \tanh^{-1}(ax) - a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax)}{2} + \frac{\text{Li}}{}\\ &= -\frac{3ax}{4} + \frac{a^3x^3}{12} + \frac{3}{4}\tanh^{-1}(ax) - a^2x^2\tanh^{-1}(ax) + \frac{1}{4}a^4x^4\tanh^{-1}(ax) - \frac{\text{Li}_2(-ax)}{2} +\end{aligned}$$

Mathematica [A] time = 0.0727434, size = 73, normalized size = 1.04

$$\frac{1}{24}\left(-12\text{PolyLog}(2,-ax) + 12\text{PolyLog}(2,ax) + 2a^3x^3 + 6a^4x^4\tanh^{-1}(ax) - 24a^2x^2\tanh^{-1}(ax) - 18ax - 9\log(1-a\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x,x]
```

```
[Out] (-18*a*x + 2*a^3*x^3 - 24*a^2*x^2*ArcTanh[a*x] + 6*a^4*x^4*ArcTanh[a*x] - 9
*Log[1 - a*x] + 9*Log[1 + a*x] - 12*PolyLog[2, -(a*x)] + 12*PolyLog[2, a*x]
)/24
```

Maple [A] time = 0.039, size = 89, normalized size = 1.3

$$\frac{a^4x^4\text{Artanh}(ax)}{4} - a^2x^2\text{Artanh}(ax) + \text{Artanh}(ax)\ln(ax) - \frac{\text{dilog}(ax)}{2} - \frac{\text{dilog}(ax+1)}{2} - \frac{\ln(ax)\ln(ax+1)}{2} + \frac{x^3a}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x,x)
```

```
[Out] 1/4*a^4*x^4*arctanh(a*x)-a^2*x^2*arctanh(a*x)+arctanh(a*x)*ln(a*x)-1/2*dilo
g(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)+1/12*x^3*a^3-3/4*a*x-3/8*ln(a
*x-1)+3/8*ln(a*x+1)
```

Maxima [A] time = 0.955372, size = 143, normalized size = 2.04

$$\frac{1}{24}\left(2\,a^2x^3 - 18\,x - \frac{12\,(\log{(ax+1)}\log{(x)} + \mathrm{Li}_2\,(-ax))}{a} + \frac{12\,(\log{(-ax+1)}\log{(x)} + \mathrm{Li}_2\,(ax))}{a} + \frac{9\,\log{(ax+1)}}{a} - \frac{9\,\log{(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/24*(2*a^2*x^3 - 18*x - 12*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 12*(log
(-a*x + 1)*log(x) + dilog(a*x))/a + 9*log(a*x + 1)/a - 9*log(a*x - 1)/a)*a
+ 1/4*(a^4*x^4 - 4*a^2*x^2 + 2*log(x^2))*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\mathrm{artanh}\,(ax)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 - 1\right)^2\,\mathrm{artanh}\,(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x, x)
```

3.198 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^2} \, dx$

Optimal. Leaf size=64

$$\frac{a^3x^2}{6} - \frac{4}{3}a\log\left(1-a^2x^2\right) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) - 2a^2x\tanh^{-1}(ax) + a\log(x) - \frac{\tanh^{-1}(ax)}{x}$$

```
[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x]
)/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

Rubi [A] time = 0.111458, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6012, 5910, 260, 5916, 266, 36, 29, 31, 43}

$$\frac{a^3x^2}{6} - \frac{4}{3}a\log\left(1-a^2x^2\right) + \frac{1}{3}a^4x^3\tanh^{-1}(ax) - 2a^2x\tanh^{-1}(ax) + a\log(x) - \frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

```
[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x]
)/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{x^2}\,dx &= \int \left(-2a^2\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{x^2}+a^4x^2\tanh^{-1}(ax)\right)dx\\ &= -\left(\left(2a^2\right)\int \tanh^{-1}(ax)\,dx\right)+a^4\int x^2\tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx+\left(2a^3\right)\int \frac{}{1}\\ &= -\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)-a\log\left(1-a^2x^2\right)+\frac{1}{2}a\,\mathrm{Subst}\left(\int\right.\\ &= -\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)-a\log\left(1-a^2x^2\right)+\frac{1}{2}a\,\mathrm{Subst}\left(\int\right.\\ &= \frac{a^3x^2}{6}-\frac{\tanh^{-1}(ax)}{x}-2a^2x\tanh^{-1}(ax)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)+a\log(x)-\frac{4}{3}a\log\left(1-a^2x\right.\end{aligned}$$

Mathematica [A] time = 0.0162693, size = 64, normalized size = 1.

$$\frac{a^3x^2}{6}-\frac{4}{3}a\log\left(1-a^2x^2\right)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)-2a^2x\tanh^{-1}(ax)+a\log(x)-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^2,x]
```

```
[Out] (a^3*x^2)/6 - ArcTanh[a*x]/x - 2*a^2*x*ArcTanh[a*x] + (a^4*x^3*ArcTanh[a*x]
)/3 + a*Log[x] - (4*a*Log[1 - a^2*x^2])/3
```

Maple [A] time = 0.039, size = 65, normalized size = 1.

$$\frac{a^4x^3\mathrm{Artanh}\,(ax)}{3}-2\,a^2x\mathrm{Artanh}\,(ax)-\frac{\mathrm{Artanh}\,(ax)}{x}+\frac{x^2a^3}{6}-\frac{4\,a\ln(ax-1)}{3}+a\ln(ax)-\frac{4\,a\ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x)
```

```
[Out] 1/3*a^4*x^3*arctanh(a*x)-2*a^2*x*arctanh(a*x)-arctanh(a*x)/x+1/6*x^2*a^3-4/
3*a*ln(a*x-1)+a*ln(a*x)-4/3*a*ln(a*x+1)
```

Maxima [A] time = 0.949371, size = 77, normalized size = 1.2

$$\frac{1}{6}\left(a^2x^2 - 8\log(ax+1) - 8\log(ax-1) + 6\log(x)\right)a + \frac{1}{3}\left(a^4x^3 - 6a^2x - \frac{3}{x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 8*log(a*x + 1) - 8*log(a*x - 1) + 6*log(x))*a + 1/3*(a^4*x^3
 - 6*a^2*x - 3/x)*arctanh(a*x)
```

Fricas [A] time = 1.99978, size = 150, normalized size = 2.34

$$\frac{a^3x^3 - 8\,ax\log\left(a^2x^2-1\right) + 6\,ax\log(x) + \left(a^4x^4 - 6\,a^2x^2 - 3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3 - 8*a*x*log(a^2*x^2 - 1) + 6*a*x*log(x) + (a^4*x^4 - 6*a^2*x^2
 - 3)*log(-(a*x + 1)/(a*x - 1)))/x
```

Sympy [A] time = 2.69881, size = 68, normalized size = 1.06

$$\begin{cases} \frac{a^4x^3\operatorname{atanh}(ax)}{3} + \frac{a^3x^2}{6} - 2a^2x\operatorname{atanh}(ax) + a\log(x) - \frac{8a\log\left(x-\frac{1}{a}\right)}{3} - \frac{8a\operatorname{atanh}(ax)}{3} - \frac{\operatorname{atanh}(ax)}{x} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**2,x)
```

```
[Out] Piecewise((a**4*x**3*atanh(a*x)/3 + a**3*x**2/6 - 2*a**2*x*atanh(a*x) + a*l
og(x) - 8*a*log(x - 1/a)/3 - 8*a*atanh(a*x)/3 - atanh(a*x)/x, Ne(a, 0)), (0
, True))
```

Giac [A] time = 1.21595, size = 89, normalized size = 1.39

$$\frac{1}{6}a^3x^2 + \frac{1}{2}a\log\left(x^2\right) + \frac{1}{6}\left(a^4x^3 - 6\,a^2x - \frac{3}{x}\right)\log\left(-\frac{ax+1}{ax-1}\right) - \frac{4}{3}a\log\left(\left|a^2x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*a^3*x^2 + 1/2*a*log(x^2) + 1/6*(a^4*x^3 - 6*a^2*x - 3/x)*log(-(a*x + 1)
/(a*x - 1)) - 4/3*a*log(abs(a^2*x^2 - 1))
```

3.199 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^3} dx$

Optimal. Leaf size=62

$$a^2\text{PolyLog}(2,-ax) - a^2\text{PolyLog}(2,ax) + \frac{1}{2}a^4x^2\tanh^{-1}(ax) + \frac{a^3x}{2} - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

```
[Out] -a/(2*x) + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 + a^
2*PolyLog[2, -(a*x)] - a^2*PolyLog[2, a*x]
```

Rubi [A] time = 0.0941136, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6012, 5916, 325, 206, 5912, 321}

$$a^2\text{PolyLog}(2,-ax) - a^2\text{PolyLog}(2,ax) + \frac{1}{2}a^4x^2\tanh^{-1}(ax) + \frac{a^3x}{2} - \frac{\tanh^{-1}(ax)}{2x^2} - \frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -a/(2*x) + (a^3*x)/2 - ArcTanh[a*x]/(2*x^2) + (a^4*x^2*ArcTanh[a*x])/2 + a^
2*PolyLog[2, -(a*x)] - a^2*PolyLog[2, a*x]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^3}\,dx = \int \left(\frac{\tanh^{-1}(ax)}{x^3}-\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)\right)dx$$
$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x}\,dx\right)+a^4\int x\tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^3}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\mathrm{Li}_2(-ax)-a^2\mathrm{Li}_2(ax)+\frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx-$$
$$= -\frac{a}{2x}+\frac{a^3x}{2}-\frac{\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a^4x^2\tanh^{-1}(ax)+a^2\mathrm{Li}_2(-ax)-a^2\mathrm{Li}_2(ax)$$

Mathematica [A] time = 0.0611303, size = 62, normalized size = 1.

$$-\frac{-2a^2x^2\mathrm{PolyLog}(2,-ax)+2a^2x^2\mathrm{PolyLog}(2,ax)-a^3x^3-a^4x^4\tanh^{-1}(ax)+ax+\tanh^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^3,x]
```

```
[Out] -(a*x - a^3*x^3 + ArcTanh[a*x] - a^4*x^4*ArcTanh[a*x] - 2*a^2*x^2*PolyLog[2
, -(a*x)] + 2*a^2*x^2*PolyLog[2, a*x])/(2*x^2)
```

Maple [A] time = 0.042, size = 80, normalized size = 1.3

$$\frac{a^4x^2\mathrm{Artanh}\,(ax)}{2}-2\,a^2\mathrm{Artanh}\,(ax)\ln(ax)-\frac{\mathrm{Artanh}\,(ax)}{2\,x^2}+a^2\mathrm{dilog}\,(ax)+a^2\mathrm{dilog}\,(ax+1)+a^2\ln(ax)\ln(ax+1)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x)
```

```
[Out] 1/2*a^4*x^2*arctanh(a*x)-2*a^2*arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)/x^2+a^
2*dilog(a*x)+a^2*dilog(a*x+1)+a^2*ln(a*x)*ln(a*x+1)+1/2*x*a^3-1/2*a/x
```

Maxima [A] time = 0.953586, size = 111, normalized size = 1.79

$$\frac{1}{2}\left(2\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)a-2\left(\log(-ax+1)\log(x)+\mathrm{Li}_2(ax)\right)a+\frac{a^2x^2-1}{x}\right)a+\frac{1}{2}\left(a^4x^2-2\,a^2\log\left(x^2\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(log(a*x + 1)*log(x) + dilog(-a*x))*a - 2*(log(-a*x + 1)*log(x) + di
log(a*x))*a + (a^2*x^2 - 1)/x)*a + 1/2*(a^4*x^2 - 2*a^2*log(x^2) - 1/x^2)*a
rctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\left(ax\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)^2\,(ax+1)^2\,\text{atanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2\text{artanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^3, x)
```

3.200 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=68

$$\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{5}{3}a^3\log(x)+a^4x\tanh^{-1}(ax)+\frac{2a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Rubi [A] time = 0.109764, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {6012, 5910, 260, 5916, 266, 44, 36, 29, 31}

$$\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{5}{3}a^3\log(x)+a^4x\tanh^{-1}(ax)+\frac{2a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^4}\,dx = \int \left(a^4\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{x^4}-\frac{2a^2\tanh^{-1}(ax)}{x^2}\right)dx$$

$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\right)+a^4\int \tanh^{-1}(ax)\,dx+\int \frac{\tanh^{-1}(ax)}{x^4}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)+\frac{1}{3}a\int \frac{1}{x^3\left(1-a^2x^2\right)}\,dx-\left(2a^3\right)\int$$

$$= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)+\frac{1}{2}a^3\log\left(1-a^2x^2\right)+\frac{1}{6}a\,\text{Subst}\Big($$

$$= -\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)+\frac{1}{2}a^3\log\left(1-a^2x^2\right)+\frac{1}{6}a\,\text{Subst}\Big($$

$$= -\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}+\frac{2a^2\tanh^{-1}(ax)}{x}+a^4x\tanh^{-1}(ax)-\frac{5}{3}a^3\log(x)+\frac{4}{3}a^3\log\big(1-$$

Mathematica [A] time = 0.0178808, size = 68, normalized size = 1.

$$\frac{4}{3}a^3\log\left(1-a^2x^2\right)-\frac{5}{3}a^3\log(x)+a^4x\tanh^{-1}(ax)+\frac{2a^2\tanh^{-1}(ax)}{x}-\frac{a}{6x^2}-\frac{\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^4,x]
```

```
[Out] -a/(6*x^2) - ArcTanh[a*x]/(3*x^3) + (2*a^2*ArcTanh[a*x])/x + a^4*x*ArcTanh[
a*x] - (5*a^3*Log[x])/3 + (4*a^3*Log[1 - a^2*x^2])/3
```

Maple [A] time = 0.036, size = 69, normalized size = 1.

$$a^4x\,\text{Artanh}\,(ax)+2\,\frac{a^2\,\text{Artanh}\,(ax)}{x}-\frac{\text{Artanh}\,(ax)}{3\,x^3}+\frac{4\,a^3\ln(ax-1)}{3}-\frac{a}{6\,x^2}-\frac{5\,a^3\ln(ax)}{3}+\frac{4\,a^3\ln(ax+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x)
```

```
[Out] a^4*x*arctanh(a*x)+2*a^2*arctanh(a*x)/x-1/3*arctanh(a*x)/x^3+4/3*a^3*ln(a*x
-1)-1/6*a/x^2-5/3*a^3*ln(a*x)+4/3*a^3*ln(a*x+1)
```

Maxima [A] time = 0.950228, size = 89, normalized size = 1.31

$$\frac{1}{6}\left(8\,a^2\log\left(ax+1\right)+8\,a^2\log\left(ax-1\right)-10\,a^2\log\left(x\right)-\frac{1}{x^2}\right)a+\frac{1}{3}\left(3\,a^4x+\frac{6\,a^2x^2-1}{x^3}\right)\text{artanh}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(8*a^2*log(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a + 1
/3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)
```

Fricas [A] time = 2.1127, size = 162, normalized size = 2.38

$$\frac{8\,a^3x^3\log\left(a^2x^2-1\right)-10\,a^3x^3\log\left(x\right)-ax+\left(3\,a^4x^4+6\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(8*a^3*x^3*log(a^2*x^2 - 1) - 10*a^3*x^3*log(x) - a*x + (3*a^4*x^4 + 6*
a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1)))/x^3
```

Sympy [A] time = 2.42279, size = 75, normalized size = 1.1

$$\begin{cases} a^4x\,\text{atanh}\left(ax\right)-\frac{5a^3\log\left(x\right)}{3}+\frac{8a^3\log\left(x-\frac{1}{a}\right)}{3}+\frac{8a^3\,\text{atanh}\left(ax\right)}{3}+\frac{2a^2\,\text{atanh}\left(ax\right)}{x}-\frac{a}{6x^2}-\frac{\text{atanh}\left(ax\right)}{3x^3} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**4,x)
```

```
[Out] Piecewise((a**4*x*atanh(a*x) - 5*a**3*log(x)/3 + 8*a**3*log(x - 1/a)/3 + 8*
a**3*atanh(a*x)/3 + 2*a**2*atanh(a*x)/x - a/(6*x**2) - atanh(a*x)/(3*x**3),
 Ne(a, 0)), (0, True))
```

Giac [A] time = 1.22223, size = 109, normalized size = 1.6

$$-\frac{5}{6}\,a^3\log\left(x^2\right)+\frac{4}{3}\,a^3\log\left(\left|a^2x^2-1\right|\right)+\frac{1}{6}\left(3\,a^4x+\frac{6\,a^2x^2-1}{x^3}\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{5\,a^3x^2-a}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] -5/6*a^3*log(x^2) + 4/3*a^3*log(abs(a^2*x^2 - 1)) + 1/6*(3*a^4*x + (6*a^2*x
^2 - 1)/x^3)*log(-(a*x + 1)/(a*x - 1)) + 1/6*(5*a^3*x^2 - a)/x^2
```

3.201 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^5} dx$

Optimal. Leaf size=77

$$-\frac{1}{2}a^4\text{PolyLog}(2,-ax)+\frac{1}{2}a^4\text{PolyLog}(2,ax)+\frac{a^2\tanh^{-1}(ax)}{x^2}+\frac{3a^3}{4x}-\frac{3}{4}a^4\tanh^{-1}(ax)-\frac{a}{12x^3}-\frac{\tanh^{-1}(ax)}{4x^4}$$

```
[Out] -a/(12*x^3) + (3*a^3)/(4*x) - (3*a^4*ArcTanh[a*x])/4 - ArcTanh[a*x]/(4*x^4)
 + (a^2*ArcTanh[a*x])/x^2 - (a^4*PolyLog[2, -(a*x)])/2 + (a^4*PolyLog[2, a*
x])/2
```

Rubi [A] time = 0.100928, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6012, 5916, 325, 206, 5912}

$$-\frac{1}{2}a^4\text{PolyLog}(2,-ax)+\frac{1}{2}a^4\text{PolyLog}(2,ax)+\frac{a^2\tanh^{-1}(ax)}{x^2}+\frac{3a^3}{4x}-\frac{3}{4}a^4\tanh^{-1}(ax)-\frac{a}{12x^3}-\frac{\tanh^{-1}(ax)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

```
[Out] -a/(12*x^3) + (3*a^3)/(4*x) - (3*a^4*ArcTanh[a*x])/4 - ArcTanh[a*x]/(4*x^4)
 + (a^2*ArcTanh[a*x])/x^2 - (a^4*PolyLog[2, -(a*x)])/2 + (a^4*PolyLog[2, a*
x])/2
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^5}\,dx = \int \left(\frac{\tanh^{-1}(ax)}{x^5} - \frac{2a^2\tanh^{-1}(ax)}{x^3} + \frac{a^4\tanh^{-1}(ax)}{x}\right)dx$$
$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x^3}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)}{x}\,dx + \int \frac{\tanh^{-1}(ax)}{x^5}\,dx$$
$$= -\frac{\tanh^{-1}(ax)}{4x^4} + \frac{a^2\tanh^{-1}(ax)}{x^2} - \frac{1}{2}a^4\mathrm{Li}_2(-ax) + \frac{1}{2}a^4\mathrm{Li}_2(ax) + \frac{1}{4}a\int \frac{1}{x^4\left(1-a^2x^2\right)}\,d$$
$$= -\frac{a}{12x^3} + \frac{a^3}{x} - \frac{\tanh^{-1}(ax)}{4x^4} + \frac{a^2\tanh^{-1}(ax)}{x^2} - \frac{1}{2}a^4\mathrm{Li}_2(-ax) + \frac{1}{2}a^4\mathrm{Li}_2(ax) + \frac{1}{4}a^3\int$$
$$= -\frac{a}{12x^3} + \frac{3a^3}{4x} - a^4\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{4x^4} + \frac{a^2\tanh^{-1}(ax)}{x^2} - \frac{1}{2}a^4\mathrm{Li}_2(-ax) + \frac{1}{2}a^4$$
$$= -\frac{a}{12x^3} + \frac{3a^3}{4x} - \frac{3}{4}a^4\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{4x^4} + \frac{a^2\tanh^{-1}(ax)}{x^2} - \frac{1}{2}a^4\mathrm{Li}_2(-ax) + \frac{1}{2}a$$

Mathematica [A] time = 0.0815758, size = 84, normalized size = 1.09

$$\frac{1}{24}\left(-12a^4\mathrm{PolyLog}(2,-ax) + 12a^4\mathrm{PolyLog}(2,ax) + \frac{24a^2\tanh^{-1}(ax)}{x^2} + \frac{18a^3}{x} + 9a^4\log(1-ax) - 9a^4\log(ax+1) -\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^5,x]
```

```
[Out] ((-2*a)/x^3 + (18*a^3)/x - (6*ArcTanh[a*x])/x^4 + (24*a^2*ArcTanh[a*x])/x^2
 + 9*a^4*Log[1 - a*x] - 9*a^4*Log[1 + a*x] - 12*a^4*PolyLog[2, -(a*x)] + 12
*a^4*PolyLog[2, a*x])/24
```

Maple [A] time = 0.047, size = 105, normalized size = 1.4

$$-\frac{\mathrm{Artanh}(ax)}{4x^4} + a^4\mathrm{Artanh}(ax)\ln(ax) + \frac{a^2\mathrm{Artanh}(ax)}{x^2} - \frac{a^4\mathrm{dilog}(ax)}{2} - \frac{a^4\mathrm{dilog}(ax+1)}{2} - \frac{a^4\ln(ax)\ln(ax+1)}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x)
```

```
[Out] -1/4*arctanh(a*x)/x^4+a^4*arctanh(a*x)*ln(a*x)+a^2*arctanh(a*x)/x^2-1/2*a^4
*dilog(a*x)-1/2*a^4*dilog(a*x+1)-1/2*a^4*ln(a*x)*ln(a*x+1)+3/8*a^4*ln(a*x-1
)-1/12*a/x^3+3/4*a^3/x-3/8*a^4*ln(a*x+1)
```

Maxima [A] time = 0.964769, size = 151, normalized size = 1.96

$$-\frac{1}{24}\left(12\left(\log(ax+1)\log(x) + \mathrm{Li}_2(-ax)\right)a^3 - 12\left(\log(-ax+1)\log(x) + \mathrm{Li}_2(ax)\right)a^3 + 9a^3\log(ax+1) - 9a^3\log(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 - 12*(log(-a*x + 1)*log(x
) + dilog(a*x))*a^3 + 9*a^3*log(a*x + 1) - 9*a^3*log(a*x - 1) - 2*(9*a^2*x^
2 - 1)/x^3)*a + 1/4*(2*a^4*log(x^2) + (4*a^2*x^2 - 1)/x^4)*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\left(ax\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax - 1\right)^2 \left(ax + 1\right)^2 \text{atanh}\left(ax\right)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**5,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 - 1\right)^2 \text{artanh}\left(ax\right)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)/x^5, x)
```

3.202 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)}{x^6} dx$

Optimal. Leaf size=83

$$\frac{7a^3}{30x^2} - \frac{4}{15}a^5 \log\left(1 - a^2x^2\right) + \frac{2a^2 \tanh^{-1}(ax)}{3x^3} + \frac{8}{15}a^5 \log(x) - \frac{a^4 \tanh^{-1}(ax)}{x} - \frac{a}{20x^4} - \frac{\tanh^{-1}(ax)}{5x^5}$$

```
[Out] -a/(20*x^4) + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

Rubi [A] time = 0.137952, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6012, 5916, 266, 44, 36, 29, 31}

$$\frac{7a^3}{30x^2} - \frac{4}{15}a^5 \log\left(1 - a^2x^2\right) + \frac{2a^2 \tanh^{-1}(ax)}{3x^3} + \frac{8}{15}a^5 \log(x) - \frac{a^4 \tanh^{-1}(ax)}{x} - \frac{a}{20x^4} - \frac{\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

```
[Out] -a/(20*x^4) + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{x^6}\,dx &= \int \left(\frac{\tanh^{-1}(ax)}{x^6}-\frac{2a^2\tanh^{-1}(ax)}{x^4}+\frac{a^4\tanh^{-1}(ax)}{x^2}\right)dx\\ &= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)}{x^4}\,dx\right)+a^4\int \frac{\tanh^{-1}(ax)}{x^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^6}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{1}{5}a\int \frac{1}{x^5\left(1-a^2x^2\right)}\,dx-\frac{1}{3}\left(2a^3\right)\int\\ &= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{1}{x^3\left(1-a^2x\right)}\,dx,x,x^2\right.\\ &= -\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{1}{10}a\,\text{Subst}\left(\int\left(\frac{1}{x^3}+\frac{a^2}{x^2}+\frac{a^4}{x}-\frac{}{-1}\right.\right.\\ &= -\frac{a}{20x^4}+\frac{7a^3}{30x^2}-\frac{\tanh^{-1}(ax)}{5x^5}+\frac{2a^2\tanh^{-1}(ax)}{3x^3}-\frac{a^4\tanh^{-1}(ax)}{x}+\frac{8}{15}a^5\log(x)-\frac{4}{15}a^5\end{aligned}$$

Mathematica [A] time = 0.0228139, size = 83, normalized size = 1.

$$\frac{7a^3}{30x^2}-\frac{4}{15}a^5\log\left(1-a^2x^2\right)+\frac{2a^2\tanh^{-1}(ax)}{3x^3}+\frac{8}{15}a^5\log(x)-\frac{a^4\tanh^{-1}(ax)}{x}-\frac{a}{20x^4}-\frac{\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x])/x^6,x]
```

```
[Out] -a/(20*x^4) + (7*a^3)/(30*x^2) - ArcTanh[a*x]/(5*x^5) + (2*a^2*ArcTanh[a*x]
)/(3*x^3) - (a^4*ArcTanh[a*x])/x + (8*a^5*Log[x])/15 - (4*a^5*Log[1 - a^2*x
^2])/15
```

Maple [A] time = 0.038, size = 80, normalized size = 1.

$$-\frac{a^4\operatorname{Artanh}(ax)}{x}-\frac{\operatorname{Artanh}(ax)}{5x^5}+\frac{2a^2\operatorname{Artanh}(ax)}{3x^3}-\frac{4a^5\ln(ax-1)}{15}-\frac{a}{20x^4}+\frac{7a^3}{30x^2}+\frac{8a^5\ln(ax)}{15}-\frac{4a^5\ln(ax+1)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x)
```

```
[Out] -a^4*arctanh(a*x)/x-1/5*arctanh(a*x)/x^5+2/3*a^2*arctanh(a*x)/x^3-4/15*a^5*
ln(a*x-1)-1/20*a/x^4+7/30*a^3/x^2+8/15*a^5*ln(a*x)-4/15*a^5*ln(a*x+1)
```

Maxima [A] time = 0.949418, size = 96, normalized size = 1.16

$$-\frac{1}{60}\left(16\,a^4\log\left(a^2x^2-1\right)-16\,a^4\log\left(x^2\right)-\frac{14\,a^2x^2-3}{x^4}\right)a-\frac{\left(15\,a^4x^4-10\,a^2x^2+3\right)\operatorname{artanh}\left(ax\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2) - (14*a^2*x^2 - 3)/x^4)*a
- 1/15*(15*a^4*x^4 - 10*a^2*x^2 + 3)*arctanh(a*x)/x^5
```

Fricas [A] time = 1.96243, size = 192, normalized size = 2.31

$$-\frac{16\,a^5x^5\log\left(a^2x^2-1\right)-32\,a^5x^5\log\left(x\right)-14\,a^3x^3+3\,ax+2\left(15\,a^4x^4-10\,a^2x^2+3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/60*(16*a^5*x^5*log(a^2*x^2 - 1) - 32*a^5*x^5*log(x) - 14*a^3*x^3 + 3*a*x
 + 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)*log(-(a*x + 1)/(a*x - 1)))/x^5
```

Sympy [A] time = 3.41173, size = 88, normalized size = 1.06

$$\begin{cases} \frac{8a^5\log\left(x\right)}{15}-\frac{8a^5\log\left(x-\frac{1}{a}\right)}{15}-\frac{8a^5\operatorname{atanh}\left(ax\right)}{15}-\frac{a^4\operatorname{atanh}\left(ax\right)}{x}+\frac{7a^3}{30x^2}+\frac{2a^2\operatorname{atanh}\left(ax\right)}{3x^3}-\frac{a}{20x^4}-\frac{\operatorname{atanh}\left(ax\right)}{5x^5} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)/x**6,x)
```

```
[Out] Piecewise((8*a**5*log(x)/15 - 8*a**5*log(x - 1/a)/15 - 8*a**5*atanh(a*x)/15
 - a**4*atanh(a*x)/x + 7*a**3/(30*x**2) + 2*a**2*atanh(a*x)/(3*x**3) - a/(2
0*x**4) - atanh(a*x)/(5*x**5), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.17547, size = 120, normalized size = 1.45

$$\frac{4}{15}\,a^5\log\left(x^2\right)-\frac{4}{15}\,a^5\log\left(\left|a^2x^2-1\right|\right)-\frac{24\,a^5x^4-14\,a^3x^2+3\,a}{60\,x^4}-\frac{\left(15\,a^4x^4-10\,a^2x^2+3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] 4/15*a^5*log(x^2) - 4/15*a^5*log(abs(a^2*x^2 - 1)) - 1/60*(24*a^5*x^4 - 14*
a^3*x^2 + 3*a)/x^4 - 1/30*(15*a^4*x^4 - 10*a^2*x^2 + 3)*log(-(a*x + 1)/(a*x
 - 1))/x^5
```

3.203 $\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=202

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{315a^5}+\frac{a^2x^7}{252}-\frac{67x^3}{11340a^2}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+\frac{1}{36}a^3x^8\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{8x^2}{}$$

```
[Out] (29*x)/(3780*a^4) - (67*x^3)/(11340*a^2) - (23*x^5)/3780 + (a^2*x^7)/252 -
(29*ArcTanh[a*x])/(3780*a^5) + (8*x^2*ArcTanh[a*x])/(315*a^3) + (4*x^4*ArcT
anh[a*x])/(315*a) - (11*a*x^6*ArcTanh[a*x])/189 + (a^3*x^8*ArcTanh[a*x])/36
 + (8*ArcTanh[a*x]^2)/(315*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (2*a^2*x^7*ArcTa
nh[a*x]^2)/7 + (a^4*x^9*ArcTanh[a*x]^2)/9 - (16*ArcTanh[a*x]*Log[2/(1 - a*x
)])/(315*a^5) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(315*a^5)
```

Rubi [A] time = 1.02455, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 59, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6012, 5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{315a^5}+\frac{a^2x^7}{252}-\frac{67x^3}{11340a^2}+\frac{1}{9}a^4x^9\tanh^{-1}(ax)^2+\frac{1}{36}a^3x^8\tanh^{-1}(ax)-\frac{2}{7}a^2x^7\tanh^{-1}(ax)^2+\frac{8x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (29*x)/(3780*a^4) - (67*x^3)/(11340*a^2) - (23*x^5)/3780 + (a^2*x^7)/252 -
(29*ArcTanh[a*x])/(3780*a^5) + (8*x^2*ArcTanh[a*x])/(315*a^3) + (4*x^4*ArcT
anh[a*x])/(315*a) - (11*a*x^6*ArcTanh[a*x])/189 + (a^3*x^8*ArcTanh[a*x])/36
 + (8*ArcTanh[a*x]^2)/(315*a^5) + (x^5*ArcTanh[a*x]^2)/5 - (2*a^2*x^7*ArcTa
nh[a*x]^2)/7 + (a^4*x^9*ArcTanh[a*x]^2)/9 - (16*ArcTanh[a*x]*Log[2/(1 - a*x
)])/(315*a^5) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(315*a^5)
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int x^4 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx = \int \left(x^4\tanh^{-1}(ax)^2 - 2a^2x^6\tanh^{-1}(ax)^2 + a^4x^8\tanh^{-1}(ax)^2\right) dx$$
$$= -\left(\left(2a^2\right)\int x^6\tanh^{-1}(ax)^2\,dx\right) + a^4\int x^8\tanh^{-1}(ax)^2\,dx + \int x^4\tanh^{-1}(ax)^2\,dx$$
$$= \frac{1}{5}x^5\tanh^{-1}(ax)^2 - \frac{2}{7}a^2x^7\tanh^{-1}(ax)^2 + \frac{1}{9}a^4x^9\tanh^{-1}(ax)^2 - \frac{1}{5}(2a)\int \frac{x^5\tanh^{-1}}{1-a^2}$$
$$= \frac{1}{5}x^5\tanh^{-1}(ax)^2 - \frac{2}{7}a^2x^7\tanh^{-1}(ax)^2 + \frac{1}{9}a^4x^9\tanh^{-1}(ax)^2 + \frac{2\int x^3\tanh^{-1}(ax)}{5a}$$
$$= \frac{x^4\tanh^{-1}(ax)}{10a} - \frac{2}{21}ax^6\tanh^{-1}(ax) + \frac{1}{36}a^3x^8\tanh^{-1}(ax) + \frac{1}{5}x^5\tanh^{-1}(ax)^2 - \frac{2}{7}$$
$$= \frac{x^2\tanh^{-1}(ax)}{5a^3} - \frac{3x^4\tanh^{-1}(ax)}{70a} - \frac{11}{189}ax^6\tanh^{-1}(ax) + \frac{1}{36}a^3x^8\tanh^{-1}(ax) +$$
$$= \frac{293x}{1260a^4} + \frac{41x^3}{3780a^2} - \frac{17x^5}{1260} + \frac{a^2x^7}{252} - \frac{3x^2\tanh^{-1}(ax)}{35a^3} + \frac{4x^4\tanh^{-1}(ax)}{315a} - \frac{11}{189}ax^6$$
$$= -\frac{601x}{3780a^4} - \frac{277x^3}{11340a^2} - \frac{23x^5}{3780} + \frac{a^2x^7}{252} - \frac{293\tanh^{-1}(ax)}{1260a^5} + \frac{8x^2\tanh^{-1}(ax)}{315a^3} +$$
$$= \frac{29x}{3780a^4} - \frac{67x^3}{11340a^2} - \frac{23x^5}{3780} + \frac{a^2x^7}{252} + \frac{601\tanh^{-1}(ax)}{3780a^5} + \frac{8x^2\tanh^{-1}(ax)}{315a^3} +$$
$$= \frac{29x}{3780a^4} - \frac{67x^3}{11340a^2} - \frac{23x^5}{3780} + \frac{a^2x^7}{252} - \frac{29\tanh^{-1}(ax)}{3780a^5} + \frac{8x^2\tanh^{-1}(ax)}{315a^3} +$$
$$= \frac{29x}{3780a^4} - \frac{67x^3}{11340a^2} - \frac{23x^5}{3780} + \frac{a^2x^7}{252} - \frac{29\tanh^{-1}(ax)}{3780a^5} + \frac{8x^2\tanh^{-1}(ax)}{315a^3} +$$

Mathematica [A] time = 1.8055, size = 138, normalized size = 0.68

$$\frac{288\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + ax\left(45a^6x^6 - 69a^4x^4 - 67a^2x^2 + 87\right) + 36\left(35a^9x^9 - 90a^7x^7 + 63a^5x^5 - 8\right)\tanh^{-1}(}{11340a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(87 - 67*a^2*x^2 - 69*a^4*x^4 + 45*a^6*x^6) + 36*(-8 + 63*a^5*x^5 - 90
*a^7*x^7 + 35*a^9*x^9)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-29 + 96*a^2*x^2 +
48*a^4*x^4 - 220*a^6*x^6 + 105*a^8*x^8 - 192*Log[1 + E^(-2*ArcTanh[a*x])])
+ 288*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(11340*a^5)
```

Maple [A] time = 0.049, size = 259, normalized size = 1.3

$$\frac{a^4x^9(\text{Artanh}(ax))^2}{9} - \frac{2a^2x^7(\text{Artanh}(ax))^2}{7} + \frac{x^5(\text{Artanh}(ax))^2}{5} + \frac{a^3x^8\text{Artanh}(ax)}{36} - \frac{11ax^6\text{Artanh}(ax)}{189} + \frac{4x^4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/9*a^4*x^9*arctanh(a*x)^2-2/7*a^2*x^7*arctanh(a*x)^2+1/5*x^5*arctanh(a*x)^
2+1/36*a^3*x^8*arctanh(a*x)-11/189*a*x^6*arctanh(a*x)+4/315*x^4*arctanh(a*x
)/a+8/315*x^2*arctanh(a*x)/a^3+8/315/a^5*arctanh(a*x)*ln(a*x-1)+8/315/a^5*a
rctanh(a*x)*ln(a*x+1)+1/252*a^2*x^7-23/3780*x^5-67/11340*x^3/a^2+29/3780*x/
a^4+29/7560/a^5*ln(a*x-1)-29/7560/a^5*ln(a*x+1)+2/315/a^5*ln(a*x-1)^2-8/315
/a^5*dilog(1/2+1/2*a*x)-4/315/a^5*ln(a*x-1)*ln(1/2+1/2*a*x)-4/315/a^5*ln(-1
/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/315/a^5*ln(-1/2*a*x+1/2)*ln(a*x+1)-2/315/a^5*
ln(a*x+1)^2
```

Maxima [A] time = 0.981327, size = 289, normalized size = 1.43

$$\frac{1}{22680}a^2\left(\frac{90\,a^7x^7-138\,a^5x^5-134\,a^3x^3+174\,ax-144\,\log(ax+1)^2+288\,\log(ax+1)\log(ax-1)+144\,\log(ax-1)^2}{a^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/22680*a^2*((90*a^7*x^7 - 138*a^5*x^5 - 134*a^3*x^3 + 174*a*x - 144*log(a*
x + 1)^2 + 288*log(a*x + 1)*log(a*x - 1) + 144*log(a*x - 1)^2 + 87*log(a*x
- 1))/a^7 - 576*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a
^7 - 87*log(a*x + 1)/a^7) + 1/3780*a*((105*a^6*x^8 - 220*a^4*x^6 + 48*a^2*x
^4 + 96*x^2)/a^4 + 96*log(a*x + 1)/a^6 + 96*log(a*x - 1)/a^6)*arctanh(a*x)
+ 1/315*(35*a^4*x^9 - 90*a^2*x^7 + 63*x^5)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4x^8-2\,a^2x^6+x^4\right)\text{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^8 - 2*a^2*x^6 + x^4)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(ax-1\right)^2\left(ax+1\right)^2\text{atanh}^2\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral(x**4*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2x^2-1\right)^2x^4\,\text{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x^4*arctanh(a*x)^2, x)
```

3.204 $\int x^3 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=156

$$\frac{a^2x^6}{168}-\frac{5x^2}{504a^2}+\frac{2\log\left(1-a^2x^2\right)}{63a^4}+\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2+\frac{1}{28}a^3x^7\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2+\frac{x\tanh^{-1}(ax)}{12a^3}-\text{ta}$$

```
[Out] (-5*x^2)/(504*a^2) - x^4/84 + (a^2*x^6)/168 + (x*ArcTanh[a*x])/(12*a^3) + (
x^3*ArcTanh[a*x])/(36*a) - (a*x^5*ArcTanh[a*x])/12 + (a^3*x^7*ArcTanh[a*x])
/28 - ArcTanh[a*x]^2/(24*a^4) + (x^4*ArcTanh[a*x]^2)/4 - (a^2*x^6*ArcTanh[a
*x]^2)/3 + (a^4*x^8*ArcTanh[a*x]^2)/8 + (2*Log[1 - a^2*x^2])/(63*a^4)
```

Rubi [A] time = 0.819183, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6012, 5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{a^2x^6}{168}-\frac{5x^2}{504a^2}+\frac{2\log\left(1-a^2x^2\right)}{63a^4}+\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2+\frac{1}{28}a^3x^7\tanh^{-1}(ax)-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2+\frac{x\tanh^{-1}(ax)}{12a^3}-\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-5*x^2)/(504*a^2) - x^4/84 + (a^2*x^6)/168 + (x*ArcTanh[a*x])/(12*a^3) + (
x^3*ArcTanh[a*x])/(36*a) - (a*x^5*ArcTanh[a*x])/12 + (a^3*x^7*ArcTanh[a*x])
/28 - ArcTanh[a*x]^2/(24*a^4) + (x^4*ArcTanh[a*x]^2)/4 - (a^2*x^6*ArcTanh[a
*x]^2)/3 + (a^4*x^8*ArcTanh[a*x]^2)/8 + (2*Log[1 - a^2*x^2])/(63*a^4)
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx = \int\left(x^3\tanh^{-1}(ax)^2-2a^2x^5\tanh^{-1}(ax)^2+a^4x^7\tanh^{-1}(ax)^2\right)dx$$

$$= -\left(\left(2a^2\right)\int x^5\tanh^{-1}(ax)^2\,dx\right)+a^4\int x^7\tanh^{-1}(ax)^2\,dx+\int x^3\tanh^{-1}(ax)^2\,dx$$

$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2+\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2-\frac{1}{2}a\int\frac{x^4\tanh^{-1}(a}{1-a^2x^2}$$

$$= \frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{3}a^2x^6\tanh^{-1}(ax)^2+\frac{1}{8}a^4x^8\tanh^{-1}(ax)^2+\frac{\int x^2\tanh^{-1}(ax)\,d}{2a}$$

$$= \frac{x^3\tanh^{-1}(ax)}{6a}-\frac{2}{15}ax^5\tanh^{-1}(ax)+\frac{1}{28}a^3x^7\tanh^{-1}(ax)+\frac{1}{4}x^4\tanh^{-1}(ax)^2-\frac{1}{3}$$

$$= \frac{x\tanh^{-1}(ax)}{2a^3}-\frac{x^3\tanh^{-1}(ax)}{18a}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{28}a^3x^7\tanh^{-1}(ax)-\frac{\tanh}{4}$$

$$= -\frac{x\tanh^{-1}(ax)}{6a^3}+\frac{x^3\tanh^{-1}(ax)}{36a}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{28}a^3x^7\tanh^{-1}(ax)+\frac{\tan}{}$$

$$= \frac{29x^2}{840a^2}-\frac{41x^4}{1680}+\frac{a^2x^6}{168}+\frac{x\tanh^{-1}(ax)}{12a^3}+\frac{x^3\tanh^{-1}(ax)}{36a}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{28}$$

$$= -\frac{13x^2}{252a^2}-\frac{x^4}{84}+\frac{a^2x^6}{168}+\frac{x\tanh^{-1}(ax)}{12a^3}+\frac{x^3\tanh^{-1}(ax)}{36a}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{28}$$

$$= -\frac{5x^2}{504a^2}-\frac{x^4}{84}+\frac{a^2x^6}{168}+\frac{x\tanh^{-1}(ax)}{12a^3}+\frac{x^3\tanh^{-1}(ax)}{36a}-\frac{1}{12}ax^5\tanh^{-1}(ax)+\frac{1}{28}$$

Mathematica [A] time = 0.0630342, size = 108, normalized size = 0.69

$$\frac{3a^6x^6-6a^4x^4-5a^2x^2+16\log\left(1-a^2x^2\right)+2ax\left(9a^6x^6-21a^4x^4+7a^2x^2+21\right)\tanh^{-1}(ax)+21\left(a^2x^2-1\right)^3\left(3a^2x^2+\right.}{504a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-5*a^2*x^2 - 6*a^4*x^4 + 3*a^6*x^6 + 2*a*x*(21 + 7*a^2*x^2 - 21*a^4*x^4 +
9*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a^2*x^2)^3*(1 + 3*a^2*x^2)*ArcTanh[a*x]^
2 + 16*Log[1 - a^2*x^2])/(504*a^4)
```

Maple [A] time = 0.047, size = 239, normalized size = 1.5

$$\frac{a^4x^8\left(\operatorname{Artanh}(ax)\right)^2}{8}-\frac{a^2x^6\left(\operatorname{Artanh}(ax)\right)^2}{3}+\frac{x^4\left(\operatorname{Artanh}(ax)\right)^2}{4}+\frac{a^3x^7\operatorname{Artanh}(ax)}{28}-\frac{ax^5\operatorname{Artanh}(ax)}{12}+\frac{x^3\operatorname{Artanh}(\ldots}{36a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/8*a^4*x^8*arctanh(a*x)^2-1/3*a^2*x^6*arctanh(a*x)^2+1/4*x^4*arctanh(a*x)^
2+1/28*a^3*x^7*arctanh(a*x)-1/12*a*x^5*arctanh(a*x)+1/36*x^3*arctanh(a*x)/a
+1/12*x*arctanh(a*x)/a^3+1/24/a^4*arctanh(a*x)*ln(a*x-1)-1/24/a^4*arctanh(a
*x)*ln(a*x+1)+1/96/a^4*ln(a*x-1)^2-1/48/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/48/
a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/48/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/
96/a^4*ln(a*x+1)^2+1/168*x^6*a^2-1/84*x^4-5/504*x^2/a^2+2/63/a^4*ln(a*x-1)+
2/63/a^4*ln(a*x+1)
```

Maxima [A] time = 0.959537, size = 230, normalized size = 1.47

$$\frac{1}{504}a\left(\frac{2\left(9a^6x^7-21a^4x^5+7a^2x^3+21x\right)}{a^4}-\frac{21\log(ax+1)}{a^5}+\frac{21\log(ax-1)}{a^5}\right)\operatorname{artanh}(ax)+\frac{1}{24}\left(3a^4x^8-8a^2x^6+6x^4\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/504*a*(2*(9*a^6*x^7 - 21*a^4*x^5 + 7*a^2*x^3 + 21*x)/a^4 - 21*log(a*x + 1
)/a^5 + 21*log(a*x - 1)/a^5)*arctanh(a*x) + 1/24*(3*a^4*x^8 - 8*a^2*x^6 + 6
*x^4)*arctanh(a*x)^2 + 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 - 2*(21
*log(a*x - 1) - 32)*log(a*x + 1) + 21*log(a*x + 1)^2 + 21*log(a*x - 1)^2 +
64*log(a*x - 1))/a^4
```

Fricas [A] time = 1.88257, size = 300, normalized size = 1.92

$$\frac{12a^6x^6-24a^4x^4-20a^2x^2+21\left(3a^8x^8-8a^6x^6+6a^4x^4-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\left(9a^7x^7-21a^5x^5+7a^3x^3+21ax\right)\log\left(\ldots\right.}{2016a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2016*(12*a^6*x^6 - 24*a^4*x^4 - 20*a^2*x^2 + 21*(3*a^8*x^8 - 8*a^6*x^6 +
6*a^4*x^4 - 1)*log(-(a*x + 1)/(a*x - 1))^2 + 4*(9*a^7*x^7 - 21*a^5*x^5 + 7*
```

```
a^3*x^3 + 21*a*x)*log(-(a*x + 1)/(a*x - 1)) + 64*log(a^2*x^2 - 1))/a^4
```

Sympy [A] time = 5.25159, size = 153, normalized size = 0.98

$$\begin{cases} \frac{a^4x^8\operatorname{atanh}^2(ax)}{8} + \frac{a^3x^7\operatorname{atanh}(ax)}{28} - \frac{a^2x^6\operatorname{atanh}^2(ax)}{3} + \frac{a^2x^6}{168} - \frac{ax^5\operatorname{atanh}(ax)}{12} + \frac{x^4\operatorname{atanh}^2(ax)}{4} - \frac{x^4}{84} + \frac{x^3\operatorname{atanh}(ax)}{36a} - \frac{5x^2}{504a^2} + \frac{x\operatorname{atanh}}{12a^3} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*x**8*atanh(a*x)**2/8 + a**3*x**7*atanh(a*x)/28 - a**2*x**6*
atanh(a*x)**2/3 + a**2*x**6/168 - a*x**5*atanh(a*x)/12 + x**4*atanh(a*x)**2
/4 - x**4/84 + x**3*atanh(a*x)/(36*a) - 5*x**2/(504*a**2) + x*atanh(a*x)/(1
2*a**3) + 4*log(x - 1/a)/(63*a**4) - atanh(a*x)**2/(24*a**4) + 4*atanh(a*x)
/(63*a**4), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18168, size = 174, normalized size = 1.12

$$\frac{1}{168}a^2x^6 - \frac{1}{84}x^4 + \frac{1}{96}\left(3a^4x^8 - 8a^2x^6 + 6x^4 - \frac{1}{a^4}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + \frac{1}{504}\left(9a^3x^7 - 21ax^5 + \frac{7x^3}{a} + \frac{21x}{a^3}\right)\log\left(-\frac{ax}{ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/168*a^2*x^6 - 1/84*x^4 + 1/96*(3*a^4*x^8 - 8*a^2*x^6 + 6*x^4 - 1/a^4)*log
(-(a*x + 1)/(a*x - 1))^2 + 1/504*(9*a^3*x^7 - 21*a*x^5 + 7*x^3/a + 21*x/a^3
)*log(-(a*x + 1)/(a*x - 1)) - 5/504*x^2/a^2 + 2/63*log(a^2*x^2 - 1)/a^4
```

3.205 $\int x^2 \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=178

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{105a^3}+\frac{a^2x^5}{105}+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2+\frac{1}{21}a^3x^6\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2-\frac{x}{210a^2}+\frac{8\tanh^{-1}(}{105a^3}$$

```
[Out] -x/(210*a^2) - (17*x^3)/630 + (a^2*x^5)/105 + ArcTanh[a*x]/(210*a^3) + (8*x
^2*ArcTanh[a*x])/(105*a) - (9*a*x^4*ArcTanh[a*x])/70 + (a^3*x^6*ArcTanh[a*x
])/21 + (8*ArcTanh[a*x]^2)/(105*a^3) + (x^3*ArcTanh[a*x]^2)/3 - (2*a^2*x^5*
ArcTanh[a*x]^2)/5 + (a^4*x^7*ArcTanh[a*x]^2)/7 - (16*ArcTanh[a*x]*Log[2/(1
- a*x)])/(105*a^3) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(105*a^3)
```

Rubi [A] time = 0.779231, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6012, 5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 302}

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{105a^3}+\frac{a^2x^5}{105}+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2+\frac{1}{21}a^3x^6\tanh^{-1}(ax)-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2-\frac{x}{210a^2}+\frac{8\tanh^{-1}(}{105a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```


```
[Out] -x/(210*a^2) - (17*x^3)/630 + (a^2*x^5)/105 + ArcTanh[a*x]/(210*a^3) + (8*x
^2*ArcTanh[a*x])/(105*a) - (9*a*x^4*ArcTanh[a*x])/70 + (a^3*x^6*ArcTanh[a*x
])/21 + (8*ArcTanh[a*x]^2)/(105*a^3) + (x^3*ArcTanh[a*x]^2)/3 - (2*a^2*x^5*
ArcTanh[a*x]^2)/5 + (a^4*x^7*ArcTanh[a*x]^2)/7 - (16*ArcTanh[a*x]*Log[2/(1
- a*x)])/(105*a^3) - (8*PolyLog[2, 1 - 2/(1 - a*x)])/(105*a^3)
```


Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx=\int\left(x^2\tanh^{-1}(ax)^2-2a^2x^4\tanh^{-1}(ax)^2+a^4x^6\tanh^{-1}(ax)^2\right)dx$$

$$=-\left(\left(2a^2\right)\int x^4\tanh^{-1}(ax)^2\,dx\right)+a^4\int x^6\tanh^{-1}(ax)^2\,dx+\int x^2\tanh^{-1}(ax)^2\,dx$$

$$=\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2-\frac{1}{3}(2a)\int\frac{x^3\tanh^{-1}(a}{1-a^2x^2}$$

$$=\frac{1}{3}x^3\tanh^{-1}(ax)^2-\frac{2}{5}a^2x^5\tanh^{-1}(ax)^2+\frac{1}{7}a^4x^7\tanh^{-1}(ax)^2+\frac{2\int x\tanh^{-1}(ax)\,dx}{3a}-$$

$$=\frac{x^2\tanh^{-1}(ax)}{3a}-\frac{1}{5}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)^2}{3a^3}+\frac{1}{3}x^3\tanh$$

$$=\frac{x}{3a^2}-\frac{x^2\tanh^{-1}(ax)}{15a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3x^6\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)^2}{15a^3}+\frac{1}{3}x$$

$$=-\frac{23x}{105a^2}-\frac{16x^3}{315}+\frac{a^2x^5}{105}-\frac{\tanh^{-1}(ax)}{3a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3$$

$$=-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{23\tanh^{-1}(ax)}{105a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{2}$$

$$=-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3$$

$$=-\frac{x}{210a^2}-\frac{17x^3}{630}+\frac{a^2x^5}{105}+\frac{\tanh^{-1}(ax)}{210a^3}+\frac{8x^2\tanh^{-1}(ax)}{105a}-\frac{9}{70}ax^4\tanh^{-1}(ax)+\frac{1}{21}a^3$$

Mathematica [A] time = 1.09864, size = 121, normalized size = 0.68

$$\frac{48\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+ax\left(6a^4x^4-17a^2x^2-3\right)+6\left(15a^7x^7-42a^5x^5+35a^3x^3-8\right)\tanh^{-1}(ax)^2+\tanh^{-1}(ax)}{630a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(-3 - 17*a^2*x^2 + 6*a^4*x^4) + 6*(-8 + 35*a^3*x^3 - 42*a^5*x^5 + 15*a
^7*x^7)*ArcTanh[a*x]^2 + ArcTanh[a*x]*(3 + 48*a^2*x^2 - 81*a^4*x^4 + 30*a^6
*x^6 - 96*Log[1 + E^(-2*ArcTanh[a*x])]) + 48*PolyLog[2, -E^(-2*ArcTanh[a*x]
)])/(630*a^3)
```

Maple [A] time = 0.05, size = 239, normalized size = 1.3

$$\frac{a^4x^7\left(\text{Artanh}\,(ax)\right)^2}{7}-\frac{2\,a^2x^5\left(\text{Artanh}\,(ax)\right)^2}{5}+\frac{x^3\left(\text{Artanh}\,(ax)\right)^2}{3}+\frac{a^3x^6\text{Artanh}\,(ax)}{21}-\frac{9\,ax^4\text{Artanh}\,(ax)}{70}+\frac{8\,x^2\text{Arta}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/7*a^4*x^7*arctanh(a*x)^2-2/5*a^2*x^5*arctanh(a*x)^2+1/3*x^3*arctanh(a*x)^
2+1/21*a^3*x^6*arctanh(a*x)-9/70*a*x^4*arctanh(a*x)+8/105*x^2*arctanh(a*x)/
```

```
a+8/105/a^3*arctanh(a*x)*ln(a*x-1)+8/105/a^3*arctanh(a*x)*ln(a*x+1)+1/105*x
^5*a^2-17/630*x^3-1/210*x/a^2-1/420/a^3*ln(a*x-1)+1/420/a^3*ln(a*x+1)+2/105
/a^3*ln(a*x-1)^2-8/105/a^3*dilog(1/2+1/2*a*x)-4/105/a^3*ln(a*x-1)*ln(1/2+1/
2*a*x)-4/105/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/105/a^3*ln(-1/2*a*x+1/2
)*ln(a*x+1)-2/105/a^3*ln(a*x+1)^2
```

Maxima [A] time = 0.983677, size = 267, normalized size = 1.5

$$\frac{1}{1260}a^2\left(\frac{12\,a^5x^5 - 34\,a^3x^3 - 6\,ax - 24\,\log(ax+1)^2 + 48\,\log(ax+1)\log(ax-1) + 24\,\log(ax-1)^2 - 3\,\log(ax-1)}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/1260*a^2*((12*a^5*x^5 - 34*a^3*x^3 - 6*a*x - 24*log(a*x + 1)^2 + 48*log(a
*x + 1)*log(a*x - 1) + 24*log(a*x - 1)^2 - 3*log(a*x - 1))/a^5 - 96*(log(a*
x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^5 + 3*log(a*x + 1)/a^5
) + 1/210*a*((10*a^4*x^6 - 27*a^2*x^4 + 16*x^2)/a^2 + 16*log(a*x + 1)/a^4 +
 16*log(a*x - 1)/a^4)*arctanh(a*x) + 1/105*(15*a^4*x^7 - 42*a^2*x^5 + 35*x^
3)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4x^6 - 2\,a^2x^4 + x^2\right)\text{artanh}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^6 - 2*a^2*x^4 + x^2)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\,(ax-1)^2\,(ax+1)^2\,\text{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral(x**2*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2x^2-1\right)^2x^2\,\text{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")

[Out] integrate((a^2*x^2 - 1)^2*x^2*arctanh(a*x)^2, x)
```

3.206 $\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=138

$$\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{4\log\left(1-a^2x^2\right)}{45a^2}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}+\frac{4x\left(1-a^2x^2\right)}{45}$$

```
[Out] (2*(1 - a^2*x^2))/(45*a^2) + (1 - a^2*x^2)^2/(60*a^2) + (8*x*ArcTanh[a*x])/
(45*a) + (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/(45*a) + (x*(1 - a^2*x^2)^2*ArcTa
nh[a*x])/(15*a) - ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(6*a^2) + (4*Log[1 - a^2
*x^2])/(45*a^2)
```

Rubi [A] time = 0.088121, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5994, 5942, 5910, 260}

$$\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{4\log\left(1-a^2x^2\right)}{45a^2}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}+\frac{4x\left(1-a^2x^2\right)}{45}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (2*(1 - a^2*x^2))/(45*a^2) + (1 - a^2*x^2)^2/(60*a^2) + (8*x*ArcTanh[a*x])/
(45*a) + (4*x*(1 - a^2*x^2)*ArcTanh[a*x])/(45*a) + (x*(1 - a^2*x^2)^2*ArcTa
nh[a*x])/(15*a) - ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(6*a^2) + (4*Log[1 - a^2
*x^2])/(45*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2\,dx=-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{\int\left(1-a^2x^2\right)^2\tanh^{-1}(ax)\,dx}{3a}$$
$$=\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}-\frac{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}{6a^2}+\frac{4\int\left(1-a^2x^2\right)\text{ta}}{15a}$$
$$=\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{15a}-$$
$$=\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x\left(1-a^2x^2\right.}{}$$
$$=\frac{2\left(1-a^2x^2\right)}{45a^2}+\frac{\left(1-a^2x^2\right)^2}{60a^2}+\frac{8x\tanh^{-1}(ax)}{45a}+\frac{4x\left(1-a^2x^2\right)\tanh^{-1}(ax)}{45a}+\frac{x\left(1-a^2x^2\right.}{}$$

Mathematica [A] time = 0.053743, size = 82, normalized size = 0.59

$$\frac{3a^4x^4-14a^2x^2+16\log\left(1-a^2x^2\right)+4ax\left(3a^4x^4-10a^2x^2+15\right)\tanh^{-1}(ax)+30\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2}{180a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-14*a^2*x^2 + 3*a^4*x^4 + 4*a*x*(15 - 10*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x]
 + 30*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2 + 16*Log[1 - a^2*x^2])/(180*a^2)
```

Maple [A] time = 0.049, size = 219, normalized size = 1.6

$$\frac{a^4(\mathrm{Artanh}(ax))^2x^6}{6}-\frac{a^2(\mathrm{Artanh}(ax))^2x^4}{2}+\frac{(\mathrm{Artanh}(ax))^2x^2}{2}+\frac{a^3\mathrm{Artanh}(ax)x^5}{15}-\frac{2a\mathrm{Artanh}(ax)x^3}{9}+\frac{x\mathrm{Artanh}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/6*a^4*arctanh(a*x)^2*x^6-1/2*a^2*arctanh(a*x)^2*x^4+1/2*arctanh(a*x)^2*x^
2+1/15*a^3*arctanh(a*x)*x^5-2/9*a*arctanh(a*x)*x^3+1/3*x*arctanh(a*x)/a+1/6
/a^2*arctanh(a*x)*ln(a*x-1)-1/6/a^2*arctanh(a*x)*ln(a*x+1)+1/24/a^2*ln(a*x-
1)^2-1/12/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/12/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/
2*a*x)-1/12/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/24/a^2*ln(a*x+1)^2+1/60*a^2*x^
4-7/90*x^2+4/45/a^2*ln(a*x-1)+4/45/a^2*ln(a*x+1)
```

Maxima [A] time = 0.964607, size = 126, normalized size = 0.91

$$\frac{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^2}{6a^2}+\frac{\left(3a^2x^4-14x^2+\frac{16\log(ax+1)}{a^2}+\frac{16\log(ax-1)}{a^2}\right)a+4\left(3a^4x^5-10a^2x^3+15x\right)\operatorname{artanh}(ax)}{180a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*x^2 - 1)^3*arctanh(a*x)^2/a^2 + 1/180*((3*a^2*x^4 - 14*x^2 + 16*lo
g(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a + 4*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)
*arctanh(a*x))/a
```

Fricas [A] time = 2.00645, size = 261, normalized size = 1.89

$$\frac{6\,a^4x^4 - 28\,a^2x^2 + 15\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 4\left(3\,a^5x^5 - 10\,a^3x^3 + 15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) + 32\,\log\left(a\right.}{360\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/360*(6*a^4*x^4 - 28*a^2*x^2 + 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*lo
g(-(a*x + 1)/(a*x - 1))^2 + 4*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*log(-(a*x +
 1)/(a*x - 1)) + 32*log(a^2*x^2 - 1))/a^2
```

Sympy [A] time = 3.57061, size = 133, normalized size = 0.96

$$\begin{cases} \frac{a^4x^6\operatorname{atanh}^2(ax)}{6} + \frac{a^3x^5\operatorname{atanh}(ax)}{15} - \frac{a^2x^4\operatorname{atanh}^2(ax)}{2} + \frac{a^2x^4}{60} - \frac{2ax^3\operatorname{atanh}(ax)}{9} + \frac{x^2\operatorname{atanh}^2(ax)}{2} - \frac{7x^2}{90} + \frac{x\operatorname{atanh}(ax)}{3a} + \frac{8\log\left(x-\frac{1}{a}\right)}{45a^2} - \mathrm{a} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*x**6*atanh(a*x)**2/6 + a**3*x**5*atanh(a*x)/15 - a**2*x**4*
atanh(a*x)**2/2 + a**2*x**4/60 - 2*a*x**3*atanh(a*x)/9 + x**2*atanh(a*x)**2
/2 - 7*x**2/90 + x*atanh(a*x)/(3*a) + 8*log(x - 1/a)/(45*a**2) - atanh(a*x)
**2/(6*a**2) + 8*atanh(a*x)/(45*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.20939, size = 151, normalized size = 1.09

$$\frac{1}{60}\,a^2x^4 + \frac{1}{24}\left(a^4x^6 - 3\,a^2x^4 + 3\,x^2 - \frac{1}{a^2}\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - \frac{7}{90}\,x^2 + \frac{1}{90}\left(3\,a^3x^5 - 10\,ax^3 + \frac{15\,x}{a}\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{4\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/60*a^2*x^4 + 1/24*(a^4*x^6 - 3*a^2*x^4 + 3*x^2 - 1/a^2)*log(-(a*x + 1)/(a
*x - 1))^2 - 7/90*x^2 + 1/90*(3*a^3*x^5 - 10*a*x^3 + 15*x/a)*log(-(a*x + 1)
/(a*x - 1)) + 4/45*log(a^2*x^2 - 1)/a^2
```

3.207 $\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 \, dx$

Optimal. Leaf size=171

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a}+\frac{a^2x^3}{30}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}$$

```
[Out] (-11*x)/30 + (a^2*x^3)/30 + (4*(1 - a^2*x^2)*ArcTanh[a*x])/(15*a) + ((1 - a
^2*x^2)^2*ArcTanh[a*x])/(10*a) + (8*ArcTanh[a*x]^2)/(15*a) + (8*x*ArcTanh[a
*x]^2)/15 + (4*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/15 + (x*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/5 - (16*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a) - (8*PolyLog[2, 1
 - 2/(1 - a*x)])/(15*a)
```

Rubi [A] time = 0.133453, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5944, 5910, 5984, 5918, 2402, 2315, 8}

$$-\frac{8\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{15a}+\frac{a^2x^3}{30}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{4}{15}x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2+\frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (-11*x)/30 + (a^2*x^3)/30 + (4*(1 - a^2*x^2)*ArcTanh[a*x])/(15*a) + ((1 - a
^2*x^2)^2*ArcTanh[a*x])/(10*a) + (8*ArcTanh[a*x]^2)/(15*a) + (8*x*ArcTanh[a
*x]^2)/15 + (4*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/15 + (x*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/5 - (16*ArcTanh[a*x]*Log[2/(1 - a*x)])/(15*a) - (8*PolyLog[2, 1
 - 2/(1 - a*x)])/(15*a)
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2\,dx = \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{10a} + \frac{1}{5}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 - \frac{1}{10}\int \left(1-a^2x^2\right)dx + \frac{4}{5}\int \big($$

$$= -\frac{x}{10} + \frac{a^2x^3}{30} + \frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{4}{15}x\left(1-a^2x^2\right)\mathrm{t}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8}{15}x\tanh^{-1}(ax$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tanh^{-1}(ax)^2}{15a}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tanh^{-1}(ax)^2}{15a}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tanh^{-1}(ax)^2}{15a}$$

$$= -\frac{11x}{30} + \frac{a^2x^3}{30} + \frac{4\left(1-a^2x^2\right)\tanh^{-1}(ax)}{15a} + \frac{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}{10a} + \frac{8\tanh^{-1}(ax)^2}{15a}$$

Mathematica [A] time = 0.653835, size = 99, normalized size = 0.58

$$\frac{16\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + ax\left(a^2x^2-11\right) + 2\left(3a^2x^2+9ax+8\right)(ax-1)^3\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\left(3a^4x^4-1\right.}{30a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^2,x]
```

```
[Out] (a*x*(-11 + a^2*x^2) + 2*(-1 + a*x)^3*(8 + 9*a*x + 3*a^2*x^2)*ArcTanh[a*x]^
2 + ArcTanh[a*x]*(11 - 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^(-2*ArcTanh[a*
x])]) + 16*PolyLog[2, -E^(-2*ArcTanh[a*x])])/(30*a)
```

Maple [A] time = 0.047, size = 216, normalized size = 1.3

$$\frac{a^4 (\mathrm{Artanh}\,(ax))^2 x^5}{5} - \frac{2 a^2 (\mathrm{Artanh}\,(ax))^2 x^3}{3} + x (\mathrm{Artanh}\,(ax))^2 + \frac{a^3 \mathrm{Artanh}\,(ax)\, x^4}{10} - \frac{7 a \mathrm{Artanh}\,(ax)\, x^2}{15} + \frac{8\ \mathrm{Artanl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2,x)
```

```
[Out] 1/5*a^4*arctanh(a*x)^2*x^5-2/3*a^2*arctanh(a*x)^2*x^3+x*arctanh(a*x)^2+1/10
*a^3*arctanh(a*x)*x^4-7/15*a*arctanh(a*x)*x^2+8/15/a*arctanh(a*x)*ln(a*x-1)
+8/15/a*arctanh(a*x)*ln(a*x+1)+1/30*x^3*a^2-11/30*x-11/60/a*ln(a*x-1)+11/60
/a*ln(a*x+1)+2/15/a*ln(a*x-1)^2-8/15/a*dilog(1/2+1/2*a*x)-4/15/a*ln(a*x-1)*
ln(1/2+1/2*a*x)-4/15/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/15/a*ln(-1/2*a*x+
1/2)*ln(a*x+1)-2/15/a*ln(a*x+1)^2
```

Maxima [A] time = 0.963282, size = 236, normalized size = 1.38

$$\frac{1}{60} a^2 \left(\frac{2 a^3 x^3 - 22 ax - 8 \log(ax+1)^2 + 16 \log(ax+1) \log(ax-1) + 8 \log(ax-1)^2 - 11 \log(ax-1)}{a^3} - \frac{32 \left(\log(ax-1\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/60*a^2*((2*a^3*x^3 - 22*a*x - 8*log(a*x + 1)^2 + 16*log(a*x + 1)*log(a*x
 - 1) + 8*log(a*x - 1)^2 - 11*log(a*x - 1))/a^3 - 32*(log(a*x - 1)*log(1/2*a
*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 + 11*log(a*x + 1)/a^3) + 1/30*(3*a^2
*x^4 - 14*x^2 + 16*log(a*x + 1)/a^2 + 16*log(a*x - 1)/a^2)*a*arctanh(a*x) +
 1/15*(3*a^4*x^5 - 10*a^2*x^3 + 15*x)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4 x^4 - 2 a^2 x^2 + 1 \right) \mathrm{artanh}\,(ax)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2, x)
```

3.208 $\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x}\, dx$

Optimal. Leaf size=186

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\tanh^{-1}(ax)\text{PolyLog}\left(2,\right.$$

```
[Out] (a^2*x^2)/12 - (3*a*x*ArcTanh[a*x])/2 + (a^3*x^3*ArcTanh[a*x])/6 + (3*ArcTa
nh[a*x]^2)/4 - a^2*x^2*ArcTanh[a*x]^2 + (a^4*x^4*ArcTanh[a*x]^2)/4 + 2*ArcT
anh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 - ArcTanh[a*x]
*PolyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] +
PolyLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rubi [A] time = 0.52983, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6012, 5914, 6052, 5948, 6058, 6610, 5916, 5980, 5910, 260, 266, 43}

$$\frac{1}{2}\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\tanh^{-1}(ax)\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

```
[Out] (a^2*x^2)/12 - (3*a*x*ArcTanh[a*x])/2 + (a^3*x^3*ArcTanh[a*x])/6 + (3*ArcTa
nh[a*x]^2)/4 - a^2*x^2*ArcTanh[a*x]^2 + (a^4*x^4*ArcTanh[a*x]^2)/4 + 2*ArcT
anh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 - ArcTanh[a*x]
*PolyLog[2, 1 - 2/(1 - a*x)] + ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] +
PolyLog[3, 1 - 2/(1 - a*x)]/2 - PolyLog[3, -1 + 2/(1 - a*x)]/2
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x} - 2a^2x\tanh^{-1}(ax)^2 + a^4x^3\tanh^{-1}(ax)^2\right)dx$$
$$= -\left(\left(2a^2\right)\int x\tanh^{-1}(ax)^2\,dx\right) + a^4\int x^3\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x}\,dx$$
$$= -a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) - (4a)\int$$
$$= -a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh^{-1}(ax)^2 + 2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) - (2a)\int$$
$$= -2ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\tanh$$
$$= -\frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\,\mathrm{ta}$$
$$= -\frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a^4x^4\,\mathrm{ta}$$
$$= \frac{a^2x^2}{12} - \frac{3}{2}ax\tanh^{-1}(ax) + \frac{1}{6}a^3x^3\tanh^{-1}(ax) + \frac{3}{4}\tanh^{-1}(ax)^2 - a^2x^2\tanh^{-1}(ax)^2 + \frac{1}{4}a$$

Mathematica [A] time = 0.0632203, size = 191, normalized size = 1.03

$$-\frac{1}{2}\mathrm{PolyLog}\left(3,\frac{-ax-1}{ax-1}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,\frac{ax+1}{ax-1}\right) + \tanh^{-1}(ax)\mathrm{PolyLog}\left(2,\frac{-ax-1}{ax-1}\right) - \tanh^{-1}(ax)\mathrm{PolyLog}\left(2,\frac{ax+1}{ax-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x,x]
```

```
[Out] (a^2*x^2)/12 - 2*a*x*ArcTanh[a*x] + (a*x*(3 + a^2*x^2)*ArcTanh[a*x])/6 - (-
1 + a^2*x^2)*ArcTanh[a*x]^2 + ((-1 + a^4*x^4)*ArcTanh[a*x]^2)/4 + 2*ArcTanh
[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (2*Log[1 - a^2*x^2])/3 + ArcTanh[a*x]*Po
lyLog[2, (-1 - a*x)/(-1 + a*x)] - ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a
*x)] - PolyLog[3, (-1 - a*x)/(-1 + a*x)]/2 + PolyLog[3, (1 + a*x)/(-1 + a*x
)]/2
```

Maple [C] time = 1.165, size = 728, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x)
```

```
[Out] -a^2*x^2*arctanh(a*x)^2+1/4*a^4*x^4*arctanh(a*x)^2+1/12*a^2*x^2-1/12+3/4*ar
ctanh(a*x)^2+1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^3*arctanh(a*x)^2-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+4/3*ln((a
*x+1)^2/(-a^2*x^2+1)+1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a
rctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+
1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)-arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+arctanh(a*x)^2*ln(a*x)-ar
ctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*
```

```
x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*
arctanh(a*x)^2-(a*x+1)*arctanh(a*x)-2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/2*(a*x-3)*(a*x+1)*arctanh(a*x)+1/2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+1
/6*(a^2*x^2-4*a*x+7)*(a*x+1)*arctanh(a*x)-1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*
x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*
arctanh(a*x)^2+1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\left(a^4x^4 - 4\,a^2x^2\right)\log\left(-ax+1\right)^2 - \frac{1}{4}\int -\frac{2\left(a^5x^5 - a^4x^4 - 2\,a^3x^3 + 2\,a^2x^2 + ax - 1\right)\log\left(ax+1\right)^2 - \left(a^5x^5 - 4\,a^3x^3 + \right.}{2\left(ax^2 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/16*(a^4*x^4 - 4*a^2*x^2)*log(-a*x + 1)^2 - 1/4*integrate(-1/2*(2*(a^5*x^5
 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (a^5*x^5 - 4
*a^3*x^3 + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x
+ 1))*log(-a*x + 1))/(a*x^2 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\left(ax\right)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^2\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2\text{artanh}\left(ax\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x, x)
```

3.209 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=156

$$\frac{5}{3}a\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3x^2\tanh^{-1}(ax)+\frac{a^2x}{3}-2a^2x\tanh$$

```
[Out] (a^2*x)/3 - (a*ArcTanh[a*x])/3 + (a^3*x^2*ArcTanh[a*x])/3 - (2*a*ArcTanh[a*
x]^2)/3 - ArcTanh[a*x]^2/x - 2*a^2*x*ArcTanh[a*x]^2 + (a^4*x^3*ArcTanh[a*x]
^2)/3 + (10*a*ArcTanh[a*x]*Log[2/(1 - a*x)])/3 + 2*a*ArcTanh[a*x]*Log[2 - 2
/(1 + a*x)] + (5*a*PolyLog[2, 1 - 2/(1 - a*x)])/3 - a*PolyLog[2, -1 + 2/(1
+ a*x)]
```

Rubi [A] time = 0.418459, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6012, 5910, 5984, 5918, 2402, 2315, 5916, 5988, 5932, 2447, 5980, 321, 206}

$$\frac{5}{3}a\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}a^4x^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3x^2\tanh^{-1}(ax)+\frac{a^2x}{3}-2a^2x\tanh$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] (a^2*x)/3 - (a*ArcTanh[a*x])/3 + (a^3*x^2*ArcTanh[a*x])/3 - (2*a*ArcTanh[a*
x]^2)/3 - ArcTanh[a*x]^2/x - 2*a^2*x*ArcTanh[a*x]^2 + (a^4*x^3*ArcTanh[a*x]
^2)/3 + (10*a*ArcTanh[a*x]*Log[2/(1 - a*x)])/3 + 2*a*ArcTanh[a*x]*Log[2 - 2
/(1 + a*x)] + (5*a*PolyLog[2, 1 - 2/(1 - a*x)])/3 - a*PolyLog[2, -1 + 2/(1
+ a*x)]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)]/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^2}\,dx = \int \left(-2a^2 \tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{x^2} + a^4x^2 \tanh^{-1}(ax)^2\right) dx$$
$$= -\left(\left(2a^2\right) \int \tanh^{-1}(ax)^2\,dx\right) + a^4 \int x^2 \tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{x} - 2a^2x \tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3 \tanh^{-1}(ax)^2 + (2a) \int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx +$$
$$= -a \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x \tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3 \tanh^{-1}(ax)^2 + (2a) \int$$
$$= \frac{1}{3}a^3x^2 \tanh^{-1}(ax) - \frac{2}{3}a \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x \tanh^{-1}(ax)^2 + \frac{1}{3}a^4x^3 \text{ta}$$
$$= \frac{a^2x}{3} + \frac{1}{3}a^3x^2 \tanh^{-1}(ax) - \frac{2}{3}a \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x \tanh^{-1}(ax)^2 + \frac{1}{3}$$
$$= \frac{a^2x}{3} - \frac{1}{3}a \tanh^{-1}(ax) + \frac{1}{3}a^3x^2 \tanh^{-1}(ax) - \frac{2}{3}a \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x$$
$$= \frac{a^2x}{3} - \frac{1}{3}a \tanh^{-1}(ax) + \frac{1}{3}a^3x^2 \tanh^{-1}(ax) - \frac{2}{3}a \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} - 2a^2x$$

Mathematica [A] time = 0.056467, size = 182, normalized size = 1.17

$$\frac{1}{3}a\left(\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) - \left(1-a^2x^2\right)\left(ax\tanh^{-1}(ax)+1\right)\tanh^{-1}(ax) + ax + ax\tanh^{-1}(ax)^2 - \tanh^{-1}(ax)^2 -\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] -2*a*ArcTanh[a*x]*(-ArcTanh[a*x] + a*x*ArcTanh[a*x] - 2*Log[1 + E^(-2*ArcTa
nh[a*x])]) - 2*a*PolyLog[2, -E^(-2*ArcTanh[a*x])] + (a*(a*x - ArcTanh[a*x]^
2 + a*x*ArcTanh[a*x]^2 - (1 - a^2*x^2)*ArcTanh[a*x]*(1 + a*x*ArcTanh[a*x])
- 2*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] + PolyLog[2, -E^(-2*ArcTanh[a
*x])]))/3 + a*(ArcTanh[a*x]*(ArcTanh[a*x] - ArcTanh[a*x]/(a*x) + 2*Log[1 -
E^(-2*ArcTanh[a*x])]) - PolyLog[2, E^(-2*ArcTanh[a*x])])
```

Maple [A] time = 0.056, size = 222, normalized size = 1.4

$$\frac{a^4x^3\left(\text{Artanh}\,(ax)\right)^2}{3} - 2\,a^2x\left(\text{Artanh}\,(ax)\right)^2 - \frac{\left(\text{Artanh}\,(ax)\right)^2}{x} + \frac{a^3x^2\text{Artanh}\,(ax)}{3} - \frac{8\,a\text{Artanh}\,(ax)\ln(ax-1)}{3} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x)
```

```
[Out] 1/3*a^4*x^3*arctanh(a*x)^2-2*a^2*x*arctanh(a*x)^2-arctanh(a*x)^2/x+1/3*a^3*
x^2*arctanh(a*x)-8/3*a*arctanh(a*x)*ln(a*x-1)+2*a*arctanh(a*x)*ln(a*x)-8/3*
a*arctanh(a*x)*ln(a*x+1)+1/3*a^2*x+1/6*a*ln(a*x-1)-1/6*a*ln(a*x+1)-a*dilog(
a*x)-a*dilog(a*x+1)-a*ln(a*x)*ln(a*x+1)-2/3*a*ln(a*x-1)^2+8/3*a*dilog(1/2+1
/2*a*x)+4/3*a*ln(a*x-1)*ln(1/2+1/2*a*x)+4/3*a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a
*x)-4/3*a*ln(-1/2*a*x+1/2)*ln(a*x+1)+2/3*a*ln(a*x+1)^2
```

Maxima [A] time = 0.976506, size = 270, normalized size = 1.73

$$\frac{1}{6}a^2\left(\frac{2\left(ax+2\log(ax+1)^2-4\log(ax+1)\log(ax-1)-2\log(ax-1)^2\right)}{a}+\frac{16\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}a\right.\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*(2*(a*x + 2*log(a*x + 1)^2 - 4*log(a*x + 1)*log(a*x - 1) - 2*log(a*
x - 1)^2)/a + 16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/
a - 6*(log(a*x + 1)*log(x) + dilog(-a*x))/a + 6*(log(-a*x + 1)*log(x) + dil
og(a*x))/a - log(a*x + 1)/a + log(a*x - 1)/a) + 1/3*(a^2*x^2 - 8*log(a*x +
1) - 8*log(a*x - 1) + 6*log(x))*a*arctanh(a*x) + 1/3*(a^4*x^3 - 6*a^2*x - 3
/x)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\,(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^2\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^2, x)
```

3.210 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=162

$$-a^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)+a^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)+2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-2a^2\tanh^{-1}(ax)\text{Pc}$$

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] - ArcTanh[a*x]^2/(2*x^2) + (a^4*
x^2*ArcTanh[a*x]^2)/2 - 4*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2
*Log[x] + 2*a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - 2*a^2*ArcTanh[a*
x]*PolyLog[2, -1 + 2/(1 - a*x)] - a^2*PolyLog[3, 1 - 2/(1 - a*x)] + a^2*Pol
yLog[3, -1 + 2/(1 - a*x)]
```

Rubi [A] time = 0.461116, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {6012, 5916, 5982, 266, 36, 29, 31, 5948, 5914, 6052, 6058, 6610, 5980, 5910, 260}

$$-a^2\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)+a^2\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)+2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)-2a^2\tanh^{-1}(ax)\text{Pc}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] - ArcTanh[a*x]^2/(2*x^2) + (a^4*
x^2*ArcTanh[a*x]^2)/2 - 4*a^2*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] + a^2
*Log[x] + 2*a^2*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] - 2*a^2*ArcTanh[a*
x]*PolyLog[2, -1 + 2/(1 - a*x)] - a^2*PolyLog[3, 1 - 2/(1 - a*x)] + a^2*Pol
yLog[3, -1 + 2/(1 - a*x)]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
```

]

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^3}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^3} - \frac{2a^2\tanh^{-1}(ax)^2}{x} + a^4x\tanh^{-1}(ax)^2\right)dx$$

$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x}\,dx\right) + a^4\int x\tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a\int \frac{\tan}{x^2(}$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + a\int \frac{\tan}{}$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax$$

$$= -\frac{a\tanh^{-1}(ax)}{x} + a^3x\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^4x^2\tanh^{-1}(ax)^2 - 4a^2\tanh^{-1}(ax$$

Mathematica [A] time = 0.0699802, size = 183, normalized size = 1.13

$$a^2\text{PolyLog}\left(3,\frac{-ax-1}{ax-1}\right) - a^2\text{PolyLog}\left(3,\frac{ax+1}{ax-1}\right) - 2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{-ax-1}{ax-1}\right) + 2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] -((a*ArcTanh[a*x])/x) + a^3*x*ArcTanh[a*x] + (a^2*(-1 + a^2*x^2)*ArcTanh[a*
x]^2)/2 + ((-1 + a^2*x^2)*ArcTanh[a*x]^2)/(2*x^2) - 4*a^2*ArcTanh[a*x]^2*Ar
cTanh[1 - 2/(1 - a*x)] + a^2*Log[x] - 2*a^2*ArcTanh[a*x]*PolyLog[2, (-1 - a
*x)/(-1 + a*x)] + 2*a^2*ArcTanh[a*x]*PolyLog[2, (1 + a*x)/(-1 + a*x)] + a^2
*PolyLog[3, (-1 - a*x)/(-1 + a*x)] - a^2*PolyLog[3, (1 + a*x)/(-1 + a*x)]
```

Maple [C] time = 1.15, size = 774, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x)
```

```
[Out] a^3*x*arctanh(a*x)+1/2*a^4*x^2*arctanh(a*x)^2-1/2*arctanh(a*x)^2/x^2-I*a^2*
Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-
a*arctanh(a*x)/x+4*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2*polylog(
3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-2*a^2
*arctanh(a*x)^2*ln(a*x)+2*a^2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)-2
*a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*arctanh(a*x)^2*l
n(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-4*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*
x^2+1)^(1/2))-4*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a
^2*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+I*a^2*Pi*csgn(I*((a*x+1)
^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2*arctanh(a*x)^2-a^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+a^2*ln(1+(a*x+1)/(
-a^2*x^2+1)^(1/2))+a^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-I*a^2*Pi*csgn(I*((a
*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+I*a^2*
arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^
2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16}\left(2x^2\log(-ax+1) - a\left(\frac{ax^2+2x}{a^2} + \frac{2\log(ax-1)}{a^3}\right)\right)a^4 - \frac{1}{2}a^4\int x\log(ax+1)\log(-ax+1)\,dx + \frac{1}{4}a^3\int ax\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*x^2*log(-a*x + 1) - a*((a*x^2 + 2*x)/a^2 + 2*log(a*x - 1)/a^3))*a^
4 - 1/2*a^4*integrate(x*log(a*x + 1)*log(-a*x + 1), x) + 1/4*a^3*integrate(
a*x*log(a*x + 1)^2, x) + 1/4*a^3*integrate(log(a*x + 1)^2/(a^3*x^3), x) + 1
/4*(a*x - (a*x - 1)*log(-a*x + 1) - 1)*a^2 - 1/2*a^2*integrate(log(a*x + 1)
^2/x, x) + a^2*integrate(log(a*x + 1)*log(-a*x + 1)/x, x) - 1/4*a^2*integra
te(log(-a*x + 1)/x, x) - 1/4*(a*(log(a*x - 1) - log(x)) - log(-a*x + 1)/x)*
a + 1/8*(a^4*x^4 - 1)*log(-a*x + 1)^2/x^2 - 1/2*integrate(log(a*x + 1)*log(
-a*x + 1)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^4 - 2a^2x^2 + 1\right)\text{artanh}(ax)^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^2(ax)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^2}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^3, x)
```

3.211 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^4} dx$

Optimal. Leaf size=167

$$-a^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\frac{5}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}+a^4x\tanh^{-1}(ax)^2-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-}$$

```
[Out] -a^2/(3*x) + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (2*a^2*ArcTanh[a*x]^2)/x + a^4*x*Ar
cTanh[a*x]^2 - 2*a^3*ArcTanh[a*x]*Log[2/(1 - a*x)] - (10*a^3*ArcTanh[a*x]*L
og[2 - 2/(1 + a*x)])/3 - a^3*PolyLog[2, 1 - 2/(1 - a*x)] + (5*a^3*PolyLog[2
, -1 + 2/(1 + a*x)])/3
```

Rubi [A] time = 0.430452, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6012, 5910, 5984, 5918, 2402, 2315, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$-a^3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+\frac{5}{3}a^3\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{3x}+a^4x\tanh^{-1}(ax)^2-\frac{2}{3}a^3\tanh^{-1}(ax)^2+\frac{1}{3}a^3\tanh^{-}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```


```
[Out] -a^2/(3*x) + (a^3*ArcTanh[a*x])/3 - (a*ArcTanh[a*x])/(3*x^2) - (2*a^3*ArcTa
nh[a*x]^2)/3 - ArcTanh[a*x]^2/(3*x^3) + (2*a^2*ArcTanh[a*x]^2)/x + a^4*x*Ar
cTanh[a*x]^2 - 2*a^3*ArcTanh[a*x]*Log[2/(1 - a*x)] - (10*a^3*ArcTanh[a*x]*L
og[2 - 2/(1 + a*x)])/3 - a^3*PolyLog[2, 1 - 2/(1 - a*x)] + (5*a^3*PolyLog[2
, -1 + 2/(1 + a*x)])/3
```


Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
```

```
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^4}\,dx = \int \left(a^4 \tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{x^4} - \frac{2a^2 \tanh^{-1}(ax)^2}{x^2}\right) dx$$

$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\right) + a^4 \int \tanh^{-1}(ax)^2\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x \tanh^{-1}(ax)^2 + \frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx -$$

$$= -a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x \tanh^{-1}(ax)^2 + \frac{1}{3}(2a)\int$$

$$= -\frac{a \tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x \tanh^{-1}$$

$$= -\frac{a^2}{3x} - \frac{a \tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2 \tanh^{-1}(ax)^2}{x} + a^4 x$$

$$= -\frac{a^2}{3x} + \frac{1}{3}a^3 \tanh^{-1}(ax) - \frac{a \tanh^{-1}(ax)}{3x^2} - \frac{2}{3}a^3 \tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{3x^3} + \frac{2a^2}{}$$

Mathematica [A] time = 0.0688928, size = 153, normalized size = 0.92

$$\frac{1}{3}\left(3a^3\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 5a^3\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right) - \frac{a^2}{x} + 3a^4 x \tanh^{-1}(ax)^2 - 8a^3 \tanh^{-1}(ax)^2 + a^3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] (-(a^2/x) + a^3*ArcTanh[a*x] - (a*ArcTanh[a*x])/x^2 - 8*a^3*ArcTanh[a*x]^2
- ArcTanh[a*x]^2/x^3 + (6*a^2*ArcTanh[a*x]^2)/x + 3*a^4*x*ArcTanh[a*x]^2 -
10*a^3*ArcTanh[a*x]*Log[1 - E^(-2*ArcTanh[a*x])] - 6*a^3*ArcTanh[a*x]*Log[1
 + E^(-2*ArcTanh[a*x])] + 3*a^3*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 5*a^3*Po
lyLog[2, E^(-2*ArcTanh[a*x])])/3
```

Maple [A] time = 0.061, size = 249, normalized size = 1.5

$$a^4 x\left(\text{Artanh}\,(ax)\right)^2 + 2\,\frac{a^2\left(\text{Artanh}\,(ax)\right)^2}{x} - \frac{\left(\text{Artanh}\,(ax)\right)^2}{3\,x^3} + \frac{8\,a^3\text{Artanh}\,(ax)\ln(ax-1)}{3} - \frac{a\text{Artanh}\,(ax)}{3\,x^2} - \frac{10\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x)
```

```
[Out] a^4*x*arctanh(a*x)^2+2*a^2*arctanh(a*x)^2/x-1/3*arctanh(a*x)^2/x^3+8/3*a^3*
arctanh(a*x)*ln(a*x-1)-1/3*a*arctanh(a*x)/x^2-10/3*a^3*arctanh(a*x)*ln(a*x)
+8/3*a^3*arctanh(a*x)*ln(a*x+1)-1/3*a^2/x-1/6*a^3*ln(a*x-1)+1/6*a^3*ln(a*x+
1)+5/3*a^3*dilog(a*x)+5/3*a^3*dilog(a*x+1)+5/3*a^3*ln(a*x)*ln(a*x+1)+2/3*a^
3*ln(a*x-1)^2-8/3*a^3*dilog(1/2+1/2*a*x)-4/3*a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-
2/3*a^3*ln(a*x+1)^2-4/3*a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+4/3*a^3*ln(-1/
2*a*x+1/2)*ln(a*x+1)
```

Maxima [A] time = 0.989153, size = 274, normalized size = 1.64

$$-\frac{1}{6}\left(16\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-10\left(\log(ax+1)\log(x)+\mathrm{Li}_2(-ax)\right)a+10\left(\log(-ax+1)\log(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 10*(
log(a*x + 1)*log(x) + dilog(-a*x))*a + 10*(log(-a*x + 1)*log(x) + dilog(a*x
))*a - a*log(a*x + 1) + a*log(a*x - 1) + 2*(2*a*x*log(a*x + 1)^2 - 4*a*x*lo
g(a*x + 1)*log(a*x - 1) - 2*a*x*log(a*x - 1)^2 + 1)/x)*a^2 + 1/3*(8*a^2*log
(a*x + 1) + 8*a^2*log(a*x - 1) - 10*a^2*log(x) - 1/x^2)*a*arctanh(a*x) + 1/
3*(3*a^4*x + (6*a^2*x^2 - 1)/x^3)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\,(ax)^2}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^2\,(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**4,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2x^2-1\right)^2\mathrm{artanh}\,(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^4, x)
```

3.212 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^5} dx$

Optimal. Leaf size=214

$$\frac{1}{2}a^4\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}a^4\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+a^4\tanh^{-1}(ax)\text{Po}$$

```
[Out] -a^2/(12*x^2) - (a*ArcTanh[a*x])/(6*x^3) + (3*a^3*ArcTanh[a*x])/(2*x) - (3*
a^4*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(4*x^4) + (a^2*ArcTanh[a*x]^2)/x^2 +
 2*a^4*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (4*a^4*Log[x])/3 + (2*a^4*
Log[1 - a^2*x^2])/3 - a^4*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] + a^4*Ar
cTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + (a^4*PolyLog[3, 1 - 2/(1 - a*x)])
/2 - (a^4*PolyLog[3, -1 + 2/(1 - a*x)])/2
```

Rubi [A] time = 0.549401, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6012, 5916, 5982, 266, 44, 36, 29, 31, 5948, 5914, 6052, 6058, 6610}

$$\frac{1}{2}a^4\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)-\frac{1}{2}a^4\text{PolyLog}\left(3,\frac{2}{1-ax}-1\right)-a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)+a^4\tanh^{-1}(ax)\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```


```
[Out] -a^2/(12*x^2) - (a*ArcTanh[a*x])/(6*x^3) + (3*a^3*ArcTanh[a*x])/(2*x) - (3*
a^4*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(4*x^4) + (a^2*ArcTanh[a*x]^2)/x^2 +
 2*a^4*ArcTanh[a*x]^2*ArcTanh[1 - 2/(1 - a*x)] - (4*a^4*Log[x])/3 + (2*a^4*
Log[1 - a^2*x^2])/3 - a^4*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)] + a^4*Ar
cTanh[a*x]*PolyLog[2, -1 + 2/(1 - a*x)] + (a^4*PolyLog[3, 1 - 2/(1 - a*x)])
/2 - (a^4*PolyLog[3, -1 + 2/(1 - a*x)])/2
```


Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)])/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
 e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^5}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^5} - \frac{2a^2\tanh^{-1}(ax)^2}{x^3} + \frac{a^4\tanh^{-1}(ax)^2}{x}\right)dx$$

$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^5}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2} + 2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + \frac{1}{2}a\int \frac{\tanh}{x^4\left(1\right.}$$

$$= -\frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2} + 2a^4\tanh^{-1}(ax)^2\tanh^{-1}\left(1-\frac{2}{1-ax}\right) + \frac{1}{2}a\int \frac{\tanh}{x}$$

$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{2a^3\tanh^{-1}(ax)}{x} - a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2} +$$

$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2}$$

$$= -\frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh^{-1}(ax)^2}{x^2}$$

$$= -\frac{a^2}{12x^2} - \frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh}{x}$$

$$= -\frac{a^2}{12x^2} - \frac{a\tanh^{-1}(ax)}{6x^3} + \frac{3a^3\tanh^{-1}(ax)}{2x} - \frac{3}{4}a^4\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{4x^4} + \frac{a^2\tanh}{x}$$

Mathematica [C] time = 0.359733, size = 238, normalized size = 1.11

$$\frac{1}{24}\left(24a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 24a^4\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) + 12a^4\text{PolyLog}\left(3,-e^{-2\tan}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^5,x]
```

```
[Out] (2*a^4 + I*a^4*Pi^3 - (2*a^2)/x^2 - (4*a*ArcTanh[a*x])/x^3 + (36*a^3*ArcTan
h[a*x])/x - 18*a^4*ArcTanh[a*x]^2 - (6*ArcTanh[a*x]^2)/x^4 + (24*a^2*ArcTan
h[a*x]^2)/x^2 - 16*a^4*ArcTanh[a*x]^3 - 24*a^4*ArcTanh[a*x]^2*Log[1 + E^(-2
*ArcTanh[a*x])] + 24*a^4*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 32*a^
4*Log[(a*x)/Sqrt[1 - a^2*x^2]] + 24*a^4*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcT
anh[a*x])] + 24*a^4*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + 12*a^4*Po
lyLog[3, -E^(-2*ArcTanh[a*x])] - 12*a^4*PolyLog[3, E^(2*ArcTanh[a*x])])/24
```

Maple [C] time = 2.052, size = 927, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x)
```

```
[Out] 4/3*a^4*arctanh(a*x)-3/4*a^4*arctanh(a*x)^2-1/4*arctanh(a*x)^2/x^4-1/6*a*ar
ctanh(a*x)/x^3+3/2*a^3*arctanh(a*x)/x-1/12*a^4/(a*x+1-(-a^2*x^2+1)^(1/2))*(
-a^2*x^2+1)^(1/2)+1/12*a^4/((-a^2*x^2+1)^(1/2)+a*x+1)*(-a^2*x^2+1)^(1/2)+a^
2*arctanh(a*x)^2/x^2+a^4*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*
a^4*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a^4*arctanh(a*
x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I*a^4*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)
-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctan
h(a*x)^2+a^4*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^4*arctanh(
a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^4*arctanh(a*x)*polylog(2,-(a*
x+1)^2/(-a^2*x^2+1))+a^4*arctanh(a*x)^2*ln(a*x)-a^4*arctanh(a*x)^2*ln((a*x+
1)^2/(-a^2*x^2+1)-1)-1/24*a^5/((-a^2*x^2+1)^(1/2)-1)*x+1/24*a^5/((-a^2*x^2+
1)^(1/2)+1)*x-2*a^4*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-4/3*a^4*ln(1+(a*x
+1)/(-a^2*x^2+1)^(1/2))-2*a^4*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^
4*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-4/3*a^4*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-
1)+1/24*a^4/((-a^2*x^2+1)^(1/2)-1)-1/24*a^4/((-a^2*x^2+1)^(1/2)+1)+1/2*I*a^
4*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctan
h(a*x)^2+1/2*I*a^4*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(4\,a^2x^2-1\right)\log\left(-ax+1\right)^2}{16\,x^4}-\frac{1}{4}\int-\frac{2\left(a^5x^5-a^4x^4-2\,a^3x^3+2\,a^2x^2+ax-1\right)\log\left(ax+1\right)^2-\left(4\,a^3x^3-ax+4\left(a^5x^5\right.\right.}{2\left(ax^6-x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*(4*a^2*x^2 - 1)*log(-a*x + 1)^2/x^4 - 1/4*integrate(-1/2*(2*(a^5*x^5 -
 a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 - (4*a^3*x^3 - a
*x + 4*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1))*
log(-a*x + 1))/(a*x^6 - x^5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\left(ax\right)^2}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ax-1\right)^2\left(ax+1\right)^2\mathrm{atanh}^2\left(ax\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**5,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^5, x)
```

3.213 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^6} \, dx$

Optimal. Leaf size=157

$$-\frac{8}{15}a^5\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{30x^3}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{11a^4}{30x}+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{11}{30}a^5\tan$$

```
[Out] -a^2/(30*x^3) + (11*a^4)/(30*x) - (11*a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x
])/(10*x^4) + (7*a^3*ArcTanh[a*x])/(15*x^2) + (8*a^5*ArcTanh[a*x]^2)/15 - A
rcTanh[a*x]^2/(5*x^5) + (2*a^2*ArcTanh[a*x]^2)/(3*x^3) - (a^4*ArcTanh[a*x]^
2)/x + (16*a^5*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/15 - (8*a^5*PolyLog[2, -1
 + 2/(1 + a*x)])/15
```

Rubi [A] time = 0.594464, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6012, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{8}{15}a^5\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^2}{30x^3}+\frac{7a^3\tanh^{-1}(ax)}{15x^2}+\frac{2a^2\tanh^{-1}(ax)^2}{3x^3}+\frac{11a^4}{30x}+\frac{8}{15}a^5\tanh^{-1}(ax)^2-\frac{11}{30}a^5\tan$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] -a^2/(30*x^3) + (11*a^4)/(30*x) - (11*a^5*ArcTanh[a*x])/30 - (a*ArcTanh[a*x
])/(10*x^4) + (7*a^3*ArcTanh[a*x])/(15*x^2) + (8*a^5*ArcTanh[a*x]^2)/15 - A
rcTanh[a*x]^2/(5*x^5) + (2*a^2*ArcTanh[a*x]^2)/(3*x^3) - (a^4*ArcTanh[a*x]^
2)/x + (16*a^5*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)])/15 - (8*a^5*PolyLog[2, -1
 + 2/(1 + a*x)])/15
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
```

```
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^6}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^6} - \frac{2a^2\tanh^{-1}(ax)^2}{x^4} + \frac{a^4\tanh^{-1}(ax)^2}{x^2}\right)dx$$

$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{2a^2\tanh^{-1}(ax)^2}{3x^3} - \frac{a^4\tanh^{-1}(ax)^2}{x} + \frac{1}{5}(2a)\int \frac{\tanh^{-1}(ax)}{x^5\left(1-a^2x^2\right)}\,dx - \frac{1}{3}\,(4$$

$$= a^5\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{2a^2\tanh^{-1}(ax)^2}{3x^3} - \frac{a^4\tanh^{-1}(ax)^2}{x} + \frac{1}{5}(2a)\int \frac{\tanh^{-1}}{x^5}$$

$$= -\frac{a\tanh^{-1}(ax)}{10x^4} + \frac{2a^3\tanh^{-1}(ax)}{3x^2} + \frac{1}{3}a^5\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{5x^5} + \frac{2a^2\tanh^{-1}(ax)}{3x^3}$$

$$= -\frac{a^2}{30x^3} + \frac{2a^4}{3x} - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{7a^3\tanh^{-1}(ax)}{15x^2} + \frac{8}{15}a^5\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{5x^5} +$$

$$= -\frac{a^2}{30x^3} + \frac{11a^4}{30x} - \frac{2}{3}a^5\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{7a^3\tanh^{-1}(ax)}{15x^2} + \frac{8}{15}a^5\tanh^{-1}(ax)$$

$$= -\frac{a^2}{30x^3} + \frac{11a^4}{30x} - \frac{11}{30}a^5\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{10x^4} + \frac{7a^3\tanh^{-1}(ax)}{15x^2} + \frac{8}{15}a^5\tanh^{-1}(a$$

Mathematica [A] time = 0.761504, size = 118, normalized size = 0.75

$$\frac{-16a^5x^5\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)+a^2x^2\left(11a^2x^2-1\right)+2(ax-1)^3\left(8a^2x^2+9ax+3\right)\tanh^{-1}(ax)^2+ax\tanh^{-1}(ax)\left(\right.}{30x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^6,x]
```

```
[Out] (a^2*x^2*(-1 + 11*a^2*x^2) + 2*(-1 + a*x)^3*(3 + 9*a*x + 8*a^2*x^2)*ArcTanh
[a*x]^2 + a*x*ArcTanh[a*x]*(-3 + 14*a^2*x^2 - 11*a^4*x^4 + 32*a^4*x^4*Log[1
 - E^(-2*ArcTanh[a*x])]) - 16*a^5*x^5*PolyLog[2, E^(-2*ArcTanh[a*x])])/(30*
x^5)
```

Maple [A] time = 0.066, size = 272, normalized size = 1.7

$$-\frac{a^4(\text{Artanh}(ax))^2}{x}-\frac{(\text{Artanh}(ax))^2}{5x^5}+\frac{2a^2(\text{Artanh}(ax))^2}{3x^3}-\frac{8a^5\text{Artanh}(ax)\ln(ax-1)}{15}-\frac{a\text{Artanh}(ax)}{10x^4}+\frac{7a^3\text{A}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x)
```

```
[Out] -a^4*arctanh(a*x)^2/x-1/5*arctanh(a*x)^2/x^5+2/3*a^2*arctanh(a*x)^2/x^3-8/1
5*a^5*arctanh(a*x)*ln(a*x-1)-1/10*a*arctanh(a*x)/x^4+7/15*a^3*arctanh(a*x)/
x^2+16/15*a^5*arctanh(a*x)*ln(a*x)-8/15*a^5*arctanh(a*x)*ln(a*x+1)+11/60*a^
5*ln(a*x-1)-1/30*a^2/x^3+11/30*a^4/x-11/60*a^5*ln(a*x+1)-8/15*a^5*dilog(a*x
)-8/15*a^5*dilog(a*x+1)-8/15*a^5*ln(a*x)*ln(a*x+1)-2/15*a^5*ln(a*x-1)^2+8/1
5*a^5*dilog(1/2+1/2*a*x)+4/15*a^5*ln(a*x-1)*ln(1/2+1/2*a*x)+2/15*a^5*ln(a*x
+1)^2+4/15*a^5*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-4/15*a^5*ln(-1/2*a*x+1/2)*l
n(a*x+1)
```

Maxima [A] time = 0.988413, size = 323, normalized size = 2.06

$$\frac{1}{60}\left(32\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^3-32\left(\log(ax+1)\log(x)+\text{Li}_2(-ax)\right)a^3+32\left(\log(-ax+1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/60*(32*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^3 - 32
*(log(a*x + 1)*log(x) + dilog(-a*x))*a^3 + 32*(log(-a*x + 1)*log(x) + dilog
(a*x))*a^3 - 11*a^3*log(a*x + 1) + 11*a^3*log(a*x - 1) + 2*(4*a^3*x^3*log(a
*x + 1)^2 - 8*a^3*x^3*log(a*x + 1)*log(a*x - 1) - 4*a^3*x^3*log(a*x - 1)^2
+ 11*a^2*x^2 - 1)/x^3)*a^2 - 1/30*(16*a^4*log(a^2*x^2 - 1) - 16*a^4*log(x^2
) - (14*a^2*x^2 - 3)/x^4)*a*arctanh(a*x) - 1/15*(15*a^4*x^4 - 10*a^2*x^2 +
3)*arctanh(a*x)^2/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\,(ax)^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)^2\,(ax+1)^2\,\text{atanh}^2\,(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**6,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 - 1\right)^2 \text{artanh}\,(ax)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^6, x)
```

3.214 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^7} dx$

Optimal. Leaf size=113

$$\frac{7a^4}{90x^2} - \frac{a^2}{60x^4} - \frac{4}{45}a^6 \log\left(1 - a^2x^2\right) + \frac{2a^3 \tanh^{-1}(ax)}{9x^3} - \frac{\left(1 - a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{8}{45}a^6 \log(x) - \frac{a^5 \tanh^{-1}(ax)}{3x} - \frac{a}{}$$

```
[Out] -a^2/(60*x^4) + (7*a^4)/(90*x^2) - (a*ArcTanh[a*x])/(15*x^5) + (2*a^3*ArcTa
nh[a*x])/(9*x^3) - (a^5*ArcTanh[a*x])/(3*x) - ((1 - a^2*x^2)^3*ArcTanh[a*x]
^2)/(6*x^6) + (8*a^6*Log[x])/45 - (4*a^6*Log[1 - a^2*x^2])/45
```

Rubi [A] time = 0.194307, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6008, 6012, 5916, 266, 44, 36, 29, 31}

$$\frac{7a^4}{90x^2} - \frac{a^2}{60x^4} - \frac{4}{45}a^6 \log\left(1 - a^2x^2\right) + \frac{2a^3 \tanh^{-1}(ax)}{9x^3} - \frac{\left(1 - a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{8}{45}a^6 \log(x) - \frac{a^5 \tanh^{-1}(ax)}{3x} - \frac{a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

```
[Out] -a^2/(60*x^4) + (7*a^4)/(90*x^2) - (a*ArcTanh[a*x])/(15*x^5) + (2*a^3*ArcTa
nh[a*x])/(9*x^3) - (a^5*ArcTanh[a*x])/(3*x) - ((1 - a^2*x^2)^3*ArcTanh[a*x]
^2)/(6*x^6) + (8*a^6*Log[x])/45 - (4*a^6*Log[1 - a^2*x^2])/45
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^7}\,dx = -\frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{x^6}\,dx$$

$$= -\frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a\int \left(\frac{\tanh^{-1}(ax)}{x^6} - \frac{2a^2\tanh^{-1}(ax)}{x^4} + \frac{a^4\tanh^{-1}(ax)}{x^2}\right) d$$

$$= -\frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{3}a\int \frac{\tanh^{-1}(ax)}{x^6}\,dx - \frac{1}{3}\left(2a^3\right)\int \frac{\tanh^{-1}(ax)}{x^4}\,dx + \frac{1}{3}a^5\int$$

$$= -\frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{15}a^2$$

$$= -\frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{30}a^2$$

$$= -\frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{6x^6} + \frac{1}{30}a^2$$

$$= -\frac{a^2}{60x^4} + \frac{7a^4}{90x^2} - \frac{a\tanh^{-1}(ax)}{15x^5} + \frac{2a^3\tanh^{-1}(ax)}{9x^3} - \frac{a^5\tanh^{-1}(ax)}{3x} - \frac{\left(1-a^2x^2\right)^3 \tanh}{6x^6}$$

Mathematica [A] time = 0.0592525, size = 99, normalized size = 0.88

$$\frac{a^2x^2\left(14a^2x^2+32a^4x^4\log(x)-16a^4x^4\log\left(1-a^2x^2\right)-3\right)+30\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2-4ax\left(15a^4x^4-10a^2x^2+3\right)\tan}{180x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^7,x]
```

```
[Out] (-4*a*x*(3 - 10*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x] + 30*(-1 + a^2*x^2)^3*Ar
cTanh[a*x]^2 + a^2*x^2*(-3 + 14*a^2*x^2 + 32*a^4*x^4*Log[x] - 16*a^4*x^4*Lo
```

```
g[1 - a^2*x^2]))/(180*x^6)
```

Maple [B] time = 0.06, size = 233, normalized size = 2.1

$$\frac{a^2 (\text{Artanh}(ax))^2}{2x^4} - \frac{(\text{Artanh}(ax))^2}{6x^6} - \frac{a^4 (\text{Artanh}(ax))^2}{2x^2} - \frac{a^6\text{Artanh}(ax)\ln(ax-1)}{6} - \frac{a\text{Artanh}(ax)}{15x^5} + \frac{2a^3\text{Artan}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x)
```

```
[Out] 1/2*a^2*arctanh(a*x)^2/x^4-1/6*arctanh(a*x)^2/x^6-1/2*a^4*arctanh(a*x)^2/x^
2-1/6*a^6*arctanh(a*x)*ln(a*x-1)-1/15*a*arctanh(a*x)/x^5+2/9*a^3*arctanh(a*
x)/x^3-1/3*a^5*arctanh(a*x)/x+1/6*a^6*arctanh(a*x)*ln(a*x+1)-1/24*a^6*ln(a*
x-1)^2+1/12*a^6*ln(a*x-1)*ln(1/2+1/2*a*x)-1/24*a^6*ln(a*x+1)^2-1/12*a^6*ln(
-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/12*a^6*ln(-1/2*a*x+1/2)*ln(a*x+1)-4/45*a^6*
ln(a*x-1)-1/60*a^2/x^4+7/90*a^4/x^2+8/45*a^6*ln(a*x)-4/45*a^6*ln(a*x+1)
```

Maxima [A] time = 0.96366, size = 254, normalized size = 2.25

$$\frac{1}{360}\left(64\,a^4\log(x) - \frac{15\,a^4x^4\log(ax+1)^2 + 15\,a^4x^4\log(ax-1)^2 + 32\,a^4x^4\log(ax-1) - 28\,a^2x^2 - 2\left(15\,a^4x^4\log(ax - \right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/360*(64*a^4*log(x) - (15*a^4*x^4*log(a*x + 1)^2 + 15*a^4*x^4*log(a*x - 1)
^2 + 32*a^4*x^4*log(a*x - 1) - 28*a^2*x^2 - 2*(15*a^4*x^4*log(a*x - 1) - 16
*a^4*x^4)*log(a*x + 1) + 6)/x^4)*a^2 + 1/90*(15*a^5*log(a*x + 1) - 15*a^5*l
og(a*x - 1) - 2*(15*a^4*x^4 - 10*a^2*x^2 + 3)/x^5)*a*arctanh(a*x) - 1/6*(3*
a^4*x^4 - 3*a^2*x^2 + 1)*arctanh(a*x)^2/x^6
```

Fricas [A] time = 2.11722, size = 300, normalized size = 2.65

$$-\frac{32\,a^6x^6\log\left(a^2x^2-1\right) - 64\,a^6x^6\log(x) - 28\,a^4x^4 + 6\,a^2x^2 - 15\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 + 4\left(15\,a^5x\right.}{360\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/360*(32*a^6*x^6*log(a^2*x^2 - 1) - 64*a^6*x^6*log(x) - 28*a^4*x^4 + 6*a^
2*x^2 - 15*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))^
2 + 4*(15*a^5*x^5 - 10*a^3*x^3 + 3*a*x)*log(-(a*x + 1)/(a*x - 1)))/x^6
```

Sympy [A] time = 4.79153, size = 148, normalized size = 1.31

$$\begin{cases} \frac{8a^6\log(x)}{45} - \frac{8a^6\log\left(x-\frac{1}{a}\right)}{45} + \frac{a^6\operatorname{atanh}^2(ax)}{6} - \frac{8a^6\operatorname{atanh}(ax)}{45} - \frac{a^5\operatorname{atanh}(ax)}{3x} - \frac{a^4\operatorname{atanh}^2(ax)}{2x^2} + \frac{7a^4}{90x^2} + \frac{2a^3\operatorname{atanh}(ax)}{9x^3} + \frac{a^2\operatorname{atanh}^2(ax)}{2x^4} - \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**7,x)
```

```
[Out] Piecewise((8*a**6*log(x)/45 - 8*a**6*log(x - 1/a)/45 + a**6*atanh(a*x)**2/6
 - 8*a**6*atanh(a*x)/45 - a**5*atanh(a*x)/(3*x) - a**4*atanh(a*x)**2/(2*x**
2) + 7*a**4/(90*x**2) + 2*a**3*atanh(a*x)/(9*x**3) + a**2*atanh(a*x)**2/(2*
x**4) - a**2/(60*x**4) - a*atanh(a*x)/(15*x**5) - atanh(a*x)**2/(6*x**6), N
e(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^7, x)
```

3.215 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^8} dx$

Optimal. Leaf size=183

$$-\frac{8}{105}a^7\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{17a^4}{630x^3}-\frac{a^2}{105x^5}-\frac{8a^5\tanh^{-1}(ax)}{105x^2}-\frac{a^4\tanh^{-1}(ax)^2}{3x^3}+\frac{9a^3\tanh^{-1}(ax)}{70x^4}+\frac{2a^2\tanh}{5x}$$

```
[Out] -a^2/(105*x^5) + (17*a^4)/(630*x^3) + a^6/(210*x) - (a^7*ArcTanh[a*x])/210
- (a*ArcTanh[a*x])/(21*x^6) + (9*a^3*ArcTanh[a*x])/(70*x^4) - (8*a^5*ArcTan
h[a*x])/(105*x^2) + (8*a^7*ArcTanh[a*x]^2)/105 - ArcTanh[a*x]^2/(7*x^7) + (
2*a^2*ArcTanh[a*x]^2)/(5*x^5) - (a^4*ArcTanh[a*x]^2)/(3*x^3) + (16*a^7*ArcT
anh[a*x]*Log[2 - 2/(1 + a*x)])/105 - (8*a^7*PolyLog[2, -1 + 2/(1 + a*x)])/1
05
```

Rubi [A] time = 0.81617, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6012, 5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{8}{105}a^7\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{17a^4}{630x^3}-\frac{a^2}{105x^5}-\frac{8a^5\tanh^{-1}(ax)}{105x^2}-\frac{a^4\tanh^{-1}(ax)^2}{3x^3}+\frac{9a^3\tanh^{-1}(ax)}{70x^4}+\frac{2a^2\tanh}{5x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

```
[Out] -a^2/(105*x^5) + (17*a^4)/(630*x^3) + a^6/(210*x) - (a^7*ArcTanh[a*x])/210
- (a*ArcTanh[a*x])/(21*x^6) + (9*a^3*ArcTanh[a*x])/(70*x^4) - (8*a^5*ArcTan
h[a*x])/(105*x^2) + (8*a^7*ArcTanh[a*x]^2)/105 - ArcTanh[a*x]^2/(7*x^7) + (
2*a^2*ArcTanh[a*x]^2)/(5*x^5) - (a^4*ArcTanh[a*x]^2)/(3*x^3) + (16*a^7*ArcT
anh[a*x]*Log[2 - 2/(1 + a*x)])/105 - (8*a^7*PolyLog[2, -1 + 2/(1 + a*x)])/1
05
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{x^8}\,dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^8} - \frac{2a^2\tanh^{-1}(ax)^2}{x^6} + \frac{a^4\tanh^{-1}(ax)^2}{x^4}\right)dx$$
$$= -\left(\left(2a^2\right)\int \frac{\tanh^{-1}(ax)^2}{x^6}\,dx\right) + a^4\int \frac{\tanh^{-1}(ax)^2}{x^4}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^8}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{7x^7} + \frac{2a^2\tanh^{-1}(ax)^2}{5x^5} - \frac{a^4\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{7}(2a)\int \frac{\tanh^{-1}(ax)}{x^7\left(1-a^2x^2\right)}\,dx - \frac{1}{5}$$
$$= -\frac{\tanh^{-1}(ax)^2}{7x^7} + \frac{2a^2\tanh^{-1}(ax)^2}{5x^5} - \frac{a^4\tanh^{-1}(ax)^2}{3x^3} + \frac{1}{7}(2a)\int \frac{\tanh^{-1}(ax)}{x^7}\,dx + \frac{1}{7}$$
$$= -\frac{a\tanh^{-1}(ax)}{21x^6} + \frac{a^3\tanh^{-1}(ax)}{5x^4} - \frac{a^5\tanh^{-1}(ax)}{3x^2} + \frac{1}{3}a^7\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{7x^7}$$
$$= -\frac{a^2}{105x^5} + \frac{a^4}{15x^3} - \frac{a^6}{3x} - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} + \frac{a^5\tanh^{-1}(ax)}{15x^2} - \frac{1}{15}a^7\,\mathrm{ta}$$
$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{4a^6}{15x} + \frac{1}{3}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} - \frac{8a^5}{}$$
$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{a^6}{210x} - \frac{4}{15}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} - 8$$
$$= -\frac{a^2}{105x^5} + \frac{17a^4}{630x^3} + \frac{a^6}{210x} - \frac{1}{210}a^7\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{21x^6} + \frac{9a^3\tanh^{-1}(ax)}{70x^4} -$$

Mathematica [A] time = 1.38135, size = 140, normalized size = 0.77

$$\frac{-48a^7x^7\mathrm{PolyLog}\left(2, e^{-2\tanh^{-1}(ax)}\right) + a^2x^2\left(3a^4x^4+17a^2x^2-6\right) + 6\left(8a^7x^7-35a^4x^4+42a^2x^2-15\right)\tanh^{-1}(ax)^2 + 3a}{630x^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^8,x]
```

```
[Out] (a^2*x^2*(-6 + 17*a^2*x^2 + 3*a^4*x^4) + 6*(-15 + 42*a^2*x^2 - 35*a^4*x^4 +
 8*a^7*x^7)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[a*x]*(-10 + 27*a^2*x^2 - 16*a^4*
x^4 - a^6*x^6 + 32*a^6*x^6*Log[1 - E^(-2*ArcTanh[a*x])]) - 48*a^7*x^7*PolyL
og[2, E^(-2*ArcTanh[a*x])])/(630*x^7)
```

Maple [A] time = 0.064, size = 292, normalized size = 1.6

$$-\frac{(\mathrm{Artanh}\,(ax))^2}{7\,x^7} + \frac{2\,a^2\,(\mathrm{Artanh}\,(ax))^2}{5\,x^5} - \frac{a^4\,(\mathrm{Artanh}\,(ax))^2}{3\,x^3} - \frac{8\,a^7\mathrm{Artanh}\,(ax)\ln(ax-1)}{105} - \frac{a\mathrm{Artanh}\,(ax)}{21\,x^6} + \frac{9\,a^3\mathrm{A}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x)
```

```
[Out] -1/7*arctanh(a*x)^2/x^7+2/5*a^2*arctanh(a*x)^2/x^5-1/3*a^4*arctanh(a*x)^2/x
^3-8/105*a^7*arctanh(a*x)*ln(a*x-1)-1/21*a*arctanh(a*x)/x^6+9/70*a^3*arctan
h(a*x)/x^4-8/105*a^5*arctanh(a*x)/x^2+16/105*a^7*arctanh(a*x)*ln(a*x)-8/105
*a^7*arctanh(a*x)*ln(a*x+1)-8/105*a^7*dilog(a*x)-8/105*a^7*dilog(a*x+1)-8/1
```

```
05*a^7*ln(a*x)*ln(a*x+1)-2/105*a^7*ln(a*x-1)^2+8/105*a^7*dilog(1/2+1/2*a*x)
+4/105*a^7*ln(a*x-1)*ln(1/2+1/2*a*x)+2/105*a^7*ln(a*x+1)^2+4/105*a^7*ln(-1/
2*a*x+1/2)*ln(1/2+1/2*a*x)-4/105*a^7*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/420*a^7*l
n(a*x-1)+1/210*a^6/x-1/105*a^2/x^5+17/630*a^4/x^3-1/420*a^7*ln(a*x+1)
```

Maxima [A] time = 0.994082, size = 343, normalized size = 1.87

$$\frac{1}{1260}\left(96\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a^5-96\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)a^5+96\left(\log\left(-ax+1\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="maxima")
```

```
[Out] 1/1260*(96*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a^5 -
96*(log(a*x + 1)*log(x) + dilog(-a*x))*a^5 + 96*(log(-a*x + 1)*log(x) + dil
og(a*x))*a^5 - 3*a^5*log(a*x + 1) + 3*a^5*log(a*x - 1) + 2*(12*a^5*x^5*log(
a*x + 1)^2 - 24*a^5*x^5*log(a*x + 1)*log(a*x - 1) - 12*a^5*x^5*log(a*x - 1)
^2 + 3*a^4*x^4 + 17*a^2*x^2 - 6)/x^5)*a^2 - 1/210*(16*a^6*log(a^2*x^2 - 1)
- 16*a^6*log(x^2) + (16*a^4*x^4 - 27*a^2*x^2 + 10)/x^6)*a*arctanh(a*x) - 1/
105*(35*a^4*x^4 - 42*a^2*x^2 + 15)*arctanh(a*x)^2/x^7
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\left(ax\right)^2}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2/x^8, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ax-1\right)^2\left(ax+1\right)^2\mathrm{atanh}^2\left(ax\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**8,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2/x**8, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a^2x^2-1\right)^2\mathrm{artanh}\left(ax\right)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^8,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^8, x)
```

3.216 $\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^9} \, dx$

Optimal. Leaf size=170

$$\frac{5a^6}{504x^2} + \frac{a^4}{84x^4} - \frac{a^2}{168x^6} - \frac{2}{63}a^8 \log\left(1-a^2x^2\right) - \frac{a^5 \tanh^{-1}(ax)}{36x^3} - \frac{a^4 \tanh^{-1}(ax)^2}{4x^4} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} + \frac{a^2 \tanh^{-1}(ax)^2}{3x^6} +$$

```
[Out] -a^2/(168*x^6) + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a*ArcTanh[a*x])/(28*x^
7) + (a^3*ArcTanh[a*x])/(12*x^5) - (a^5*ArcTanh[a*x])/(36*x^3) - (a^7*ArcTa
nh[a*x])/(12*x) + (a^8*ArcTanh[a*x]^2)/24 - ArcTanh[a*x]^2/(8*x^8) + (a^2*A
rcTanh[a*x]^2)/(3*x^6) - (a^4*ArcTanh[a*x]^2)/(4*x^4) + (4*a^8*Log[x])/63 -
 (2*a^8*Log[1 - a^2*x^2])/63
```

Rubi [A] time = 0.840865, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 56, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6012, 5916, 5982, 266, 44, 36, 29, 31, 5948}

$$\frac{5a^6}{504x^2} + \frac{a^4}{84x^4} - \frac{a^2}{168x^6} - \frac{2}{63}a^8 \log\left(1-a^2x^2\right) - \frac{a^5 \tanh^{-1}(ax)}{36x^3} - \frac{a^4 \tanh^{-1}(ax)^2}{4x^4} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} + \frac{a^2 \tanh^{-1}(ax)^2}{3x^6} +$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

```
[Out] -a^2/(168*x^6) + a^4/(84*x^4) + (5*a^6)/(504*x^2) - (a*ArcTanh[a*x])/(28*x^
7) + (a^3*ArcTanh[a*x])/(12*x^5) - (a^5*ArcTanh[a*x])/(36*x^3) - (a^7*ArcTa
nh[a*x])/(12*x) + (a^8*ArcTanh[a*x]^2)/24 - ArcTanh[a*x]^2/(8*x^8) + (a^2*A
rcTanh[a*x]^2)/(3*x^6) - (a^4*ArcTanh[a*x]^2)/(4*x^4) + (4*a^8*Log[x])/63 -
 (2*a^8*Log[1 - a^2*x^2])/63
```

Rule 6012

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d
 + e, 0] && IGtQ[p, 0] && IGtQ[q, 1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}{x^9} dx = \int \left(\frac{\tanh^{-1}(ax)^2}{x^9} - \frac{2a^2 \tanh^{-1}(ax)^2}{x^7} + \frac{a^4 \tanh^{-1}(ax)^2}{x^5}\right) dx$$

$$= -\left((2a^2)\int \frac{\tanh^{-1}(ax)^2}{x^7} dx\right) + a^4 \int \frac{\tanh^{-1}(ax)^2}{x^5} dx + \int \frac{\tanh^{-1}(ax)^2}{x^9} dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2 \tanh^{-1}(ax)^2}{3x^6} - \frac{a^4 \tanh^{-1}(ax)^2}{4x^4} + \frac{1}{4}a \int \frac{\tanh^{-1}(ax)}{x^8(1-a^2x^2)} dx - \frac{1}{3}(2a^3)$$

$$= -\frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2 \tanh^{-1}(ax)^2}{3x^6} - \frac{a^4 \tanh^{-1}(ax)^2}{4x^4} + \frac{1}{4}a \int \frac{\tanh^{-1}(ax)}{x^8} dx + \frac{1}{4}a^3 \int$$

$$= -\frac{a \tanh^{-1}(ax)}{28x^7} + \frac{2a^3 \tanh^{-1}(ax)}{15x^5} - \frac{a^5 \tanh^{-1}(ax)}{6x^3} - \frac{\tanh^{-1}(ax)^2}{8x^8} + \frac{a^2 \tanh^{-1}(ax)^2}{3x^6} -$$

$$= -\frac{a \tanh^{-1}(ax)}{28x^7} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} + \frac{a^5 \tanh^{-1}(ax)}{18x^3} - \frac{a^7 \tanh^{-1}(ax)}{2x} + \frac{1}{4}a^8 \tanh^{-1}(ax)^2$$

$$= -\frac{a \tanh^{-1}(ax)}{28x^7} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} - \frac{a^5 \tanh^{-1}(ax)}{36x^3} + \frac{a^7 \tanh^{-1}(ax)}{6x} - \frac{1}{12}a^8 \tanh^{-1}(ax)^2$$

$$= -\frac{a^2}{168x^6} + \frac{41a^4}{1680x^4} - \frac{29a^6}{840x^2} - \frac{a \tanh^{-1}(ax)}{28x^7} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} - \frac{a^5 \tanh^{-1}(ax)}{36x^3} - \frac{a^7 \tanh}{1}$$

$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{13a^6}{252x^2} - \frac{a \tanh^{-1}(ax)}{28x^7} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} - \frac{a^5 \tanh^{-1}(ax)}{36x^3} - \frac{a^7 \tanh}{12x}$$

$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a \tanh^{-1}(ax)}{28x^7} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} - \frac{a^5 \tanh^{-1}(ax)}{36x^3} - \frac{a^7 \tanh}{12x}$$

$$= -\frac{a^2}{168x^6} + \frac{a^4}{84x^4} + \frac{5a^6}{504x^2} - \frac{a \tanh^{-1}(ax)}{28x^7} + \frac{a^3 \tanh^{-1}(ax)}{12x^5} - \frac{a^5 \tanh^{-1}(ax)}{36x^3} - \frac{a^7 \tanh}{12x}$$

Mathematica [A] time = 0.064794, size = 124, normalized size = 0.73

$$\frac{a^2x^2\left(5a^4x^4 + 6a^2x^2 + 32a^6x^6\log(x) - 16a^6x^6\log\left(1-a^2x^2\right) - 3\right) + 21\left(a^2x^2+3\right)\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2 - 2ax\left(21a^6x^6\right.}{504x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^2*ArcTanh[a*x]^2)/x^9,x]
```

```
[Out] (-2*a*x*(9 - 21*a^2*x^2 + 7*a^4*x^4 + 21*a^6*x^6)*ArcTanh[a*x] + 21*(-1 + a
^2*x^2)^3*(3 + a^2*x^2)*ArcTanh[a*x]^2 + a^2*x^2*(-3 + 6*a^2*x^2 + 5*a^4*x^
4 + 32*a^6*x^6*Log[x] - 16*a^6*x^6*Log[1 - a^2*x^2]))/(504*x^8)
```

Maple [A] time = 0.062, size = 253, normalized size = 1.5

$$-\frac{(\mathrm{Artanh}(ax))^2}{8x^8} - \frac{a^4(\mathrm{Artanh}(ax))^2}{4x^4} + \frac{a^2(\mathrm{Artanh}(ax))^2}{3x^6} - \frac{a^8\mathrm{Artanh}(ax)\ln(ax-1)}{24} - \frac{a\mathrm{Artanh}(ax)}{28x^7} + \frac{a^3\mathrm{Artanh}(}{12x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x)
```

```
[Out] -1/8*arctanh(a*x)^2/x^8-1/4*a^4*arctanh(a*x)^2/x^4+1/3*a^2*arctanh(a*x)^2/x
^6-1/24*a^8*arctanh(a*x)*ln(a*x-1)-1/28*a*arctanh(a*x)/x^7+1/12*a^3*arctanh
```

```
(a*x)/x^5-1/36*a^5*arctanh(a*x)/x^3-1/12*a^7*arctanh(a*x)/x+1/24*a^8*arctan
h(a*x)*ln(a*x+1)-1/96*a^8*ln(a*x-1)^2+1/48*a^8*ln(a*x-1)*ln(1/2+1/2*a*x)-1/
96*a^8*ln(a*x+1)^2+1/48*a^8*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/48*a^8*ln(-1/2*a*x
+1/2)*ln(1/2+1/2*a*x)-2/63*a^8*ln(a*x-1)-1/168*a^2/x^6+1/84*a^4/x^4+5/504*a
^6/x^2+4/63*a^8*ln(a*x)-2/63*a^8*ln(a*x+1)
```

Maxima [A] time = 0.974479, size = 275, normalized size = 1.62

$$\frac{1}{2016}\left(128\,a^6\log(x) - \frac{21\,a^6x^6\log(ax+1)^2 + 21\,a^6x^6\log(ax-1)^2 + 64\,a^6x^6\log(ax-1) - 20\,a^4x^4 - 24\,a^2x^2 - 2\left(21\,a\right.}{x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2016*(128*a^6*log(x) - (21*a^6*x^6*log(a*x + 1)^2 + 21*a^6*x^6*log(a*x -
1)^2 + 64*a^6*x^6*log(a*x - 1) - 20*a^4*x^4 - 24*a^2*x^2 - 2*(21*a^6*x^6*lo
g(a*x - 1) - 32*a^6*x^6)*log(a*x + 1) + 12)/x^6)*a^2 + 1/504*(21*a^7*log(a*
x + 1) - 21*a^7*log(a*x - 1) - 2*(21*a^6*x^6 + 7*a^4*x^4 - 21*a^2*x^2 + 9)/
x^7)*a*arctanh(a*x) - 1/24*(6*a^4*x^4 - 8*a^2*x^2 + 3)*arctanh(a*x)^2/x^8
```

Fricas [A] time = 1.99544, size = 338, normalized size = 1.99

$$-\frac{64\,a^8x^8\log\left(a^2x^2-1\right) - 128\,a^8x^8\log(x) - 20\,a^6x^6 - 24\,a^4x^4 + 12\,a^2x^2 - 21\left(a^8x^8 - 6\,a^4x^4 + 8\,a^2x^2 - 3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{2016\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="fricas")
```

```
[Out] -1/2016*(64*a^8*x^8*log(a^2*x^2 - 1) - 128*a^8*x^8*log(x) - 20*a^6*x^6 - 24
*a^4*x^4 + 12*a^2*x^2 - 21*(a^8*x^8 - 6*a^4*x^4 + 8*a^2*x^2 - 3)*log(-(a*x
+ 1)/(a*x - 1))^2 + 4*(21*a^7*x^7 + 7*a^5*x^5 - 21*a^3*x^3 + 9*a*x)*log(-(a
*x + 1)/(a*x - 1)))/x^8
```

Sympy [A] time = 5.916, size = 168, normalized size = 0.99

$$\begin{cases} \frac{4a^8\log(x)}{63} - \frac{4a^8\log\left(x-\frac{1}{a}\right)}{63} + \frac{a^8\operatorname{atanh}^2(ax)}{24} - \frac{4a^8\operatorname{atanh}(ax)}{63} - \frac{a^7\operatorname{atanh}(ax)}{12x} + \frac{5a^6}{504x^2} - \frac{a^5\operatorname{atanh}(ax)}{36x^3} - \frac{a^4\operatorname{atanh}^2(ax)}{4x^4} + \frac{a^4}{84x^4} + \frac{a^3\operatorname{atanh}}{12x^5} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**2/x**9,x)
```

```
[Out] Piecewise((4*a**8*log(x)/63 - 4*a**8*log(x - 1/a)/63 + a**8*atanh(a*x)**2/2
4 - 4*a**8*atanh(a*x)/63 - a**7*atanh(a*x)/(12*x) + 5*a**6/(504*x**2) - a**
5*atanh(a*x)/(36*x**3) - a**4*atanh(a*x)**2/(4*x**4) + a**4/(84*x**4) + a**
3*atanh(a*x)/(12*x**5) + a**2*atanh(a*x)**2/(3*x**6) - a**2/(168*x**6) - a*
atanh(a*x)/(28*x**7) - atanh(a*x)**2/(8*x**8), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2x^2-1)^2 \operatorname{artanh}(ax)^2}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^2/x^9,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^2/x^9, x)
```

3.217 $\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3\,dx$

Optimal. Leaf size=248

$$\frac{4\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{5a}-\frac{8\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a}-\frac{1-a^2x^2}{20a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(a$$

```
[Out] -(1 - a^2*x^2)/(20*a) - x*ArcTanh[a*x] - (x*(1 - a^2*x^2)*ArcTanh[a*x])/10
+ (2*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(5*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x]^
2)/(20*a) + (8*ArcTanh[a*x]^3)/(15*a) + (8*x*ArcTanh[a*x]^3)/15 + (4*x*(1 -
 a^2*x^2)*ArcTanh[a*x]^3)/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/5 - (8*Ar
cTanh[a*x]^2*Log[2/(1 - a*x)])/(5*a) - Log[1 - a^2*x^2]/(2*a) - (8*ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a) + (4*PolyLog[3, 1 - 2/(1 - a*x)])/(
5*a)
```

Rubi [A] time = 0.251565, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5944, 5910, 5984, 5918, 5948, 6058, 6610, 260, 5942}

$$\frac{4\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{5a}-\frac{8\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{5a}-\frac{1-a^2x^2}{20a}-\frac{\log\left(1-a^2x^2\right)}{2a}+\frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

```
[Out] -(1 - a^2*x^2)/(20*a) - x*ArcTanh[a*x] - (x*(1 - a^2*x^2)*ArcTanh[a*x])/10
+ (2*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(5*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x]^
2)/(20*a) + (8*ArcTanh[a*x]^3)/(15*a) + (8*x*ArcTanh[a*x]^3)/15 + (4*x*(1 -
 a^2*x^2)*ArcTanh[a*x]^3)/15 + (x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/5 - (8*Ar
cTanh[a*x]^2*Log[2/(1 - a*x)])/(5*a) - Log[1 - a^2*x^2]/(2*a) - (8*ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(5*a) + (4*PolyLog[3, 1 - 2/(1 - a*x)])/(
5*a)
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
```

```
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3\,dx = \frac{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}{20a} + \frac{1}{5}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3 - \frac{3}{10}\int \left(1-a^2x^2\right)\tanh^{-1}($$

$$= -\frac{1-a^2x^2}{20a} - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} + \frac{3\left(1-a^2x^2\right)^2 \text{t}}{20}$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} +$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} +$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} +$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} +$$

$$= -\frac{1-a^2x^2}{20a} - x\tanh^{-1}(ax) - \frac{1}{10}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{5a} +$$

Mathematica [A] time = 0.602464, size = 183, normalized size = 0.74

$$\frac{96\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 48\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 3a^2x^2 - 30\log\left(1-a^2x^2\right) + 12a^5x^5\tan}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^2*ArcTanh[a*x]^3,x]
```

```
[Out] (-3 + 3*a^2*x^2 - 66*a*x*ArcTanh[a*x] + 6*a^3*x^3*ArcTanh[a*x] + 33*ArcTanh
[a*x]^2 - 42*a^2*x^2*ArcTanh[a*x]^2 + 9*a^4*x^4*ArcTanh[a*x]^2 - 32*ArcTanh
[a*x]^3 + 60*a*x*ArcTanh[a*x]^3 - 40*a^3*x^3*ArcTanh[a*x]^3 + 12*a^5*x^5*Ar
cTanh[a*x]^3 - 96*ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - 30*Log[1 -
a^2*x^2] + 96*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 48*PolyLog[3,
 -E^(-2*ArcTanh[a*x])])/(60*a)
```

Maple [C] time = 1.285, size = 883, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2*arctanh(a*x)^3,x)
```

```
[Out] -1/20/a+11/20*arctanh(a*x)^2/a+8/15*arctanh(a*x)^3/a+x*arctanh(a*x)^3-11/10
*x*arctanh(a*x)+1/10*a^2*arctanh(a*x)*x^3+1/20*a*x^2-2/3*a^2*arctanh(a*x)^3
*x^3-7/10*a*arctanh(a*x)^2*x^2+4/5/a*arctanh(a*x)^2*ln(a*x-1)+4/5/a*arctanh
(a*x)^2*ln(a*x+1)-8/5/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+3/20*
a^3*arctanh(a*x)^2*x^4+1/5*a^4*arctanh(a*x)^3*x^5-8/5/a*arctanh(a*x)^2*ln(2
)-8/5/a*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-arctanh(a*x)/a+2/5*
I/a*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi+1/a*ln
((a*x+1)^2/(-a^2*x^2+1)+1)+4/5/a*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-4/5*I/a
*arctanh(a*x)^2*Pi-2/5*I/a*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
```

```
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi-2/5*I/a*arc
tanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*Pi-4/5*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1
)/(-a^2*x^2+1)^(1/2))*Pi+2/5*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi-4/5*I/a*ar
ctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi-2/5*I/a*arctanh(a*x)^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi+4/5*I/a*arctanh(a*x)^2*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*Pi-2/5*I/a*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2400*(36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 + 480*a*x - 60
*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/
8*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a -
 1/1440000*(288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x + 1
) - 6)*(a*x - 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*log
(-a*x + 1) - 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2
+ 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-a*x
+ 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 - 3*log
(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/432*(4*(9*log(-a*x + 1
)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x
 + 1)^3 - 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-
a*x + 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a - 1/8*in
tegrate(-1/150*(150*(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*l
og(a*x + 1)^3 + (36*a^5*x^5 - 45*a^4*x^4 - 140*a^3*x^3 + 210*a^2*x^2 - 450*
(a^5*x^5 - a^4*x^4 - 2*a^3*x^3 + 2*a^2*x^2 + a*x - 1)*log(a*x + 1)^2 + 480*
a*x - 60*(3*a^5*x^5 - 10*a^3*x^3 + 15*a*x + 8)*log(a*x + 1))*log(-a*x + 1))
/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{artanh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (ax-1)^2\,(ax+1)^2\,\text{atanh}^3\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2*atanh(a*x)**3,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*arctanh(a*x)^3, x)
```

3.218 $\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Rubi [A] time = 0.036213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx = \int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 0.903532, size = 0, normalized size = 0.

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x],x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x], x]
```

Maple [A] time = 0.32, size = 0, normalized size = 0.

$$\int \frac{x\left(-a^2x^2+1\right)^2}{\operatorname{Artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x*(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2x}{\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4x^5-2\,a^2x^3+x}{\operatorname{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(ax-1\right)^2\left(ax+1\right)^2}{\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2x}{\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x), x)
```

3.219 $\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)} dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Rubi [A] time = 0.0226321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)} dx = \int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)} dx$$

Mathematica [A] time = 0.477367, size = 0, normalized size = 0.

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x],x]
```

```
[Out] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x], x]
```

Maple [A] time = 0.328, size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^2}{\text{Artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4x^4-2\,a^2x^2+1}{\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)^2\,(ax+1)^2}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2-1\right)^2}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x), x)
```

3.220 $\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0514779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx = \int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.44732, size = 0, normalized size = 0.

$$\int \frac{(1-a^2x^2)^2}{x\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]),x]
```

```
[Out] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]), x]
```

Maple [A] time = 0.38, size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^2}{x\text{Artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

```
[Out] int((-a^2*x^2+1)^2/x/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2x^2-1)^2}{x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^4x^4-2\,a^2x^2+1}{x\,\mathrm{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax-1)^2\,(ax+1)^2}{x\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/x/atanh(a*x),x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a^2x^2-1)^2}{x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)), x)
```

3.221 $\int \frac{x(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Rubi [A] time = 0.0357664, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} \, dx = \int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} \, dx$$

Mathematica [A] time = 0.753182, size = 0, normalized size = 0.

$$\int \frac{x\left(1-a^2x^2\right)^2}{\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(x*(1 - a^2*x^2)^2)/ArcTanh[a*x]^2, x]
```

Maple [A] time = 0.379, size = 0, normalized size = 0.

$$\int \frac{x\left(-a^2x^2+1\right)^2}{\left(\text{Artanh}\,(ax)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(a^6x^7 - 3a^4x^5 + 3a^2x^3 - x\right)}{a\log(ax+1) - a\log(-ax+1)} + \int -\frac{2\left(7a^6x^6 - 15a^4x^4 + 9a^2x^2 - 1\right)}{a\log(ax+1) - a\log(-ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)/(a*log(a*x + 1) - a*log(-a*x + 1))
+ integrate(-2*(7*a^6*x^6 - 15*a^4*x^4 + 9*a^2*x^2 - 1)/(a*log(a*x + 1) - a
*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4x^5 - 2a^2x^3 + x}{\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 - 2*a^2*x^3 + x)/arctanh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(ax-1)^2(ax+1)^2}{\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x*(a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 - 1\right)^2 x}{\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2*x/arctanh(a*x)^2, x)
```

3.222 $\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]
```

Rubi [A] time = 0.0209154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} dx = \int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} dx$$

Mathematica [A] time = 1.10547, size = 0, normalized size = 0.

$$\int \frac{(1-a^2x^2)^2}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[(1 - a^2*x^2)^2/ArcTanh[a*x]^2, x]
```

Maple [A] time = 0.313, size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^2}{(\operatorname{Artanh}(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)}{a\log\left(ax+1\right) - a\log\left(-ax+1\right)} + \int -\frac{12\left(a^5x^5 - 2\,a^3x^3 + ax\right)}{\log\left(ax+1\right) - \log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*log(a*x + 1) - a*log(-a*x + 1))
+ integrate(-12*(a^5*x^5 - 2*a^3*x^3 + a*x)/(log(a*x + 1) - log(-a*x + 1)),
 x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4x^4 - 2\,a^2x^2 + 1}{\text{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2}{\text{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/atanh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 - 1\right)^2}{\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/arctanh(a*x)^2, x)
```

3.223 $\int \frac{(1-a^2x^2)^2}{x \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{(1-a^2x^2)^2}{x \tanh^{-1}(ax)^2}, x\right)$$

```
[Out] Unintegrable[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.0480228, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(1-a^2x^2)^2}{x \tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{(1-a^2x^2)^2}{x \tanh^{-1}(ax)^2} dx = \int \frac{(1-a^2x^2)^2}{x \tanh^{-1}(ax)^2} dx$$

Mathematica [A] time = 1.00985, size = 0, normalized size = 0.

$$\int \frac{(1-a^2x^2)^2}{x \tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[(1 - a^2*x^2)^2/(x*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.32, size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^2}{x (\text{Artanh}(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)}{ax\log\left(ax+1\right) - ax\log\left(-ax+1\right)} + \int -\frac{2\left(5\,a^6x^6 - 9\,a^4x^4 + 3\,a^2x^2 + 1\right)}{ax^2\log\left(ax+1\right) - ax^2\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)/(a*x*log(a*x + 1) - a*x*log(-a*x +
1)) + integrate(-2*(5*a^6*x^6 - 9*a^4*x^4 + 3*a^2*x^2 + 1)/(a*x^2*log(a*x +
 1) - a*x^2*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4x^4 - 2\,a^2x^2 + 1}{x\,\text{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)/(x*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax-1\right)^2\left(ax+1\right)^2}{x\,\text{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**2/x/atanh(a*x)**2,x)
```

```
[Out] Integral((a*x - 1)**2*(a*x + 1)**2/(x*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a^2x^2 - 1\right)^2}{x\,\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^2/x/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*x^2 - 1)^2/(x*arctanh(a*x)^2), x)
```

3.224 $\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=144

$$\frac{\left(1-a^2x^2\right)^3}{42a}+\frac{3\left(1-a^2x^2\right)^2}{70a}+\frac{4\left(1-a^2x^2\right)}{35a}+\frac{8\log\left(1-a^2x^2\right)}{35a}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax$$

```
[Out] (4*(1 - a^2*x^2))/(35*a) + (3*(1 - a^2*x^2)^2)/(70*a) + (1 - a^2*x^2)^3/(42
*a) + (16*x*ArcTanh[a*x])/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x])/35 + (6*x*(
1 - a^2*x^2)^2*ArcTanh[a*x])/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x])/7 + (8*L
og[1 - a^2*x^2])/(35*a)
```

Rubi [A] time = 0.0674981, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5942, 5910, 260}

$$\frac{\left(1-a^2x^2\right)^3}{42a}+\frac{3\left(1-a^2x^2\right)^2}{70a}+\frac{4\left(1-a^2x^2\right)}{35a}+\frac{8\log\left(1-a^2x^2\right)}{35a}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (4*(1 - a^2*x^2))/(35*a) + (3*(1 - a^2*x^2)^2)/(70*a) + (1 - a^2*x^2)^3/(42
*a) + (16*x*ArcTanh[a*x])/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x])/35 + (6*x*(
1 - a^2*x^2)^2*ArcTanh[a*x])/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x])/7 + (8*L
og[1 - a^2*x^2])/(35*a)
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax) + \frac{6}{7}\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax)\,dx$$
$$= \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{6}{35}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax) + \frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(a$$
$$= \frac{4\left(1-a^2x^2\right)}{35a} + \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{8}{35}x\left(1-a^2x^2\right)\tanh^{-1}(ax) + \frac{6}{35}x\left(1-a\right.$$
$$= \frac{4\left(1-a^2x^2\right)}{35a} + \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{16}{35}x\tanh^{-1}(ax) + \frac{8}{35}x\left(1-a^2x^2\right)\tanh$$
$$= \frac{4\left(1-a^2x^2\right)}{35a} + \frac{3\left(1-a^2x^2\right)^2}{70a} + \frac{\left(1-a^2x^2\right)^3}{42a} + \frac{16}{35}x\tanh^{-1}(ax) + \frac{8}{35}x\left(1-a^2x^2\right)\tanh$$

Mathematica [A] time = 0.0458864, size = 79, normalized size = 0.55

$$\frac{-5a^6x^6+24a^4x^4-57a^2x^2+48\log\left(1-a^2x^2\right)-6ax\left(5a^6x^6-21a^4x^4+35a^2x^2-35\right)\tanh^{-1}(ax)}{210a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (-57*a^2*x^2 + 24*a^4*x^4 - 5*a^6*x^6 - 6*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^
4 + 5*a^6*x^6)*ArcTanh[a*x] + 48*Log[1 - a^2*x^2])/(210*a)
```

Maple [A] time = 0.029, size = 88, normalized size = 0.6

$$-\frac{a^6\mathrm{Artanh}\,(ax)\,x^7}{7} + \frac{3\,a^4\mathrm{Artanh}\,(ax)\,x^5}{5} - a^2\mathrm{Artanh}\,(ax)\,x^3 + x\mathrm{Artanh}\,(ax) - \frac{a^5x^6}{42} + \frac{4\,x^4a^3}{35} - \frac{19\,ax^2}{70} + \frac{8\,\ln(ax\,-}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x),x)
```

```
[Out] -1/7*a^6*arctanh(a*x)*x^7+3/5*a^4*arctanh(a*x)*x^5-a^2*arctanh(a*x)*x^3+x*a
rctanh(a*x)-1/42*a^5*x^6+4/35*x^4*a^3-19/70*a*x^2+8/35/a*ln(a*x-1)+8/35/a*l
n(a*x+1)
```

Maxima [A] time = 0.966892, size = 111, normalized size = 0.77

$$-\frac{1}{210}\left(5\,a^4x^6 - 24\,a^2x^4 + 57\,x^2 - \frac{48\,\log(ax+1)}{a^2} - \frac{48\,\log(ax-1)}{a^2}\right)a - \frac{1}{35}\left(5\,a^6x^7 - 21\,a^4x^5 + 35\,a^2x^3 - 35\,x\right)\mathrm{artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/210*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*log(a*x
- 1)/a^2)*a - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3 - 35*x)*arctanh(a*x
)
```

Fricas [A] time = 2.39057, size = 198, normalized size = 1.38

$$-\frac{5\,a^{6}x^{6}-24\,a^{4}x^{4}+57\,a^{2}x^{2}+3\left(5\,a^{7}x^{7}-21\,a^{5}x^{5}+35\,a^{3}x^{3}-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-48\,\log\left(a^{2}x^{2}-1\right)}{210\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/210*(5*a^6*x^6 - 24*a^4*x^4 + 57*a^2*x^2 + 3*(5*a^7*x^7 - 21*a^5*x^5 + 3
5*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 48*log(a^2*x^2 - 1))/a
```

Sympy [A] time = 4.30289, size = 97, normalized size = 0.67

$$\begin{cases} -\frac{a^{6}x^{7}\operatorname{atanh}(ax)}{7}-\frac{a^{5}x^{6}}{42}+\frac{3a^{4}x^{5}\operatorname{atanh}(ax)}{5}+\frac{4a^{3}x^{4}}{35}-a^{2}x^{3}\operatorname{atanh}(ax)-\frac{19ax^{2}}{70}+x\operatorname{atanh}(ax)+\frac{16\log\left(x-\frac{1}{a}\right)}{35a}+\frac{16\operatorname{atanh}(ax)}{35a} & \text{for} \\ 0 & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x),x)
```

```
[Out] Piecewise((-a**6*x**7*atanh(a*x)/7 - a**5*x**6/42 + 3*a**4*x**5*atanh(a*x)/
5 + 4*a**3*x**4/35 - a**2*x**3*atanh(a*x) - 19*a*x**2/70 + x*atanh(a*x) + 1
6*log(x - 1/a)/(35*a) + 16*atanh(a*x)/(35*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.20726, size = 124, normalized size = 0.86

$$-\frac{1}{70}\left(5\,a^{6}x^{7}-21\,a^{4}x^{5}+35\,a^{2}x^{3}-35\,x\right)\log\left(-\frac{ax+1}{ax-1}\right)+\frac{8\,\log\left(\left|a^{2}x^{2}-1\right|\right)}{35\,a}-\frac{5\,a^{11}x^{6}-24\,a^{9}x^{4}+57\,a^{7}x^{2}}{210\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/70*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3 - 35*x)*log(-(a*x + 1)/(a*x - 1
) + 8/35*log(abs(a^2*x^2 - 1))/a - 1/210*(5*a^11*x^6 - 24*a^9*x^4 + 57*a^7*
x^2)/a^6
```

3.225 $\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=227

$$-\frac{16\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}-\frac{1}{105}a^4x^5+\frac{19a^2x^3}{315}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{8}{35}x($$

```
[Out] (-38*x)/105 + (19*a^2*x^3)/315 - (a^4*x^5)/105 + (8*(1 - a^2*x^2)*ArcTanh[a
*x])/(35*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x])/(35*a) + ((1 - a^2*x^2)^3*Ar
cTanh[a*x])/(21*a) + (16*ArcTanh[a*x]^2)/(35*a) + (16*x*ArcTanh[a*x]^2)/35
+ (8*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]
^2)/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2)/7 - (32*ArcTanh[a*x]*Log[2/(1 -
 a*x)])/(35*a) - (16*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a)
```

Rubi [A] time = 0.171278, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5944, 5910, 5984, 5918, 2402, 2315, 8, 194}

$$-\frac{16\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}-\frac{1}{105}a^4x^5+\frac{19a^2x^3}{315}+\frac{1}{7}x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2+\frac{6}{35}x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2+\frac{8}{35}x($$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

```
[Out] (-38*x)/105 + (19*a^2*x^3)/315 - (a^4*x^5)/105 + (8*(1 - a^2*x^2)*ArcTanh[a
*x])/(35*a) + (3*(1 - a^2*x^2)^2*ArcTanh[a*x])/(35*a) + ((1 - a^2*x^2)^3*Ar
cTanh[a*x])/(21*a) + (16*ArcTanh[a*x]^2)/(35*a) + (16*x*ArcTanh[a*x]^2)/35
+ (8*x*(1 - a^2*x^2)*ArcTanh[a*x]^2)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]
^2)/35 + (x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2)/7 - (32*ArcTanh[a*x]*Log[2/(1 -
 a*x)])/(35*a) - (16*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a)
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2\,dx = \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}{21a} + \frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2 - \frac{1}{21}\int \left(1-a^2x^2\right)^2\,dx + \frac{6}{7}\int (1$$

$$= \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}{21a} + \frac{6}{35}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2 + \frac{1}{7}$$

$$= -\frac{2x}{15} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{(1-a^2x}{}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{(1-a^2}{}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{(1-a^2}{}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{(1-a^2}{}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{(1-a^2}{}$$

$$= -\frac{38x}{105} + \frac{19a^2x^3}{315} - \frac{a^4x^5}{105} + \frac{8\left(1-a^2x^2\right)\tanh^{-1}(ax)}{35a} + \frac{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}{35a} + \frac{(1-a^2}{}$$

Mathematica [A] time = 1.18337, size = 124, normalized size = 0.55

$$-\frac{-144\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 3a^5x^5 - 19a^3x^3 + 9(ax-1)^4\left(5a^3x^3+20a^2x^2+29ax+16\right)\tanh^{-1}(ax)^2 + 3\tanh^{-1}}{315a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^2,x]
```

```
[Out] -(114*a*x - 19*a^3*x^3 + 3*a^5*x^5 + 9*(-1 + a*x)^4*(16 + 29*a*x + 20*a^2*x
^2 + 5*a^3*x^3)*ArcTanh[a*x]^2 + 3*ArcTanh[a*x]*(-38 + 57*a^2*x^2 - 24*a^4*
x^4 + 5*a^6*x^6 + 96*Log[1 + E^(-2*ArcTanh[a*x])]) - 144*PolyLog[2, -E^(-2*
ArcTanh[a*x])])/(315*a)
```

Maple [A] time = 0.049, size = 250, normalized size = 1.1

$$-\frac{a^6 (\mathrm{Artanh}(ax))^2 x^7}{7}+\frac{3 a^4 (\mathrm{Artanh}(ax))^2 x^5}{5}-a^2 (\mathrm{Artanh}(ax))^2 x^3+x (\mathrm{Artanh}(ax))^2-\frac{a^5 \mathrm{Artanh}(ax) x^6}{21}+\frac{8 a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x)^2,x)
```

```
[Out] -1/7*a^6*arctanh(a*x)^2*x^7+3/5*a^4*arctanh(a*x)^2*x^5-a^2*arctanh(a*x)^2*x
^3+x*arctanh(a*x)^2-1/21*a^5*arctanh(a*x)*x^6+8/35*a^3*arctanh(a*x)*x^4-19/
35*a*arctanh(a*x)*x^2+16/35/a*arctanh(a*x)*ln(a*x-1)+16/35/a*arctanh(a*x)*l
n(a*x+1)+4/35/a*ln(a*x-1)^2-16/35/a*dilog(1/2+1/2*a*x)-8/35/a*ln(a*x-1)*ln(
1/2+1/2*a*x)-4/35/a*ln(a*x+1)^2-8/35/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+8/3
5/a*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/105*a^4*x^5+19/315*x^3*a^2-38/105*x-19/105
/a*ln(a*x-1)+19/105/a*ln(a*x+1)
```

Maxima [A] time = 0.976557, size = 269, normalized size = 1.19

$$-\frac{1}{315}\,a^2\left(\frac{3 a^5 x^5-19 a^3 x^3+114 ax+36 \log(ax+1)^2-72 \log(ax+1)\log(ax-1)-36 \log(ax-1)^2+57 \log(ax-}{a^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/315*a^2*((3*a^5*x^5 - 19*a^3*x^3 + 114*a*x + 36*log(a*x + 1)^2 - 72*log(
a*x + 1)*log(a*x - 1) - 36*log(a*x - 1)^2 + 57*log(a*x - 1))/a^3 + 144*(log
(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3 - 57*log(a*x + 1)
/a^3) - 1/105*(5*a^4*x^6 - 24*a^2*x^4 + 57*x^2 - 48*log(a*x + 1)/a^2 - 48*l
og(a*x - 1)/a^2)*a*arctanh(a*x) - 1/35*(5*a^6*x^7 - 21*a^4*x^5 + 35*a^2*x^3
 - 35*x)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^6x^6-3a^4x^4+3a^2x^2-1\right)\mathrm{artanh}(ax)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int 3a^2x^2\operatorname{atanh}^2(ax)\,dx-\int -3a^4x^4\operatorname{atanh}^2(ax)\,dx-\int a^6x^6\operatorname{atanh}^2(ax)\,dx-\int -\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x)**2,x)
```

```
[Out] -Integral(3*a**2*x**2*atanh(a*x)**2, x) - Integral(-3*a**4*x**4*atanh(a*x)*
*2, x) - Integral(a**6*x**6*atanh(a*x)**2, x) - Integral(-atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^2, x)
```

3.226 $\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3\,dx$

Optimal. Leaf size=338

$$\frac{24\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{35a}-\frac{48\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{13\left(1-a^2x^2\right)}{210a}-\frac{7\log\left(1-a^2x^2\right)}{15a}+$$

```
[Out] (-13*(1 - a^2*x^2))/(210*a) - (1 - a^2*x^2)^2/(140*a) - (14*x*ArcTanh[a*x])
/15 - (13*x*(1 - a^2*x^2)*ArcTanh[a*x])/105 - (x*(1 - a^2*x^2)^2*ArcTanh[a*
x])/35 + (12*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(35*a) + (9*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/(70*a) + ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(14*a) + (16*ArcTanh[
a*x]^3)/(35*a) + (16*x*ArcTanh[a*x]^3)/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x]
^3)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/35 + (x*(1 - a^2*x^2)^3*ArcTa
nh[a*x]^3)/7 - (48*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(35*a) - (7*Log[1 - a^2
*x^2])/(15*a) - (48*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a) + (24*
PolyLog[3, 1 - 2/(1 - a*x)])/(35*a)
```

Rubi [A] time = 0.333408, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {5944, 5910, 5984, 5918, 5948, 6058, 6610, 260, 5942}

$$\frac{24\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{35a}-\frac{48\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{35a}-\frac{\left(1-a^2x^2\right)^2}{140a}-\frac{13\left(1-a^2x^2\right)}{210a}-\frac{7\log\left(1-a^2x^2\right)}{15a}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```


```
[Out] (-13*(1 - a^2*x^2))/(210*a) - (1 - a^2*x^2)^2/(140*a) - (14*x*ArcTanh[a*x])
/15 - (13*x*(1 - a^2*x^2)*ArcTanh[a*x])/105 - (x*(1 - a^2*x^2)^2*ArcTanh[a*
x])/35 + (12*(1 - a^2*x^2)*ArcTanh[a*x]^2)/(35*a) + (9*(1 - a^2*x^2)^2*ArcT
anh[a*x]^2)/(70*a) + ((1 - a^2*x^2)^3*ArcTanh[a*x]^2)/(14*a) + (16*ArcTanh[
a*x]^3)/(35*a) + (16*x*ArcTanh[a*x]^3)/35 + (8*x*(1 - a^2*x^2)*ArcTanh[a*x]
^3)/35 + (6*x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3)/35 + (x*(1 - a^2*x^2)^3*ArcTa
nh[a*x]^3)/7 - (48*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/(35*a) - (7*Log[1 - a^2
*x^2])/(15*a) - (48*ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/(35*a) + (24*
PolyLog[3, 1 - 2/(1 - a*x)])/(35*a)
```


Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3\,dx = \frac{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}{14a} + \frac{1}{7}x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3 - \frac{1}{7}\int \left(1-a^2x^2\right)^2 \tanh^{-1}(ax$$

$$= -\frac{\left(1-a^2x^2\right)^2}{140a} - \frac{1}{35}x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax) + \frac{9\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}{70a} + \frac{\left(1-a^2x^2\right.}{}$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}x\left(1-a^2x^2\right)^2 \tanh$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}$$

$$= -\frac{13\left(1-a^2x^2\right)}{210a} - \frac{\left(1-a^2x^2\right)^2}{140a} - \frac{14}{15}x\tanh^{-1}(ax) - \frac{13}{105}x\left(1-a^2x^2\right)\tanh^{-1}(ax) - \frac{1}{35}$$

Mathematica [A] time = 1.09682, size = 231, normalized size = 0.68

$$-\frac{-576\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) - 288\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 3a^4x^4 - 32a^2x^2 + 196\log\left(1-a^2x\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^3*ArcTanh[a*x]^3,x]
```

```
[Out] -(29 - 32*a^2*x^2 + 3*a^4*x^4 + 456*a*x*ArcTanh[a*x] - 76*a^3*x^3*ArcTanh[a
*x] + 12*a^5*x^5*ArcTanh[a*x] - 228*ArcTanh[a*x]^2 + 342*a^2*x^2*ArcTanh[a*
x]^2 - 144*a^4*x^4*ArcTanh[a*x]^2 + 30*a^6*x^6*ArcTanh[a*x]^2 + 192*ArcTanh
[a*x]^3 - 420*a*x*ArcTanh[a*x]^3 + 420*a^3*x^3*ArcTanh[a*x]^3 - 252*a^5*x^5
*ArcTanh[a*x]^3 + 60*a^7*x^7*ArcTanh[a*x]^3 + 576*ArcTanh[a*x]^2*Log[1 + E^
(-2*ArcTanh[a*x])] + 196*Log[1 - a^2*x^2] - 576*ArcTanh[a*x]*PolyLog[2, -E^
(-2*ArcTanh[a*x])] - 288*PolyLog[3, -E^(-2*ArcTanh[a*x])])/(420*a)
```

Maple [C] time = 2.487, size = 932, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^3*arctanh(a*x)^3,x)
```

```
[Out] -29/420/a+19/35*arctanh(a*x)^2/a+16/35*arctanh(a*x)^3/a+x*arctanh(a*x)^3-38
/35*x*arctanh(a*x)-1/7*a^6*arctanh(a*x)^3*x^7-1/14*a^5*arctanh(a*x)^2*x^6-1
/35*a^4*arctanh(a*x)*x^5+19/105*a^2*arctanh(a*x)*x^3+8/105*a*x^2-a^2*arctan
h(a*x)^3*x^3-57/70*a*arctanh(a*x)^2*x^2+24/35/a*arctanh(a*x)^2*ln(a*x-1)+24
/35/a*arctanh(a*x)^2*ln(a*x+1)-48/35/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+
1)^(1/2))+12/35*a^3*arctanh(a*x)^2*x^4+3/5*a^4*arctanh(a*x)^3*x^5-48/35/a*a
```

```
rctanh(a*x)^2*ln(2)-48/35/a*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))
-1/140*x^4*a^3-14/15*arctanh(a*x)/a+14/15/a*ln((a*x+1)^2/(-a^2*x^2+1)+1)+24
/35/a*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-24/35*I/a*Pi*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-12/35*I/a*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a
*x)^2-12/35*I/a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+12/35*I/a*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2*arctanh(a*x)^2-24/35*I/a*Pi*arctanh(a*x)^2+12/35*I/a*Pi*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-12/35*I/a*Pi*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2-12/35*I/a*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2-24/35*I/a*Pi*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+24/35*I/a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2
+1)+1))^2*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/19600*(150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 840*a^4*x^4 + 1330*a^3*x
^3 - 1995*a^2*x^2 - 3360*a*x - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 - 3
5*a*x - 16)*log(a*x + 1))*log(-a*x + 1)^2/a - 1/8*(log(-a*x + 1)^3 - 3*log(
-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1)/a + 1/691488000*(36000*(343*lo
g(-a*x + 1)^3 - 147*log(-a*x + 1)^2 + 42*log(-a*x + 1) - 6)*(a*x - 1)^7 + 2
401000*(36*log(-a*x + 1)^3 - 18*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 1)*(a*x
 - 1)^6 + 2074464*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x +
 1) - 6)*(a*x - 1)^5 + 13505625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 +
12*log(-a*x + 1) - 3)*(a*x - 1)^4 + 48020000*(9*log(-a*x + 1)^3 - 9*log(-a*
x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 64827000*(4*log(-a*x + 1)^3 -
 6*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 86436000*(log(-a*x
+ 1)^3 - 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a - 1/480000*(
288*(125*log(-a*x + 1)^3 - 75*log(-a*x + 1)^2 + 30*log(-a*x + 1) - 6)*(a*x
- 1)^5 + 5625*(32*log(-a*x + 1)^3 - 24*log(-a*x + 1)^2 + 12*log(-a*x + 1) -
 3)*(a*x - 1)^4 + 40000*(9*log(-a*x + 1)^3 - 9*log(-a*x + 1)^2 + 6*log(-a*x
 + 1) - 2)*(a*x - 1)^3 + 90000*(4*log(-a*x + 1)^3 - 6*log(-a*x + 1)^2 + 6*l
og(-a*x + 1) - 3)*(a*x - 1)^2 + 180000*(log(-a*x + 1)^3 - 3*log(-a*x + 1)^2
 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/288*(4*(9*log(-a*x + 1)^3 - 9*log(
-a*x + 1)^2 + 6*log(-a*x + 1) - 2)*(a*x - 1)^3 + 27*(4*log(-a*x + 1)^3 - 6*
log(-a*x + 1)^2 + 6*log(-a*x + 1) - 3)*(a*x - 1)^2 + 108*(log(-a*x + 1)^3 -
 3*log(-a*x + 1)^2 + 6*log(-a*x + 1) - 6)*(a*x - 1))/a + 1/8*integrate(-1/1
225*(1225*(a^7*x^7 - a^6*x^6 - 3*a^5*x^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x^
2 - a*x + 1)*log(a*x + 1)^3 + (150*a^7*x^7 - 175*a^6*x^6 - 672*a^5*x^5 + 84
0*a^4*x^4 + 1330*a^3*x^3 - 1995*a^2*x^2 - 3675*(a^7*x^7 - a^6*x^6 - 3*a^5*x
^5 + 3*a^4*x^4 + 3*a^3*x^3 - 3*a^2*x^2 - a*x + 1)*log(a*x + 1)^2 - 3360*a*x
 - 210*(5*a^7*x^7 - 21*a^5*x^5 + 35*a^3*x^3 - 35*a*x - 16)*log(a*x + 1))*lo
g(-a*x + 1))/(a*x - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{artanh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int 3a^2x^2 \operatorname{atanh}^3(ax)\,dx - \int -3a^4x^4 \operatorname{atanh}^3(ax)\,dx - \int a^6x^6 \operatorname{atanh}^3(ax)\,dx - \int -\operatorname{atanh}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**3*atanh(a*x)**3,x)
```

```
[Out] -Integral(3*a**2*x**2*atanh(a*x)**3, x) - Integral(-3*a**4*x**4*atanh(a*x)*
*3, x) - Integral(a**6*x**6*atanh(a*x)**3, x) - Integral(-atanh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(a^2x^2 - 1\right)^3 \operatorname{artanh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^3*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*x^2 - 1)^3*arctanh(a*x)^3, x)
```

3.227 $\int \frac{x^3 \tanh^{-1}(ax)}{1-a^2x^2} dx$

Optimal. Leaf size=87

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{x}{2a^3}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}$$

```
[Out] -x/(2*a^3) + ArcTanh[a*x]/(2*a^4) - (x^2*ArcTanh[a*x])/(2*a^2) - ArcTanh[a*
x]^2/(2*a^4) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 + PolyLog[2, 1 - 2/(1 -
a*x)]/(2*a^4)
```

Rubi [A] time = 0.132883, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5980, 5916, 321, 206, 5984, 5918, 2402, 2315}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{x}{2a^3}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)}{2a^4}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -x/(2*a^3) + ArcTanh[a*x]/(2*a^4) - (x^2*ArcTanh[a*x])/(2*a^2) - ArcTanh[a*
x]^2/(2*a^4) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 + PolyLog[2, 1 - 2/(1 -
a*x)]/(2*a^4)
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\tanh^{-1}(ax)}{1-a^2x^2}\,dx &= -\frac{\int x\tanh^{-1}(ax)\,dx}{a^2}+\frac{\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^3}+\frac{\int \frac{x^2}{1-a^2x^2}\,dx}{2a}\\
&= -\frac{x}{2a^3}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}+\frac{\int \frac{1}{1-a^2x^2}\,dx}{2a^3}-\frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a^3}\\
&= -\frac{x}{2a^3}+\frac{\tanh^{-1}(ax)}{2a^4}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}+\frac{\text{Subst}\left(\int \frac{\log}{1-}\right.}{}\\
&= -\frac{x}{2a^3}+\frac{\tanh^{-1}(ax)}{2a^4}-\frac{x^2\tanh^{-1}(ax)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^4}+\frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4}+\frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right.}{2a^4}
\end{aligned}$$

Mathematica [A] time = 0.126848, size = 60, normalized size = 0.69

$$\frac{-\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(-a^2x^2+2\log\left(e^{-2\tanh^{-1}(ax)}+1\right)+1\right)-ax+\tanh^{-1}(ax)^2}{2a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] (-(a*x) + ArcTanh[a*x]^2 + ArcTanh[a*x]*(1 - a^2*x^2 + 2*Log[1 + E^(-2*ArcT
anh[a*x])]) - PolyLog[2, -E^(-2*ArcTanh[a*x])])/(2*a^4)
```

Maple [B] time = 0.049, size = 165, normalized size = 1.9

$$-\frac{x^2\mathrm{Artanh}\,(ax)}{2\,a^2}-\frac{\mathrm{Artanh}\,(ax)\ln\,(ax-1)}{2\,a^4}-\frac{\mathrm{Artanh}\,(ax)\ln\,(ax+1)}{2\,a^4}-\frac{x}{2\,a^3}-\frac{\ln\,(ax-1)}{4\,a^4}+\frac{\ln\,(ax+1)}{4\,a^4}-\frac{(\ln\,(ax-1))^2}{8\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] -1/2*x^2*arctanh(a*x)/a^2-1/2/a^4*arctanh(a*x)*ln(a*x-1)-1/2/a^4*arctanh(a*
x)*ln(a*x+1)-1/2*x/a^3-1/4/a^4*ln(a*x-1)+1/4/a^4*ln(a*x+1)-1/8/a^4*ln(a*x-1
)^2+1/2/a^4*dilog(1/2+1/2*a*x)+1/4/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)+1/4/a^4*ln
(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/4/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/8/a^4*l
n(a*x+1)^2
```

Maxima [A] time = 0.969405, size = 162, normalized size = 1.86

$$-\frac{1}{8}\,a\left(\frac{4\,ax-\log\,(ax+1)^2+2\,\log\,(ax+1)\log\,(ax-1)+\log\,(ax-1)^2+2\,\log\,(ax-1)}{a^5}-\frac{4\left(\log\,(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\right.}{a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*a*((4*a*x - log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1
)^2 + 2*log(a*x - 1))/a^5 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2
*a*x + 1/2))/a^5 - 2*log(a*x + 1)/a^5) - 1/2*(x^2/a^2 + log(a^2*x^2 - 1)/a^
4)*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{x^3\,\mathrm{artanh}\,(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3\,\mathrm{atanh}\,(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 \operatorname{artanh}(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)/(a^2*x^2 - 1), x)
```

3.228 $\int \frac{x^2 \tanh^{-1}(ax)}{1-a^2x^2} \, dx$

Optimal. Leaf size=42

$$-\frac{\log\left(1-a^2x^2\right)}{2a^3}+\frac{\tanh^{-1}(ax)^2}{2a^3}-\frac{x\tanh^{-1}(ax)}{a^2}$$

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

Rubi [A] time = 0.0692734, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5980, 5910, 260, 5948}

$$-\frac{\log\left(1-a^2x^2\right)}{2a^3}+\frac{\tanh^{-1}(ax)^2}{2a^3}-\frac{x\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{1-a^2x^2}\,dx = -\frac{\int \tanh^{-1}(ax)\,dx}{a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}$$
$$= -\frac{x\tanh^{-1}(ax)}{a^2} + \frac{\tanh^{-1}(ax)^2}{2a^3} + \frac{\int \frac{x}{1-a^2x^2}\,dx}{a}$$
$$= -\frac{x\tanh^{-1}(ax)}{a^2} + \frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{\log\left(1-a^2x^2\right)}{2a^3}$$

Mathematica [A] time = 0.0370791, size = 42, normalized size = 1.

$$-\frac{\log\left(1-a^2x^2\right)}{2a^3} + \frac{\tanh^{-1}(ax)^2}{2a^3} - \frac{x\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^2) + ArcTanh[a*x]^2/(2*a^3) - Log[1 - a^2*x^2]/(2*a^3)
```

Maple [B] time = 0.049, size = 145, normalized size = 3.5

$$-\frac{x\operatorname{Artanh}(ax)}{a^2} - \frac{\operatorname{Artanh}(ax)\ln(ax-1)}{2a^3} + \frac{\operatorname{Artanh}(ax)\ln(ax+1)}{2a^3} - \frac{(\ln(ax-1))^2}{8a^3} + \frac{\ln(ax-1)}{4a^3}\ln\left(\frac{1}{2}+\frac{ax}{2}\right) - \ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] -x*arctanh(a*x)/a^2-1/2/a^3*arctanh(a*x)*ln(a*x-1)+1/2/a^3*arctanh(a*x)*ln(
a*x+1)-1/8/a^3*ln(a*x-1)^2+1/4/a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-1/2/a^3*ln(a*x
-1)-1/2/a^3*ln(a*x+1)-1/4/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/4/a^3*ln(-
1/2*a*x+1/2)*ln(a*x+1)-1/8/a^3*ln(a*x+1)^2
```

Maxima [B] time = 0.970101, size = 115, normalized size = 2.74

$$-\frac{1}{2}\left(\frac{2x}{a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax) + \frac{2(\log(ax-1)-2)\log(ax+1) - \log(ax+1)^2 - \log(ax-1)^2 -}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x) + 1/8*(2*
(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log(a
*x - 1))/a^3
```

Fricas [A] time = 2.09577, size = 128, normalized size = 3.05

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right) - \log\left(-\frac{ax+1}{ax-1}\right)^2 + 4\,\log\left(a^2x^2-1\right)}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a*x - 1))^2 + 4*log
(a^2*x^2 - 1))/a^3
```

Sympy [A] time = 1.61955, size = 41, normalized size = 0.98

$$\begin{cases} -\frac{x \operatorname{atanh}(ax)}{a^2} - \frac{\log\left(x - \frac{1}{a}\right)}{a^3} + \frac{\operatorname{atanh}^2(ax)}{2a^3} - \frac{\operatorname{atanh}(ax)}{a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((-x*atanh(a*x)/a**2 - log(x - 1/a)/a**3 + atanh(a*x)**2/(2*a**3)
- atanh(a*x)/a**3, Ne(a, 0)), (0, True))
```

Giac [A] time = 1.19701, size = 80, normalized size = 1.9

$$-\frac{x \log\left(-\frac{ax+1}{ax-1}\right)}{2\,a^2} + \frac{\log\left(-\frac{ax+1}{ax-1}\right)^2}{8\,a^3} - \frac{\log\left(a^2x^2 - 1\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*x*log(-(a*x + 1)/(a*x - 1))/a^2 + 1/8*log(-(a*x + 1)/(a*x - 1))^2/a^3
- 1/2*log(a^2*x^2 - 1)/a^3
```

3.229 $\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx$

Optimal. Leaf size=54

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}$$

```
[Out] -ArcTanh[a*x]^2/(2*a^2) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^2 + PolyLog[2,
1 - 2/(1 - a*x)]/(2*a^2)
```

Rubi [A] time = 0.0706783, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5984, 5918, 2402, 2315}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^2}{2a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -ArcTanh[a*x]^2/(2*a^2) + (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^2 + PolyLog[2,
1 - 2/(1 - a*x)]/(2*a^2)
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^2}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^2}{2a^2} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} - \frac{\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^2}{2a^2} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1-ax}\right)}{a^2}$$
$$= -\frac{\tanh^{-1}(ax)^2}{2a^2} + \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2}$$

Mathematica [A] time = 0.0485874, size = 44, normalized size = 0.81

$$-\frac{\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\left(\tanh^{-1}(ax)+2\log\left(e^{-2\tanh^{-1}(ax)}+1\right)\right)}{2a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2),x]
```

```
[Out] -(-(ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 + E^(-2*ArcTanh[a*x])])) + PolyLog
[2, -E^(-2*ArcTanh[a*x])])/(2*a^2)
```

Maple [B] time = 0.046, size = 125, normalized size = 2.3

$$-\frac{\text{Artanh}(ax)\ln(ax-1)}{2a^2}-\frac{\text{Artanh}(ax)\ln(ax+1)}{2a^2}-\frac{(\ln(ax-1))^2}{8a^2}+\frac{1}{2a^2}\text{dilog}\left(\frac{1}{2}+\frac{ax}{2}\right)+\frac{\ln(ax-1)}{4a^2}\ln\left(\frac{1}{2}+\frac{ax}{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] -1/2/a^2*arctanh(a*x)*ln(a*x-1)-1/2/a^2*arctanh(a*x)*ln(a*x+1)-1/8/a^2*ln(a
*x-1)^2+1/2/a^2*dilog(1/2+1/2*a*x)+1/4/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)-1/4/a^
2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/4/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/8/a
^2*ln(a*x+1)^2
```

Maxima [B] time = 0.967308, size = 169, normalized size = 3.13

$$-\frac{1}{8}a\left(\frac{\log(ax+1)^2+2\log(ax+1)\log(ax-1)-\log(ax-1)^2}{a^3}-\frac{4\left(\log(ax-1)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\text{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)}{a^3}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*a*((log(a*x + 1)^2 + 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a^3
 - 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a^3) + 1/4*(
```

```
log(a*x + 1)/a - log(a*x - 1)/a)*log(a^2*x^2 - 1)/a - 1/2*arctanh(a*x)*log(
a^2*x^2 - 1)/a^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\,\text{artanh}\,(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\,\text{atanh}\,(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x\,\text{artanh}\,(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)/(a^2*x^2 - 1), x)
```

3.230 $\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

```
[Out] ArcTanh[a*x]^2/(2*a)
```

Rubi [A] time = 0.0157754, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^2/(2*a)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx = \frac{\tanh^{-1}(ax)^2}{2a}$$

Mathematica [A] time = 0.0042339, size = 13, normalized size = 1.

$$\frac{\tanh^{-1}(ax)^2}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^2/(2*a)
```

Maple [A] time = 0.025, size = 12, normalized size = 0.9

$$\frac{(\operatorname{Artanh}(ax))^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1),x)
```

```
[Out] 1/2*arctanh(a*x)^2/a
```

Maxima [B] time = 0.975797, size = 88, normalized size = 6.77

$$\frac{1}{2}\left(\frac{\log\left(ax+1\right)}{a}-\frac{\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)-\frac{\log\left(ax+1\right)^{2}-2\,\log\left(ax+1\right)\log\left(ax-1\right)+\log\left(ax-1\right)^{2}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x) - 1/8*(log(a*x + 1)^2 -
2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/a
```

Fricas [A] time = 1.87087, size = 47, normalized size = 3.62

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^{2}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

Sympy [A] time = 1.45779, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^{2}{\left(ax \right)}}{2a} & \text{for}\: a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**2/(2*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.13503, size = 30, normalized size = 2.31

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^{2}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/8*log(-(a*x + 1)/(a*x - 1))^2/a
```

3.231 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

```
[Out] ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1
 + a*x)]/2
```

Rubi [A] time = 0.0870035, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5988, 5932, 2447}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}\tanh^{-1}(ax)^2+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(1
 + a*x)]/2
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{2}\tanh^{-1}(ax)^2 + \int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx$$
$$= \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - a\int \frac{\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx$$
$$= \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{2}\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right)$$

Mathematica [A] time = 0.0669345, size = 42, normalized size = 0.93

$$\frac{1}{2}\left(\tanh^{-1}(ax)\left(\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)-\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)),x]
```

```
[Out] (ArcTanh[a*x]*(ArcTanh[a*x] + 2*Log[1 - E^(-2*ArcTanh[a*x])]) - PolyLog[2,
E^(-2*ArcTanh[a*x])])/2
```

Maple [B] time = 0.052, size = 130, normalized size = 2.9

$$-\frac{\mathrm{Artanh}\,(ax)\ln\,(ax-1)}{2}+\mathrm{Artanh}\,(ax)\ln\,(ax)-\frac{\mathrm{Artanh}\,(ax)\ln\,(ax+1)}{2}-\frac{(\ln\,(ax-1))^2}{8}+\frac{1}{2}\mathrm{dilog}\left(\frac{1}{2}+\frac{ax}{2}\right)+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1),x)
```

```
[Out] -1/2*arctanh(a*x)*ln(a*x-1)+arctanh(a*x)*ln(a*x)-1/2*arctanh(a*x)*ln(a*x+1)
-1/8*ln(a*x-1)^2+1/2*dilog(1/2+1/2*a*x)+1/4*ln(a*x-1)*ln(1/2+1/2*a*x)-1/4*(
ln(a*x+1)-ln(1/2+1/2*a*x))*ln(-1/2*a*x+1/2)+1/8*ln(a*x+1)^2-1/2*dilog(a*x)-
1/2*dilog(a*x+1)-1/2*ln(a*x)*ln(a*x+1)
```

Maxima [B] time = 0.970119, size = 178, normalized size = 3.96

$$\frac{1}{8}a\left(\frac{\log\,(ax+1)^2-2\,\log\,(ax+1)\log\,(ax-1)-\log\,(ax-1)^2}{a}+\frac{4\left(\log\,(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)}{a}-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*a*((log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) - log(a*x - 1)^2)/a +
4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x
+ 1)*log(x) + dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a) - 1
/2*(log(a^2*x^2 - 1) - log(x^2))*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\text{atanh}\,(ax)}{a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)/(a**2*x**3 - x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\text{artanh}\,(ax)}{(a^2x^2-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x), x)
```

3.232 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/
2
```

Rubi [A] time = 0.077544, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5982, 5916, 266, 36, 29, 31, 5948}

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/
2
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)+\frac{1}{2}a^3\,\mathrm{Subst}\left(\int \frac{1}{1-a^2x}\,dx,x,x^2\right)\\
&= -\frac{\tanh^{-1}(ax)}{x}+\frac{1}{2}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0402725, size = 41, normalized size = 1.

$$-\frac{1}{2}a\log\left(1-a^2x^2\right)+a\log(x)+\frac{1}{2}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(ArcTanh[a*x]/x) + (a*ArcTanh[a*x]^2)/2 + a*Log[x] - (a*Log[1 - a^2*x^2])/
2
```

Maple [B] time = 0.056, size = 132, normalized size = 3.2

$$-\frac{\mathrm{Artanh}\,(ax)}{x}-\frac{a\mathrm{Artanh}\,(ax)\ln(ax-1)}{2}+\frac{a\mathrm{Artanh}\,(ax)\ln(ax+1)}{2}-\frac{a\left(\ln(ax-1)\right)^2}{8}+\frac{a\ln(ax-1)}{4}\ln\left(\frac{1}{2}+\frac{ax}{2}\right)-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1),x)
```

```
[Out] -arctanh(a*x)/x-1/2*a*arctanh(a*x)*ln(a*x-1)+1/2*a*arctanh(a*x)*ln(a*x+1)-1
/8*a*ln(a*x-1)^2+1/4*a*ln(a*x-1)*ln(1/2+1/2*a*x)-1/2*a*ln(a*x-1)+a*ln(a*x)-
1/2*a*ln(a*x+1)+1/4*a*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/4*a*ln(-1/2*a*x+1/2)*ln(
1/2+1/2*a*x)-1/8*a*ln(a*x+1)^2
```

Maxima [B] time = 0.959937, size = 111, normalized size = 2.71

$$\frac{1}{8}\left(2\left(\log\left(ax-1\right)-2\right)\log\left(ax+1\right)-\log\left(ax+1\right)^{2}-\log\left(ax-1\right)^{2}-4\log\left(ax-1\right)+8\log\left(x\right)\right)a+\frac{1}{2}\left(a\log\left(ax+1\right)-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 -
4*log(a*x - 1) + 8*log(x))*a + 1/2*(a*log(a*x + 1) - a*log(a*x - 1) - 2/x)*
arctanh(a*x)
```

Fricas [A] time = 2.1324, size = 150, normalized size = 3.66

$$\frac{ax\log\left(-\frac{ax+1}{ax-1}\right)^{2}-4\,ax\log\left(a^{2}x^{2}-1\right)+8\,ax\log\left(x\right)-4\,\log\left(-\frac{ax+1}{ax-1}\right)}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^2 - 4*a*x*log(a^2*x^2 - 1) + 8*a*x*log(x
) - 4*log(-(a*x + 1)/(a*x - 1)))/x
```

Sympy [A] time = 3.38263, size = 37, normalized size = 0.9

$$\begin{cases} a\log\left(x\right)-a\log\left(x-\frac{1}{a}\right)+\frac{a\operatorname{atanh}^{2}\left(ax\right)}{2}-a\operatorname{atanh}\left(ax\right)-\frac{\operatorname{atanh}\left(ax\right)}{x} & \text{for}\ a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((a*log(x) - a*log(x - 1/a) + a*atanh(a*x)**2/2 - a*atanh(a*x) - a
tanh(a*x)/x, Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}\left(ax\right)}{\left(a^{2}x^{2}-1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^2), x)
```

3.233 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=84

$$-\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a^2\tanh^{-1}(ax)+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

```
[Out] -a/(2*x) + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*ArcTanh[a*x]^
2)/2 + a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - (a^2*PolyLog[2, -1 + 2/(1 +
a*x)])/2
```

Rubi [A] time = 0.154584, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5982, 5916, 325, 206, 5988, 5932, 2447}

$$-\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{2}a^2\tanh^{-1}(ax)^2+\frac{1}{2}a^2\tanh^{-1}(ax)+a^2\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)-\frac{\tanh^{-1}(ax)}{2x^2}-\frac{a}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -a/(2*x) + (a^2*ArcTanh[a*x])/2 - ArcTanh[a*x]/(2*x^2) + (a^2*ArcTanh[a*x]^
2)/2 + a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - (a^2*PolyLog[2, -1 + 2/(1 +
a*x)])/2
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3}\,dx\\ &= -\frac{\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 + \frac{1}{2}a\int \frac{1}{x^2(1-a^2x^2)}\,dx + a^2\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\ &= -\frac{a}{2x} - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 + a^2\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) + \frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\,dx \cdots\\ &= -\frac{a}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 + a^2\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{2}\cdots\end{aligned}$$

Mathematica [A] time = 0.258635, size = 60, normalized size = 0.71

$$-\frac{1}{2}a^2\left(\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-\tanh^{-1}(ax)\left(-\frac{1}{a^2x^2}+\tanh^{-1}(ax)+2\log\left(1-e^{-2\tanh^{-1}(ax)}\right)+1\right)+\frac{1}{ax}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -(a^2*(1/(a*x) - ArcTanh[a*x]*(1 - 1/(a^2*x^2) + ArcTanh[a*x] + 2*Log[1 - E
^(-2*ArcTanh[a*x])]) + PolyLog[2, E^(-2*ArcTanh[a*x])]))/2
```

Maple [B] time = 0.058, size = 209, normalized size = 2.5

$$-\frac{a^2\text{Artanh}(ax)\ln(ax-1)}{2} - \frac{\text{Artanh}(ax)}{2x^2} + a^2\text{Artanh}(ax)\ln(ax) - \frac{a^2\text{Artanh}(ax)\ln(ax+1)}{2} - \frac{a}{2x} - \frac{a^2\ln(ax-\cdots}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1),x)
```

```
[Out] -1/2*a^2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*x)/x^2+a^2*arctanh(a*x)*ln(a*
x)-1/2*a^2*arctanh(a*x)*ln(a*x+1)-1/2*a/x-1/4*a^2*ln(a*x-1)+1/4*a^2*ln(a*x+
1)-1/8*a^2*ln(a*x-1)^2+1/2*a^2*dilog(1/2+1/2*a*x)+1/4*a^2*ln(a*x-1)*ln(1/2+
1/2*a*x)-1/4*a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/4*a^2*ln(-1/2*a*x+1/2)*ln(1/2
+1/2*a*x)+1/8*a^2*ln(a*x+1)^2-1/2*a^2*dilog(a*x)-1/2*a^2*dilog(a*x+1)-1/2*a
^2*ln(a*x)*ln(a*x+1)
```

Maxima [B] time = 0.969706, size = 219, normalized size = 2.61

$$\frac{1}{8}\left(4\left(\log\left(ax-1\right)\log\left(\frac{1}{2}ax+\frac{1}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{2}ax+\frac{1}{2}\right)\right)a-4\left(\log\left(ax+1\right)\log\left(x\right)+\mathrm{Li}_2\left(-ax\right)\right)a+4\left(\log\left(-ax+1\right)\log\left(x\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 4*(log
(a*x + 1)*log(x) + dilog(-a*x))*a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))*a
 + 2*a*log(a*x + 1) - 2*a*log(a*x - 1) + (a*x*log(a*x + 1)^2 - 2*a*x*log(a*
x + 1)*log(a*x - 1) - a*x*log(a*x - 1)^2 - 4)/x)*a - 1/2*(a^2*log(a^2*x^2 -
 1) - a^2*log(x^2) + 1/x^2)*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\left(ax\right)}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\mathrm{atanh}\left(ax\right)}{a^2x^5-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)/(a**2*x**5 - x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\mathrm{artanh}\left(ax\right)}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)*x^3), x)
```

3.234 $\int \frac{x^3 \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=135

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}-\frac{\log\left(1-a^2x^2\right)}{2a^4}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}}{2}$$

```
[Out] -((x*ArcTanh[a*x])/a^3) + ArcTanh[a*x]^2/(2*a^4) - (x^2*ArcTanh[a*x]^2)/(2*
a^2) - ArcTanh[a*x]^3/(3*a^4) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - Log
[1 - a^2*x^2]/(2*a^4) + (ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^4 - Po
lyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rubi [A] time = 0.302269, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5980, 5916, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}-\frac{\log\left(1-a^2x^2\right)}{2a^4}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((x*ArcTanh[a*x])/a^3) + ArcTanh[a*x]^2/(2*a^4) - (x^2*ArcTanh[a*x]^2)/(2*
a^2) - ArcTanh[a*x]^3/(3*a^4) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - Log
[1 - a^2*x^2]/(2*a^4) + (ArcTanh[a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^4 - Po
lyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{1-a^2x^2}\,dx = -\frac{\int x\tanh^{-1}(ax)^2\,dx}{a^2}+\frac{\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^3}+\frac{\int \frac{x^2\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4}-\frac{\int \tanh^{-1}(ax)\,dx}{a^3}+\frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}}{a^3}$$

$$= -\frac{x\tanh^{-1}(ax)}{a^3}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4}+$$

$$= -\frac{x\tanh^{-1}(ax)}{a^3}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{x^2\tanh^{-1}(ax)^2}{2a^2}-\frac{\tanh^{-1}(ax)^3}{3a^4}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4}-$$

Mathematica [A] time = 0.121508, size = 112, normalized size = 0.83

$$-\frac{\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)-\log\left(\frac{1}{\sqrt{1-a^2x^2}}\right)-\frac{1}{2}\left(1-a^2x^2\right)\tanh^{-1}(ax)^2 -}{a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((a*x*ArcTanh[a*x] - ((1 - a^2*x^2)*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^3/3 -
 ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] - Log[1/Sqrt[1 - a^2*x^2]] + A
rcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*
x])]/2)/a^4)
```

Maple [C] time = 0.292, size = 812, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] -1/2*x^2*arctanh(a*x)^2/a^2-1/2/a^4*arctanh(a*x)^2*ln(a*x-1)-1/2/a^4*arctan
h(a*x)^2*ln(a*x+1)+1/a^4*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/a^
4*arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2/a^4*polylog(3,-(a*x+1
)^2/(-a^2*x^2+1))+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2-1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))^2-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*I
/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^3+1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))+1
/2*I/a^4*arctanh(a*x)^2*Pi+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))^3+1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3-1/3*arctanh(a*x)^3/a^4+1/a^4*arctanh(a*x)^2*ln(2)-x*arctanh(a*x)/a^3+1/2*
arctanh(a*x)^2/a^4-arctanh(a*x)/a^4+1/a^4*ln((a*x+1)^2/(-a^2*x^2+1)+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\left(a^{2} x^{2}+\log(ax+1)\right)\log(-ax+1)^{2}+\log(-ax+1)^{3}}{24\,a^{4}}+\frac{1}{4}\int -\frac{a^{3} x^{3}\log(ax+1)^{2}-\left(a^{3} x^{3}+a^{2} x^{2}+\left(2\,a^{3} x^{3}+ax+1\right)\mathrm{l}\right.}{a^{5} x^{2}-a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^4 +
1/4*integrate(-(a^3*x^3*log(a*x + 1)^2 - (a^3*x^3 + a^2*x^2 + (2*a^3*x^3 +
a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^5*x^2 - a^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^{3}\,\text{artanh}\,(ax)^{2}}{a^{2} x^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3 \operatorname{atanh}^2(ax)}{a^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 \operatorname{artanh}(ax)^2}{a^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

3.235 $\int \frac{x^2 \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=75

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^3}{3a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}-\frac{\tanh^{-1}(ax)^2}{a^3}+\frac{2\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}$$

```
[Out] -(ArcTanh[a*x]^2/a^3) - (x*ArcTanh[a*x]^2)/a^2 + ArcTanh[a*x]^3/(3*a^3) + (
2*ArcTanh[a*x]*Log[2/(1 - a*x)])/a^3 + PolyLog[2, 1 - 2/(1 - a*x)]/a^3
```

Rubi [A] time = 0.168586, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5980, 5910, 5984, 5918, 2402, 2315, 5948}

$$\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^3}{3a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}-\frac{\tanh^{-1}(ax)^2}{a^3}+\frac{2\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```



```
[Out] -(ArcTanh[a*x]^2/a^3) - (x*ArcTanh[a*x]^2)/a^2 + ArcTanh[a*x]^3/(3*a^3) + (
2*ArcTanh[a*x]*Log[2/(1 - a*x)])/a^3 + PolyLog[2, 1 - 2/(1 - a*x)]/a^3
```


Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx &= -\frac{\int \tanh^{-1}(ax)^2\,dx}{a^2}+\frac{\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^2}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}-\frac{2\int \frac{\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a^2}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{2\,\mathrm{Subst}\left(\int \frac{\log(2}{1-2}\right.}{a^3}\\
&= -\frac{\tanh^{-1}(ax)^2}{a^3}-\frac{x\tanh^{-1}(ax)^2}{a^2}+\frac{\tanh^{-1}(ax)^3}{3a^3}+\frac{2\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{\mathrm{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.172413, size = 59, normalized size = 0.79

$$-\frac{\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)\left(-3ax\tanh^{-1}(ax)+\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)+6\log\left(e^{-2\tanh^{-1}(ax)}\right.\right.}{a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((-(ArcTanh[a*x]*(-3*a*x*ArcTanh[a*x] + ArcTanh[a*x]*(3 + ArcTanh[a*x]) +
6*Log[1 + E^(-2*ArcTanh[a*x])]))/3 + PolyLog[2, -E^(-2*ArcTanh[a*x])])/a^3)
```

Maple [C] time = 0.394, size = 5573, normalized size = 74.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] result too large to display
```

Maxima [B] time = 0.99029, size = 270, normalized size = 3.6

$$-\frac{1}{2}\left(\frac{2\,x}{a^2}-\frac{\log\left(ax+1\right)}{a^3}+\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\frac{3\,(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\left(\log(ax-1)^2-4\,\log(ax-1)\right)\log}{a}}{24\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(2*x/a^2 - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)*arctanh(a*x)^2 - 1/24*
((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x - 1)^3 - 3
*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^2)/a - 24*
(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a)/a^2 + 1/4*(2*(
log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x - 1)^2 - 4*log(a*
x - 1))*arctanh(a*x)/a^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{x^2\operatorname{artanh}\left(ax\right)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2\operatorname{atanh}^2\left(ax\right)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^2\operatorname{artanh}\left(ax\right)^2}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

3.236 $\int \frac{x \tanh^{-1}(ax)^2}{1-a^2x^2} dx$

Optimal. Leaf size=78

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2}-\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2}$$

```
[Out] -ArcTanh[a*x]^3/(3*a^2) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^2 + (ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^2 - PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^2)
```

Rubi [A] time = 0.157278, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5984, 5918, 5948, 6058, 6610}

$$-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}+\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^2}-\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)^2}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -ArcTanh[a*x]^3/(3*a^2) + (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^2 + (ArcTanh[
a*x]*PolyLog[2, 1 - 2/(1 - a*x)])/a^2 - PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^2)
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2}-\frac{2\int \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2}-\frac{\int \frac{\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$
$$= -\frac{\tanh^{-1}(ax)^3}{3a^2}+\frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^2}+\frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{a^2}-\frac{\text{Li}_3\left(1-\frac{2}{1-ax}\right)}{2a^2}$$

Mathematica [A] time = 0.0647912, size = 68, normalized size = 0.87

$$-\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)^3-\tanh^{-1}(ax)^2\log\left(e^{-2\tanh^{-1}(a
}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2),x]
```

```
[Out] -((-ArcTanh[a*x]^3/3 - ArcTanh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTan
h[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])]/
2)/a^2)
```

Maple [C] time = 0.278, size = 741, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] -1/2/a^2*arctanh(a*x)^2*ln(a*x-1)-1/2/a^2*arctanh(a*x)^2*ln(a*x+1)+1/a^2*ar
ctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*arctanh(a*x)^3/a^2+1/a^2*ar
ctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-1/2/a^2*polylog(3,-(a*x+1)^2/
(-a^2*x^2+1))+1/4*I/a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi+1
/4*I/a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))^2*Pi+1/4*I/a^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+1/2*I/a^2
*arctanh(a*x)^2*Pi-1/4*I/a^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))*Pi-1/4*I/a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+1/2*I/a^2*arct
anh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi+1/2*I/a^2*arctanh(a*x)^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi+1/4*
I/a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^3*Pi-1/2*I/a^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+1/
```

```
a^2*arctanh(a*x)^2*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3 \log(ax+1)\log(-ax+1)^2 + \log(-ax+1)^3}{24a^2} + \frac{1}{4}\int -\frac{ax\log(ax+1)^2 - (3ax+1)\log(ax+1)\log(-ax+1)}{a^3x^2 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(3*log(a*x + 1)*log(-a*x + 1)^2 + log(-a*x + 1)^3)/a^2 + 1/4*integrat
e(-(a*x*log(a*x + 1)^2 - (3*a*x + 1)*log(a*x + 1)*log(-a*x + 1))/(a^3*x^2 -
 a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\,\text{artanh}(ax)^2}{a^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\,\text{atanh}^2(ax)}{a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)**2/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\,\text{artanh}(ax)^2}{a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

3.237 $\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

```
[Out] ArcTanh[a*x]^3/(3*a)
```

Rubi [A] time = 0.0261026, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^3/(3*a)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx = \frac{\tanh^{-1}(ax)^3}{3a}$$

Mathematica [A] time = 0.0048607, size = 13, normalized size = 1.

$$\frac{\tanh^{-1}(ax)^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^3/(3*a)
```

Maple [A] time = 0.023, size = 12, normalized size = 0.9

$$\frac{(\text{Artanh}\,(ax))^3}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1),x)
```

```
[Out] 1/3*arctanh(a*x)^3/a
```

Maxima [B] time = 0.975451, size = 171, normalized size = 13.15

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a}-\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2-\frac{\left(\log(ax+1)^2-2\log(ax+1)\log(ax-1)+\log(ax-1)^2\right)\operatorname{artanh}(ax}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^2 - 1/4*(log(a*x + 1)^2
- 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)*arctanh(a*x)/a + 1/24*(log(
a*x + 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2
- log(a*x - 1)^3)/a
```

Fricas [A] time = 2.27843, size = 49, normalized size = 3.77

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

Sympy [A] time = 2.23219, size = 10, normalized size = 0.77

$$\begin{cases}\frac{\operatorname{atanh}^{3}(ax)}{3a} & \text{for } a \neq 0\\ 0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**3/(3*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.163, size = 30, normalized size = 2.31

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^3}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/24*log(-(a*x + 1)/(a*x - 1))^3/a
```

3.238 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}\tanh^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

```
[Out] ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyL
og[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rubi [A] time = 0.180857, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5988, 5932, 5948, 6056, 6610}

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}\tanh^{-1}(ax)^3+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyL
og[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{3}\tanh^{-1}(ax)^3 + \int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$

$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - (2a)\int \frac{\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx$$

$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right) + a\int \frac{\mathrm{Li}_2\left(-1+\frac{2}{1+}\right.}{1-a^2x^2}$$

$$= \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \tanh^{-1}(ax)\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{1}{2}\mathrm{Li}_3\left(-1+\frac{2}{1+a}\right.$$

Mathematica [A] time = 0.0626553, size = 60, normalized size = 0.91

$$\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - \frac{1}{2}\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) - \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)),x]
```

```
[Out] -ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + ArcTanh[a*
x]*PolyLog[2, E^(2*ArcTanh[a*x])] - PolyLog[3, E^(2*ArcTanh[a*x])]/2
```

Maple [C] time = 0.268, size = 1188, normalized size = 18.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1),x)
```

```
[Out] 1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2-1/2*I*Pi*csgn(
I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+arctanh(a*x)^2*ln(a*x)-arcta
nh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^
2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*
polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a
^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))*arctanh(a*x)^2-1/3*arctanh(a*x)^3+arctanh(a*x)^2*ln(2)-1/2*arctanh(
a*x)^2*ln(a*x-1)-1/2*arctanh(a*x)^2*ln(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a
^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^3*arctanh(a*x)^2-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))*arctanh(a*x)^2+1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+1/4*I*Pi*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+1/4*I*Pi*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2-2*polylog(3,-(a*x+1)/(-a^2*x
```

```
^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^
2-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh
(a*x)^2-1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2
*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\log\left(ax+1\right)\log\left(-ax+1\right)^2-\frac{1}{24}\log\left(-ax+1\right)^3+\frac{1}{4}\int\frac{\left(a^2x^2+ax+2\right)\log\left(ax+1\right)\log\left(-ax+1\right)-\log\left(ax+1\right)^2}{a^2x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/8*log(a*x + 1)*log(-a*x + 1)^2 - 1/24*log(-a*x + 1)^3 + 1/4*integrate(((
a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1) - log(a*x + 1)^2)/(a^2*x^3 -
x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}\left(ax\right)^2}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{atanh}^2\left(ax\right)}{a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**2*x**3 - x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\operatorname{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x), x)
```

3.239 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx$

Optimal. Leaf size=66

$$-a\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

```
[Out] a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a*ArcTanh[a*x]^3)/3 + 2*a*ArcTanh[a*
x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.214768, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5982, 5916, 5988, 5932, 2447, 5948}

$$-a\,\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{3}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+2a\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a*ArcTanh[a*x]^3)/3 + 2*a*ArcTanh[a*
x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx &= a^2\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx+\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&= a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+(2a)\int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx\\
&= a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-\left(2a^2\right)\int \frac{\log(}{1}\\
&= a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)^2}{x}+\frac{1}{3}a\tanh^{-1}(ax)^3+2a\tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right)-a\text{Li}_2\left(-1+\frac{}{1}\right.\end{aligned}$$

Mathematica [A] time = 0.222912, size = 61, normalized size = 0.92

$$-a\left(\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)-\frac{1}{3}\tanh^{-1}(ax)\left(\left(\tanh^{-1}(ax)+3\right)\tanh^{-1}(ax)-\frac{3\tanh^{-1}(ax)}{ax}+6\log\left(1-e^{-2\tanh^{-1}(ax)}\right)\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(a*(-(ArcTanh[a*x]*((-3*ArcTanh[a*x])/(a*x) + ArcTanh[a*x]*(3 + ArcTanh[a*
x]) + 6*Log[1 - E^(-2*ArcTanh[a*x])]))/3 + PolyLog[2, E^(-2*ArcTanh[a*x])])
)
```

Maple [C] time = 0.374, size = 4449, normalized size = 67.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x)
```

```
[Out] -a*arctanh(a*x)^2+1/2*I*a*Pi*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*polylog(2,(a*x+1)/(-a^2*
x^2+1)^(1/2))-1/2*I*a*Pi*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*arcta
nh(a*x)^2-1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh
```

```
(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2/x+1/2*a*arctanh(a*x)^
2*ln(a*x+1)-a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*x
)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/3*a*arctanh(a*x)^3+1/4*I*a*Pi*csgn(I*(
a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))^2*arctanh(a*x)^2-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1
/2))-1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/2*I*a*Pi*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-1/4*I*a
*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arct
anh(a*x)^2+1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1
/2))+1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1
/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a
*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^
2+1)^(1/2))-1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2
))+1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*
I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x
+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(
-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn
(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,(a
*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2
*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-a*dilog((a*x+1)/(-a^2*x
^2+1)^(1/2))+a*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,-(a*x+1)/(-a
^2*x^2+1)^(1/2))+a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a*arctanh(a*x)*ln(
1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a*arctanh(a*x)^2*ln(a*x-1)+1/2*I*a*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2
*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(
1/2))-1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2
*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*
arctanh(a*x)^2+1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(
a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+
1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2+1/2*I*a
```

```
*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+
1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*arctanh(a*x)*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*polylog(2,(a*x+1)
/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(
a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))*arctanh(a*x)^2+1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*arctanh(a*x)^2-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog((a*x+1)/(-a^2*
x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog((a*x+1)
/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog((a*x+
1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog(1+(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+
1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))-1/2*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,(a*x+
1)/(-a^2*x^2+1)^(1/2))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3*arctanh(a*x)^2
```

Maxima [B] time = 0.993476, size = 320, normalized size = 4.85

$$-\frac{1}{24}a^2\left(\frac{3(\log(ax-1)-2)\log(ax+1)^2-\log(ax+1)^3+\log(ax-1)^3-3\left(\log(ax-1)^2-4\log(ax-1)\right)\log(ax+1)+}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*a^2*((3*(log(a*x - 1) - 2)*log(a*x + 1)^2 - log(a*x + 1)^3 + log(a*x
 - 1)^3 - 3*(log(a*x - 1)^2 - 4*log(a*x - 1))*log(a*x + 1) + 6*log(a*x - 1)^
2)/a - 24*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a + 24*
(log(a*x + 1)*log(x) + dilog(-a*x))/a - 24*(log(-a*x + 1)*log(x) + dilog(a*
x))/a) + 1/4*(2*(log(a*x - 1) - 2)*log(a*x + 1) - log(a*x + 1)^2 - log(a*x
 - 1)^2 - 4*log(a*x - 1) + 8*log(x))*a*arctanh(a*x) + 1/2*(a*log(a*x + 1) -
a*log(a*x - 1) - 2/x)*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^2}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^4 - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**2*x**4 - x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2 - 1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^2), x)
```

3.240 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=138

$$-\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{3}a^2\tanh^{-1}(a$$

```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) + (
a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2 + a^2*ArcTanh
[a*x]^2*Log[2 - 2/(1 + a*x)] - a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)
] - (a^2*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.357181, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5982, 5916, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610}

$$-\frac{1}{2}a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{1}{2}a^2\log\left(1-a^2x^2\right)+a^2\log(x)+\frac{1}{3}a^2\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```



```
[Out] -((a*ArcTanh[a*x])/x) + (a^2*ArcTanh[a*x]^2)/2 - ArcTanh[a*x]^2/(2*x^2) + (
a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log[1 - a^2*x^2])/2 + a^2*ArcTanh
[a*x]^2*Log[2 - 2/(1 + a*x)] - a^2*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)
] - (a^2*PolyLog[3, -1 + 2/(1 + a*x)])/2
```


Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx + a^2\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$
$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) + a\int \frac{\tanh^{-1}(ax)}{x^2}\,dx + a^3\int$$
$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log\left(2-\frac{}{1}\right.$$
$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log\left(2-\frac{}{1}\right.$$
$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\tanh^{-1}(ax)^2\log\left(2-\frac{}{1}\right.$$
$$= -\frac{a\tanh^{-1}(ax)}{x} + \frac{1}{2}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\log(x) - \frac{1}{2}a^2\log\left(1-a^2\right.$$

Mathematica [C] time = 0.340868, size = 133, normalized size = 0.96

$$-a^2\left(-\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{1}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right)+\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}{2a^2x^2}+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -(a^2*((-I/24)*Pi^3 + ArcTanh[a*x]/(a*x) + ((1 - a^2*x^2)*ArcTanh[a*x]^2)/(
2*a^2*x^2) + ArcTanh[a*x]^3/3 - ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])]
- Log[(a*x)/Sqrt[1 - a^2*x^2]] - ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])
] + PolyLog[3, E^(2*ArcTanh[a*x])]/2))
```

Maple [C] time = 0.41, size = 1360, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x)
```

```
[Out] -2*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*a^2*polylog(3,(a*x+1)/(-a^2
*x^2+1)^(1/2))-1/4*I*a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi-1/2*I*a^2*arctanh(
a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+1/4*I*a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*Pi-1/2*I*a^2*arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+1/4*I*a^2*arctanh
(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*
Pi+1/2*I*a^2*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+
1)^2/(a^2*x^2-1))^2*Pi-a^2*arctanh(a*x)+1/2*a^2*arctanh(a*x)^2-1/2*arctanh(
```

```
a*x)^2/x^2+a^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ln((a*x+1)/(-a^2*x^2+1)
^(1/2)-1)+a^2*arctanh(a*x)^2*ln(a*x)-a^2*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*
x^2+1)-1)+2*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*a
rctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*a^2*arctanh(a*x)^2*
Pi-a*arctanh(a*x)/x+1/2*I*a^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))*Pi-1/3*a^2*arctanh(a*x)^3+a^2*arctanh(a*x)^2*ln(
1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^
(1/2))-1/2*a^2*arctanh(a*x)^2*ln(a*x-1)-1/2*a^2*arctanh(a*x)^2*ln(a*x+1)+a^
2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)^2*ln(2)-1/
4*I*a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi+1/
4*I*a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^3*Pi-1/2*I*a^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+
1/2*I*a^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi+1/2*I*a^2*
arctanh(a*x)^2*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3*Pi+1/4*I*a^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2x^2 \log(-ax+1)^3 + 3\left(a^2x^2\log(ax+1)+1\right)\log(-ax+1)^2}{24\,x^2} + \frac{1}{4}\int -\frac{\log(ax+1)^2 - \left(a^2x^2 + ax + \left(a^4x^4 + a^3x^3 + 2\right.\right.}{a^2x^5 - x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/24*(a^2*x^2*log(-a*x + 1)^3 + 3*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)
^2)/x^2 + 1/4*integrate(-(log(a*x + 1)^2 - (a^2*x^2 + a*x + (a^4*x^4 + a^3*
x^3 + 2)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^5 - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^2}{a^2x^5 - x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^2x^5 - x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**2*x**5 - x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)*x^3), x)
```

3.241 $\int \frac{x^3 \tanh^{-1}(ax)^3}{1-a^2x^2} dx$

Optimal. Leaf size=205

$$\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(\right.}{2a^4}$$

```
[Out] (-3*ArcTanh[a*x]^2)/(2*a^4) - (3*x*ArcTanh[a*x]^2)/(2*a^3) + ArcTanh[a*x]^3
/(2*a^4) - (x^2*ArcTanh[a*x]^3)/(2*a^2) - ArcTanh[a*x]^4/(4*a^4) + (3*ArcTa
nh[a*x]*Log[2/(1 - a*x)])/a^4 + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 + (3*
PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(2*a^4) - (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^4) +
 (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rubi [A] time = 0.469477, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {5980, 5916, 5910, 5984, 5918, 2402, 2315, 5948, 6058, 6062, 6610}

$$\frac{3\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(\right.}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] (-3*ArcTanh[a*x]^2)/(2*a^4) - (3*x*ArcTanh[a*x]^2)/(2*a^3) + ArcTanh[a*x]^3
/(2*a^4) - (x^2*ArcTanh[a*x]^3)/(2*a^2) - ArcTanh[a*x]^4/(4*a^4) + (3*ArcTa
nh[a*x]*Log[2/(1 - a*x)])/a^4 + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 + (3*
PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]^2*PolyLog[2, 1 - 2/(
1 - a*x)])/(2*a^4) - (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^4) +
 (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = -\frac{\int x\tanh^{-1}(ax)^3\,dx}{a^2} + \frac{\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a^3} + \frac{3\int \frac{x^2\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{2a}$$

$$= -\frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{3\int \tanh^{-1}(ax)^2\,dx}{2a^3} + \frac{3\int \frac{\tanh^{-1}}{1-}}{2}$$

$$= -\frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^4}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{\tanh}{}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{3\tan}{}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{3\tan}{}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{2a^4} - \frac{3x\tanh^{-1}(ax)^2}{2a^3} + \frac{\tanh^{-1}(ax)^3}{2a^4} - \frac{x^2\tanh^{-1}(ax)^3}{2a^2} - \frac{\tanh^{-1}(ax)^4}{4a^4} + \frac{3\tan}{}$$

Mathematica [A] time = 0.2687, size = 142, normalized size = 0.69

$$-\frac{6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 6\left(\tanh^{-1}(ax)^2+1\right)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 3\text{PolyLog}\left(4,-e^{-2\tan}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -(-6*ArcTanh[a*x]^2 + 6*a*x*ArcTanh[a*x]^2 - 2*(1 - a^2*x^2)*ArcTanh[a*x]^3
 - ArcTanh[a*x]^4 - 12*ArcTanh[a*x]*Log[1 + E^(-2*ArcTanh[a*x])] - 4*ArcTan
h[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*(1 + ArcTanh[a*x]^2)*PolyLog[2, -
E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(-2*ArcTanh[a*x])] + 3*
PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a^4)
```

Maple [A] time = 0.895, size = 248, normalized size = 1.2

$$-\frac{(\text{Artanh}\,(ax))^4}{4\,a^4} - \frac{x^2\,(\text{Artanh}\,(ax))^3}{2\,a^2} - \frac{3\,x\,(\text{Artanh}\,(ax))^2}{2\,a^3} + \frac{(\text{Artanh}\,(ax))^3}{2\,a^4} - \frac{3\,(\text{Artanh}\,(ax))^2}{2\,a^4} + \frac{(\text{Artanh}\,(ax))^3}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x)
```

```
[Out] -1/4*arctanh(a*x)^4/a^4-1/2*x^2*arctanh(a*x)^3/a^2-3/2*x*arctanh(a*x)^2/a^3
+1/2*arctanh(a*x)^3/a^4-3/2*arctanh(a*x)^2/a^4+1/a^4*arctanh(a*x)^3*ln((a*x
+1)^2/(-a^2*x^2+1)+1)+3/2/a^4*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2
+1))-3/2/a^4*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4/a^4*polylo
g(4,-(a*x+1)^2/(-a^2*x^2+1))+3/a^4*arctanh(a*x)*ln((a*x+1)^2/(-a^2*x^2+1)+1
```

```
)+3/2/a^4*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\left(a^2x^2+\log(ax+1)\right)\log(-ax+1)^3+\log(-ax+1)^4}{64\,a^4}-\frac{1}{8}\int\frac{2\,a^3x^3\log(ax+1)^3-6\,a^3x^3\log(ax+1)^2\log(-ax+1)+}{2\left(a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(4*(a^2*x^2 + log(a*x + 1))*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^4 - 1
/8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)^3 - 6*a^3*x^3*log(a*x + 1)^2*log(-
a*x + 1) + 3*(a^3*x^3 + a^2*x^2 + (2*a^3*x^3 + a*x + 1)*log(a*x + 1))*log(-
a*x + 1)^2)/(a^5*x^2 - a^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^3\,\text{artanh}(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3\,\text{atanh}^3(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**3*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^3\,\text{artanh}(ax)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

3.242 $\int \frac{x^2 \tanh^{-1}(ax)^3}{1-a^2x^2}\,dx$

Optimal. Leaf size=103

$$-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^4}{4a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}-\frac{\tanh^{-1}(ax)^3}{a^3}+\frac{3}{}$$

```
[Out] -(ArcTanh[a*x]^3/a^3) - (x*ArcTanh[a*x]^3)/a^2 + ArcTanh[a*x]^4/(4*a^3) + (
3*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^3 + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(
1 - a*x)])/a^3 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3)
```

Rubi [A] time = 0.269013, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5980, 5910, 5984, 5918, 5948, 6058, 6610}

$$-\frac{3\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^3}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^3}+\frac{\tanh^{-1}(ax)^4}{4a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}-\frac{\tanh^{-1}(ax)^3}{a^3}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -(ArcTanh[a*x]^3/a^3) - (x*ArcTanh[a*x]^3)/a^2 + ArcTanh[a*x]^4/(4*a^3) + (
3*ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^3 + (3*ArcTanh[a*x]*PolyLog[2, 1 - 2/(
1 - a*x)])/a^3 - (3*PolyLog[3, 1 - 2/(1 - a*x)])/(2*a^3)
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = -\frac{\int \tanh^{-1}(ax)^3\,dx}{a^2}+\frac{\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}-\frac{6\int \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\tanh^{-1}(ax)\mathrm{Li}_2\left(\dots\right.}{a^3}$$

$$= -\frac{\tanh^{-1}(ax)^3}{a^3}-\frac{x\tanh^{-1}(ax)^3}{a^2}+\frac{\tanh^{-1}(ax)^4}{4a^3}+\frac{3\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^3}+\frac{3\tanh^{-1}(ax)\mathrm{Li}_2\left(\dots\right.}{a^3}$$

Mathematica [A] time = 0.242113, size = 78, normalized size = 0.76

$$\frac{-12\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)-6\mathrm{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)^2\left(\tanh^{-1}(ax)^2+(4-4ax)\tan\dots\right.}{4a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] (ArcTanh[a*x]^2*((4 - 4*a*x)*ArcTanh[a*x] + ArcTanh[a*x]^2 + 12*Log[1 + E^(
-2*ArcTanh[a*x])]) - 12*ArcTanh[a*x]*PolyLog[2, -E^(-2*ArcTanh[a*x])] - 6*P
olyLog[3, -E^(-2*ArcTanh[a*x])])/(4*a^3)
```

Maple [C] time = 0.327, size = 788, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x)
```

```
[Out] -x*arctanh(a*x)^3/a^2-1/2/a^3*arctanh(a*x)^3*ln(a*x-1)+1/2/a^3*arctanh(a*x)
^3*ln(a*x+1)-1/a^3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*arctan
h(a*x)^4/a^3-1/2*I/a^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^2*arctanh(a*x)^3-1/4*I/a^3*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^3-1/4*I/a^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^3*arctanh(a*x)^3-1/4*I/a^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^
3+1/4*I/a^3*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)+3/a^3*arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)+1)+3/a^3*arctanh(a*x)*poly
log(2,-(a*x+1)^2/(-a^2*x^2+1))+1/2*I/a^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))^3*arctanh(a*x)^3-1/4*I/a^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3+1/2*I/a^3*Pi*arctanh(a*x)^3-1/2*I/
a^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+1/4*I/a^3*Pi*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^2*arctanh(a*x)^3-arctanh(a*x)^3/a^3-3/2/a^3*polylog(3,-(a*x+1)^2/(
-a^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\,(2\,ax - \log(ax+1) - 2)\log(-ax+1)^3 + \log(-ax+1)^4 - 6\left(4\,(ax+1)\log(ax+1) - \log(ax+1)^2\right)\log(-ax+1)^2}{64\,a^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(4*(2*a*x - log(a*x + 1) - 2)*log(-a*x + 1)^3 + log(-a*x + 1)^4 - 6*(4
*(a*x + 1)*log(a*x + 1) - log(a*x + 1)^2)*log(-a*x + 1)^2)/a^3 + 1/8*integr
ate(-1/2*(2*a^2*x^2*log(a*x + 1)^3 - 3*((2*a^2*x^2 - a*x - 1)*log(a*x + 1)^
2 + 4*(a^2*x^2 + 2*a*x + 1)*log(a*x + 1))*log(-a*x + 1))/(a^4*x^2 - a^2), x
)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^2\,\text{artanh}(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\,\text{atanh}^3(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x**2*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 \operatorname{artanh}(ax)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

3.243 $\int \frac{x \tanh^{-1}(ax)^3}{1-a^2x^2} \, dx$

Optimal. Leaf size=108

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^2}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^2}$$

```
[Out] -ArcTanh[a*x]^4/(4*a^2) + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^2 + (3*ArcTan
h[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2) - (3*ArcTanh[a*x]*PolyLog[3,
1 - 2/(1 - a*x)])/(2*a^2) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^2)
```

Rubi [A] time = 0.206464, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {5984, 5918, 5948, 6058, 6062, 6610}

$$\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^2}+\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^2}-\frac{\tanh^{-1}(ax)^4}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -ArcTanh[a*x]^4/(4*a^2) + (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^2 + (3*ArcTan
h[a*x]^2*PolyLog[2, 1 - 2/(1 - a*x)])/(2*a^2) - (3*ArcTanh[a*x]*PolyLog[3,
1 - 2/(1 - a*x)])/(2*a^2) + (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^2)
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = -\frac{\tanh^{-1}(ax)^4}{4a^2} + \frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2} - \frac{3\int \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{1-a^2x^2}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2} - \frac{3\int \frac{\tanh^{-1}(ax)\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{1-a^2x^2}}{a}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(1-\right.}{2a^2}$$

$$= -\frac{\tanh^{-1}(ax)^4}{4a^2} + \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^2} + \frac{3\tanh^{-1}(ax)^2\text{Li}_2\left(1-\frac{2}{1-ax}\right)}{2a^2} - \frac{3\tanh^{-1}(ax)\text{Li}_3\left(1-\right.}{2a^2}$$

Mathematica [A] time = 0.0640732, size = 87, normalized size = 0.81

$$-\frac{6\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right)+3\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right)-}{4a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2),x]
```

```
[Out] -(-ArcTanh[a*x]^4 - 4*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 6*ArcTa
nh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*PolyLog[3, -E^(
-2*ArcTanh[a*x])] + 3*PolyLog[4, -E^(-2*ArcTanh[a*x])])/(4*a^2)
```

Maple [C] time = 0.285, size = 776, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1),x)
```

```
[Out] -1/2/a^2*arctanh(a*x)^3*ln(a*x-1)-1/2/a^2*arctanh(a*x)^3*ln(a*x+1)+1/a^2*ar
ctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*arctanh(a*x)^4/a^2+3/2/a^2*
```

```
arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))-3/2/a^2*arctanh(a*x)*poly
log(3,-(a*x+1)^2/(-a^2*x^2+1))+3/4/a^2*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))+1
/2*I/a^2*arctanh(a*x)^3*Pi-1/4*I/a^2*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))*Pi-1/2*I/a^2*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*Pi+1/4*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*P
i+1/4*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^3*Pi+1/2*I/a^2*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3
*Pi+1/2*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))*Pi+1/4*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi+1/4*I/a^2*arctanh(a*x)^3*csgn(I
/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*
x^2+1)+1))^2*Pi-1/4*I/a^2*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi+1/a^2*arctanh(a*x
)^3*ln(2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{4\log(ax+1)\log(-ax+1)^3+\log(-ax+1)^4}{64\,a^2}-\frac{1}{8}\int\frac{2\,ax\log(ax+1)^3-6\,ax\log(ax+1)^2\log(-ax+1)+3\,(3\,ax+1)}{2\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(4*log(a*x + 1)*log(-a*x + 1)^3 + log(-a*x + 1)^4)/a^2 - 1/8*integrate
(1/2*(2*a*x*log(a*x + 1)^3 - 6*a*x*log(a*x + 1)^2*log(-a*x + 1) + 3*(3*a*x
+ 1)*log(a*x + 1)*log(-a*x + 1)^2)/(a^3*x^2 - a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\,\text{artanh}(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\,\text{atanh}^3(ax)}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(x*atanh(a*x)**3/(a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x\,\text{artanh}\,(ax)^3}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

3.244 $\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

```
[Out] ArcTanh[a*x]^4/(4*a)
```

Rubi [A] time = 0.0234723, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^4/(4*a)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx = \frac{\tanh^{-1}(ax)^4}{4a}$$

Mathematica [A] time = 0.005276, size = 13, normalized size = 1.

$$\frac{\tanh^{-1}(ax)^4}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^4/(4*a)
```

Maple [A] time = 0.021, size = 12, normalized size = 0.9

$$\frac{(\operatorname{Artanh}(ax))^4}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1),x)
```

```
[Out] 1/4*arctanh(a*x)^4/a
```

Maxima [B] time = 0.982012, size = 282, normalized size = 21.69

$$\frac{1}{2}\left(\frac{\log(ax+1)}{a} - \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3 + \frac{1}{64}a\left(\frac{8\left(\log(ax+1)^3 - 3\log(ax+1)^2\log(ax-1) + 3\log(ax+1)\log(a\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1)/a - log(a*x - 1)/a)*arctanh(a*x)^3 + 1/64*a*(8*(log(a*x +
 1)^3 - 3*log(a*x + 1)^2*log(a*x - 1) + 3*log(a*x + 1)*log(a*x - 1)^2 - log
(a*x - 1)^3)*arctanh(a*x)/a^2 - (log(a*x + 1)^4 - 4*log(a*x + 1)^3*log(a*x
- 1) + 6*log(a*x + 1)^2*log(a*x - 1)^2 - 4*log(a*x + 1)*log(a*x - 1)^3 + lo
g(a*x - 1)^4)/a^2) - 3/8*(log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + lo
g(a*x - 1)^2)*arctanh(a*x)^2/a
```

Fricas [A] time = 2.20808, size = 49, normalized size = 3.77

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

Sympy [A] time = 1.12115, size = 10, normalized size = 0.77

$$\begin{cases} \frac{\operatorname{atanh}^4(ax)}{4a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((atanh(a*x)**4/(4*a), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.18429, size = 30, normalized size = 2.31

$$\frac{\log\left(-\frac{ax+1}{ax-1}\right)^4}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/64*log(-(a*x + 1)/(a*x - 1))^4/a
```

3.245 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)} dx$

Optimal. Leaf size=91

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)+\frac{1}{4}\tanh$$

```
[Out] ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*
PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*
x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])/4
```

Rubi [A] time = 0.221187, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5988, 5932, 5948, 6056, 6060, 6610}

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)+\frac{1}{4}\tanh$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

```
[Out] ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*
PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*
x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])/4
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx = \frac{1}{4}\tanh^{-1}(ax)^4 + \int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - (3a)\int \frac{\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right)}{1-a^2x^2}\,dx$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right) + (3a)\int \frac{\tan}{}$$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}($$

$$= \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}(ax)^2\mathrm{Li}_2\left(-1+\frac{2}{1+ax}\right) - \frac{3}{2}\tanh^{-1}($$

Mathematica [A] time = 0.0734301, size = 83, normalized size = 0.91

$$\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - \frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + \frac{3}{4}\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right) - \frac{1}{4}\text{ta}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)),x]
```

```
[Out] -ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] + (3*ArcTanh
[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*PolyLog[3, E^(2
*ArcTanh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcTanh[a*x])])/4
```

Maple [C] time = 0.349, size = 1245, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1),x)
```

```
[Out] -1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+
1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)
^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3+1/2*I*Pi*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3-1/2*
```

```
I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-
a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arc
tanh(a*x)^3+1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3+1/
2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^3+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3*arctanh(a*x)^3+1/4*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+
3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*P
i*arctanh(a*x)^3+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*
x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+ln(2)*arctanh(a*x)^3-1/4*arctanh(a*x)
^4-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+6*polylog(4
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*I
*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+1/2*I*arc
tanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/2
*arctanh(a*x)^3*ln(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*ln((a
*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*arctanh(a*x)^3*Pi*csgn(I*((a*x+1)^2/(-a^2*x
^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-a
rctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(a*x)^3*ln(a*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{16}\log(ax+1)\log(-ax+1)^3+\frac{1}{64}\log(-ax+1)^4-\frac{1}{8}\int\frac{3\left(a^2x^2+ax+2\right)\log(ax+1)\log(-ax+1)^2+2\log(ax+1)^3}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/16*log(a*x + 1)*log(-a*x + 1)^3 + 1/64*log(-a*x + 1)^4 - 1/8*integrate(1/
2*(3*(a^2*x^2 + a*x + 2)*log(a*x + 1)*log(-a*x + 1)^2 + 2*log(a*x + 1)^3 -
6*log(a*x + 1)^2*log(-a*x + 1))/(a^2*x^3 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}(ax)^3}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{atanh}^3(ax)}{a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**3 - x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x), x)
```

3.246 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)} \, dx$

Optimal. Leaf size=90

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{4}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/x + (a*ArcTanh[a*x]^4)/4 + 3*a*ArcTanh[a*
x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] -
 (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.272736, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5982, 5916, 5988, 5932, 5948, 6056, 6610}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{4}a\tanh^{-1}(ax)^4+a\tanh^{-1}(ax)^3-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/x + (a*ArcTanh[a*x]^4)/4 + 3*a*ArcTanh[a*
x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] -
 (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx$$

$$= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx$$

$$= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + 3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - \left(6a^2\right)\int$$

$$= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + 3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - 3a\tanh^{-}$$

$$= a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{1}{4}a\tanh^{-1}(ax)^4 + 3a\tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+ax}\right) - 3a\tanh^{-}$$

Mathematica [C] time = 0.193665, size = 93, normalized size = 1.03

$$-a\left(-3\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) - \frac{1}{4}\tanh^{-1}(ax)^4 + \frac{\tanh^{-1}(ax)^3}{ax} + \tanh^{-1}(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)),x]
```

```
[Out] -(a*((-I/8)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - ArcTanh[a*x]^4/4
 - 3*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[2,
 E^(2*ArcTanh[a*x])] + (3*PolyLog[3, E^(2*ArcTanh[a*x])])/2))
```

Maple [C] time = 0.343, size = 826, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x)
```

```
[Out] -arctanh(a*x)^3/x-1/2*a*arctanh(a*x)^3*ln(a*x-1)+1/2*a*arctanh(a*x)^3*ln(a*
x+1)+3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*a
*arctanh(a*x)^4-a*arctanh(a*x)^3-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-
6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*a*Pi*csgn(I*(a*x+1)/(-a^2*
x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^3+1/2*I*a*Pi*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3+1/4*I*a*arctanh(a*x)^3*Pi*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/4*I*a*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3-1/4*I*a*Pi*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3-
1/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+1/2*I*a*Pi*arctanh(a*x)^3-1/2*
I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-a*arctanh(a*x)^3
*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3-1/4
*I*a*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ax\log\left(-ax+1\right)^4-4\left(ax\log\left(ax+1\right)+2\,ax-2\right)\log\left(-ax+1\right)^3+6\left(ax\log\left(ax+1\right)^2-4\left(ax+1\right)\log\left(ax+1\right)\right)\log\left(-ax+\right.}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(a*x*log(-a*x + 1)^4 - 4*(a*x*log(a*x + 1) + 2*a*x - 2)*log(-a*x + 1)^
3 + 6*(a*x*log(a*x + 1)^2 - 4*(a*x + 1)*log(a*x + 1))*log(-a*x + 1)^2)/x -
1/8*integrate(1/2*(2*log(a*x + 1)^3 + 3*((a^3*x^3 + a^2*x^2 - 2)*log(a*x +
1)^2 - 4*(a^3*x^3 + 2*a^2*x^2 + a*x)*log(a*x + 1))*log(-a*x + 1))/(a^2*x^4
- x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}\left(ax\right)^3}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\text{atanh}^3\left(ax\right)}{a^2x^4-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**4 - x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^2), x)
```

3.247 $\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)}\,dx$

Optimal. Leaf size=200

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{4}a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\tanh^{-1}($$

```
[Out] (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3
)/2 - ArcTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^4)/4 + 3*a^2*ArcTanh[a*x]*
Log[2 - 2/(1 + a*x)] + a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*a^2*Pol
yLog[2, -1 + 2/(1 + a*x)])/2 - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 +
 a*x)])/2 - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*a^2*Po
lyLog[4, -1 + 2/(1 + a*x)])/4
```

Rubi [A] time = 0.49564, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5982, 5916, 5988, 5932, 2447, 5948, 6056, 6060, 6610}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{4}a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```


```
[Out] (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3
)/2 - ArcTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^4)/4 + 3*a^2*ArcTanh[a*x]*
Log[2 - 2/(1 + a*x)] + a^2*ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*a^2*Pol
yLog[2, -1 + 2/(1 + a*x)])/2 - (3*a^2*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 +
 a*x)])/2 - (3*a^2*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*a^2*Po
lyLog[4, -1 + 2/(1 + a*x)])/4
```


Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)}\,dx + a^2\int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tanh^{-1}(ax)^3\log\left(2-\frac{2}{1+ax}\right) + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2}\,dx +$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tanh^{-1}(ax)^3\log\Bigg(2-$$

$$= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\tan$$

$$= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + 3a^2\,\mathrm{ta}$$

$$= \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} + \frac{1}{2}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{4}a^2\tanh^{-1}(ax)^4 + 3a^2\,\mathrm{ta}$$

Mathematica [A] time = 0.444073, size = 165, normalized size = 0.82

$$-\frac{1}{64}a^2\Bigg(-96\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) + 96\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 96\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(a}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)),x]
```

```
[Out] -(a^2*(-Pi^4 - 96*ArcTanh[a*x]^2 + (96*ArcTanh[a*x]^2)/(a*x) + (32*(1 - a^2
*x^2)*ArcTanh[a*x]^3)/(a^2*x^2) + 16*ArcTanh[a*x]^4 - 192*ArcTanh[a*x]*Log[
1 - E^(-2*ArcTanh[a*x])] - 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] +
96*PolyLog[2, E^(-2*ArcTanh[a*x])] - 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcT
anh[a*x])] + 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] - 48*PolyLog[4,
 E^(2*ArcTanh[a*x])]))/64
```

Maple [B] time = 1.251, size = 406, normalized size = 2.

$$-\frac{a^2(\mathrm{Artanh}\,(ax))^4}{4} + \frac{a^2(\mathrm{Artanh}\,(ax))^3}{2} - \frac{3\,a^2(\mathrm{Artanh}\,(ax))^2}{2} - \frac{3\,a\,(\mathrm{Artanh}\,(ax))^2}{2\,x} - \frac{(\mathrm{Artanh}\,(ax))^3}{2\,x^2} + a^2(\mathrm{Artanh}\,(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x)
```

```
[Out] -1/4*a^2*arctanh(a*x)^4+1/2*a^2*arctanh(a*x)^3-3/2*a^2*arctanh(a*x)^2-3/2*a
*arctanh(a*x)^2/x-1/2*arctanh(a*x)^3/x^2+a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2
))-6*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2*polylog(
4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)
^(1/2))+3*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a^2*ar
ctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2*polylog(4,(a*x+1)/(-
```

```
a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a
^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2x^2\log(-ax+1)^4+4\left(a^2x^2\log(ax+1)+1\right)\log(-ax+1)^3}{64\,x^2}-\frac{1}{8}\int\frac{2\,\log(ax+1)^3-6\,\log(ax+1)^2\log(-ax+1)+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/64*(a^2*x^2*log(-a*x + 1)^4 + 4*(a^2*x^2*log(a*x + 1) + 1)*log(-a*x + 1)^
3)/x^2 - 1/8*integrate(1/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x +
1) + 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 + 2)*log(a*x + 1))*log(-a*x + 1)
^2)/(a^2*x^5 - x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\mathrm{artanh}\,(ax)^3}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\mathrm{atanh}^3\,(ax)}{a^2x^5-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1),x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**2*x**5 - x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\mathrm{artanh}\,(ax)^3}{\left(a^2x^2-1\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)*x^3), x)
```

3.248 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{1-a^2x^2} \, dx$

Optimal. Leaf size=15

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

```
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

Rubi [A] time = 0.024674, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5948}

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{1-a^2x^2} \, dx = \frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Mathematica [A] time = 0.0080679, size = 15, normalized size = 1.

$$\frac{2\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2),x]
```

```
[Out] (2*ArcTanh[a*x]^(3/2))/(3*a)
```

Maple [A] time = 0.036, size = 12, normalized size = 0.8

$$\frac{2}{3\,a} \left(\operatorname{Artanh}\left(ax \right) \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x)
```

```
[Out] 2/3*arctanh(a*x)^(3/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{\operatorname{artanh}(ax)}}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1), x)
```

Fricas [B] time = 2.42955, size = 63, normalized size = 4.2

$$\frac{\sqrt{2}\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

Sympy [A] time = 1.76586, size = 14, normalized size = 0.93

$$\begin{cases} \frac{2\operatorname{atanh}^{\frac{3}{2}}(ax)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((2*atanh(a*x)**(3/2)/(3*a), Ne(a, 0)), (0, True))
```

Giac [B] time = 1.15293, size = 34, normalized size = 2.27

$$\frac{\sqrt{2}\log\left(-\frac{ax+1}{ax-1}\right)^{\frac{3}{2}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(2)*log(-(a*x + 1)/(a*x - 1))^(3/2)/a
```

3.249 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0440452, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx = \int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.31281, size = 0, normalized size = 0.

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.132, size = 0, normalized size = 0.

$$\int \frac{x}{(-a^2x^2+1)\,\text{Artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{a^2x^2\operatorname{atanh}(ax)-\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x) - atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

3.250 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)} dx$

Optimal. Leaf size=9

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] Log[ArcTanh[a*x]]/a
```

Rubi [A] time = 0.0275559, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5946}

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]

[Out] Log[ArcTanh[a*x]]/a
```

Rule 5946

```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} \, dx=\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.0286535, size = 9, normalized size = 1.

$$\frac{\log\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]),x]

[Out] Log[ArcTanh[a*x]]/a
```

Maple [A] time = 0.023, size = 10, normalized size = 1.1

$$\frac{\ln\left(\operatorname{Artanh}(ax)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] ln(arctanh(a*x))/a
```

Maxima [B] time = 0.955185, size = 28, normalized size = 3.11

$$\frac{\log\left(-\log\left(ax+1\right)+\log\left(-ax+1\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] log(-log(a*x + 1) + log(-a*x + 1))/a
```

Fricas [B] time = 2.26805, size = 46, normalized size = 5.11

$$\frac{\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] log(log(-(a*x + 1)/(a*x - 1)))/a
```

Sympy [A] time = 0.891838, size = 7, normalized size = 0.78

$$\frac{\log\left(\operatorname{atanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] log(atanh(a*x))/a
```

Giac [B] time = 1.17304, size = 28, normalized size = 3.11

$$\frac{\log\left(\left|\log\left(-\frac{ax+1}{ax-1}\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] log(abs(log(-(a*x + 1)/(a*x - 1))))/a
```

3.251 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0643657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx = \int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.226681, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)\operatorname{Artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(a^2x^2-1)x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{(a^2x^3-x)\,\mathrm{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^2x^3\,\mathrm{atanh}\,(ax)-x\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x),x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x) - x*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2-1)x\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)), x)
```

3.252 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=25

$$\frac{\text{Unintegrable}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a}-\frac{x}{a\tanh^{-1}(ax)}$$

```
[Out] -(x/(a*ArcTanh[a*x])) + Unintegrable[ArcTanh[a*x]^(-1), x]/a
```

Rubi [A] time = 0.0467729, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*ArcTanh[a*x])) + Defer[Int][ArcTanh[a*x]^(-1), x]/a
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx=-\frac{x}{a\tanh^{-1}(ax)}+\frac{\int \frac{1}{\tanh^{-1}(ax)}\,dx}{a}$$

Mathematica [A] time = 0.128207, size = 0, normalized size = 0.

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.117, size = 0, normalized size = 0.

$$\int \frac{x}{(-a^2x^2+1)(\text{Artanh}(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2x}{a\log(ax+1)-a\log(-ax+1)}-2\int -\frac{1}{a\log(ax+1)-a\log(-ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x/(a*log(a*x + 1) - a*log(-a*x + 1)) - 2*integrate(-1/(a*log(a*x + 1) -
a*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{a^2x^2\operatorname{atanh}^2(ax)-\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

3.253 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{1}{a\tanh^{-1}(ax)}$$

```
[Out] -(1/(a*ArcTanh[a*x]))
```

Rubi [A] time = 0.0247837, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$-\frac{1}{a\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]

[Out] -(1/(a*ArcTanh[a*x]))
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\tanh^{-1}(ax)}$$

Mathematica [A] time = 0.0047918, size = 11, normalized size = 1.

$$-\frac{1}{a\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^2),x]

[Out] -(1/(a*ArcTanh[a*x]))
```

Maple [A] time = 0.023, size = 12, normalized size = 1.1

$$-\frac{1}{a\operatorname{Artanh}(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] -1/a/arctanh(a*x)
```

Maxima [B] time = 0.970097, size = 31, normalized size = 2.82

$$-\frac{2}{a\log\left(ax+1\right)-a\log\left(-ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/(a*log(a*x + 1) - a*log(-a*x + 1))
```

Fricas [A] time = 2.10922, size = 46, normalized size = 4.18

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [A] time = 1.21073, size = 8, normalized size = 0.73

$$-\frac{1}{a\,\mathrm{atanh}\left(ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -1/(a*atanh(a*x))
```

Giac [A] time = 1.14018, size = 30, normalized size = 2.73

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1)))
```

3.254 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=32

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}-\frac{1}{ax\tanh^{-1}(ax)}$$

```
[Out] -(1/(a*x*ArcTanh[a*x])) - Unintegrable[1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi [A] time = 0.0777517, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2} dx = -\frac{1}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} dx}{a}$$

Mathematica [A] time = 0.140771, size = 0, normalized size = 0.

$$\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.118, size = 0, normalized size = 0.

$$\int \frac{1}{x(-a^2x^2+1)(\text{Artanh}(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ax\log(ax+1) - ax\log(-ax+1)} + 2\int -\frac{1}{ax^2\log(ax+1) - ax^2\log(-ax+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/(a*x*log(a*x + 1) - a*x*log(-a*x + 1)) + 2*integrate(-1/(a*x^2*log(a*x +
 1) - a*x^2*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(a^2x^3 - x\right)\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^2x^3\operatorname{atanh}^2(ax) - x\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2 - 1\right)x\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^2), x)
```

3.255 $\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=30

$$\frac{\text{Unintegrable}\left(\frac{1}{\tanh^{-1}(ax)^2}, x\right)}{2a} - \frac{x}{2a\tanh^{-1}(ax)^2}$$

```
[Out] -x/(2*a*ArcTanh[a*x]^2) + Unintegrable[ArcTanh[a*x]^(-2), x]/(2*a)
```

Rubi [A] time = 0.0486874, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*ArcTanh[a*x]^2) + Defer[Int][ArcTanh[a*x]^(-2), x]/(2*a)
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3} dx = -\frac{x}{2a\tanh^{-1}(ax)^2} + \frac{\int \frac{1}{\tanh^{-1}(ax)^2} dx}{2a}$$

Mathematica [A] time = 0.56179, size = 0, normalized size = 0.

$$\int \frac{x}{(1-a^2x^2)\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[x/((1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.115, size = 0, normalized size = 0.

$$\int \frac{x}{(-a^2x^2+1)(\text{Artanh}(ax))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int(x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ax-\left(a^2x^2-1\right)\log\left(ax+1\right)+\left(a^2x^2-1\right)\log\left(-ax+1\right)}{a^2\log\left(ax+1\right)^2-2\,a^2\log\left(ax+1\right)\log\left(-ax+1\right)+a^2\log\left(-ax+1\right)^2}+2\int-\frac{x}{\log\left(ax+1\right)-\log\left(-ax+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*lo
g(a*x + 1)^2 - 2*a^2*log(a*x + 1)*log(-a*x + 1) + a^2*log(-a*x + 1)^2) + 2*
integrate(-x/(log(a*x + 1) - log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{a^2x^2\operatorname{atanh}^3\left(ax\right)-\operatorname{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(x/(a**2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x}{\left(a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

3.256 $\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

```
[Out] -1/(2*a*ArcTanh[a*x]^2)
```

Rubi [A] time = 0.0257797, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*ArcTanh[a*x]^2)
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\tanh^{-1}(ax)^2}$$

Mathematica [A] time = 0.005255, size = 13, normalized size = 1.

$$-\frac{1}{2a\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*ArcTanh[a*x]^2)
```

Maple [A] time = 0.024, size = 12, normalized size = 0.9

$$-\frac{1}{2\,a\,(\mathrm{Artanh}\,(ax))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] -1/2/a/arctanh(a*x)^2
```

Maxima [B] time = 0.973626, size = 57, normalized size = 4.38

$$-\frac{2}{a\log(ax+1)^2 - 2\,a\log(ax+1)\log(-ax+1) + a\log(-ax+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2/(a*log(a*x + 1)^2 - 2*a*log(a*x + 1)*log(-a*x + 1) + a*log(-a*x + 1)^2)
```

Fricas [A] time = 2.24365, size = 49, normalized size = 3.77

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [A] time = 1.4995, size = 12, normalized size = 0.92

$$-\frac{1}{2a\,\mathrm{atanh}^2\,(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -1/(2*a*atanh(a*x)**2)
```

Giac [A] time = 1.19174, size = 30, normalized size = 2.31

$$-\frac{2}{a\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] -2/(a*log(-(a*x + 1)/(a*x - 1))^2)
```

3.257 $\int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tanh^{-1}(ax)^2},x\right)}{2a}-\frac{1}{2ax\tanh^{-1}(ax)^2}$$

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - Unintegrable[1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi [A] time = 0.0784743, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - Defer[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}\,dx=-\frac{1}{2ax\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)^2}\,dx}{2a}$$

Mathematica [A] time = 0.721442, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.127, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)\left(\text{Artanh}\,(ax)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ax+\left(a^2x^2-1\right)\log\left(ax+1\right)-\left(a^2x^2-1\right)\log\left(-ax+1\right)}{a^2x^2\log\left(ax+1\right)^2-2\,a^2x^2\log\left(ax+1\right)\log\left(-ax+1\right)+a^2x^2\log\left(-ax+1\right)^2}-2\int-\frac{1}{a^2x^3\log\left(ax+1\right)-a^2x^3\log\left(-ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(-a*x + 1))/(a^2*x^
2*log(a*x + 1)^2 - 2*a^2*x^2*log(a*x + 1)*log(-a*x + 1) + a^2*x^2*log(-a*x
+ 1)^2) - 2*integrate(-1/(a^2*x^3*log(a*x + 1) - a^2*x^3*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(a^2x^3-x\right)\mathrm{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*x^3 - x)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{a^2x^3\,\mathrm{atanh}^3\left(ax\right)-x\,\mathrm{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**2*x**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int-\frac{1}{\left(a^2x^2-1\right)x\,\mathrm{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*x*arctanh(a*x)^3), x)
```

3.258 $\int \frac{\tanh^{-1}(ax)^p}{1-a^2x^2} dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

```
[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

Rubi [A] time = 0.0309873, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5948}

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^p/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^p}{1-a^2x^2} dx = \frac{\tanh^{-1}(ax)^{1+p}}{a(1+p)}$$

Mathematica [A] time = 0.0112008, size = 17, normalized size = 1.

$$\frac{\tanh^{-1}(ax)^{p+1}}{a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^p/(1 - a^2*x^2),x]
```

```
[Out] ArcTanh[a*x]^(1 + p)/(a*(1 + p))
```

Maple [A] time = 0.024, size = 18, normalized size = 1.1

$$\frac{(\text{Artanh}\,(ax))^{1+p}}{a\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^p/(-a^2*x^2+1),x)
```

```
[Out] arctanh(a*x)^(1+p)/a/(1+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{artanh}(ax)^p}{a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(arctanh(a*x)^p/(a^2*x^2 - 1), x)
```

Fricas [B] time = 2.15072, size = 212, normalized size = 12.47

$$\frac{\cosh\left(p\log\left(\frac{1}{2}\,\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+\log\left(-\frac{ax+1}{ax-1}\right)\sinh\left(p\log\left(\frac{1}{2}\,\log\left(-\frac{ax+1}{ax-1}\right)\right)\right)}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(p*log(1/2*log(-(a*x + 1)/(a*x - 1))))*log(-(a*x + 1)/(a*x - 1)) +
 log(-(a*x + 1)/(a*x - 1))*sinh(p*log(1/2*log(-(a*x + 1)/(a*x - 1)))))/(a*p
 + a)
```

Sympy [A] time = 2.97073, size = 26, normalized size = 1.53

$$\begin{cases} \frac{\begin{cases} \frac{\operatorname{atanh}^{p+1}(ax)}{p+1} & \text{for}\ p \neq -1 \\ \log(\operatorname{atanh}(ax)) & \text{otherwise} \end{cases}}{a} & \text{for}\ a \neq 0 \\ 0^p x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**p/(-a**2*x**2+1),x)
```

```
[Out] Piecewise((Piecewise((atanh(a*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(atanh(a
*x)), True))/a, Ne(a, 0)), (0**p*x, True))
```

Giac [A] time = 1.1937, size = 41, normalized size = 2.41

$$\frac{\left(\frac{1}{2}\,\log\left(-\frac{ax+1}{ax-1}\right)\right)^{p+1}}{a\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^p/(-a^2*x^2+1),x, algorithm="giac")
```

```
[Out] (1/2*log(-(a*x + 1)/(a*x - 1)))^(p + 1)/(a*(p + 1))
```

3.259 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=109

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{x}{4a^3\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{\tanh^{-1}(ax)}{4a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}$$

```
[Out] -x/(4*a^3*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^4) + ArcTanh[a*x]/(2*a^4*(1 -
a^2*x^2)) + ArcTanh[a*x]^2/(2*a^4) - (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 -
PolyLog[2, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rubi [A] time = 0.160454, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6028, 5984, 5918, 2402, 2315, 5994, 199, 206}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{x}{4a^3\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^4}-\frac{\tanh^{-1}(ax)}{4a^4}-\frac{\log\left(\frac{2}{1-ax}\right)\tanh^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -x/(4*a^3*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^4) + ArcTanh[a*x]/(2*a^4*(1 -
a^2*x^2)) + ArcTanh[a*x]^2/(2*a^4) - (ArcTanh[a*x]*Log[2/(1 - a*x)])/a^4 -
PolyLog[2, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)}{1-a^2x^2}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx}{2a^3} - \frac{\int \frac{\tanh^{-1}(ax)}{1-ax}\,dx}{a^3}$$

$$= -\frac{x}{4a^3\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a^3} + \frac{\int \frac{\log(}{1-}}{}$$

$$= -\frac{x}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\text{Subs}}{}$$

$$= -\frac{x}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{4a^4} + \frac{\tanh^{-1}(ax)}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^2}{2a^4} - \frac{\tanh^{-1}(ax)\log\left(\frac{2}{1-ax}\right)}{a^4} - \frac{\text{Li}_2(1}{}$$

Mathematica [A] time = 0.159252, size = 64, normalized size = 0.59

$$-\frac{-4\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 4\tanh^{-1}(ax)^2 + \sinh\left(2\tanh^{-1}(ax)\right) - 2\tanh^{-1}(ax)\left(\cosh\left(2\tanh^{-1}(ax)\right) - 4\log\left(e^{-}\right.\right.}{8a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -(4*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 4*Log[1 + E^(-2
*ArcTanh[a*x])]) - 4*PolyLog[2, -E^(-2*ArcTanh[a*x])] + Sinh[2*ArcTanh[a*x]
])/(8*a^4)
```

Maple [B] time = 0.057, size = 203, normalized size = 1.9

$$-\frac{\text{Artanh}\,(ax)}{4\,a^4\,(ax-1)}+\frac{\text{Artanh}\,(ax)\ln(ax-1)}{2\,a^4}+\frac{\text{Artanh}\,(ax)}{4\,a^4\,(ax+1)}+\frac{\text{Artanh}\,(ax)\ln(ax+1)}{2\,a^4}+\frac{(\ln(ax-1))^2}{8\,a^4}-\frac{1}{2\,a^4}\text{dilog}\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/a^4*arctanh(a*x)/(a*x-1)+1/2/a^4*arctanh(a*x)*ln(a*x-1)+1/4/a^4*arctan
h(a*x)/(a*x+1)+1/2/a^4*arctanh(a*x)*ln(a*x+1)+1/8/a^4*ln(a*x-1)^2-1/2/a^4*d
ilog(1/2+1/2*a*x)-1/4/a^4*ln(a*x-1)*ln(1/2+1/2*a*x)-1/8/a^4*ln(a*x+1)^2+1/4
/a^4*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/4/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/
8/a^4/(a*x-1)+1/8/a^4*ln(a*x-1)+1/8/a^4/(a*x+1)-1/8/a^4*ln(a*x+1)
```

Maxima [A] time = 0.968799, size = 239, normalized size = 2.19

$$-\frac{1}{8}\,a\left(\frac{\left(a^2x^2-1\right)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax+1)\log(ax-1)-\left(a^2x^2-1\right)\log(ax-1)^2-2\,ax-\left(a^2x^2-1\right)\log}{a^7x^2-a^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2*a*x - (a^2*x^2 - 1)*log(a*x - 1))
/(a^7*x^2 - a^5) + 4*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/
2))/a^5 + log(a*x + 1)/a^5) - 1/2*(1/(a^6*x^2 - a^4) - log(a^2*x^2 - 1)/a^4
)*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\,\text{artanh}\,(ax)}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atanh}(ax)}{(ax-1)^2 (ax+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)}{(a^2x^2-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

3.260 $\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=57

$$-\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^3}$$

```
[Out] -1/(4*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*a^2*(1 - a^2*x^2)) - ArcTanh
[a*x]^2/(4*a^3)
```

Rubi [A] time = 0.0628238, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5998, 5948}

$$-\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/(4*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*a^2*(1 - a^2*x^2)) - ArcTanh
[a*x]^2/(4*a^3)
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}dx &= -\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}dx}{2a^2}\\ &= -\frac{1}{4a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^2}{4a^3}\end{aligned}$$

Mathematica [A] time = 0.0961048, size = 45, normalized size = 0.79

$$\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2-2ax\tanh^{-1}(ax)+1}{4a^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (1 - a^2*x^2)*ArcTanh[a*x]^2)/(4*a^3*(-1 + a^2*x^
2))
```

Maple [B] time = 0.053, size = 169, normalized size = 3.

$$-\frac{\operatorname{Artanh}(ax)}{4\,a^3\,(ax-1)}+\frac{\operatorname{Artanh}(ax)\ln(ax-1)}{4\,a^3}-\frac{\operatorname{Artanh}(ax)}{4\,a^3\,(ax+1)}-\frac{\operatorname{Artanh}(ax)\ln(ax+1)}{4\,a^3}+\frac{(\ln(ax+1))^2}{16\,a^3}-\frac{\ln(ax+1)}{8\,a^3}\ln\Big(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/a^3*arctanh(a*x)/(a*x-1)+1/4/a^3*arctanh(a*x)*ln(a*x-1)-1/4/a^3*arctan
h(a*x)/(a*x+1)-1/4/a^3*arctanh(a*x)*ln(a*x+1)+1/16/a^3*ln(a*x+1)^2-1/8/a^3*
ln(-1/2*a*x+1/2)*ln(a*x+1)+1/8/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/16/a^
3*ln(a*x-1)^2-1/8/a^3*ln(a*x-1)*ln(1/2+1/2*a*x)+1/8/a^3/(a*x-1)-1/8/a^3/(a*
x+1)
```

Maxima [B] time = 0.964538, size = 170, normalized size = 2.98

$$-\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)+\frac{\left((a^2x^2-1)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax+1)\log(ax-\right.}{16\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x) + 1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(
a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a/(a^6*x^2 - a^4)
```

Fricas [A] time = 2.00204, size = 142, normalized size = 2.49

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^5*x^2 - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}(ax)}{(ax-1)^2 (ax+1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

3.261 $\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=55

$$-\frac{x}{4a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{4a^2}$$

```
[Out] -x/(4*a*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^2) + ArcTanh[a*x]/(2*a^2*(1 - a^
2*x^2))
```

Rubi [A] time = 0.0376937, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5994, 199, 206}

$$-\frac{x}{4a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] -x/(4*a*(1 - a^2*x^2)) - ArcTanh[a*x]/(4*a^2) + ArcTanh[a*x]/(2*a^2*(1 - a^
2*x^2))
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx = \frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)} - \frac{\int \frac{1}{(1-a^2x^2)^2}\,dx}{2a}$$
$$= -\frac{x}{4a(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)} - \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a}$$
$$= -\frac{x}{4a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{4a^2} + \frac{\tanh^{-1}(ax)}{2a^2(1-a^2x^2)}$$

Mathematica [A] time = 0.0542891, size = 66, normalized size = 1.2

$$\frac{-a^2x^2\log(ax+1) + (a^2x^2-1)\log(1-ax) + 2ax + \log(ax+1) - 4\tanh^{-1}(ax)}{8a^2(a^2x^2-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^2,x]
```

```
[Out] (2*a*x - 4*ArcTanh[a*x] + (-1 + a^2*x^2)*Log[1 - a*x] + Log[1 + a*x] - a^2*
x^2*Log[1 + a*x])/(8*a^2*(-1 + a^2*x^2))
```

Maple [A] time = 0.034, size = 68, normalized size = 1.2

$$-\frac{\mathrm{Artanh}(ax)}{2a^2(a^2x^2-1)} + \frac{1}{8a^2(ax-1)} + \frac{\ln(ax-1)}{8a^2} + \frac{1}{8a^2(ax+1)} - \frac{\ln(ax+1)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2/a^2/(a^2*x^2-1)*arctanh(a*x)+1/8/a^2/(a*x-1)+1/8/a^2*ln(a*x-1)+1/8/a^2
/(a*x+1)-1/8/a^2*ln(a*x+1)
```

Maxima [A] time = 0.947538, size = 84, normalized size = 1.53

$$\frac{\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}}{8a} - \frac{\mathrm{artanh}(ax)}{2(a^2x^2-1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)/a - 1/2*arctanh(a
*x)/((a^2*x^2 - 1)*a^2)
```

Fricas [A] time = 1.9274, size = 96, normalized size = 1.75

$$\frac{2\,ax - \left(a^2x^2 + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{8\left(a^4x^2 - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a*x - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [A] time = 1.91601, size = 61, normalized size = 1.11

$$\begin{cases} -\frac{a^2x^2\,\mathrm{atanh}\,(ax)}{4a^4x^2-4a^2} + \frac{ax}{4a^4x^2-4a^2} - \frac{\mathrm{atanh}\,(ax)}{4a^4x^2-4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Piecewise((-a**2*x**2*atanh(a*x)/(4*a**4*x**2 - 4*a**2) + a*x/(4*a**4*x**2
- 4*a**2) - atanh(a*x)/(4*a**4*x**2 - 4*a**2), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.11388, size = 99, normalized size = 1.8

$$\frac{x}{4\left(a^2x^2-1\right)a} - \frac{\log\left(|ax+1|\right)}{8\,a^2} + \frac{\log\left(|ax-1|\right)}{8\,a^2} - \frac{\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^2x^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((a^2*x^2 - 1)*a) - 1/8*log(abs(a*x + 1))/a^2 + 1/8*log(abs(a*x - 1))
/a^2 - 1/4*log(-(a*x + 1)/(a*x - 1))/((a^2*x^2 - 1)*a^2)
```

3.262 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4a}$$

```
[Out] -1/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^
2/(4*a)
```

Rubi [A] time = 0.0221565, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5956, 261}

$$-\frac{1}{4a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^2}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

```
[Out] -1/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^
2/(4*a)
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx &= \frac{x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4a}-\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx\\ &= -\frac{1}{4a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{4a}\end{aligned}$$

Mathematica [A] time = 0.0715913, size = 44, normalized size = 0.81

$$\frac{\left(a^2x^2-1\right)\tanh^{-1}(ax)^2-2ax\tanh^{-1}(ax)+1}{4a\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (-1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a*(-1 + a^2*x^2
))
```

Maple [B] time = 0.052, size = 169, normalized size = 3.1

$$-\frac{\operatorname{Artanh}(ax)}{4a(ax-1)}-\frac{\operatorname{Artanh}(ax)\ln(ax-1)}{4a}-\frac{\operatorname{Artanh}(ax)}{4a(ax+1)}+\frac{\operatorname{Artanh}(ax)\ln(ax+1)}{4a}-\frac{(\ln(ax-1))^2}{16a}+\frac{\ln(ax-1)}{8a}\ln\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/a*arctanh(a*x)/(a*x-1)-1/4/a*arctanh(a*x)*ln(a*x-1)-1/4/a*arctanh(a*x)
/(a*x+1)+1/4/a*arctanh(a*x)*ln(a*x+1)-1/16/a*ln(a*x-1)^2+1/8/a*ln(a*x-1)*ln
(1/2+1/2*a*x)-1/16/a*ln(a*x+1)^2+1/8/a*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/8/a*ln(
-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/8/a/(a*x-1)-1/8/a/(a*x+1)
```

Maxima [B] time = 0.960022, size = 165, normalized size = 3.06

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)-\frac{\left(\left(a^2x^2-1\right)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax+1)\log(ax-\right.}{16\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x) - 1
/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x -
1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a/(a^4*x^2 - a^2)
```

Fricas [A] time = 1.85949, size = 139, normalized size = 2.57

$$-\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x
- 1))^2 - 4)/(a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a^2*x^2 - 1)^2, x)
```

3.263 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{ax}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{4}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(a$$

```
[Out] -(a*x)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]/4 + ArcTanh[a*x]/(2*(1 - a^2*x^2))
+ ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/
(1 + a*x)]/2
```

Rubi [A] time = 0.164075, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6030, 5988, 5932, 2447, 5994, 199, 206}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{ax}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{4}\tanh^{-1}(ax)+\log\left(2-\frac{2}{ax+1}\right)\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] -(a*x)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]/4 + ArcTanh[a*x]/(2*(1 - a^2*x^2))
+ ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/
(1 + a*x)]/2
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```

```
x][[2]], Expon[Pq, x]]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx \\
&= \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 - \frac{1}{2}a\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x(1+ax)}\,dx \\
&= -\frac{ax}{4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) - \frac{1}{4}a\int \frac{1}{1-a^2x^2} \\
&= -\frac{ax}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax) + \frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}\tanh^{-1}(ax)^2 + \tanh^{-1}(ax)\log\left(2-\frac{2}{1+ax}\right) -
\end{aligned}
$$

Mathematica [A] time = 0.168184, size = 63, normalized size = 0.69

$$\frac{1}{8}\left(-4\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right) + 4\tanh^{-1}(ax)^2 - \sinh\left(2\tanh^{-1}(ax)\right) + 2\tanh^{-1}(ax)\left(4\log\left(1-e^{-2\tanh^{-1}(ax)}\right) + \text{co}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (4*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] + 4*Log[1 - E^(-2*
ArcTanh[a*x])]) - 4*PolyLog[2, E^(-2*ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]])
/8
```

Maple [B] time = 0.064, size = 190, normalized size = 2.1

$$-\frac{\text{Artanh}(ax)}{4ax-4} - \frac{\text{Artanh}(ax)\ln(ax-1)}{2} + \text{Artanh}(ax)\ln(ax) + \frac{\text{Artanh}(ax)}{4ax+4} - \frac{\text{Artanh}(ax)\ln(ax+1)}{2} - \frac{\text{dilog}(a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4*arctanh(a*x)/(a*x-1)-1/2*arctanh(a*x)*ln(a*x-1)+arctanh(a*x)*ln(a*x)+1
/4*arctanh(a*x)/(a*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*dilog(a*x)-1/2*dilog
(a*x+1)-1/2*ln(a*x)*ln(a*x+1)-1/8*ln(a*x-1)^2+1/2*dilog(1/2+1/2*a*x)+1/4*ln
(a*x-1)*ln(1/2+1/2*a*x)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2+1/2*a*x))*ln(
-1/2*a*x+1/2)+1/8/(a*x-1)+1/8*ln(a*x-1)+1/8/(a*x+1)-1/8*ln(a*x+1)
```

Maxima [B] time = 0.981946, size = 278, normalized size = 3.05

$$\frac{1}{8}a\left(\frac{\left(a^2x^2-1\right)\log\left(ax+1\right)^2-2\left(a^2x^2-1\right)\log\left(ax+1\right)\log\left(ax-1\right)-\left(a^2x^2-1\right)\log\left(ax-1\right)^2+2\,ax}{a^3x^2-a}+\frac{4\left(\log\left(ax-1\right)\log\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a*(((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
 - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 2*a*x)/(a^3*x^2 - a) + 4*(log(a*x -
1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 4*(log(a*x + 1)*log(x) +
 dilog(-a*x))/a + 4*(log(-a*x + 1)*log(x) + dilog(a*x))/a - log(a*x + 1)/a
+ log(a*x - 1)/a) - 1/2*(1/(a^2*x^2 - 1) + log(a^2*x^2 - 1) - log(x^2))*arc
tanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(ax\right)}{a^4x^5-2\,a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{atanh}\left(ax\right)}{x\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x), x)
```

3.264 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^2} dx$

Optimal. Leaf size=82

$$-\frac{a}{4(1-a^2x^2)}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)}+a\log(x)+\frac{3}{4}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

```
[Out] -a/(4*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*x*ArcTanh[a*x])/(2*(1 - a^2*x^
2)) + (3*a*ArcTanh[a*x]^2)/4 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rubi [A] time = 0.145359, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6030, 5982, 5916, 266, 36, 29, 31, 5948, 5956, 261}

$$-\frac{a}{4(1-a^2x^2)}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{a^2x\tanh^{-1}(ax)}{2(1-a^2x^2)}+a\log(x)+\frac{3}{4}a\tanh^{-1}(ax)^2-\frac{\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] -a/(4*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*x*ArcTanh[a*x])/(2*(1 - a^2*x^
2)) + (3*a*ArcTanh[a*x]^2)/4 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx\\
&= \frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{4}a\tanh^{-1}(ax)^2+a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx-\frac{1}{2}a^3\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}
\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+a\int \frac{1}{x\left(1-a^2x^2\right)}\,dx\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\left(1-a^2x\right)}\right.\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)\\
&= -\frac{a}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{x}+\frac{a^2x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{3}{4}a\tanh^{-1}(ax)^2+a\log(x)-\frac{1}{2}a\log\left(1-a^2x
\end{aligned}
$$

Mathematica [A] time = 0.141376, size = 77, normalized size = 0.94

$$
\frac{1}{4}\left(a\left(\frac{1}{a^2x^2-1}-2\log\left(1-a^2x^2\right)+4\log(ax)\right)-\frac{2\left(3a^2x^2-2\right)\tanh^{-1}(ax)}{x\left(a^2x^2-1\right)}+3a\tanh^{-1}(ax)^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] ((-2*(-2 + 3*a^2*x^2)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)) + 3*a*ArcTanh[a*x]^2
 + a*((-1 + a^2*x^2)^(-1) + 4*Log[a*x] - 2*Log[1 - a^2*x^2]))/4
```

Maple [B] time = 0.062, size = 180, normalized size = 2.2

$$-\frac{a\text{Artanh}\,(ax)}{4\,ax-4}-\frac{3\,a\text{Artanh}\,(ax)\ln\,(ax-1)}{4}-\frac{\text{Artanh}\,(ax)}{x}-\frac{a\text{Artanh}\,(ax)}{4\,ax+4}+\frac{3\,a\text{Artanh}\,(ax)\ln\,(ax+1)}{4}-\frac{3\,a\,(\ln\,(ax}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4*a*arctanh(a*x)/(a*x-1)-3/4*a*arctanh(a*x)*ln(a*x-1)-arctanh(a*x)/x-1/4
*a*arctanh(a*x)/(a*x+1)+3/4*a*arctanh(a*x)*ln(a*x+1)-3/16*a*ln(a*x-1)^2+3/8
*a*ln(a*x-1)*ln(1/2+1/2*a*x)-3/16*a*ln(a*x+1)^2-3/8*a*ln(-1/2*a*x+1/2)*ln(1
/2+1/2*a*x)+3/8*a*ln(-1/2*a*x+1/2)*ln(a*x+1)-1/2*a*ln(a*x-1)+1/8*a/(a*x-1)+
a*ln(a*x)-1/2*a*ln(a*x+1)-1/8*a/(a*x+1)
```

Maxima [B] time = 0.98279, size = 203, normalized size = 2.48

$$-\frac{1}{16}\,a\left(\frac{3\left(a^2x^2-1\right)\log\,(ax+1)^2-6\left(a^2x^2-1\right)\log\,(ax+1)\log\,(ax-1)+3\left(a^2x^2-1\right)\log\,(ax-1)^2-4}{a^2x^2-1}+8\,\log\,(ax+1)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/16*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*x + 1)*log
(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8*log(a*x +
 1) + 8*log(a*x - 1) - 16*log(x)) + 1/4*(3*a*log(a*x + 1) - 3*a*log(a*x - 1
) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)
```

Fricas [A] time = 2.12923, size = 252, normalized size = 3.07

$$\frac{3\left(a^3x^3-ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,ax-8\left(a^3x^3-ax\right)\log\left(a^2x^2-1\right)+16\left(a^3x^3-ax\right)\log\,(x)-4\left(3\,a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)}{16\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3*x^3 - a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x - 8*(a^3*x^3 -
a*x)*log(a^2*x^2 - 1) + 16*(a^3*x^3 - a*x)*log(x) - 4*(3*a^2*x^2 - 2)*log(-
(a*x + 1)/(a*x - 1)))/(a^2*x^3 - x)
```

Sympy [A] time = 4.4774, size = 253, normalized size = 3.09

$$\begin{cases} \frac{4a^3x^3\log(x)}{4a^2x^3-4x} - \frac{4a^3x^3\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x} + \frac{3a^3x^3\,\mathrm{atanh}^2(ax)}{4a^2x^3-4x} - \frac{4a^3x^3\,\mathrm{atanh}(ax)}{4a^2x^3-4x} - \frac{6a^2x^2\,\mathrm{atanh}(ax)}{4a^2x^3-4x} - \frac{4ax\log(x)}{4a^2x^3-4x} + \frac{4ax\log\left(x-\frac{1}{a}\right)}{4a^2x^3-4x} - \frac{3ax\,\mathrm{atanh}^2(ax)}{4a^2x^3-4x} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Piecewise((4*a**3*x**3*log(x)/(4*a**2*x**3 - 4*x) - 4*a**3*x**3*log(x - 1/a
)/(4*a**2*x**3 - 4*x) + 3*a**3*x**3*atanh(a*x)**2/(4*a**2*x**3 - 4*x) - 4*a
**3*x**3*atanh(a*x)/(4*a**2*x**3 - 4*x) - 6*a**2*x**2*atanh(a*x)/(4*a**2*x*
*3 - 4*x) - 4*a*x*log(x)/(4*a**2*x**3 - 4*x) + 4*a*x*log(x - 1/a)/(4*a**2*x
**3 - 4*x) - 3*a*x*atanh(a*x)**2/(4*a**2*x**3 - 4*x) + 4*a*x*atanh(a*x)/(4*
a**2*x**3 - 4*x) + a*x/(4*a**2*x**3 - 4*x) + 4*atanh(a*x)/(4*a**2*x**3 - 4*
x), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^2), x)
```

3.265 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=123

$$-a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^3x}{4\left(1-a^2x^2\right)}+\frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a^2\tanh^{-1}(ax)^2+\frac{1}{4}a^2\tanh^{-1}(ax)+2a^2\log\left(2-\frac{2}{ax+1}\right)$$

```
[Out] -a/(2*x) - (a^3*x)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x])/4 - ArcTanh[a*x]/
(2*x^2) + (a^2*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + a^2*ArcTanh[a*x]^2 + 2*a^2
*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a^2*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.358372, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6030, 5982, 5916, 325, 206, 5988, 5932, 2447, 5994, 199}

$$-a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{a^3x}{4\left(1-a^2x^2\right)}+\frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+a^2\tanh^{-1}(ax)^2+\frac{1}{4}a^2\tanh^{-1}(ax)+2a^2\log\left(2-\frac{2}{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] -a/(2*x) - (a^3*x)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x])/4 - ArcTanh[a*x]/
(2*x^2) + (a^2*ArcTanh[a*x])/(2*(1 - a^2*x^2)) + a^2*ArcTanh[a*x]^2 + 2*a^2
*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a^2*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\right) + a^4\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{2}a\int \frac{1}{x^2\left(1-a^2x^2\right)}\,dx + 2\left(\frac{1}{2}a^2\tanh^{-1}(ax)^2 + a^2\int \frac{\tanh^{-1}(ax}{x(1+ax)}\right.$$

$$= -\frac{a}{2x} - \frac{a^3x}{4\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}a^3\int \frac{1}{1-a^2x^2}\,dx + \frac{1}{2}a^3\int \frac{1}{1-a^2x^2}\,dx +$$

$$= -\frac{a}{2x} - \frac{a^3x}{4\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax) - \frac{\tanh^{-1}(ax)}{2x^2} + \frac{a^2\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + 2\left(\frac{1}{2}a^2\tanh^{-1}(ax)^2 + a^2\right.$$

Mathematica [A] time = 0.429802, size = 83, normalized size = 0.67

$$\frac{1}{8}a^2\left(-8\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\left(-\frac{2}{a^2x^2} + 8\log\left(1-e^{-2\tanh^{-1}(ax)}\right) + \cosh\left(2\tanh^{-1}(ax)\right) + 2\right) - \frac{4}{ax} +\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*(-4/(a*x) + 8*ArcTanh[a*x]^2 + 2*ArcTanh[a*x]*(2 - 2/(a^2*x^2) + Cosh[
2*ArcTanh[a*x]] + 8*Log[1 - E^(-2*ArcTanh[a*x])]) - 8*PolyLog[2, E^(-2*ArcT
anh[a*x])] - Sinh[2*ArcTanh[a*x]]))/8
```

Maple [B] time = 0.066, size = 265, normalized size = 2.2

$$-\frac{a^2\text{Artanh}\,(ax)}{4\,ax-4} - a^2\text{Artanh}\,(ax)\ln\,(ax-1) - \frac{\text{Artanh}\,(ax)}{2\,x^2} + 2\,a^2\text{Artanh}\,(ax)\ln\,(ax) + \frac{a^2\text{Artanh}\,(ax)}{4\,ax+4} - a^2\text{Artanh}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4*a^2*arctanh(a*x)/(a*x-1)-a^2*arctanh(a*x)*ln(a*x-1)-1/2*arctanh(a*x)/x
^2+2*a^2*arctanh(a*x)*ln(a*x)+1/4*a^2*arctanh(a*x)/(a*x+1)-a^2*arctanh(a*x)
*ln(a*x+1)-a^2*dilog(a*x)-a^2*dilog(a*x+1)-a^2*ln(a*x)*ln(a*x+1)-1/4*a^2*ln
(a*x-1)^2+a^2*dilog(1/2+1/2*a*x)+1/2*a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+1/4*a^2*
ln(a*x+1)^2-1/2*a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/2*a^2*ln(-1/2*a*x+1/2)*ln(
1/2+1/2*a*x)+1/8*a^2/(a*x-1)-1/8*a^2*ln(a*x-1)-1/2*a/x+1/8*a^2/(a*x+1)+1/8*
a^2*ln(a*x+1)
```

Maxima [B] time = 0.990467, size = 315, normalized size = 2.56

$$\frac{1}{8}\left(8\left(\log\,(ax-1)\log\left(\frac{1}{2}\,ax+\frac{1}{2}\right) + \text{Li}_2\left(-\frac{1}{2}\,ax+\frac{1}{2}\right)\right)a - 8\,(\log\,(ax+1)\log\,(x) + \text{Li}_2\,(-ax))a + 8\,(\log\,(-ax+1)\log\,(x) +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))*a - 8*(log
(a*x + 1)*log(x) + dilog(-a*x))*a + 8*(log(-a*x + 1)*log(x) + dilog(a*x))*a
 + a*log(a*x + 1) - a*log(a*x - 1) - 2*(a^2*x^2 - (a^3*x^3 - a*x)*log(a*x +
 1)^2 + 2*(a^3*x^3 - a*x)*log(a*x + 1)*log(a*x - 1) + (a^3*x^3 - a*x)*log(a
*x - 1)^2 - 2)/(a^2*x^3 - x))*a - 1/2*(2*a^2*log(a^2*x^2 - 1) - 2*a^2*log(x
^2) + (2*a^2*x^2 - 1)/(a^2*x^4 - x^2))*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}(ax)}{a^4x^7 - 2\,a^2x^5 + x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{x^3\,(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((a^2*x^2 - 1)^2*x^3), x)
```

3.266 $\int \frac{x^3 \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=161

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}+\frac{1}{4a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)}+\frac{\tanh^{-}}{3a}$$

```
[Out] 1/(4*a^4*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a^3*(1 - a^2*x^2)) - ArcTanh[
a*x]^2/(4*a^4) + ArcTanh[a*x]^2/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(3*a
^4) - (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - (ArcTanh[a*x]*PolyLog[2, 1 -
2/(1 - a*x)])/a^4 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rubi [A] time = 0.291178, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6028, 5984, 5918, 5948, 6058, 6610, 5994, 5956, 261}

$$\frac{\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{a^4}+\frac{1}{4a^4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)}+\frac{\tanh^{-}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] 1/(4*a^4*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a^3*(1 - a^2*x^2)) - ArcTanh[
a*x]^2/(4*a^4) + ArcTanh[a*x]^2/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(3*a
^4) - (ArcTanh[a*x]^2*Log[2/(1 - a*x)])/a^4 - (ArcTanh[a*x]*PolyLog[2, 1 -
2/(1 - a*x)])/a^4 + PolyLog[3, 1 - 2/(1 - a*x)]/(2*a^4)
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a^3} - \frac{\int \frac{\tanh^{-1}(ax)^2}{1-ax}\,dx}{a^3}$$

$$= -\frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2\log\left(\frac{2}{1-ax}\right)}{a^4} + \frac{2}{}$$

$$= \frac{1}{4a^4\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2}{a^4}$$

$$= \frac{1}{4a^4\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^4} + \frac{\tanh^{-1}(ax)^2}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^3}{3a^4} - \frac{\tanh^{-1}(ax)^2}{a^4}$$

Mathematica [A] time = 0.178218, size = 103, normalized size = 0.64

$$\frac{\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) - \frac{1}{3}\tanh^{-1}(ax)^3 - \tanh^{-1}(ax)^2\log\left(e^{-2\tanh^{-1}(ax)}\right.}{a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-ArcTanh[a*x]^3/3 + ((1 + 2*ArcTanh[a*x]^2)*Cosh[2*ArcTanh[a*x]])/8 - ArcT
anh[a*x]^2*Log[1 + E^(-2*ArcTanh[a*x])] + ArcTanh[a*x]*PolyLog[2, -E^(-2*Ar
cTanh[a*x])] + PolyLog[3, -E^(-2*ArcTanh[a*x])]/2 - (ArcTanh[a*x]*Sinh[2*Ar
cTanh[a*x]])/4)/a^4
```

Maple [C] time = 0.348, size = 907, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/16/a^3/(a*x+1)*x-1/16/a^3*x/(a*x-1)-1/4/a^4*arctanh(a*x)^2/(a*x-1)+1/4/a
^4*arctanh(a*x)^2/(a*x+1)+1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))-1/4*arctanh(a*x)^2/a^4+1/2/a^4*polylog(3,-(a*x+1)
^2/(-a^2*x^2+1))-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^
3-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^3-1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3+1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/8/a^
3*arctanh(a*x)/(a*x+1)*x-1/2*I/a^4*arctanh(a*x)^2*Pi+1/8/a^3*arctanh(a*x)/(
a*x-1)*x-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/4*I/a^4*arctanh(a*x)^2*P
```

```
i*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2-1/4*I/a^4*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))-1/16/a^4/(a*
x-1)+1/16/a^4/(a*x+1)+1/3*arctanh(a*x)^3/a^4-1/a^4*arctanh(a*x)^2*ln(2)+1/8
/a^4*arctanh(a*x)/(a*x-1)+1/2/a^4*arctanh(a*x)^2*ln(a*x-1)+1/2/a^4*arctanh(
a*x)^2*ln(a*x+1)-1/a^4*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/a^4*
arctanh(a*x)*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+1/8/a^4*arctanh(a*x)/(a*x+1
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{4}a^3\int\frac{x^3\log(ax+1)\log(-ax+1)}{a^7x^4-2a^5x^2+a^3}\,dx-\frac{1}{4}a^2\int\frac{x^2\log(ax+1)\log(-ax+1)}{a^7x^4-2a^5x^2+a^3}\,dx-\frac{1}{32}\left(a\left(\frac{2}{a^7x-a^6}-\frac{\log(ax+1)}{a^6}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -3/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^
3), x) - 1/4*a^2*integrate(x^2*log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*
x^2 + a^3), x) - 1/32*(a*(2/(a^7*x - a^6) - log(a*x + 1)/a^6 + log(a*x - 1)
/a^6) + 4*log(-a*x + 1)/(a^7*x^2 - a^5))*a + 1/4*a*integrate(x*log(a*x + 1)
*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x) + 1/24*((a^2*x^2 - 1)*log(-a
*x + 1)^3 + 3*((a^2*x^2 - 1)*log(a*x + 1) - 1)*log(-a*x + 1)^2)/(a^6*x^2 -
a^4) + 1/4*integrate(a^3*x^3*log(a*x + 1)^2/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
 + 1/4*integrate(log(a*x + 1)*log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
 + 1/4*integrate(log(-a*x + 1)/(a^7*x^4 - 2*a^5*x^2 + a^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3\,\mathrm{artanh}(ax)^2}{a^4x^4-2a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\,\mathrm{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

3.267 $\int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=94

$$\frac{x}{4a^2(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\tanh^{-1}(ax)}{4a^3}$$

```
[Out] x/(4*a^2*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a^3) - ArcTanh[a*x]/(2*a^3*(1 - a
^2*x^2)) + (x*ArcTanh[a*x]^2)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(6*a^3
)
```

Rubi [A] time = 0.102659, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6000, 5994, 199, 206}

$$\frac{x}{4a^2(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{\tanh^{-1}(ax)}{2a^3(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\tanh^{-1}(ax)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] x/(4*a^2*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a^3) - ArcTanh[a*x]/(2*a^3*(1 - a
^2*x^2)) + (x*ArcTanh[a*x]^2)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(6*a^3
)
```

Rule 6000

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> -Simp[(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^
2, x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} - \frac{\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx}{2a^2}$$

$$= \frac{x}{4a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{1-a^2x^2}\,dx}{4a^2}$$

$$= \frac{x}{4a^2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)}{4a^3} - \frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3}$$

Mathematica [A] time = 0.130132, size = 93, normalized size = 0.99

$$\frac{-3\left(\left(a^2x^2-1\right)\log(1-ax)+\left(1-a^2x^2\right)\log(ax+1)+2ax\right)+\left(4-4a^2x^2\right)\tanh^{-1}(ax)^3-12ax\tanh^{-1}(ax)^2+12\tanh^{-1}(a}{24a^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + (4 - 4*a^2*x^2)*ArcTanh[a*x]^3 -
 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(24*
a^3*(-1 + a^2*x^2))
```

Maple [C] time = 0.406, size = 1722, normalized size = 18.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] 1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2
*Pi*x^2-1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arct
anh(a*x)^2*Pi-1/4/a^3*arctanh(a*x)^2/(a*x-1)+1/4/a^3*arctanh(a*x)^2*ln(a*x-
1)-1/4/a^3*arctanh(a*x)^2/(a*x+1)-1/4/a^3*arctanh(a*x)^2*ln(a*x+1)+1/2/a^3*
arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I/a^3/(a*x-1)/(a*x+1)*arc
tanh(a*x)^2*Pi-1/6/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*x^2+1/4/a/(a*x-1)/(a*x+
1)*arctanh(a*x)*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1))^3*Pi*x^2-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+
1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2-1/4*I/a^3/(a*x
-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*arctanh(a*x)^2*Pi+1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^
2*Pi-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
```

```
/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^2*Pi-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))*arctanh(a*x)^2*Pi+1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi*x^2-1/8*I/a/
(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)
*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*
x^2-1))*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi
*x^2+1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*arctanh(a*x)^2*Pi-1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)
^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-1/4*I/a/
(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*x^2-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x
+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi+1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-1/4/a^2/(a*x-1)/(a*x+1)*x+1/4/
a^3/(a*x-1)/(a*x+1)*arctanh(a*x)+1/6/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)^3
```

Maxima [B] time = 0.980199, size = 369, normalized size = 3.93

$$-\frac{1}{4}\left(\frac{2\,x}{a^4x^2-a^2}+\frac{\log\left(ax+1\right)}{a^3}-\frac{\log\left(ax-1\right)}{a^3}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(\left(a^2x^2-1\right)\log\left(ax+1\right)^3-3\left(a^2x^2-1\right)\log\left(ax+1\right)^2\log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x)^2 - 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*
log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*(2*a^2*x^2 - (a^2*
x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 - 1)*log(a*x - 1))*a
^2/(a^7*x^2 - a^5) + 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*lo
g(a*x + 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)/
(a^6*x^2 - a^4)
```

Fricas [A] time = 1.92203, size = 211, normalized size = 2.24

$$-\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^2 -
a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{(ax-1)^2 (ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^2}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

3.268 $\int \frac{x \tanh^{-1}(ax)^2}{(1-a^2x^2)^2} \, dx$

Optimal. Leaf size=82

$$\frac{1}{4a^2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{x\tanh^{-1}(ax)}{2a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^2}{4a^2}$$

```
[Out] 1/(4*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^2/(4*a^2) + ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2))
```

Rubi [A] time = 0.0679388, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5994, 5956, 261}

$$\frac{1}{4a^2(1-a^2x^2)} + \frac{\tanh^{-1}(ax)^2}{2a^2(1-a^2x^2)} - \frac{x\tanh^{-1}(ax)}{2a(1-a^2x^2)} - \frac{\tanh^{-1}(ax)^2}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] 1/(4*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(2*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^2/(4*a^2) + ArcTanh[a*x]^2/(2*a^2*(1 - a^2*x^2))
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx = \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{a}$$
$$= -\frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^2} + \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} + \frac{1}{2}\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx$$
$$= \frac{1}{4a^2\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{4a^2} + \frac{\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.0512578, size = 43, normalized size = 0.52

$$\frac{\left(a^2x^2+1\right)\tanh^{-1}(ax)^2 - 2ax\tanh^{-1}(ax) + 1}{4a^2 - 4a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^2,x]
```

```
[Out] (1 - 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2)/(4*a^2 - 4*a^4*x^2)
```

Maple [B] time = 0.055, size = 191, normalized size = 2.3

$$-\frac{(\mathrm{Artanh}\,(ax))^2}{2\,a^2\left(a^2x^2-1\right)} + \frac{\mathrm{Artanh}\,(ax)}{4\,a^2\,(ax-1)} + \frac{\mathrm{Artanh}\,(ax)\ln(ax-1)}{4\,a^2} + \frac{\mathrm{Artanh}\,(ax)}{4\,a^2\,(ax+1)} - \frac{\mathrm{Artanh}\,(ax)\ln(ax+1)}{4\,a^2} + \frac{(\ln(ax+1))^2}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2/a^2/(a^2*x^2-1)*arctanh(a*x)^2+1/4/a^2*arctanh(a*x)/(a*x-1)+1/4/a^2*ar
ctanh(a*x)*ln(a*x-1)+1/4/a^2*arctanh(a*x)/(a*x+1)-1/4/a^2*arctanh(a*x)*ln(a
*x+1)+1/16/a^2*ln(a*x+1)^2+1/8/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/8/a^2
*ln(-1/2*a*x+1/2)*ln(a*x+1)+1/16/a^2*ln(a*x-1)^2-1/8/a^2*ln(a*x-1)*ln(1/2+1
/2*a*x)-1/8/a^2/(a*x-1)+1/8/a^2/(a*x+1)
```

Maxima [B] time = 0.966322, size = 197, normalized size = 2.4

$$\frac{\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\mathrm{artanh}\,(ax)}{4\,a} + \frac{\left(a^2x^2-1\right)\log(ax+1)^2 - 2\left(a^2x^2-1\right)\log(ax+1)\log(ax-1) + \left(a^2x^2-1\right)}{16\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)/a +
1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x -
 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^4*x^2 - a^2) - 1/2*arctanh(a*x)^
2/((a^2*x^2 - 1)*a^2)
```

Fricas [A] time = 2.09665, size = 140, normalized size = 1.71

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4}{16\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1))^2 - 4)/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,\mathrm{atanh}^2\,(ax)}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x*atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,\mathrm{artanh}\,(ax)^2}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

3.269 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=88

$$\frac{x}{4(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{\tanh^{-1}(ax)}{4a}$$

```
[Out] x/(4*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a) - ArcTanh[a*x]/(2*a*(1 - a^2*x^2))
 + (x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a)
```

Rubi [A] time = 0.0639225, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5956, 5994, 199, 206}

$$\frac{x}{4(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{2a(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{\tanh^{-1}(ax)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

```
[Out] x/(4*(1 - a^2*x^2)) + ArcTanh[a*x]/(4*a) - ArcTanh[a*x]/(2*a*(1 - a^2*x^2))
 + (x*ArcTanh[a*x]^2)/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/(6*a)
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx &= \frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}-a\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx\\
&= -\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{1}{2}\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx\\
&= \frac{x}{4\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}+\frac{1}{4}\int \frac{1}{1-a^2x^2}\,dx\\
&= \frac{x}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4a}-\frac{\tanh^{-1}(ax)}{2a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{6a}
\end{aligned}
$$

Mathematica [A] time = 0.11065, size = 93, normalized size = 1.06

$$
\frac{-3\left(\left(a^2x^2-1\right)\log(1-ax)+\left(1-a^2x^2\right)\log(ax+1)+2ax\right)+4\left(a^2x^2-1\right)\tanh^{-1}(ax)^3-12ax\tanh^{-1}(ax)^2+12\tanh^{-1}}{24a\left(a^2x^2-1\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^2,x]
```

```
[Out] (12*ArcTanh[a*x] - 12*a*x*ArcTanh[a*x]^2 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^3
- 3*(2*a*x + (-1 + a^2*x^2)*Log[1 - a*x] + (1 - a^2*x^2)*Log[1 + a*x]))/(24
*a*(-1 + a^2*x^2))
```

Maple [C] time = 0.413, size = 1695, normalized size = 19.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/4/(a*x-1)/(a*x+1)*x-1/4/a*arctanh(a*x)^2/(a*x-1)-1/4/a*arctanh(a*x)^2/(a
*x+1)-1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))^2*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*Pi-
1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*
x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arc
tanh(a*x)^2*Pi+1/4*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn
(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2*Pi+1/4*I*a/(a*x-1)/(a*x+1)*ar
ctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2-1/8*I*a/(a*x-1)/(a
*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1
))^3*Pi*x^2-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^3*Pi*x^2-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi+1/4*a/(a*x-1)/(a*x
+1)*arctanh(a*x)*x^2+1/6*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*x^2+1/8*I/a/(a*x-
1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arcta
nh(a*x)^2*Pi+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctan
h(a*x)^2*Pi+1/4*I/a/(a*x-1)/(a*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*ar
ctanh(a*x)^2*Pi+1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*x^2-1/4*I/a/(a*x-
1)/(a*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-1/8*I*a/(
```

```
a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2+1/8*I*a/(a*x-1)/(a*
x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*
x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*
x^2-1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2
))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi*x^2-1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*Pi-1/6/a/(a*x-1)/(a*x+
1)*arctanh(a*x)^3+1/4/a/(a*x-1)/(a*x+1)*arctanh(a*x)-1/4/a*arctanh(a*x)^2*l
n(a*x-1)+1/4/a*arctanh(a*x)^2*ln(a*x+1)-1/2/a*arctanh(a*x)^2*ln((a*x+1)/(-a
^2*x^2+1)^(1/2))+1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(
-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2
```

Maxima [B] time = 0.986465, size = 362, normalized size = 4.11

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2+\frac{\left(\left(a^2x^2-1\right)\log(ax+1)^3-3\left(a^2x^2-1\right)\log(ax+1)^2\log(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2 +
 1/48*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(a*
x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2 -
1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^
5*x^2 - a^3) - 1/8*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x
+ 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/(a^4*x
^2 - a^2)
```

Fricas [A] time = 2.05194, size = 208, normalized size = 2.36

$$-\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\,ax-6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{48\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a*
x - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^3*x^2 -
a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^2(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(a^2*x^2 - 1)^2, x)
```

3.270 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{3}\text{t}$$

```
[Out] 1/(4*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) - ArcTanh[a*x]^2
/4 + ArcTanh[a*x]^2/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*L
og[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3
, -1 + 2/(1 + a*x)]/2
```

Rubi [A] time = 0.292017, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {6030, 5988, 5932, 5948, 6056, 6610, 5994, 5956, 261}

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{1}{4\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}-\frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{3}\text{t}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] 1/(4*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(2*(1 - a^2*x^2)) - ArcTanh[a*x]^2
/4 + ArcTanh[a*x]^2/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*L
og[2 - 2/(1 + a*x)] - ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3
, -1 + 2/(1 + a*x)]/2
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

```
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx \\
&= \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{3}\tanh^{-1}(ax)^3 - a\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x(1+ax)}\,dx \\
&= -\frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2\log\left(2-\frac{2}{1+\right. \\
&= \frac{1}{4\left(1-a^2x^2\right)} - \frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2 \\
&= \frac{1}{4\left(1-a^2x^2\right)} - \frac{ax\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{3}\tanh^{-1}(ax)^3 + \tanh^{-1}(ax)^2
\end{aligned}
$$

Mathematica [C] time = 0.183503, size = 106, normalized size = 0.78

$$\frac{1}{24}\left(24\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-12\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-8\tanh^{-1}(ax)^3+24\tanh^{-1}(ax)^2\log\left(1-e^{2\tanh^{-1}(ax)}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (I*Pi^3 - 8*ArcTanh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*Cosh
[2*ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 24*ArcTa
nh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] - 12*PolyLog[3, E^(2*ArcTanh[a*x])]
- 6*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/24
```

Maple [C] time = 0.431, size = 1290, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x)
```

```
[Out] arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(
a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x
^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctan
h(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^2+1/8*(a
*x+1)*arctanh(a*x)/(a*x-1)-1/8*(a*x-1)*arctanh(a*x)/(a*x+1)+1/2*I*Pi*csgn(I
*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*
x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-1/16*(a*x
-1)/(a*x+1)+arctanh(a*x)^2*ln(2)-1/16*(a*x+1)/(a*x-1)-1/3*arctanh(a*x)^3-1/
4*arctanh(a*x)^2-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)
^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arct
anh(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2+1/2*I*Pi
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+1/2*I*Pi*csgn(I*((a*x+
1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2-2*polylog
(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4
*I*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/4*arctanh(a*x)^2/(a*x-1)-1/2*arct
anh(a*x)^2*ln(a*x-1)+1/4*arctanh(a*x)^2/(a*x+1)-1/2*arctanh(a*x)^2*ln(a*x+1
)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+1/2*I*Pi*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*
x)^2-1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2
+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arc
tanh(a*x)^2-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(
-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}a^4\int\frac{x^4\log(ax+1)\log(-ax+1)}{a^4x^5-2a^2x^3+x}\,dx+\frac{1}{4}a^3\int\frac{x^3\log(ax+1)\log(-ax+1)}{a^4x^5-2a^2x^3+x}\,dx-\frac{1}{32}\left(a\left(\frac{2}{a^4x-a^3}-\frac{\log(ax+1)}{a^3}+\frac{\log}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^4*integrate(x^4*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x),
 x) + 1/4*a^3*integrate(x^3*log(a*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3
 + x), x) - 1/32*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/a^3)
 + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^2 - 1/4*a^2*integrate(x^2*log(a*x + 1
)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/4*a*integrate(x*log(a*x +
 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) + 1/4*a*integrate(x*log(-a*
x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/24*((a^2*x^2 - 1)*log(-a*x + 1)^3
+ 3*((a^2*x^2 - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^2*x^2 - 1) + 1/4*i
ntegrate(log(a*x + 1)^2/(a^4*x^5 - 2*a^2*x^3 + x), x) - 1/2*integrate(log(a
*x + 1)*log(-a*x + 1)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\,(ax)^2}{a^4x^5-2\,a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{atanh}^2\,(ax)}{x\,(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{artanh}\,(ax)^2}{\left(a^2x^2-1\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x), x)
```

3.271 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^2} dx$

Optimal. Leaf size=142

$$-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{a^2x}{4(1-a^2x^2)}+\frac{a^2x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{a\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\tan$$

```
[Out] (a^2*x)/(4*(1 - a^2*x^2)) + (a*ArcTanh[a*x])/4 - (a*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2)/(2*
(1 - a^2*x^2)) + (a*ArcTanh[a*x]^3)/2 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x
)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.315069, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6030, 5982, 5916, 5988, 5932, 2447, 5948, 5956, 5994, 199, 206}

$$-a\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{a^2x}{4(1-a^2x^2)}+\frac{a^2x\tanh^{-1}(ax)^2}{2(1-a^2x^2)}-\frac{a\tanh^{-1}(ax)}{2(1-a^2x^2)}+\frac{1}{2}a\tanh^{-1}(ax)^3+a\tanh^{-1}(ax)^2-\tan$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*x)/(4*(1 - a^2*x^2)) + (a*ArcTanh[a*x])/4 - (a*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2)/(2*
(1 - a^2*x^2)) + (a*ArcTanh[a*x]^3)/2 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x
)] - a*PolyLog[2, -1 + 2/(1 + a*x)]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
```

```
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx$$

$$= \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{6}a\tanh^{-1}(ax)^3 + a^2\int \frac{\tanh^{-1}(ax)^2}{1-a^2x^2}\,dx - a^3\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}}{x^2}$$

$$= -\frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{2}a\tanh^{-1}(ax)^3 + (2a)\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx +$$

$$= \frac{a^2x}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{2}a\tanh^{-1}(ax$$

$$= \frac{a^2x}{4\left(1-a^2x^2\right)} + \frac{1}{4}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}$$

$$= \frac{a^2x}{4\left(1-a^2x^2\right)} + \frac{1}{4}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}$$

Mathematica [A] time = 0.34847, size = 97, normalized size = 0.68

$$\frac{-8ax\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right) + 4ax\tanh^{-1}(ax)^3 + 2\tanh^{-1}(ax)^2\left(4ax + ax\sinh\left(2\tanh^{-1}(ax)\right) - 4\right) + ax\sinh\left(2\tan\right.}{8x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (4*a*x*ArcTanh[a*x]^3 - 2*a*x*ArcTanh[a*x]*(Cosh[2*ArcTanh[a*x]] - 8*Log[1
- E^(-2*ArcTanh[a*x])]) - 8*a*x*PolyLog[2, E^(-2*ArcTanh[a*x])] + a*x*Sinh[
2*ArcTanh[a*x]] + 2*ArcTanh[a*x]^2*(-4 + 4*a*x + a*x*Sinh[2*ArcTanh[a*x]]))
/(8*x)
```

Maple [C] time = 0.55, size = 4589, normalized size = 32.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x)
```

```
[Out] 3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+3/8*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2
+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(
1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a
*x+1)^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I/
((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2
*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)
/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctan
h(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*
```

```
x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a*arctanh(a*x)^
2-a*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a*dilog(1+(a*x+1)/(-a^2*x^2+1)^
(1/2))+3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-
3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-3/4*I*a*Pi*csgn(I/((a*x+1)^2/(
-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a
*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/
(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a
*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2
+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x
^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*
a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*pol
ylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(
1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-3/8*I*a*Pi*csgn(I*(a*x
+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+3/8*
I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*a*ar
ctanh(a*x)^2*ln(a*x+1)-3/2*a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+
2*a*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*arctanh(a*x)*ln(1-(a*x+
1)/(-a^2*x^2+1)^(1/2))+1/16/(a*x+1)*a^2*x-1/16*a^2*x/(a*x-1)-1/4*a*arctanh(
a*x)^2/(a*x-1)-3/4*a*arctanh(a*x)^2*ln(a*x-1)-1/4*a*arctanh(a*x)^2/(a*x+1)+
1/8*arctanh(a*x)/(a*x-1)*a^2*x+1/8*arctanh(a*x)/(a*x+1)*a^2*x-3/4*I*a*Pi*po
lylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*polylog(2,(a*x+1)/(-a^2*x^2
+1)^(1/2))+3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog
(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))^3*arctanh(a*x)^2+3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog
(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*arctanh(a*x)^2-3/4*I*a*Pi*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x
^2+1)^(1/2))+3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/
(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1
+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/
(a^2*x^2-1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2
*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2
))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^
```

```
(1/2))+3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)^3*arctanh(a*x)^2-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^3*arctanh(a*x)^2-3/4*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)^2/x+3/4*I*a*Pi*arctanh(a*x
)^2+3/4*I*a*Pi*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/4*I*a*Pi*dilog((a*x+1)
/(-a^2*x^2+1)^(1/2))-1/16*a/(a*x-1)-1/16*a/(a*x+1)+1/2*a*arctanh(a*x)^3-3/8
*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2,(a*x+1)/(-
a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*
x^2+1)^(1/2))+3/4*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^
2/(a^2*x^2-1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*c
sgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a
*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2
+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(
a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*a*Pi*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1
/8*a*arctanh(a*x)/(a*x-1)-1/8*a*arctanh(a*x)/(a*x+1)
```

Maxima [B] time = 1.01456, size = 548, normalized size = 3.86

$$\frac{1}{16}a^2\left(\frac{\left(a^2x^2-1\right)\log\left(ax+1\right)^3-\left(a^2x^2-1\right)\log\left(ax-1\right)^3+\left(4\,a^2x^2-3\left(a^2x^2-1\right)\log\left(ax-1\right)-4\right)\log\left(ax+1\right)^2-4\left(a^2x^2\right.}{a^3x^2-a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/16*a^2*(((a^2*x^2 - 1)*log(a*x + 1)^3 - (a^2*x^2 - 1)*log(a*x - 1)^3 + (4
*a^2*x^2 - 3*(a^2*x^2 - 1)*log(a*x - 1) - 4)*log(a*x + 1)^2 - 4*(a^2*x^2 -
1)*log(a*x - 1)^2 - 4*a*x + (3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 8*(a^2*x^2 -
1)*log(a*x - 1))*log(a*x + 1))/(a^3*x^2 - a) + 16*(log(a*x - 1)*log(1/2*a*x
 + 1/2) + dilog(-1/2*a*x + 1/2))/a - 16*(log(a*x + 1)*log(x) + dilog(-a*x))
/a + 16*(log(-a*x + 1)*log(x) + dilog(a*x))/a + 2*log(a*x + 1)/a - 2*log(a*
x - 1)/a) - 1/8*a*((3*(a^2*x^2 - 1)*log(a*x + 1)^2 - 6*(a^2*x^2 - 1)*log(a*
x + 1)*log(a*x - 1) + 3*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4)/(a^2*x^2 - 1) + 8
*log(a*x + 1) + 8*log(a*x - 1) - 16*log(x))*arctanh(a*x) + 1/4*(3*a*log(a*x
 + 1) - 3*a*log(a*x - 1) - 2*(3*a^2*x^2 - 2)/(a^2*x^3 - x))*arctanh(a*x)^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(ax\right)^2}{a^4x^6-2\,a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^2), x)
```

3.272 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=205

$$-a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{a^2}{4(1-a^2x^2)}-\frac{1}{2}a^2\log\left(1-a^2x^2\right)-\frac{a^3x\tanh^{-1}}{2(1-a^2}$$

```
[Out] a^2/(4*(1 - a^2*x^2)) - (a*ArcTanh[a*x])/x - (a^3*x*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + (a^2*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(2*x^2) + (a^2*ArcTanh[a
*x]^2)/(2*(1 - a^2*x^2)) + (2*a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log
[1 - a^2*x^2])/2 + 2*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a^2*ArcTan
h[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - a^2*PolyLog[3, -1 + 2/(1 + a*x)]
```

Rubi [A] time = 0.695633, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {6030, 5982, 5916, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610, 5994, 5956, 261}

$$-a^2\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-2a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{a^2}{4(1-a^2x^2)}-\frac{1}{2}a^2\log\left(1-a^2x^2\right)-\frac{a^3x\tanh^{-1}}{2(1-a^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```


```
[Out] a^2/(4*(1 - a^2*x^2)) - (a*ArcTanh[a*x])/x - (a^3*x*ArcTanh[a*x])/(2*(1 - a
^2*x^2)) + (a^2*ArcTanh[a*x]^2)/4 - ArcTanh[a*x]^2/(2*x^2) + (a^2*ArcTanh[a
*x]^2)/(2*(1 - a^2*x^2)) + (2*a^2*ArcTanh[a*x]^3)/3 + a^2*Log[x] - (a^2*Log
[1 - a^2*x^2])/2 + 2*a^2*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 2*a^2*ArcTan
h[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - a^2*PolyLog[3, -1 + 2/(1 + a*x)]
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
```

```
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx\right) + a^4\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + a\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)}\,dx + 2\left(\frac{1}{3}a^2\tanh^{-1}(ax)^3 + a^2\int \frac{\tanh^{-1}(ax}{x(1+ax}\right.$$

$$= -\frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} - \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + a\int \frac{\tanh^{-1}(ax)}{x^2}\,dx + a^3$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(a}{2\left(1-a^2x^2\right.}$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(a}{2\left(1-a^2x^2\right.}$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(a}{2\left(1-a^2x^2\right.}$$

$$= \frac{a^2}{4\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{x} - \frac{a^3x\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^2 - \frac{\tanh^{-1}(ax)^2}{2x^2} + \frac{a^2\tanh^{-1}(a}{2\left(1-a^2x^2\right.}$$

Mathematica [C] time = 1.02408, size = 146, normalized size = 0.71

$$a^2\left(2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) + \frac{1}{24}\left(-24\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 24\log\left(\frac{ax}{\sqrt{1-a^2x^2}}\right) + 6\tanh^{-1}(ax)^2\left(-\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] a^2*(2*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((2*I)*Pi^3 - 16*ArcTa
nh[a*x]^3 + 3*Cosh[2*ArcTanh[a*x]] + 6*ArcTanh[a*x]^2*(2 - 2/(a^2*x^2) + Co
sh[2*ArcTanh[a*x]] + 8*Log[1 - E^(2*ArcTanh[a*x])]) + 24*Log[(a*x)/Sqrt[1 -
```

```
a^2*x^2]] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] - (6*ArcTanh[a*x]*(4 + a*x*S
inh[2*ArcTanh[a*x]]))/(a*x))/24)
```

Maple [C] time = 1.34, size = 3040, normalized size = 14.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x)
```

```
[Out] I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-
1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))-1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-4*a^2*polylog(3,-(a*x+1)/(-a^
2*x^2+1)^(1/2))-4*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*arctanh(a*x
)^2/x^2+I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^
2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/2*I*a^2/(a*x-1)/(a*x+1)*arcta
nh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-I*a^2/(a*x-1)/(
a*x+1)*arctanh(a*x)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)
^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+
1)+1))-1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*
a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x
)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^2-I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(
I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))-I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*((a*x
+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x
^2+1)+1))^2-1/16*a^4/(a*x-1)/(a*x+1)*x^2+2/3*a^2/(a*x-1)/(a*x+1)*arctanh(a*
x)^3-1/4*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2+a^2/(a*x-1)/(a*x+1)*arctanh(a*x
)+4*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+4*a^2*arctanh(a
*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x
)^2*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn
(I/((a*x+1)^2/(-a^2*x^2+1)+1))+I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^2+I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))^3+I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(
I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-I*a^2/(a*x-1)/(a
*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2
/(a^2*x^2-1))^2-1/2*I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/
(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I*a^2/(a*x-1)/(a*x+
1)*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*arcta
nh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/2*I*a^4*x^2/(a*x-1)/(a*x+1)*
arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
3-I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^2+a^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*ln((a*x+1)/(-a^2*x^2+1)^(1
/2)-1)-3/16*a^2/(a*x-1)/(a*x+1)-1/4*a^2*arctanh(a*x)^2/(a*x-1)-a^2*arctanh(
a*x)^2*ln(a*x-1)+2*a^2*arctanh(a*x)^2*ln(a*x)-2*a^2*arctanh(a*x)^2*ln((a*x+
1)^2/(-a^2*x^2+1)-1)+1/4*a^2*arctanh(a*x)^2/(a*x+1)-a^2*arctanh(a*x)^2*ln(a
*x+1)+2*a^2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x
)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))+1/2*I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^
```

```
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-2
*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*ln(2)-2/3*a^4*x^2/(a*x-1)/(a*x+1)*arcta
nh(a*x)^3+1/4*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2-a^4*x^2/(a*x-1)/(a*x+1
)*arctanh(a*x)-1/2*a^3*x/(a*x-1)/(a*x+1)*arctanh(a*x)+a/x/(a*x-1)/(a*x+1)*a
rctanh(a*x)+2*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*ln(2)-I*a^2/(a*x-1)/(a
*x+1)*arctanh(a*x)^2*Pi+I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))^2+I*a^4*x^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi-1/2*I
*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-I*a^
2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/
2*I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))^3-I*a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a^6\int\frac{x^6\log(ax+1)\log(-ax+1)}{a^4x^7-2a^2x^5+x^3}\,dx+\frac{1}{2}a^5\int\frac{x^5\log(ax+1)\log(-ax+1)}{a^4x^7-2a^2x^5+x^3}\,dx-\frac{1}{16}\left(a\left(\frac{2}{a^4x-a^3}-\frac{\log(ax+1)}{a^3}+\log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^6*integrate(x^6*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3
), x) + 1/2*a^5*integrate(x^5*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x
^5 + x^3), x) - 1/16*(a*(2/(a^4*x - a^3) - log(a*x + 1)/a^3 + log(a*x - 1)/
a^3) + 4*log(-a*x + 1)/(a^4*x^2 - a^2))*a^4 - 1/2*a^4*integrate(x^4*log(a*x
 + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/2*a^3*integrate(x^3
*log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) + 1/2*a^3*integ
rate(x^3*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a^2*integrate(
x^2*log(-a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/4*a*integrate(x*log(-
a*x + 1)/(a^4*x^7 - 2*a^2*x^5 + x^3), x) - 1/24*(2*(a^4*x^4 - a^2*x^2)*log(
-a*x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1) - 1)*log(-a
*x + 1)^2)/(a^2*x^4 - x^2) + 1/4*integrate(log(a*x + 1)^2/(a^4*x^7 - 2*a^2*
x^5 + x^3), x) - 1/2*integrate(log(a*x + 1)*log(-a*x + 1)/(a^4*x^7 - 2*a^2*
x^5 + x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(ax)^2}{a^4x^7-2a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{atanh}^2(ax)}{x^3(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((a^2*x^2 - 1)^2*x^3), x)
```

3.273 $\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=227

$$-\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{3x}{8a^3(1-a^2x^2)}$$

```
[Out] (-3*x)/(8*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^4) + (3*ArcTanh[a*x])/
(4*a^4*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) - ArcTan
h[a*x]^3/(4*a^4) + ArcTanh[a*x]^3/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(4
*a^4) - (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 - (3*ArcTanh[a*x]^2*PolyLog[2
, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/
(2*a^4) - (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rubi [A] time = 0.402791, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6028, 5984, 5918, 5948, 6058, 6062, 6610, 5994, 5956, 199, 206}

$$-\frac{3\text{PolyLog}\left(4,1-\frac{2}{1-ax}\right)}{4a^4}-\frac{3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,1-\frac{2}{1-ax}\right)}{2a^4}+\frac{3\tanh^{-1}(ax)\text{PolyLog}\left(3,1-\frac{2}{1-ax}\right)}{2a^4}-\frac{3x}{8a^3(1-a^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^4) + (3*ArcTanh[a*x])/
(4*a^4*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) - ArcTan
h[a*x]^3/(4*a^4) + ArcTanh[a*x]^3/(2*a^4*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(4
*a^4) - (ArcTanh[a*x]^3*Log[2/(1 - a*x)])/a^4 - (3*ArcTanh[a*x]^2*PolyLog[2
, 1 - 2/(1 - a*x)])/(2*a^4) + (3*ArcTanh[a*x]*PolyLog[3, 1 - 2/(1 - a*x)])/
(2*a^4) - (3*PolyLog[4, 1 - 2/(1 - a*x)])/(4*a^4)
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

]

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^3}{1-a^2x^2}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\int \frac{\tanh^{-1}(ax)^3}{1-ax}\,dx}{a^3} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{2a^3}$$

$$= -\frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \frac{\tanh^{-1}(ax)^3\log\left(\frac{2}{1-ax}\right)}{a^4} + \cdots$$

$$= \frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{4a^4} - \cdots$$

$$= -\frac{3x}{8a^3\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \frac{\tanh^{-1}(ax)^3}{2a^4\left(1-a^2x^2\right)} + \cdots$$

$$= -\frac{3x}{8a^3\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{8a^4} + \frac{3\tanh^{-1}(ax)}{4a^4\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^4} + \cdots$$

Mathematica [A] time = 0.1808, size = 139, normalized size = 0.61

$$24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(ax)}\right) + 12\text{PolyLog}\left(4,-e^{-2\tanh^{-1}(ax)}\right) \cdots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-4*ArcTanh[a*x]^4 + 6*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3
*Cosh[2*ArcTanh[a*x]] - 16*ArcTanh[a*x]^3*Log[1 + E^(-2*ArcTanh[a*x])] + 24
*ArcTanh[a*x]^2*PolyLog[2, -E^(-2*ArcTanh[a*x])] + 24*ArcTanh[a*x]*PolyLog[
3, -E^(-2*ArcTanh[a*x])] + 12*PolyLog[4, -E^(-2*ArcTanh[a*x])] - 3*Sinh[2*A
rcTanh[a*x]] - 6*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/(16*a^4)
```

Maple [C] time = 0.348, size = 1015, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] -3/32/a^3/(a*x+1)*x+3/32/a^3*x/(a*x-1)+3/16/a^4*arctanh(a*x)^2/(a*x-1)+3/16
/a^4*arctanh(a*x)^2/(a*x+1)-1/4/a^4*arctanh(a*x)^3/(a*x-1)+1/2/a^4*arctanh(
a*x)^3*ln(a*x-1)+1/4/a^4*arctanh(a*x)^3/(a*x+1)+1/2/a^4*arctanh(a*x)^3*ln(a
*x+1)-1/a^4*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/a^4*arctanh(a*x
)^3*ln(2)+1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x
```

```
^2+1)+1))-3/2/a^4*arctanh(a*x)^2*polylog(2,-(a*x+1)^2/(-a^2*x^2+1))+3/2/a^4
*arctanh(a*x)*polylog(3,-(a*x+1)^2/(-a^2*x^2+1))-3/16/a^3*arctanh(a*x)/(a*x
+1)*x-3/16/a^3*arctanh(a*x)/(a*x-1)*x-1/2*I/a^4*arctanh(a*x)^3*Pi-3/16/a^3*
arctanh(a*x)^2/(a*x+1)*x+3/16/a^3*arctanh(a*x)^2/(a*x-1)*x+3/32/a^4/(a*x-1)
+3/32/a^4/(a*x+1)-1/4*arctanh(a*x)^3/a^4+1/4*arctanh(a*x)^4/a^4-1/4*I/a^4*a
rctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))^2-1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I/a^4*arctanh(a
*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))
+1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I/a^4*arctanh(a*x)^3*Pi*csgn
(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/2*I/a^4*arctanh(a*x)^3*Pi*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))^3-1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^3-1/4*I/a^4*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))^3-3/4/a^4*polylog(4,-(a*x+1)^2/(-a^2*x^2+1))-3/16/a^4*arc
tanh(a*x)/(a*x-1)+3/16/a^4*arctanh(a*x)/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(a^2x^2-1\right)\log\left(-ax+1\right)^4+4\left(\left(a^2x^2-1\right)\log\left(ax+1\right)-1\right)\log\left(-ax+1\right)^3}{64\left(a^6x^2-a^4\right)}+\frac{1}{8}\int\frac{2\,a^3x^3\log\left(ax+1\right)^3-6\,a^3x^3\log\left(ax\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) - 1)*l
og(-a*x + 1)^3)/(a^6*x^2 - a^4) + 1/8*integrate(1/2*(2*a^3*x^3*log(a*x + 1)
^3 - 6*a^3*x^3*log(a*x + 1)^2*log(-a*x + 1) - 3*(a*x - (3*a^3*x^3 + a^2*x^2
 - a*x - 1)*log(a*x + 1) + 1)*log(-a*x + 1)^2)/(a^7*x^4 - 2*a^5*x^2 + a^3),
 x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3\operatorname{artanh}\left(ax\right)^3}{a^4x^4-2\,a^2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\operatorname{atanh}^3\left(ax\right)}{\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^3}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

3.274 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=121

$$-\frac{3}{8a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^4}{8a^3}+\frac{3\tanh^{-1}(ax)^2}{8a^3}$$

```
[Out] -3/(8*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*a^2*(1 - a^2*x^2)) + (3*Ar
cTanh[a*x]^2)/(8*a^3) - (3*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) + (x*ArcTa
nh[a*x]^3)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(8*a^3)
```

Rubi [A] time = 0.133575, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6000, 5994, 5956, 261}

$$-\frac{3}{8a^3\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^4}{8a^3}+\frac{3\tanh^{-1}(ax)^2}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] -3/(8*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*a^2*(1 - a^2*x^2)) + (3*Ar
cTanh[a*x]^2)/(8*a^3) - (3*ArcTanh[a*x]^2)/(4*a^3*(1 - a^2*x^2)) + (x*ArcTa
nh[a*x]^3)/(2*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(8*a^3)
```

Rule 6000

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> -Simp[(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
 x] + (-Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^
2, x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} - \frac{3\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{2a}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} + \frac{3\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{2a^2}$$

$$= \frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a^3} - \frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} - \frac{3\int \frac{x}{\left(1-a^2x^2\right)}}{4a}$$

$$= -\frac{3}{8a^3\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a^3} - \frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}}{8a^3}$$

Mathematica [A] time = 0.0811536, size = 72, normalized size = 0.6

$$\frac{\left(1-a^2x^2\right)\tanh^{-1}(ax)^4 + 3\left(a^2x^2+1\right)\tanh^{-1}(ax)^2 - 4ax\tanh^{-1}(ax)^3 - 6ax\tanh^{-1}(ax) + 3}{8a^3\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a*
x]^3 + (1 - a^2*x^2)*ArcTanh[a*x]^4)/(8*a^3*(-1 + a^2*x^2))
```

Maple [C] time = 0.442, size = 1771, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] 3/16/a^3/(a*x-1)/(a*x+1)-1/4/a^3*arctanh(a*x)^3/(a*x-1)-1/4/a^3*arctanh(a*x
)^3/(a*x+1)-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)^3*Pi+1/8*I/a^3/(a*x-1)/(a*x+1)*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*arctanh(a*x)^3*Pi-1/8*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*arctanh(a*x)^3*Pi-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))^3*Pi*x^2+1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I/((
a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^
3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2-1/8/a/(
a*x-1)/(a*x+1)*arctanh(a*x)^4*x^2+3/8/a/(a*x-1)/(a*x+1)*arctanh(a*x)^2*x^2-
3/4/a^2/(a*x-1)/(a*x+1)*arctanh(a*x)*x+1/4/a^3*arctanh(a*x)^3*ln(a*x-1)-1/4
/a^3*arctanh(a*x)^3*ln(a*x+1)+1/2/a^3*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1
)^(1/2))-1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(
```

```
a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3*Pi+1/8*I/a^3/(a*x-1)/(a*x+1)*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*
x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi+1/4*I/a/(a*x-1
)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a
^2*x^2+1)^(1/2))*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+
1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^2-1/8*I/a/(a*x-
1)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2+1/8*I/a/(a*x-1)/(a*x+1)*arct
anh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(
I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2-1/8*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2+3/16/a/(a*x-1)/(a*x+
1)*x^2+1/8/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)^4+3/8/a^3/(a*x-1)/(a*x+1)*arcta
nh(a*x)^2-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi*x^2-1/8*I/a^3/(a*x-1)/(
a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^3*Pi-1/8*I/a^3/(a*x-1)/
(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(
a*x)^3*Pi+1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*ar
ctanh(a*x)^3*Pi-1/4*I/a^3/(a*x-1)/(a*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)^2*arctanh(a*x)^3*Pi+1/4*I/a^3/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi
```

Maxima [B] time = 1.03771, size = 628, normalized size = 5.19

$$-\frac{1}{4}\left(\frac{2x}{a^4x^2-a^2}+\frac{\log(ax+1)}{a^3}-\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^3+\frac{3\left((a^2x^2-1)\log(ax+1)^2-2(a^2x^2-1)\log(ax+1)\log\cdots\right.}{16\left(a^6x^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^4*x^2 - a^2) + log(a*x + 1)/a^3 - log(a*x - 1)/a^3)*arctanh(a*
x)^3 + 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*lo
g(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 + 4)*a*arctanh(a*x)^2/(a^6*x^2 -
a^4) + 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^
3*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 - 6*(2*a^2*x^2 - (a^2*x^2 - 1
)*log(a*x - 1)^2 - 2)*log(a*x + 1)^2 - 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*
((a^2*x^2 - 1)*log(a*x - 1)^3 - 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1)
+ 48)*a^2/(a^8*x^2 - a^6) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 -
1)*log(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 + 12*a*x - 3*
(2*a^2*x^2 - (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) + 6*(a^2*x^2 -
1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^2 - a^5))*a
```

Fricas [A] time = 1.91611, size = 258, normalized size = 2.13

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+48\,ax\log\left(-\frac{ax+1}{ax-1}\right)-12\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-48}{128\left(a^5x^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*x
```

```
+ 1)/(a*x - 1))^2 - 48)/(a^5*x^2 - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

3.275 $\int \frac{x \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2} dx$

Optimal. Leaf size=119

$$-\frac{3x}{8a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^3}{4a^2}-\frac{3\tanh^{-1}(ax)}{8a^2}$$

```
[Out] (-3*x)/(8*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^2) + (3*ArcTanh[a*x])/(4
*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^3/(4*a^2) + ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2))
```

Rubi [A] time = 0.111713, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5994, 5956, 199, 206}

$$-\frac{3x}{8a\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)}-\frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)^3}{4a^2}-\frac{3\tanh^{-1}(ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (-3*x)/(8*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(8*a^2) + (3*ArcTanh[a*x])/(4
*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) - ArcTanh[a*
x]^3/(4*a^2) + ArcTanh[a*x]^3/(2*a^2*(1 - a^2*x^2))
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{2a}$$

$$= -\frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} + \frac{3}{2}\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{3\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx}{4a}$$

$$= -\frac{3x}{8a\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{3\int \frac{1}{1-a^2x^2}\,d}{8a}$$

$$= -\frac{3x}{8a\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{8a^2} + \frac{3\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{4a^2} + \frac{\tanh^{-1}(ax}{2a^2\left(1-a^2x\right.}$$

Mathematica [A] time = 0.0577841, size = 91, normalized size = 0.76

$$\frac{3\left(a^2x^2-1\right)\log(1-ax) - 3\left(a^2x^2-1\right)\log(ax+1) - 4\left(a^2x^2+1\right)\tanh^{-1}(ax)^3 + 6ax + 12ax\tanh^{-1}(ax)^2 - 12\tanh^{-1}(ax)}{16a^2\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^2,x]
```

```
[Out] (6*a*x - 12*ArcTanh[a*x] + 12*a*x*ArcTanh[a*x]^2 - 4*(1 + a^2*x^2)*ArcTanh[
a*x]^3 + 3*(-1 + a^2*x^2)*Log[1 - a*x] - 3*(-1 + a^2*x^2)*Log[1 + a*x])/(16
*a^2*(-1 + a^2*x^2))
```

Maple [C] time = 0.407, size = 1708, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] -1/2/a^2/(a^2*x^2-1)*arctanh(a*x)^3+3/8/a^2*arctanh(a*x)^2/(a*x-1)+3/8/a^2*
arctanh(a*x)^2*ln(a*x-1)+3/8/a^2*arctanh(a*x)^2/(a*x+1)-3/8/a^2*arctanh(a*x
)^2*ln(a*x+1)+3/4/a^2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/8/a^2
/(a*x-1)/(a*x+1)*arctanh(a*x)+3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2*Pi-3/16*I/(a*x-1)/(a*x+1)*arcta
nh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*
csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2+3/16*I/(a*x
-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*Pi*x^2-3/16*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
```

```
^2*Pi*x^2+3/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*Pi*x^2+3/16*I/(a*x-1)/(a*x+1)*arctan
h(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))*arctanh(a*x)^2*Pi+3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^
2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*
x)^2*Pi-3/8*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a
*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^2*Pi-3/16*I/a^2/(a*x-1)/(a*x+1)*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arctanh(a*x)
^2*Pi+3/8/a/(a*x-1)/(a*x+1)*x+3/8*I/a^2/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi-3
/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*Pi*x^2+1/4/a^2/(a*x-1)/(a*x+1)*arctanh(
a*x)^3-1/4/(a*x-1)/(a*x+1)*arctanh(a*x)^3*x^2-3/8/(a*x-1)/(a*x+1)*arctanh(a
*x)*x^2+3/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))^2*Pi*x^2-3/8*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))^3*Pi*x^2-3/16*I/a^2/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)
/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-3/8*I/a^2/(a*x-1)/(a*x+1)*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2*Pi+3/8*I/a^2/(a*x-1)/(a
*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2*Pi-3/16*I/a^2/(a*
x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2*Pi+3/16*I/(a*x-
1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^3*Pi*x^2+3/16*I/(a*x-1)/(a*x+1)*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))^3*Pi*x^2
```

Maxima [B] time = 1.00284, size = 402, normalized size = 3.38

$$\frac{3\left(\frac{2x}{a^2x^2-1} - \frac{\log(ax+1)}{a} + \frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2}{8a} - \frac{\frac{\left((a^2x^2-1)\log(ax+1)^3-3\left(a^2x^2-1\right)\log(ax+1)^2\log(ax-1)-\left(a^2x^2-1\right)\log(ax-1)^3-12\,ax+3\left(2\,a\right.}{a^5x^2-a^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 3/8*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^2/a
- 1/32*(((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log(a*x + 1)^2*log(
a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2*x^2 + (a^2*x^2
- 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log(a*x - 1))*a^2/(
a^5*x^2 - a^3) - 6*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x
+ 1)*log(a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)/(a^4*x
^2 - a^2))/a - 1/2*arctanh(a*x)^3/((a^2*x^2 - 1)*a^2)
```

Fricas [A] time = 1.94574, size = 209, normalized size = 1.76

$$\frac{6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2 - \left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 12\,ax - 6\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{32\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - (a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1))^3 + 12*a*x - 6*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a
```

```
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{atanh}^3(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(x*atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{artanh}(ax)^3}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

3.276 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^2} dx$

Optimal. Leaf size=115

$$-\frac{3}{8a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}+\frac{3\tanh^{-1}(ax)^2}{8a}$$

```
[Out] -3/(8*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*ArcTanh[
a*x]^2)/(8*a) - (3*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)
/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a)
```

Rubi [A] time = 0.0962471, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5956, 5994, 261}

$$-\frac{3}{8a(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{2(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{4a(1-a^2x^2)}+\frac{3x\tanh^{-1}(ax)}{4(1-a^2x^2)}+\frac{\tanh^{-1}(ax)^4}{8a}+\frac{3\tanh^{-1}(ax)^2}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

```
[Out] -3/(8*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*ArcTanh[
a*x]^2)/(8*a) - (3*ArcTanh[a*x]^2)/(4*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)
/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/(8*a)
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{8a} - \frac{1}{2}(3a)\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{8a} + \frac{3}{2}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{3x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a} - \frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax)^4}{8a} - \frac{1}{4}(3a)\int \frac{}{(1-}$$

$$= -\frac{3}{8a\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{8a} - \frac{3\tanh^{-1}(ax)^2}{4a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{\tanh^{-1}(ax}{8a}$$

Mathematica [A] time = 0.0577629, size = 71, normalized size = 0.62

$$\frac{\left(a^2x^2-1\right)\tanh^{-1}(ax)^4 + 3\left(a^2x^2+1\right)\tanh^{-1}(ax)^2 - 4ax\tanh^{-1}(ax)^3 - 6ax\tanh^{-1}(ax) + 3}{8a\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^2,x]
```

```
[Out] (3 - 6*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 - 4*a*x*ArcTanh[a*
x]^3 + (-1 + a^2*x^2)*ArcTanh[a*x]^4)/(8*a*(-1 + a^2*x^2))
```

Maple [C] time = 0.408, size = 1742, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^2,x)
```

```
[Out] 3/16/(a*x-1)/(a*x+1)/a+1/8*a/(a*x-1)/(a*x+1)*arctanh(a*x)^4*x^2+3/8*a/(a*x-
1)/(a*x+1)*arctanh(a*x)^2*x^2-1/4/a*arctanh(a*x)^3/(a*x-1)-1/4/a*arctanh(a*
x)^3*ln(a*x-1)-1/4/a*arctanh(a*x)^3/(a*x+1)+1/4/a*arctanh(a*x)^3*ln(a*x+1)-
1/2/a*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I/a/(a*x-1)/(a*x+1)
*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3*Pi-3/4/(a*x-1)/(a*x+1)
*arctanh(a*x)*x-1/4*I/a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*Pi+1/8*I*a/(a*x-1)/(
a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x
^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2
+1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-1/8*I*a/(a*x-1
)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2-1/4*I*a/(a*x-1)/(a*x+1)*
arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^
(1/2))*Pi*x^2-1/8*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*
x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^2-1/8*I/a/(a*x-1)/(a*x+1)
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi+1/8*I/a/(
a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^3*Pi-1/8*I*a/(a*x-1)/(a*x+1)
*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2+1/8*I/a/(a*x-1)/(a*x
+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*arct
```

```
anh(a*x)^3*Pi-1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^3*Pi-1/8*I
*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3*Pi*x^2+1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csgn(I/((a*
x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2-1/4*I*a/(a*x-1)/(a*x+1)*arctanh(a*x)^3*csg
n(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2+1/4*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)^3*Pi+
3/16/(a*x-1)/(a*x+1)*a*x^2-1/8/a/(a*x-1)/(a*x+1)*arctanh(a*x)^4+3/8/a/(a*x-
1)/(a*x+1)*arctanh(a*x)^2+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2
-1))^3*arctanh(a*x)^3*Pi+1/8*I/a/(a*x-1)/(a*x+1)*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3*Pi+1/4*I*a/(a*x-1)/(a*x+1)*
arctanh(a*x)^3*Pi*x^2-1/4*I/a/(a*x-1)/(a*x+1)*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^3*arctanh(a*x)^3*Pi
```

Maxima [B] time = 1.03263, size = 620, normalized size = 5.39

$$-\frac{1}{4}\left(\frac{2x}{a^2x^2-1}-\frac{\log(ax+1)}{a}+\frac{\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^3-\frac{3\left(\left(a^2x^2-1\right)\log(ax+1)^2-2\left(a^2x^2-1\right)\log(ax+1)\log\right.}{16\left(a^4x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*x/(a^2*x^2 - 1) - log(a*x + 1)/a + log(a*x - 1)/a)*arctanh(a*x)^3 -
 3/16*((a^2*x^2 - 1)*log(a*x + 1)^2 - 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
- 1) + (a^2*x^2 - 1)*log(a*x - 1)^2 - 4)*a*arctanh(a*x)^2/(a^4*x^2 - a^2) -
 1/128*(((a^2*x^2 - 1)*log(a*x + 1)^4 - 4*(a^2*x^2 - 1)*log(a*x + 1)^3*log(
a*x - 1) + (a^2*x^2 - 1)*log(a*x - 1)^4 + 6*(2*a^2*x^2 + (a^2*x^2 - 1)*log(
a*x - 1)^2 - 2)*log(a*x + 1)^2 + 12*(a^2*x^2 - 1)*log(a*x - 1)^2 - 4*((a^2*
x^2 - 1)*log(a*x - 1)^3 + 6*(a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1) - 48)*
a^2/(a^6*x^2 - a^4) - 8*((a^2*x^2 - 1)*log(a*x + 1)^3 - 3*(a^2*x^2 - 1)*log
(a*x + 1)^2*log(a*x - 1) - (a^2*x^2 - 1)*log(a*x - 1)^3 - 12*a*x + 3*(2*a^2
*x^2 + (a^2*x^2 - 1)*log(a*x - 1)^2 - 2)*log(a*x + 1) - 6*(a^2*x^2 - 1)*log
(a*x - 1))*a*arctanh(a*x)/(a^5*x^2 - a^3))*a
```

Fricas [A] time = 1.9237, size = 255, normalized size = 2.22

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)^3-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+48\,ax\log\left(-\frac{ax+1}{ax-1}\right)-12\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-48}{128\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/128*(8*a*x*log(-(a*x + 1)/(a*x - 1))^3 - (a^2*x^2 - 1)*log(-(a*x + 1)/(a
*x - 1))^4 + 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 12*(a^2*x^2 + 1)*log(-(a*x
+ 1)/(a*x - 1))^2 - 48)/(a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}^3(ax)}{(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a^2*x^2 - 1)^2, x)
```

3.277 $\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=193

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\frac{}{8(1}$$

```
[Out] (-3*a*x)/(8*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/8 + (3*ArcTanh[a*x])/(4*(1 -
a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]^3/4 + A
rcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 -
 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcT
anh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])
/4
```

Rubi [A] time = 0.388259, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6030, 5988, 5932, 5948, 6056, 6060, 6610, 5994, 5956, 199, 206}

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\frac{}{8(1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a*x)/(8*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/8 + (3*ArcTanh[a*x])/(4*(1 -
a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) - ArcTanh[a*x]^3/4 + A
rcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 + ArcTanh[a*x]^3*Log[2 -
 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - (3*ArcT
anh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLog[4, -1 + 2/(1 + a*x)])
/4
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 - \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x(1+ax)}\,dx$$

$$= -\frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}(ax)^3\log\left(2-\frac{}{1}\right.$$

$$= \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}(ax)^4 + \tanh^{-1}($$

$$= -\frac{3ax}{8\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{4}\tanh^{-1}$$

$$= -\frac{3ax}{8\left(1-a^2x^2\right)} - \frac{3}{8}\tanh^{-1}(ax) + \frac{3\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3ax\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}\tanh^{-1}(ax)^3 + \frac{\tanh^{-1}(}{2\left(1-a^2\right.}$$

Mathematica [A] time = 0.185497, size = 135, normalized size = 0.7

$$\frac{1}{64}\left(96\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 96\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 48\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax)}\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^2),x]
```

```
[Out] (Pi^4 - 16*ArcTanh[a*x]^4 + 24*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 16*ArcTa
nh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x
])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 96*ArcTanh[a*x]*Po
lyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4, E^(2*ArcTanh[a*x])] - 12*Sinh[
2*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]])/64
```

Maple [C] time = 0.501, size = 1387, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x)
```

```
[Out] 3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh
(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+ln(2)*arctanh(a*x
)^3+arctanh(a*x)^3*ln(a*x)-3/16*(a*x+1)*arctanh(a*x)/(a*x-1)-3/16*(a*x-1)*a
rctanh(a*x)/(a*x+1)+3/16*(a*x+1)*arctanh(a*x)^2/(a*x-1)+1/2*I*Pi*arctanh(a*
x)^3-3/32*(a*x-1)/(a*x+1)+3/32*(a*x+1)/(a*x-1)-1/4*arctanh(a*x)^4-1/4*arcta
nh(a*x)^3+1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^3-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctan
h(a*x)^3+1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^3+1/4*I
```

```
*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-3/16*arctanh(a*x)^2*(a*x
-1)/(a*x+1)+6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)/(-
a^2*x^2+1)^(1/2))+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^2-1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2*I*arctanh(a*x)^3*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/4*I*Pi*arcta
nh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))^2-1/4*arctanh(a*x)^3/(a*x-1)-1/2*arctanh(a*x)^3*
ln(a*x-1)+1/4*arctanh(a*x)^3/(a*x+1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a
*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))^2+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3-1/4*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))+1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*cs
gn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^2x^2-1\right)\log\left(-ax+1\right)^4+4\left(\left(a^2x^2-1\right)\log\left(ax+1\right)+1\right)\log\left(-ax+1\right)^3}{64\left(a^2x^2-1\right)}-\frac{1}{8}\int-\frac{2\log\left(ax+1\right)^3-6\log\left(ax+1\right)^2\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/64*((a^2*x^2 - 1)*log(-a*x + 1)^4 + 4*((a^2*x^2 - 1)*log(a*x + 1) + 1)*lo
g(-a*x + 1)^3)/(a^2*x^2 - 1) - 1/8*integrate(-1/2*(2*log(a*x + 1)^3 - 6*log
(a*x + 1)^2*log(-a*x + 1) - 3*(a^2*x^2 + a*x + (a^4*x^4 + a^3*x^3 - a^2*x^2
 - a*x - 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\left(ax\right)^3}{a^4x^5-2\,a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{atanh}^3\left(ax\right)}{x\left(ax-1\right)^2\left(ax+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(a^2x^2-1)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x), x)
```

3.278 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)^2}\,dx$

Optimal. Leaf size=191

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3a}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(a}{4\left(1-a^2x^2\right.}$$

```
[Out] (-3*a)/(8*(1 - a^2*x^2)) + (3*a^2*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*a*
ArcTanh[a*x]^2)/8 - (3*a*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + a*ArcTanh[a*x]
^3 - ArcTanh[a*x]^3/x + (a^2*x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (3*a*Arc
Tanh[a*x]^4)/8 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.435097, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6030, 5982, 5916, 5988, 5932, 5948, 6056, 6610, 5956, 5994, 261}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3a}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}-\frac{3a\tanh^{-1}(a}{4\left(1-a^2x^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (-3*a)/(8*(1 - a^2*x^2)) + (3*a^2*x*ArcTanh[a*x])/(4*(1 - a^2*x^2)) + (3*a*
ArcTanh[a*x]^2)/8 - (3*a*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + a*ArcTanh[a*x]
^3 - ArcTanh[a*x]^3/x + (a^2*x*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (3*a*Arc
Tanh[a*x]^4)/8 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - 3*a*ArcTanh[a*x]
*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3, -1 + 2/(1 + a*x)])/2
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)}dx$$

$$= \frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{8}a\tanh^{-1}(ax)^4 + a^2\int \frac{\tanh^{-1}(ax)^3}{1-a^2x^2}dx - \frac{1}{2}\left(3a^3\right)\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}dx + \int \frac{\mathrm{t}}{-}$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{x} + \frac{a^2x\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^4 + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}dx$$

$$= \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{x} + \frac{a^2x\tanh}{2\left(1-\right.}$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}}{x}$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}}{x}$$

$$= -\frac{3a}{8\left(1-a^2x^2\right)} + \frac{3a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{8}a\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + a\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}}{x}$$

Mathematica [C] time = 0.341374, size = 144, normalized size = 0.75

$$\frac{1}{16}a\left(48\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) - 24\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) + 6\tanh^{-1}(ax)^4 - \frac{16\tanh^{-1}(ax)^3}{ax} - 16\tanh\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^2),x]
```

```
[Out] (a*((2*I)*Pi^3 - 16*ArcTanh[a*x]^3 - (16*ArcTanh[a*x]^3)/(a*x) + 6*ArcTanh[
a*x]^4 - 3*Cosh[2*ArcTanh[a*x]] - 6*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]] + 4
8*ArcTanh[a*x]^2*Log[1 - E^(2*ArcTanh[a*x])] + 48*ArcTanh[a*x]*PolyLog[2, E
^(2*ArcTanh[a*x])] - 24*PolyLog[3, E^(2*ArcTanh[a*x])] + 6*ArcTanh[a*x]*Sin
h[2*ArcTanh[a*x]] + 4*ArcTanh[a*x]^3*Sinh[2*ArcTanh[a*x]]))/16
```

Maple [B] time = 0.871, size = 442, normalized size = 2.3

$$-6\,a\mathrm{polylog}\left(3,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) - 6\,a\mathrm{polylog}\left(3,\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) + \frac{3\,a^2x}{32\,ax+32} + \frac{3\,a^2x}{32\,ax-32} + 3\,a\left(\mathrm{Artanh}\,(ax)\right)^2\ln\left(1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x)
```

```
[Out] -arctanh(a*x)^3/x-6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,
(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/32/(a*x+1
)*a^2*x+3/32*a^2*x/(a*x-1)+3/16*a*arctanh(a*x)^2/(a*x-1)-3/16*a*arctanh(a*x
)^2/(a*x+1)-3/16*arctanh(a*x)/(a*x-1)*a^2*x+3/16*arctanh(a*x)/(a*x+1)*a^2*x
+3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(a*x)^2*ln(
1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a/(a*x-1)*arctanh(a*x)^3+3/32*a/(a*x-1)-3
/32*a/(a*x+1)-a*arctanh(a*x)^3+3/8*a*arctanh(a*x)^4-1/8*a/(a*x+1)*arctanh(a
```

```
*x)^3-1/8/(a*x-1)*arctanh(a*x)^3*x*a^2+3/16/(a*x-1)*arctanh(a*x)^2*x*a^2+1/
8/(a*x+1)*arctanh(a*x)^3*x*a^2+3/16/(a*x+1)*arctanh(a*x)^2*x*a^2-3/16*a*arc
tanh(a*x)/(a*x-1)-3/16*a*arctanh(a*x)/(a*x+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(ax)^3}{a^4x^6 - 2a^2x^4 + x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atanh}^3(ax)}{x^2(ax-1)^2(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax)^3}{(a^2x^2-1)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^2), x)
```

3.279 $\int \frac{\tanh^{-1}(ax)^3}{x^3(1-a^2x^2)^2} dx$

Optimal. Leaf size=302

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-3a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-3a^2\tanh^{-1}(a$$

```
[Out] (-3*a^3*x)/(8*(1 - a^2*x^2)) - (3*a^2*ArcTanh[a*x])/8 + (3*a^2*ArcTanh[a*x]
)/(4*(1 - a^2*x^2)) + (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x)
 - (3*a^3*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x]^3)/4 - Ar
cTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (a^2*ArcTan
h[a*x]^4)/2 + 3*a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + 2*a^2*ArcTanh[a*x]^
3*Log[2 - 2/(1 + a*x)] - (3*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - 3*a^2*Arc
Tanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)] - 3*a^2*ArcTanh[a*x]*PolyLog[3, -1
 + 2/(1 + a*x)] - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.955259, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {6030, 5982, 5916, 5988, 5932, 2447, 5948, 6056, 6060, 6610, 5994, 5956, 199, 206}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}a^2\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-3a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-3a^2\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```


```
[Out] (-3*a^3*x)/(8*(1 - a^2*x^2)) - (3*a^2*ArcTanh[a*x])/8 + (3*a^2*ArcTanh[a*x]
)/(4*(1 - a^2*x^2)) + (3*a^2*ArcTanh[a*x]^2)/2 - (3*a*ArcTanh[a*x]^2)/(2*x)
 - (3*a^3*x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)) + (a^2*ArcTanh[a*x]^3)/4 - Ar
cTanh[a*x]^3/(2*x^2) + (a^2*ArcTanh[a*x]^3)/(2*(1 - a^2*x^2)) + (a^2*ArcTan
h[a*x]^4)/2 + 3*a^2*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] + 2*a^2*ArcTanh[a*x]^
3*Log[2 - 2/(1 + a*x)] - (3*a^2*PolyLog[2, -1 + 2/(1 + a*x)])/2 - 3*a^2*Arc
Tanh[a*x]^2*PolyLog[2, -1 + 2/(1 + a*x)] - 3*a^2*ArcTanh[a*x]*PolyLog[3, -1
 + 2/(1 + a*x)] - (3*a^2*PolyLog[4, -1 + 2/(1 + a*x)])/2
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
```

```
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^2}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)}\,dx$$

$$= 2\left(a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx\right) + a^4\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3}\,dx$$

$$= -\frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{a^2\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)}\,dx + 2\left(\frac{1}{4}a^2\tanh^{-1}(ax)^4 + a^2\int \frac{\tanh^{-1}}{x(1}\right.$$

$$= -\frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} - \frac{1}{4}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{a^2\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)} + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2}$$

$$= \frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} - \frac{3a\tanh^{-1}(ax)^2}{2x} - \frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + \frac{1}{4}a^2\tanh^{-1}(ax)^3 - \frac{\tanh^{-1}(ax)^3}{2x^2} + \frac{a^2\,\mathrm{t}}{2(}$$

$$= -\frac{3a^3x}{8\left(1-a^2x^2\right)} + \frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} - \frac{3a^3x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)} + \frac{1}{4}a^2$$

$$= -\frac{3a^3x}{8\left(1-a^2x^2\right)} - \frac{3}{8}a^2\tanh^{-1}(ax) + \frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} - \frac{3a^3x\,\mathrm{t}}{4(}$$

$$= -\frac{3a^3x}{8\left(1-a^2x^2\right)} - \frac{3}{8}a^2\tanh^{-1}(ax) + \frac{3a^2\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)} + \frac{3}{2}a^2\tanh^{-1}(ax)^2 - \frac{3a\tanh^{-1}(ax)^2}{2x} - \frac{3a^3x\,\mathrm{t}}{4(}$$

Mathematica [A] time = 0.74957, size = 215, normalized size = 0.71

$$\frac{1}{32}a^2\left(96\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-96\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-48\mathrm{PolyLog}\left(2,e^{-2\tanh^{-1}(a}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^2),x]
```

```
[Out] (a^2*(Pi^4 + 48*ArcTanh[a*x]^2 - (48*ArcTanh[a*x]^2)/(a*x) - (16*(1 - a^2*x
^2)*ArcTanh[a*x]^3)/(a^2*x^2) - 16*ArcTanh[a*x]^4 + 12*ArcTanh[a*x]*Cosh[2*
ArcTanh[a*x]] + 8*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 96*ArcTanh[a*x]*Log
[1 - E^(-2*ArcTanh[a*x])] + 64*ArcTanh[a*x]^3*Log[1 - E^(2*ArcTanh[a*x])] -
 48*PolyLog[2, E^(-2*ArcTanh[a*x])] + 96*ArcTanh[a*x]^2*PolyLog[2, E^(2*Arc
Tanh[a*x])] - 96*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 48*PolyLog[4
, E^(2*ArcTanh[a*x])] - 6*Sinh[2*ArcTanh[a*x]] - 12*ArcTanh[a*x]^2*Sinh[2*A
rcTanh[a*x]]))/32
```

Maple [B] time = 1.29, size = 672, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x)
```

```
[Out] 12*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+12*a^2*polylog(4,(a*x+1)/(-a^
2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2-3/2*a*arctanh(a*x)^2/x+3/16*a^2*arct
anh(a*x)/(a*x+1)-3/16*a^2*arctanh(a*x)/(a*x-1)-12*a^2*arctanh(a*x)*polylog(
3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2
*x^2+1)^(1/2))+6*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-1
2*a^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-3/16*a^3/(a*x+1)*
arctanh(a*x)*x-1/8*a^3/(a*x-1)*arctanh(a*x)^3*x+3/16*a^3/(a*x-1)*arctanh(a*
x)^2*x-3/16*a^3/(a*x-1)*arctanh(a*x)*x-1/8*a^3/(a*x+1)*arctanh(a*x)^3*x-3/1
6*a^3/(a*x+1)*arctanh(a*x)^2*x+3/32*a^3*x/(a*x-1)-3/32*a^3/(a*x+1)*x-1/8*a^
2/(a*x-1)*arctanh(a*x)^3+1/8*a^2/(a*x+1)*arctanh(a*x)^3+2*a^2*arctanh(a*x)^
3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*
x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arc
tanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/32*a^2/(a*x-1)+3/32*a^2/(a*x+1
)+1/2*a^2*arctanh(a*x)^3-1/2*arctanh(a*x)^3/x^2-1/2*a^2*arctanh(a*x)^4+3*a^
2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+
1)^(1/2))+3/16*a^2*arctanh(a*x)^2/(a*x-1)+3/16*a^2*arctanh(a*x)^2/(a*x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^4x^4-a^2x^2\right)\log\left(-ax+1\right)^4+2\left(2\,a^2x^2+2\left(a^4x^4-a^2x^2\right)\log\left(ax+1\right)-1\right)\log\left(-ax+1\right)^3}{32\left(a^2x^4-x^2\right)}-\frac{1}{8}\int-\frac{2\,\log\left(ax+1\right)^3-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/32*((a^4*x^4 - a^2*x^2)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4 - a^2
*x^2)*log(a*x + 1) - 1)*log(-a*x + 1)^3)/(a^2*x^4 - x^2) - 1/8*integrate(-1
```

```
/2*(2*log(a*x + 1)^3 - 6*log(a*x + 1)^2*log(-a*x + 1) - 3*(2*a^4*x^4 + 2*a^
3*x^3 - a^2*x^2 - a*x + 2*(a^6*x^6 + a^5*x^5 - a^4*x^4 - a^3*x^3 - 1)*log(a
*x + 1))*log(-a*x + 1)^2)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\,(ax)^3}{a^4x^7 - 2\,a^2x^5 + x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atanh}^3\,(ax)}{x^3\,(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(atanh(a*x)**3/(x**3*(a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\,(ax)^3}{\left(a^2x^2-1\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((a^2*x^2 - 1)^2*x^3), x)
```

3.280 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}$$

```
[Out] (x*Sqrt[ArcTanh[a*x]])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^(3/2)/(3*a) + (Sqrt
[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*S
qrt[ArcTanh[a*x]]])/(16*a)
```

Rubi [A] time = 0.144336, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5956, 6034, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

```
[Out] (x*Sqrt[ArcTanh[a*x]])/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^(3/2)/(3*a) + (Sqrt
[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*S
qrt[ArcTanh[a*x]]])/(16*a)
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^2}\,dx &= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{1}{4}a\int \frac{x}{\left(1-a^2x^2\right)^2\sqrt{\tanh^{-1}(ax)}}\,dx\\
&= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{4a}\\
&= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{4a}\\
&= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{8a}\\
&= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\tanh^{-1}(ax)\right)}{16a}-\frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx,x,\tanh^{-1}\right.}{16a}\\
&= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\tanh^{-1}(ax)}\right)}{8a}-\frac{\text{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\text{t}}\right.}{8a}\\
&= \frac{x\sqrt{\tanh^{-1}(ax)}}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^{3/2}}{3a}+\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}
\end{aligned}
$$

Mathematica [A] time = 0.223862, size = 87, normalized size = 0.84

$$\sqrt{\tanh^{-1}(ax)}\left(\frac{\tanh^{-1}(ax)}{3a}-\frac{x}{2\left(a^2x^2-1\right)}\right)-\frac{\sqrt{\frac{\pi}{2}}\left(\text{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)-\text{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^2,x]
```

```
[Out] Sqrt[ArcTanh[a*x]]*(-x/(2*(-1 + a^2*x^2)) + ArcTanh[a*x]/(3*a)) - (Sqrt[Pi/
2]*(-Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]] + Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]]))/(
16*a)
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^2}\sqrt{\text{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\text{artanh}\,(ax)}}{\left(a^2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\text{atanh}\,(ax)}}{(ax-1)^2\,(ax+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**2,x)
```

```
[Out] Integral(sqrt(atanh(a*x))/((a*x - 1)**2*(a*x + 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{artanh}(ax)}}{(a^2x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^2, x)
```

3.281 $\int \frac{x^4}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=39

$$\frac{\text{Unintegrable}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a^4}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}-\frac{3\log\left(\tanh^{-1}(ax)\right)}{2a^5}$$

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^5) - (3*Log[ArcTanh[a*x]])/(2*a^5) + Unin
tegrable[ArcTanh[a*x]^(-1), x]/a^4
```

Rubi [A] time = 0.0635334, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^4}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx = \int \frac{x^4}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Mathematica [A] time = 4.57327, size = 0, normalized size = 0.

$$\int \frac{x^4}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^4/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Maple [A] time = 0.191, size = 0, normalized size = 0.

$$\int \frac{x^4}{(-a^2x^2+1)^2 \operatorname{Artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x^4}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^4/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(ax-1)^2\,(ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**4/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

3.282 $\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=42

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}-\frac{\text{Unintegrable}\left(\frac{x}{(1-a^2x^2)\tanh^{-1}(ax)},x\right)}{a^2}$$

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^4) - Unintegrable[x/((1 - a^2*x^2)*ArcTan
h[a*x]), x]/a^2
```

Rubi [A] time = 0.0629171, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx = \int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Mathematica [A] time = 1.59783, size = 0, normalized size = 0.

$$\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{x^3}{(-a^2x^2+1)^2 \text{Artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

3.283 $\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

Rubi [A] time = 0.10304, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 3312, 3301}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)} \, dx = \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{2a^3}$$

$$= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Mathematica [A] time = 0.112589, size = 27, normalized size = 1.

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a^3) - Log[ArcTanh[a*x]]/(2*a^3)
```

Maple [A] time = 0.064, size = 24, normalized size = 0.9

$$\frac{\text{Chi}\left(2\,\text{Artanh}\,(ax)\right)}{2\,a^3} - \frac{\ln\left(\text{Artanh}\,(ax)\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] 1/2*Chi(2*arctanh(a*x))/a^3-1/2*ln(arctanh(a*x))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a^2x^2-1)^2 \,\text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [B] time = 2.01163, size = 161, normalized size = 5.96

$$-\frac{2\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a*x + 1)/(a*x - 1))
 - log_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

3.284 $\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=14

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Rubi [A] time = 0.0696786, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^2\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^2}$$
$$= \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.0637272, size = 14, normalized size = 1.

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(2*a^2)
```

Maple [A] time = 0.058, size = 13, normalized size = 0.9

$$\frac{\text{Shi}\left(2\,\text{Artanh}\,(ax)\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] 1/2*Shi(2*arctanh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2x^2-1)^2\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [B] time = 1.96954, size = 112, normalized size = 8.

$$\frac{\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(log_integral(-(a*x + 1)/(a*x - 1)) - log_integral(-(a*x - 1)/(a*x + 1)
))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

3.285 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=27

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

Rubi [A] time = 0.0662723, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5968, 3312, 3301}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + Log[ArcTanh[a*x]]/(2*a)
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} \, dx &= \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right) \, dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\log\left(\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{2a}\\
&= \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\log\left(\tanh^{-1}(ax)\right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.0895055, size = 20, normalized size = 0.74

$$\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)+\log\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] (CoshIntegral[2*ArcTanh[a*x]] + Log[ArcTanh[a*x]])/(2*a)
```

Maple [A] time = 0.057, size = 24, normalized size = 0.9

$$\frac{\mathrm{Chi}\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{2\,a}+\frac{\ln\left(\mathrm{Artanh}\left(ax\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/2*ln(arctanh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a^2x^2-1\right)^2\,\mathrm{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

Fricas [B] time = 2.02823, size = 157, normalized size = 5.81

$$\frac{2\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a*x + 1)/(a*x - 1))
+ log_integral(-(a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(ax-1\right)^2\left(ax+1\right)^2\,\mathrm{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)), x)
```

3.286 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)},x\right)+\frac{1}{2}\text{Shi}\left(2\tanh^{-1}(ax)\right)$$

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/2 + Unintegrable[1/(x*(1 - a^2*x^2)*ArcTanh[a*
x]), x]
```

Rubi [A] time = 0.0748869, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx = \int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx$$

Mathematica [A] time = 0.998146, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]), x]
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^2 \text{Artanh}\,(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^4x^5-2\,a^2x^3+x\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(ax-1)^2\,(ax+1)^2 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x),x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)), x)
```

3.287 $\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=60

$$-\frac{\text{Unintegrable}\left(\frac{1}{\tanh^{-1}(ax)},x\right)}{a^3}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^4}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{x}{a^3\tanh^{-1}(ax)}$$

```
[Out] x/(a^3*ArcTanh[a*x]) - x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*
ArcTanh[a*x]]/a^4 - Unintegrable[ArcTanh[a*x]^(-1), x]/a^3
```

Rubi [A] time = 0.31626, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] x/(a^3*ArcTanh[a*x]) - x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*
ArcTanh[a*x]]/a^4 - Defer[Int][ArcTanh[a*x]^(-1), x]/a^3
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx=\frac{\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx}{a^2}-\frac{\int \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$=\frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\int \frac{1}{\tanh^{-1}(ax)}\,dx}{a^3}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx}{a^3}+$$

$$=\frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^4}+\text{Su}$$

$$=\frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\text{Subst}\left(\int\left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx,x,\tanh^{-1}(ax\right.}{a^4}$$

$$=\frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{2a^4}-\int$$

$$=\frac{x}{a^3\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^4}-\frac{\int \frac{1}{\tanh^{-1}(ax)}\,dx}{a^3}$$

Mathematica [A] time = 3.33926, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.209, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-a^2x^2+1\right)^2 (\text{Artanh}\,(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^3}{\left(a^3x^2-a\right)\log\,(ax+1)-\left(a^3x^2-a\right)\log\,(-ax+1)}+\int -\frac{2\left(a^2x^4-3\,x^2\right)}{\left(a^5x^4-2\,a^3x^2+a\right)\log\,(ax+1)-\left(a^5x^4-2\,a^3x^2+a\right)\log\,(-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x^3/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) + integrat
e(-2*(a^2*x^4 - 3*x^2)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 -
 2*a^3*x^2 + a)*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax-1)^2\,(ax+1)^2\,\text{atanh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

3.288 $\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=38

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -(x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Rubi [A] time = 0.133315, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6006, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(x^2/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx = -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx}{a}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{x^2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3}$$

Mathematica [A] time = 0.139602, size = 36, normalized size = 0.95

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^3} + \frac{x^2}{a\left(a^2x^2-1\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] x^2/(a*(-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^3
```

Maple [A] time = 0.063, size = 36, normalized size = 1.

$$\frac{1}{a^3}\left(\frac{1}{2\,\text{Artanh}\,(ax)} - \frac{\cosh\left(2\,\text{Artanh}\,(ax)\right)}{2\,\text{Artanh}\,(ax)} + \text{Shi}\left(2\,\text{Artanh}\,(ax)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh
(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^2}{\left(a^3x^2-a\right)\log(ax+1)-\left(a^3x^2-a\right)\log(-ax+1)} - 4\int -\frac{x}{\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)-\left(a^5x^4-2\,a^3x^2+a\right)\log(-ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x^2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - 4*integr
ate(-x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)), x)
```

Fricas [B] time = 2.13977, size = 258, normalized size = 6.79

$$\frac{4\,a^2x^2 + \left(\left(a^2x^2 - 1\right)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right) - \left(a^2x^2 - 1\right)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^5x^2 - a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2 + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x
^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^
5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax-1)^2\,(ax+1)^2\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2x^2 - 1\right)^2\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

3.289 $\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=36

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2} - \frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/a^2
```

Rubi [A] time = 0.207256, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6032, 6034, 3312, 3301, 5968}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2} - \frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/a^2
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx}{a} + a\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{\text{Subst}\left(\int \ldots\right)}{a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0715305, size = 32, normalized size = 0.89

$$\frac{\frac{ax}{\left(a^2x^2-1\right)\tanh^{-1}(ax)} + \text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] ((a*x)/((-1 + a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]])/a^2
```

Maple [A] time = 0.063, size = 28, normalized size = 0.8

$$\frac{1}{a^2}\left(-\frac{\sinh(2\,\text{Artanh}(ax))}{2\,\text{Artanh}(ax)} + \text{Chi}(2\,\text{Artanh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/2/arctanh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2x}{\left(a^3x^2-a\right)\log(ax+1)-\left(a^3x^2-a\right)\log(-ax+1)} - \int -\frac{2\left(a^2x^2+1\right)}{\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)-\left(a^5x^4-2\,a^3x^2+a\right)\log(-a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 - a)*log(-a*x + 1)) - integrate(
-2*(a^2*x^2 + 1)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3
*x^2 + a)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.0095, size = 252, normalized size = 7.

$$\frac{4\,ax+\left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^4x^2-a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x + ((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 -
 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^
2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

3.290 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=35

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a} - \frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a
```

Rubi [A] time = 0.0945495, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5966, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a} - \frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx &= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+(2a)\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx\\ &= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{2\,\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{2\,\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\ &= -\frac{1}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0912557, size = 30, normalized size = 0.86

$$\frac{\frac{1}{\left(a^2x^2-1\right)\tanh^{-1}(ax)}+\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] (1/((-1 + a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]])/a
```

Maple [A] time = 0.066, size = 36, normalized size = 1.

$$\frac{1}{a}\left(-\frac{1}{2\,\text{Artanh}\,(ax)}-\frac{\cosh\left(2\,\text{Artanh}\,(ax)\right)}{2\,\text{Artanh}\,(ax)}+\text{Shi}\left(2\,\text{Artanh}\,(ax)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(-1/2/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh(
a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-4\,a\int -\frac{x}{\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)-\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-ax+1\right)}\,dx+\frac{2}{\left(a^3x^2-a\right)\log\left(ax+1\right)-\left(a^3x^2-a\right)\log}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -4*a*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^
2*x^2 + 1)*log(-a*x + 1)), x) + 2/((a^3*x^2 - a)*log(a*x + 1) - (a^3*x^2 -
a)*log(-a*x + 1))
```

Fricas [B] time = 1.99044, size = 244, normalized size = 6.97

$$\frac{\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)+4}{2\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) + 4)/((a^3*x^2 -
a)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^2(ax+1)^2\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^2), x)
```

3.291 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=61

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}-\frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\text{Chi}\left(2\tanh^{-1}(ax)\right)-\frac{1}{ax\tanh^{-1}(ax)}$$

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral
[2*ArcTanh[a*x]] - Unintegrable[1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi [A] time = 0.341975, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral
[2*ArcTanh[a*x]] - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} \, dx &= a^2\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} \, dx+\int \frac{1}{x\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} \, dx \\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}+a\int \frac{1}{\left(1-a^2x^2\right)^2\tanh
\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}+\text{Subst}\left(\int \frac{\cosh^2(x)}{x} \, d
\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}-\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh}{2}
\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}+2\left(\frac{1}{2}\text{Subst}\left(\int \frac{\cosh(2x)}{x} \, dx,x,\tanh^{-1}(a
\\
&= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\text{Chi}\left(2\tanh^{-1}(ax)\right)-\frac{\int \frac{1}{x^2\tanh^{-1}(ax)} \, dx}{a}
\end{aligned}$$

Mathematica [A] time = 3.96287, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^2\left(\operatorname{Artanh}(ax)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2}{\left(a^3x^3-ax\right)\log(ax+1)-\left(a^3x^3-ax\right)\log(-ax+1)} - \int -\frac{2\left(3\,a^2x^2-1\right)}{\left(a^5x^6-2\,a^3x^4+ax^2\right)\log(ax+1)-\left(a^5x^6-2\,a^3x^4+ax^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2/((a^3*x^3 - a*x)*log(a*x + 1) - (a^3*x^3 - a*x)*log(-a*x + 1)) - integrat
e(-2*(3*a^2*x^2 - 1)/((a^5*x^6 - 2*a^3*x^4 + a*x^2)*log(a*x + 1) - (a^5*x^6
 - 2*a^3*x^4 + a*x^2)*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4x^5-2\,a^2x^3+x\right)\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 x \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^2), x)
```

3.292 $\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=101

$$-\frac{\text{Unintegrable}\left(\frac{1}{\tanh^{-1}(ax)^2},x\right)}{2a^3}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^4}-\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{}{2}$$

```
[Out] x/(2*a^3*ArcTanh[a*x]^2) - x/(2*a^3*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^
2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^
4 - Unintegrable[ArcTanh[a*x]^(-2), x]/(2*a^3)
```

Rubi [A] time = 0.22487, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] x/(2*a^3*ArcTanh[a*x]^2) - x/(2*a^3*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^
2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^
4 - Defer[Int][ArcTanh[a*x]^(-2), x]/(2*a^3)
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx = \frac{\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx}{a^2} - \frac{\int \frac{x}{\left(1-a^2x^2\right) \tanh^{-1}(ax)^3} dx}{a^2}$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\int \frac{}{\tan}}{}$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\text{Sul}}{}$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\text{Sul}}{}$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subs}}{}$$

$$= \frac{x}{2a^3 \tanh^{-1}(ax)^2} - \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Shi}(}{}$$

Mathematica [A] time = 10.8905, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[x^3/((1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.227, size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(-a^2x^2+1\right)^2 (\mathrm{Artanh}\,(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] int(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax^3-\left(a^2x^4-3\,x^2\right)\log\left(ax+1\right)+\left(a^2x^4-3\,x^2\right)\log\left(-ax+1\right)}{\left(a^4x^2-a^2\right)\log\left(ax+1\right)^2-2\left(a^4x^2-a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^4x^2-a^2\right)\log\left(-ax+1\right)^2}-\int-\frac{}{\left(a^6x^4-2\,a^4x^2+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x^3 - (a^2*x^4 - 3*x^2)*log(a*x + 1) + (a^2*x^4 - 3*x^2)*log(-a*x + 1)
)/((a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x
 + 1) + (a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-2*(a^4*x^5 - 2*a^2*x^
3 + 3*x)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1) - (a^6*x^4 - 2*a^4*x^2 +
 a^2)*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3}{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\,(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x**3/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

3.293 $\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=64

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -x^2/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(a^2*(1 - a^2*x^2)*ArcTanh[a*x]
) + CoshIntegral[2*ArcTanh[a*x]]/a^3
```

Rubi [A] time = 0.271811, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6006, 6032, 6034, 3312, 3301, 5968}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -x^2/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(a^2*(1 - a^2*x^2)*ArcTanh[a*x]
) + CoshIntegral[2*ArcTanh[a*x]]/a^3
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3}\,dx = -\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} + \frac{\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2}\,dx}{a}$$

$$= -\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)}\,dx}{a^2} +$$

$$= -\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \text{ta}\right.}{a^3}$$

$$= -\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)\right.}{a^3}$$

$$= -\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{t}\right.}{2a^3}$$

$$= -\frac{x^2}{2a(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{x}{a^2(1-a^2x^2)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^3}$$

Mathematica [A] time = 0.112581, size = 47, normalized size = 0.73

$$\frac{\frac{ax\left(ax+2\tanh^{-1}(ax)\right)}{(a^2x^2-1)\tanh^{-1}(ax)^2} + 2\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] ((a*x*(a*x + 2*ArcTanh[a*x]))/((-1 + a^2*x^2)*ArcTanh[a*x]^2) + 2*CoshInteg
ral[2*ArcTanh[a*x]])/(2*a^3)
```

Maple [A] time = 0.063, size = 51, normalized size = 0.8

$$\frac{1}{a^3}\left(\frac{1}{4\,(\text{Artanh}\,(ax))^2} - \frac{\cosh(2\,\text{Artanh}\,(ax))}{4\,(\text{Artanh}\,(ax))^2} - \frac{\sinh(2\,\text{Artanh}\,(ax))}{2\,\text{Artanh}\,(ax)} + \text{Chi}\,(2\,\text{Artanh}\,(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2/arcta
nh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(ax^2 + x\log(ax+1) - x\log(-ax+1)\right)}{\left(a^4x^2 - a^2\right)\log(ax+1)^2 - 2\left(a^4x^2 - a^2\right)\log(ax+1)\log(-ax+1) + \left(a^4x^2 - a^2\right)\log(-ax+1)^2} - \int -\frac{}{\left(a^6x^4 - 2\,a^4x^2 + a^2 \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(a*x^2 + x*log(a*x + 1) - x*log(-a*x + 1))/((a^4*x^2 - a^2)*log(a*x + 1)^
2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^4*x^2 - a^2)*log(-a*x
 + 1)^2) - integrate(-2*(a^2*x^2 + 1)/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x
+ 1) - (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.08127, size = 309, normalized size = 4.83

$$\frac{4\,a^2x^2 + 4\,ax\log\left(-\frac{ax+1}{ax-1}\right) + \left(\left(a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + \left(a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{2\left(a^5x^2 - a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*x^2 + 4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integ
ral(-(a*x + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)
))*log(-(a*x + 1)/(a*x - 1))^2)/((a^5*x^2 - a^3)*log(-(a*x + 1)/(a*x - 1))^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax-1)^2\,(ax+1)^2\,\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x**2/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

3.294 $\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=72

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

```
[Out] -x/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^2)*
ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2
```

Rubi [A] time = 0.112101, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5996, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a^2*x^2)/(2*a^2*(1 - a^2*x^2)*
ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/a^2
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} dx = -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(\ldots$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x} \ldots\right)}{a^2}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x \ldots\right)}{a^2}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \mathrm{t}\ldots\right)}{a^2}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0614714, size = 66, normalized size = 0.92

$$\frac{2\left(a^2x^2-1\right)\tanh^{-1}(ax)^2\mathrm{Shi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2+1\right)\tanh^{-1}(ax)+ax}{2a^2\left(a^2x^2-1\right)\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] (a*x + (1 + a^2*x^2)*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*SinhInt
egral[2*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

Maple [A] time = 0.06, size = 43, normalized size = 0.6

$$\frac{1}{a^2}\left(-\frac{\sinh\left(2\,\mathrm{Artanh}\,(ax)\right)}{4\,\left(\mathrm{Artanh}\,(ax)\right)^2}-\frac{\cosh\left(2\,\mathrm{Artanh}\,(ax)\right)}{2\,\mathrm{Artanh}\,(ax)}+\mathrm{Shi}\left(2\,\mathrm{Artanh}\,(ax)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] 1/a^2*(-1/4/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/2/arctanh(a*x)*cosh(2*arc
tanh(a*x))+Shi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax+\left(a^2x^2+1\right)\log\left(ax+1\right)-\left(a^2x^2+1\right)\log\left(-ax+1\right)}{\left(a^4x^2-a^2\right)\log\left(ax+1\right)^2-2\left(a^4x^2-a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^4x^2-a^2\right)\log\left(-ax+1\right)^2}-4\int -\frac{\ldots}{\left(a^4x^4-2\,a^2x\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (a^2*x^2 + 1)*log(a*x + 1) - (a^2*x^2 + 1)*log(-a*x + 1))/((a^4*x^
2 - a^2)*log(a*x + 1)^2 - 2*(a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a
^4*x^2 - a^2)*log(-a*x + 1)^2) - 4*integrate(-x/((a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 1.95548, size = 317, normalized size = 4.4

$$\frac{\left(\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4\,ax+2\left(a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^4x^2-a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*a*x + 2*(a^
2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/((a^4*x^2 - a^2)*log(-(a*x + 1)/(a*x
- 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax-1)^2\,(ax+1)^2\,\text{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2x^2-1\right)^2\,\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

3.295 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=58

$$-\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

```
[Out] -1/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/((1 - a^2*x^2)*ArcTanh[a*x]) + Co
shIntegral[2*ArcTanh[a*x]]/a
```

Rubi [A] time = 0.23446, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 6032, 6034, 3312, 3301, 5968}

$$-\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/((1 - a^2*x^2)*ArcTanh[a*x]) + Co
shIntegral[2*ArcTanh[a*x]]/a
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} + a\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + a^2\int \frac{x^2}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \tanh^{-1}(\right.}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)dx, x,\right.}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + 2\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-}\right.}{2a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.0731106, size = 58, normalized size = 1.

$$\frac{2\left(a^2x^2-1\right)\tanh^{-1}(ax)^2\text{Chi}\left(2\tanh^{-1}(ax)\right) + 2ax\tanh^{-1}(ax) + 1}{2a\left(a^2x^2-1\right)\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] (1 + 2*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*CoshIntegral[2*Ar
cTanh[a*x]])/(2*a*(-1 + a^2*x^2)*ArcTanh[a*x]^2)
```

Maple [A] time = 0.064, size = 51, normalized size = 0.9

$$\frac{1}{a}\left(-\frac{1}{4\left(\text{Artanh}(ax)\right)^2} - \frac{\cosh(2\,\text{Artanh}(ax))}{4\left(\text{Artanh}(ax)\right)^2} - \frac{\sinh(2\,\text{Artanh}(ax))}{2\,\text{Artanh}(ax)} + \text{Chi}(2\,\text{Artanh}(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] 1/a*(-1/4/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2/arctan
h(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,(ax\log{(ax+1)}-ax\log{(-ax+1)}+1)}{\left(a^3x^2-a\right)\log{(ax+1)}^2-2\left(a^3x^2-a\right)\log{(ax+1)}\log{(-ax+1)}+\left(a^3x^2-a\right)\log{(-ax+1)}^2}-\int-\frac{}{\left(a^4x^4-2\,a^2x^2+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(a*x*log(a*x + 1) - a*x*log(-a*x + 1) + 1)/((a^3*x^2 - a)*log(a*x + 1)^2
 - 2*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1) + (a^3*x^2 - a)*log(-a*x + 1)^
2) - integrate(-2*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (
a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 1.96564, size = 296, normalized size = 5.1

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)+\left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+4}{2\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*x*log(-(a*x + 1)/(a*x - 1)) + ((a^2*x^2 - 1)*log_integral(-(a*x +
1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x
 + 1)/(a*x - 1))^2 + 4)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(ax-1)^2\,(ax+1)^2\,\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a^2x^2-1\right)^2\,\mathrm{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^3), x)
```

3.296 $\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=98

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tanh^{-1}(ax)^2}, x\right)}{2a} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{a^2x^2+1}{2(1-a^2x^2)\tanh^{-1}(ax)} + \text{Shi}\left(2\tanh^{-1}(ax)\right) - \frac{}{2ax}$$

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a
^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]] - Uni
ntegrable[1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi [A] time = 0.247191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (1 + a
^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]] - Def
er[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx = a^2 \int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx + \int \frac{1}{x(1-a^2x^2)\tanh^{-1}(ax)^3} dx$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(ax)} - \frac{\int \frac{}{x^2\,\mathrm{ta}}}{}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(ax)} + 2\,\text{Subs}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(ax)} + 2\,\text{Subs}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(ax)} - \frac{\int \frac{}{x^2\,\mathrm{ta}}}{}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{1+a^2x^2}{2(1-a^2x^2)\tanh^{-1}(ax)} + \text{Shi}\,(2$$

Mathematica [A] time = 3.42441, size = 0, normalized size = 0.

$$\int \frac{1}{x(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^2*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.169, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^2 (\operatorname{Artanh}(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax+\left(3\,a^2x^2-1\right)\log(ax+1)-\left(3\,a^2x^2-1\right)\log(-ax+1)}{\left(a^4x^4-a^2x^2\right)\log(ax+1)^2-2\left(a^4x^4-a^2x^2\right)\log(ax+1)\log(-ax+1)+\left(a^4x^4-a^2x^2\right)\log(-ax+1)^2}-\int -\frac{}{\left(a^6x^7-2\,a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (3*a^2*x^2 - 1)*log(a*x + 1) - (3*a^2*x^2 - 1)*log(-a*x + 1))/((a^
4*x^4 - a^2*x^2)*log(a*x + 1)^2 - 2*(a^4*x^4 - a^2*x^2)*log(a*x + 1)*log(-a
*x + 1) + (a^4*x^4 - a^2*x^2)*log(-a*x + 1)^2) - integrate(-2*(3*a^4*x^4 -
2*a^2*x^2 + 1)/((a^6*x^7 - 2*a^4*x^5 + a^2*x^3)*log(a*x + 1) - (a^6*x^7 - 2
*a^4*x^5 + a^2*x^3)*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^4x^5-2\,a^2x^3+x\right)\operatorname{artanh}(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\,(ax-1)^2\,(ax+1)^2\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**2/atanh(a*x)**3,x)
```

```
[Out] Integral(1/(x*(a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 x \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^2/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*x*arctanh(a*x)^3), x)
```

3.297 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^4} \, dx$

Optimal. Leaf size=97

$$-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}+\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}$$

```
[Out] -1/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(3*(1 - a^2*x^2)*ArcTanh[a*x]^2)
- (1 + a^2*x^2)/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*SinhIntegral[2*ArcTan
h[a*x]])/(3*a)
```

Rubi [A] time = 0.143065, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 5996, 6034, 5448, 12, 3298}

$$-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}+\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

```
[Out] -1/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(3*(1 - a^2*x^2)*ArcTanh[a*x]^2)
- (1 + a^2*x^2)/(3*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*SinhIntegral[2*ArcTan
h[a*x]])/(3*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}\,dx &= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}+\frac{1}{3}(2a)\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}\,dx\\ &= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}\\ &= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}\\ &= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}\\ &= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}\\ &= -\frac{1}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{x}{3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{1+a^2x^2}{3a\left(1-a^2x^2\right)\tanh^{-1}(ax)}\end{aligned}$$

Mathematica [A] time = 0.133063, size = 73, normalized size = 0.75

$$\frac{2\left(a^2x^2-1\right)\tanh^{-1}(ax)^3\operatorname{Shi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+ax\tanh^{-1}(ax)+1}{3a\left(a^2x^2-1\right)\tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^4),x]
```

```
[Out] (1 + a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*Arc
Tanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]])/(3*a*(-1 + a^2*x^2)*ArcTanh[a*x]^
3)
```

Maple [A] time = 0.064, size = 68, normalized size = 0.7

$$\frac{1}{a}\left(-\frac{1}{6\left(\operatorname{Artanh}(ax)\right)^3}-\frac{\cosh(2\operatorname{Artanh}(ax))}{6\left(\operatorname{Artanh}(ax)\right)^3}-\frac{\sinh(2\operatorname{Artanh}(ax))}{6\left(\operatorname{Artanh}(ax)\right)^2}-\frac{\cosh(2\operatorname{Artanh}(ax))}{3\operatorname{Artanh}(ax)}+\frac{2\operatorname{Shi}(2\operatorname{Artanh}(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x)
```

```
[Out] 1/a*(-1/6/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6/arctan
h(a*x)^2*sinh(2*arctanh(a*x))-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*Shi
(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8a\int -\frac{x}{3\left(\left(a^4x^4-2a^2x^2+1\right)\log\left(ax+1\right)-\left(a^4x^4-2a^2x^2+1\right)\log\left(-ax+1\right)\right)}\,dx+\frac{2\left(2ax\log\left(ax+1\right)+\left(a^2x^2+\right.\right.}{3\left(\left(a^3x^2-a\right)\log\left(ax+1\right)^3-3\left(a^3x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -8*a*integrate(-1/3*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/3*(2*a*x*log(a*x + 1) + (a^2*x^2 + 1)
*log(a*x + 1)^2 + (a^2*x^2 + 1)*log(-a*x + 1)^2 - 2*(a*x + (a^2*x^2 + 1)*lo
g(a*x + 1))*log(-a*x + 1) + 4)/((a^3*x^2 - a)*log(a*x + 1)^3 - 3*(a^3*x^2 -
 a)*log(a*x + 1)^2*log(-a*x + 1) + 3*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x +
1)^2 - (a^3*x^2 - a)*log(-a*x + 1)^3)
```

Fricas [A] time = 1.96973, size = 358, normalized size = 3.69

$$\frac{\left(\left(a^2x^2-1\right)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^2x^2-1\right)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+4ax\log\left(-\frac{ax+1}{ax-1}\right)+2\left(a^2x^2+1\right)\log}{3\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log_
integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^3 + 4*a*x*log(-(a
*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 8)/((a^3
*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax-1\right)^2\left(ax+1\right)^2\operatorname{atanh}^4\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**4,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}\left(ax\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^4), x)
```

3.298 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^5} dx$

Optimal. Leaf size=120

$$-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{a^2x^2+1}{12a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4}+\frac{\text{Chi}(}{}$$

```
[Out] -1/(4*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(6*(1 - a^2*x^2)*ArcTanh[a*x]^3)
- (1 + a^2*x^2)/(12*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(3*(1 - a^2*x^2)*Ar
cTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a)
```

Rubi [A] time = 0.289994, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 5996, 6032, 6034, 3312, 3301, 5968}

$$-\frac{x}{3(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{6(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{a^2x^2+1}{12a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{1}{4a(1-a^2x^2)\tanh^{-1}(ax)^4}+\frac{\text{Chi}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

```
[Out] -1/(4*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(6*(1 - a^2*x^2)*ArcTanh[a*x]^3)
- (1 + a^2*x^2)/(12*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - x/(3*(1 - a^2*x^2)*Ar
cTanh[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^5}\,dx = -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}+\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}\,dx$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}(ax\right.}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}(ax\right.}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}(ax\right.}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}(ax\right.}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}(ax\right.}$$

$$= -\frac{1}{4a\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{x}{6\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{1+a^2x^2}{12a\left(1-a^2x^2\right)\tanh^{-1}(ax\right.}$$

Mathematica [A] time = 0.0916459, size = 84, normalized size = 0.7

$$\frac{4\left(a^2x^2-1\right)\tanh^{-1}(ax)^4\text{Chi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+4ax\tanh^{-1}(ax)^3+2ax\tanh^{-1}(ax)+3}{12a\left(a^2x^2-1\right)\tanh^{-1}(ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^5),x]
```

```
[Out] (3 + 2*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[a*x]
^3 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^4*CoshIntegral[2*ArcTanh[a*x]])/(12*a*(-
1 + a^2*x^2)*ArcTanh[a*x]^4)
```

Maple [A] time = 0.065, size = 83, normalized size = 0.7

$$\frac{1}{a}\left(-\frac{1}{8\left(\mathrm{Artanh}\left(ax\right)\right)^{4}}-\frac{\cosh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{8\left(\mathrm{Artanh}\left(ax\right)\right)^{4}}-\frac{\sinh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{12\left(\mathrm{Artanh}\left(ax\right)\right)^{3}}-\frac{\cosh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{12\left(\mathrm{Artanh}\left(ax\right)\right)^{2}}-\frac{\sinh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{6\,\mathrm{Artanh}\left(ax\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x)
```

```
[Out] 1/a*(-1/8/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12/arcta
nh(a*x)^3*sinh(2*arctanh(a*x))-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/6
/arctanh(a*x)*sinh(2*arctanh(a*x))+1/3*Chi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax\log\left(ax+1\right)^{3}-2\,ax\log\left(-ax+1\right)^{3}+4\,ax\log\left(ax+1\right)+\left(a^{2}x^{2}+1\right)\log\left(ax+1\right)^{2}+\left(a^{2}x^{2}+6\,ax\log\left(ax+1\right)+1\right)\log\left(\right.}{3\left(\left(a^{3}x^{2}-a\right)\log\left(ax+1\right)^{4}-4\left(a^{3}x^{2}-a\right)\log\left(ax+1\right)^{3}\log\left(-ax+1\right)+6\left(a^{3}x^{2}-a\right)\log\left(ax+1\right)^{2}\log\left(-ax+1\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*x*log(a*x + 1)^3 - 2*a*x*log(-a*x + 1)^3 + 4*a*x*log(a*x + 1) + (a
^2*x^2 + 1)*log(a*x + 1)^2 + (a^2*x^2 + 6*a*x*log(a*x + 1) + 1)*log(-a*x +
1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*a*x + (a^2*x^2 + 1)*log(a*x + 1))*log(-a
*x + 1) + 12)/((a^3*x^2 - a)*log(a*x + 1)^4 - 4*(a^3*x^2 - a)*log(a*x + 1)^
3*log(-a*x + 1) + 6*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^2 - 4*(a^3*x
^2 - a)*log(a*x + 1)*log(-a*x + 1)^3 + (a^3*x^2 - a)*log(-a*x + 1)^4) - int
egrate(-2/3*(a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^
4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [A] time = 2.07711, size = 408, normalized size = 3.4

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{3}+\left(\left(a^{2}x^{2}-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^{2}x^{2}-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}+8\,ax\log\left(-\frac{ax+1}{ax-1}\right.}{6\left(a^{3}x^{2}-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(4*a*x*log(-(a*x + 1)/(a*x - 1))^3 + ((a^2*x^2 - 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a
*x + 1)/(a*x - 1))^4 + 8*a*x*log(-(a*x + 1)/(a*x - 1)) + 2*(a^2*x^2 + 1)*lo
g(-(a*x + 1)/(a*x - 1))^2 + 24)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^5(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**5,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^5,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^5), x)
```

3.299 $\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^6} dx$

Optimal. Leaf size=154

$$-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{a^2x^2+1}{15a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{a^2x^2+1}{30a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{}{5a}$$

```
[Out] -1/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(10*(1 - a^2*x^2)*ArcTanh[a*x]^4)
 - (1 + a^2*x^2)/(30*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(15*(1 - a^2*x^2)*
ArcTanh[a*x]^2) - (1 + a^2*x^2)/(15*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Sinh
Integral[2*ArcTanh[a*x]])/(15*a)
```

Rubi [A] time = 0.195573, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 5996, 6034, 5448, 12, 3298}

$$-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4}-\frac{a^2x^2+1}{15a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{a^2x^2+1}{30a\left(1-a^2x^2\right)\tanh^{-1}(ax)^3}-\frac{}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```


```
[Out] -1/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(10*(1 - a^2*x^2)*ArcTanh[a*x]^4)
 - (1 + a^2*x^2)/(30*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - x/(15*(1 - a^2*x^2)*
ArcTanh[a*x]^2) - (1 + a^2*x^2)/(15*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Sinh
Integral[2*ArcTanh[a*x]])/(15*a)
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^6}\,dx = -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} + \frac{1}{5}(2a)\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5}\,dx$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)^5} - \frac{x}{10\left(1-a^2x^2\right)\tanh^{-1}(ax)^4} - \frac{1+a^2x^2}{30a\left(1-a^2x^2\right)\tanh^{-1}(a$$

Mathematica [A] time = 0.123685, size = 101, normalized size = 0.66

$$\frac{4\left(a^2x^2-1\right)\tanh^{-1}(ax)^5\text{Shi}\left(2\tanh^{-1}(ax)\right)+2\left(a^2x^2+1\right)\tanh^{-1}(ax)^4+\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+2ax\tanh^{-1}(ax)^3}{30a\left(a^2x^2-1\right)\tanh^{-1}(ax)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^6),x]
```

```
[Out] (6 + 3*a*x*ArcTanh[a*x] + (1 + a^2*x^2)*ArcTanh[a*x]^2 + 2*a*x*ArcTanh[a*x]
^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^5*SinhI
ntegral[2*ArcTanh[a*x]])/(30*a*(-1 + a^2*x^2)*ArcTanh[a*x]^5)
```

Maple [A] time = 0.067, size = 98, normalized size = 0.6

$$\frac{1}{a}\left(-\frac{1}{10\left(\operatorname{Artanh}(ax)\right)^{5}}-\frac{\cosh(2\operatorname{Artanh}(ax))}{10\left(\operatorname{Artanh}(ax)\right)^{5}}-\frac{\sinh(2\operatorname{Artanh}(ax))}{20\left(\operatorname{Artanh}(ax)\right)^{4}}-\frac{\cosh(2\operatorname{Artanh}(ax))}{30\left(\operatorname{Artanh}(ax)\right)^{3}}-\frac{\sinh(2\operatorname{Artanh}(ax))}{30\left(\operatorname{Artanh}(ax)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x)
```

```
[Out] 1/a*(-1/10/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/arc
tanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1
/30/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/15/arctanh(a*x)*cosh(2*arctanh(a*
x))+2/15*Shi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8a\int-\frac{x}{15\left(\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log(ax+1)-\left(a^{4}x^{4}-2a^{2}x^{2}+1\right)\log(-ax+1)\right)}\,dx+\frac{2\left(2ax\log(ax+1)^{3}+\left(a^{2}x^{2}+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="maxima")
```

```
[Out] -8*a*integrate(-1/15*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 -
 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 2/15*(2*a*x*log(a*x + 1)^3 + (a^2*x^2
+ 1)*log(a*x + 1)^4 + (a^2*x^2 + 1)*log(-a*x + 1)^4 - 2*(a*x + 2*(a^2*x^2 +
 1)*log(a*x + 1))*log(-a*x + 1)^3 + 12*a*x*log(a*x + 1) + 2*(a^2*x^2 + 1)*l
og(a*x + 1)^2 + 2*(a^2*x^2 + 3*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(a*x +
 1)^2 + 1)*log(-a*x + 1)^2 - 2*(3*a*x*log(a*x + 1)^2 + 2*(a^2*x^2 + 1)*log(
a*x + 1)^3 + 6*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 48)/((a^
3*x^2 - a)*log(a*x + 1)^5 - 5*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1) +
10*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^3*x^2 - a)*log(a*x
+ 1)^2*log(-a*x + 1)^3 + 5*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^4 - (a^
3*x^2 - a)*log(-a*x + 1)^5)
```

Fricas [A] time = 2.01265, size = 473, normalized size = 3.07

$$\frac{\left(\left(a^{2}x^{2}-1\right)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{2}x^{2}-1\right)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{5}+4ax\log\left(-\frac{ax+1}{ax-1}\right)^{3}+2\left(a^{2}x^{2}+1\right)\log}{15\left(a^{3}x^{2}-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*log
_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^5 + 4*a*x*log(-(
a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 24*a*
x*log(-(a*x + 1)/(a*x - 1)) + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 +
 96)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^2(ax+1)^2\operatorname{atanh}^6(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**6,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2\operatorname{artanh}(ax)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^6,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^6), x)
```

3.300 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^7} dx$

Optimal. Leaf size=177

$$-\frac{2x}{45(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{45(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5}-\frac{a^2x^2+1}{90a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{}{60}$$

```
[Out] -1/(6*a*(1 - a^2*x^2)*ArcTanh[a*x]^6) - x/(15*(1 - a^2*x^2)*ArcTanh[a*x]^5)
 - (1 + a^2*x^2)/(60*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(45*(1 - a^2*x^2)*
ArcTanh[a*x]^3) - (1 + a^2*x^2)/(90*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*x)
/(45*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*CoshIntegral[2*ArcTanh[a*x]])/(45*a)
```

Rubi [A] time = 0.344156, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 5996, 6032, 6034, 3312, 3301, 5968}

$$-\frac{2x}{45(1-a^2x^2)\tanh^{-1}(ax)}-\frac{x}{45(1-a^2x^2)\tanh^{-1}(ax)^3}-\frac{x}{15(1-a^2x^2)\tanh^{-1}(ax)^5}-\frac{a^2x^2+1}{90a(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{}{60}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```


```
[Out] -1/(6*a*(1 - a^2*x^2)*ArcTanh[a*x]^6) - x/(15*(1 - a^2*x^2)*ArcTanh[a*x]^5)
 - (1 + a^2*x^2)/(60*a*(1 - a^2*x^2)*ArcTanh[a*x]^4) - x/(45*(1 - a^2*x^2)*
ArcTanh[a*x]^3) - (1 + a^2*x^2)/(90*a*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*x)
/(45*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*CoshIntegral[2*ArcTanh[a*x]])/(45*a)
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^7}\,dx = -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}+\frac{1}{3}a\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^6}\,dx$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

$$= -\frac{1}{6a\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{x}{15\left(1-a^2x^2\right)\tanh^{-1}(ax)^5}-\frac{1+a^2x^2}{60a\left(1-a^2x^2\right)\tanh^{-1}(a\text{)}}$$

Mathematica [A] time = 0.0950506, size = 112, normalized size = 0.63

$$\frac{8\left(a^2x^2-1\right)\tanh^{-1}(ax)^6\text{Chi}\left(2\tanh^{-1}(ax)\right)+2\left(a^2x^2+1\right)\tanh^{-1}(ax)^4+3\left(a^2x^2+1\right)\tanh^{-1}(ax)^2+8ax\tanh^{-1}(ax\text{)}}{180a\left(a^2x^2-1\right)\tanh^{-1}(ax)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^7),x]
```

```
[Out] (30 + 12*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 4*a*x*ArcTanh[
a*x]^3 + 2*(1 + a^2*x^2)*ArcTanh[a*x]^4 + 8*a*x*ArcTanh[a*x]^5 + 8*(-1 + a^
2*x^2)*ArcTanh[a*x]^6*CoshIntegral[2*ArcTanh[a*x]])/(180*a*(-1 + a^2*x^2)*A
rcTanh[a*x]^6)
```

Maple [A] time = 0.063, size = 113, normalized size = 0.6

$$\frac{1}{a}\left(-\frac{1}{12\,(\mathrm{Artanh}\,(ax))^6}-\frac{\cosh(2\,\mathrm{Artanh}\,(ax))}{12\,(\mathrm{Artanh}\,(ax))^6}-\frac{\sinh(2\,\mathrm{Artanh}\,(ax))}{30\,(\mathrm{Artanh}\,(ax))^5}-\frac{\cosh(2\,\mathrm{Artanh}\,(ax))}{60\,(\mathrm{Artanh}\,(ax))^4}-\frac{\sinh(2\,\mathrm{Artanh}\,(ax))}{90\,(\mathrm{Artanh}\,(ax))^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x)
```

```
[Out] 1/a*(-1/12/arctanh(a*x)^6-1/12/arctanh(a*x)^6*cosh(2*arctanh(a*x))-1/30/arc
tanh(a*x)^5*sinh(2*arctanh(a*x))-1/60/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1
/90/arctanh(a*x)^3*sinh(2*arctanh(a*x))-1/90/arctanh(a*x)^2*cosh(2*arctanh(
a*x))-1/45/arctanh(a*x)*sinh(2*arctanh(a*x))+2/45*Chi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(2\,ax\log(ax+1)^5-2\,ax\log(-ax+1)^5+4\,ax\log(ax+1)^3+\left(a^2x^2+1\right)\log(ax+1)^4+\left(a^2x^2+10\,ax\log(ax+1)+1\right)\cdots}{45\left(\left(a^3x^2-a\right)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="maxima")
```

```
[Out] 2/45*(2*a*x*log(a*x + 1)^5 - 2*a*x*log(-a*x + 1)^5 + 4*a*x*log(a*x + 1)^3 +
 (a^2*x^2 + 1)*log(a*x + 1)^4 + (a^2*x^2 + 10*a*x*log(a*x + 1) + 1)*log(-a*
x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + a*x + (a^2*x^2 + 1)*log(a*x + 1))*log(
-a*x + 1)^3 + 48*a*x*log(a*x + 1) + 6*(a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(10*
a*x*log(a*x + 1)^3 + 3*a^2*x^2 + 6*a*x*log(a*x + 1) + 3*(a^2*x^2 + 1)*log(a
*x + 1)^2 + 3)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x + 1)^4 + 6*a*x*log(a*x +
1)^2 + 2*(a^2*x^2 + 1)*log(a*x + 1)^3 + 24*a*x + 6*(a^2*x^2 + 1)*log(a*x +
1))*log(-a*x + 1) + 240)/((a^3*x^2 - a)*log(a*x + 1)^6 - 6*(a^3*x^2 - a)*lo
g(a*x + 1)^5*log(-a*x + 1) + 15*(a^3*x^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^
2 - 20*(a^3*x^2 - a)*log(a*x + 1)^3*log(-a*x + 1)^3 + 15*(a^3*x^2 - a)*log(
a*x + 1)^2*log(-a*x + 1)^4 - 6*(a^3*x^2 - a)*log(a*x + 1)*log(-a*x + 1)^5 +
 (a^3*x^2 - a)*log(-a*x + 1)^6) - integrate(-4/45*(a^2*x^2 + 1)/((a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [A] time = 2.07757, size = 524, normalized size = 2.96

$$\frac{4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^5+\left(\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+\left(a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^6+8\,ax\log\left(-\frac{ax+1}{ax-1}\right.\cdots}{45\left(a^3x^2-a\right)\log\left(-\frac{ax+1}{ax-1}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="fricas")
```

```
[Out] 1/45*(4*a*x*log(-(a*x + 1)/(a*x - 1))^5 + ((a^2*x^2 - 1)*log_integral(-(a*x
 + 1)/(a*x - 1)) + (a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(
a*x + 1)/(a*x - 1))^6 + 8*a*x*log(-(a*x + 1)/(a*x - 1))^3 + 2*(a^2*x^2 + 1)
*log(-(a*x + 1)/(a*x - 1))^4 + 96*a*x*log(-(a*x + 1)/(a*x - 1)) + 12*(a^2*x
^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 + 480)/((a^3*x^2 - a)*log(-(a*x + 1)/(a
*x - 1))^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^7(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**7,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**7), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^7,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^7), x)
```

3.301 $\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^8} dx$

Optimal. Leaf size=211

$$-\frac{2x}{315(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{105(1-a^2x^2)\tanh^{-1}(ax)^4}-\frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6}-\frac{2(a^2x^2+1)}{315a(1-a^2x^2)\tanh^{-1}(ax)}-$$

```
[Out] -1/(7*a*(1 - a^2*x^2)*ArcTanh[a*x]^7) - x/(21*(1 - a^2*x^2)*ArcTanh[a*x]^6)
 - (1 + a^2*x^2)/(105*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(105*(1 - a^2*x^2
)*ArcTanh[a*x]^4) - (1 + a^2*x^2)/(315*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - (2
*x)/(315*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*(1 + a^2*x^2))/(315*a*(1 - a^2*
x^2)*ArcTanh[a*x]) + (4*SinhIntegral[2*ArcTanh[a*x]])/(315*a)
```

Rubi [A] time = 0.255907, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5966, 5996, 6034, 5448, 12, 3298}

$$-\frac{2x}{315(1-a^2x^2)\tanh^{-1}(ax)^2}-\frac{x}{105(1-a^2x^2)\tanh^{-1}(ax)^4}-\frac{x}{21(1-a^2x^2)\tanh^{-1}(ax)^6}-\frac{2(a^2x^2+1)}{315a(1-a^2x^2)\tanh^{-1}(ax)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

```
[Out] -1/(7*a*(1 - a^2*x^2)*ArcTanh[a*x]^7) - x/(21*(1 - a^2*x^2)*ArcTanh[a*x]^6)
 - (1 + a^2*x^2)/(105*a*(1 - a^2*x^2)*ArcTanh[a*x]^5) - x/(105*(1 - a^2*x^2
)*ArcTanh[a*x]^4) - (1 + a^2*x^2)/(315*a*(1 - a^2*x^2)*ArcTanh[a*x]^3) - (2
*x)/(315*(1 - a^2*x^2)*ArcTanh[a*x]^2) - (2*(1 + a^2*x^2))/(315*a*(1 - a^2*
x^2)*ArcTanh[a*x]) + (4*SinhIntegral[2*ArcTanh[a*x]])/(315*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^8}\,dx = -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}+\frac{1}{7}(2a)\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^7}\,dx$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

$$= -\frac{1}{7a\left(1-a^2x^2\right)\tanh^{-1}(ax)^7}-\frac{x}{21\left(1-a^2x^2\right)\tanh^{-1}(ax)^6}-\frac{1+a^2x^2}{105a\left(1-a^2x^2\right)\tanh^{-1}(a$$

Mathematica [A] time = 0.125017, size = 128, normalized size = 0.61

$$\frac{4\left(a^2x^2-1\right)\tanh^{-1}(ax)^7\text{Shi}\left(2\tanh^{-1}(ax)\right)+2\left(a^2x^2+1\right)\tanh^{-1}(ax)^6+\left(a^2x^2+1\right)\tanh^{-1}(ax)^4+3\left(a^2x^2+1\right)\tanh}{315a\left(a^2x^2-1\right)\tanh^{-1}(ax)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^2*ArcTanh[a*x]^8),x]
```

```
[Out] (45 + 15*a*x*ArcTanh[a*x] + 3*(1 + a^2*x^2)*ArcTanh[a*x]^2 + 3*a*x*ArcTanh[
a*x]^3 + (1 + a^2*x^2)*ArcTanh[a*x]^4 + 2*a*x*ArcTanh[a*x]^5 + 2*(1 + a^2*x
^2)*ArcTanh[a*x]^6 + 4*(-1 + a^2*x^2)*ArcTanh[a*x]^7*SinhIntegral[2*ArcTanh
```

```
[a*x]])/(315*a*(-1 + a^2*x^2)*ArcTanh[a*x]^7)
```

Maple [A] time = 0.066, size = 128, normalized size = 0.6

$$\frac{1}{a}\left(-\frac{1}{14\left(\operatorname{Artanh}(ax)\right)^{7}}-\frac{\cosh\left(2\operatorname{Artanh}(ax)\right)}{14\left(\operatorname{Artanh}(ax)\right)^{7}}-\frac{\sinh\left(2\operatorname{Artanh}(ax)\right)}{42\left(\operatorname{Artanh}(ax)\right)^{6}}-\frac{\cosh\left(2\operatorname{Artanh}(ax)\right)}{105\left(\operatorname{Artanh}(ax)\right)^{5}}-\frac{\sinh\left(2\operatorname{Artanh}(ax)\right)}{210\left(\operatorname{Artanh}(ax)\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x)
```

```
[Out] 1/a*(-1/14/arctanh(a*x)^7-1/14/arctanh(a*x)^7*cosh(2*arctanh(a*x))-1/42/arc
tanh(a*x)^6*sinh(2*arctanh(a*x))-1/105/arctanh(a*x)^5*cosh(2*arctanh(a*x))-
1/210/arctanh(a*x)^4*sinh(2*arctanh(a*x))-1/315/arctanh(a*x)^3*cosh(2*arcta
nh(a*x))-1/315/arctanh(a*x)^2*sinh(2*arctanh(a*x))-2/315/arctanh(a*x)*cosh(
2*arctanh(a*x))+4/315*Shi(2*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="maxima")
```

```
[Out] -16*a*integrate(-1/315*x/((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - (a^4*x^4
 - 2*a^2*x^2 + 1)*log(-a*x + 1)), x) + 4/315*(2*a*x*log(a*x + 1)^5 + (a^2*x
^2 + 1)*log(a*x + 1)^6 + (a^2*x^2 + 1)*log(-a*x + 1)^6 - 2*(a*x + 3*(a^2*x^
2 + 1)*log(a*x + 1))*log(-a*x + 1)^5 + 12*a*x*log(a*x + 1)^3 + 2*(a^2*x^2 +
 1)*log(a*x + 1)^4 + (2*a^2*x^2 + 10*a*x*log(a*x + 1) + 15*(a^2*x^2 + 1)*lo
g(a*x + 1)^2 + 2)*log(-a*x + 1)^4 - 4*(5*a*x*log(a*x + 1)^2 + 5*(a^2*x^2 +
1)*log(a*x + 1)^3 + 3*a*x + 2*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)^3 +
 240*a*x*log(a*x + 1) + 24*(a^2*x^2 + 1)*log(a*x + 1)^2 + (20*a*x*log(a*x +
 1)^3 + 15*(a^2*x^2 + 1)*log(a*x + 1)^4 + 24*a^2*x^2 + 36*a*x*log(a*x + 1)
+ 12*(a^2*x^2 + 1)*log(a*x + 1)^2 + 24)*log(-a*x + 1)^2 - 2*(5*a*x*log(a*x
+ 1)^4 + 3*(a^2*x^2 + 1)*log(a*x + 1)^5 + 18*a*x*log(a*x + 1)^2 + 4*(a^2*x^
2 + 1)*log(a*x + 1)^3 + 120*a*x + 24*(a^2*x^2 + 1)*log(a*x + 1))*log(-a*x +
 1) + 1440)/((a^3*x^2 - a)*log(a*x + 1)^7 - 7*(a^3*x^2 - a)*log(a*x + 1)^6*
log(-a*x + 1) + 21*(a^3*x^2 - a)*log(a*x + 1)^5*log(-a*x + 1)^2 - 35*(a^3*x
^2 - a)*log(a*x + 1)^4*log(-a*x + 1)^3 + 35*(a^3*x^2 - a)*log(a*x + 1)^3*lo
g(-a*x + 1)^4 - 21*(a^3*x^2 - a)*log(a*x + 1)^2*log(-a*x + 1)^5 + 7*(a^3*x^
2 - a)*log(a*x + 1)*log(-a*x + 1)^6 - (a^3*x^2 - a)*log(-a*x + 1)^7)
```

Fricas [A] time = 2.3043, size = 591, normalized size = 2.8

$$\frac{2\left(\left(\left(\left(a^{2}x^{2}-1\right)\operatorname{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-\left(a^{2}x^{2}-1\right)\operatorname{log\_integral}\left(-\frac{ax-1}{ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)^{7}+4\,ax\log\left(-\frac{ax+1}{ax-1}\right)^{5}+2\left(a^{2}x^{2}+1\right)\right.\right.}{315\left(a^{3}x^{2}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="fricas")
```

```
[Out] 2/315*(((a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^2*x^2 - 1)*lo
g_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^7 + 4*a*x*log(-
(a*x + 1)/(a*x - 1))^5 + 2*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^6 + 24*a
*x*log(-(a*x + 1)/(a*x - 1))^3 + 4*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^
4 + 480*a*x*log(-(a*x + 1)/(a*x - 1)) + 48*(a^2*x^2 + 1)*log(-(a*x + 1)/(a*
x - 1))^2 + 2880)/((a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^7)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^2 (ax+1)^2 \operatorname{atanh}^8(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**2/atanh(a*x)**8,x)
```

```
[Out] Integral(1/((a*x - 1)**2*(a*x + 1)**2*atanh(a*x)**8), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^2 \operatorname{artanh}(ax)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^2/arctanh(a*x)^8,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^2*arctanh(a*x)^8), x)
```

3.302 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=77

$$-\frac{x^3}{16a\left(1-a^2x^2\right)^2}+\frac{3x}{32a^3\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{32a^4}$$

```
[Out] -x^3/(16*a*(1 - a^2*x^2)^2) + (3*x)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x
])/(32*a^4) + (x^4*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2)
```

Rubi [A] time = 0.0627185, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6008, 288, 206}

$$-\frac{x^3}{16a\left(1-a^2x^2\right)^2}+\frac{3x}{32a^3\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -x^3/(16*a*(1 - a^2*x^2)^2) + (3*x)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x
])/(32*a^4) + (x^4*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2)
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx = \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}a\int \frac{x^4}{\left(1-a^2x^2\right)^3}\,dx$$

$$= -\frac{x^3}{16a\left(1-a^2x^2\right)^2} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3\int \frac{x^2}{\left(1-a^2x^2\right)^2}\,dx}{16a}$$

$$= -\frac{x^3}{16a\left(1-a^2x^2\right)^2} + \frac{3x}{32a^3\left(1-a^2x^2\right)} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} - \frac{3\int \frac{1}{1-a^2x^2}\,dx}{32a^3}$$

$$= -\frac{x^3}{16a\left(1-a^2x^2\right)^2} + \frac{3x}{32a^3\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{32a^4} + \frac{x^4 \tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}$$

Mathematica [A] time = 0.0830387, size = 98, normalized size = 1.27

$$-\frac{5x}{32a^3\left(a^2x^2-1\right)} - \frac{x}{16a^3\left(a^2x^2-1\right)^2} + \frac{\left(2a^2x^2-1\right)\tanh^{-1}(ax)}{4a^4\left(a^2x^2-1\right)^2} - \frac{5\log(1-ax)}{64a^4} + \frac{5\log(ax+1)}{64a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -x/(16*a^3*(-1 + a^2*x^2)^2) - (5*x)/(32*a^3*(-1 + a^2*x^2)) + ((-1 + 2*a^2
*x^2)*ArcTanh[a*x])/(4*a^4*(-1 + a^2*x^2)^2) - (5*Log[1 - a*x])/(64*a^4) +
(5*Log[1 + a*x])/(64*a^4)
```

Maple [A] time = 0.043, size = 136, normalized size = 1.8

$$\frac{\mathrm{Artanh}\,(ax)}{16\,a^4\,(ax-1)^2} + \frac{3\,\mathrm{Artanh}\,(ax)}{16\,a^4\,(ax-1)} + \frac{\mathrm{Artanh}\,(ax)}{16\,a^4\,(ax+1)^2} - \frac{3\,\mathrm{Artanh}\,(ax)}{16\,a^4\,(ax+1)} - \frac{1}{64\,a^4\,(ax-1)^2} - \frac{5}{64\,a^4\,(ax-1)} - \frac{5\,\ln\,(ax-1)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^4*arctanh(a*x)/(a*x-1)^2+3/16/a^4*arctanh(a*x)/(a*x-1)+1/16/a^4*arct
anh(a*x)/(a*x+1)^2-3/16/a^4*arctanh(a*x)/(a*x+1)-1/64/a^4/(a*x-1)^2-5/64/a^
4/(a*x-1)-5/64/a^4*ln(a*x-1)+1/64/a^4/(a*x+1)^2-5/64/a^4/(a*x+1)+5/64/a^4*l
n(a*x+1)
```

Maxima [A] time = 0.96068, size = 134, normalized size = 1.74

$$-\frac{1}{64}a\left(\frac{2\left(5\,a^2x^3-3\,x\right)}{a^8x^4-2\,a^6x^2+a^4} - \frac{5\,\log\,(ax+1)}{a^5} + \frac{5\,\log\,(ax-1)}{a^5}\right) + \frac{\left(2\,a^2x^2-1\right)\mathrm{artanh}\,(ax)}{4\left(a^8x^4-2\,a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)/(a^8*x^4 - 2*a^
6*x^2 + a^4)
```

Fricas [A] time = 2.02289, size = 151, normalized size = 1.96

$$-\frac{10\,a^3x^3-6\,ax-\left(5\,a^4x^4+6\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\,\left(a^8x^4-2\,a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(10*a^3*x^3 - 6*a*x - (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x
 - 1)))/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Sympy [A] time = 4.0009, size = 158, normalized size = 2.05

$$\begin{cases} \frac{5a^4x^4\operatorname{atanh}(ax)}{32a^8x^4-64a^6x^2+32a^4} - \frac{5a^3x^3}{32a^8x^4-64a^6x^2+32a^4} + \frac{6a^2x^2\operatorname{atanh}(ax)}{32a^8x^4-64a^6x^2+32a^4} + \frac{3ax}{32a^8x^4-64a^6x^2+32a^4} - \frac{3\operatorname{atanh}(ax)}{32a^8x^4-64a^6x^2+32a^4} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((5*a**4*x**4*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) -
 5*a**3*x**3/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 6*a**2*x**2*atanh(a*
x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4) + 3*a*x/(32*a**8*x**4 - 64*a**6*
x**2 + 32*a**4) - 3*atanh(a*x)/(32*a**8*x**4 - 64*a**6*x**2 + 32*a**4), Ne(
a, 0)), (0, True))
```

Giac [A] time = 1.23055, size = 127, normalized size = 1.65

$$\frac{5\,\log\left(|ax+1|\right)}{64\,a^4}-\frac{5\,\log\left(|ax-1|\right)}{64\,a^4}-\frac{5\,a^2x^3-3\,x}{32\,\left(a^2x^2-1\right)^2a^3}+\frac{\left(2\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{8\,\left(a^2x^2-1\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] 5/64*log(abs(a*x + 1))/a^4 - 5/64*log(abs(a*x - 1))/a^4 - 1/32*(5*a^2*x^3 -
 3*x)/((a^2*x^2 - 1)^2*a^3) + 1/8*(2*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))
/((a^2*x^2 - 1)^2*a^4)
```

3.303 $\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=100

$$\frac{1}{16a^3(1-a^2x^2)} - \frac{1}{16a^3(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{\tanh^{-1}(ax)^2}{16a^3}$$

```
[Out] -1/(16*a^3*(1 - a^2*x^2)^2) + 1/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(
4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x])/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a
*x]^2/(16*a^3)
```

Rubi [A] time = 0.071921, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5998, 5956, 261}

$$\frac{1}{16a^3(1-a^2x^2)} - \frac{1}{16a^3(1-a^2x^2)^2} - \frac{x\tanh^{-1}(ax)}{8a^2(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{\tanh^{-1}(ax)^2}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -1/(16*a^3*(1 - a^2*x^2)^2) + 1/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(
4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x])/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a
*x]^2/(16*a^3)
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx = -\frac{1}{16a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{4a^2}$$
$$= -\frac{1}{16a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{16a^3} + \frac{\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx}{8a}$$
$$= -\frac{1}{16a^3\left(1-a^2x^2\right)^2} + \frac{1}{16a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{16a^3}$$

Mathematica [A] time = 0.0840135, size = 61, normalized size = 0.61

$$-\frac{a^2x^2 + \left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2 - 2\left(a^3x^3+ax\right)\tanh^{-1}(ax)}{16a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -(a^2*x^2 - 2*(a*x + a^3*x^3)*ArcTanh[a*x] + (-1 + a^2*x^2)^2*ArcTanh[a*x]^
2)/(16*a^3*(-1 + a^2*x^2)^2)
```

Maple [B] time = 0.057, size = 225, normalized size = 2.3

$$\frac{\mathrm{Artanh}\,(ax)}{16\,a^3\,(ax-1)^2} + \frac{\mathrm{Artanh}\,(ax)}{16\,a^3\,(ax-1)} + \frac{\mathrm{Artanh}\,(ax)\ln(ax-1)}{16\,a^3} - \frac{\mathrm{Artanh}\,(ax)}{16\,a^3\,(ax+1)^2} + \frac{\mathrm{Artanh}\,(ax)}{16\,a^3\,(ax+1)} - \frac{\mathrm{Artanh}\,(ax)\ln(ax+1)}{16\,a^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^3*arctanh(a*x)/(a*x-1)^2+1/16/a^3*arctanh(a*x)/(a*x-1)+1/16/a^3*arct
anh(a*x)*ln(a*x-1)-1/16/a^3*arctanh(a*x)/(a*x+1)^2+1/16/a^3*arctanh(a*x)/(a
*x+1)-1/16/a^3*arctanh(a*x)*ln(a*x+1)+1/64/a^3*ln(a*x+1)^2-1/32/a^3*ln(-1/2
*a*x+1/2)*ln(a*x+1)+1/32/a^3*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+1/64/a^3*ln(a
*x-1)^2-1/32/a^3*ln(a*x-1)*ln(1/2+1/2*a*x)-1/64/a^3/(a*x-1)^2-1/64/a^3/(a*x
-1)-1/64/a^3/(a*x+1)^2+1/64/a^3/(a*x+1)
```

Maxima [B] time = 0.985418, size = 242, normalized size = 2.42

$$\frac{1}{16}\left(\frac{2\left(a^2x^3+x\right)}{a^6x^4-2\,a^4x^2+a^2} - \frac{\log(ax+1)}{a^3} + \frac{\log(ax-1)}{a^3}\right)\mathrm{artanh}\,(ax) - \frac{\left(4\,a^2x^2 - \left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)^2 + 2\left(a^4x^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x) - 1/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*lo
```

```
g(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Fricas [A] time = 1.93931, size = 201, normalized size = 2.01

$$-\frac{4\,a^2x^2 + \left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 4\left(a^3x^3 + ax\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\left(a^7x^4 - 2\,a^5x^2 + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(4*a^2*x^2 + (a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 -
4*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 \operatorname{atanh}(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 \operatorname{artanh}(ax)}{\left(a^2x^2 - 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

3.304 $\int \frac{x \tanh^{-1}(ax)}{(1-a^2x^2)^3} \, dx$

Optimal. Leaf size=75

$$-\frac{3x}{32a(1-a^2x^2)}-\frac{x}{16a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{32a^2}$$

```
[Out] -x/(16*a*(1 - a^2*x^2)^2) - (3*x)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(
32*a^2) + ArcTanh[a*x]/(4*a^2*(1 - a^2*x^2)^2)
```

Rubi [A] time = 0.0469405, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5994, 199, 206}

$$-\frac{3x}{32a(1-a^2x^2)}-\frac{x}{16a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -x/(16*a*(1 - a^2*x^2)^2) - (3*x)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x])/(
32*a^2) + ArcTanh[a*x]/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx = \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{\int \frac{1}{(1-a^2x^2)^3}\,dx}{4a}$$

$$= -\frac{x}{16a(1-a^2x^2)^2} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{1}{(1-a^2x^2)^2}\,dx}{16a}$$

$$= -\frac{x}{16a(1-a^2x^2)^2} - \frac{3x}{32a(1-a^2x^2)} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2} - \frac{3\int \frac{1}{1-a^2x^2}\,dx}{32a}$$

$$= -\frac{x}{16a(1-a^2x^2)^2} - \frac{3x}{32a(1-a^2x^2)} - \frac{3\tanh^{-1}(ax)}{32a^2} + \frac{\tanh^{-1}(ax)}{4a^2(1-a^2x^2)^2}$$

Mathematica [A] time = 0.0566228, size = 88, normalized size = 1.17

$$\frac{3x}{32a(a^2x^2-1)} - \frac{x}{16a(a^2x^2-1)^2} + \frac{\tanh^{-1}(ax)}{4a^2(a^2x^2-1)^2} + \frac{3\log(1-ax)}{64a^2} - \frac{3\log(ax+1)}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^3,x]
```

```
[Out] -x/(16*a*(-1 + a^2*x^2)^2) + (3*x)/(32*a*(-1 + a^2*x^2)) + ArcTanh[a*x]/(4*
a^2*(-1 + a^2*x^2)^2) + (3*Log[1 - a*x])/(64*a^2) - (3*Log[1 + a*x])/(64*a^
2)
```

Maple [A] time = 0.035, size = 92, normalized size = 1.2

$$\frac{\mathrm{Artanh}(ax)}{4a^2(a^2x^2-1)^2} - \frac{1}{64a^2(ax-1)^2} + \frac{3}{64a^2(ax-1)} + \frac{3\ln(ax-1)}{64a^2} + \frac{1}{64a^2(ax+1)^2} + \frac{3}{64a^2(ax+1)} - \frac{3\ln(ax+1)}{64a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/4/a^2/(a^2*x^2-1)^2*arctanh(a*x)-1/64/a^2/(a*x-1)^2+3/64/a^2/(a*x-1)+3/64
/a^2*ln(a*x-1)+1/64/a^2/(a*x+1)^2+3/64/a^2/(a*x+1)-3/64/a^2*ln(a*x+1)
```

Maxima [A] time = 0.968843, size = 111, normalized size = 1.48

$$\frac{\frac{2(3a^2x^3-5x)}{a^4x^4-2a^2x^2+1} - \frac{3\log(ax+1)}{a} + \frac{3\log(ax-1)}{a}}{64a} + \frac{\mathrm{artanh}(ax)}{4(a^2x^2-1)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)/a + 1/4*arctanh(a*x)/((a^2*x^2 - 1)^2*a^2)
```

Fricas [A] time = 1.99264, size = 150, normalized size = 2.

$$\frac{6\,a^{3}x^{3} - 10\,ax - \left(3\,a^{4}x^{4} - 6\,a^{2}x^{2} - 5\right)\log\left(-\frac{ax+1}{ax-1}\right)}{64\left(a^{6}x^{4} - 2\,a^{4}x^{2} + a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(6*a^3*x^3 - 10*a*x - (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x
- 1)))/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

Sympy [A] time = 4.02812, size = 158, normalized size = 2.11

$$\begin{cases} - \frac{3 a^{4} x^{4} \operatorname{atanh}{\left(a x \right)}}{32 a^{6} x^{4} - 64 a^{4} x^{2} + 32 a^{2}} + \frac{3 a^{3} x^{3}}{32 a^{6} x^{4} - 64 a^{4} x^{2} + 32 a^{2}} + \frac{6 a^{2} x^{2} \operatorname{atanh}{\left(a x \right)}}{32 a^{6} x^{4} - 64 a^{4} x^{2} + 32 a^{2}} - \frac{5 a x}{32 a^{6} x^{4} - 64 a^{4} x^{2} + 32 a^{2}} + \frac{5 \operatorname{atanh}{\left(a x \right)}}{32 a^{6} x^{4} - 64 a^{4} x^{2} + 32 a^{2}} & \text{for}\: a \neq 0 \\0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((-3*a**4*x**4*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2)
+ 3*a**3*x**3/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) + 6*a**2*x**2*atanh(a
*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2) - 5*a*x/(32*a**6*x**4 - 64*a**4
*x**2 + 32*a**2) + 5*atanh(a*x)/(32*a**6*x**4 - 64*a**4*x**2 + 32*a**2), Ne
(a, 0)), (0, True))
```

Giac [A] time = 1.18839, size = 113, normalized size = 1.51

$$-\frac{3\,\log\left(|ax+1|\right)}{64\,a^{2}} + \frac{3\,\log\left(|ax-1|\right)}{64\,a^{2}} + \frac{3\,a^{2}x^{3} - 5\,x}{32\left(a^{2}x^{2}-1\right)^{2}a} + \frac{\log\left(-\frac{ax+1}{ax-1}\right)}{8\left(a^{2}x^{2}-1\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] -3/64*log(abs(a*x + 1))/a^2 + 3/64*log(abs(a*x - 1))/a^2 + 1/32*(3*a^2*x^3
- 5*x)/((a^2*x^2 - 1)^2*a) + 1/8*log(-(a*x + 1)/(a*x - 1))/((a^2*x^2 - 1)^2
*a^2)
```

3.305 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=94

$$-\frac{3}{16a\left(1-a^2x^2\right)}-\frac{1}{16a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

```
[Out] -1/(16*a*(1 - a^2*x^2)^2) - 3/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(4*(1
 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^2)/
(16*a)
```

Rubi [A] time = 0.0448151, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5960, 5956, 261}

$$-\frac{3}{16a\left(1-a^2x^2\right)}-\frac{1}{16a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)^2}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```


```
[Out] -1/(16*a*(1 - a^2*x^2)^2) - 3/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x])/(4*(1
 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^2)/
(16*a)
```


Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx = -\frac{1}{16a\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3}{4}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx$$
$$= -\frac{1}{16a\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{16a} - \frac{1}{8}(3a)\int \frac{x}{\left(1-a^2x^2\right)^2}\,dx$$
$$= -\frac{1}{16a\left(1-a^2x^2\right)^2} - \frac{3}{16a\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)^2}{16a}$$

Mathematica [A] time = 0.0634113, size = 65, normalized size = 0.69

$$\frac{3a^2x^2+3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2+\left(10ax-6a^3x^3\right)\tanh^{-1}(ax)-4}{16a\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^3,x]
```

```
[Out] (-4 + 3*a^2*x^2 + (10*a*x - 6*a^3*x^3)*ArcTanh[a*x] + 3*(-1 + a^2*x^2)^2*Ar
cTanh[a*x]^2)/(16*a*(-1 + a^2*x^2)^2)
```

Maple [B] time = 0.061, size = 225, normalized size = 2.4

$$\frac{\mathrm{Artanh}\,(ax)}{16\,a\,(ax-1)^2} - \frac{3\,\mathrm{Artanh}\,(ax)}{16\,a\,(ax-1)} - \frac{3\,\mathrm{Artanh}\,(ax)\ln(ax-1)}{16\,a} - \frac{\mathrm{Artanh}\,(ax)}{16\,a\,(ax+1)^2} - \frac{3\,\mathrm{Artanh}\,(ax)}{16\,a\,(ax+1)} + \frac{3\,\mathrm{Artanh}\,(ax)\ln(ax+1}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a*arctanh(a*x)/(a*x-1)^2-3/16/a*arctanh(a*x)/(a*x-1)-3/16/a*arctanh(a*
x)*ln(a*x-1)-1/16/a*arctanh(a*x)/(a*x+1)^2-3/16/a*arctanh(a*x)/(a*x+1)+3/16
/a*arctanh(a*x)*ln(a*x+1)-3/64/a*ln(a*x-1)^2+3/32/a*ln(a*x-1)*ln(1/2+1/2*a*
x)-3/64/a*ln(a*x+1)^2-3/32/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+3/32/a*ln(-1/
2*a*x+1/2)*ln(a*x+1)-1/64/a/(a*x-1)^2+7/64/a/(a*x-1)-1/64/a/(a*x+1)^2-7/64/
a/(a*x+1)
```

Maxima [B] time = 0.979753, size = 246, normalized size = 2.62

$$-\frac{1}{16}\left(\frac{2\left(3\,a^2x^3-5\,x\right)}{a^4x^4-2\,a^2x^2+1} - \frac{3\,\log(ax+1)}{a} + \frac{3\,\log(ax-1)}{a}\right)\mathrm{artanh}\,(ax) + \frac{\left(12\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x) + 1/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 +
 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1)
```

```
- 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a/(a^6*x^4 - 2*a^4*x^2 +
 a^2)
```

Fricas [A] time = 2.02996, size = 213, normalized size = 2.27

$$\frac{12\,a^{2}x^{2}+3\,\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^{2}-4\,\left(3\,a^{3}x^{3}-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-16}{64\,\left(a^{5}x^{4}-2\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2
- 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^5*x^4 - 2*a^3*x^
2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{atanh}\left(ax\right)}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\operatorname{artanh}\left(ax\right)}{\left(a^{2}x^{2}-1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/(a^2*x^2 - 1)^3, x)
```

3.306 $\int \frac{\tanh^{-1}(ax)}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=129

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{11ax}{32\left(1-a^2x^2\right)}-\frac{ax}{16\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{11}{32}\tanh$$

```
[Out] -(a*x)/(16*(1 - a^2*x^2)^2) - (11*a*x)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x
])/32 + ArcTanh[a*x]/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(2*(1 - a^2*x^2)) +
 ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(
1 + a*x)]/2
```

Rubi [A] time = 0.249641, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6030, 5988, 5932, 2447, 5994, 199, 206}

$$-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{11ax}{32\left(1-a^2x^2\right)}-\frac{ax}{16\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{11}{32}\tanh$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*x)/(16*(1 - a^2*x^2)^2) - (11*a*x)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x
])/32 + ArcTanh[a*x]/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(2*(1 - a^2*x^2)) +
 ArcTanh[a*x]^2/2 + ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - PolyLog[2, -1 + 2/(
1 + a*x)]/2
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^3}\,dx &= a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx+\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^2}\,dx\\
&= \frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}-\frac{1}{4}a\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx+a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)}\,dx\\
&= -\frac{ax}{16\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}\tanh^{-1}(ax)^2-\frac{1}{16}(3a)\int \frac{1}{\left(1-a^2x^2\right)^2}\,dx-\\
&= -\frac{ax}{16\left(1-a^2x^2\right)^2}-\frac{11ax}{32\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}\tanh^{-1}(ax)^2+\tanh^{-1}(ax)\\
&= -\frac{ax}{16\left(1-a^2x^2\right)^2}-\frac{11ax}{32\left(1-a^2x^2\right)}-\frac{11}{32}\tanh^{-1}(ax)+\frac{\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)}{2\left(1-a^2x^2\right)}+\frac{1}{2}\tanh^{-1}(a
\end{aligned}$$

Mathematica [A] time = 0.200269, size = 81, normalized size = 0.63

$$\frac{1}{128}\left(-64\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right)+64\tanh^{-1}(ax)^2-24\sinh\left(2\tanh^{-1}(ax)\right)-\sinh\left(4\tanh^{-1}(ax)\right)+4\tanh^{-1}(ax\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (64*ArcTanh[a*x]^2 + 4*ArcTanh[a*x]*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTa
nh[a*x]] + 32*Log[1 - E^(-2*ArcTanh[a*x])]) - 64*PolyLog[2, E^(-2*ArcTanh[a
*x])] - 24*Sinh[2*ArcTanh[a*x]] - Sinh[4*ArcTanh[a*x]])/128
```

Maple [B] time = 0.063, size = 234, normalized size = 1.8

$$\frac{\operatorname{Artanh}(ax)}{16\,(ax-1)^2}-\frac{5\operatorname{Artanh}(ax)}{16ax-16}-\frac{\operatorname{Artanh}(ax)\ln(ax-1)}{2}+\operatorname{Artanh}(ax)\ln(ax)+\frac{\operatorname{Artanh}(ax)}{16\,(ax+1)^2}+\frac{5\operatorname{Artanh}(ax)}{16ax+16}-\frac{\operatorname{Art}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16*arctanh(a*x)/(a*x-1)^2-5/16*arctanh(a*x)/(a*x-1)-1/2*arctanh(a*x)*ln(a
*x-1)+arctanh(a*x)*ln(a*x)+1/16*arctanh(a*x)/(a*x+1)^2+5/16*arctanh(a*x)/(a
*x+1)-1/2*arctanh(a*x)*ln(a*x+1)-1/2*dilog(a*x)-1/2*dilog(a*x+1)-1/2*ln(a*x
)*ln(a*x+1)-1/8*ln(a*x-1)^2+1/2*dilog(1/2+1/2*a*x)+1/4*ln(a*x-1)*ln(1/2+1/2
*a*x)+1/8*ln(a*x+1)^2-1/4*(ln(a*x+1)-ln(1/2+1/2*a*x))*ln(-1/2*a*x+1/2)-1/64
/(a*x-1)^2+11/64/(a*x-1)+11/64*ln(a*x-1)+1/64/(a*x+1)^2+11/64/(a*x+1)-11/64
*ln(a*x+1)
```

Maxima [B] time = 0.998892, size = 362, normalized size = 2.81

$$\frac{1}{64}a\left(\frac{2\left(11\,a^3x^3+4\left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)^2-8\left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)\log(ax-1)-4\left(a^4x^4-2\,a^2x^2+\right.\right.}{a^5x^4-2\,a^3x^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a*(2*(11*a^3*x^3 + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 - 8*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 4*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x - 1)^2 - 13*a*x)/(a^5*x^4 - 2*a^3*x^2 + a) + 32*(log(a*x - 1)*lo
g(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/a - 32*(log(a*x + 1)*log(x) + dil
og(-a*x))/a + 32*(log(-a*x + 1)*log(x) + dilog(a*x))/a - 11*log(a*x + 1)/a
+ 11*log(a*x - 1)/a) - 1/4*((2*a^2*x^2 - 3)/(a^4*x^4 - 2*a^2*x^2 + 1) + 2*l
og(a^2*x^2 - 1) - 2*log(x^2))*arctanh(a*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\operatorname{artanh}(ax)}{a^6x^7-3\,a^4x^5+3\,a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{atanh}(ax)}{a^6x^7-3a^4x^5+3a^2x^3-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)}{(a^2x^2-1)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x), x)
```

3.307 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+a\log(x)+\frac{15}{16}a\tanh^{-1}(ax)^2$$

```
[Out] -a/(16*(1 - a^2*x^2)^2) - (7*a)/(16*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*
x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x])/(8*(1 - a^2*x^
2)) + (15*a*ArcTanh[a*x]^2)/16 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```

Rubi [A] time = 0.235148, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.55, Rules used = {6030, 5982, 5916, 266, 36, 29, 31, 5948, 5956, 261, 5960}

$$-\frac{7a}{16\left(1-a^2x^2\right)}-\frac{a}{16\left(1-a^2x^2\right)^2}-\frac{1}{2}a\log\left(1-a^2x^2\right)+\frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2}+a\log(x)+\frac{15}{16}a\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```


```
[Out] -a/(16*(1 - a^2*x^2)^2) - (7*a)/(16*(1 - a^2*x^2)) - ArcTanh[a*x]/x + (a^2*
x*ArcTanh[a*x])/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x])/(8*(1 - a^2*x^
2)) + (15*a*ArcTanh[a*x]^2)/16 + a*Log[x] - (a*Log[1 - a^2*x^2])/2
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)}{x^2\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + a^2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\mathrm{tanh}}{x^2(1}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{7}{16}a\tanh^{-1}(ax)^2 + a^2\int \frac{\tanh^{-1}(ax)}{1-a^2x^2}\,dx$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2} - \frac{7a}{16\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{15}{16}a\,\mathrm{tanh}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2} - \frac{7a}{16\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{15}{16}a\,\mathrm{tanh}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2} - \frac{7a}{16\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{15}{16}a\,\mathrm{tanh}$$

$$= -\frac{a}{16\left(1-a^2x^2\right)^2} - \frac{7a}{16\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{x} + \frac{a^2x\tanh^{-1}(ax)}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + \frac{15}{16}a\,\mathrm{tanh}$$

Mathematica [A] time = 0.161525, size = 94, normalized size = 0.76

$$\frac{1}{16}\left(a\left(\frac{7a^2x^2-8}{\left(a^2x^2-1\right)^2} - 8\log\left(1-a^2x^2\right) + 16\log(x)\right) - \frac{2\left(15a^4x^4-25a^2x^2+8\right)\tanh^{-1}(ax)}{x\left(a^2x^2-1\right)^2} + 15a\tanh^{-1}(ax)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] ((-2*(8 - 25*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x])/(x*(-1 + a^2*x^2)^2) + 15*
a*ArcTanh[a*x]^2 + a*((-8 + 7*a^2*x^2)/(-1 + a^2*x^2)^2 + 16*Log[x] - 8*Log
[1 - a^2*x^2]))/16
```

Maple [B] time = 0.06, size = 228, normalized size = 1.9

$$\frac{a\mathrm{Artanh}\,(ax)}{16\,(ax-1)^2} - \frac{7\,a\mathrm{Artanh}\,(ax)}{16\,ax-16} - \frac{15\,a\mathrm{Artanh}\,(ax)\ln(ax-1)}{16} - \frac{\mathrm{Artanh}\,(ax)}{x} - \frac{a\mathrm{Artanh}\,(ax)}{16\,(ax+1)^2} - \frac{7\,a\mathrm{Artanh}\,(ax)}{16\,ax+16} + 15$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16*a*arctanh(a*x)/(a*x-1)^2-7/16*a*arctanh(a*x)/(a*x-1)-15/16*a*arctanh(a
*x)*ln(a*x-1)-arctanh(a*x)/x-1/16*a*arctanh(a*x)/(a*x+1)^2-7/16*a*arctanh(a
*x)/(a*x+1)+15/16*a*arctanh(a*x)*ln(a*x+1)-15/64*a*ln(a*x-1)^2+15/32*a*ln(a
*x-1)*ln(1/2+1/2*a*x)-15/64*a*ln(a*x+1)^2+15/32*a*ln(-1/2*a*x+1/2)*ln(a*x+1
)-15/32*a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)-1/64*a/(a*x-1)^2+15/64*a/(a*x-1)
-1/2*a*ln(a*x-1)+a*ln(a*x)-1/64*a/(a*x+1)^2-15/64*a/(a*x+1)-1/2*a*ln(a*x+1)
```

Maxima [A] time = 0.99672, size = 275, normalized size = 2.24

$$\frac{1}{64}a\left(\frac{28\,a^2x^2 - 15\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(ax+1\right)^2 + 30\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(ax+1\right)\log\left(ax-1\right) - 15\left(a^4x^4 - 2\,a^2x^2\right.}{a^4x^4 - 2\,a^2x^2 + 1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a*((28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2 +
 1)*log(a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 32*l
og(a*x - 1) + 64*log(x)) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1) - 2*
(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)
```

Fricas [A] time = 2.06662, size = 358, normalized size = 2.91

$$\frac{28\,a^3x^3 + 15\left(a^5x^5 - 2\,a^3x^3 + ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 32\,ax - 32\left(a^5x^5 - 2\,a^3x^3 + ax\right)\log\left(a^2x^2 - 1\right) + 64\left(a^5x^5 - 2\,a^3x^3 + \right.}{64\left(a^4x^5 - 2\,a^2x^3 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(28*a^3*x^3 + 15*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))
^2 - 32*a*x - 32*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a^2*x^2 - 1) + 64*(a^5*x^5
 - 2*a^3*x^3 + a*x)*log(x) - 4*(15*a^4*x^4 - 25*a^2*x^2 + 8)*log(-(a*x + 1)
/(a*x - 1)))/(a^4*x^5 - 2*a^2*x^3 + x)
```

Sympy [A] time = 8.62253, size = 578, normalized size = 4.7

$$\begin{cases} \frac{64a^5x^5\log\left(x\right)}{64a^4x^5-128a^2x^3+64x} - \frac{64a^5x^5\log\left(x-\frac{1}{a}\right)}{64a^4x^5-128a^2x^3+64x} + \frac{60a^5x^5\operatorname{atanh}^2\left(ax\right)}{64a^4x^5-128a^2x^3+64x} - \frac{64a^5x^5\operatorname{atanh}\left(ax\right)}{64a^4x^5-128a^2x^3+64x} + \frac{7a^5x^5}{64a^4x^5-128a^2x^3+64x} - \frac{120a^4x^4\operatorname{atanh}\left(ax\right)}{64a^4x^5-128a^2x^3+64x} \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] Piecewise((64*a**5*x**5*log(x)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) - 64*a
**5*x**5*log(x - 1/a)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) + 60*a**5*x**5*
atanh(a*x)**2/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) - 64*a**5*x**5*atanh(a*
x)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) + 7*a**5*x**5/(64*a**4*x**5 - 128*
a**2*x**3 + 64*x) - 120*a**4*x**4*atanh(a*x)/(64*a**4*x**5 - 128*a**2*x**3
+ 64*x) - 128*a**3*x**3*log(x)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) + 128*
a**3*x**3*log(x - 1/a)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) - 120*a**3*x**
3*atanh(a*x)**2/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) + 128*a**3*x**3*atanh
(a*x)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) + 14*a**3*x**3/(64*a**4*x**5 -
128*a**2*x**3 + 64*x) + 200*a**2*x**2*atanh(a*x)/(64*a**4*x**5 - 128*a**2*x
**3 + 64*x) + 64*a*x*log(x)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) - 64*a*x*
log(x - 1/a)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) + 60*a*x*atanh(a*x)**2/(
64*a**4*x**5 - 128*a**2*x**3 + 64*x) - 64*a*x*atanh(a*x)/(64*a**4*x**5 - 12
```

```
8*a**2*x**3 + 64*x) - 25*a*x/(64*a**4*x**5 - 128*a**2*x**3 + 64*x) - 64*ata
nh(a*x)/(64*a**4*x**5 - 128*a**2*x**3 + 64*x), Ne(a, 0)), (0, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)}{\left(a^2x^2-1\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)/((a^2*x^2 - 1)^3*x^2), x)
```

3.308 $\int \frac{x^3 \tanh^{-1}(ax)^2}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=127

$$\frac{x^4}{32\left(1-a^2x^2\right)^2}-\frac{3}{32a^4\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{x^3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{32a^4}$$

```
[Out] x^4/(32*(1 - a^2*x^2)^2) - 3/(32*a^4*(1 - a^2*x^2)) - (x^3*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(16*a^3*(1 - a^2*x^2)) - (3*ArcTan
h[a*x]^2)/(32*a^4) + (x^4*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2)
```

Rubi [A] time = 0.183527, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6008, 6002, 5998, 5948}

$$\frac{x^4}{32\left(1-a^2x^2\right)^2}-\frac{3}{32a^4\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{x^3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```


```
[Out] x^4/(32*(1 - a^2*x^2)^2) - 3/(32*a^4*(1 - a^2*x^2)) - (x^3*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x])/(16*a^3*(1 - a^2*x^2)) - (3*ArcTan
h[a*x]^2)/(32*a^4) + (x^4*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2)
```


Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6002

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(q_), x_Symbol] :> -Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x]
 + (-Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a
 + b*ArcTanh[c*x]), x], x] + Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTanh[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2
*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3} dx = \frac{x^4 \tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} - \frac{1}{2}a \int \frac{x^4 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3} dx$$

$$= \frac{x^4}{32\left(1-a^2x^2\right)^2} - \frac{x^3 \tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{x^4 \tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{3\int \frac{x^2 \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2} dx}{8a}$$

$$= \frac{x^4}{32\left(1-a^2x^2\right)^2} - \frac{3}{32a^4\left(1-a^2x^2\right)} - \frac{x^3 \tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{3x \tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)} + \frac{x^4 \tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} - \frac{3\int \frac{\tan}{1}}{1}$$

$$= \frac{x^4}{32\left(1-a^2x^2\right)^2} - \frac{3}{32a^4\left(1-a^2x^2\right)} - \frac{x^3 \tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{3x \tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right)} - \frac{3 \tanh^{-1}(ax)^2}{32a^4} + \frac{x^4 \tanh}{4(1-}$$

Mathematica [A] time = 0.0927196, size = 71, normalized size = 0.56

$$\frac{5a^2x^2 + \left(5a^4x^4 + 6a^2x^2 - 3\right) \tanh^{-1}(ax)^2 + \left(6ax - 10a^3x^3\right) \tanh^{-1}(ax) - 4}{32a^4\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-4 + 5*a^2*x^2 + (6*a*x - 10*a^3*x^3)*ArcTanh[a*x] + (-3 + 6*a^2*x^2 + 5*a
^4*x^4)*ArcTanh[a*x]^2)/(32*a^4*(-1 + a^2*x^2)^2)
```

Maple [B] time = 0.07, size = 297, normalized size = 2.3

$$\frac{(\operatorname{Artanh}(ax))^2}{16\,a^4\,(ax-1)^2} + \frac{3\,(\operatorname{Artanh}(ax))^2}{16\,a^4\,(ax-1)} + \frac{(\operatorname{Artanh}(ax))^2}{16\,a^4\,(ax+1)^2} - \frac{3\,(\operatorname{Artanh}(ax))^2}{16\,a^4\,(ax+1)} - \frac{\operatorname{Artanh}(ax)}{32\,a^4\,(ax-1)^2} - \frac{5\operatorname{Artanh}(ax)}{32\,a^4\,(ax-1)} - \frac{5\operatorname{Arta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^4*arctanh(a*x)^2/(a*x-1)^2+3/16/a^4*arctanh(a*x)^2/(a*x-1)+1/16/a^4*
arctanh(a*x)^2/(a*x+1)^2-3/16/a^4*arctanh(a*x)^2/(a*x+1)-1/32/a^4*arctanh(a
*x)/(a*x-1)^2-5/32/a^4*arctanh(a*x)/(a*x-1)-5/32/a^4*arctanh(a*x)*ln(a*x-1)
+1/32/a^4*arctanh(a*x)/(a*x+1)^2-5/32/a^4*arctanh(a*x)/(a*x+1)+5/32/a^4*arc
tanh(a*x)*ln(a*x+1)-5/128/a^4*ln(a*x-1)^2+5/64/a^4*ln(a*x-1)*ln(1/2+1/2*a*x
)-5/128/a^4*ln(a*x+1)^2-5/64/a^4*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+5/64/a^4*
ln(-1/2*a*x+1/2)*ln(a*x+1)+1/128/a^4/(a*x-1)^2+9/128/a^4/(a*x-1)+1/128/a^4/
(a*x+1)^2-9/128/a^4/(a*x+1)
```

Maxima [B] time = 0.985992, size = 305, normalized size = 2.4

$$-\frac{1}{32}\,a\left(\frac{2\left(5\,a^2x^3-3\,x\right)}{a^8x^4-2\,a^6x^2+a^4}-\frac{5\,\log\left(ax+1\right)}{a^5}+\frac{5\,\log\left(ax-1\right)}{a^5}\right)\mathrm{artanh}\left(ax\right)+\frac{\left(20\,a^2x^2-5\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/32*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x) + 1/128*(20*a^2*x^2 - 5*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x + 1)^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log
(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a^2/(a^10*x^4
- 2*a^8*x^2 + a^6) + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^2/(a^8*x^4 - 2*a^6*x^
2 + a^4)
```

Fricas [A] time = 1.9317, size = 217, normalized size = 1.71

$$\frac{20\,a^2x^2+\left(5\,a^4x^4+6\,a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\left(5\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-16}{128\left(a^8x^4-2\,a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*a^2*x^2 + (5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1))^2
 - 4*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^8*x^4 - 2*a^6*x
^2 + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3\,\mathrm{atanh}^2\left(ax\right)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**3*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^3\,\mathrm{artanh}\left(ax\right)^2}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

3.309 $\int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^3} dx$

Optimal. Leaf size=163

$$-\frac{x}{64a^2(1-a^2x^2)}+\frac{x}{32a^2(1-a^2x^2)^2}-\frac{x\tanh^{-1}(ax)^2}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2}-\frac{\tanh^{-1}(a}{24a^3}$$

```
[Out] x/(32*a^2*(1 - a^2*x^2)^2) - x/(64*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]/(64*a^
3) - ArcTanh[a*x]/(8*a^3*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(8*a^3*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x]^2)/(8*a^
2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(24*a^3)
```

Rubi [A] time = 0.250392, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6028, 5956, 5994, 199, 206, 5964}

$$-\frac{x}{64a^2(1-a^2x^2)}+\frac{x}{32a^2(1-a^2x^2)^2}-\frac{x\tanh^{-1}(ax)^2}{8a^2(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4a^2(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a^3(1-a^2x^2)^2}-\frac{\tanh^{-1}(a}{24a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```


```
[Out] x/(32*a^2*(1 - a^2*x^2)^2) - x/(64*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]/(64*a^
3) - ArcTanh[a*x]/(8*a^3*(1 - a^2*x^2)^2) + ArcTanh[a*x]/(8*a^3*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*a^2*(1 - a^2*x^2)^2) - (x*ArcTanh[a*x]^2)/(8*a^
2*(1 - a^2*x^2)) - ArcTanh[a*x]^3/(24*a^3)
```


Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{a^2}$$

$$= -\frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)^2}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^3}{6a^3} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx}{8a^2} + \frac{3\int \text{ta}}{}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{2a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)^2}{8a^2\left(1-a^2x^2\right)} - \text{ta}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{13x}{64a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{x}{8}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{x}{64a^2\left(1-a^2x^2\right)} - \frac{13\tanh^{-1}(ax)}{64a^3} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)} + \frac{x}{4a}$$

$$= \frac{x}{32a^2\left(1-a^2x^2\right)^2} - \frac{x}{64a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)}{64a^3} - \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)}{8a^3\left(1-a^2x^2\right)} + \frac{x\,\text{ta}}{4a^2\,(}$$

Mathematica [A] time = 0.116072, size = 121, normalized size = 0.74

$$\frac{6ax\left(a^2x^2+1\right)+3\left(a^2x^2-1\right)^2\log(1-ax)-3\left(a^2x^2-1\right)^2\log(ax+1)-16\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^3+48\left(a^3x^3+ax\right)\text{ta}}{384a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (6*a*x*(1 + a^2*x^2) - 48*a^2*x^2*ArcTanh[a*x] + 48*(a*x + a^3*x^3)*ArcTanh
[a*x]^2 - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^3 + 3*(-1 + a^2*x^2)^2*Log[1 - a
*x] - 3*(-1 + a^2*x^2)^2*Log[1 + a*x])/(384*a^3*(-1 + a^2*x^2)^2)
```

Maple [C] time = 0.441, size = 2571, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/64/(a*x-1)^2/(a*x+1)^2*x^3+1/16/a^3*arctanh(a*x)^2/(a*x-1)^2-1/16/a^3*arc
tanh(a*x)^2/(a*x+1)^2+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^4+1/16*I*a/(
a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csg
n(I*(a*x+1)^2/(a^2*x^2-1))^2*x^4+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^4+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arcta
nh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4
-1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2
+1)+1))^3*x^4+1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*x^4-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*c
sgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x
)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^2+1/8*I
/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
^3*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))^2*x^2+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+1/16*I/a^3/(a*x
-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I
*(a*x+1)^2/(a^2*x^2-1))^2-1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*
csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2
*x^2+1)+1))^2+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^2+1/16/a^3*arctanh(a*x)^2/(a*x-1)+1/16/a^3*arctanh(a*x)^2*ln(a*x-1)+1/
16/a^3*arctanh(a*x)^2/(a*x+1)-1/16/a^3*arctanh(a*x)^2*ln(a*x+1)+1/8/a^3*arc
tanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*P
i*arctanh(a*x)^2-1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
^2*x^4+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^
4-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1
)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2*ar
ctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^
2-1))^2*x^2+1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^
2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2-1/3
2*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2
/(-a^2*x^2+1)+1))+1/64/a^2/(a*x-1)^2/(a*x+1)^2*x-1/24/a^3/(a*x-1)^2/(a*x+1)
^2*arctanh(a*x)^3-1/64/a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)-1/32*I*a/(a*x-1
)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(
a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+
1))*x^4+1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-
a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))*x^2-1/24*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^
4-1/64*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^4+1/12/a/(a*x-1)^2/(a*x+1)^2*ar
```

```
ctanh(a*x)^3*x^2-3/32/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^2+1/32*I/a^3/(a*
x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/32*I/a
^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^3-1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arcta
nh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/16*I*a/(a*x-1)^2/(a*x+1)^2
*arctanh(a*x)^2*Pi*x^4+1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2
```

Maxima [B] time = 1.00358, size = 524, normalized size = 3.21

$$\frac{1}{16}\left(\frac{2\left(a^2x^3+x\right)}{a^6x^4-2a^4x^2+a^2}-\frac{\log(ax+1)}{a^3}+\frac{\log(ax-1)}{a^3}\right)\operatorname{artanh}(ax)^2+\frac{\left(6a^3x^3-2\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^3+6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x)^2 + 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 +
1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1)
 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*
x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4
*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^9*x^4 - 2*a^7*x^2 + a^5) - 1/32*
(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)
^2)*a*arctanh(a*x)/(a^8*x^4 - 2*a^6*x^2 + a^4)
```

Fricas [A] time = 2.05374, size = 294, normalized size = 1.8

$$\frac{6a^3x^3-2\left(a^4x^4-2a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\left(a^3x^3+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+6ax-3\left(a^4x^4+6a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)}{384\left(a^7x^4-2a^5x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^3
+ 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 6*a*x - 3*(a^4*x^4 + 6*a
^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 \operatorname{artanh}(ax)^2}{(a^2x^2-1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

3.310 $\int \frac{x\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=125

$$\frac{3}{32a^2\left(1-a^2x^2\right)}+\frac{1}{32a^2\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2}-\frac{3x\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^2}{32a^2}$$

```
[Out] 1/(32*a^2*(1 - a^2*x^2)^2) + 3/(32*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(16*a*(1 - a^2*x^2)) - (3*ArcTanh[
a*x]^2)/(32*a^2) + ArcTanh[a*x]^2/(4*a^2*(1 - a^2*x^2)^2)
```

Rubi [A] time = 0.093837, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5994, 5960, 5956, 261}

$$\frac{3}{32a^2\left(1-a^2x^2\right)}+\frac{1}{32a^2\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2}-\frac{3x\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)}-\frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^2}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] 1/(32*a^2*(1 - a^2*x^2)^2) + 3/(32*a^2*(1 - a^2*x^2)) - (x*ArcTanh[a*x])/(8
*a*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(16*a*(1 - a^2*x^2)) - (3*ArcTanh[
a*x]^2)/(32*a^2) + ArcTanh[a*x]^2/(4*a^2*(1 - a^2*x^2)^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx = \frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx}{2a}$$

$$= \frac{1}{32a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} - \frac{3\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^2}\,dx}{8a}$$

$$= \frac{1}{32a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)^2}{32a^2} + \frac{\tanh^{-1}(ax)^2}{4a^2\left(1-a^2x^2\right)^2} + \frac{3}{16}\int$$

$$= \frac{1}{32a^2\left(1-a^2x^2\right)^2} + \frac{3}{32a^2\left(1-a^2x^2\right)} - \frac{x\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)}{16a\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)^2}{32a^2} + \frac{\tanh}{4a^2(1}$$

Mathematica [A] time = 0.0698054, size = 71, normalized size = 0.57

$$\frac{-3a^2x^2+2ax\left(3a^2x^2-5\right)\tanh^{-1}(ax)+\left(-3a^4x^4+6a^2x^2+5\right)\tanh^{-1}(ax)^2+4}{32a^2\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^3,x]
```

```
[Out] (4 - 3*a^2*x^2 + 2*a*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x] + (5 + 6*a^2*x^2 - 3*a
^4*x^4)*ArcTanh[a*x]^2)/(32*a^2*(-1 + a^2*x^2)^2)
```

Maple [B] time = 0.058, size = 247, normalized size = 2.

$$\frac{(\mathrm{Artanh}\,(ax))^2}{4\,a^2\left(a^2x^2-1\right)^2} - \frac{\mathrm{Artanh}\,(ax)}{32\,a^2\,(ax-1)^2} + \frac{3\,\mathrm{Artanh}\,(ax)}{32\,a^2\,(ax-1)} + \frac{3\,\mathrm{Artanh}\,(ax)\ln(ax-1)}{32\,a^2} + \frac{\mathrm{Artanh}\,(ax)}{32\,a^2\,(ax+1)^2} + \frac{3\,\mathrm{Artanh}\,(ax)}{32\,a^2\,(ax+1)} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/4/a^2/(a^2*x^2-1)^2*arctanh(a*x)^2-1/32/a^2*arctanh(a*x)/(a*x-1)^2+3/32/a
^2*arctanh(a*x)/(a*x-1)+3/32/a^2*arctanh(a*x)*ln(a*x-1)+1/32/a^2*arctanh(a*
x)/(a*x+1)^2+3/32/a^2*arctanh(a*x)/(a*x+1)-3/32/a^2*arctanh(a*x)*ln(a*x+1)+
3/128/a^2*ln(a*x-1)^2-3/64/a^2*ln(a*x-1)*ln(1/2+1/2*a*x)+3/128/a^2*ln(a*x+1
)^2-3/64/a^2*ln(-1/2*a*x+1/2)*ln(a*x+1)+3/64/a^2*ln(-1/2*a*x+1/2)*ln(1/2+1/
2*a*x)+1/128/a^2/(a*x-1)^2-7/128/a^2/(a*x-1)+1/128/a^2/(a*x+1)^2+7/128/a^2/
(a*x+1)
```

Maxima [A] time = 0.982664, size = 278, normalized size = 2.22

$$\frac{\left(\frac{2\left(3a^2x^3-5x\right)}{a^4x^4-2a^2x^2+1} - \frac{3\log(ax+1)}{a} + \frac{3\log(ax-1)}{a}\right)\mathrm{artanh}\,(ax)}{32\,a} - \frac{12\,a^2x^2-3\left(a^4x^4-2\,a^2x^2+1\right)\log(ax+1)^2+6\left(a^4x^4-2\,a^2x^2+1\right.}{128\left(a^6x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)*arctanh(a*x)/a - 1/128*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1
) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)/(a^6*x^4 - 2*a^4*x^2 +
 a^2) + 1/4*arctanh(a*x)^2/((a^2*x^2 - 1)^2*a^2)
```

Fricas [A] time = 2.0753, size = 219, normalized size = 1.75

$$-\frac{12\,a^2x^2+\left(3\,a^4x^4-6\,a^2x^2-5\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-4\left(3\,a^3x^3-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-16}{128\left(a^6x^4-2\,a^4x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(12*a^2*x^2 + (3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))^
2 - 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1)) - 16)/(a^6*x^4 - 2*a^4*
x^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\,\mathrm{atanh}^2\,(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x*atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x\,\mathrm{artanh}\,(ax)^2}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

3.311 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=151

$$\frac{15x}{64(1-a^2x^2)}+\frac{x}{32(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^2}{8(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^3}{8a}+$$

```
[Out] x/(32*(1 - a^2*x^2)^2) + (15*x)/(64*(1 - a^2*x^2)) + (15*ArcTanh[a*x])/(64*
a) - ArcTanh[a*x]/(8*a*(1 - a^2*x^2)^2) - (3*ArcTanh[a*x])/(8*a*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x]^2)/(8*(1 -
 a^2*x^2)) + ArcTanh[a*x]^3/(8*a)
```

Rubi [A] time = 0.110423, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 199, 206}

$$\frac{15x}{64(1-a^2x^2)}+\frac{x}{32(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^2}{8(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^2}{4(1-a^2x^2)^2}-\frac{3\tanh^{-1}(ax)}{8a(1-a^2x^2)}-\frac{\tanh^{-1}(ax)}{8a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^3}{8a}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```


```
[Out] x/(32*(1 - a^2*x^2)^2) + (15*x)/(64*(1 - a^2*x^2)) + (15*ArcTanh[a*x])/(64*
a) - ArcTanh[a*x]/(8*a*(1 - a^2*x^2)^2) - (3*ArcTanh[a*x])/(8*a*(1 - a^2*x^
2)) + (x*ArcTanh[a*x]^2)/(4*(1 - a^2*x^2)^2) + (3*x*ArcTanh[a*x]^2)/(8*(1 -
 a^2*x^2)) + ArcTanh[a*x]^3/(8*a)
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx = -\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{1}{8}\int\frac{1}{\left(1-a^2x^2\right)^3}\,dx+\frac{3}{4}\int\frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{\tanh^{-1}(ax)^3}{8a}+\frac{3}{32}\int\frac{}{(1}$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{3x}{64\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(}{8\left(1-a^2\right.}$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{15x}{64\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax}{4\left(1-a^2x^2\right.}$$

$$=\frac{x}{32\left(1-a^2x^2\right)^2}+\frac{15x}{64\left(1-a^2x^2\right)}+\frac{15\tanh^{-1}(ax)}{64a}-\frac{\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)}{8a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(a}{4\left(1-a^2x\right.}$$

Mathematica [A] time = 0.107171, size = 127, normalized size = 0.84

$$\frac{1}{128}\left(-\frac{30x}{a^2x^2-1}+\frac{4x}{\left(a^2x^2-1\right)^2}-\frac{16x\left(3a^2x^2-5\right)\tanh^{-1}(ax)^2}{\left(a^2x^2-1\right)^2}+\frac{16\left(3a^2x^2-4\right)\tanh^{-1}(ax)}{a\left(a^2x^2-1\right)^2}-\frac{15\log(1-ax)}{a}+\frac{15\log}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^3,x]
```

```
[Out] ((4*x)/(-1 + a^2*x^2)^2 - (30*x)/(-1 + a^2*x^2) + (16*(-4 + 3*a^2*x^2)*ArcT
anh[a*x])/(a*(-1 + a^2*x^2)^2) - (16*x*(-5 + 3*a^2*x^2)*ArcTanh[a*x]^2)/(-1
 + a^2*x^2)^2 + (16*ArcTanh[a*x]^3)/a - (15*Log[1 - a*x])/a + (15*Log[1 + a
*x])/a)/128
```

Maple [C] time = 0.443, size = 2571, normalized size = 17.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^3,x)
```

```
[Out] -3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4+3/
16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1
/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^2+3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctan
h(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)^2*x^2-3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2+3/
16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2-3/32*I*
a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x
^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x
^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4+3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arct
anh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4-3/16*I*a^3/(a*x-1)^2
/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4+3/16*
I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x
^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^2-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a
*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^2+3/8*I*a/(a*x-1)^2/(a*x+1)
^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2-3/32*I/a/(a*x
-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn
(I*(a*x+1)^2/(a^2*x^2-1))-3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*cs
gn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+3/32*I/a/(
a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(
a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-3/32*I/a/(a*x-1)^2/(a*x+
1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*a
rctanh(a*x)^2+1/8*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^4+15/64*a^3/(a*x
-1)^2/(a*x+1)^2*arctanh(a*x)*x^4-1/4*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x
^2-3/32*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x^2-3/16*I*a/(a*x-1)^2/(a*x+1)^2
*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*x^2+3/32*
I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))*x^4+3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arcta
nh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*x^4-3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-
a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*x^4+3/32*I*a^3/(a*x-1)^2/
(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4+17/64/(a*x-1)^2/(a*x+1)^2*x+1
/16/a*arctanh(a*x)^2/(a*x-1)^2-3/16/a*arctanh(a*x)^2/(a*x-1)-3/16/a*arctanh
(a*x)^2*ln(a*x-1)-1/16/a*arctanh(a*x)^2/(a*x+1)^2-3/16/a*arctanh(a*x)^2/(a*
x+1)+3/16/a*arctanh(a*x)^2*ln(a*x+1)-3/8/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*
x^2+1)^(1/2))-15/64*a^2/(a*x-1)^2/(a*x+1)^2*x^3-17/64/a/(a*x-1)^2/(a*x+1)^2
*arctanh(a*x)+1/8/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3-3/32*I/a/(a*x-1)^2/(
a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-3/32*I/a/(a*x-1)
^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^
2*x^2+1)+1))^3+3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+
1)^2/(-a^2*x^2+1)+1))^3-3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn
(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+3/16*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x
)^2*Pi*x^4-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2
```

Maxima [B] time = 1.02304, size = 529, normalized size = 3.5

$$-\frac{1}{16}\left(\frac{2\left(3\,a^{2}x^{3}-5\,x\right)}{a^{4}x^{4}-2\,a^{2}x^{2}+1}-\frac{3\,\log\left(ax+1\right)}{a}+\frac{3\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^{2}-\frac{\left(30\,a^{3}x^{3}-2\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\log\left(ax+1\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^
2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x
- 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 -
 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)
+ 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2 + a^3
) + 1/32*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2)
```

Fricas [A] time = 1.90494, size = 302, normalized size = 2.

$$-\frac{30\,a^3x^3-2\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+4\left(3\,a^3x^3-5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-34\,ax-\left(15\,a^4x^4-6\,a^2x^2-17\right)\log\left(-\right.}{128\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^
3 + 4*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 34*a*x - (15*a^4*x^
4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/(a^2*x^2 - 1)^3, x)
```

3.312 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^3} dx$

Optimal. Leaf size=196

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{11}{32\left(1-a^2x^2\right)}+\frac{1}{32\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{t}{4}$$

```
[Out] 1/(32*(1 - a^2*x^2)^2) + 11/(32*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(8*(1 -
 a^2*x^2)^2) - (11*a*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^
2)/32 + ArcTanh[a*x]^2/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^2/(2*(1 - a^2*x^2
)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*
PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```

Rubi [A] time = 0.449244, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {6030, 5988, 5932, 5948, 6056, 6610, 5994, 5956, 261, 5960}

$$-\frac{1}{2}\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{11}{32\left(1-a^2x^2\right)}+\frac{1}{32\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)}+\frac{t}{4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```


```
[Out] 1/(32*(1 - a^2*x^2)^2) + 11/(32*(1 - a^2*x^2)) - (a*x*ArcTanh[a*x])/(8*(1 -
 a^2*x^2)^2) - (11*a*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^
2)/32 + ArcTanh[a*x]^2/(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^2/(2*(1 - a^2*x^2
)) + ArcTanh[a*x]^3/3 + ArcTanh[a*x]^2*Log[2 - 2/(1 + a*x)] - ArcTanh[a*x]*
PolyLog[2, -1 + 2/(1 + a*x)] - PolyLog[3, -1 + 2/(1 + a*x)]/2
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} - \frac{1}{2}a\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx + a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^2}{2\left(1-a^2x^2\right)} + \frac{1}{3}\tanh^{-1}(ax)^3 - \frac{1}{8}(3a)\int \frac{\tanh}{(1-}$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(a}{2\left(1-a^2\right.}$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} + \frac{11}{32\left(1-a^2x^2\right)} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(a}{4\left(1-a^2\right.}$$

$$= \frac{1}{32\left(1-a^2x^2\right)^2} + \frac{11}{32\left(1-a^2x^2\right)} - \frac{ax\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{11ax\tanh^{-1}(ax)}{16\left(1-a^2x^2\right)} - \frac{11}{32}\tanh^{-1}(ax)^2 + \frac{\tanh^{-1}(a}{4\left(1-a^2\right.}$$

Mathematica [C] time = 0.396813, size = 129, normalized size = 0.66

$$\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right) + \frac{1}{768}\left(-384\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right) - 256\tanh^{-1}(ax)^3 - 12\tanh^{-1}(ax)\left(24\sinh\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + ((32*I)*Pi^3 - 256*ArcTanh[a*
x]^3 + 144*Cosh[2*ArcTanh[a*x]] + 3*Cosh[4*ArcTanh[a*x]] + 24*ArcTanh[a*x]^
2*(12*Cosh[2*ArcTanh[a*x]] + Cosh[4*ArcTanh[a*x]] + 32*Log[1 - E^(2*ArcTanh
[a*x])]) - 384*PolyLog[3, E^(2*ArcTanh[a*x])] - 12*ArcTanh[a*x]*(24*Sinh[2*
ArcTanh[a*x]] + Sinh[4*ArcTanh[a*x]]))/768
```

Maple [C] time = 0.506, size = 1392, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x)
```

```
[Out] arctanh(a*x)^2*ln(a*x)-arctanh(a*x)^2*ln((a*x+1)^2/(-a^2*x^2+1)-1)+arctanh(
a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x
^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctan
h(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I*Pi*arctanh(a*x)^2+3/16*(
a*x+1)*arctanh(a*x)/(a*x-1)-3/16*(a*x-1)*arctanh(a*x)/(a*x+1)+1/2*I*Pi*csgn
(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((
a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2-1/128*a
rctanh(a*x)*(a*x+1)^2/(a*x-1)^2-3/32*(a*x-1)/(a*x+1)+arctanh(a*x)^2*ln(2)-3
/32*(a*x+1)/(a*x-1)+1/16*arctanh(a*x)^2/(a*x-1)^2+1/16*arctanh(a*x)^2/(a*x+
1)^2+1/512*(a*x-1)^2/(a*x+1)^2+1/512*(a*x+1)^2/(a*x-1)^2-1/3*arctanh(a*x)^3
-11/32*arctanh(a*x)^2-1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh
(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3
```

```
*arctanh(a*x)^2+1/4*I*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2+1/2
*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2+1/2*I*Pi*csgn(I*(
(a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2-2*po
lylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/4*I*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)
^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-5/16*arctanh(a*x)^2/(a*x-1)-1/
2*arctanh(a*x)^2*ln(a*x-1)+5/16*arctanh(a*x)^2/(a*x+1)-1/2*arctanh(a*x)^2*l
n(a*x+1)+arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/128*(a*x-1)^2*arct
anh(a*x)/(a*x+1)^2+1/4*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2+1/2*I*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-1/4*I*Pi*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*a
rctanh(a*x)^2-1/4*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arcta
nh(a*x)^2-1/2*I*Pi*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I*((a*x+1)^2/(-a
^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-1/2*I*Pi*csgn(I/(
(a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*arctanh(a*x)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^6*integrate(1/2*x^6*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 +
 3*a^2*x^3 - x), x) + 1/2*a^5*integrate(1/2*x^5*log(a*x + 1)*log(-a*x + 1)/
(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/256*(a*(2*(5*a^2*x^2 + 3*a*x
- 6)/(a^8*x^3 - a^7*x^2 - a^6*x + a^5) - 5*log(a*x + 1)/a^5 + 5*log(a*x - 1
)/a^5) + 16*(2*a^2*x^2 - 1)*log(-a*x + 1)/(a^8*x^4 - 2*a^6*x^2 + a^4))*a^4
- a^4*integrate(1/2*x^4*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3
*a^2*x^3 - x), x) - a^3*integrate(1/2*x^3*log(a*x + 1)*log(-a*x + 1)/(a^6*x
^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + 1/2*a^3*integrate(1/2*x^3*log(-a*x +
1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 3/512*(a*(2*(3*a^2*x^2 - 3*a
*x - 2)/(a^6*x^3 - a^5*x^2 - a^4*x + a^3) - 3*log(a*x + 1)/a^3 + 3*log(a*x
- 1)/a^3) - 16*log(-a*x + 1)/(a^6*x^4 - 2*a^4*x^2 + a^2))*a^2 + 1/2*a^2*int
egrate(1/2*x^2*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3
- x), x) + 1/2*a*integrate(1/2*x*log(a*x + 1)*log(-a*x + 1)/(a^6*x^7 - 3*a^
4*x^5 + 3*a^2*x^3 - x), x) - 3/4*a*integrate(1/2*x*log(-a*x + 1)/(a^6*x^7 -
 3*a^4*x^5 + 3*a^2*x^3 - x), x) - 1/48*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*
x + 1)^3 + 3*(2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log
(-a*x + 1)^2)/(a^4*x^4 - 2*a^2*x^2 + 1) - 1/2*integrate(1/2*log(a*x + 1)^2/
(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x) + integrate(1/2*log(a*x + 1)*log(
-a*x + 1)/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^2}{a^6x^7 - 3\,a^4x^5 + 3\,a^2x^3 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}^2(ax)}{a^6x^7 - 3a^4x^5 + 3a^2x^3 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^2}{\left(a^2x^2 - 1\right)^3 x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x), x)
```

3.313 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=209

$$-a\mathrm{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}$$

```
[Out] (a^2*x)/(32*(1 - a^2*x^2)^2) + (31*a^2*x)/(64*(1 - a^2*x^2)) + (31*a*ArcTan
h[a*x])/64 - (a*ArcTanh[a*x])/(8*(1 - a^2*x^2)^2) - (7*a*ArcTanh[a*x])/(8*(
1 - a^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2
)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^2)/(8*(1 - a^2*x^2)) + (5*a*A
rcTanh[a*x]^3)/8 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1
+ 2/(1 + a*x)]
```

Rubi [A] time = 0.492116, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6030, 5982, 5916, 5988, 5932, 2447, 5948, 5956, 5994, 199, 206, 5964}

$$-a\mathrm{PolyLog}\left(2,\frac{2}{ax+1}-1\right)+\frac{31a^2x}{64\left(1-a^2x^2\right)}+\frac{a^2x}{32\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)}+\frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2}-\frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] (a^2*x)/(32*(1 - a^2*x^2)^2) + (31*a^2*x)/(64*(1 - a^2*x^2)) + (31*a*ArcTan
h[a*x])/64 - (a*ArcTanh[a*x])/(8*(1 - a^2*x^2)^2) - (7*a*ArcTanh[a*x])/(8*(
1 - a^2*x^2)) + a*ArcTanh[a*x]^2 - ArcTanh[a*x]^2/x + (a^2*x*ArcTanh[a*x]^2
)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^2)/(8*(1 - a^2*x^2)) + (5*a*A
rcTanh[a*x]^3)/8 + 2*a*ArcTanh[a*x]*Log[2 - 2/(1 + a*x)] - a*PolyLog[2, -1
+ 2/(1 + a*x)]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

```
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{1}{8}a^2\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx + \frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx + a^2$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} + \frac{a^2x\tanh^{-1}(ax)^2}{4\left(1-a^2x^2\right)^2} + \frac{7a^2x\tanh^{-1}(ax)^2}{8\left(1-a^2x^2\right)} + \frac{7}{24}a\tanh^{-1}(ax)^3 +$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{3a^2x}{64\left(1-a^2x^2\right)} - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^2}{x} + \frac{a^2x\tanh^{-1}}{4\left(1-a^2x\right.}$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{31a^2x}{64\left(1-a^2x^2\right)} + \frac{3}{64}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + a\tanh^{-}$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{31a^2x}{64\left(1-a^2x^2\right)} + \frac{31}{64}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + a\tanh^{-}$$

$$= \frac{a^2x}{32\left(1-a^2x^2\right)^2} + \frac{31a^2x}{64\left(1-a^2x^2\right)} + \frac{31}{64}a\tanh^{-1}(ax) - \frac{a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)^2} - \frac{7a\tanh^{-1}(ax)}{8\left(1-a^2x^2\right)} + a\tanh^{-}$$

Mathematica [A] time = 0.567019, size = 127, normalized size = 0.61

$$-a\left(\text{PolyLog}\left(2,e^{-2\tanh^{-1}(ax)}\right) + \tanh^{-1}(ax)^2\left(\frac{ax}{a^2x^2-1} + \frac{1}{ax} - \frac{1}{32}\sinh\left(4\tanh^{-1}(ax)\right) - 1\right) - \frac{5}{8}\tanh^{-1}(ax)^3 - \frac{1}{4}\sin$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*((-5*ArcTanh[a*x]^3)/8 + (ArcTanh[a*x]*(32*Cosh[2*ArcTanh[a*x]] + Cosh[
4*ArcTanh[a*x]] - 128*Log[1 - E^(-2*ArcTanh[a*x])]))/64 + PolyLog[2, E^(-2*
ArcTanh[a*x])] - Sinh[2*ArcTanh[a*x]]/4 + ArcTanh[a*x]^2*(-1 + 1/(a*x) + (a
*x)/(-1 + a^2*x^2) - Sinh[4*ArcTanh[a*x]]/32) - Sinh[4*ArcTanh[a*x]]/256))
```

Maple [C] time = 0.602, size = 4797, normalized size = 23.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x)

[Out] 15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-a*arctanh(a*x)^2-a*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2-15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*arctanh(a*x)^2-15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)^2-15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*arctanh(a*x)^2+15/16*I*a*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/128*a*arctanh(a*x)/(a*x-1)^2-1/128*a*arctanh(a*x)/(a*x+1)^2+15/16*a*arctanh(a*x)^2*ln(a*x+1)-15/8*a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/8/(a*x+1)*a^2*x-1/8*a^2*x/(a*x-1)-7/16*a*arctanh(a*x)^2/(a*x-1)-15/16*a*arctanh(a*x)^2*ln(a*x-1)-7/16*a*arctanh(a*x)^2/(a*x+1)+1/4*arctanh(a*x)/(a

```
*x-1)*a^2*x+1/4*arctanh(a*x)/(a*x+1)*a^2*x+1/512*a/(a*x-1)^2-1/512*a/(a*x+1
)^2-arctanh(a*x)^2/x-15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(
-a^2*x^2+1)+1))^3*arctanh(a*x)^2+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I
*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1
5/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilo
g((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*
dilog((a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I*(a*x
+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))+15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*arctanh(a*x)^2-1
5/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^2+15/32*I*a*P
i*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*polylog(2,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*
polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))^2*dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/
16*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*dilog((a*x+1)/(-a^2*x^2+1)^(
1/2))-15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2
*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a/(a*x-1)-1/8*a/(a*x+1)+5/8*a*arctanh(a*x)
^3-1/128*arctanh(a*x)/(a*x-1)^2*a^3*x^2-1/64*arctanh(a*x)/(a*x-1)^2*a^2*x-1
/128*arctanh(a*x)/(a*x+1)^2*a^3*x^2+1/64*arctanh(a*x)/(a*x+1)^2*a^2*x+15/16
*I*a*Pi*arctanh(a*x)^2-15/16*I*a*Pi*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-
15/16*I*a*Pi*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-15/16*I*a*Pi*dilog((a*x+
1)/(-a^2*x^2+1)^(1/2))+15/16*I*a*Pi*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/5
12/(a*x+1)^2*a^3*x^2+1/256/(a*x+1)^2*a^2*x+1/512/(a*x-1)^2*a^3*x^2+1/256/(a
*x-1)^2*a^2*x+1/16*a*arctanh(a*x)^2/(a*x-1)^2-1/16*a*arctanh(a*x)^2/(a*x+1)
^2-15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^
2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*dilog((a*x+1
)/(-a^2*x^2+1)^(1/2))-15/32*I*a*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+15/32*I*a*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))*arctanh(a*x)^2+15/32*I*a*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-15/32*I*
a*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/
(-a^2*x^2+1)+1))^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*a*arct
anh(a*x)/(a*x-1)-1/4*a*arctanh(a*x)/(a*x+1)+15/32*I*a*Pi*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2
*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.03817, size = 721, normalized size = 3.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/128*a^2*(2*(31*a^3*x^3 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 5*
(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - (16*a^4*x^4 - 32*a^2*x^2 - 15*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1) + 16)*log(a*x + 1)^2 + 16*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x - 1)^2 - 33*a*x - (15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a
```

```
*x - 1)^2 - 32*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))/(a^5*x
^4 - 2*a^3*x^2 + a) - 128*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x
 + 1/2))/a + 128*(log(a*x + 1)*log(x) + dilog(-a*x))/a - 128*(log(-a*x + 1)
*log(x) + dilog(a*x))/a - 31*log(a*x + 1)/a + 31*log(a*x - 1)/a) + 1/32*a*(
(28*a^2*x^2 - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 30*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(
a*x - 1)^2 - 32)/(a^4*x^4 - 2*a^2*x^2 + 1) - 32*log(a*x + 1) - 32*log(a*x -
 1) + 64*log(x))*arctanh(a*x) + 1/16*(15*a*log(a*x + 1) - 15*a*log(a*x - 1)
 - 2*(15*a^4*x^4 - 25*a^2*x^2 + 8)/(a^4*x^5 - 2*a^2*x^3 + x))*arctanh(a*x)^
2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^2}{a^6x^8 - 3\,a^4x^6 + 3\,a^2x^4 - x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^2/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\text{atanh}^2\,(ax)}{a^6x^8 - 3a^4x^6 + 3a^2x^4 - x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**2/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\text{artanh}\,(ax)^2}{\left(a^2x^2 - 1\right)^3 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^2/((a^2*x^2 - 1)^3*x^2), x)
```

3.314 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3} \, dx$

Optimal. Leaf size=192

$$-\frac{3x^3}{128a\left(1-a^2x^2\right)^2}+\frac{45x}{256a^3\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3x^4\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3x^3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{9x\tanh^{-1}(ax)^2}{32a^3\left(1-a^2x^2\right)}-\frac{3\cdots}{}$$

```
[Out] (-3*x^3)/(128*a*(1 - a^2*x^2)^2) + (45*x)/(256*a^3*(1 - a^2*x^2)) + (27*Arc
Tanh[a*x])/(256*a^4) + (3*x^4*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) - (9*ArcTa
nh[a*x])/(32*a^4*(1 - a^2*x^2)) - (3*x^3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2
)^2) + (9*x*ArcTanh[a*x]^2)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32
*a^4) + (x^4*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2)
```

Rubi [A] time = 0.267907, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6008, 6004, 6000, 5994, 199, 206, 288}

$$-\frac{3x^3}{128a\left(1-a^2x^2\right)^2}+\frac{45x}{256a^3\left(1-a^2x^2\right)}+\frac{x^4\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3x^4\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3x^3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{9x\tanh^{-1}(ax)^2}{32a^3\left(1-a^2x^2\right)}-\frac{3\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*x^3)/(128*a*(1 - a^2*x^2)^2) + (45*x)/(256*a^3*(1 - a^2*x^2)) + (27*Arc
Tanh[a*x])/(256*a^4) + (3*x^4*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) - (9*ArcTa
nh[a*x])/(32*a^4*(1 - a^2*x^2)) - (3*x^3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2
)^2) + (9*x*ArcTanh[a*x]^2)/(32*a^3*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32
*a^4) + (x^4*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2)
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6004

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> -Simp[(b*p*(f*x)^m*(d + e*x^2)^(q + 1)*(a + b*
ArcTanh[c*x])^(p - 1))/(c*d*m^2), x] + (-Dist[(f^2*(m - 1))/(c^2*d*m), Int[
(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^
2*p*(p - 1))/m^2, Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x
], x] + Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(
c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && Eq
Q[m + 2*q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

Rule 6000

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2
)^2, x_Symbol] :> -Simp[(a + b*ArcTanh[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)),
```

```
 x] + (-Dist[(b*p)/(2*c), Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^
2, x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*c^2*d*(d + e*x^2)), x]) /; F
reeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = \frac{x^4 \tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} - \frac{1}{4}(3a)\int \frac{x^4 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx$$

$$= \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \frac{x^4 \tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2} + \frac{9\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{16a} - \frac{1}{32}(3a)\int \frac{x^4}{\left(1-a^2\right.}\cdots$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \frac{9x \tanh^{-1}(ax)^2}{32a^3\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)^3}{32a^4} + \cdots$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{9x}{256a^3\left(1-a^2x^2\right)} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{9\tanh^{-1}(ax)}{32a^4\left(1-a^2x^2\right)} - \frac{3x^3 \tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \cdots$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{45x}{256a^3\left(1-a^2x^2\right)} - \frac{9\tanh^{-1}(ax)}{256a^4} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{9\tanh^{-1}(ax)}{32a^4\left(1-a^2x^2\right)} - \cdots$$

$$= -\frac{3x^3}{128a\left(1-a^2x^2\right)^2} + \frac{45x}{256a^3\left(1-a^2x^2\right)} + \frac{27\tanh^{-1}(ax)}{256a^4} + \frac{3x^4 \tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2} - \frac{9\tanh^{-1}(ax)}{32a^4\left(1-a^2x^2\right)} - \cdots$$

Mathematica [A] time = 0.127838, size = 135, normalized size = 0.7

$$\frac{3\left(-34a^3x^3-17\left(a^2x^2-1\right)^2\log(1-ax)+17\left(a^2x^2-1\right)^2\log(ax+1)+30ax\right)+16\left(5a^4x^4+6a^2x^2-3\right)\tanh^{-1}(ax)^3-}{512a^4\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (48*(-4 + 5*a^2*x^2)*ArcTanh[a*x] - 48*a*x*(-3 + 5*a^2*x^2)*ArcTanh[a*x]^2
+ 16*(-3 + 6*a^2*x^2 + 5*a^4*x^4)*ArcTanh[a*x]^3 + 3*(30*a*x - 34*a^3*x^3 -
 17*(-1 + a^2*x^2)^2*Log[1 - a*x] + 17*(-1 + a^2*x^2)^2*Log[1 + a*x]))/(512
*a^4*(-1 + a^2*x^2)^2)
```

Maple [C] time = 0.457, size = 2634, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^4*arctanh(a*x)^3/(a*x-1)^2+1/16/a^4*arctanh(a*x)^3/(a*x+1)^2-3/64/a^
4*arctanh(a*x)^2/(a*x-1)^2+3/64/a^4*arctanh(a*x)^2/(a*x+1)^2-51/256/a/(a*x-
1)^2/(a*x+1)^2*x^3+45/256/a^3/(a*x-1)^2/(a*x+1)^2*x+5/32/a^4/(a*x-1)^2/(a*x
+1)^2*arctanh(a*x)^3-45/256/a^4/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)+5/32/(a*x-
1)^2/(a*x+1)^2*arctanh(a*x)^3*x^4+51/256/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x
^4+15/64*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2+15/64*I/a^2/(a*x-1)^2/(a*x+1)^2*ar
ctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2-15/32*I/a^2/(a*x-1)^2/(
a*x+1)^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2+15/32*I
/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
2*Pi*x^2+15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1
5/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-15/64*
I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arc
tanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-
1))+15/128*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4-15/128
*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^4-15/64*I/(a*
x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(
a*x+1)^2/(a^2*x^2-1))^2*Pi*x^4-15/128*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^4-1
5/64/a^4*arctanh(a*x)^2/(a*x-1)-15/64/a^4*arctanh(a*x)^2/(a*x+1)-15/64*I/a^
2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^
2+1)+1))*Pi*x^2+3/16/a^4*arctanh(a*x)^3/(a*x-1)-3/16/a^4*arctanh(a*x)^3/(a*
x+1)+15/128*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-
1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))*Pi*x^4+15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x
)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-15/64*I/a^2/(a*x-1)^2/(a
```

```
*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2+15/64*I/a^2/(a*x-1)^2/(a*x+1)^2
*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*
csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2+15/32*I/a^2/(a*x-1)^2/(a*x+1)^2*a
rctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-
1))^2*Pi*x^2+15/64*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)/
(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*Pi*x^2-5/16/a^2/(a*x-1)
^2/(a*x+1)^2*arctanh(a*x)^3*x^2+9/128/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*
x^2+15/64*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2+15/64*I/(a*x-1)^2/(a*
x+1)^2*arctanh(a*x)^2*Pi*x^4-15/128*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*cs
gn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^4-15/128*I/(a
*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^4+15/
64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*Pi*x^4-15/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))^2*Pi*x^4-15/128*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-15/128*I/a^4/(a*x-1
)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+15/64*I/a^4
/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3
-15/64*I/a^4/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))^2-15/32*I/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2-15/64/a^
4*arctanh(a*x)^2*ln(a*x-1)+15/64/a^4*arctanh(a*x)^2*ln(a*x+1)-15/32/a^4*arc
tanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [B] time = 1.03131, size = 590, normalized size = 3.07

$$-\frac{3}{64}a\left(\frac{2\left(5a^2x^3-3x\right)}{a^8x^4-2a^6x^2+a^4}-\frac{5\log\left(ax+1\right)}{a^5}+\frac{5\log\left(ax-1\right)}{a^5}\right)\operatorname{artanh}\left(ax\right)^2+\frac{\left(2a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^3}{4\left(a^8x^4-2a^6x^2+a^4\right)}-\frac{1}{512}\left(\frac{\left(102a^3x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -3/64*a*(2*(5*a^2*x^3 - 3*x)/(a^8*x^4 - 2*a^6*x^2 + a^4) - 5*log(a*x + 1)/a
^5 + 5*log(a*x - 1)/a^5)*arctanh(a*x)^2 + 1/4*(2*a^2*x^2 - 1)*arctanh(a*x)^
3/(a^8*x^4 - 2*a^6*x^2 + a^4) - 1/512*((102*a^3*x^3 - 10*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x + 1)^3 + 30*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*
x - 1) + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 90*a*x - 3*(17*a^4*x
^4 - 34*a^2*x^2 + 10*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 17)*log(a*x
 + 1) + 51*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^11*x^4 - 2*a^9*x^
2 + a^7) - 12*(20*a^2*x^2 - 5*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 10
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 5*(a^4*x^4 - 2*a^2*x
^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^10*x^4 - 2*a^8*x^2 + a^6))*a
```

Fricas [A] time = 2.17544, size = 313, normalized size = 1.63

$$-\frac{102a^3x^3-2\left(5a^4x^4+6a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\left(5a^3x^3-3ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-90ax-3\left(17a^4x^4+6a^2x^2-15\right)\log}{512\left(a^8x^4-2a^6x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/512*(102*a^3*x^3 - 2*(5*a^4*x^4 + 6*a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1
))^3 + 12*(5*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 90*a*x - 3*(17*
```

```
a^4*x^4 + 6*a^2*x^2 - 15)*log(-(a*x + 1)/(a*x - 1)))/(a^8*x^4 - 2*a^6*x^2 +
 a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3 \operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**3*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3 \operatorname{artanh}(ax)^3}{(a^2x^2 - 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^3*arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

3.315 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx$

Optimal. Leaf size=215

$$\frac{3}{128a^3\left(1-a^2x^2\right)}-\frac{3}{128a^3\left(1-a^2x^2\right)^2}-\frac{x\tanh^{-1}(ax)^3}{8a^2\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)^2}-\frac{3x\tan}{64a^2(1}$$

```
[Out] -3/(128*a^3*(1 - a^2*x^2)^2) + 3/(128*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x
])/(32*a^2*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(64*a^2*(1 - a^2*x^2)) - (
3*ArcTanh[a*x]^2)/(128*a^3) - (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)^2) +
 (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*a^2*(1 -
 a^2*x^2)^2) - (x*ArcTanh[a*x]^3)/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(3
2*a^3)
```

Rubi [A] time = 0.358817, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6028, 5956, 5994, 261, 5964, 5960}

$$\frac{3}{128a^3\left(1-a^2x^2\right)}-\frac{3}{128a^3\left(1-a^2x^2\right)^2}-\frac{x\tanh^{-1}(ax)^3}{8a^2\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)}-\frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)^2}-\frac{3x\tan}{64a^2(1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] -3/(128*a^3*(1 - a^2*x^2)^2) + 3/(128*a^3*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x
])/(32*a^2*(1 - a^2*x^2)^2) - (3*x*ArcTanh[a*x])/(64*a^2*(1 - a^2*x^2)) - (
3*ArcTanh[a*x]^2)/(128*a^3) - (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)^2) +
 (3*ArcTanh[a*x]^2)/(16*a^3*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*a^2*(1 -
 a^2*x^2)^2) - (x*ArcTanh[a*x]^3)/(8*a^2*(1 - a^2*x^2)) - ArcTanh[a*x]^4/(3
2*a^3)
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx}{a^2}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)^2} + \frac{x\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2} - \frac{x\tanh^{-1}(ax)^3}{2a^2\left(1-a^2x^2\right)} - \frac{\tanh^{-1}(ax)^4}{8a^3} + \frac{3\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx}{8a^2} + -\ 3$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3\tanh^{-1}(ax)^2}{16a^3\left(1-a^2x^2\right)^2} + \frac{3\tanh^{-1}(ax)^2}{4a^3\left(1-a^2x^2\right)} + \frac{x\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2}$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{39x\tanh^{-1}(ax)}{64a^2\left(1-a^2x^2\right)} - \frac{39\tanh^{-1}(ax)^2}{128a^3} - \frac{3\tanh^{-1}(ax)}{16a^3\left(1-a^2x^2\right.}$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{39}{128a^3\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)}{64a^2\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{128a^3}$$

$$= -\frac{3}{128a^3\left(1-a^2x^2\right)^2} + \frac{3}{128a^3\left(1-a^2x^2\right)} + \frac{3x\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)}{64a^2\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)}{128a^3}$$

Mathematica [A] time = 0.122521, size = 107, normalized size = 0.5

$$\frac{-3a^2x^2 - 4\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^4 + 16\left(a^3x^3+ax\right)\tanh^{-1}(ax)^3 - 3\left(a^4x^4+6a^2x^2+1\right)\tanh^{-1}(ax)^2 + 6\left(a^3x^3+ax\right)}{128a^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-3*a^2*x^2 + 6*(a*x + a^3*x^3)*ArcTanh[a*x] - 3*(1 + 6*a^2*x^2 + a^4*x^4)*
ArcTanh[a*x]^2 + 16*(a*x + a^3*x^3)*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*Arc
Tanh[a*x]^4)/(128*a^3*(-1 + a^2*x^2)^2)
```

Maple [C] time = 0.432, size = 2646, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/16/a^3*arctanh(a*x)^3*ln(a*x-1)-1/16/a^3*arctanh(a*x)^3/(a*x+1)^2+1/16/a^
3*arctanh(a*x)^3/(a*x+1)-1/16/a^3*arctanh(a*x)^3*ln(a*x+1)+1/8/a^3*arctanh(
a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/1024/a^3/(a*x-1)^2/(a*x+1)^2+1/16/a
^3*arctanh(a*x)^3/(a*x-1)^2+1/16/a^3*arctanh(a*x)^3/(a*x-1)-1/16*I*a/(a*x-1
)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4+1/
32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^4+1/16*I*a/(a*x-1)^2/(a*x+1)^2*arcta
nh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4+1/8*I/a/(a*x-1)^2/(a*
x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^2-1/16*I/a/
(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)
^2/(-a^2*x^2+1)+1))^3*x^2-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*cs
gn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2+1/32*I/a^3/(a*x-1)^2/(a*x+1)^
2*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*ar
ctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))^2+1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3
*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+1/32*I/
a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/32*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^4
*x^4-3/128*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^4+1/16/a/(a*x-1)^2/(a*x+1
)^2*arctanh(a*x)^4*x^2-9/64/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^2+3/64/a
^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x+1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(
a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/(
(a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4-1/32*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^
3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x
+1)^2/(a^2*x^2-1))*x^4+1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(
I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^4+1/32*I*a/
(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^4-1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)
^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*x^2+1/16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^
2/(a^2*x^2-1))*x^2-1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*x^2-1/16*I/a/(a*x-
1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+
1)/(-a^2*x^2+1)^(1/2))^2*x^2-1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)
^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-1/32*I*a/(a*x-1)^2/(a*x+1
)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(
a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^4+1/
16*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
```

```
+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*x^2+1/8*I/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*x^2-1/16
*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))^3+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+1/32*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*a
rctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+1/16*I/a^3/(a*x-1)^2/(a*x+1)^
2*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-3/1024*a/(a*x-1)^2
/(a*x+1)^2*x^4-9/512/a/(a*x-1)^2/(a*x+1)^2*x^2+3/64/(a*x-1)^2/(a*x+1)^2*arc
tanh(a*x)*x^3-1/32/a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^4-3/128/a^3/(a*x-1)
^2/(a*x+1)^2*arctanh(a*x)^2-1/16*I/a^3/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^
3-1/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*x^4
```

Maxima [B] time = 1.06039, size = 887, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(a^2*x^3 + x)/(a^6*x^4 - 2*a^4*x^2 + a^2) - log(a*x + 1)/a^3 + log(
a*x - 1)/a^3)*arctanh(a*x)^3 - 3/64*(4*a^2*x^2 - (a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x + 1)^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - (a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2)*a*arctanh(a*x)^2/(a^8*x^4 - 2*a^6*x
^2 + a^4) + 1/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 - 4*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log
(a*x - 1)^4 - 12*a^2*x^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2
+ 1)*log(a*x - 1)^2 + 1)*log(a*x + 1)^2 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a
*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 + 3*(a^4*x^4 - 2*
a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1))*a^2/(a^10*x^4 - 2*a^8*x^2 + a^6) +
 4*(6*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2
*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log
(a*x - 1)^3 + 6*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*
log(a*x - 1)^2 + 1)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)
)*a*arctanh(a*x)/(a^9*x^4 - 2*a^7*x^2 + a^5))*a
```

Fricas [A] time = 2.03667, size = 348, normalized size = 1.62

$$-\frac{\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4+12\,a^2x^2-8\left(a^3x^3+ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+3\left(a^4x^4+6\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-12\,(a}{512\left(a^7x^4-2\,a^5x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/512*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 12*a^2*x^2
- 8*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(a^4*x^4 + 6*a^2*x^2 +
1)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(a^3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1
)))/(a^7*x^4 - 2*a^5*x^2 + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 \operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x**2*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 \operatorname{artanh}(ax)^3}{(a^2x^2 - 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2*arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

3.316 $\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=188

$$-\frac{45x}{256a(1-a^2x^2)}-\frac{3x}{128a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{9x\tanh^{-1}(ax)^2}{32a(1-a^2x^2)}-\frac{3x\tanh^{-1}(ax)^2}{16a(1-a^2x^2)^2}+\frac{9\tanh^{-1}(ax)}{32a^2(1-a^2x^2)}+\frac{3}{32a}$$

```
[Out] (-3*x)/(128*a*(1 - a^2*x^2)^2) - (45*x)/(256*a*(1 - a^2*x^2)) - (45*ArcTanh
[a*x])/(256*a^2) + (3*ArcTanh[a*x])/(32*a^2*(1 - a^2*x^2)^2) + (9*ArcTanh[a
*x])/(32*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) -
 (9*x*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32*a^2) +
ArcTanh[a*x]^3/(4*a^2*(1 - a^2*x^2)^2)
```

Rubi [A] time = 0.163647, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5994, 5964, 5956, 199, 206}

$$-\frac{45x}{256a(1-a^2x^2)}-\frac{3x}{128a(1-a^2x^2)^2}+\frac{\tanh^{-1}(ax)^3}{4a^2(1-a^2x^2)^2}-\frac{9x\tanh^{-1}(ax)^2}{32a(1-a^2x^2)}-\frac{3x\tanh^{-1}(ax)^2}{16a(1-a^2x^2)^2}+\frac{9\tanh^{-1}(ax)}{32a^2(1-a^2x^2)}+\frac{3}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```


```
[Out] (-3*x)/(128*a*(1 - a^2*x^2)^2) - (45*x)/(256*a*(1 - a^2*x^2)) - (45*ArcTanh
[a*x])/(256*a^2) + (3*ArcTanh[a*x])/(32*a^2*(1 - a^2*x^2)^2) + (9*ArcTanh[a
*x])/(32*a^2*(1 - a^2*x^2)) - (3*x*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) -
 (9*x*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) - (3*ArcTanh[a*x]^3)/(32*a^2) +
ArcTanh[a*x]^3/(4*a^2*(1 - a^2*x^2)^2)
```


Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = \frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx}{4a}$$

$$= \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} + \frac{\tanh^{-1}(ax)^3}{4a^2\left(1-a^2x^2\right)^2} - \frac{3\int \frac{1}{\left(1-a^2x^2\right)^3}\,dx}{32a} - \frac{9\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^2}\,dx}{16a}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} - \frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} - \frac{9x\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)} - \frac{3\tanh^{-1}(ax)^3}{32a^2} + \frac{\text{t}}{4a}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} - \frac{9x}{256a\left(1-a^2x^2\right)} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} + \frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)} - \frac{3x\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2} - \frac{9}{}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} - \frac{45x}{256a\left(1-a^2x^2\right)} - \frac{9\tanh^{-1}(ax)}{256a^2} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} + \frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)} - \frac{3x}{16a}$$

$$= -\frac{3x}{128a\left(1-a^2x^2\right)^2} - \frac{45x}{256a\left(1-a^2x^2\right)} - \frac{45\tanh^{-1}(ax)}{256a^2} + \frac{3\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)^2} + \frac{9\tanh^{-1}(ax)}{32a^2\left(1-a^2x^2\right)} - \frac{3}{16}$$

Mathematica [A] time = 0.0864559, size = 148, normalized size = 0.79

$$\frac{90a^3x^3 - 45a^4x^4\log(ax+1) + 90a^2x^2\log(ax+1) + 45\left(a^2x^2-1\right)^2\log(1-ax) + 48ax\left(3a^2x^2-5\right)\tanh^{-1}(ax)^2 + \left(-48a^4\right.}{512a^2\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^3,x]
```

```
[Out] (-102*a*x + 90*a^3*x^3 - 48*(-4 + 3*a^2*x^2)*ArcTanh[a*x] + 48*a*x*(-5 + 3*
a^2*x^2)*ArcTanh[a*x]^2 + (80 + 96*a^2*x^2 - 48*a^4*x^4)*ArcTanh[a*x]^3 + 4
5*(-1 + a^2*x^2)^2*Log[1 - a*x] - 45*Log[1 + a*x] + 90*a^2*x^2*Log[1 + a*x]
 - 45*a^4*x^4*Log[1 + a*x])/(512*a^2*(-1 + a^2*x^2)^2)
```

Maple [C] time = 0.348, size = 2582, normalized size = 13.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] 1/4/a^2/(a^2*x^2-1)^2*arctanh(a*x)^3-3/64/a^2*arctanh(a*x)^2/(a*x-1)^2+9/64
/a^2*arctanh(a*x)^2/(a*x-1)+9/64/a^2*arctanh(a*x)^2*ln(a*x-1)+3/64/a^2*arct
anh(a*x)^2/(a*x+1)^2+9/64/a^2*arctanh(a*x)^2/(a*x+1)-9/64/a^2*arctanh(a*x)^
2*ln(a*x+1)+9/32/a^2*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-9/64*I/a
^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
^3+9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-9/6
4*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^2-9/32*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*
x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*
x^2+9/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2-9/64*I/(
a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2+9/128*I*a^2/(a*
x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^4+9/12
8*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/
((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4-9/64*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*
x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4+9/64*I*a^2/(a*x-1)^2/(a*x+
1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4+9/128*I/a^2
/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2
*csgn(I*(a*x+1)^2/(a^2*x^2-1))+9/64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*
x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-9/1
28*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-9/128*I/a^2/(a
*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2
+1)+1))+9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2
/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^
2*x^4+9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^
2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^4+9/64*I*a^2/(a*x-1)^2/(a
*x+1)^2*arctanh(a*x)^2*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)
^2/(a^2*x^2-1))^2*x^4-9/128*I*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csg
n(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^
2+1)+1))^2*x^4+9/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)
^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*x^2-9/64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arcta
nh(a*x)^2+9/32*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*x^2-9/128*I*a^2/(a*x
-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1
)+1))*x^4-3/32*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^4-45/256*a^2/(a*x-1
)^2/(a*x+1)^2*arctanh(a*x)*x^4+9/64*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*
x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-9/64*I*a^2/(a*x-1)^2/(a*x+1)^2*ar
ctanh(a*x)^2*Pi*x^4+45/256*a/(a*x-1)^2/(a*x+1)^2*x^3-51/256/a/(a*x-1)^2/(a*
x+1)^2*x-3/32/a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3+51/256/a^2/(a*x-1)^2/(
a*x+1)^2*arctanh(a*x)+3/16/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*x^2+9/128/(a*
x-1)^2/(a*x+1)^2*arctanh(a*x)*x^2-9/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2
*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2-9/64*I/(a*x-1)^2/(a*x+1)^2*arctanh(
a*x)^2*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^2+9/
32*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1
))^3*x^2-9/32*I/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*Pi*csgn(I/((a*x+1)^2/(-a
^2*x^2+1)+1))^2*x^2+9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^2*csgn(
```

```
I*(a*x+1)^2/(a^2*x^2-1))^3+9/128*I/a^2/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3
```

Maxima [B] time = 1.01383, size = 570, normalized size = 3.03

$$\frac{3\left(\frac{2\left(3a^2x^3-5x\right)}{a^4x^4-2a^2x^2+1}-\frac{3\log(ax+1)}{a}+\frac{3\log(ax-1)}{a}\right)\operatorname{artanh}(ax)^2}{64a}+\frac{3\left(\frac{\left(30a^3x^3-2\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^3+6\left(a^4x^4-2a^2x^2+1\right)\log(ax+1)^2\log(ax-\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 3/64*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3*
log(a*x - 1)/a)*arctanh(a*x)^2/a + 3/512*((30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*
x^2 + 1)*log(a*x + 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*
x - 1) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4
 - 10*a^2*x^2 + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1
) + 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*a^2/(a^7*x^4 - 2*a^5*x^2 + a
^3) - 4*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)
*log(a*x - 1)^2 - 16)*a*arctanh(a*x)/(a^6*x^4 - 2*a^4*x^2 + a^2))/a + 1/4*a
rctanh(a*x)^3/((a^2*x^2 - 1)^2*a^2)
```

Fricas [A] time = 2.05961, size = 312, normalized size = 1.66

$$\frac{90a^3x^3-2\left(3a^4x^4-6a^2x^2-5\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+12\left(3a^3x^3-5ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-102ax-3\left(15a^4x^4-6a^2x^2-17\right)\log\left(\right.}{512\left(a^6x^4-2a^4x^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(90*a^3*x^3 - 2*(3*a^4*x^4 - 6*a^2*x^2 - 5)*log(-(a*x + 1)/(a*x - 1))
^3 + 12*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 102*a*x - 3*(15*a
^4*x^4 - 6*a^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1)))/(a^6*x^4 - 2*a^4*x^2 +
a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x\operatorname{atanh}^3(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(x*atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x \operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-x*arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

3.317 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=203

$$-\frac{45}{128a(1-a^2x^2)}-\frac{3}{128a(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^3}{8(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{4(1-a^2x^2)^2}-\frac{9\tanh^{-1}(ax)^2}{16a(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{16a(1-a^2x^2)^2}+\frac{45x\tanh}{64(1-}$$

```
[Out] -3/(128*a*(1 - a^2*x^2)^2) - 45/(128*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/
(32*(1 - a^2*x^2)^2) + (45*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) + (45*ArcTanh
[a*x]^2)/(128*a) - (3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) - (9*ArcTanh[a
*x]^2)/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (3*x
*ArcTanh[a*x]^3)/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^4)/(32*a)
```

Rubi [A] time = 0.177304, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 261, 5960}

$$-\frac{45}{128a(1-a^2x^2)}-\frac{3}{128a(1-a^2x^2)^2}+\frac{3x\tanh^{-1}(ax)^3}{8(1-a^2x^2)}+\frac{x\tanh^{-1}(ax)^3}{4(1-a^2x^2)^2}-\frac{9\tanh^{-1}(ax)^2}{16a(1-a^2x^2)}-\frac{3\tanh^{-1}(ax)^2}{16a(1-a^2x^2)^2}+\frac{45x\tanh}{64(1-}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```


```
[Out] -3/(128*a*(1 - a^2*x^2)^2) - 45/(128*a*(1 - a^2*x^2)) + (3*x*ArcTanh[a*x])/
(32*(1 - a^2*x^2)^2) + (45*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) + (45*ArcTanh
[a*x]^2)/(128*a) - (3*ArcTanh[a*x]^2)/(16*a*(1 - a^2*x^2)^2) - (9*ArcTanh[a
*x]^2)/(16*a*(1 - a^2*x^2)) + (x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (3*x
*ArcTanh[a*x]^3)/(8*(1 - a^2*x^2)) + (3*ArcTanh[a*x]^4)/(32*a)
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx = -\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3}{8}\int\frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx+\frac{3}{4}\int\frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)^3}{8\left(1-a^2x^2\right)}+ \frac{3\text{ t}}{}$$

$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{9x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{9\tanh^{-1}(ax)^2}{128a}-\frac{3\tanh^{-1}(ax)^2}{16a\left(1-a^2x^2\right)^2}-\frac{9\text{ ta}}{16a}$$

$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}-\frac{9}{128a\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{45x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{45\tanh^{-1}(ax)^2}{128a}-$$

$$= -\frac{3}{128a\left(1-a^2x^2\right)^2}-\frac{45}{128a\left(1-a^2x^2\right)}+\frac{3x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{45x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{45\tanh^{-1}(ax)^2}{128a}-$$

Mathematica [A] time = 0.153703, size = 111, normalized size = 0.55

$$\frac{45a^2x^2+12\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^4+\left(80ax-48a^3x^3\right)\tanh^{-1}(ax)^3+3\left(15a^4x^4-6a^2x^2-17\right)\tanh^{-1}(ax)^2+\left(102ax\right.}{128a\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^3,x]
```

```
[Out] (-48 + 45*a^2*x^2 + (102*a*x - 90*a^3*x^3)*ArcTanh[a*x] + 3*(-17 - 6*a^2*x^
2 + 15*a^4*x^4)*ArcTanh[a*x]^2 + (80*a*x - 48*a^3*x^3)*ArcTanh[a*x]^3 + 12*
(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)/(128*a*(-1 + a^2*x^2)^2)
```

Maple [C] time = 0.452, size = 2646, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^3,x)
```

```
[Out] 3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1
)+1))^2*x^2-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
^2+3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2
-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-3/16*I/a/(a
*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn
(I*(a*x+1)^2/(a^2*x^2-1))^2-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*
csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))+3/16*I*a
^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
^3*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a
^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*
arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^4-3/16*I*a^3/(a*x-1)^2/
(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4-3/8*I*
a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^
3*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*
x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arcta
nh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*x^2+3/32*I/a/(a*x-1)^2/(a*x+1)
^2*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a
^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-3/32*I*
a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1)
)*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^4+3/32*I*a^3
/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^4-3/16*I*a^3/(a*x-1)
^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+
1)/(-a^2*x^2+1)^(1/2))*x^4-3/32*I*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi
*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^4+3/1
6*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*x^2-3/16*I*a
/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*x^2+3/8*I*a/(a*x-1)^2/
(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/
(-a^2*x^2+1)^(1/2))*x^2+3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn
(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*x^2-195/1024
/a/(a*x-1)^2/(a*x+1)^2+3/16/a*arctanh(a*x)^3*ln(a*x+1)-3/8/a*arctanh(a*x)^3
*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+1/16/a*arctanh(a*x)^3/(a*x-1)^2-3/16/a*arct
anh(a*x)^3/(a*x-1)-3/16/a*arctanh(a*x)^3*ln(a*x-1)-1/16/a*arctanh(a*x)^3/(a
*x+1)^2-3/16/a*arctanh(a*x)^3/(a*x+1)+3/32*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(
a*x)^4*x^4+45/128*a^3/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^4-3/16*a/(a*x-1)
^2/(a*x+1)^2*arctanh(a*x)^4*x^2-45/64*a^2/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*
x^3-9/64*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2*x^2+3/16*I*a^3/(a*x-1)^2/(a*x
+1)^2*arctanh(a*x)^3*Pi*x^4-3/8*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*x
^2+3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x
^2+1)+1))^3-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2
/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-3/32*I/a/(a*x-1)^2/(a*x+1)^2*Pi*
arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-3/16*I/a/(a*x-1)^2/(a*x+1)^2
*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2+3/32*I*a^3/(a*x-1)^
2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*
x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)
)*x^4-3/16*I*a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*x^2+189/1024*a^3/(a*x-1)^2/(a*x+1)^2*x^4-9/512*a/(
a*x-1)^2/(a*x+1)^2*x^2+51/64/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)*x+3/32/a/(a*x
-1)^2/(a*x+1)^2*arctanh(a*x)^4-51/128/a/(a*x-1)^2/(a*x+1)^2*arctanh(a*x)^2+
3/16*I/a/(a*x-1)^2/(a*x+1)^2*Pi*arctanh(a*x)^3
```

Maxima [B] time = 1.06578, size = 895, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(3*a^2*x^3 - 5*x)/(a^4*x^4 - 2*a^2*x^2 + 1) - 3*log(a*x + 1)/a + 3
*log(a*x - 1)/a)*arctanh(a*x)^3 + 3/64*(12*a^2*x^2 - 3*(a^4*x^4 - 2*a^2*x^2
 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1
) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 - 16)*a*arctanh(a*x)^2/(a^6*
x^4 - 2*a^4*x^2 + a^2) - 3/512*(((a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^4 -
 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^3*log(a*x - 1) + (a^4*x^4 - 2*a^2
*x^2 + 1)*log(a*x - 1)^4 - 60*a^2*x^2 + 3*(5*a^4*x^4 - 10*a^2*x^2 + 2*(a^4*
x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1)^2 + 15*(a^4*x^4 - 2*a
^2*x^2 + 1)*log(a*x - 1)^2 - 2*(2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3
+ 15*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + 1) + 64)*a^2/(a^8*x^
4 - 2*a^6*x^2 + a^4) + 4*(30*a^3*x^3 - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x
+ 1)^3 + 6*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2*log(a*x - 1) + 2*(a^4*x
^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^3 - 34*a*x - 3*(5*a^4*x^4 - 10*a^2*x^2 + 2
*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1)^2 + 5)*log(a*x + 1) + 15*(a^4*x^4 -
 2*a^2*x^2 + 1)*log(a*x - 1))*a*arctanh(a*x)/(a^7*x^4 - 2*a^5*x^2 + a^3))*a
```

Fricas [A] time = 2.12718, size = 375, normalized size = 1.85

$$\frac{3\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4 + 180\,a^2x^2 - 8\left(3\,a^3x^3 - 5\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 3\left(15\,a^4x^4 - 6\,a^2x^2 - 17\right)\log\left(-\frac{ax+1}{ax-1}\right.}{512\left(a^5x^4 - 2\,a^3x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^4 + 180*a^2*x^
2 - 8*(3*a^3*x^3 - 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 3*(15*a^4*x^4 - 6*a
^2*x^2 - 17)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^3*x^3 - 17*a*x)*log(-(a
*x + 1)/(a*x - 1)) - 192)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}^3(ax)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2 - 1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/(a^2*x^2 - 1)^3, x)
```

3.318 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=277

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\frac{}{256}$$

```
[Out] (-3*a*x)/(128*(1 - a^2*x^2)^2) - (141*a*x)/(256*(1 - a^2*x^2)) - (141*ArcTa
nh[a*x])/256 + (3*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) + (33*ArcTanh[a*x])/(3
2*(1 - a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (33*a*x*Ar
cTanh[a*x]^2)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^3)/32 + ArcTanh[a*x]^3/
(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 +
 ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/
(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLo
g[4, -1 + 2/(1 + a*x)])/4
```

Rubi [A] time = 0.626081, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {6030, 5988, 5932, 5948, 6056, 6060, 6610, 5994, 5956, 199, 206, 5964}

$$-\frac{3}{4}\text{PolyLog}\left(4,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{3}{2}\tanh^{-1}(ax)\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-\frac{}{256}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```


```
[Out] (-3*a*x)/(128*(1 - a^2*x^2)^2) - (141*a*x)/(256*(1 - a^2*x^2)) - (141*ArcTa
nh[a*x])/256 + (3*ArcTanh[a*x])/(32*(1 - a^2*x^2)^2) + (33*ArcTanh[a*x])/(3
2*(1 - a^2*x^2)) - (3*a*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (33*a*x*Ar
cTanh[a*x]^2)/(32*(1 - a^2*x^2)) - (11*ArcTanh[a*x]^3)/32 + ArcTanh[a*x]^3/
(4*(1 - a^2*x^2)^2) + ArcTanh[a*x]^3/(2*(1 - a^2*x^2)) + ArcTanh[a*x]^4/4 +
 ArcTanh[a*x]^3*Log[2 - 2/(1 + a*x)] - (3*ArcTanh[a*x]^2*PolyLog[2, -1 + 2/
(1 + a*x)])/2 - (3*ArcTanh[a*x]*PolyLog[3, -1 + 2/(1 + a*x)])/2 - (3*PolyLo
g[4, -1 + 2/(1 + a*x)])/4
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
```

```
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6060

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(
2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
 u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^2}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}-\frac{1}{4}(3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx+a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)}\,dx$$

$$= \frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{\tanh^{-1}(ax)^3}{2\left(1-a^2x^2\right)}+\frac{1}{4}\tanh^{-1}(ax)^4-\frac{1}{32}(3a)\int$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2}+\frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}-\frac{33ax\tanh^{-1}(ax)^2}{32\left(1-a^2x^2\right)}-\frac{11}{32}\tanh^{-1}(ax)^3+$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2}-\frac{9ax}{256\left(1-a^2x^2\right)}+\frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}-\frac{3ax\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}-\frac{33a}{3}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2}-\frac{141ax}{256\left(1-a^2x^2\right)}-\frac{9}{256}\tanh^{-1}(ax)+\frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}-\frac{3ax}{16}$$

$$= -\frac{3ax}{128\left(1-a^2x^2\right)^2}-\frac{141ax}{256\left(1-a^2x^2\right)}-\frac{141}{256}\tanh^{-1}(ax)+\frac{3\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{33\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)}-\frac{3ax}{16}$$

Mathematica [A] time = 0.28276, size = 189, normalized size = 0.68

$$1536\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)-1536\tanh^{-1}(ax)\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)+768\text{PolyLog}\left(4,e^{2\tanh^{-1}(ax}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^3),x]
```

```
[Out] (16*Pi^4 - 256*ArcTanh[a*x]^4 + 576*ArcTanh[a*x]*Cosh[2*ArcTanh[a*x]] + 384
*ArcTanh[a*x]^3*Cosh[2*ArcTanh[a*x]] + 12*ArcTanh[a*x]*Cosh[4*ArcTanh[a*x]]
 + 32*ArcTanh[a*x]^3*Cosh[4*ArcTanh[a*x]] + 1024*ArcTanh[a*x]^3*Log[1 - E^(
2*ArcTanh[a*x])] + 1536*ArcTanh[a*x]^2*PolyLog[2, E^(2*ArcTanh[a*x])] - 153
6*ArcTanh[a*x]*PolyLog[3, E^(2*ArcTanh[a*x])] + 768*PolyLog[4, E^(2*ArcTanh
[a*x])] - 288*Sinh[2*ArcTanh[a*x]] - 576*ArcTanh[a*x]^2*Sinh[2*ArcTanh[a*x]
] - 3*Sinh[4*ArcTanh[a*x]] - 24*ArcTanh[a*x]^2*Sinh[4*ArcTanh[a*x]])/1024
```

Maple [C] time = 0.477, size = 1533, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x)
```

```
[Out] 3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*po
lylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2
*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh
(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))-arctanh(a*x)^3*ln((a*x+1)^2/(-a^2*x^2+1)-1)+ln(2)*arctanh(a*x
)^3+arctanh(a*x)^3*ln(a*x)-9/32*(a*x+1)*arctanh(a*x)/(a*x-1)-9/32*(a*x-1)*a
rctanh(a*x)/(a*x+1)+9/32*(a*x+1)*arctanh(a*x)^2/(a*x-1)+1/2*I*Pi*arctanh(a*
x)^3+3/512*arctanh(a*x)*(a*x+1)^2/(a*x-1)^2-9/64*(a*x-1)/(a*x+1)+9/64*(a*x+
1)/(a*x-1)+3/2048*(a*x-1)^2/(a*x+1)^2-3/256*arctanh(a*x)^2*(a*x+1)^2/(a*x-1
)^2+3/256*arctanh(a*x)^2*(a*x-1)^2/(a*x+1)^2-3/2048*(a*x+1)^2/(a*x-1)^2-1/4
*arctanh(a*x)^4-11/32*arctanh(a*x)^3+1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1
)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/2*I*Pi*csgn(I/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*arctanh(a*x)^3+1/2*I*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)
+1))^3*arctanh(a*x)^3+1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))
^3-9/32*arctanh(a*x)^2*(a*x-1)/(a*x+1)+6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1
/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))+1/4*I*Pi*arctanh(a*x)^3*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))-1/2*I*arctanh(
a*x)^3*Pi*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)
-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2-1/4*I*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/2
*I*arctanh(a*x)^3*Pi*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a
^2*x^2-1))^2+1/4*I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/16*arctanh(a*x)^3
/(a*x-1)^2+1/16*arctanh(a*x)^3/(a*x+1)^2+3/512*(a*x-1)^2*arctanh(a*x)/(a*x+
1)^2-5/16*arctanh(a*x)^3/(a*x-1)-1/2*arctanh(a*x)^3*ln(a*x-1)+5/16*arctanh(
a*x)^3/(a*x+1)-1/2*arctanh(a*x)^3*ln(a*x+1)+arctanh(a*x)^3*ln((a*x+1)/(-a^2
*x^2+1)^(1/2))-1/2*I*Pi*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*c
sgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+1/4*I*Pi*arc
tanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3-1/4*
I*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^
2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))+1/2*I*Pi
*arctanh(a*x)^3*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1))*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))*csgn(I*((a*x+1)^2/(-a^2*x^2+1)-1)/((a*x+1)^2/(-a^2*x^2+1)+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(-ax + 1\right)^4 + 2\left(2\,a^2x^2 + 2\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log\left(ax + 1\right) - 3\right)\log\left(-ax + 1\right)^3}{64\left(a^4x^4 - 2\,a^2x^2 + 1\right)} - \frac{1}{8}\int \frac{4\log\left(ax + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/64*((a^4*x^4 - 2*a^2*x^2 + 1)*log(-a*x + 1)^4 + 2*(2*a^2*x^2 + 2*(a^4*x^4
 - 2*a^2*x^2 + 1)*log(a*x + 1) - 3)*log(-a*x + 1)^3)/(a^4*x^4 - 2*a^2*x^2 +
 1) - 1/8*integrate(1/4*(4*log(a*x + 1)^3 - 12*log(a*x + 1)^2*log(-a*x + 1)
 + 3*(2*a^4*x^4 + 2*a^3*x^3 - 3*a^2*x^2 - 3*a*x + 2*(a^6*x^6 + a^5*x^5 - 2*
a^4*x^4 - 2*a^3*x^3 + a^2*x^2 + a*x + 2)*log(a*x + 1))*log(-a*x + 1)^2)/(a^
```

```
6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}\,(ax)^3}{a^6x^7 - 3\,a^4x^5 + 3\,a^2x^3 - x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\text{atanh}^3\,(ax)}{a^6x^7 - 3a^4x^5 + 3a^2x^3 - x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**7 - 3*a**4*x**5 + 3*a**2*x**3 - x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\text{artanh}\,(ax)^3}{\left(a^2x^2 - 1\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x), x)
```

3.319 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=281

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{93a}{128(1-a^2x^2)}-\frac{3a}{128(1-a^2x^2)^2}+\frac{7a^2x\tanh^{-1}}{8(1-a^2}$$

```
[Out] (-3*a)/(128*(1 - a^2*x^2)^2) - (93*a)/(128*(1 - a^2*x^2)) + (3*a^2*x*ArcTan
h[a*x])/(32*(1 - a^2*x^2)^2) + (93*a^2*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) +
 (93*a*ArcTanh[a*x]^2)/128 - (3*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (2
1*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/
x + (a^2*x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^3)/(
8*(1 - a^2*x^2)) + (15*a*ArcTanh[a*x]^4)/32 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/
(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3
, -1 + 2/(1 + a*x)])/2
```

Rubi [A] time = 0.690237, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {6030, 5982, 5916, 5988, 5932, 5948, 6056, 6610, 5956, 5994, 261, 5964, 5960}

$$-\frac{3}{2}a\text{PolyLog}\left(3,\frac{2}{ax+1}-1\right)-3a\tanh^{-1}(ax)\text{PolyLog}\left(2,\frac{2}{ax+1}-1\right)-\frac{93a}{128(1-a^2x^2)}-\frac{3a}{128(1-a^2x^2)^2}+\frac{7a^2x\tanh^{-1}}{8(1-a^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```


```
[Out] (-3*a)/(128*(1 - a^2*x^2)^2) - (93*a)/(128*(1 - a^2*x^2)) + (3*a^2*x*ArcTan
h[a*x])/(32*(1 - a^2*x^2)^2) + (93*a^2*x*ArcTanh[a*x])/(64*(1 - a^2*x^2)) +
 (93*a*ArcTanh[a*x]^2)/128 - (3*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)^2) - (2
1*a*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + a*ArcTanh[a*x]^3 - ArcTanh[a*x]^3/
x + (a^2*x*ArcTanh[a*x]^3)/(4*(1 - a^2*x^2)^2) + (7*a^2*x*ArcTanh[a*x]^3)/(
8*(1 - a^2*x^2)) + (15*a*ArcTanh[a*x]^4)/32 + 3*a*ArcTanh[a*x]^2*Log[2 - 2/
(1 + a*x)] - 3*a*ArcTanh[a*x]*PolyLog[2, -1 + 2/(1 + a*x)] - (3*a*PolyLog[3
, -1 + 2/(1 + a*x)])/2
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6056

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x
] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e
, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
```

```
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^3}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx+\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^2}\,dx$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{1}{8}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^3}\,dx+\frac{1}{4}\left(3a^2\right)\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^2}\,dx+$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}-\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}+\frac{a^2x\tanh^{-1}(ax)^3}{4\left(1-a^2x^2\right)^2}+\frac{7a^2x\tanh^{-1}(ax)^3}{8\left(1-a^2x^2\right)}+$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{9a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{9}{128}a\tanh^{-1}(ax)^2-\frac{3a\tanh^{-1}(ax)^2}{16\left(1-a^2x^2\right)^2}-$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2}-\frac{9a}{128\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{93}{128}a\tanh^{-1}(ax)^2-$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2}-\frac{93a}{128\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{93}{128}a\tanh^{-1}(ax)^2-$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2}-\frac{93a}{128\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{93}{128}a\tanh^{-1}(ax)^2-$$

$$= -\frac{3a}{128\left(1-a^2x^2\right)^2}-\frac{93a}{128\left(1-a^2x^2\right)}+\frac{3a^2x\tanh^{-1}(ax)}{32\left(1-a^2x^2\right)^2}+\frac{93a^2x\tanh^{-1}(ax)}{64\left(1-a^2x^2\right)}+\frac{93}{128}a\tanh^{-1}(ax)^2-$$

Mathematica [C] time = 0.690842, size = 218, normalized size = 0.78

$$-a\left(-3\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{2\tanh^{-1}(ax)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2\tanh^{-1}(ax)}\right)-\frac{ax\tanh^{-1}(ax)^3}{1-a^2x^2}-\frac{15}{32}\tanh^{-1}(ax)^4+\frac{\tanh^{-1}}{ax}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^3),x]
```

```
[Out] -(a*((-I/8)*Pi^3 + ArcTanh[a*x]^3 + ArcTanh[a*x]^3/(a*x) - (a*x*ArcTanh[a*x
]^3)/(1 - a^2*x^2) - (15*ArcTanh[a*x]^4)/32 + (3*Cosh[2*ArcTanh[a*x]])/8 +
(3*ArcTanh[a*x]^2*Cosh[2*ArcTanh[a*x]])/4 + (3*Cosh[4*ArcTanh[a*x]])/1024 +
 (3*ArcTanh[a*x]^2*Cosh[4*ArcTanh[a*x]])/128 - 3*ArcTanh[a*x]^2*Log[1 - E^(
2*ArcTanh[a*x])] - 3*ArcTanh[a*x]*PolyLog[2, E^(2*ArcTanh[a*x])] + (3*PolyL
og[3, E^(2*ArcTanh[a*x])])/2 - (3*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/4 - (3
*ArcTanh[a*x]*Sinh[4*ArcTanh[a*x]])/256 - (ArcTanh[a*x]^3*Sinh[4*ArcTanh[a*
x]])/32))
```

Maple [B] time = 0.714, size = 842, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x)
```

```
[Out] -arctanh(a*x)^3/x-6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,
(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1
)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+3/512*a*arc
tanh(a*x)/(a*x-1)^2-3/512*a*arctanh(a*x)/(a*x+1)^2-1/64*a/(a*x+1)^2*arctanh
(a*x)^3+1/64*a/(a*x-1)^2*arctanh(a*x)^3+3/16/(a*x+1)*a^2*x+3/16*a^2*x/(a*x-
1)+3/8*a*arctanh(a*x)^2/(a*x-1)-3/8*a*arctanh(a*x)^2/(a*x+1)-3/8*arctanh(a*
x)/(a*x-1)*a^2*x+3/8*arctanh(a*x)/(a*x+1)*a^2*x-3/2048*a/(a*x-1)^2-3/2048*a
/(a*x+1)^2+3*a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(
a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*a/(a*x-1)*arctanh(a*x)^3-1/64/(
a*x+1)^2*arctanh(a*x)^3*x^2*a^3-3/256/(a*x+1)^2*arctanh(a*x)^2*x^2*a^3+1/32
/(a*x+1)^2*arctanh(a*x)^3*x*a^2+3/128/(a*x+1)^2*arctanh(a*x)^2*x*a^2+1/64/(
a*x-1)^2*arctanh(a*x)^3*x^2*a^3-3/256/(a*x-1)^2*arctanh(a*x)^2*x^2*a^3+1/32
/(a*x-1)^2*arctanh(a*x)^3*x*a^2-3/128/(a*x-1)^2*arctanh(a*x)^2*x*a^2+3/16*a
/(a*x-1)-3/16*a/(a*x+1)-a*arctanh(a*x)^3+15/32*a*arctanh(a*x)^4-1/4*a/(a*x+
1)*arctanh(a*x)^3+3/512*arctanh(a*x)/(a*x-1)^2*a^3*x^2+3/256*arctanh(a*x)/(
a*x-1)^2*a^2*x-3/512*arctanh(a*x)/(a*x+1)^2*a^3*x^2+3/256*arctanh(a*x)/(a*x
+1)^2*a^2*x-3/2048/(a*x+1)^2*a^3*x^2+3/1024/(a*x+1)^2*a^2*x-3/2048/(a*x-1)^
2*a^3*x^2-3/1024/(a*x-1)^2*a^2*x-3/256*a*arctanh(a*x)^2/(a*x-1)^2-3/256*a*a
rctanh(a*x)^2/(a*x+1)^2-1/4/(a*x-1)*arctanh(a*x)^3*x*a^2+3/8/(a*x-1)*arctan
h(a*x)^2*x*a^2+1/4/(a*x+1)*arctanh(a*x)^3*x*a^2+3/8/(a*x+1)*arctanh(a*x)^2*
x*a^2-3/8*a*arctanh(a*x)/(a*x-1)-3/8*a*arctanh(a*x)/(a*x+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\operatorname{artanh}(ax)^3}{a^6x^8 - 3\,a^4x^6 + 3\,a^2x^4 - x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] integral(-arctanh(a*x)^3/(a^6*x^8 - 3*a^4*x^6 + 3*a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atanh}^{3}(ax)}{a^{6}x^{8}-3a^{4}x^{6}+3a^{2}x^{4}-x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(atanh(a*x)**3/(a**6*x**8 - 3*a**4*x**6 + 3*a**2*x**4 - x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\operatorname{artanh}(ax)^{3}}{\left(a^{2}x^{2}-1\right)^{3}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-arctanh(a*x)^3/((a^2*x^2 - 1)^3*x^2), x)
```

3.320 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^3}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\cdots$$

```
[Out] ArcTanh[a*x]^(3/2)/(4*a) + (Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(256*a) + (
Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi]*Erfi[2*Sqrt[
ArcTanh[a*x]]])/(256*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16
*a) + (Sqrt[ArcTanh[a*x]]*Sinh[2*ArcTanh[a*x]])/(4*a) + (Sqrt[ArcTanh[a*x]]
*Sinh[4*ArcTanh[a*x]])/(32*a)
```

Rubi [A] time = 0.195244, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5968, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,\text{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\text{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}-\frac{\sqrt{\frac{\pi}{2}}\,\text{Erfi}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```


```
[Out] ArcTanh[a*x]^(3/2)/(4*a) + (Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(256*a) + (
Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16*a) - (Sqrt[Pi]*Erfi[2*Sqrt[
ArcTanh[a*x]]])/(256*a) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(16
*a) + (Sqrt[ArcTanh[a*x]]*Sinh[2*ArcTanh[a*x]])/(4*a) + (Sqrt[ArcTanh[a*x]]
*Sinh[4*ArcTanh[a*x]])/(32*a)
```


Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^3}\,dx &= \frac{\text{Subst}\left(\int \sqrt{x}\cosh^4(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3\sqrt{x}}{8}+\frac{1}{2}\sqrt{x}\cosh(2x)+\frac{1}{8}\sqrt{x}\cosh(4x)\right)dx, x, \tanh^{-1}(ax)\right)}{a}\\
&= \frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\text{Subst}\left(\int \sqrt{x}\cosh(4x)\,dx, x, \tanh^{-1}(ax)\right)}{8a}+\frac{\text{Subst}\left(\int \sqrt{x}\cosh(2x)\,dx, x, \tanh^{-1}\right.}{2a}\\
&= \frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{32a}-\frac{\text{Subs}}{}\\
&= \frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{32a}+\frac{\text{Subs}}{}\\
&= \frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{4a}+\frac{\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{32a}+\frac{\text{Subs}}{}\\
&= \frac{\tanh^{-1}(ax)^{3/2}}{4a}+\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{16a}-\frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right.}{256a}
\end{aligned}
$$

Mathematica [A] time = 0.48492, size = 152, normalized size = 0.9

$$
\frac{\frac{\sqrt{\tanh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4\tanh^{-1}(ax)\right)}{\sqrt{-\tanh^{-1}(ax)}}+\frac{8\sqrt{2}\sqrt{\tanh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2\tanh^{-1}(ax)\right)}{\sqrt{-\tanh^{-1}(ax)}}-8\sqrt{2}\text{Gamma}\left(\frac{1}{2},2\tanh^{-1}(ax)\right)-\text{Gamma}\left(\frac{1}{2},4\right.}{256a}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^3,x]
```

```
[Out] ((32*Sqrt[ArcTanh[a*x]]*(5*a*x - 3*a^3*x^3 + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x
]))/(-1 + a^2*x^2)^2 + (Sqrt[ArcTanh[a*x]]*Gamma[1/2, -4*ArcTanh[a*x]])/Sqr
t[-ArcTanh[a*x]] + (8*Sqrt[2]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -2*ArcTanh[a*x]
```

```
])/Sqrt[-ArcTanh[a*x]] - 8*Sqrt[2]*Gamma[1/2, 2*ArcTanh[a*x]] - Gamma[1/2,
4*ArcTanh[a*x]])/(256*a)
```

Maple [F] time = 0.469, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right)^{3}} \sqrt{\operatorname{Artanh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{\operatorname{artanh}\left(a x\right)}}{\left(a^{2} x^{2}-1\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{\operatorname{atanh}\left(a x\right)}}{a^{6} x^{6} - 3 a^{4} x^{4} + 3 a^{2} x^{2} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**3,x)
```

```
[Out] -Integral(sqrt(atanh(a*x))/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(-sqrt(arctanh(a*x))/(a^2*x^2 - 1)^3, x)
```

3.321 $\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0688041, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Mathematica [A] time = 7.70756, size = 0, normalized size = 0.

$$\int \frac{x^6}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^6/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Maple [A] time = 0.196, size = 0, normalized size = 0.

$$\int \frac{x^6}{(-a^2x^2+1)^3 \operatorname{Artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^6}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^6}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-x^6/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^6}{a^6x^6\operatorname{atanh}(ax)-3a^4x^4\operatorname{atanh}(ax)+3a^2x^2\operatorname{atanh}(ax)-\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**6/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^6}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^6/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.322 $\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0688522, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx = \int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Mathematica [A] time = 14.6586, size = 0, normalized size = 0.

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int \frac{x^5}{(-a^2x^2+1)^3 \operatorname{Artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^5}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**5/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^5}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.323 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=41

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(2*a^5) + CoshIntegral[4*ArcTanh[a*x]]/(8*a^5
) + (3*Log[ArcTanh[a*x]])/(8*a^5)
```

Rubi [A] time = 0.121805, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 3312, 3301}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(2*a^5) + CoshIntegral[4*ArcTanh[a*x]]/(8*a^5
) + (3*Log[ArcTanh[a*x]])/(8*a^5)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x} - \frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^5}$$
$$= -\frac{\mathbf{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^5} + \frac{\mathbf{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^5} + \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Mathematica [A] time = 0.119849, size = 31, normalized size = 0.76

$$\frac{-4\mathbf{Chi}\left(2\tanh^{-1}(ax)\right) + \mathbf{Chi}\left(4\tanh^{-1}(ax)\right) + 3\log\left(\tanh^{-1}(ax)\right)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (-4*CoshIntegral[2*ArcTanh[a*x]] + CoshIntegral[4*ArcTanh[a*x]] + 3*Log[Arc
Tanh[a*x]])/(8*a^5)
```

Maple [A] time = 0.059, size = 36, normalized size = 0.9

$$-\frac{\text{Chi}\left(2\,\text{Artanh}\left(ax\right)\right)}{2\,a^5} + \frac{\text{Chi}\left(4\,\text{Artanh}\left(ax\right)\right)}{8\,a^5} + \frac{3\,\ln\left(\text{Artanh}\left(ax\right)\right)}{8\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] -1/2*Chi(2*arctanh(a*x))/a^5+1/8*Chi(4*arctanh(a*x))/a^5+3/8*ln(arctanh(a*x
))/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{\left(a^2x^2-1\right)^3 \text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] time = 2.29574, size = 328, normalized size = 8.

$$\frac{6\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-4\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-4\,\text{log\_inte}}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a
*x + 1)) - 4*log_integral(-(a*x + 1)/(a*x - 1)) - 4*log_integral(-(a*x - 1)
/(a*x + 1)))/a^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^4}{a^6x^6\,\text{atanh}\,(ax)-3a^4x^4\,\text{atanh}\,(ax)+3a^2x^2\,\text{atanh}\,(ax)-\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^4}{\left(a^2x^2-1\right)^3\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.324 $\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4}-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4}$$

```
[Out] -SinhIntegral[2*ArcTanh[a*x]]/(4*a^4) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^4
)
```

Rubi [A] time = 0.11822, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3298}

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4}-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -SinhIntegral[2*ArcTanh[a*x]]/(4*a^4) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^4
)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a^4}$$
$$= -\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^4} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^4}$$

Mathematica [A] time = 0.111425, size = 24, normalized size = 0.83

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right) - 2\text{Shi}\left(2\tanh^{-1}(ax)\right)}{8a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (-2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^4)
```

Maple [A] time = 0.062, size = 24, normalized size = 0.8

$$\frac{1}{a^4}\left(-\frac{\text{Shi}\left(2\,\text{Artanh}\,(ax)\right)}{4} + \frac{\text{Shi}\left(4\,\text{Artanh}\,(ax)\right)}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] 1/a^4*(-1/4*Shi(2*arctanh(a*x))+1/8*Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{(a^2x^2-1)^3\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] time = 2.27262, size = 281, normalized size = 9.69

$$\frac{\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - \text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - 2\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right) + 2\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*log_integral(-(a*x + 1
)/(a*x - 1)) + 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3}{\left(a^2x^2 - 1\right)^3 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.325 $\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=27

$$\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

[Out] CoshIntegral[4*ArcTanh[a*x]]/(8*a^3) - Log[ArcTanh[a*x]]/(8*a^3)

Rubi [A] time = 0.11257, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3301}

$$\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]

[Out] CoshIntegral[4*ArcTanh[a*x]]/(8*a^3) - Log[ArcTanh[a*x]]/(8*a^3)

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^3}$$

$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^3}$$

$$= \frac{\textbf{Chi}\left(4\tanh^{-1}(ax)\right)}{8a^3} - \frac{\log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Mathematica [A] time = 0.0966547, size = 22, normalized size = 0.81

$$\frac{\textbf{Chi}\left(4\tanh^{-1}(ax)\right) - \log\left(\tanh^{-1}(ax)\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]

[Out] (CoshIntegral[4*ArcTanh[a*x]] - Log[ArcTanh[a*x]])/(8*a^3)
```

Maple [A] time = 0.066, size = 24, normalized size = 0.9

$$\frac{\text{Chi}\,(4\,\text{Artanh}\,(ax))}{8\,a^3} - \frac{\ln\,(\text{Artanh}\,(ax))}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x)

[Out] 1/8*Chi(4*arctanh(a*x))/a^3-1/8*ln(arctanh(a*x))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\left(a^2x^2-1\right)^3\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")

[Out] -integrate(x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] time = 2.34905, size = 224, normalized size = 8.3

$$-\frac{2\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - \text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/16*(2*log(log(-(a*x + 1)/(a*x - 1))) - log_integral((a^2*x^2 + 2*a*x + 1
)/(a^2*x^2 - 2*a*x + 1)) - log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*
a*x + 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{a^6x^6 \operatorname{atanh}(ax) - 3a^4x^4 \operatorname{atanh}(ax) + 3a^2x^2 \operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2
*atanh(a*x) - atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.326 $\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(4*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^2)
```

Rubi [A] time = 0.0941661, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6034, 5448, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[2*ArcTanh[a*x]]/(4*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8*a^2)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a^2}$$
$$= \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{4a^2} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Mathematica [A] time = 0.107331, size = 24, normalized size = 0.83

$$\frac{2\text{Shi}\left(2\tanh^{-1}(ax)\right)+\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] (2*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[4*ArcTanh[a*x]])/(8*a^2)
```

Maple [A] time = 0.06, size = 24, normalized size = 0.8

$$\frac{1}{a^2}\left(\frac{\text{Shi}\left(2\,\text{Artanh}\left(ax\right)\right)}{4}+\frac{\text{Shi}\left(4\,\text{Artanh}\left(ax\right)\right)}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] 1/a^2*(1/4*Shi(2*arctanh(a*x))+1/8*Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{(a^2x^2-1)^3\,\text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] time = 2.56244, size = 281, normalized size = 9.69

$$\frac{\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+2\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)-2\,\text{log\_integral}\left(-\frac{ax-1}{ax+1}\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - log_integ
ral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 2*log_integral(-(a*x + 1
)/(a*x - 1)) - 2*log_integral(-(a*x - 1)/(a*x + 1)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{a^6x^6\operatorname{atanh}(ax) - 3a^4x^4\operatorname{atanh}(ax) + 3a^2x^2\operatorname{atanh}(ax) - \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*at
anh(a*x) - atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{(a^2x^2-1)^3\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.327 $\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=41

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + CoshIntegral[4*ArcTanh[a*x]]/(8*a) + (
3*Log[ArcTanh[a*x]])/(8*a)
```

Rubi [A] time = 0.0831106, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5968, 3312, 3301}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[2*ArcTanh[a*x]]/(2*a) + CoshIntegral[4*ArcTanh[a*x]]/(8*a) + (
3*Log[ArcTanh[a*x]])/(8*a)
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}+\frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a}+\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tan\right.}{2a}$$
$$= \frac{\textbf{Chi}\left(2\tanh^{-1}(ax)\right)}{2a}+\frac{\textbf{Chi}\left(4\tanh^{-1}(ax)\right)}{8a}+\frac{3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Mathematica [A] time = 0.0690022, size = 33, normalized size = 0.8

$$-\frac{-4\textbf{Chi}\left(2\tanh^{-1}(ax)\right)-\textbf{Chi}\left(4\tanh^{-1}(ax)\right)-3\log\left(\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] -(-4*CoshIntegral[2*ArcTanh[a*x]] - CoshIntegral[4*ArcTanh[a*x]] - 3*Log[Ar
cTanh[a*x]])/(8*a)
```

Maple [A] time = 0.066, size = 36, normalized size = 0.9

$$\frac{\text{Chi}(2\,\text{Artanh}(ax))}{2a}+\frac{\text{Chi}(4\,\text{Artanh}(ax))}{8a}+\frac{3\ln(\text{Artanh}(ax))}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] 1/2*Chi(2*arctanh(a*x))/a+1/8*Chi(4*arctanh(a*x))/a+3/8*ln(arctanh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(a^2x^2-1)^3 \,\text{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

Fricas [B] time = 2.27058, size = 325, normalized size = 7.93

$$\frac{6\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)+\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+4\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+4\,\text{log\_integr}}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/16*(6*log(log(-(a*x + 1)/(a*x - 1))) + log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a
*x + 1)) + 4*log_integral(-(a*x + 1)/(a*x - 1)) + 4*log_integral(-(a*x - 1)
/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^{6}x^{6}\operatorname{atanh}(ax)-3a^{4}x^{4}\operatorname{atanh}(ax)+3a^{2}x^{2}\operatorname{atanh}(ax)-\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x) - 3*a**4*x**4*atanh(a*x) + 3*a**2*x**2*at
anh(a*x) - atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^{2}x^{2}-1\right)^{3}\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)), x)
```

3.328 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=48

$$-\text{Unintegrable}\left(\frac{1}{x\left(a^2x^2-1\right)\tanh^{-1}(ax)},x\right)+\frac{3}{4}\text{Shi}\left(2\tanh^{-1}(ax)\right)+\frac{1}{8}\text{Shi}\left(4\tanh^{-1}(ax)\right)$$

```
[Out] (3*SinhIntegral[2*ArcTanh[a*x]])/4 + SinhIntegral[4*ArcTanh[a*x]]/8 - Unint
egrable[1/(x*(-1 + a^2*x^2)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0636374, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx=\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.23197, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]), x]
```

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3\text{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{(a^6x^7-3\,a^4x^5+3\,a^2x^3-x)\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^6x^7\operatorname{atanh}(ax)-3a^4x^5\operatorname{atanh}(ax)+3a^2x^3\operatorname{atanh}(ax)-x\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x),x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x) - 3*a**4*x**5*atanh(a*x) + 3*a**2*x**3*at
anh(a*x) - x*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)), x)
```

3.329 $\int \frac{x^5}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=101

$$\frac{\text{Unintegrable}\left(\frac{1}{\tanh^{-1}(ax)}, x\right)}{a^5} - \frac{3\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^6} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^6} + \frac{2x}{a^5\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{x}{a^5\left(1-a^2x^2\right)^2 \ldots}$$

```
[Out] -(x/(a^5*ArcTanh[a*x])) - x/(a^5*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (2*x)/(a^5
*(1 - a^2*x^2)*ArcTanh[a*x]) - (3*CoshIntegral[2*ArcTanh[a*x]])/(2*a^6) + C
oshIntegral[4*ArcTanh[a*x]]/(2*a^6) + Unintegrable[ArcTanh[a*x]^(-1), x]/a^
5
```

Rubi [A] time = 0.904577, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^5}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a^5*ArcTanh[a*x])) - x/(a^5*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (2*x)/(a^5
*(1 - a^2*x^2)*ArcTanh[a*x]) - (3*CoshIntegral[2*ArcTanh[a*x]])/(2*a^6) + C
oshIntegral[4*ArcTanh[a*x]]/(2*a^6) + Defer[Int][ArcTanh[a*x]^(-1), x]/a^5
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx = \frac{\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx}{a^2} - \frac{\int \frac{x^3}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx}{a^2}$$

$$= \frac{\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx}{a^4} - 2\frac{\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx}{a^4} + \frac{\int \frac{x}{(1-a^2x^2) \tanh^{-1}(ax)^2} dx}{a^4}$$

$$= -\frac{x}{a^5 \tanh^{-1}(ax)} - \frac{x}{a^5 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\int \frac{1}{\tanh^{-1}(ax)} dx}{a^5} + \frac{\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax}}{a^5}$$

$$= -\frac{x}{a^5 \tanh^{-1}(ax)} - \frac{x}{a^5 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^4(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^6}$$

$$= -\frac{x}{a^5 \tanh^{-1}(ax)} - \frac{x}{a^5 (1-a^2x^2)^2 \tanh^{-1}(ax)} - 2\left(-\frac{x}{a^5 (1-a^2x^2) \tanh^{-1}(ax)} - \frac{\text{Sub}}{}\right.$$

$$= -\frac{x}{a^5 \tanh^{-1}(ax)} - \frac{x}{a^5 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x} dx, x, \tanh^{-1}(ax)\right)}{8a^6}$$

$$= -\frac{x}{a^5 \tanh^{-1}(ax)} - \frac{x}{a^5 (1-a^2x^2)^2 \tanh^{-1}(ax)} + \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^6} - 2\left(-\frac{}{a^5 (1-a^2}\right.$$

Mathematica [A] time = 11.9583, size = 0, normalized size = 0.

$$\int \frac{x^5}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x^5/((1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.199, size = 0, normalized size = 0.

$$\int \frac{x^5}{(-a^2x^2+1)^3 (\text{Artanh}(ax))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] int(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2x^5}{(a^5x^4 - 2a^3x^2 + a)\log(ax+1) - (a^5x^4 - 2a^3x^2 + a)\log(-ax+1)} - \int -\frac{2(a^2x^6}{(a^7x^6 - 3a^5x^4 + 3a^3x^2 - a)\log(ax+1) -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^5/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) - integrate(-2*(a^2*x^6 - 5*x^4)/((a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x^5}{\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x^5/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5}{a^6x^6\,\text{atanh}^2\left(ax\right) - 3a^4x^4\,\text{atanh}^2\left(ax\right) + 3a^2x^2\,\text{atanh}^2\left(ax\right) - \text{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**5/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^5}{\left(a^2x^2 - 1\right)^3\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^5/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

3.330 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=53

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}-\frac{x^4}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -(x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) - SinhIntegral[2*ArcTanh[a*x]]/a^5
+ SinhIntegral[4*ArcTanh[a*x]]/(2*a^5)
```

Rubi [A] time = 0.186329, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6006, 6034, 5448, 3298}

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}-\frac{x^4}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x^4/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) - SinhIntegral[2*ArcTanh[a*x]]/a^5
+ SinhIntegral[4*ArcTanh[a*x]]/(2*a^5)
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx = -\frac{x^4}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{4\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx}{a}$$

$$= -\frac{x^4}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^5}$$

$$= -\frac{x^4}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a^5} - \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,d\right.}{a^5}$$

$$= -\frac{x^4}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} - \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a^5} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Mathematica [A] time = 0.185935, size = 49, normalized size = 0.92

$$\frac{-\frac{2a^4x^4}{\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)} - 2\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] ((-2*a^4*x^4)/((-1 + a^2*x^2)^2*ArcTanh[a*x]) - 2*SinhIntegral[2*ArcTanh[a*
x]] + SinhIntegral[4*ArcTanh[a*x]])/(2*a^5)
```

Maple [A] time = 0.069, size = 62, normalized size = 1.2

$$\frac{1}{a^5}\left(-\frac{3}{8\,\text{Artanh}\,(ax)} + \frac{\cosh\,(2\,\text{Artanh}\,(ax))}{2\,\text{Artanh}\,(ax)} - \text{Shi}\,(2\,\text{Artanh}\,(ax)) - \frac{\cosh\,(4\,\text{Artanh}\,(ax))}{8\,\text{Artanh}\,(ax)} + \frac{\text{Shi}\,(4\,\text{Artanh}\,(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^5*(-3/8/arctanh(a*x)+1/2/arctanh(a*x)*cosh(2*arctanh(a*x))-Shi(2*arctan
h(a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^4}{\left(a^5x^4-2\,a^3x^2+a\right)\log\,(ax+1) - \left(a^5x^4-2\,a^3x^2+a\right)\log\,(-ax+1)} + 8\int -\frac{x}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\,(ax+1) -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^4/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + 8*integrate(-x^3/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*lo
g(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.32265, size = 545, normalized size = 10.28

$$-\frac{8\,a^4x^4 - \left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - \left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - 2\left(a^4x^4 - \right.\right.}{4\left(a^9x^4 - 2\,a^7x^2 + a^5\right)\log\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^4*x^4 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_in
tegral(-(a*x + 1)/(a*x - 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^9*x^4 - 2*a^7*x^2 + a^5)
*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{a^6x^6\operatorname{atanh}^2(ax) - 3a^4x^4\operatorname{atanh}^2(ax) + 3a^2x^2\operatorname{atanh}^2(ax) - \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^4}{\left(a^2x^2 - 1\right)^3 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

3.331 $\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4}-\frac{x^3}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -(x^3/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) - CoshIntegral[2*ArcTanh[a*x]]/(2*a
^4) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^4)
```

Rubi [A] time = 0.520601, antiderivative size = 76, normalized size of antiderivative = 1.38, number of steps used = 20, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6028, 6032, 6034, 3312, 3301, 5968, 5448}

$$-\frac{\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^4}+\frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x])) + x/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]
) - CoshIntegral[2*ArcTanh[a*x]]/(2*a^4) + CoshIntegral[4*ArcTanh[a*x]]/(2*
a^4)
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx}{a^2} - \frac{\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}\,dx}{a^3} -$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \text{ta}\right.}{a^4}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \left(\frac{1}{2x} - \frac{\cosh(2x)}{2x}\right)\right.}{a^4}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \text{ta}\right.}{8a^4}$$

$$= -\frac{x}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{x}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^4} + \frac{\text{Ch}}{}$$

Mathematica [A] time = 0.114223, size = 80, normalized size = 1.45

$$\frac{-\left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Chi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2-1\right)^2\tanh^{-1}(ax)\text{Chi}\left(4\tanh^{-1}(ax)\right)-2a^3x^3}{2a^4\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a^3*x^3 - (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] +
(-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^4*(-1 + a^
2*x^2)^2*ArcTanh[a*x])
```

Maple [A] time = 0.06, size = 54, normalized size = 1.

$$\frac{1}{a^4}\left(\frac{\sinh(2\,\mathrm{Artanh}\,(ax))}{4\,\mathrm{Artanh}\,(ax)}-\frac{\mathrm{Chi}\,(2\,\mathrm{Artanh}\,(ax))}{2}-\frac{\sinh(4\,\mathrm{Artanh}\,(ax))}{8\,\mathrm{Artanh}\,(ax)}+\frac{\mathrm{Chi}\,(4\,\mathrm{Artanh}\,(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^4*(1/4/arctanh(a*x)*sinh(2*arctanh(a*x))-1/2*Chi(2*arctanh(a*x))-1/8/ar
ctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,x^3}{\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)-\left(a^5x^4-2\,a^3x^2+a\right)\log(-ax+1)}+\int-\frac{2\left(a^2x^4+\right.}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log(ax+1)-\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^3/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + integrate(-2*(a^2*x^4 + 3*x^2)/((a^7*x^6 - 3*a^5*x^4 + 3*a
^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x +
 1)), x)
```

Fricas [B] time = 2.35858, size = 540, normalized size = 9.82

$$-\frac{8\,a^3x^3-\left(\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)-\left(a^4x^4-2\,a^2\right.\right.}{4\left(a^8x^4-2\,a^6x^2+a^4\right)\log\left(-\frac{ax+1}{ax-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*x^3 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_inte
gral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x -
 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log
(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^3}{a^6x^6\,\mathrm{atanh}^2\,(ax)-3a^4x^4\,\mathrm{atanh}^2\,(ax)+3a^2x^2\,\mathrm{atanh}^2\,(ax)-\mathrm{atanh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

3.332 $\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=41

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3} - \frac{x^2}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

```
[Out] -(x^2/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + SinhIntegral[4*ArcTanh[a*x]]/(2*a
^3)
```

Rubi [A] time = 0.287269, antiderivative size = 60, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6028, 5966, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{1}{a^3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a^3*(1 - a^2*x^2)^2*ArcTanh[a*x])) + 1/(a^3*(1 - a^2*x^2)*ArcTanh[a*x]
) + SinhIntegral[4*ArcTanh[a*x]]/(2*a^3)
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx = \frac{\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx}{a^2} - \frac{\int \frac{1}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}\,dx}{a^2}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{2\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}\,dx}{a}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,d}{a^3}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} - \frac{2\,\mathrm{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx,x,t}{a^3}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx,x,ta}{2a^3}$$

$$= -\frac{1}{a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)} + \frac{1}{a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)} + \frac{\mathrm{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^3}$$

Mathematica [A] time = 0.202416, size = 56, normalized size = 1.37

$$\frac{\left(a^2x^2-1\right)^2\tanh^{-1}(ax)\mathrm{Shi}\left(4\tanh^{-1}(ax)\right)-2a^2x^2}{2a^3\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a^2*x^2 + (-1 + a^2*x^2)^2*ArcTanh[a*x]*SinhIntegral[4*ArcTanh[a*x]])/(
2*a^3*(-1 + a^2*x^2)^2*ArcTanh[a*x])
```

Maple [A] time = 0.067, size = 38, normalized size = 0.9

$$\frac{1}{a^3}\left(\frac{1}{8\,\mathrm{Artanh}\,(ax)} - \frac{\cosh\left(4\,\mathrm{Artanh}\,(ax)\right)}{8\,\mathrm{Artanh}\,(ax)} + \frac{\mathrm{Shi}\left(4\,\mathrm{Artanh}\,(ax)\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^3*(1/8/arctanh(a*x)-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arc
tanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2x^2}{(a^5x^4-2a^3x^2+a)\log(ax+1)-(a^5x^4-2a^3x^2+a)\log(-ax+1)}+\int-\frac{4(a^2x^3+\ldots}{(a^7x^6-3a^5x^4+3a^3x^2-a)\log(ax+1)-(a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x^2/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*
log(-a*x + 1)) + integrate(-4*(a^2*x^3 + x)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
, x)
```

Fricas [B] time = 2.33203, size = 370, normalized size = 9.02

$$-\frac{8a^2x^2-\left(\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right)}{4\left(a^7x^4-2a^5x^2+a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^
4 - 2*a^5*x^2 + a^3)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{a^6x^6\operatorname{atanh}^2(ax)-3a^4x^4\operatorname{atanh}^2(ax)+3a^2x^2\operatorname{atanh}^2(ax)-\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**
2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

3.333 $\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=53

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/(2*a^2
) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2)
```

Rubi [A] time = 0.245151, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6032, 6034, 5448, 3301, 5968, 3312}

$$\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + CoshIntegral[2*ArcTanh[a*x]]/(2*a^2
) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2)
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx}{a} + (3a)\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(a\ldots}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\operatorname{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2} + \frac{3\operatorname{Subst}\left(\int \frac{\cosh\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\operatorname{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^2}\ldots$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\operatorname{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^2} + \frac{3\operatorname{Subst}\left(\int \frac{\cosh\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\operatorname{Chi}\left(2\tanh^{-1}(ax)\right)}{2a^2} + \frac{\operatorname{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.0994367, size = 75, normalized size = 1.42

$$\frac{\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)\operatorname{Chi}\left(2\tanh^{-1}(ax)\right)+\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)\operatorname{Chi}\left(4\tanh^{-1}(ax)\right)-2ax}{2a^2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2*a*x + (-1 + a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[2*ArcTanh[a*x]] + (-1
+ a^2*x^2)^2*ArcTanh[a*x]*CoshIntegral[4*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^
2)^2*ArcTanh[a*x])
```

Maple [A] time = 0.062, size = 54, normalized size = 1.

$$\frac{1}{a^2}\left(-\frac{\sinh(2\operatorname{Artanh}(ax))}{4\operatorname{Artanh}(ax)}+\frac{\operatorname{Chi}(2\operatorname{Artanh}(ax))}{2}-\frac{\sinh(4\operatorname{Artanh}(ax))}{8\operatorname{Artanh}(ax)}+\frac{\operatorname{Chi}(4\operatorname{Artanh}(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a^2*(-1/4/arctanh(a*x)*sinh(2*arctanh(a*x))+1/2*Chi(2*arctanh(a*x))-1/8/a
rctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2x}{(a^5x^4-2a^3x^2+a)\log(ax+1)-(a^5x^4-2a^3x^2+a)\log(-ax+1)}+\int-\frac{2(3a^2x^2}{(a^7x^6-3a^5x^4+3a^3x^2-a)\log(ax+1)-(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*x/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*lo
g(-a*x + 1)) + integrate(-2*(3*a^2*x^2 + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x
^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
, x)
```

Fricas [B] time = 2.37035, size = 535, normalized size = 10.09

$$-\frac{8ax-\left(\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)+\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+\left(a^4x^4-2a^2x^2\right.}{4\left(a^6x^4-2a^4x^2+a^2\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*x - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)
/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 -
 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral
(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a
*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{a^6x^6\operatorname{atanh}^2(ax)-3a^4x^4\operatorname{atanh}^2(ax)+3a^2x^2\operatorname{atanh}^2(ax)-\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*x
**2*atanh(a*x)**2 - atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)
```

3.334 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] -(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a + Si
nhIntegral[4*ArcTanh[a*x]]/(2*a)
```

Rubi [A] time = 0.120532, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}+\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^2*ArcTanh[a*x])) + SinhIntegral[2*ArcTanh[a*x]]/a + Si
nhIntegral[4*ArcTanh[a*x]]/(2*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + (4a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{4\,\text{Subst}\left(\int \left(\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right) dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{\sinh(2}{x}\right.}{}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{a} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.131599, size = 43, normalized size = 0.88

$$\frac{-\frac{2}{\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)} + 2\text{Shi}\left(2\tanh^{-1}(ax)\right) + \text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] (-2/((-1 + a^2*x^2)^2*ArcTanh[a*x]) + 2*SinhIntegral[2*ArcTanh[a*x]] + Sinh
Integral[4*ArcTanh[a*x]])/(2*a)
```

Maple [A] time = 0.061, size = 60, normalized size = 1.2

$$\frac{1}{a}\left(-\frac{3}{8\,\text{Artanh}\,(ax)} - \frac{\cosh\,(2\,\text{Artanh}\,(ax))}{2\,\text{Artanh}\,(ax)} + \text{Shi}\,(2\,\text{Artanh}\,(ax)) - \frac{\cosh\,(4\,\text{Artanh}\,(ax))}{8\,\text{Artanh}\,(ax)} + \frac{\text{Shi}\,(4\,\text{Artanh}\,(ax))}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(-3/8/arctanh(a*x)-1/2/arctanh(a*x)*cosh(2*arctanh(a*x))+Shi(2*arctanh(
a*x))-1/8/arctanh(a*x)*cosh(4*arctanh(a*x))+1/2*Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8\,a\int -\frac{x}{\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\,(ax+1) - \left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\,(-ax+1)}\,dx - \frac{}{\left(a^5x^4 - 2\,a^3x^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

[Out] 8*a*integrate(-x/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x) - 2/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1) - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1))

Fricas [B] time = 2.01794, size = 531, normalized size = 10.84

$$\frac{\left(\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+2\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{lo}\right.}{4\left(a^5x^4-2\,a^3x^2+a\right)\log\left(-\frac{ax+1}{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")

[Out] 1/4*(((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)) - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1)))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^6x^6\operatorname{atanh}^2(ax)-3a^4x^4\operatorname{atanh}^2(ax)+3a^2x^2\operatorname{atanh}^2(ax)-\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**2,x)

[Out] -Integral(1/(a**6*x**6*atanh(a*x)**2 - 3*a**4*x**4*atanh(a*x)**2 + 3*a**2*x**2*atanh(a*x)**2 - atanh(a*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")

[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^2), x)

3.335 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=98

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\tanh^{-1}(ax)},x\right)}{a}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}+\frac{3}{2}\text{Chi}\left(2\tanh^{-1}(ax)\right)+\frac{1}{2}\text{Chi}$$

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)^2*ArcTanh[a*x]) - (a*x)/((1
- a^2*x^2)*ArcTanh[a*x]) + (3*CoshIntegral[2*ArcTanh[a*x]])/2 + CoshIntegra
l[4*ArcTanh[a*x]]/2 - Unintegrable[1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi [A] time = 0.650164, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*x*ArcTanh[a*x])) - (a*x)/((1 - a^2*x^2)^2*ArcTanh[a*x]) - (a*x)/((1
- a^2*x^2)*ArcTanh[a*x]) + (3*CoshIntegral[2*ArcTanh[a*x]])/2 + CoshIntegra
l[4*ArcTanh[a*x]]/2 - Defer[Int][1/(x^2*ArcTanh[a*x]), x]/a
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx = a^2\int \frac{x}{(1-a^2x^2)^3\tanh^{-1}(ax)^2}\,dx+\int \frac{1}{x(1-a^2x^2)^2\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}+a\int \frac{1}{(1-a^2x^2)^3\tanh^{-1}(ax)}\,dx+a^2\int \frac{x}{(1-a^2x^2)^2\tan}$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+3\,\text{Subst}\Big(\int$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+3\,\text{Subst}\Big(\int$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\frac{1}{8}\,\text{Subst}\Big(\int$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\frac{1}{2}\text{Chi}\big(2\tan$$

$$= -\frac{1}{ax\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)^2\tanh^{-1}(ax)}-\frac{ax}{(1-a^2x^2)\tanh^{-1}(ax)}+\frac{3}{2}\text{Chi}\big(2\tan$$

Mathematica [A] time = 4.26807, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.249, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3 (\mathrm{Artanh}\,(ax))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{\left(a^5x^5-2\,a^3x^3+ax\right)\log\left(ax+1\right)-\left(a^5x^5-2\,a^3x^3+ax\right)\log\left(-ax+1\right)}+\int -\frac{2\,(5}{\left(a^7x^8-3\,a^5x^6+3\,a^3x^4-ax^2\right)\log\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/((a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x + 1) - (a^5*x^5 - 2*a^3*x^3 + a*x)*
log(-a*x + 1)) + integrate(-2*(5*a^2*x^2 - 1)/((a^7*x^8 - 3*a^5*x^6 + 3*a^3
*x^4 - a*x^2)*log(a*x + 1) - (a^7*x^8 - 3*a^5*x^6 + 3*a^3*x^4 - a*x^2)*log(
-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\left(a^6x^7-3\,a^4x^5+3\,a^2x^3-x\right)\mathrm{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^6x^7 \operatorname{atanh}^2(ax) - 3a^4x^5 \operatorname{atanh}^2(ax) + 3a^2x^3 \operatorname{atanh}^2(ax) - x \operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**2,x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x)**2 - 3*a**4*x**5*atanh(a*x)**2 + 3*a**2*x
**3*atanh(a*x)**2 - x*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{(a^2x^2-1)^3 x \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^2), x)
```

3.336 $\int \frac{x^4}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=100

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^5}-\frac{x^4}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2x}{a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^4\left(1-a^2x^2\right)^2\text{t}}$$

```
[Out] -x^4/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/(a^4*(1 - a^2*x^2)^2*ArcT
anh[a*x]) + (2*x)/(a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - CoshIntegral[2*ArcTanh
[a*x]]/a^5 + CoshIntegral[4*ArcTanh[a*x]]/a^5
```

Rubi [A] time = 0.5942, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6006, 6028, 6032, 6034, 3312, 3301, 5968, 5448}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a^5}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^5}-\frac{x^4}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{2x}{a^4\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^4\left(1-a^2x^2\right)^2\text{t}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -x^4/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/(a^4*(1 - a^2*x^2)^2*ArcT
anh[a*x]) + (2*x)/(a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - CoshIntegral[2*ArcTanh
[a*x]]/a^5 + CoshIntegral[4*ArcTanh[a*x]]/a^5
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} dx = -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{2\int \frac{x^3}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} dx}{a}$$

$$= -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{2\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} dx}{a^3} - \frac{2\int \frac{x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} dx}{a^3}$$

$$= -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4\left(1-a^2x^2\right) \tanh^{-1}(ax)} +$$

$$= -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4\left(1-a^2x^2\right) \tanh^{-1}(ax)} -$$

$$= -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4\left(1-a^2x^2\right) \tanh^{-1}(ax)} +$$

$$= -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4\left(1-a^2x^2\right) \tanh^{-1}(ax)} +$$

$$= -\frac{x^4}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{a^4\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2x}{a^4\left(1-a^2x^2\right) \tanh^{-1}(ax)} -$$

Mathematica [A] time = 0.209843, size = 60, normalized size = 0.6

$$-\frac{\frac{a^3x^3\left(ax+4\tanh^{-1}(ax)\right)}{\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2} + 2\text{Chi}\left(2\tanh^{-1}(ax)\right) - 2\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -((a^3*x^3*(a*x + 4*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTanh[a*x]^2) + 2*Co
shIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]])/(2*a^5)
```

Maple [A] time = 0.07, size = 90, normalized size = 0.9

$$\frac{1}{a^5}\left(-\frac{3}{16\left(\text{Artanh}(ax)\right)^2} + \frac{\cosh(2\,\text{Artanh}(ax))}{4\left(\text{Artanh}(ax)\right)^2} + \frac{\sinh(2\,\text{Artanh}(ax))}{2\,\text{Artanh}(ax)} - \text{Chi}(2\,\text{Artanh}(ax)) - \frac{\cosh(4\,\text{Artanh}(ax)}{16\left(\text{Artanh}(ax)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^5*(-3/16/arctanh(a*x)^2+1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))+1/2/arc
tanh(a*x)*sinh(2*arctanh(a*x))-Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh
(4*arctanh(a*x))-1/4/arctanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(ax^4 + 2x^3\log(ax+1) - 2x^3\log(-ax+1)\right)}{\left(a^6x^4 - 2a^4x^2 + a^2\right)\log(ax+1)^2 - 2\left(a^6x^4 - 2a^4x^2 + a^2\right)\log(ax+1)\log(-ax+1) + \left(a^6x^4 - 2a^4x^2 + a^2\right)\log(-ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a*x^4 + 2*x^3*log(a*x + 1) - 2*x^3*log(-a*x + 1))/((a^6*x^4 - 2*a^4*x^2
 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)*log(-a*
x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + integrate(-4*(a^2*x
^4 + 3*x^2)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1) - (a^8*x^
6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.02812, size = 597, normalized size = 5.97

$$-\frac{4a^4x^4 + 8a^3x^3\log\left(-\frac{ax+1}{ax-1}\right) - \left(\left(a^4x^4 - 2a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right) + \left(a^4x^4 - 2a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2}{a^2}\right.\right.}{2\left(a^9x^4 - 2a^7x^2 + a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*x^4 + 8*a^3*x^3*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x
^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^
4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)
) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) - (a^4*x^4
 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x -
 1))^2)/((a^9*x^4 - 2*a^7*x^2 + a^5)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^4}{a^6x^6\operatorname{atanh}^3(ax) - 3a^4x^4\operatorname{atanh}^3(ax) + 3a^2x^2\operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**4/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^4}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^4/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

3.337 $\int \frac{x^3}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^4}-\frac{x^4}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x^3}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{3}{2a^2\left(1-a^2x^2\right.}$$

```
[Out] -x^3/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (3*x^2)/(2*a^2*(1 - a^2*x^2)^2*
ArcTanh[a*x]) - x^4/(2*(1 - a^2*x^2)^2*ArcTanh[a*x]) - SinhIntegral[2*ArcTa
nh[a*x]]/(2*a^4) + SinhIntegral[4*ArcTanh[a*x]]/a^4
```

Rubi [A] time = 0.65472, antiderivative size = 160, normalized size of antiderivative = 1.5, number of steps used = 25, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6028, 5996, 6034, 5448, 12, 3298, 6032, 5966}

$$-\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^4}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^4}+\frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{a}{2a^4\left(1-a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(2*a^3*(1 - a^2*x^2)*ArcTanh[
a*x]^2) - 2/(a^4*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 3/(2*a^4*(1 - a^2*x^2)*Arc
Tanh[a*x]) + (1 + a^2*x^2)/(2*a^4*(1 - a^2*x^2)*ArcTanh[a*x]) - SinhIntegra
l[2*ArcTanh[a*x]]/(2*a^4) + SinhIntegral[4*ArcTanh[a*x]]/a^4
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx = \frac{\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}dx}{a^2} - \frac{\int \frac{x}{\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}dx}{a^2}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} + \frac{1+a^2x^2}{2a^4\left(1-a^2x^2\right)\tanh^{-}}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{1}{2a^4\left(1-a^2x^2\right)^2\tanh}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2\tanh^{-1}}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2\tanh^{-1}}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2\tanh^{-1}}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2\tanh^{-1}}$$

$$= -\frac{x}{2a^3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{x}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{2}{a^4\left(1-a^2x^2\right)^2\tanh^{-1}}$$

Mathematica [A] time = 0.259509, size = 66, normalized size = 0.62

$$-\frac{\frac{a^2x^2\left(\left(a^2x^2+3\right)\tanh^{-1}(ax)+ax\right)}{\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2} + \text{Shi}\left(2\tanh^{-1}(ax)\right) - 2\text{Shi}\left(4\tanh^{-1}(ax)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -((a^2*x^2*(a*x + (3 + a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^2*ArcTanh[a*
x]^2) + SinhIntegral[2*ArcTanh[a*x]] - 2*SinhIntegral[4*ArcTanh[a*x]])/(2*a
^4)
```

Maple [A] time = 0.072, size = 82, normalized size = 0.8

$$\frac{1}{a^4}\left(\frac{\sinh(2\,\text{Artanh}(ax))}{8\,(\text{Artanh}(ax))^2} + \frac{\cosh(2\,\text{Artanh}(ax))}{4\,\text{Artanh}(ax)} - \frac{\text{Shi}(2\,\text{Artanh}(ax))}{2} - \frac{\sinh(4\,\text{Artanh}(ax))}{16\,(\text{Artanh}(ax))^2} - \frac{\cosh(4\,\text{Artanh}}{4\,\text{Artanh}(ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^4*(1/8/arctanh(a*x)^2*sinh(2*arctanh(a*x))+1/4/arctanh(a*x)*cosh(2*arct
anh(a*x))-1/2*Shi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))-
```

```
1/4/arctanh(a*x)*cosh(4*arctanh(a*x))+Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ax^3 + \left(a^2x^4 + 3\,x^2\right)\log\left(ax+1\right) - \left(a^2x^4 + 3\,x^2\right)\log\left(-ax+1\right)}{\left(a^6x^4 - 2\,a^4x^2 + a^2\right)\log\left(ax+1\right)^2 - 2\left(a^6x^4 - 2\,a^4x^2 + a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right) + \left(a^6x^4 - 2\,a^4x^2 + a^2\right)\log\left(-ax+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x^3 + (a^2*x^4 + 3*x^2)*log(a*x + 1) - (a^2*x^4 + 3*x^2)*log(-a*x + 1
))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a
^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^
2) + integrate(-2*(5*a^2*x^3 + 3*x)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2
)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Fricas [B] time = 1.99358, size = 621, normalized size = 5.8

$$-\frac{8\,a^3x^3 - \left(2\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) - 2\left(a^4x^4 - 2\,a^2x^2 + 1\right)\text{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right) - \left(a^4x^4 - 2\right.\right.}{4\left(a^8x^4 - 2\,a^6x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*x^3 - (2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*
x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a
^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_
integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^4*x^4 + 3*a^2*x^2)*l
og(-(a*x + 1)/(a*x - 1)))/((a^8*x^4 - 2*a^6*x^2 + a^4)*log(-(a*x + 1)/(a*x
- 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{a^6x^6\operatorname{atanh}^3(ax) - 3a^4x^4\operatorname{atanh}^3(ax) + 3a^2x^2\operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**3/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^3}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^3/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

3.338 $\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=86

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}-\frac{x^2}{2a\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}+\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}$$

```
[Out] -x^2/(2*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/(a^2*(1 - a^2*x^2)^2*Arc
Tanh[a*x]) + x/(a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + CoshIntegral[4*ArcTanh[a*
x]]/a^3
```

Rubi [A] time = 0.585663, antiderivative size = 109, normalized size of antiderivative = 1.27, number of steps used = 22, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {6028, 5966, 6032, 6034, 3312, 3301, 5968, 5448}

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3}+\frac{x}{a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2x}{a^2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}+\frac{1}{2a^3\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{}{2a^3\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a^3*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a^3*(1 - a^2*x^2)*ArcTanh[
a*x]^2) - (2*x)/(a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/(a^2*(1 - a^2*x^2)*A
rcTanh[a*x]) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx = \frac{\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx}{a^2} - \frac{\int \frac{1}{(1-a^2x^2)^2 \tanh^{-1}(ax)^3} dx}{a^2}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{\int \frac{x}{(1-a^2x^2)^2 \tanh^{-1}(ax)^2} dx}{a}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{2x}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{2x}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{2x}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{2x}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{2a^3(1-a^2x^2)^2 \tanh^{-1}(ax)^2} + \frac{1}{2a^3(1-a^2x^2)\tanh^{-1}(ax)^2} - \frac{2x}{a^2(1-a^2x^2)^2 \tanh^{-1}(ax)}$$

Mathematica [A] time = 0.154842, size = 56, normalized size = 0.65

$$\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a^3} - \frac{x\left(2\left(a^2x^2+1\right)\tanh^{-1}(ax)+ax\right)}{2a^2\left(a^2x^2-1\right)^2\tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -(x*(a*x + 2*(1 + a^2*x^2)*ArcTanh[a*x]))/(2*a^2*(-1 + a^2*x^2)^2*ArcTanh[a
*x]^2) + CoshIntegral[4*ArcTanh[a*x]]/a^3
```

Maple [A] time = 0.067, size = 51, normalized size = 0.6

$$\frac{1}{a^3}\left(\frac{1}{16\,(\text{Artanh}\,(ax))^2} - \frac{\cosh(4\,\text{Artanh}\,(ax))}{16\,(\text{Artanh}\,(ax))^2} - \frac{\sinh(4\,\text{Artanh}\,(ax))}{4\,\text{Artanh}\,(ax)} + \text{Chi}\,(4\,\text{Artanh}\,(ax))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^3*(1/16/arctanh(a*x)^2-1/16/arctanh(a*x)^2*cosh(4*arctanh(a*x))-1/4/arc
tanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(ax^2+\left(a^2x^3+x\right)\log(ax+1)-\left(a^2x^3+x\right)\log(-ax+1)\right)}{\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)^2-2\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)\log(-ax+1)+\left(a^6x^4-2\,a^4x^2+a^2\right)\log(-ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(a*x^2 + (a^2*x^3 + x)*log(a*x + 1) - (a^2*x^3 + x)*log(-a*x + 1))/((a^6
*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*log(
a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + int
egrate(-2*(a^4*x^4 + 6*a^2*x^2 + 1)/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2
)*log(a*x + 1) - (a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-a*x + 1)), x)
```

Fricas [B] time = 1.98297, size = 437, normalized size = 5.08

$$-\frac{4\,a^2x^2 - \left(\left(a^4x^4 - 2\,a^2x^2 + 1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right) + \left(a^4x^4 - 2\,a^2x^2 + 1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)\right)\log\left(-\frac{ax+1}{ax-1}\right.}{2\left(a^7x^4 - 2\,a^5x^2 + a^3\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*x^2 - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x
+ 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x
^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4*(a^
3*x^3 + a*x)*log(-(a*x + 1)/(a*x - 1)))/((a^7*x^4 - 2*a^5*x^2 + a^3)*log(-(
a*x + 1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{a^6x^6\operatorname{atanh}^3(ax) - 3a^4x^4\operatorname{atanh}^3(ax) + 3a^2x^2\operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x**2/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**
2*x**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\left(a^2x^2-1\right)^3\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x^2/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

3.339 $\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=100

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{3}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2}{a^2\left(1-a^2x^2\right)^2\text{ta}}$$

```
[Out] -x/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - 2/(a^2*(1 - a^2*x^2)^2*ArcTanh[a*
x]) + 3/(2*a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/(
2*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2
```

Rubi [A] time = 0.464066, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {6032, 6028, 5966, 6034, 5448, 12, 3298}

$$\frac{\text{Shi}\left(2\tanh^{-1}(ax)\right)}{2a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{2a\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}+\frac{3}{2a^2\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{2}{a^2\left(1-a^2x^2\right)^2\text{ta}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - 2/(a^2*(1 - a^2*x^2)^2*ArcTanh[a*
x]) + 3/(2*a^2*(1 - a^2*x^2)*ArcTanh[a*x]) + SinhIntegral[2*ArcTanh[a*x]]/(
2*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx}{2a} + \frac{1}{2}(3a)\int \frac{x^2}{\left(1-a^2x^2\right)^3 \tanh}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{1}{2a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + 2\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{3}{2a^2\left(1-a^2x^2\right) \tanh^{-1}(a}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{3}{2a^2\left(1-a^2x^2\right) \tanh^{-1}(a}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{3}{2a^2\left(1-a^2x^2\right) \tanh^{-1}(a}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{3}{2a^2\left(1-a^2x^2\right) \tanh^{-1}(a}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2}{a^2\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{3}{2a^2\left(1-a^2x^2\right) \tanh^{-1}(a}$$

Mathematica [A] time = 0.189889, size = 96, normalized size = 0.96

$$-\frac{-\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2 \text{Shi}\left(2\tanh^{-1}(ax)\right) - 2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2 \text{Shi}\left(4\tanh^{-1}(ax)\right) + 3a^2x^2 \tanh^{-1}(ax) + }{2a^2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -(a*x + ArcTanh[a*x] + 3*a^2*x^2*ArcTanh[a*x] - (-1 + a^2*x^2)^2*ArcTanh[a*
x]^2*SinhIntegral[2*ArcTanh[a*x]] - 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*SinhI
ntegral[4*ArcTanh[a*x]])/(2*a^2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

Maple [A] time = 0.062, size = 82, normalized size = 0.8

$$\frac{1}{a^2}\left(-\frac{\sinh(2\operatorname{Artanh}(ax))}{8(\operatorname{Artanh}(ax))^2}-\frac{\cosh(2\operatorname{Artanh}(ax))}{4\operatorname{Artanh}(ax)}+\frac{\operatorname{Shi}(2\operatorname{Artanh}(ax))}{2}-\frac{\sinh(4\operatorname{Artanh}(ax))}{16(\operatorname{Artanh}(ax))^2}-\frac{\cosh(4\operatorname{Artanh}(a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a^2*(-1/8/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/4/arctanh(a*x)*cosh(2*arc
tanh(a*x))+1/2*Shi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*x))
-1/4/arctanh(a*x)*cosh(4*arctanh(a*x))+Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ax+\left(3\,a^2x^2+1\right)\log(ax+1)-\left(3\,a^2x^2+1\right)\log(-ax+1)}{\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)^2-2\left(a^6x^4-2\,a^4x^2+a^2\right)\log(ax+1)\log(-ax+1)+\left(a^6x^4-2\,a^4x^2+a^2\right)\log(-ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (3*a^2*x^2 + 1)*log(a*x + 1) - (3*a^2*x^2 + 1)*log(-a*x + 1))/((a
^6*x^4 - 2*a^4*x^2 + a^2)*log(a*x + 1)^2 - 2*(a^6*x^4 - 2*a^4*x^2 + a^2)*lo
g(a*x + 1)*log(-a*x + 1) + (a^6*x^4 - 2*a^4*x^2 + a^2)*log(-a*x + 1)^2) + i
ntegrate(-2*(3*a^2*x^3 + 5*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*
x + 1) - (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 1.99856, size = 606, normalized size = 6.06

$$\frac{\left(2\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)-2\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\right.}{4\left(a^6x^4-2\,a^4x^2+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
```

```
+ 1)))*log(-(a*x + 1)/(a*x - 1))^2 - 8*a*x - 4*(3*a^2*x^2 + 1)*log(-(a*x +
1)/(a*x - 1)))/((a^6*x^4 - 2*a^4*x^2 + a^2)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(x/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*x
**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-x/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

3.340 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=69

$$-\frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a}$$

```
[Out] -1/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/((1 - a^2*x^2)^2*ArcTanh[a*
x]) + CoshIntegral[2*ArcTanh[a*x]]/a + CoshIntegral[4*ArcTanh[a*x]]/a
```

Rubi [A] time = 0.27001, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{a}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```


```
[Out] -1/(2*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (2*x)/((1 - a^2*x^2)^2*ArcTanh[a*
x]) + CoshIntegral[2*ArcTanh[a*x]]/a + CoshIntegral[4*ArcTanh[a*x]]/a
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} + (2a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + 2\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(a$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \mathrm{t}\right.}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{2\,\mathrm{Subst}\left(\int \left(\frac{3}{8x} + \frac{\cosh(2x)}{2x}\right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \mathrm{ta}\right.}{4a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2} - \frac{2x}{\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)} + \frac{\mathbf{Chi}\left(2\tanh^{-1}(ax)\right)}{a} + \frac{\mathrm{Ch}}{}$$

Mathematica [A] time = 0.177831, size = 86, normalized size = 1.25

$$\frac{2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2 \mathbf{Chi}\left(2\tanh^{-1}(ax)\right) + 2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2 \mathbf{Chi}\left(4\tanh^{-1}(ax)\right) - 4ax\tanh^{-1}(ax) - 1}{2a\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] (-1 - 4*a*x*ArcTanh[a*x] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[2
*ArcTanh[a*x]] + 2*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2*CoshIntegral[4*ArcTanh[a
```

```
*x]])/(2*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2)
```

Maple [A] time = 0.064, size = 88, normalized size = 1.3

$$\frac{1}{a}\left(-\frac{3}{16\left(\operatorname{Artanh}(ax)\right)^2}-\frac{\cosh(2\operatorname{Artanh}(ax))}{4\left(\operatorname{Artanh}(ax)\right)^2}-\frac{\sinh(2\operatorname{Artanh}(ax))}{2\operatorname{Artanh}(ax)}+\operatorname{Chi}(2\operatorname{Artanh}(ax))-\frac{\cosh(4\operatorname{Artanh}(ax))}{16\left(\operatorname{Artanh}(ax)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] 1/a*(-3/16/arctanh(a*x)^2-1/4/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/2/arcta
nh(a*x)*sinh(2*arctanh(a*x))+Chi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*cosh(4
*arctanh(a*x))-1/4/arctanh(a*x)*sinh(4*arctanh(a*x))+Chi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(2\,ax\log(ax+1)-2\,ax\log(-ax+1)+1\right)}{\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)^2-2\left(a^5x^4-2\,a^3x^2+a\right)\log(ax+1)\log(-ax+1)+\left(a^5x^4-2\,a^3x^2+a\right)\log(-ax+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a*x*log(a*x + 1) - 2*a*x*log(-a*x + 1) + 1)/((a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^2 - 2*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1) +
 (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^2) + integrate(-4*(3*a^2*x^2 + 1)/
((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6*x^6 - 3*a^4*x^4
+ 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.01353, size = 578, normalized size = 8.38

$$-\frac{8\,ax\log\left(-\frac{ax+1}{ax-1}\right)-\left(\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a\right.\right.}{2\left(a^5x^4-2\,a^3x^2+a\right)\log\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*x*log(-(a*x + 1)/(a*x - 1)) - ((a^4*x^4 - 2*a^2*x^2 + 1)*log_inte
gral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 +
1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2
*a^2*x^2 + 1)*log_integral(-(a*x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1
)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 4)/((a^
5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^6x^6 \operatorname{atanh}^3(ax) - 3a^4x^4 \operatorname{atanh}^3(ax) + 3a^2x^2 \operatorname{atanh}^3(ax) - \operatorname{atanh}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**3 - 3*a**4*x**4*atanh(a*x)**3 + 3*a**2*x
**2*atanh(a*x)**3 - atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2 - 1\right)^3 \operatorname{artanh}(ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^3), x)
```

3.341 $\int \frac{1}{x(1-a^2x^2)^3 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=175

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2 \tanh^{-1}(ax)^2},x\right)}{2a}-\frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{a^2x^2+1}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)}+$$

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (a*x
)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - 2/((1 - a^2*x^2)^2*ArcTanh[a*x]) + 3/(
2*(1 - a^2*x^2)*ArcTanh[a*x]) - (1 + a^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]
) + (3*SinhIntegral[2*ArcTanh[a*x]])/2 + SinhIntegral[4*ArcTanh[a*x]] - Uni
ntegrable[1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi [A] time = 0.766691, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*x*ArcTanh[a*x]^2) - (a*x)/(2*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) - (a*x
)/(2*(1 - a^2*x^2)*ArcTanh[a*x]^2) - 2/((1 - a^2*x^2)^2*ArcTanh[a*x]) + 3/(
2*(1 - a^2*x^2)*ArcTanh[a*x]) - (1 + a^2*x^2)/(2*(1 - a^2*x^2)*ArcTanh[a*x]
) + (3*SinhIntegral[2*ArcTanh[a*x]])/2 + SinhIntegral[4*ArcTanh[a*x]] - Def
er[Int][1/(x^2*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx = a^2\int \frac{x}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx + \int \frac{1}{x\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} + \frac{1}{2}a\int \frac{1}{\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}\,dx + a^2\int \frac{}{\left(1-a^2x^2\right.}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{}{2(1}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{}{(1-}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{}{(1-}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{}{(1-}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{}{(1-}$$

$$= -\frac{1}{2ax\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2} - \frac{ax}{2\left(1-a^2x^2\right)\tanh^{-1}(ax)^2} - \frac{}{(1-}$$

Mathematica [A] time = 5.38553, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^3*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.221, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^3(\mathrm{Artanh}\,(ax))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,ax + \left(5\,a^2x^2-1\right)\log(ax+1) - \left(5\,a^2x^2-1\right)\log(-ax+1)}{\left(a^6x^6-2\,a^4x^4+a^2x^2\right)\log(ax+1)^2 - 2\left(a^6x^6-2\,a^4x^4+a^2x^2\right)\log(ax+1)\log(-ax+1) + \left(a^6x^6-2\,a^4x^4+a^2x^2\right)\mathrm{lo}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -(2*a*x + (5*a^2*x^2 - 1)*log(a*x + 1) - (5*a^2*x^2 - 1)*log(-a*x + 1))/((a
^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(a*x + 1)^2 - 2*(a^6*x^6 - 2*a^4*x^4 + a^2
*x^2)*log(a*x + 1)*log(-a*x + 1) + (a^6*x^6 - 2*a^4*x^4 + a^2*x^2)*log(-a*x
 + 1)^2) + integrate(-2*(10*a^4*x^4 - 3*a^2*x^2 + 1)/((a^8*x^9 - 3*a^6*x^7
+ 3*a^4*x^5 - a^2*x^3)*log(a*x + 1) - (a^8*x^9 - 3*a^6*x^7 + 3*a^4*x^5 - a^
2*x^3)*log(-a*x + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{1}{\left(a^6x^7 - 3\,a^4x^5 + 3\,a^2x^3 - x\right)\text{artanh}\left(ax\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^6*x^7 - 3*a^4*x^5 + 3*a^2*x^3 - x)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^6x^7\,\text{atanh}^3\left(ax\right) - 3a^4x^5\,\text{atanh}^3\left(ax\right) + 3a^2x^3\,\text{atanh}^3\left(ax\right) - x\,\text{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**3/atanh(a*x)**3,x)
```

```
[Out] -Integral(1/(a**6*x**7*atanh(a*x)**3 - 3*a**4*x**5*atanh(a*x)**3 + 3*a**2*x
**3*atanh(a*x)**3 - x*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2 - 1\right)^3 x\,\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^3/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*x*arctanh(a*x)^3), x)
```

3.342 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^4} dx$

Optimal. Leaf size=125

$$-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}+\frac{2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}+$$

```
[Out] -1/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) - (2*x)/(3*(1 - a^2*x^2)^2*ArcTanh[
a*x]^2) - 8/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 2/(a*(1 - a^2*x^2)*ArcTanh
[a*x]) + (2*SinhIntegral[2*ArcTanh[a*x]])/(3*a) + (4*SinhIntegral[4*ArcTanh
[a*x]])/(3*a)
```

Rubi [A] time = 0.499948, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 6032, 6028, 6034, 5448, 12, 3298}

$$-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}+\frac{2}{a\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}-\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```


```
[Out] -1/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) - (2*x)/(3*(1 - a^2*x^2)^2*ArcTanh[
a*x]^2) - 8/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 2/(a*(1 - a^2*x^2)*ArcTanh
[a*x]) + (2*SinhIntegral[2*ArcTanh[a*x]])/(3*a) + (4*SinhIntegral[4*ArcTanh
[a*x]])/(3*a)
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^4}\,dx = -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}+\frac{1}{3}(4a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^3}\,dx$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}+\frac{2}{3}\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(a}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{2}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

$$= -\frac{1}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^3}-\frac{2x}{3\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^2}-\frac{8}{3a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)}$$

Mathematica [A] time = 0.247077, size = 108, normalized size = 0.86

$$-\frac{-2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^3\mathbf{Shi}\left(2\tanh^{-1}(ax)\right)-4\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^3\mathbf{Shi}\left(4\tanh^{-1}(ax)\right)+6a^2x^2\tanh^{-1}(ax)^2+2}{3a\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^4),x]
```

```
[Out] -(1 + 2*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2 - 2*
(-1 + a^2*x^2)^2*ArcTanh[a*x]^3*SinhIntegral[2*ArcTanh[a*x]] - 4*(-1 + a^2*
x^2)^2*ArcTanh[a*x]^3*SinhIntegral[4*ArcTanh[a*x]])/(3*a*(-1 + a^2*x^2)^2*A
rcTanh[a*x]^3)
```

Maple [A] time = 0.073, size = 122, normalized size = 1.

$$\frac{1}{a}\left(-\frac{1}{8\left(\operatorname{Artanh}(ax)\right)^{3}}-\frac{\cosh(2\operatorname{Artanh}(ax))}{6\left(\operatorname{Artanh}(ax)\right)^{3}}-\frac{\sinh(2\operatorname{Artanh}(ax))}{6\left(\operatorname{Artanh}(ax)\right)^{2}}-\frac{\cosh(2\operatorname{Artanh}(ax))}{3\operatorname{Artanh}(ax)}+\frac{2\operatorname{Shi}(2\operatorname{Artanh}(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x)
```

```
[Out] 1/a*(-1/8/arctanh(a*x)^3-1/6/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1/6/arctan
h(a*x)^2*sinh(2*arctanh(a*x))-1/3/arctanh(a*x)*cosh(2*arctanh(a*x))+2/3*Shi
(2*arctanh(a*x))-1/24/arctanh(a*x)^3*cosh(4*arctanh(a*x))-1/12/arctanh(a*x)
^2*sinh(4*arctanh(a*x))-1/3/arctanh(a*x)*cosh(4*arctanh(a*x))+4/3*Shi(4*arc
tanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(2\,ax\log(ax+1)+\left(3\,a^{2}x^{2}+1\right)\log(ax+1)^{2}+\left(3\,a^{2}x^{2}+1\right)\log(-ax+1)^{2}-2\left(ax+\left(3\,a^{2}x^{2}\right.\right.\right.}{3\left(\left(a^{5}x^{4}-2\,a^{3}x^{2}+a\right)\log(ax+1)^{3}-3\left(a^{5}x^{4}-2\,a^{3}x^{2}+a\right)\log(ax+1)^{2}\log(-ax+1)+3\left(a^{5}x^{4}-2\,a^{3}x^{2}+a\right)\log(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -4/3*(2*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 1)
*log(-a*x + 1)^2 - 2*(a*x + (3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1) + 2
)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^3 - 3*(a^5*x^4 - 2*a^3*x^2 + a)*l
og(a*x + 1)^2*log(-a*x + 1) + 3*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(
-a*x + 1)^2 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^3) + integrate(-8/3*(
3*a^3*x^3 + 5*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a
^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.05897, size = 647, normalized size = 5.18

$$\frac{\left(2\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\operatorname{log\_integral}\left(\frac{a^{2}x^{2}+2\,ax+1}{a^{2}x^{2}-2\,ax+1}\right)-2\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\operatorname{log\_integral}\left(\frac{a^{2}x^{2}-2\,ax+1}{a^{2}x^{2}+2\,ax+1}\right)+\left(a^{4}x^{4}-2\,a^{2}x^{2}+\right.\right.}{3\left(a^{5}x^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) - 2*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^3 - 8*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3
*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))^2 - 8)/((a^5*x^4 - 2*a^3*x^2 + a)*l
og(-(a*x + 1)/(a*x - 1))^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^{6}x^{6}\operatorname{atanh}^{4}(ax) - 3a^{4}x^{4}\operatorname{atanh}^{4}(ax) + 3a^{2}x^{2}\operatorname{atanh}^{4}(ax) - \operatorname{atanh}^{4}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**4,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**4 - 3*a**4*x**4*atanh(a*x)**4 + 3*a**2*x
**2*atanh(a*x)**4 - atanh(a*x)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^{2}x^{2}-1\right)^{3}\operatorname{artanh}(ax)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^4), x)
```

3.343 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx$

Optimal. Leaf size=170

$$\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}+\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{}{3a\left(1\right.}$$

```
[Out] -1/(4*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^4) - x/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]
^3) - 2/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a*(1 - a^2*x^2)*ArcTanh
[a*x]^2) - (8*x)/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/((1 - a^2*x^2)*ArcTan
h[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a) + (4*CoshIntegral[4*ArcTanh[a*
x]])/(3*a)
```

Rubi [A] time = 0.963358, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5966, 6032, 6028, 6034, 3312, 3301, 5968, 5448}

$$\frac{x}{\left(1-a^2x^2\right)\tanh^{-1}(ax)}-\frac{8x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)}-\frac{x}{3\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^3}+\frac{1}{2a\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{}{3a\left(1-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```


```
[Out] -1/(4*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^4) - x/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]
^3) - 2/(3*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + 1/(2*a*(1 - a^2*x^2)*ArcTanh
[a*x]^2) - (8*x)/(3*(1 - a^2*x^2)^2*ArcTanh[a*x]) + x/((1 - a^2*x^2)*ArcTan
h[a*x]) + CoshIntegral[2*ArcTanh[a*x]]/(3*a) + (4*CoshIntegral[4*ArcTanh[a*
x]])/(3*a)
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
```

```
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}(ax)^5}\,dx = -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} + a\int \frac{x}{(1-a^2x^2)^3 \tanh^{-1}(ax)^4}\,dx$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} + \frac{1}{3}\int \frac{1}{(1-a^2x^2)^3 \tanh^{-1}}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{1}{6a(1-a^2x^2)^2 \tanh^{-1}(a}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \tanh^{-1}(a}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \tanh^{-1}(a}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \tanh^{-1}(a}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \tanh^{-1}(a}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \tanh^{-1}(a}$$
$$= -\frac{1}{4a(1-a^2x^2)^2 \tanh^{-1}(ax)^4} - \frac{x}{3(1-a^2x^2)^2 \tanh^{-1}(ax)^3} - \frac{2}{3a(1-a^2x^2)^2 \tanh^{-1}(a}$$

Mathematica [A] time = 0.164388, size = 132, normalized size = 0.78

$$-\frac{-4(a^2x^2-1)^2 \tanh^{-1}(ax)^4 \mathrm{Chi}(2\tanh^{-1}(ax)) - 16(a^2x^2-1)^2 \tanh^{-1}(ax)^4 \mathrm{Chi}(4\tanh^{-1}(ax)) + 12a^3x^3 \tanh^{-1}(ax}{12a(a^2x^2-1)^2 \tanh^{-1}(ax)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^5),x]
```

```
[Out] -(3 + 4*a*x*ArcTanh[a*x] + 2*ArcTanh[a*x]^2 + 6*a^2*x^2*ArcTanh[a*x]^2 + 20
*a*x*ArcTanh[a*x]^3 + 12*a^3*x^3*ArcTanh[a*x]^3 - 4*(-1 + a^2*x^2)^2*ArcTan
h[a*x]^4*CoshIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^4*
CoshIntegral[4*ArcTanh[a*x]])/(12*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^4)
```

Maple [A] time = 0.071, size = 152, normalized size = 0.9

$$\frac{1}{a}\left(-\frac{3}{32(\mathrm{Artanh}(ax))^4} - \frac{\cosh(2\,\mathrm{Artanh}(ax))}{8(\mathrm{Artanh}(ax))^4} - \frac{\sinh(2\,\mathrm{Artanh}(ax))}{12(\mathrm{Artanh}(ax))^3} - \frac{\cosh(2\,\mathrm{Artanh}(ax))}{12(\mathrm{Artanh}(ax))^2} - \frac{\sinh(2\,\mathrm{Artanh}(}{6\,\mathrm{Artanh}(ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x)
```

```
[Out] 1/a*(-3/32/arctanh(a*x)^4-1/8/arctanh(a*x)^4*cosh(2*arctanh(a*x))-1/12/arct
anh(a*x)^3*sinh(2*arctanh(a*x))-1/12/arctanh(a*x)^2*cosh(2*arctanh(a*x))-1/
```

```
6/arctanh(a*x)*sinh(2*arctanh(a*x))+1/3*Chi(2*arctanh(a*x))-1/32/arctanh(a*
x)^4*cosh(4*arctanh(a*x))-1/24/arctanh(a*x)^3*sinh(4*arctanh(a*x))-1/12/arc
tanh(a*x)^2*cosh(4*arctanh(a*x))-1/3/arctanh(a*x)*sinh(4*arctanh(a*x))+4/3*
Chi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(3\,a^3x^3+5\,ax\right)\log\left(ax+1\right)^3-\left(3\,a^3x^3+5\,ax\right)\log\left(-ax+1\right)^3+4\,ax\log\left(ax+1\right)+\left(3\,a^2x^2+1\right)\log\left(ax+1\right)^2+\left(3\,a^2x^2\right.\right.}{3\left(\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)^4-4\left(a^5x^4-2\,a^3x^2+a\right)\log\left(ax+1\right)^3\log\left(-ax+1\right)+6\left(a^5x^4-2\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="maxima")
```

```
[Out] -2/3*((3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - (3*a^3*x^3 + 5*a*x)*log(-a*x + 1
)^3 + 4*a*x*log(a*x + 1) + (3*a^2*x^2 + 1)*log(a*x + 1)^2 + (3*a^2*x^2 + 3*
(3*a^3*x^3 + 5*a*x)*log(a*x + 1) + 1)*log(-a*x + 1)^2 - (3*(3*a^3*x^3 + 5*a
*x)*log(a*x + 1)^2 + 4*a*x + 2*(3*a^2*x^2 + 1)*log(a*x + 1))*log(-a*x + 1)
+ 6)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4 - 4*(a^5*x^4 - 2*a^3*x^2 + a
)*log(a*x + 1)^3*log(-a*x + 1) + 6*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^2
*log(-a*x + 1)^2 - 4*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log(-a*x + 1)^3
 + (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^4) + integrate(-2/3*(3*a^4*x^4 +
 24*a^2*x^2 + 5)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) - (a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.03757, size = 720, normalized size = 4.24

$$\frac{\left(4\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+4\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{log\_integral}\left(\frac{a^2x^2-2\,ax+1}{a^2x^2+2\,ax+1}\right)+\left(a^4x^4-2\,a^2x^2+1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="fricas")
```

```
[Out] 1/6*((4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x
^2 - 2*a*x + 1)) + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a*
x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*
x + 1)/(a*x - 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
+ 1)))*log(-(a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(
a*x - 1))^3 - 16*a*x*log(-(a*x + 1)/(a*x - 1)) - 4*(3*a^2*x^2 + 1)*log(-(a*
x + 1)/(a*x - 1))^2 - 24)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x -
1))^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{a^6x^6\,\mathrm{atanh}^5\left(ax\right)-3a^4x^4\,\mathrm{atanh}^5\left(ax\right)+3a^2x^2\,\mathrm{atanh}^5\left(ax\right)-\mathrm{atanh}^5\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**5,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**5 - 3*a**4*x**4*atanh(a*x)**5 + 3*a**2*x
**2*atanh(a*x)**5 - atanh(a*x)**5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 \operatorname{artanh}\left(ax\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^5,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^5), x)
```

3.344 $\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^6} \, dx$

Optimal. Leaf size=257

$$\frac{x}{5\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{8x}{15\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{x}{5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}+\frac{a^2x^2+1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{}{5a\left(1\right.}$$

```
[Out] -1/(5*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^5) - x/(5*(1 - a^2*x^2)^2*ArcTanh[a*x]
^4) - 4/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + 1/(5*a*(1 - a^2*x^2)*ArcTan
h[a*x]^3) - (8*x)/(15*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(5*(1 - a^2*x^2)*
ArcTanh[a*x]^2) - 32/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 8/(5*a*(1 - a^2*
x^2)*ArcTanh[a*x]) + (1 + a^2*x^2)/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Si
nhIntegral[2*ArcTanh[a*x]])/(15*a) + (16*SinhIntegral[4*ArcTanh[a*x]])/(15*
a)
```

Rubi [A] time = 1.34905, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5966, 6032, 6028, 5996, 6034, 5448, 12, 3298}

$$\frac{x}{5\left(1-a^2x^2\right)\tanh^{-1}(ax)^2}-\frac{8x}{15\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^2}-\frac{x}{5\left(1-a^2x^2\right)^2\tanh^{-1}(ax)^4}+\frac{a^2x^2+1}{5a\left(1-a^2x^2\right)\tanh^{-1}(ax)}+\frac{}{5a\left(1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

```
[Out] -1/(5*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^5) - x/(5*(1 - a^2*x^2)^2*ArcTanh[a*x]
^4) - 4/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]^3) + 1/(5*a*(1 - a^2*x^2)*ArcTan
h[a*x]^3) - (8*x)/(15*(1 - a^2*x^2)^2*ArcTanh[a*x]^2) + x/(5*(1 - a^2*x^2)*
ArcTanh[a*x]^2) - 32/(15*a*(1 - a^2*x^2)^2*ArcTanh[a*x]) + 8/(5*a*(1 - a^2*
x^2)*ArcTanh[a*x]) + (1 + a^2*x^2)/(5*a*(1 - a^2*x^2)*ArcTanh[a*x]) + (2*Si
nhIntegral[2*ArcTanh[a*x]])/(15*a) + (16*SinhIntegral[4*ArcTanh[a*x]])/(15*
a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5996

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[(x*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(p + 1)*(d + e*
x^2)), x] + (Dist[4/(b^2*(p + 1)*(p + 2)), Int[(x*(a + b*ArcTanh[c*x])^(p +
 2))/(d + e*x^2)^2, x], x] + Simp[((1 + c^2*x^2)*(a + b*ArcTanh[c*x])^(p +
2))/(b^2*e*(p + 1)*(p + 2)*(d + e*x^2)), x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[c^2*d + e, 0] && LtQ[p, -1] && NeQ[p, -2]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^6}\,dx = -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} + \frac{1}{5}(4a)\int \frac{x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^5}\,dx$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} + \frac{1}{5}\int \frac{1}{\left(1-a^2x^2\right)^3 \tanh^{-1}(a$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{1}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

$$= -\frac{1}{5a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^5} - \frac{x}{5\left(1-a^2x^2\right)^2 \tanh^{-1}(ax)^4} - \frac{4}{15a\left(1-a^2x^2\right)^2 \tanh^{-1}(ax}$$

Mathematica [A] time = 0.300222, size = 166, normalized size = 0.65

$$-\frac{-2\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^5 \mathrm{Shi}\left(2\tanh^{-1}(ax)\right) - 16\left(a^2x^2-1\right)^2 \tanh^{-1}(ax)^5 \mathrm{Shi}\left(4\tanh^{-1}(ax)\right) + 3a^4x^4\tanh^{-1}(ax)^4 + }{15a\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^3*ArcTanh[a*x]^6),x]
```

```
[Out] -(3 + 3*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2 + 3*a^2*x^2*ArcTanh[a*x]^2 + 5*a*
x*ArcTanh[a*x]^3 + 3*a^3*x^3*ArcTanh[a*x]^3 + 5*ArcTanh[a*x]^4 + 24*a^2*x^2
*ArcTanh[a*x]^4 + 3*a^4*x^4*ArcTanh[a*x]^4 - 2*(-1 + a^2*x^2)^2*ArcTanh[a*x
]^5*SinhIntegral[2*ArcTanh[a*x]] - 16*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5*SinhI
ntegral[4*ArcTanh[a*x]])/(15*a*(-1 + a^2*x^2)^2*ArcTanh[a*x]^5)
```

Maple [A] time = 0.07, size = 182, normalized size = 0.7

$$\frac{1}{a}\left(-\frac{3}{40\left(\mathrm{Artanh}(ax)\right)^5} - \frac{\cosh(2\,\mathrm{Artanh}(ax))}{10\left(\mathrm{Artanh}(ax)\right)^5} - \frac{\sinh(2\,\mathrm{Artanh}(ax))}{20\left(\mathrm{Artanh}(ax)\right)^4} - \frac{\cosh(2\,\mathrm{Artanh}(ax))}{30\left(\mathrm{Artanh}(ax)\right)^3} - \frac{\sinh(2\,\mathrm{Artanh}(ax))}{30\left(\mathrm{Artanh}(ax)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x)
```

```
[Out] 1/a*(-3/40/arctanh(a*x)^5-1/10/arctanh(a*x)^5*cosh(2*arctanh(a*x))-1/20/arc
tanh(a*x)^4*sinh(2*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(2*arctanh(a*x))-1
/30/arctanh(a*x)^2*sinh(2*arctanh(a*x))-1/15/arctanh(a*x)*cosh(2*arctanh(a*
x))+2/15*Shi(2*arctanh(a*x))-1/40/arctanh(a*x)^5*cosh(4*arctanh(a*x))-1/40/
arctanh(a*x)^4*sinh(4*arctanh(a*x))-1/30/arctanh(a*x)^3*cosh(4*arctanh(a*x)
)-1/15/arctanh(a*x)^2*sinh(4*arctanh(a*x))-4/15/arctanh(a*x)*cosh(4*arctanh
(a*x))+16/15*Shi(4*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="maxima")
```

```
[Out] -2/15*((3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^4 + (3*a^4*x^4 + 24*a^2*x^
2 + 5)*log(-a*x + 1)^4 + 2*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^3 - 2*(3*a^3*x^
3 + 5*a*x + 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1))*log(-a*x + 1)^3 +
24*a*x*log(a*x + 1) + 4*(3*a^2*x^2 + 1)*log(a*x + 1)^2 + 2*(6*a^2*x^2 + 3*(
3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^2 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x
+ 1) + 2)*log(-a*x + 1)^2 - 2*(2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(a*x + 1)^
3 + 3*(3*a^3*x^3 + 5*a*x)*log(a*x + 1)^2 + 12*a*x + 4*(3*a^2*x^2 + 1)*log(a
*x + 1))*log(-a*x + 1) + 48)/((a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^5 - 5*
(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)^4*log(-a*x + 1) + 10*(a^5*x^4 - 2*a^
3*x^2 + a)*log(a*x + 1)^3*log(-a*x + 1)^2 - 10*(a^5*x^4 - 2*a^3*x^2 + a)*lo
g(a*x + 1)^2*log(-a*x + 1)^3 + 5*(a^5*x^4 - 2*a^3*x^2 + a)*log(a*x + 1)*log
(-a*x + 1)^4 - (a^5*x^4 - 2*a^3*x^2 + a)*log(-a*x + 1)^5) + integrate(-8/15
*(15*a^3*x^3 + 17*a*x)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)
- (a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-a*x + 1)), x)
```

Fricas [A] time = 1.99863, size = 803, normalized size = 3.12

$$\frac{\left(8\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-8\left(a^4x^4-2a^2x^2+1\right)\text{log\_integral}\left(\frac{a^2x^2-2ax+1}{a^2x^2+2ax+1}\right)+\left(a^4x^4-2a^2x^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) - 8*(a^4*x^4 - 2*a^2*x^2 + 1)*log_integral((a^2*x^2 - 2*a
*x + 1)/(a^2*x^2 + 2*a*x + 1)) + (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a
*x + 1)/(a*x - 1)) - (a^4*x^4 - 2*a^2*x^2 + 1)*log_integral(-(a*x - 1)/(a*x
 + 1)))*log(-(a*x + 1)/(a*x - 1))^5 - 2*(3*a^4*x^4 + 24*a^2*x^2 + 5)*log(-(
a*x + 1)/(a*x - 1))^4 - 4*(3*a^3*x^3 + 5*a*x)*log(-(a*x + 1)/(a*x - 1))^3 -
 48*a*x*log(-(a*x + 1)/(a*x - 1)) - 8*(3*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x -
 1))^2 - 96)/((a^5*x^4 - 2*a^3*x^2 + a)*log(-(a*x + 1)/(a*x - 1))^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{a^6x^6 \operatorname{atanh}^6(ax) - 3a^4x^4 \operatorname{atanh}^6(ax) + 3a^2x^2 \operatorname{atanh}^6(ax) - \operatorname{atanh}^6(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**3/atanh(a*x)**6,x)
```

```
[Out] -Integral(1/(a**6*x**6*atanh(a*x)**6 - 3*a**4*x**4*atanh(a*x)**6 + 3*a**2*x
**2*atanh(a*x)**6 - atanh(a*x)**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(a^2x^2-1\right)^3 \operatorname{artanh}(ax)^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^3/arctanh(a*x)^6,x, algorithm="giac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)^3*arctanh(a*x)^6), x)
```

3.345 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^4} dx$

Optimal. Leaf size=134

$$-\frac{5}{32a(1-a^2x^2)}-\frac{5}{96a(1-a^2x^2)^2}-\frac{1}{36a(1-a^2x^2)^3}+\frac{5x\tanh^{-1}(ax)}{16(1-a^2x^2)}+\frac{5x\tanh^{-1}(ax)}{24(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{6(1-a^2x^2)^3}+\frac{5\tanh^{-1}(}{32a}$$

```
[Out] -1/(36*a*(1 - a^2*x^2)^3) - 5/(96*a*(1 - a^2*x^2)^2) - 5/(32*a*(1 - a^2*x^2
)) + (x*ArcTanh[a*x])/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x])/(24*(1 - a^2
*x^2)^2) + (5*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^2)/(32*a
)
```

Rubi [A] time = 0.0738376, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5960, 5956, 261}

$$-\frac{5}{32a(1-a^2x^2)}-\frac{5}{96a(1-a^2x^2)^2}-\frac{1}{36a(1-a^2x^2)^3}+\frac{5x\tanh^{-1}(ax)}{16(1-a^2x^2)}+\frac{5x\tanh^{-1}(ax)}{24(1-a^2x^2)^2}+\frac{x\tanh^{-1}(ax)}{6(1-a^2x^2)^3}+\frac{5\tanh^{-1}(}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
```

```
[Out] -1/(36*a*(1 - a^2*x^2)^3) - 5/(96*a*(1 - a^2*x^2)^2) - 5/(32*a*(1 - a^2*x^2
)) + (x*ArcTanh[a*x])/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x])/(24*(1 - a^2
*x^2)^2) + (5*x*ArcTanh[a*x])/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^2)/(32*a
)
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^4}\,dx = -\frac{1}{36a(1-a^2x^2)^3} + \frac{x\tanh^{-1}(ax)}{6(1-a^2x^2)^3} + \frac{5}{6}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^3}\,dx$$
$$= -\frac{1}{36a(1-a^2x^2)^3} - \frac{5}{96a(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)}{6(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)}{24(1-a^2x^2)^2} + \frac{5}{8}\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^2}\,dx$$
$$= -\frac{1}{36a(1-a^2x^2)^3} - \frac{5}{96a(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)}{6(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)}{24(1-a^2x^2)^2} + \frac{5x\tanh^{-1}(ax)}{16(1-a^2x^2)} + \frac{5\tanh^{-1}(}{32a}$$
$$= -\frac{1}{36a(1-a^2x^2)^3} - \frac{5}{96a(1-a^2x^2)^2} - \frac{5}{32a(1-a^2x^2)} + \frac{x\tanh^{-1}(ax)}{6(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)}{24(1-a^2x^2)^2} + \frac{5x\tanh^{-1}}{16(1-a^2}$$

Mathematica [A] time = 0.182143, size = 81, normalized size = 0.6

$$\frac{45a^4x^4 - 105a^2x^2 - 6ax(15a^4x^4 - 40a^2x^2 + 33)\tanh^{-1}(ax) + 45(a^2x^2-1)^3\tanh^{-1}(ax)^2 + 68}{288a(a^2x^2-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^4,x]
```

```
[Out] (68 - 105*a^2*x^2 + 45*a^4*x^4 - 6*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTa
nh[a*x] + 45*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2)/(288*a*(-1 + a^2*x^2)^3)
```

Maple [B] time = 0.059, size = 281, normalized size = 2.1

$$-\frac{\operatorname{Artanh}(ax)}{48a(ax-1)^3} + \frac{\operatorname{Artanh}(ax)}{16a(ax-1)^2} - \frac{5\operatorname{Artanh}(ax)}{32a(ax-1)} - \frac{5\operatorname{Artanh}(ax)\ln(ax-1)}{32a} - \frac{\operatorname{Artanh}(ax)}{48a(ax+1)^3} - \frac{\operatorname{Artanh}(ax)}{16a(ax+1)^2} - \frac{5\operatorname{Arta}}{32a(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^4,x)
```

```
[Out] -1/48/a*arctanh(a*x)/(a*x-1)^3+1/16/a*arctanh(a*x)/(a*x-1)^2-5/32/a*arctanh
(a*x)/(a*x-1)-5/32/a*arctanh(a*x)*ln(a*x-1)-1/48/a*arctanh(a*x)/(a*x+1)^3-1
/16/a*arctanh(a*x)/(a*x+1)^2-5/32/a*arctanh(a*x)/(a*x+1)+5/32/a*arctanh(a*x
)*ln(a*x+1)-5/128/a*ln(a*x-1)^2+5/64/a*ln(a*x-1)*ln(1/2+1/2*a*x)-5/128/a*ln
(a*x+1)^2-5/64/a*ln(-1/2*a*x+1/2)*ln(1/2+1/2*a*x)+5/64/a*ln(-1/2*a*x+1/2)*l
n(a*x+1)+1/288/a/(a*x-1)^3-7/384/a/(a*x-1)^2+37/384/a/(a*x-1)-1/288/a/(a*x+
1)^3-7/384/a/(a*x+1)^2-37/384/a/(a*x+1)
```

Maxima [B] time = 1.00347, size = 324, normalized size = 2.42

$$-\frac{1}{96}\left(\frac{2(15a^4x^5 - 40a^2x^3 + 33x)}{a^6x^6 - 3a^4x^4 + 3a^2x^2 - 1} - \frac{15\log(ax+1)}{a} + \frac{15\log(ax-1)}{a}\right)\operatorname{artanh}(ax) + \frac{(180a^4x^4 - 420a^2x^2 - 45(a^6x^6 -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x) + 1/1152*(180*a^
4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)
^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1) - 4
5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a/(a^8*x^6 -
3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

Fricas [A] time = 1.94932, size = 293, normalized size = 2.19

$$\frac{180\,a^4x^4 - 420\,a^2x^2 + 45\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 12\left(15\,a^5x^5 - 40\,a^3x^3 + 33\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right) + 272}{1152\left(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/1152*(180*a^4*x^4 - 420*a^2*x^2 + 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1
)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*log(-
(a*x + 1)/(a*x - 1)) + 272)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}\left(ax\right)}{\left(ax - 1\right)^4\left(ax + 1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}\left(ax\right)}{\left(a^2x^2 - 1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(a^2*x^2 - 1)^4, x)
```

3.346 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^4} dx$

Optimal. Leaf size=214

$$\frac{245x}{1152(1-a^2x^2)} + \frac{65x}{1728(1-a^2x^2)^2} + \frac{x}{108(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)^2}{16(1-a^2x^2)} + \frac{5x\tanh^{-1}(ax)^2}{24(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)^2}{6(1-a^2x^2)^3} - \frac{5\tanh^{-1}}{16a(1-}$$

```
[Out] x/(108*(1 - a^2*x^2)^3) + (65*x)/(1728*(1 - a^2*x^2)^2) + (245*x)/(1152*(1
- a^2*x^2)) + (245*ArcTanh[a*x])/(1152*a) - ArcTanh[a*x]/(18*a*(1 - a^2*x^2
)^3) - (5*ArcTanh[a*x])/(48*a*(1 - a^2*x^2)^2) - (5*ArcTanh[a*x])/(16*a*(1
- a^2*x^2)) + (x*ArcTanh[a*x]^2)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^2)
/(24*(1 - a^2*x^2)^2) + (5*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + (5*ArcTan
h[a*x]^3)/(48*a)
```

Rubi [A] time = 0.168887, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 199, 206}

$$\frac{245x}{1152(1-a^2x^2)} + \frac{65x}{1728(1-a^2x^2)^2} + \frac{x}{108(1-a^2x^2)^3} + \frac{5x\tanh^{-1}(ax)^2}{16(1-a^2x^2)} + \frac{5x\tanh^{-1}(ax)^2}{24(1-a^2x^2)^2} + \frac{x\tanh^{-1}(ax)^2}{6(1-a^2x^2)^3} - \frac{5\tanh^{-1}}{16a(1-}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

```
[Out] x/(108*(1 - a^2*x^2)^3) + (65*x)/(1728*(1 - a^2*x^2)^2) + (245*x)/(1152*(1
- a^2*x^2)) + (245*ArcTanh[a*x])/(1152*a) - ArcTanh[a*x]/(18*a*(1 - a^2*x^2
)^3) - (5*ArcTanh[a*x])/(48*a*(1 - a^2*x^2)^2) - (5*ArcTanh[a*x])/(16*a*(1
- a^2*x^2)) + (x*ArcTanh[a*x]^2)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^2)
/(24*(1 - a^2*x^2)^2) + (5*x*ArcTanh[a*x]^2)/(16*(1 - a^2*x^2)) + (5*ArcTan
h[a*x]^3)/(48*a)
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
```

```
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^4}\,dx = -\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}+\frac{1}{18}\int \frac{1}{\left(1-a^2x^2\right)^4}\,dx+\frac{5}{6}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^3}\,dx$$

$$= \frac{x}{108\left(1-a^2x^2\right)^3}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^2}{24\left(1-a^2x^2\right)^2}+\frac{5}{108}$$

$$= \frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^2}{6\left(1-a^2x^2\right)^3}+\frac{5x}{24}$$

$$= \frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{65x}{1152\left(1-a^2x^2\right)}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)}{48a\left(1-a^2x^2\right)^2}-\frac{5}{16}$$

$$= \frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{65\tanh^{-1}(ax)}{1152a}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5}{48}$$

$$= \frac{x}{108\left(1-a^2x^2\right)^3}+\frac{65x}{1728\left(1-a^2x^2\right)^2}+\frac{245x}{1152\left(1-a^2x^2\right)}+\frac{245\tanh^{-1}(ax)}{1152a}-\frac{\tanh^{-1}(ax)}{18a\left(1-a^2x^2\right)^3}-\frac{5}{48}$$

Mathematica [A] time = 0.288938, size = 157, normalized size = 0.73

$$\frac{-\frac{1470x}{a^2x^2-1}+\frac{260x}{\left(a^2x^2-1\right)^2}-\frac{64x}{\left(a^2x^2-1\right)^3}-\frac{144x\left(15a^4x^4-40a^2x^2+33\right)\tanh^{-1}(ax)^2}{\left(a^2x^2-1\right)^3}+\frac{48\left(45a^4x^4-105a^2x^2+68\right)\tanh^{-1}(ax)}{a\left(a^2x^2-1\right)^3}-\frac{735\log(1-ax)}{a}+\frac{735\log(a}{a}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^4,x]
```

```
[Out] ((-64*x)/(-1 + a^2*x^2)^3 + (260*x)/(-1 + a^2*x^2)^2 - (1470*x)/(-1 + a^2*x
^2) + (48*(68 - 105*a^2*x^2 + 45*a^4*x^4)*ArcTanh[a*x])/(a*(-1 + a^2*x^2)^3
) - (144*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^2)/(-1 + a^2*x^2)^3
+ (720*ArcTanh[a*x]^3)/a - (735*Log[1 - a*x])/a + (735*Log[1 + a*x])/a)/691
2
```

Maple [C] time = 0.512, size = 3447, normalized size = 16.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^4,x)
```

```
[Out] 5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*Pi*x^6-15/32*I*a^3/(a*x-1)^3/
(a*x+1)^3*arctanh(a*x)^2*Pi*x^4+15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^
2*Pi*x^2-5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(
-a^2*x^2+1)+1))^3+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I/((a
*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^2+5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^
2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2+5/64*I/a/(a*x-1)^3/(a*x+1)^
3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(
a^2*x^2-1))-5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+5/
32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))^3*Pi*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2
/(a^2*x^2-1))^3*Pi*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^6-5/32*I*a^5/(a*x
-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^6-
15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+
1)+1))^3*Pi*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+
1)^2/(a^2*x^2-1))^3*Pi*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^4+15/32*I*a^
3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*P
i*x^4+15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*
x^2+1)+1))^3*Pi*x^2-15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*
x+1)^2/(a^2*x^2-1))^3*Pi*x^2-15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2-15/32*I*a/
(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*
x^2+15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-
1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-5/64*
I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^6-5/32*I*a
^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(
I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^6+5/48*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a
*x)^3*x^6+245/1152*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^6-5/16*a^3/(a*x-1
)^3/(a*x+1)^3*arctanh(a*x)^3*x^4-125/384*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*
x)*x^4+5/16*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*x^2-35/384*a/(a*x-1)^3/(a*
x+1)^3*arctanh(a*x)*x^2-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(
I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^6+5/64*I
*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(
I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^6+15/64*I*a^3/(a
*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^4+15/32*I*a^3/(a*x
-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+
1)/(-a^2*x^2+1)^(1/2))*Pi*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^
2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^4
-15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^4-15/64*
I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*c
sgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-15/32*I*a/
(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(
a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^2-5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a
*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn
(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))-15/64*I*a/(a*x-1)^3/(a
```

```
*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)/(-a^2*x
^2+1)^(1/2))^2*Pi*x^2-245/1152*a^4/(a*x-1)^3/(a*x+1)^3*x^5+25/54*a^2/(a*x-1
)^3/(a*x+1)^3*x^3-5/48/a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3+299/1152/a/(a*x
-1)^3/(a*x+1)^3*arctanh(a*x)-1/48/a*arctanh(a*x)^2/(a*x+1)^3-15/64*I*a^3/(a
*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)
+1))*Pi*x^4+5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2*csgn(I*(a*x+1)^2/
(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1
)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^6+15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(
a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csg
n(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^2+1/16/a*arctanh
(a*x)^2/(a*x-1)^2-5/32/a*arctanh(a*x)^2/(a*x-1)-5/32/a*arctanh(a*x)^2*ln(a*
x-1)-1/16/a*arctanh(a*x)^2/(a*x+1)^2-5/32/a*arctanh(a*x)^2/(a*x+1)+5/32/a*a
rctanh(a*x)^2*ln(a*x+1)-5/16/a*arctanh(a*x)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2)
)-299/1152/(a*x-1)^3/(a*x+1)^3*x-1/48/a*arctanh(a*x)^2/(a*x-1)^3-5/32*I/a/(
a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctan
h(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*ar
ctanh(a*x)^2*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^3+5/3
2*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^2*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+
1))^2
```

Maxima [B] time = 1.03495, size = 697, normalized size = 3.26

$$-\frac{1}{96}\left(\frac{2\left(15\,a^4x^5-40\,a^2x^3+33\,x\right)}{a^6x^6-3\,a^4x^4+3\,a^2x^2-1}-\frac{15\,\log\left(ax+1\right)}{a}+\frac{15\,\log\left(ax-1\right)}{a}\right)\operatorname{artanh}\left(ax\right)^2-\frac{\left(1470\,a^5x^5-3200\,a^3x^3-90\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^2 - 1/6912*(1470
*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
+ 1)^3 + 270*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x -
 1) + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x -
15*(49*a^6*x^6 - 147*a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^
2*x^2 - 1)*log(a*x - 1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3
*a^2*x^2 - 1)*log(a*x - 1))*a^2/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 - a^3) + 1
/576*(180*a^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*
log(a*x + 1)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(
a*x - 1) - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a
*arctanh(a*x)/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)
```

Fricas [A] time = 2.03648, size = 412, normalized size = 1.93

$$-\frac{1470\,a^5x^5-3200\,a^3x^3-90\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+36\left(15\,a^5x^5-40\,a^3x^3+33\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2}{6912\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] -1/6912*(1470*a^5*x^5 - 3200*a^3*x^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
- 1)*log(-(a*x + 1)/(a*x - 1))^3 + 36*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*lo
```

```
g(-(a*x + 1)/(a*x - 1))^2 + 1794*a*x - 3*(245*a^6*x^6 - 375*a^4*x^4 - 105*a
^2*x^2 + 299)*log(-(a*x + 1)/(a*x - 1)))/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{(ax-1)^4(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)**2/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(a^2x^2-1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(a^2*x^2 - 1)^4, x)
```

3.347 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^4}\,dx$

Optimal. Leaf size=291

$$-\frac{245}{768a\left(1-a^2x^2\right)}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^3}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^3}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}-\frac{15}{3}$$

```
[Out] -1/(216*a*(1 - a^2*x^2)^3) - 65/(2304*a*(1 - a^2*x^2)^2) - 245/(768*a*(1 -
a^2*x^2)) + (x*ArcTanh[a*x])/(36*(1 - a^2*x^2)^3) + (65*x*ArcTanh[a*x])/(57
6*(1 - a^2*x^2)^2) + (245*x*ArcTanh[a*x])/(384*(1 - a^2*x^2)) + (245*ArcTan
h[a*x]^2)/(768*a) - ArcTanh[a*x]^2/(12*a*(1 - a^2*x^2)^3) - (5*ArcTanh[a*x]
^2)/(32*a*(1 - a^2*x^2)^2) - (15*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) + (x*
ArcTanh[a*x]^3)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^3)/(24*(1 - a^2*x^2
)^2) + (5*x*ArcTanh[a*x]^3)/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^4)/(64*a)
```

Rubi [A] time = 0.326792, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5964, 5956, 5994, 261, 5960}

$$-\frac{245}{768a\left(1-a^2x^2\right)}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{5x\tanh^{-1}(ax)^3}{16\left(1-a^2x^2\right)}+\frac{5x\tanh^{-1}(ax)^3}{24\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}-\frac{15}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

```
[Out] -1/(216*a*(1 - a^2*x^2)^3) - 65/(2304*a*(1 - a^2*x^2)^2) - 245/(768*a*(1 -
a^2*x^2)) + (x*ArcTanh[a*x])/(36*(1 - a^2*x^2)^3) + (65*x*ArcTanh[a*x])/(57
6*(1 - a^2*x^2)^2) + (245*x*ArcTanh[a*x])/(384*(1 - a^2*x^2)) + (245*ArcTan
h[a*x]^2)/(768*a) - ArcTanh[a*x]^2/(12*a*(1 - a^2*x^2)^3) - (5*ArcTanh[a*x]
^2)/(32*a*(1 - a^2*x^2)^2) - (15*ArcTanh[a*x]^2)/(32*a*(1 - a^2*x^2)) + (x*
ArcTanh[a*x]^3)/(6*(1 - a^2*x^2)^3) + (5*x*ArcTanh[a*x]^3)/(24*(1 - a^2*x^2
)^2) + (5*x*ArcTanh[a*x]^3)/(16*(1 - a^2*x^2)) + (5*ArcTanh[a*x]^4)/(64*a)
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5956

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sy
mbol] :> Simp[(x*(a + b*ArcTanh[c*x])^p)/(2*d*(d + e*x^2)), x] + (-Dist[(b*
c*p)/2, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(d + e*x^2)^2, x], x] + Simp[(
a + b*ArcTanh[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x]) /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^4}\,dx = -\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right)^3}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}+\frac{1}{6}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^4}\,dx+\frac{5}{6}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^3}\,dx$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}-\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right)^3}-\frac{5\tanh^{-1}(ax)^2}{32a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)^3}{6\left(1-a^2x^2\right)^3}+\frac{5x\tan\ldots}{24(1\ldots}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}-\frac{\tanh^{-1}(ax)^2}{12a\left(1-a^2x^2\right)^3}-\frac{5\ldots}{32\ldots}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}+\frac{65x\tanh^{-1}(ax)}{384\left(1-a^2x^2\right)}+\frac{6\ldots}{\ldots}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{65}{768a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}+\frac{2\ldots}{\ldots}$$

$$= -\frac{1}{216a\left(1-a^2x^2\right)^3}-\frac{65}{2304a\left(1-a^2x^2\right)^2}-\frac{245}{768a\left(1-a^2x^2\right)}+\frac{x\tanh^{-1}(ax)}{36\left(1-a^2x^2\right)^3}+\frac{65x\tanh^{-1}(ax)}{576\left(1-a^2x^2\right)^2}+\frac{2\ldots}{\ldots}$$

Mathematica [A] time = 0.153052, size = 143, normalized size = 0.49

$$\frac{2205a^4x^4-4605a^2x^2-144ax\left(15a^4x^4-40a^2x^2+33\right)\tanh^{-1}(ax)^3-6ax\left(735a^4x^4-1600a^2x^2+897\right)\tanh^{-1}(ax)+540\ldots}{6912a\left(a^2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^4,x]
```

```
[Out] (2432 - 4605*a^2*x^2 + 2205*a^4*x^4 - 6*a*x*(897 - 1600*a^2*x^2 + 735*a^4*x
^4)*ArcTanh[a*x] + 9*(299 - 105*a^2*x^2 - 375*a^4*x^4 + 245*a^6*x^6)*ArcTan
h[a*x]^2 - 144*a*x*(33 - 40*a^2*x^2 + 15*a^4*x^4)*ArcTanh[a*x]^3 + 540*(-1
+ a^2*x^2)^3*ArcTanh[a*x]^4)/(6912*a*(-1 + a^2*x^2)^3)
```

Maple [C] time = 0.523, size = 3550, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^4,x)
```

```
[Out] 5/64*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4*x^6+245/768*a^5/(a*x-1)^3/(a*x+
1)^3*arctanh(a*x)^2*x^6+5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(
I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arcta
nh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^6-5/64*I*a^5/(a*x-1)^3/(a*x+
1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1)
)^3*Pi*x^6-5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I/((a*x+1)^2/
(-a^2*x^2+1)+1))^2*Pi*x^6-15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*cs
gn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^4+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*a
rctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^4+15/64*I*a^3/(a*x-1)^3/
(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^3*Pi*x^4+15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I/((a*x
+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^4+15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^
3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3*Pi*x^2-15/64*I*a/(a*x-1)^3/(a*x+1)^3
*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3*Pi*x^2-15/64*I*a/(a*x-1)^3/
(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+
1)+1))^3*Pi*x^2-15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I/((a*x+1
)^2/(-a^2*x^2+1)+1))^2*Pi*x^2+5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^
3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^2+5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*
(a*x+1)/(-a^2*x^2+1)^(1/2))*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2-5/64*I/a/(a*x-1
)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2+5/64*I/a/(a*x-1)^3/(a*x+1)^3*
Pi*arctanh(a*x)^3*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*csgn(I*(a*x+1)^2/(a^
2*x^2-1))+9485/55296*a^5/(a*x-1)^3/(a*x+1)^3*x^6+9971/55296/a/(a*x-1)^3/(a*
x+1)^3-1/48/a*arctanh(a*x)^3/(a*x-1)^3-1/48/a*arctanh(a*x)^3/(a*x+1)^3+5/32
/a*arctanh(a*x)^3*ln(a*x+1)-5/16/a*arctanh(a*x)^3*ln((a*x+1)/(-a^2*x^2+1)^(
1/2))+1/16/a*arctanh(a*x)^3/(a*x-1)^2-5/32/a*arctanh(a*x)^3/(a*x-1)-5/32/a*
arctanh(a*x)^3*ln(a*x-1)-1/16/a*arctanh(a*x)^3/(a*x+1)^2-5/32/a*arctanh(a*x
)^3/(a*x+1)+299/768/a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^2+15/64*I*a/(a*x-1)^
3/(a*x+1)^3*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1
)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*P
i*x^2-15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^
2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^4+5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)
^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*
(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))*Pi*x^6-299/384/(a*x-1)^3/
(a*x+1)^3*arctanh(a*x)*x-3605/18432*a^3/(a*x-1)^3/(a*x+1)^3*x^4-2795/18432*
a/(a*x-1)^3/(a*x+1)^3*x^2-5/64/a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4+5/64*I*
a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^6-5/64*I*a^5/(a*x
-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)
/(-a^2*x^2+1)^(1/2))^2*Pi*x^6+15/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^
3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^
2/(-a^2*x^2+1)+1))^2*Pi*x^4+15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*
csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x^4-1
5/64*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))
*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^4+15/64*I*
a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^4-15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh
(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x
```

```
)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi*x
^2+15/64*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1
))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^2-15/64*
I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I
*(a*x+1)/(-a^2*x^2+1)^(1/2))^2*Pi*x^2-5/64*I*a^5/(a*x-1)^3/(a*x+1)^3*arctan
h(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((
a*x+1)^2/(-a^2*x^2+1)+1))^2*Pi*x^6-5/32*I*a^5/(a*x-1)^3/(a*x+1)^3*arctanh(a
*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^2*csgn(I*(a*x+1)/(-a^2*x^2+1)^(1/2))*Pi
*x^6-5/64*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2
*x^2+1)+1))*csgn(I*(a*x+1)^2/(a^2*x^2-1))*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a*
x+1)^2/(-a^2*x^2+1)+1))-5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3+5/64
*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1)/((a
*x+1)^2/(-a^2*x^2+1)+1))^3+5/32*I/a/(a*x-1)^3/(a*x+1)^3*Pi*arctanh(a*x)^3*c
sgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^2-15/32*I*a^3/(a*x-1)^3/(a*x+1)^3*arctanh
(a*x)^3*Pi*x^4+15/32*I*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*x^2+5/32*I*a
^5/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^3*Pi*x^6-5/32*I/a/(a*x-1)^3/(a*x+1)^3*P
i*arctanh(a*x)^3*csgn(I/((a*x+1)^2/(-a^2*x^2+1)+1))^3+5/64*I/a/(a*x-1)^3/(a
*x+1)^3*Pi*arctanh(a*x)^3*csgn(I*(a*x+1)^2/(a^2*x^2-1))^3-35/256*a/(a*x-1)^
3/(a*x+1)^3*arctanh(a*x)^2*x^2-15/64*a^3/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4
*x^4-245/384*a^4/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^5-125/256*a^3/(a*x-1)^3
/(a*x+1)^3*arctanh(a*x)^2*x^4+15/64*a/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)^4*x^
2+25/18*a^2/(a*x-1)^3/(a*x+1)^3*arctanh(a*x)*x^3
```

Maxima [B] time = 1.11325, size = 1176, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(15*a^4*x^5 - 40*a^2*x^3 + 33*x)/(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1) - 15*log(a*x + 1)/a + 15*log(a*x - 1)/a)*arctanh(a*x)^3 + 1/384*(180*a
^4*x^4 - 420*a^2*x^2 - 45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1
)^2 + 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)*log(a*x - 1) -
45*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^2 + 272)*a*arctanh(a*
x)^2/(a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2) + 1/27648*((8820*a^4*x^4 - 135
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^4 + 540*(a^6*x^6 - 3*a^
4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3*log(a*x - 1) - 135*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^4 - 18420*a^2*x^2 - 45*(49*a^6*x^6 - 147*a
^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1
)^2 - 49)*log(a*x + 1)^2 - 2205*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a
*x - 1)^2 + 90*(6*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 49
*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + 1) + 9728)*a
^2/(a^10*x^6 - 3*a^8*x^4 + 3*a^6*x^2 - a^4) - 12*(1470*a^5*x^5 - 3200*a^3*x
^3 - 90*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^3 + 270*(a^6*x^6
 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1)^2*log(a*x - 1) + 90*(a^6*x^6 - 3
*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1)^3 + 1794*a*x - 15*(49*a^6*x^6 - 147*
a^4*x^4 + 147*a^2*x^2 + 18*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x -
1)^2 - 49)*log(a*x + 1) + 735*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x
 - 1))*a*arctanh(a*x)/(a^9*x^6 - 3*a^7*x^4 + 3*a^5*x^2 - a^3))*a
```

Fricas [A] time = 2.06117, size = 506, normalized size = 1.74

$$\frac{8820\,a^4x^4+135\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)^4-18420\,a^2x^2-72\left(15\,a^5x^5-40\,a^3x^3+33\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3}{27648\left(a^7x^6-3\,a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/27648*(8820*a^4*x^4 + 135*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(-(a*x
 + 1)/(a*x - 1))^4 - 18420*a^2*x^2 - 72*(15*a^5*x^5 - 40*a^3*x^3 + 33*a*x)*
log(-(a*x + 1)/(a*x - 1))^3 + 9*(245*a^6*x^6 - 375*a^4*x^4 - 105*a^2*x^2 +
299)*log(-(a*x + 1)/(a*x - 1))^2 - 12*(735*a^5*x^5 - 1600*a^3*x^3 + 897*a*x
)*log(-(a*x + 1)/(a*x - 1)) + 9728)/(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{(ax-1)^4(ax+1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(atanh(a*x)**3/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(a^2*x^2 - 1)^4, x)
```

3.348 $\int \frac{\sqrt{\tanh^{-1}(ax)}}{(1-a^2x^2)^4} \, dx$

Optimal. Leaf size=252

$$\frac{3\sqrt{\pi}\text{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\text{Erf}\left(\sqrt{6}\sqrt{\tanh^{-1}(ax)}\right)}{768a}-\frac{3\sqrt{\pi}\text{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}-$$

```
[Out] (5*ArcTanh[a*x]^(3/2))/(24*a) + (3*Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(512
*a) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a) + (Sqrt[Pi/6]
*Erf[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) - (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcTanh
[a*x]]])/(512*a) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a)
 - (Sqrt[Pi/6]*Erfi[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) + (15*Sqrt[ArcTanh
[a*x]]*Sinh[2*ArcTanh[a*x]])/(64*a) + (3*Sqrt[ArcTanh[a*x]]*Sinh[4*ArcTanh[
a*x]])/(64*a) + (Sqrt[ArcTanh[a*x]]*Sinh[6*ArcTanh[a*x]])/(192*a)
```

Rubi [A] time = 0.30064, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5968, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\text{Erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\text{Erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\text{Erf}\left(\sqrt{6}\sqrt{\tanh^{-1}(ax)}\right)}{768a}-\frac{3\sqrt{\pi}\text{Erfi}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```


```
[Out] (5*ArcTanh[a*x]^(3/2))/(24*a) + (3*Sqrt[Pi]*Erf[2*Sqrt[ArcTanh[a*x]]])/(512
*a) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a) + (Sqrt[Pi/6]
*Erf[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) - (3*Sqrt[Pi]*Erfi[2*Sqrt[ArcTanh
[a*x]]])/(512*a) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcTanh[a*x]]])/(256*a)
 - (Sqrt[Pi/6]*Erfi[Sqrt[6]*Sqrt[ArcTanh[a*x]]])/(768*a) + (15*Sqrt[ArcTanh
[a*x]]*Sinh[2*ArcTanh[a*x]])/(64*a) + (3*Sqrt[ArcTanh[a*x]]*Sinh[4*ArcTanh[
a*x]])/(64*a) + (Sqrt[ArcTanh[a*x]]*Sinh[6*ArcTanh[a*x]])/(192*a)
```


Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{\tanh^{-1}(ax)}}{\left(1-a^2x^2\right)^4}\,dx = \frac{\text{Subst}\left(\int \sqrt{x}\cosh^6(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\sqrt{x}}{16}+\frac{15}{32}\sqrt{x}\cosh(2x)+\frac{3}{16}\sqrt{x}\cosh(4x)+\frac{1}{32}\sqrt{x}\cosh(6x)\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{\text{Subst}\left(\int \sqrt{x}\cosh(6x)\,dx, x, \tanh^{-1}(ax)\right)}{32a}+\frac{3\,\text{Subst}\left(\int \sqrt{x}\cosh(4x)\,dx, x, \text{t}\right.}{16a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{15\sqrt{\tanh^{-1}(ax)}\sinh\left(2\tanh^{-1}(ax)\right)}{64a}+\frac{3\sqrt{\tanh^{-1}(ax)}\sinh\left(4\tanh^{-1}(ax)\right)}{64a}$$

$$= \frac{5\tanh^{-1}(ax)^{3/2}}{24a}+\frac{3\sqrt{\pi}\,\text{erf}\left(2\sqrt{\tanh^{-1}(ax)}\right)}{512a}+\frac{15\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\tanh^{-1}(ax)}\right)}{256a}+\frac{\sqrt{\frac{\pi}{6}}\,\text{erf}\left(\sqrt{6}\sqrt{\text{t}}\right.}{768a}$$

Mathematica [A] time = 0.764107, size = 257, normalized size = 1.02

$$\frac{\sqrt{6}\sqrt{\tanh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-6\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}}+\frac{27\sqrt{\tanh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-4\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}}+\frac{135\sqrt{2}\sqrt{\tanh^{-1}(ax)}\text{Gamma}\left(\frac{1}{2},-2\tanh^{-1}(ax)\right)}{a\sqrt{-\tanh^{-1}(ax)}}-\frac{135\sqrt{2}\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[ArcTanh[a*x]]/(1 - a^2*x^2)^4,x]
```

```
[Out] ((-3168*x*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)^3 + (3840*a^2*x^3*Sqrt[ArcTanh
[a*x]])/(-1 + a^2*x^2)^3 - (1440*a^4*x^5*Sqrt[ArcTanh[a*x]])/(-1 + a^2*x^2)
^3 + (960*ArcTanh[a*x]^(3/2))/a + (Sqrt[6]*Sqrt[ArcTanh[a*x]]*Gamma[1/2, -6
*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (27*Sqrt[ArcTanh[a*x]]*Gamma[1/2,
 -4*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) + (135*Sqrt[2]*Sqrt[ArcTanh[a*x]
]*Gamma[1/2, -2*ArcTanh[a*x]])/(a*Sqrt[-ArcTanh[a*x]]) - (135*Sqrt[2]*Gamma
[1/2, 2*ArcTanh[a*x]])/a - (27*Gamma[1/2, 4*ArcTanh[a*x]])/a - (Sqrt[6]*Gam
ma[1/2, 6*ArcTanh[a*x]])/a)/4608
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right)^{4}} \sqrt{\operatorname{Artanh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

```
[Out] int(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{artanh}\left(a x\right)}}{\left(a^{2} x^{2}-1\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{atanh}\left(a x\right)}}{\left(a x-1\right)^{4} \left(a x+1\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**(1/2)/(-a**2*x**2+1)**4,x)
```

```
[Out] Integral(sqrt(atanh(a*x))/((a*x - 1)**4*(a*x + 1)**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\operatorname{artanh}(ax)}}{\left(a^2x^2-1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^(1/2)/(-a^2*x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(arctanh(a*x))/(a^2*x^2 - 1)^4, x)
```

3.349 $\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.067944, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 8.30661, size = 0, normalized size = 0.

$$\int \frac{x^8}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^8/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A] time = 0.237, size = 0, normalized size = 0.

$$\int \frac{x^8}{(-a^2x^2+1)^4 \operatorname{Artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^8}{(a^8x^8-4a^6x^6+6a^4x^4-4a^2x^2+1)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^8/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*x
)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**8/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^8/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.350 $\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0665811, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx = \int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Mathematica [A] time = 38.7303, size = 0, normalized size = 0.

$$\int \frac{x^7}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^7/((1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A] time = 0.182, size = 0, normalized size = 0.

$$\int \frac{x^7}{(-a^2x^2+1)^4 \operatorname{Artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^7}{(a^8x^8-4\,a^6x^6+6\,a^4x^4-4\,a^2x^2+1)\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^7/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*arctanh(a*x
)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**7/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^7/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.351 $\int \frac{x^6}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}$$

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a^7) - (3*CoshIntegral[4*ArcTanh[a*x]
])/(16*a^7) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^7) - (5*Log[ArcTanh[a*x]])
/(16*a^7)
```

Rubi [A] time = 0.145157, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 3312, 3301}

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7}-\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7}-\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a^7) - (3*CoshIntegral[4*ArcTanh[a*x]
])/(16*a^7) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^7) - (5*Log[ArcTanh[a*x]])
/(16*a^7)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\sinh^6(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^7}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{5}{16x} - \frac{15\cosh(2x)}{32x} + \frac{3\cosh(4x)}{16x} - \frac{\cosh(6x)}{32x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^7}$$
$$= -\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a^7} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^7} - \frac{3\,\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx\right.}{16a^7}$$
$$= \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^7} - \frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^7} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^7} - \frac{5\log\left(\tanh^{-}\right.}{16a^7}$$

Mathematica [A] time = 0.155643, size = 40, normalized size = 0.73

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right) - 6\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) - 10\log\left(\tanh^{-1}(ax)\right)}{32a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]] - 6*CoshIntegral[4*ArcTanh[a*x]] + CoshInt
egral[6*ArcTanh[a*x]] - 10*Log[ArcTanh[a*x]])/(32*a^7)
```

Maple [A] time = 0.072, size = 48, normalized size = 0.9

$$\frac{15\,\text{Chi}\left(2\,\text{Artanh}\left(ax\right)\right)}{32\,a^7} - \frac{3\,\text{Chi}\left(4\,\text{Artanh}\left(ax\right)\right)}{16\,a^7} + \frac{\text{Chi}\left(6\,\text{Artanh}\left(ax\right)\right)}{32\,a^7} - \frac{5\,\ln\left(\text{Artanh}\left(ax\right)\right)}{16\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 15/32*Chi(2*arctanh(a*x))/a^7-3/16*Chi(4*arctanh(a*x))/a^7+1/32*Chi(6*arcta
nh(a*x))/a^7-5/16*ln(arctanh(a*x))/a^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(a^2x^2-1\right)^4 \text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 2.0381, size = 568, normalized size = 10.33

$$-\frac{20 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) - \text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) - \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) + 6\,\text{log\_integral}\left(\frac{a^2x^2+2}{a^2x^2-2}\right.}{64a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x
^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^3
 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_integr
al((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^2 -
 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - 15*log_integral(-(a*x + 1)/(a*x - 1))
- 15*log_integral(-(a*x - 1)/(a*x + 1)))/a^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**6/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(a^2x^2-1)^4 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.352 $\int \frac{x^5}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^6) - SinhIntegral[4*ArcTanh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^6)
```

Rubi [A] time = 0.143713, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3298}

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^6) - SinhIntegral[4*ArcTanh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^6)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^5}{(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^6}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} - \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^6}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{8a^6} + \frac{5\,\text{Subst}\left(\right.}{}$$

$$= \frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^6} - \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^6} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Mathematica [A] time = 0.162454, size = 33, normalized size = 0.77

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right) - 4\text{Shi}\left(4\tanh^{-1}(ax)\right) + \text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]] - 4*SinhIntegral[4*ArcTanh[a*x]] + SinhInte
gral[6*ArcTanh[a*x]])/(32*a^6)
```

Maple [A] time = 0.061, size = 33, normalized size = 0.8

$$\frac{1}{a^6}\left(\frac{5\,\text{Shi}\,(2\,\text{Artanh}\,(ax))}{32} - \frac{\text{Shi}\,(4\,\text{Artanh}\,(ax))}{8} + \frac{\text{Shi}\,(6\,\text{Artanh}\,(ax))}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 1/a^6*(5/32*Shi(2*arctanh(a*x))-1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(
a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a^2x^2-1)^4\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 2.03028, size = 516, normalized size = 12.

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)-4\,\text{log\_integral}\left(\frac{a^2x^2+2\,ax+1}{a^2x^2-2\,ax+1}\right)+4\,\text{log\_integra}}{64\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) - 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2
*a*x + 1)) + 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) +
5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x + 1))
)/a^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(ax-1)^4\,(ax+1)^4\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**5/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(a^2x^2-1\right)^4\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^5/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.353 $\int \frac{x^4}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx$

Optimal. Leaf size=55

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5}-\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5}+\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5}$$

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^5) - CoshIntegral[4*ArcTanh[a*x]]/(16*a
^5) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^5) + Log[ArcTanh[a*x]]/(16*a^5)
```

Rubi [A] time = 0.154642, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3301}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5}-\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5}+\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^5) - CoshIntegral[4*ArcTanh[a*x]]/(16*a
^5) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^5) + Log[ArcTanh[a*x]]/(16*a^5)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^4}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx = \frac{\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16x} - \frac{\cosh(2x)}{32x} - \frac{\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^5}$$
$$= \frac{\log\left(\tanh^{-1}(ax)\right)}{16a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x} dx, x, \tanh^{-1}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x} dx, x, \text{ta}\right.}{32a^5}$$
$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^5} - \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^5} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^5} + \frac{\log\left(\tanh^{-1}(ax\right.}{16a^5}$$

Mathematica [A] time = 0.134723, size = 40, normalized size = 0.73

$$\frac{-\text{Chi}\left(2\tanh^{-1}(ax)\right) - 2\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) + 2\log\left(\tanh^{-1}(ax)\right)}{32a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (-CoshIntegral[2*ArcTanh[a*x]] - 2*CoshIntegral[4*ArcTanh[a*x]] + CoshInteg
ral[6*ArcTanh[a*x]] + 2*Log[ArcTanh[a*x]])/(32*a^5)
```

Maple [A] time = 0.076, size = 48, normalized size = 0.9

$$-\frac{\text{Chi}(2\,\text{Artanh}(ax))}{32\,a^5} - \frac{\text{Chi}(4\,\text{Artanh}(ax))}{16\,a^5} + \frac{\text{Chi}(6\,\text{Artanh}(ax))}{32\,a^5} + \frac{\ln(\text{Artanh}(ax))}{16\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^5-1/16*Chi(4*arctanh(a*x))/a^5+1/32*Chi(6*arcta
nh(a*x))/a^5+1/16*ln(arctanh(a*x))/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a^2x^2-1)^4 \,\text{artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 1.95745, size = 558, normalized size = 10.15

$$\frac{4 \log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right) + \text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right) + \text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right) - 2\,\text{log\_integral}\left(\frac{a^2x^2+2ax+}{a^2x^2-2ax+}\right.}{64a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^2
 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3 -
 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integral
((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2 - 2
*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) - log_integral(-(a*x + 1)/(a*x - 1)) - log
_integral(-(a*x - 1)/(a*x + 1)))/a^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**4/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.354 $\int \frac{x^3}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=29

$$\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4} - \frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

```
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]])/(32*a^4) + SinhIntegral[6*ArcTanh[a*x]]/(
32*a^4)
```

Rubi [A] time = 0.125032, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3298}

$$\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4} - \frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]])/(32*a^4) + SinhIntegral[6*ArcTanh[a*x]]/(
32*a^4)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^3}{(1-a^2x^2)^4\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^3(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{3\sinh(2x)}{32x}+\frac{\sinh(6x)}{32x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^4}$$
$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^4} - \frac{3\,\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^4}$$
$$= -\frac{3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^4}$$

Mathematica [A] time = 0.140588, size = 24, normalized size = 0.83

$$\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)-3\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (-3*SinhIntegral[2*ArcTanh[a*x]] + SinhIntegral[6*ArcTanh[a*x]])/(32*a^4)
```

Maple [A] time = 0.066, size = 24, normalized size = 0.8

$$\frac{1}{a^4}\left(-\frac{3\,\text{Shi}\,(2\,\text{Artanh}\,(ax))}{32}+\frac{\text{Shi}\,(6\,\text{Artanh}\,(ax))}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 1/a^4*(-3/32*Shi(2*arctanh(a*x))+1/32*Shi(6*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a^2x^2-1)^4\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 1.94995, size = 348, normalized size = 12.

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)-3\,\text{log\_integral}\left(-\frac{ax+1}{ax-1}\right)+3\,\text{log\_integral}\left(-\frac{ax-}{ax+}\right.}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) - 3*log_integral(-(a*x + 1)/(a*x - 1)) + 3*log_inte
gral(-(a*x - 1)/(a*x + 1)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**3/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^3/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.355 $\int \frac{x^2}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=55

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^3) + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Rubi [A] time = 0.142112, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6034, 5448, 3301}

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^3) + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{16x}-\frac{\cosh(2x)}{32x}+\frac{\cosh(4x)}{16x}+\frac{\cosh(6x)}{32x}\right)dx, x, \tanh^{-1}(ax)\right)}{a^3}$$
$$= -\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x,\right.}{32a^3}$$
$$= -\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(a\right.}{16a^3}$$

Mathematica [A] time = 0.159147, size = 55, normalized size = 1.

$$-\frac{\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a^3}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a^3}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a^3}-\frac{\log\left(\tanh^{-1}(ax)\right)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] -CoshIntegral[2*ArcTanh[a*x]]/(32*a^3) + CoshIntegral[4*ArcTanh[a*x]]/(16*a
^3) + CoshIntegral[6*ArcTanh[a*x]]/(32*a^3) - Log[ArcTanh[a*x]]/(16*a^3)
```

Maple [A] time = 0.076, size = 48, normalized size = 0.9

$$-\frac{\text{Chi}\,(2\,\text{Artanh}\,(ax))}{32\,a^3}+\frac{\text{Chi}\,(4\,\text{Artanh}\,(ax))}{16\,a^3}+\frac{\text{Chi}\,(6\,\text{Artanh}\,(ax))}{32\,a^3}-\frac{\ln\,(\text{Artanh}\,(ax))}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] -1/32*Chi(2*arctanh(a*x))/a^3+1/16*Chi(4*arctanh(a*x))/a^3+1/32*Chi(6*arcta
nh(a*x))/a^3-1/16*ln(arctanh(a*x))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2x^2-1\right)^4\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 1.96815, size = 559, normalized size = 10.16

$$-\frac{4\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)-\mathrm{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right)-\mathrm{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right)-2\,\mathrm{log\_integral}\left(\frac{a^2x^2+2ax}{a^2x^2-2ax}\right.}{64a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/64*(4*log(log(-(a*x + 1)/(a*x - 1))) - log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - log_integral(-(a^3*x^3
- 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) - 2*log_integra
l((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 2*log_integral((a^2*x^2 -
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + log_integral(-(a*x + 1)/(a*x - 1)) + lo
g_integral(-(a*x - 1)/(a*x + 1)))/a^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ax-1)^4\,(ax+1)^4\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x**2/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a^2x^2-1\right)^4\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.356 $\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=43

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Rubi [A] time = 0.112312, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {6034, 5448, 3298}

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2}+\frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)} \, dx = \frac{\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \frac{\sinh(6x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{32a^2} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x} \, dx, x, \tanh^{-1}(ax)\right)}{8a^2} + \frac{5\,\text{Subst}\left(\right.}{}$$

$$= \frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Mathematica [A] time = 0.198329, size = 43, normalized size = 1.

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{32a^2} + \frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{8a^2} + \frac{\text{Shi}\left(6\tanh^{-1}(ax)\right)}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (5*SinhIntegral[2*ArcTanh[a*x]])/(32*a^2) + SinhIntegral[4*ArcTanh[a*x]]/(8
*a^2) + SinhIntegral[6*ArcTanh[a*x]]/(32*a^2)
```

Maple [A] time = 0.063, size = 33, normalized size = 0.8

$$\frac{1}{a^2}\left(\frac{5\,\text{Shi}\,(2\,\text{Artanh}\,(ax))}{32} + \frac{\text{Shi}\,(4\,\text{Artanh}\,(ax))}{8} + \frac{\text{Shi}\,(6\,\text{Artanh}\,(ax))}{32}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 1/a^2*(5/32*Shi(2*arctanh(a*x))+1/8*Shi(4*arctanh(a*x))+1/32*Shi(6*arctanh(
a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a^2x^2-1)^4 \,\text{artanh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 1.91039, size = 516, normalized size = 12.

$$\frac{\text{log\_integral}\left(-\frac{a^3x^3+3a^2x^2+3ax+1}{a^3x^3-3a^2x^2+3ax-1}\right)-\text{log\_integral}\left(-\frac{a^3x^3-3a^2x^2+3ax-1}{a^3x^3+3a^2x^2+3ax+1}\right)+4\,\text{log\_integral}\left(\frac{a^2x^2+2ax+1}{a^2x^2-2ax+1}\right)-4\,\text{log\_integra}}{64a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2
+ 3*a*x - 1)) - log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 +
3*a^2*x^2 + 3*a*x + 1)) + 4*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2
*a*x + 1)) - 4*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) +
5*log_integral(-(a*x + 1)/(a*x - 1)) - 5*log_integral(-(a*x - 1)/(a*x + 1))
)/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax-1)^4\,(ax+1)^4\,\text{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2x^2-1\right)^4\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.357 $\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=55

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a) + (3*CoshIntegral[4*ArcTanh[a*x]])
/(16*a) + CoshIntegral[6*ArcTanh[a*x]]/(32*a) + (5*Log[ArcTanh[a*x]])/(16*a
)
```

Rubi [A] time = 0.103471, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5968, 3312, 3301}

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a}+\frac{5\log\left(\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]])/(32*a) + (3*CoshIntegral[4*ArcTanh[a*x]])
/(16*a) + CoshIntegral[6*ArcTanh[a*x]]/(32*a) + (5*Log[ArcTanh[a*x]])/(16*a
)
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)} dx = \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5}{16x} + \frac{15\cosh(2x)}{32x} + \frac{3\cosh(4x)}{16x} + \frac{\cosh(6x)}{32x}\right) dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{5\log\left(\tanh^{-1}(ax)\right)}{16a} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x} dx, x, \tanh^{-1}(ax)\right)}{32a} + \frac{3\,\text{Subst}\left(\int \frac{\cosh(4x)}{x} dx, x\right.}{16a}$$
$$= \frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{32a} + \frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{16a} + \frac{\text{Chi}\left(6\tanh^{-1}(ax)\right)}{32a} + \frac{5\log\left(\tanh^{-1}\right.}{16a}$$

Mathematica [A] time = 0.160401, size = 40, normalized size = 0.73

$$\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right) + 6\text{Chi}\left(4\tanh^{-1}(ax)\right) + \text{Chi}\left(6\tanh^{-1}(ax)\right) + 10\log\left(\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] (15*CoshIntegral[2*ArcTanh[a*x]] + 6*CoshIntegral[4*ArcTanh[a*x]] + CoshInt
egral[6*ArcTanh[a*x]] + 10*Log[ArcTanh[a*x]])/(32*a)
```

Maple [A] time = 0.073, size = 48, normalized size = 0.9

$$\frac{15\,\text{Chi}\,(2\,\text{Artanh}\,(ax))}{32\,a} + \frac{3\,\text{Chi}\,(4\,\text{Artanh}\,(ax))}{16\,a} + \frac{\text{Chi}\,(6\,\text{Artanh}\,(ax))}{32\,a} + \frac{5\,\ln\,(\text{Artanh}\,(ax))}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] 15/32*Chi(2*arctanh(a*x))/a+3/16*Chi(4*arctanh(a*x))/a+1/32*Chi(6*arctanh(a
*x))/a+5/16*ln(arctanh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2x^2-1)^4 \operatorname{artanh}(ax)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

Fricas [B] time = 2.02222, size = 564, normalized size = 10.25

$$\frac{20\,\log\left(\log\left(-\frac{ax+1}{ax-1}\right)\right)+\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2x^2+3\,ax-1}{a^3x^3+3\,a^2x^2+3\,ax+1}\right)+6\,\mathrm{log\_integral}\left(\frac{a^2x^2+2\,a}{a^2x^2-2\,a}\right.}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/64*(20*log(log(-(a*x + 1)/(a*x - 1))) + log_integral(-(a^3*x^3 + 3*a^2*x^
2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + log_integral(-(a^3*x^3
- 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 6*log_integra
l((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 6*log_integral((a^2*x^2 -
2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 15*log_integral(-(a*x + 1)/(a*x - 1)) +
 15*log_integral(-(a*x - 1)/(a*x + 1)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^4\,(ax+1)^4\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)), x)
```

3.358 $\int \frac{1}{x(1-a^2x^2)^4 \tanh^{-1}(ax)} dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.061957, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx = \int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.44235, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A] time = 0.202, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-a^2x^2+1\right)^4 \mathrm{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4 x \operatorname{artanh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^8x^9-4\,a^6x^7+6\,a^4x^5-4\,a^2x^3+x\right)\operatorname{artanh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^8*x^9 - 4*a^6*x^7 + 6*a^4*x^5 - 4*a^2*x^3 + x)*arctanh(a*x))
, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(ax-1\right)^4\left(ax+1\right)^4 \operatorname{atanh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(1/(x*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4 x \operatorname{artanh}\left(ax\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x*arctanh(a*x)), x)
```

3.359 $\int \frac{1}{x^2(1-a^2x^2)^4 \tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{1}{x^2\left(1-a^2x^2\right)^4\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0704338, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x^2\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx=\int \frac{1}{x^2\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 2.8913, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*(1 - a^2*x^2)^4*ArcTanh[a*x]), x]
```

Maple [A] time = 0.224, size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(-a^2x^2+1\right)^4\text{Artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

```
[Out] int(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4 x^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\left(a^8x^{10}-4\,a^6x^8+6\,a^4x^6-4\,a^2x^4+x^2\right)\operatorname{artanh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/((a^8*x^10 - 4*a^6*x^8 + 6*a^4*x^6 - 4*a^2*x^4 + x^2)*arctanh(a*
x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\,(ax-1)^4\,(ax+1)^4\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-a**2*x**2+1)**4/atanh(a*x),x)
```

```
[Out] Integral(1/(x**2*(a*x - 1)**4*(a*x + 1)**4*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4 x^2 \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-a^2*x^2+1)^4/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*x^2*arctanh(a*x)), x)
```

3.360 $\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=67

$$\frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}+\frac{3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}$$

```
[Out] -(x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (5*CoshIntegral[2*ArcTanh[a*x]])/(1
6*a^2) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2) + (3*CoshIntegral[6*ArcTanh[a
*x]])/(16*a^2)
```

Rubi [A] time = 0.306893, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6032, 6034, 5448, 3301, 5968, 3312}

$$\frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2}+\frac{3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (5*CoshIntegral[2*ArcTanh[a*x]])/(1
6*a^2) + CoshIntegral[4*ArcTanh[a*x]]/(2*a^2) + (3*CoshIntegral[6*ArcTanh[a
*x]])/(16*a^2)
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,dx}{a} + (5a)\int \frac{x^2}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)}\,a$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh^6(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^2} + \frac{5\,\text{Subst}\left(\int \frac{\cosh^4(x}{}\right.}{}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int\left(\frac{5}{16x}+\frac{15\cosh(2x)}{32x}+\frac{3\cosh(4x)}{16x}+\frac{\cosh(6x)}{32x}\right)dx,x,\text{ta}\right.}{a^2}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{32a^2} - \frac{5\,\text{Subst}\left(\int \frac{\cosh(2x}{x}\right.}{32}$$

$$= -\frac{x}{a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)} + \frac{5\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a^2} + \frac{\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a^2} + \frac{3\text{Chi}\left(6\,\text{ta}\right.}{16}$$

Mathematica [A] time = 0.185365, size = 56, normalized size = 0.84

$$\frac{\frac{16ax}{\left(a^2x^2-1\right)^3\tanh^{-1}(ax)} + 5\text{Chi}\left(2\tanh^{-1}(ax)\right) + 8\text{Chi}\left(4\tanh^{-1}(ax)\right) + 3\text{Chi}\left(6\tanh^{-1}(ax)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] ((16*a*x)/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + 5*CoshIntegral[2*ArcTanh[a*x]]
+ 8*CoshIntegral[4*ArcTanh[a*x]] + 3*CoshIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

Maple [A] time = 0.069, size = 78, normalized size = 1.2

$$\frac{1}{a^2}\left(-\frac{5\sinh(2\,\text{Artanh}(ax))}{32\,\text{Artanh}(ax)} + \frac{5\,\text{Chi}(2\,\text{Artanh}(ax))}{16} - \frac{\sinh(4\,\text{Artanh}(ax))}{8\,\text{Artanh}(ax)} + \frac{\text{Chi}(4\,\text{Artanh}(ax))}{2} - \frac{\sinh(6\,\text{Artanl}}{32\,\text{Artanh}\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x)
```

```
[Out] 1/a^2*(-5/32/arctanh(a*x)*sinh(2*arctanh(a*x))+5/16*Chi(2*arctanh(a*x))-1/8
/arctanh(a*x)*sinh(4*arctanh(a*x))+1/2*Chi(4*arctanh(a*x))-1/32/arctanh(a*x
)*sinh(6*arctanh(a*x))+3/16*Chi(6*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x}{\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(ax+1\right)-\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)\log\left(-ax+1\right)}-\int-\frac{}{\left(a^9x^8-4\,a^7x^6+6\,a^5x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2*x/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*a^5*
x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)) - integrate(-2*(5*a^2*x^2 + 1)/((a^9*x^
8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(a*x + 1) - (a^9*x^8 - 4*a^7*
x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)*log(-a*x + 1)), x)
```

Fricas [B] time = 2.01965, size = 965, normalized size = 14.4

$$\frac{64\,ax+\left(3\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\operatorname{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)+3\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\operatorname{log\_integra}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(64*a*x + (3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*
x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) + 3*(a^6*x^
6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x -
 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2
 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) + 8*(a^6*x^
6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2
+ 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x
+ 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*
x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4
*x^2 - a^2)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(ax-1\right)^4\left(ax+1\right)^4\operatorname{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

3.361 $\int \frac{1}{(1-a^2x^2)^4 \tanh^{-1}(ax)^2} \, dx$

Optimal. Leaf size=66

$$-\frac{1}{a(1-a^2x^2)^3 \tanh^{-1}(ax)}+\frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (15*SinhIntegral[2*ArcTanh[a*x]])/(
16*a) + (3*SinhIntegral[4*ArcTanh[a*x]])/(4*a) + (3*SinhIntegral[6*ArcTanh[
a*x]])/(16*a)
```

Rubi [A] time = 0.142841, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a(1-a^2x^2)^3 \tanh^{-1}(ax)}+\frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a}+\frac{3\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^3*ArcTanh[a*x])) + (15*SinhIntegral[2*ArcTanh[a*x]])/(
16*a) + (3*SinhIntegral[4*ArcTanh[a*x]])/(4*a) + (3*SinhIntegral[6*ArcTanh[
a*x]])/(16*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^2} dx = -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + (6a)\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)} dx$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{6\,\text{Subst}\left(\int \frac{\cosh^5(x)\sinh(x)}{x} dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{6\,\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32x} + \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\right) dx, x, \tanh^{-1}(a$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(6x)}{x} dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(4}{x}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{15\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a} + \frac{3\text{Shi}\left(4\tanh^{-1}(ax)\right)}{4a} + \frac{3\text{Shi}\left(6\,\text{t}\right.}{1}$$

Mathematica [A] time = 0.281275, size = 56, normalized size = 0.85

$$\frac{\frac{1}{\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)} + \frac{15}{16}\text{Shi}\left(2\tanh^{-1}(ax)\right) + \frac{3}{4}\text{Shi}\left(4\tanh^{-1}(ax)\right) + \frac{3}{16}\text{Shi}\left(6\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^2),x]
```

```
[Out] (1/((-1 + a^2*x^2)^3*ArcTanh[a*x]) + (15*SinhIntegral[2*ArcTanh[a*x]])/16 +
 (3*SinhIntegral[4*ArcTanh[a*x]])/4 + (3*SinhIntegral[6*ArcTanh[a*x]])/16)/
a
```

Maple [A] time = 0.068, size = 86, normalized size = 1.3

$$\frac{1}{a}\left(-\frac{5}{16\,\text{Artanh}\,(ax)} - \frac{15\,\cosh\,(2\,\text{Artanh}\,(ax))}{32\,\text{Artanh}\,(ax)} + \frac{15\,\text{Shi}\,(2\,\text{Artanh}\,(ax))}{16} - \frac{3\,\cosh\,(4\,\text{Artanh}\,(ax))}{16\,\text{Artanh}\,(ax)} + \frac{3\,\text{Shi}\,(4\,\text{Artanh}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(-5/16/arctanh(a*x)-15/32/arctanh(a*x)*cosh(2*arctanh(a*x))+15/16*Shi(2
*arctanh(a*x))-3/16/arctanh(a*x)*cosh(4*arctanh(a*x))+3/4*Shi(4*arctanh(a*x
))-1/32/arctanh(a*x)*cosh(6*arctanh(a*x))+3/16*Shi(6*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-12\,a\int -\frac{x}{\left(a^8x^8 - 4\,a^6x^6 + 6\,a^4x^4 - 4\,a^2x^2 + 1\right)\log\,(ax+1) - \left(a^8x^8 - 4\,a^6x^6 + 6\,a^4x^4 - 4\,a^2x^2 + 1\right)\log\,(-ax+1)}\,dx + \frac{}{(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -12*a*integrate(-x/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a
*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)),
 x) + 2/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a*x + 1) - (a^7*x^6 - 3*
a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1))
```

Fricas [B] time = 1.99132, size = 954, normalized size = 14.45

$$3\left(\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right) - \left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{log\_integral}\left(-\frac{a^3x^3-3}{a^3x^3+3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(3*((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - (a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^
3 + 3*a^2*x^2 + 3*a*x + 1)) + 4*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_i
ntegral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 4*(a^6*x^6 - 3*a^4*x
^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1
)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a*x -
 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x
 + 1)))*log(-(a*x + 1)/(a*x - 1)) + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 -
 a)*log(-(a*x + 1)/(a*x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^4\,(ax+1)^4\,\text{atanh}^2\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4 \text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^2), x)
```

3.362 $\int \frac{x}{(1-a^2x^2)^4 \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=114

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}+\frac{9\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh$$

```
[Out] -x/(2*a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - 3/(a^2*(1 - a^2*x^2)^3*ArcTanh[a*
x]) + 5/(2*a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (5*SinhIntegral[2*ArcTanh[a*
x]])/(16*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2 + (9*SinhIntegral[6*ArcTan
h[a*x]])/(16*a^2)
```

Rubi [A] time = 0.585044, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6032, 6028, 5966, 6034, 5448, 3298}

$$\frac{5\text{Shi}\left(2\tanh^{-1}(ax)\right)}{16a^2}+\frac{\text{Shi}\left(4\tanh^{-1}(ax)\right)}{a^2}+\frac{9\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}-\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```


```
[Out] -x/(2*a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - 3/(a^2*(1 - a^2*x^2)^3*ArcTanh[a*
x]) + 5/(2*a^2*(1 - a^2*x^2)^2*ArcTanh[a*x]) + (5*SinhIntegral[2*ArcTanh[a*
x]])/(16*a^2) + SinhIntegral[4*ArcTanh[a*x]]/a^2 + (9*SinhIntegral[6*ArcTan
h[a*x]])/(16*a^2)
```


Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^4\tanh^{-1}(ax)^2}\,dx}{2a}+\frac{1}{2}(5a)\int \frac{x^2}{\left(1-a^2x^2\right)^4\tanh$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{1}{2a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+3\int \frac{x}{\left(1-a^2x^2\right)^4\tanh^{-}$$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}($$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}($$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}($$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}($$

$$= -\frac{x}{2a\left(1-a^2x^2\right)^3\tanh^{-1}(ax)^2}-\frac{3}{a^2\left(1-a^2x^2\right)^3\tanh^{-1}(ax)}+\frac{5}{2a^2\left(1-a^2x^2\right)^2\tanh^{-1}($$

Mathematica [A] time = 0.358251, size = 73, normalized size = 0.64

$$\frac{\frac{8\left(\left(5a^2x^2+1\right)\tanh^{-1}(ax)+ax\right)}{\left(a^2x^2-1\right)^3\tanh^{-1}(ax)^2}+5\text{Shi}\left(2\tanh^{-1}(ax)\right)+16\text{Shi}\left(4\tanh^{-1}(ax)\right)+9\text{Shi}\left(6\tanh^{-1}(ax)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] ((8*(a*x + (1 + 5*a^2*x^2)*ArcTanh[a*x]))/((-1 + a^2*x^2)^3*ArcTanh[a*x]^2)
 + 5*SinhIntegral[2*ArcTanh[a*x]] + 16*SinhIntegral[4*ArcTanh[a*x]] + 9*Sin
hIntegral[6*ArcTanh[a*x]])/(16*a^2)
```

Maple [A] time = 0.076, size = 121, normalized size = 1.1

$$\frac{1}{a^2}\left(-\frac{5\,\sinh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{64\,\left(\mathrm{Artanh}\left(ax\right)\right)^2}-\frac{5\,\cosh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{32\,\mathrm{Artanh}\left(ax\right)}+\frac{5\,\mathrm{Shi}\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{16}-\frac{\sinh\left(4\,\mathrm{Artanh}\left(ax\right)\right)}{16\,\left(\mathrm{Artanh}\left(ax\right)\right)^2}-\frac{\cosh\left(4\,\mathrm{Arta\right.}}{4\,\mathrm{Artanl}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x)
```

```
[Out] 1/a^2*(-5/64/arctanh(a*x)^2*sinh(2*arctanh(a*x))-5/32/arctanh(a*x)*cosh(2*a
rctanh(a*x))+5/16*Shi(2*arctanh(a*x))-1/16/arctanh(a*x)^2*sinh(4*arctanh(a*
x))-1/4/arctanh(a*x)*cosh(4*arctanh(a*x))+Shi(4*arctanh(a*x))-1/64/arctanh(
a*x)^2*sinh(6*arctanh(a*x))-3/32/arctanh(a*x)*cosh(6*arctanh(a*x))+9/16*Shi
(6*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,ax+\left(5\,a^2x^2+1\right)\log\left(ax+1\right)-\left(5\,a^2x^2+1\right)\log\left(-ax+1\right)}{\left(a^8x^6-3\,a^6x^4+3\,a^4x^2-a^2\right)\log\left(ax+1\right)^2-2\left(a^8x^6-3\,a^6x^4+3\,a^4x^2-a^2\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^8x^6-3\,a^6x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] (2*a*x + (5*a^2*x^2 + 1)*log(a*x + 1) - (5*a^2*x^2 + 1)*log(-a*x + 1))/((a^
8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(a*x + 1)^2 - 2*(a^8*x^6 - 3*a^6*x^
4 + 3*a^4*x^2 - a^2)*log(a*x + 1)*log(-a*x + 1) + (a^8*x^6 - 3*a^6*x^4 + 3*
a^4*x^2 - a^2)*log(-a*x + 1)^2) - integrate(-4*(5*a^2*x^3 + 4*x)/((a^8*x^8
- 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^
6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)*log(-a*x + 1)), x)
```

Fricas [B] time = 1.84023, size = 1037, normalized size = 9.1

$$\frac{\left(9\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3+3\,a^2x^2+3\,ax+1}{a^3x^3-3\,a^2x^2+3\,ax-1}\right)-9\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{log\_integral}\left(-\frac{a^3x^3-3\,a^2}{a^3x^3+3\,a^2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((9*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a
^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)) - 9*(a^6*x^6 - 3*a^4
*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*
x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 16*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*lo
g_integral((a^2*x^2 + 2*a*x + 1)/(a^2*x^2 - 2*a*x + 1)) - 16*(a^6*x^6 - 3*a
^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x
```

```
 + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x + 1)/(a
*x - 1)) - 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(a*x - 1)/
(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2 + 64*a*x + 32*(5*a^2*x^2 + 1)*log(-
(a*x + 1)/(a*x - 1)))/((a^8*x^6 - 3*a^6*x^4 + 3*a^4*x^2 - a^2)*log(-(a*x +
1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^3(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

3.363 $\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=89

$$-\frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}+\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a}+\frac{9\text{Chi}\left(6\tanh\right.}{16a}$$

```
[Out] -1/(2*a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - (3*x)/((1 - a^2*x^2)^3*ArcTanh[a*
x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/(16*a) + (3*CoshIntegral[4*ArcTanh[
a*x]])/(2*a) + (9*CoshIntegral[6*ArcTanh[a*x]])/(16*a)
```

Rubi [A] time = 0.386036, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2}+\frac{15\text{Chi}\left(2\tanh^{-1}(ax)\right)}{16a}+\frac{3\text{Chi}\left(4\tanh^{-1}(ax)\right)}{2a}+\frac{9\text{Chi}\left(6\tanh\right.}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^3*ArcTanh[a*x]^2) - (3*x)/((1 - a^2*x^2)^3*ArcTanh[a*
x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/(16*a) + (3*CoshIntegral[4*ArcTanh[
a*x]])/(2*a) + (9*CoshIntegral[6*ArcTanh[a*x]])/(16*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} + (3a)\int \frac{x}{\left(1-a^2x^2\right)^4 \tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + 3\int \frac{1}{\left(1-a^2x^2\right)^4 \tanh^{-1}(a$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh^6(x)}{x}\,dx, x, t\right.}{a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \left(\frac{5}{16x} + \frac{15\cosh(2}{32x}\right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{3\,\mathrm{Subst}\left(\int \frac{\cosh(6x)}{x}\,dx, x, t\right.}{32a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)^2} - \frac{3x}{\left(1-a^2x^2\right)^3 \tanh^{-1}(ax)} + \frac{15\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{16a} + \frac{3}{}$$

Mathematica [A] time = 0.187294, size = 83, normalized size = 0.93

$$\frac{1}{16}\left(\frac{48x}{\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)} + \frac{8}{a\left(a^2x^2-1\right)^3 \tanh^{-1}(ax)^2} + \frac{15\mathrm{Chi}\left(2\tanh^{-1}(ax)\right)}{a} + \frac{24\mathrm{Chi}\left(4\tanh^{-1}(ax)\right)}{a} + \frac{9\mathrm{Chi}\left(\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^4*ArcTanh[a*x]^3),x]
```

```
[Out] (8/(a*(-1 + a^2*x^2)^3*ArcTanh[a*x]^2) + (48*x)/((-1 + a^2*x^2)^3*ArcTanh[a
*x]) + (15*CoshIntegral[2*ArcTanh[a*x]])/a + (24*CoshIntegral[4*ArcTanh[a*x
]])/a + (9*CoshIntegral[6*ArcTanh[a*x]])/a)/16
```

Maple [A] time = 0.075, size = 131, normalized size = 1.5

$$\frac{1}{a}\left(-\frac{5}{32\left(\mathrm{Artanh}\left(ax\right)\right)^{2}}-\frac{15\cosh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{64\left(\mathrm{Artanh}\left(ax\right)\right)^{2}}-\frac{15\sinh\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{32\,\mathrm{Artanh}\left(ax\right)}+\frac{15\,\mathrm{Chi}\left(2\,\mathrm{Artanh}\left(ax\right)\right)}{16}-\frac{3\cosh\left(4\,\mathrm{A}\right.}{32\left(\mathrm{Arta}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x)
```

```
[Out] 1/a*(-5/32/arctanh(a*x)^2-15/64/arctanh(a*x)^2*cosh(2*arctanh(a*x))-15/32/a
rctanh(a*x)*sinh(2*arctanh(a*x))+15/16*Chi(2*arctanh(a*x))-3/32/arctanh(a*x
)^2*cosh(4*arctanh(a*x))-3/8/arctanh(a*x)*sinh(4*arctanh(a*x))+3/2*Chi(4*ar
ctanh(a*x))-1/64/arctanh(a*x)^2*cosh(6*arctanh(a*x))-3/32/arctanh(a*x)*sinh
(6*arctanh(a*x))+9/16*Chi(6*arctanh(a*x)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(3\,ax\log\left(ax+1\right)-3\,ax\log\left(-ax+1\right)+1\right)}{\left(a^{7}x^{6}-3\,a^{5}x^{4}+3\,a^{3}x^{2}-a\right)\log\left(ax+1\right)^{2}-2\left(a^{7}x^{6}-3\,a^{5}x^{4}+3\,a^{3}x^{2}-a\right)\log\left(ax+1\right)\log\left(-ax+1\right)+\left(a^{7}x^{6}-3\,a^{5}x^{4}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 2*(3*a*x*log(a*x + 1) - 3*a*x*log(-a*x + 1) + 1)/((a^7*x^6 - 3*a^5*x^4 + 3*
a^3*x^2 - a)*log(a*x + 1)^2 - 2*(a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(a
*x + 1)*log(-a*x + 1) + (a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-a*x + 1)
^2) - integrate(-6*(5*a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^
2*x^2 + 1)*log(a*x + 1) - (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)
*log(-a*x + 1)), x)
```

Fricas [B] time = 1.93944, size = 1014, normalized size = 11.39

$$\frac{192\,ax\log\left(-\frac{ax+1}{ax-1}\right)+3\left(3\left(a^{6}x^{6}-3\,a^{4}x^{4}+3\,a^{2}x^{2}-1\right)\mathrm{log\_integral}\left(-\frac{a^{3}x^{3}+3\,a^{2}x^{2}+3\,ax+1}{a^{3}x^{3}-3\,a^{2}x^{2}+3\,ax-1}\right)+3\left(a^{6}x^{6}-3\,a^{4}x^{4}+3\,a^{2}x^{2}-1\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(192*a*x*log(-(a*x + 1)/(a*x - 1)) + 3*(3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral(-(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)/(a^3*x^3 - 3*a^2*
x^2 + 3*a*x - 1)) + 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral(-(
a^3*x^3 - 3*a^2*x^2 + 3*a*x - 1)/(a^3*x^3 + 3*a^2*x^2 + 3*a*x + 1)) + 8*(a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a^2*x^2 + 2*a*x + 1)/(a^2*
x^2 - 2*a*x + 1)) + 8*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log_integral((a
```

```
^2*x^2 - 2*a*x + 1)/(a^2*x^2 + 2*a*x + 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a^2
*x^2 - 1)*log_integral(-(a*x + 1)/(a*x - 1)) + 5*(a^6*x^6 - 3*a^4*x^4 + 3*a
^2*x^2 - 1)*log_integral(-(a*x - 1)/(a*x + 1)))*log(-(a*x + 1)/(a*x - 1))^2
 + 64)/((a^7*x^6 - 3*a^5*x^4 + 3*a^3*x^2 - a)*log(-(a*x + 1)/(a*x - 1))^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax-1)^4 (ax+1)^4 \operatorname{atanh}^3(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**4/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((a*x - 1)**4*(a*x + 1)**4*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2x^2-1\right)^4 \operatorname{artanh}(ax)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^4/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*x^2 - 1)^4*arctanh(a*x)^3), x)
```

3.364 $\int \frac{x^5 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=139

$$-\frac{x^3\sqrt{1-a^2x^2}}{20a^3}-\frac{5x\sqrt{1-a^2x^2}}{24a^5}-\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2}-\frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}-\frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6}+\frac{89\sin}{12}$$

```
[Out] (-5*x*Sqrt[1 - a^2*x^2])/(24*a^5) - (x^3*Sqrt[1 - a^2*x^2])/(20*a^3) + (89*
ArcSin[a*x])/(120*a^6) - (8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^6) - (4*x
^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^4*Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(5*a^2)
```

Rubi [A] time = 0.23484, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6016, 321, 216, 5994}

$$-\frac{x^3\sqrt{1-a^2x^2}}{20a^3}-\frac{5x\sqrt{1-a^2x^2}}{24a^5}-\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2}-\frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}-\frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6}+\frac{89\sin}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-5*x*Sqrt[1 - a^2*x^2])/(24*a^5) - (x^3*Sqrt[1 - a^2*x^2])/(20*a^3) + (89*
ArcSin[a*x])/(120*a^6) - (8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^6) - (4*x
^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^4*Sqrt[1 - a^2*x^2]*ArcTan
h[a*x])/(5*a^2)
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
```

```
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x^5 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2} + \frac{4\int \frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{5a^2} + \frac{\int \frac{x^4}{\sqrt{1-a^2x^2}}\,dx}{5a}$$

$$= -\frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4} - \frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5a^2} + \frac{8\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{15a^4} + \cdots$$

$$= -\frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^3\sqrt{1-a^2x^2}}{20a^3} - \frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6} - \frac{4x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4} - \frac{x^4\sqrt{1-\cdots}}{\cdots}$$

$$= -\frac{5x\sqrt{1-a^2x^2}}{24a^5} - \frac{x^3\sqrt{1-a^2x^2}}{20a^3} + \frac{89\sin^{-1}(ax)}{120a^6} - \frac{8\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^6} - \frac{4x^2\sqrt{1-a^2x^2}\tanh\cdots}{15a^4}$$

Mathematica [A] time = 0.0986469, size = 79, normalized size = 0.57

$$-\frac{ax\sqrt{1-a^2x^2}\left(6a^2x^2+25\right)+8\sqrt{1-a^2x^2}\left(3a^4x^4+4a^2x^2+8\right)\tanh^{-1}(ax)-89\sin^{-1}(ax)}{120a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(a*x*Sqrt[1 - a^2*x^2]*(25 + 6*a^2*x^2) - 89*ArcSin[a*x] + 8*Sqrt[1 - a^2*
x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/(120*a^6)
```

Maple [C] time = 0.282, size = 120, normalized size = 0.9

$$-\frac{24\,a^4x^4\mathrm{Artanh}\,(ax)+6\,x^3a^3+32\,a^2x^2\mathrm{Artanh}\,(ax)+25\,ax+64\,\mathrm{Artanh}\,(ax)}{120\,a^6}\sqrt{-(ax-1)\,(ax+1)}+\frac{\frac{89\,i}{120}}{a^6}\ln\left((ax+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/120/a^6*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*x^3*a^3+32*a
^2*x^2*arctanh(a*x)+25*a*x+64*arctanh(a*x))+89/120*I*ln((a*x+1)/(-a^2*x^2+1
)^(1/2)+I)/a^6-89/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^6
```

Maxima [A] time = 1.45784, size = 269, normalized size = 1.94

$$-\frac{1}{120}a\left(\frac{3\left(\frac{2\sqrt{-a^2x^2+1}x^3}{a^2}+\frac{3\sqrt{-a^2x^2+1}x}{a^4}-\frac{3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^4}\right)}{a^2}+\frac{16\left(\frac{\sqrt{-a^2x^2+1}x}{a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)}{a^4}-\frac{64\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^6}\right)-\frac{1}{15}\left(3\sqrt{-a^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*a*(3*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2 + 1)*x/a^4 - 3*
arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4))/a^2 + 16*(sqrt(-a^2*x^2 + 1)*x/a^2
 - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))/a^4 - 64*arcsin(a^2*x/sqrt(a^2)
)/(sqrt(a^2)*a^6)) - 1/15*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^2 +
 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*arctanh(a*x)
```

Fricas [A] time = 2.0824, size = 212, normalized size = 1.53

$$-\frac{\left(6 a^{3} x^{3} + 25 a x + 4 \left(3 a^{4} x^{4} + 4 a^{2} x^{2} + 8\right) \log\left(-\frac{a x+1}{a x-1}\right)\right) \sqrt{-a^{2} x^{2} + 1} + 178 \arctan\left(\frac{\sqrt{-a^{2} x^{2}+1}-1}{a x}\right)}{120 a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*((6*a^3*x^3 + 25*a*x + 4*(3*a^4*x^4 + 4*a^2*x^2 + 8)*log(-(a*x + 1)/
(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 178*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))
)/a^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{5} \operatorname{atanh}(a x)}{\sqrt{-(a x - 1)(a x + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.25854, size = 161, normalized size = 1.16

$$-\frac{1}{120} \sqrt{-a^{2} x^{2} + 1} x \left(\frac{6 x^{2}}{a^{3}} + \frac{25}{a^{5}}\right) - \frac{\left(3 \left(a^{2} x^{2} - 1\right)^{2} \sqrt{-a^{2} x^{2} + 1} - 10 \left(-a^{2} x^{2} + 1\right)^{\frac{3}{2}} + 15 \sqrt{-a^{2} x^{2} + 1}\right) \log\left(-\frac{a x+1}{a x-1}\right)}{30 a^{6}} + \frac{89 \arcsin ...}{12...}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*sqrt(-a^2*x^2 + 1)*x*(6*x^2/a^3 + 25/a^5) - 1/30*(3*(a^2*x^2 - 1)^2*
sqrt(-a^2*x^2 + 1) - 10*(-a^2*x^2 + 1)^(3/2) + 15*sqrt(-a^2*x^2 + 1))*log(-
(a*x + 1)/(a*x - 1))/a^6 + 89/120*arcsin(a*x)*sgn(a)/(a^5*abs(a))
```

3.365 $\int \frac{x^4 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=197

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a^5}-\frac{5\sqrt{1-a^2x^2}}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{1}}{}$$

```
[Out] (-5*Sqrt[1 - a^2*x^2])/(8*a^5) + (1 - a^2*x^2)^(3/2)/(12*a^5) - (3*x*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/(8*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4
*a^2) - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^5) - (((3
*I)/8)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (((3*I)/8)*Pol
yLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rubi [A] time = 0.216797, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6016, 266, 43, 261, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a^5}-\frac{5\sqrt{1-a^2x^2}}{8a^5}-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-5*Sqrt[1 - a^2*x^2])/(8*a^5) + (1 - a^2*x^2)^(3/2)/(12*a^5) - (3*x*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/(8*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(4
*a^2) - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^5) - (((3
*I)/8)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (((3*I)/8)*Pol
yLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{x^4 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4a^2} + \frac{3\int \frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{4a^2} + \frac{\int \frac{x^3}{\sqrt{1-a^2x^2}}\,dx}{4a}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4a^2} + \frac{3\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^4} + \frac{3\int \frac{x}{\sqrt{1-a^2x^2}}\,dx}{8a^3} + \text{Sub}$$

$$= -\frac{3\sqrt{1-a^2x^2}}{8a^5} - \frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4a^2} - \frac{3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(a}{4a^5}$$

$$= -\frac{5\sqrt{1-a^2x^2}}{8a^5} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a^5} - \frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^4} - \frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4a^2} - \frac{3\tan^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.504469, size = 160, normalized size = 0.81

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{9i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}} - 2a^2x^2 - 6ax\left(a^2x^2-1\right)\tanh^{-1}(ax) - \frac{9i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh}\right.\right.}{\sqrt{1}}\right.}{24a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-13 - 2*a^2*x^2 - 15*a*x*ArcTanh[a*x] - 6*a*x*(-1 + a^2
*x^2)*ArcTanh[a*x] - ((9*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1
 + I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((9*I)*(PolyLog[2, (-I)/E^ArcTan
h[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^5)
```

Maple [A] time = 0.262, size = 175, normalized size = 0.9

$$-\frac{6a^3x^3\text{Artanh}(ax)+2a^2x^2+9ax\text{Artanh}(ax)+13}{24a^5}\sqrt{-(ax-1)(ax+1)} - \frac{\frac{3i}{8}\text{Artanh}(ax)}{a^5}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/24/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2+9*a*x*
arctanh(a*x)+13)-3/8*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+
```

```
3/8*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5-3/8*I*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^4 \operatorname{artanh}(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.366 $\int \frac{x^3 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=87

$$-\frac{x\sqrt{1-a^2x^2}}{6a^3}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4}+\frac{5\sin^{-1}(ax)}{6a^4}$$

```
[Out] -(x*Sqrt[1 - a^2*x^2])/(6*a^3) + (5*ArcSin[a*x])/(6*a^4) - (2*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2)
```

Rubi [A] time = 0.124237, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6016, 321, 216, 5994}

$$-\frac{x\sqrt{1-a^2x^2}}{6a^3}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4}+\frac{5\sin^{-1}(ax)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x*Sqrt[1 - a^2*x^2])/(6*a^3) + (5*ArcSin[a*x])/(6*a^4) - (2*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^2)
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a^2} + \frac{\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{3a}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{6a^3} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{6a^3} + \frac{2\int \frac{1}{\sqrt{1-a}}}{3a^3}$$
$$= -\frac{x\sqrt{1-a^2x^2}}{6a^3} + \frac{5\sin^{-1}(ax)}{6a^4} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^2}$$

Mathematica [A] time = 0.0688944, size = 60, normalized size = 0.69

$$-\frac{ax\sqrt{1-a^2x^2} + 2\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\tanh^{-1}(ax) - 5\sin^{-1}(ax)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(a*x*Sqrt[1 - a^2*x^2] - 5*ArcSin[a*x] + 2*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)
*ArcTanh[a*x])/(6*a^4)
```

Maple [C] time = 0.236, size = 99, normalized size = 1.1

$$-\frac{2\,a^2x^2\mathrm{Artanh}\,(ax) + ax + 4\,\mathrm{Artanh}\,(ax)}{6\,a^4}\sqrt{-(ax-1)(ax+1)} + \frac{\frac{5\,i}{6}}{a^4}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}} + i\right) - \frac{\frac{5\,i}{6}}{a^4}\ln\left((ax+1)\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/6/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x+4*arctanh(a*x
))+5/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-5/6*I*ln((a*x+1)/(-a^2*x^2+1)
^(1/2)-I)/a^4
```

Maxima [A] time = 1.44765, size = 151, normalized size = 1.74

$$-\frac{1}{6}a\left(\frac{\frac{\sqrt{-a^2x^2+1}x}{a^2} - \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}}{a^2} - \frac{4\,\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^4}\right) - \frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*a*((sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2)
)/a^2 - 4*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^4)) - 1/3*(sqrt(-a^2*x^2 + 1
)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arctanh(a*x)
```

Fricas [A] time = 2.18607, size = 166, normalized size = 1.91

$$-\frac{\sqrt{-a^2x^2+1}\left(ax+\left(a^2x^2+2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)+10\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1))) +
10*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.22851, size = 109, normalized size = 1.25

$$-\frac{\sqrt{-a^2x^2+1}x}{6\,a^3}+\frac{\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}-3\sqrt{-a^2x^2+1}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,a^4}+\frac{5\arcsin(ax)\operatorname{sgn}(a)}{6\,a^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*x^2 + 1)*x/a^3 + 1/6*((-a^2*x^2 + 1)^(3/2) - 3*sqrt(-a^2*x^2
 + 1))*log(-(a*x + 1)/(a*x - 1))/a^4 + 5/6*arcsin(a*x)*sgn(a)/(a^3*abs(a))
```

3.367 $\int \frac{x^2 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=146

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}-\frac{\sqrt{1-a^2x^2}}{2a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}$$

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) - (
ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - ((I/2)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rubi [A] time = 0.104588, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6016, 261, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a^3}-\frac{\sqrt{1-a^2x^2}}{2a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2}-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -Sqrt[1 - a^2*x^2]/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*a^2) - (
ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - ((I/2)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}}\,dx}{2a}$$

$$= -\frac{\sqrt{1-a^2x^2}}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2a^2} - \frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a^3} + \frac{i\text{Li}_2\left(\frac{i\sqrt{1-}}{\sqrt{1+}}\right.}{2a^3}$$

Mathematica [A] time = 0.218912, size = 125, normalized size = 0.86

$$-\frac{i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right) + \sqrt{1-a^2x^2} + ax\sqrt{1-a^2x^2}\tanh^{-1}(ax) + i\tanh^{-1}(ax)\log\left(1\right.}{2a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2] + a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + I*ArcTanh[a*x]*L
og[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + I*Pol
yLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x]])/(2*a^3)
```

Maple [A] time = 0.26, size = 154, normalized size = 1.1

$$-\frac{ax\text{Artanh}(ax)+1}{2a^3}\sqrt{-(ax-1)(ax+1)} - \frac{\frac{i}{2}\text{Artanh}(ax)}{a^3}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{\frac{i}{2}\text{Artanh}(ax)}{a^3}\ln\left(1-i(ax+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3-1/2*I*ln(1+I*(a*x+1)
/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*arctanh(a*x)/a^3-1/2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/2*I*
dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.368 $\int \frac{x \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\, dx$

Optimal. Leaf size=32

$$\frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}$$

```
[Out] ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Rubi [A] time = 0.0469097, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5994, 216}

$$\frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x \tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\, dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2} + \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\, dx}{a}\\ &= \frac{\sin^{-1}(ax)}{a^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.031359, size = 29, normalized size = 0.91

$$\frac{\sin^{-1}(ax) - \sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (ArcSin[a*x] - Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^2
```

Maple [C] time = 0.222, size = 81, normalized size = 2.5

$$-\frac{\operatorname{Artanh}(ax)}{a^2}\sqrt{-(ax-1)(ax+1)}+\frac{i}{a^2}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}+i\right)-\frac{i}{a^2}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}-i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a^2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)+I*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)+I)/a^2-I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

Maxima [A] time = 1.44362, size = 57, normalized size = 1.78

$$\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a}-\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a) - sqrt(-a^2*x^2 + 1)*arctanh(a*x)/a^2
```

Fricas [A] time = 2.07488, size = 135, normalized size = 4.22

$$-\frac{\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)+4\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)) + 4*arctan((sqrt(-a^2*x^
2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [A] time = 1.21385, size = 63, normalized size = 1.97

$$\frac{\arcsin(ax)\,\mathrm{sgn}(a)}{a|a|} - \frac{\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(a*x)*sgn(a)/(a*abs(a)) - 1/2*sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
- 1))/a^2
```

3.369 $\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=95

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

```
[Out] (-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a
```

Rubi [A] time = 0.0264132, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx=-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}+\frac{i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [A] time = 0.0733041, size = 76, normalized size = 0.8

$$-\frac{i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}\right.\right.\right.}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]])
 + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a
```

Maple [B] time = 0.247, size = 366, normalized size = 3.9

$$\frac{\frac{i}{2}\operatorname{Artanh}(ax)}{a}\ln\left(-i\frac{1}{\sqrt{-a^2x^2+1}}-iax\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{i\operatorname{Artanh}(ax)}{a}\ln\left((1-i)\cosh\left(\frac{\operatorname{Artanh}(ax)}{2}\right)+(1+i)\sinh\left(\frac{\operatorname{Arta}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*I/a*arctanh(a*x)*ln(-I/(-a^2*x^2+1)^(1/2)-I*a*x/(-a^2*x^2+1)^(1/2))-I/a
*ln((1-I)*cosh(1/2*arctanh(a*x))+(1+I)*sinh(1/2*arctanh(a*x)))*arctanh(a*x)
-1/2*I/a*arctanh(a*x)*ln(I/(-a^2*x^2+1)^(1/2)+I*a*x/(-a^2*x^2+1)^(1/2))+I/a
*ln((1+I)*cosh(1/2*arctanh(a*x))+(1-I)*sinh(1/2*arctanh(a*x)))*arctanh(a*x)
+I/a*ln((1-I)*cosh(1/2*arctanh(a*x))+(1+I)*sinh(1/2*arctanh(a*x)))*ln(-I/(-
a^2*x^2+1)^(1/2)-I*a*x/(-a^2*x^2+1)^(1/2))-I/a*ln((1+I)*cosh(1/2*arctanh(a*
x))+(1-I)*sinh(1/2*arctanh(a*x)))*ln(I/(-a^2*x^2+1)^(1/2)+I*a*x/(-a^2*x^2+1
)^(1/2))+I/a*dilog(-I/(-a^2*x^2+1)^(1/2)-I*a*x/(-a^2*x^2+1)^(1/2))-I/a*dilo
g(I/(-a^2*x^2+1)^(1/2)+I*a*x/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

3.370 $\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=75

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

```
[Out] -2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1
- a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.0650753, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {6018}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1
- a*x]/Sqrt[1 + a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx=-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)$$

Mathematica [A] time = 0.0915049, size = 57, normalized size = 0.76

$$\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\log\left(e^{-\tanh^{-1}(ax)}+1\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ArcTanh[a*x]*(Log[1 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])]) + Po
lyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A] time = 0.235, size = 99, normalized size = 1.3

$$-\mathrm{Artanh}\,(ax)\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)-\mathrm{polylog}\left(2,-(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+\mathrm{Artanh}\,(ax)\ln\left(1-(ax+1)\frac{1}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-polylog(2,-(a*x+1)/(-a^2*x^2
+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.371 $\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.0758943, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6008, 266, 63, 208}

$$-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\ &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.050512, size = 48, normalized size = 1.14

$$-a\log\left(\sqrt{1-a^2x^2}+1\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+a\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + a*Log[x] - a*Log[1 + Sqrt[1 - a^2*x
^2]]
```

Maple [A] time = 0.231, size = 72, normalized size = 1.7

$$-\frac{\text{Artanh}\,(ax)}{x}\sqrt{-(ax-1)(ax+1)}-a\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+a\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)/x-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Maxima [A] time = 1.43981, size = 69, normalized size = 1.64

$$-a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*arctanh
(a*x)/x
```

Fricas [A] time = 2.09887, size = 127, normalized size = 3.02

$$\frac{2\,ax\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*x*log((sqrt(-a^2*x^2 + 1) - 1)/x) - sqrt(-a^2*x^2 + 1)*log(-(a*x +
 1)/(a*x - 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [B] time = 1.27043, size = 150, normalized size = 3.57

$$-\frac{1}{2}\,a\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{1}{2}\,a\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\log\left(-\frac{ax}{ax}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(sqrt(-a^2*x^2 + 1) + 1) + 1/2*a*log(-sqrt(-a^2*x^2 + 1) + 1) + 1
/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*ab
s(a) + a)/(x*abs(a)))*log(-(a*x + 1)/(a*x - 1))
```

3.372 $\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=137

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\left(-\tanh^{-1}(ax)\right)\text{ta}$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 - (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rubi [A] time = 0.135654, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6026, 264, 6018}

$$\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\left(-\tanh^{-1}(ax)\right)\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 - (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} - a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \frac{1}{2}a^2\mathrm{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.$$

Mathematica [A] time = 0.689746, size = 126, normalized size = 0.92

$$\frac{1}{8}a^2\left(4\mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 4\mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right) + 4\tanh^{-1}(ax)\log\left(1-e^{-\tanh\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 + 4*Arc
Tanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] + 4*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*PolyLog[2, E^(-ArcTanh[a*x])]
- ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A] time = 0.254, size = 141, normalized size = 1.

$$-\frac{ax+\mathrm{Artanh}(ax)}{2x^2}\sqrt{-(ax-1)(ax+1)} - \frac{a^2\mathrm{Artanh}(ax)}{2}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{a^2}{2}\mathrm{polylog}\left(2,-(ax+1)\frac{1}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2-1/2*a^2*arctanh(a*x)*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))+1/2*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\mathrm{artanh}(ax)}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.373 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=205

$$-\frac{5i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}+\frac{5i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}-\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(a$$

```
[Out] -Sqrt[1 - a^2*x^2]/(3*a^4) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^3) - (
10*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^4) - (2*Sqrt[1 -
a^2*x^2]*ArcTanh[a*x]^2)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(
3*a^2) - (((5*I)/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (
((5*I)/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rubi [A] time = 0.313618, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6016, 261, 5950, 5994}

$$-\frac{5i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}+\frac{5i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^4}-\frac{\sqrt{1-a^2x^2}}{3a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```


```
[Out] -Sqrt[1 - a^2*x^2]/(3*a^4) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a^3) - (
10*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^4) - (2*Sqrt[1 -
a^2*x^2]*ArcTanh[a*x]^2)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(
3*a^2) - (((5*I)/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (
((5*I)/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```


Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{3a^2} + \frac{2\int \frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a}$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^3} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,d}{3a^3}$$

$$= -\frac{\sqrt{1-a^2x^2}}{3a^4} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^3} - \frac{10\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^4} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^4}$$

Mathematica [A] time = 0.391785, size = 160, normalized size = 0.78

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{5i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}} + \left(1-a^2x^2\right)\tanh^{-1}(ax)^2 - \frac{5i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log(1\right.}{\sqrt{1-a^2x^2}}\right.}{3a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-1 - a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + (1 - a^2*x^2
)*ArcTanh[a*x]^2 - ((5*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
 I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((5*I)*(PolyLog[2, (-I)/E^ArcTanh[
a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(3*a^4)
```

Maple [A] time = 0.258, size = 175, normalized size = 0.9

$$-\frac{a^2x^2\left(\text{Artanh}(ax)\right)^2 + ax\text{Artanh}(ax) + 2\left(\text{Artanh}(ax)\right)^2 + 1}{3a^4}\sqrt{-(ax-1)(ax+1)} - \frac{\frac{5i}{3}\text{Artanh}(ax)}{a^4}\ln\left(1+i(ax+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/3/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)+
2*arctanh(a*x)^2+1)-5/3*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a
^4+5/3*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4-5/3*I*dilog(1+
I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4+5/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2
))/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^3 \operatorname{artanh}(ax)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.374 $\int \frac{x^2 \tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=161

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{i\text{PolyI}}{}$$

```
[Out] ArcSin[a*x]/a^3 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^3 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^2) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^3 -
(I*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (I*ArcTanh[a*x]*Poly
Log[2, I*E^ArcTanh[a*x]])/a^3 + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - (
I*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rubi [A] time = 0.25452, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6016, 5994, 216, 5952, 4180, 2531, 2282, 6589}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{i\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ArcSin[a*x]/a^3 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^3 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^2) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^3 -
(I*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (I*ArcTanh[a*x]*Poly
Log[2, I*E^ArcTanh[a*x]])/a^3 + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - (
I*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{\text{Subst}\left(\int x^2\mathbf{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{2a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(a}{a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(a}{a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(a}{a^3}$$

$$= \frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(a}{a^3}$$

Mathematica [A] time = 0.762677, size = 188, normalized size = 1.17

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax}\right)\right.}{\sqrt{1-a^2x^2}}\right.}{2a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2*ArcTanh[a*x] - a*x*ArcTanh[a*x]^2 - (I*((4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a^
3)
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int x^{2} \left(\text{Artanh}\left(ax\right)\right)^{2} \frac{1}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2} \operatorname{artanh}\left(ax\right)^{2}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^{2}x^{2}+1}x^{2} \operatorname{artanh}\left(ax\right)^{2}}{a^{2}x^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.375 $\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}+\frac{2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}-\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^2}$$

```
[Out] (-4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^2 - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x]^2)/a^2 - ((2*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a^2 + ((2*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rubi [A] time = 0.0988155, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5994, 5950}

$$-\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}+\frac{2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}-\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^2 - (Sqrt[1 - a^2*x
^2]*ArcTanh[a*x]^2)/a^2 - ((2*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a^2 + ((2*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx=-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}+\frac{2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a}$$

$$=-\frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^2}-\frac{2i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^2}+\frac{2i\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^2}$$

Mathematica [A] time = 0.215484, size = 104, normalized size = 0.87

$$-\frac{2i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\sqrt{1-a^2x^2}\tanh^{-1}(ax)+2i\left(\log\left(1-ie^{-\right.\right.\right.}}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + (2*I)*(Log[1 - I/E^ArcTan
h[a*x]] - Log[1 + I/E^ArcTanh[a*x]])) + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]])/a^2)
```

Maple [A] time = 0.237, size = 151, normalized size = 1.3

$$-\frac{(\text{Artanh}\,(ax))^2}{a^2}\sqrt{-(ax-1)(ax+1)}-\frac{2\,i\text{Artanh}\,(ax)}{a^2}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{2\,i\text{Artanh}\,(ax)}{a^2}\ln\left(1-i(ax\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/a^2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2-2*I*ln(1+I*(a*x+1)/(-a^2*x^2
+1)^(1/2))*arctanh(a*x)/a^2+2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(
a*x)/a^2-2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+2*I*dilog(1-I*(a*x+1
)/(-a^2*x^2+1)^(1/2))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x\,\text{artanh}\,(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{atanh}^{2}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{artanh}(ax)^{2}}{\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.376 $\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-$$

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - ((2*I)*ArcTanh[a*x]*PolyLog[2
, (-I)*E^ArcTanh[a*x]])/a + ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]
])/a + ((2*I)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((2*I)*PolyLog[3, I*E^Ar
cTanh[a*x]])/a
```

Rubi [A] time = 0.0978629, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5952, 4180, 2531, 2282, 6589}

$$-\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{2i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```


```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - ((2*I)*ArcTanh[a*x]*PolyLog[2
, (-I)*E^ArcTanh[a*x]])/a + ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]
])/a + ((2*I)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((2*I)*PolyLog[3, I*E^Ar
cTanh[a*x]])/a
```


Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^2\operatorname{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{(2i)\,\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx, x, \tanh^{-1}(ax)\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(a}\right.}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(a}\right.}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a} - \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{2i\tanh^{-1}(ax)\text{Li}_2\left(ie^{\tanh^{-1}(a}\right.}{a}$$

Mathematica [A] time = 0.0990944, size = 119, normalized size = 1.16

$$\frac{i\left(-2\tanh^{-1}(ax)\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)-2\left(\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\right.\right.}{a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (I*(-(ArcTanh[a*x]^2*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]
)) - 2*ArcTanh[a*x]*(PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTa
nh[a*x]]) - 2*(PolyLog[3, (-I)/E^ArcTanh[a*x]] - PolyLog[3, I/E^ArcTanh[a*x
]])))/a
```

Maple [F] time = 0.359, size = 0, normalized size = 0.

$$\int (\text{Artanh}\,(ax))^2\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

3.377 $\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=68

$$-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-2\text{PolyLo}$$

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*PolyLog[2, -E^Ar
cTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 2*PolyLog[3, -E^A
rcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.141289, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6020, 4182, 2531, 2282, 6589}

$$-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-2\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*PolyLog[2, -E^Ar
cTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 2*PolyLog[3, -E^A
rcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \tanh^{-1}(ax)\right) + 2\,\text{Subst}\Big($$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{Li}_2\Big(e^{\tanh^{-1}}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{Li}_2\Big(e^{\tanh^{-1}}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 2\tanh^{-1}(ax)\text{Li}_2\Big(e^{\tanh^{-1}}$$

Mathematica [A] time = 0.0838509, size = 100, normalized size = 1.47

$$2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 2\text{P}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Log[1 + E^(-ArcT
anh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 2*ArcTanh[a*x]
*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyL
og[3, E^(-ArcTanh[a*x])]
```

Maple [A] time = 0.251, size = 158, normalized size = 2.3

$$-(\text{Artanh}(ax))^2\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) - 2\,\text{Artanh}(ax)\,\text{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) + 2\,\text{polylog}\left(3,-\frac{ax+1}{\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-
(a*x+1)/(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctan
h(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/
(-a^2*x^2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{a^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atanh}^2\,(ax)}{x\sqrt{-\,(ax-1)\,(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.378 $\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=105

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)$$

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a
*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.1665, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6008, 6018}

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 -
 a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a
*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}+(2a)\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}-4a\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+2a\text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-2a\text{Li}_2\left(\frac{\sqrt{1}}{\sqrt{1}}\right.\end{aligned}$$

Mathematica [A] time = 0.489556, size = 89, normalized size = 0.85

$$2a\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-2a\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\frac{\tanh^{-1}(ax)\left(\sqrt{1-a^2x^2}\tanh^{-1}(ax)+2ax\left(\log\left(e^{-\tanh^{-1}(ax)}\right.\right.\right.}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -((ArcTanh[a*x]*(Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2*a*x*(-Log[1 - E^(-ArcTa
nh[a*x])] + Log[1 + E^(-ArcTanh[a*x])])))/x) + 2*a*PolyLog[2, -E^(-ArcTanh[
a*x])] - 2*a*PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A] time = 0.245, size = 131, normalized size = 1.3

$$-\frac{(\text{Artanh}(ax))^2}{x}\sqrt{-(ax-1)(ax+1)}-2a\text{Artanh}(ax)\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-2a\text{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+2a\text{Artar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/x-2*a*arctanh(a*x)*ln(1+(a*x+1)/(-
a^2*x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*
x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^2}{a^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.379 $\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=152

$$-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-a^2\text{Pol}$$

```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] - a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] + a^2*PolyLog[3, -E^ArcTanh[a*x]] - a^2*Poly
Log[3, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.415348, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6026, 6008, 266, 63, 208, 6020, 4182, 2531, 2282, 6589}

$$-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-a^2\text{Pol}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```


```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] - a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] + a^2*PolyLog[3, -E^ArcTanh[a*x]] - a^2*Poly
Log[3, E^ArcTanh[a*x]]
```


Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int x^2\mathbf{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 + \frac{1}{2}a^2\,\text{Su}$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - a^2\,\text{tanh}$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - a^2\,\text{tanh}$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - a^2\,\text{tanh}$$

Mathematica [A] time = 1.19724, size = 188, normalized size = 1.24

$$\frac{1}{8}a^2\left(8\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 8\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 8\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 8\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2 + 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 4*ArcTanh[a*x]^2*Lo
g[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 8*ArcTanh[a*x]*Pol
yLog[2, -E^(-ArcTanh[a*x])] - 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])]
+ 8*PolyLog[3, -E^(-ArcTanh[a*x])] - 8*PolyLog[3, E^(-ArcTanh[a*x])] - ArcT
anh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A] time = 0.283, size = 231, normalized size = 1.5

$$-\frac{\text{Artanh}(ax)(2ax+\text{Artanh}(ax))}{2x^2}\sqrt{-(ax-1)(ax+1)} - \frac{a^2(\text{Artanh}(ax))^2}{2}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) - a^2\text{Artanh}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2-1/2*a^2
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/
2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*poly
log(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atanh}^2\,(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\,(ax)^2}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.380 $\int \frac{x^3 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=219

$$-\frac{5i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{5i\text{I}}{}$$

```
[Out] ArcSin[a*x]/a^4 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^3) + (5*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^4
- (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^3)/(3*a^2) - ((5*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])
/a^4 + ((5*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 + ((5*I)*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a^4 - ((5*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^4
```

Rubi [A] time = 0.569595, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6016, 5994, 216, 5952, 4180, 2531, 2282, 6589}

$$-\frac{5i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{5i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{5i\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```


```
[Out] ArcSin[a*x]/a^4 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4 - (x*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a^3) + (5*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^4
- (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(3*a^4) - (x^2*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^3)/(3*a^2) - ((5*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])
/a^4 + ((5*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 + ((5*I)*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a^4 - ((5*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^4
```


Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{3a^2} + \frac{2\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}dx}{3a^2} + \frac{\int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}dx}{a}$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{3a^2} + \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}}{2a^3}$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{3a^4} - \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{3a^2}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^4}$$

$$= \frac{\sin^{-1}(ax)}{a^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} + \frac{5\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)}{a^4}$$

Mathematica [A] time = 0.886576, size = 215, normalized size = 0.98

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{3i\left(10\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-10\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+10\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-10\text{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)\right.}{\sqrt{1-a^2x^2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-3*a*x*ArcTanh[a*x]^2 + 2*(1 - a^2*x^2)*ArcTanh[a*x]^3
- 6*ArcTanh[a*x]*(1 + ArcTanh[a*x]^2) - ((3*I)*((4*I)*ArcTan[Tanh[ArcTanh[a
*x]/2]] + 5*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 5*ArcTanh[a*x]^2*Log
[1 + I/E^ArcTanh[a*x]] + 10*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] -
10*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 10*PolyLog[3, (-I)/E^ArcTanh
[a*x]] - 10*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(6*a^4)
```

Maple [F] time = 0.3, size = 0, normalized size = 0.

$$\int x^3 \left(\text{Artanh}\,(ax)\right)^3 \frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^3\,\text{artanh}\,(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,\text{atanh}^3\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\,\text{artanh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.381 $\int \frac{x^2 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=305

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh^{-1}(ax)^2\text{PolyLo}}{2a^3}$$

```
[Out] (-6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - (3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^2)/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(2*a^2
) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 - (((3*I)/2)*ArcTanh[a*x]^2
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2
, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 +
a*x]])/a^3 + ((3*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((3*
I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]*
PolyLog[3, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])
/a^3 + ((3*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a^3
```

Rubi [A] time = 0.339874, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6016, 5994, 5950, 5952, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{2a^3}+\frac{3i\tanh^{-1}(ax)^2\text{PolyLo}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```


```
[Out] (-6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 - (3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^2)/(2*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(2*a^2
) + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 - (((3*I)/2)*ArcTanh[a*x]^2
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2
, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 +
a*x]])/a^3 + ((3*I)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((3*
I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]*
PolyLog[3, I*E^ArcTanh[a*x]])/a^3 - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])
/a^3 + ((3*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a^3
```


Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
```

```
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{2a^2} + \frac{3\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{2a}$$

$$= -\frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\text{Subst}\left(\int x^3\mathbf{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{2a^3}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\tan^{-1}\left(e\right.}{}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\tan^{-1}\left(e\right.}{}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\tan^{-1}\left(e\right.}{}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\tan^{-1}\left(e\right.}{}$$

$$= -\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} - \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^3} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2a^2} + \frac{\tan^{-1}\left(e\right.}{}$$

Mathematica [A] time = 4.46736, size = 570, normalized size = 1.87

$$-\frac{i\left(192\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right) + 192i\pi\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right) + 384\tanh^{-1}(ax)\text{PolyLog}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-I/128)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (192
*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 - (64*I)*a*
x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + 384*ArcTanh[a*x]*L
og[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] - 384*ArcTa
nh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTan
h[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh[a
*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[
a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*Log
[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8*I
)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(Pi^2 - (4*I)*Pi*ArcTanh[
a*x] - 4*(2 + ArcTanh[a*x]^2))*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 384*PolyLo
g[2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]]
 - 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2
, I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcT
anh[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)
*E^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4,
 (-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a^3
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int x^2 (\text{Artanh}\,(ax))^3 \frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)

[Out] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)

[Out] Integral(x**2*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(x^2*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.382 $\int \frac{x \tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=128

$$-\frac{6i \tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i \tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^2}-\frac{6i\text{I}}{}$$

```
[Out] (6*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/a^2 - ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^2 + ((
6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^2 + ((6*I)*PolyLog[3, (-I
)*E^ArcTanh[a*x]])/a^2 - ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^2
```

Rubi [A] time = 0.176988, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5994, 5952, 4180, 2531, 2282, 6589}

$$-\frac{6i \tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i \tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^2}+\frac{6i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^2}-\frac{6i\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (6*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/a^2 - ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^2 + ((
6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^2 + ((6*I)*PolyLog[3, (-I
)*E^ArcTanh[a*x]])/a^2 - ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^2
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}+\frac{3\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}+\frac{3\,\text{Subst}\left(\int x^2\,\text{sech}(x)\,dx,x,\tanh^{-1}(ax)\right)}{a^2}\\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}-\frac{(6i)\,\text{Subst}\left(\int x\log\left(1-ie^x\right)\,dx,\right.}{a^2}\\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}\\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}\\
&= \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^2}-\frac{6i\tanh^{-1}(ax)\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.180252, size = 157, normalized size = 1.23

$$-\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-6i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+6i\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 + (3*I)*ArcTanh[a*x]^2*Log[1 - I/E^ArcT
anh[a*x]] - (3*I)*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*ArcTanh[
a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*PolyLog[2, I/E^Ar
cTanh[a*x]] + (6*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (6*I)*PolyLog[3, I/E^
ArcTanh[a*x]])/a^2)
```

Maple [F] time = 0.267, size = 0, normalized size = 0.

$$\int x(\operatorname{Artanh}(ax))^3 \frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}x \operatorname{artanh}(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.383 $\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=153

$$-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}$$

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - ((3*I)*ArcTanh[a*x]^2*PolyLog
[2, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[
a*x]])/a + ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((6*I)*
ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a - ((6*I)*PolyLog[4, (-I)*E^Arc
Tanh[a*x]])/a + ((6*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a
```

Rubi [A] time = 0.1331, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5952, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - ((3*I)*ArcTanh[a*x]^2*PolyLog
[2, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[
a*x]])/a + ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - ((6*I)*
ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a - ((6*I)*PolyLog[4, (-I)*E^Arc
Tanh[a*x]])/a + ((6*I)*PolyLog[4, I*E^ArcTanh[a*x]])/a
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\text{Subst}\left(\int x^3\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{(3i)\,\text{Subst}\left(\int x^2\log\left(1-ie^x\right)\,dx, x, \tanh^{-1}(ax)\right)}{a} + \frac{(3i)\,\text{Subs}\ldots}{}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tan\ldots}\right.}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tan\ldots}\right.}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tan\ldots}\right.}{a}$$

$$= \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3}{a} - \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(-ie^{\tanh^{-1}(ax)}\right)}{a} + \frac{3i\tanh^{-1}(ax)^2\text{Li}_2\left(ie^{\tan\ldots}\right.}{a}$$

Mathematica [B] time = 0.449595, size = 451, normalized size = 2.95

$$-\frac{i\left(192\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)+192i\pi\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)+384\tanh^{-1}(ax)\text{PolyLo}\ldots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((-I/64)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 - (32*I
)*Pi*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*
x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] - (96*I)*Pi*ArcTanh[a*
x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*x]
] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] + (96*I)*Pi*ArcTanh[a*x]
^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + 64*Ar
```

```
cTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[Tan[(Pi + (2*I)*Arc
Tanh[a*x])/4]] - 48*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^ArcTanh[a
*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - 48*Pi^2*PolyLog
[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]
] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcTanh[a*x]*PolyLog[3
, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*x]] -
 (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)/E^ArcTanh[a*
x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

Maple [F] time = 0.377, size = 0, normalized size = 0.

$$\int (\operatorname{Artanh}(ax))^3 \frac{1}{\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

3.384 $\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=102

$$-3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(a}\right.$$

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^
ArcTanh[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x
]*PolyLog[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] -
 6*PolyLog[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.17306, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6020, 4182, 2531, 6609, 2282, 6589}

$$-3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(a}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^
ArcTanh[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x
]*PolyLog[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] -
 6*PolyLog[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \tanh^{-1}(ax)\right) + 3\,\text{Subst}\Big($$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2\Big(e^{\tanh}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2\Big(e^{\tanh}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2\Big(e^{\tanh}$$
$$= -2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\tanh^{-1}(ax)^2\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right) + 3\tanh^{-1}(ax)^2\text{Li}_2\Big(e^{\tanh}$$

Mathematica [A] time = 0.133548, size = 146, normalized size = 1.43

$$\frac{1}{8}\Big(24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right) + 48\tanh^{-1}(ax)\text{PolyLog}\Big(3,$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (Pi^4 - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*
ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^(-
ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[
a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[
a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*x]])/
8
```

Maple [A] time = 0.275, size = 215, normalized size = 2.1

$$-\left(\text{Artanh}\,(ax)\right)^3\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) - 3\left(\text{Artanh}\,(ax)\right)^2\text{polylog}\left(2,-\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) + 6\,\text{Artanh}\,(ax)\,\text{polyl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*arctanh(a*x)^2*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1
)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^3*ln(1-(a*x+
1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2
))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+6*polylog(4,(a*x+1)
/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^3}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^3 - x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

3.385 $\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=98

$$-6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6a\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-6a\mathrm{Po}$$

```
[Out] -6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x]*P
olyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLog[3
, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.248515, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6008, 6020, 4182, 2531, 2282, 6589}

$$-6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6a\mathrm{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-6a\mathrm{Po}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] -6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x]*P
olyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLog[3
, E^ArcTanh[a*x]]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} + (3a)\,\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$
$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} - (6a)\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,d\right.$$
$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right.$$
$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right.$$
$$= -6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} - 6a\tanh^{-1}(ax)\text{Li}_2\left(-e^{\tanh^{-1}(ax)}\right.$$

Mathematica [A] time = 0.37222, size = 131, normalized size = 1.34

$$a\left(6\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 6\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 6\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(a*x)) + 3*ArcTanh[a*x]^2*Log[1 - E
^(-ArcTanh[a*x])] - 3*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 6*ArcTanh
[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*PolyLog[2, E^(-ArcTan
h[a*x])] + 6*PolyLog[3, -E^(-ArcTanh[a*x])] - 6*PolyLog[3, E^(-ArcTanh[a*x]
)])
```

Maple [A] time = 0.279, size = 190, normalized size = 1.9

$$-\frac{(\text{Artanh}\,(ax))^3}{x}\sqrt{-(ax-1)(ax+1)} - 3a(\text{Artanh}\,(ax))^2\ln\left(1+\frac{ax+1}{\sqrt{-a^2x^2+1}}\right) - 6a\text{Artanh}\,(ax)\,\text{polylog}\left(2,-\frac{ax}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^3/x-3*a*arctanh(a*x)^2*ln(1+(a*x+1)/
(-a^2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a*arctanh(a*x)^2*ln(1-(a*x+1)
/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^3}{a^2x^4-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^4 - x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

3.386 $\int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=267

$$3a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+\frac{3}{2}a^2\tanh^{-1}(ax)^2$$

```
[Out] (-3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a^2*ArcTan
h[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*ArcTanh[a*x]^2*PolyLog
[2, -E^ArcTanh[a*x]])/2 + (3*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]])
/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*PolyLog[2, Sq
rt[1 - a*x]/Sqrt[1 + a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]]
 - 3*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 3*a^2*PolyLog[4, -E^ArcT
anh[a*x]] + 3*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.447053, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6026, 6008, 6018, 6020, 4182, 2531, 6609, 2282, 6589}

$$3a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+\frac{3}{2}a^2\tanh^{-1}(ax)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```


```
[Out] (-3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3)/(2*x^2) - a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a^2*ArcTan
h[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*ArcTanh[a*x]^2*PolyLog
[2, -E^ArcTanh[a*x]])/2 + (3*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]])
/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*PolyLog[2, Sq
rt[1 - a*x]/Sqrt[1 + a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcTanh[a*x]]
 - 3*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 3*a^2*PolyLog[4, -E^ArcT
anh[a*x]] + 3*a^2*PolyLog[4, E^ArcTanh[a*x]]
```


Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^2\,\text{Subst}\left(\int x^3\mathbf{csch}(x)\,dx, x, \tanh^{-1}(ax)\right.$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 6a^2\,\text{t}$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 6a^2\,\text{t}$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 6a^2\,\text{t}$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 6a^2\,\text{t}$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} - a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 6a^2\,\text{t}$$

Mathematica [A] time = 8.7494, size = 416, normalized size = 1.56

$$\frac{1}{16}a^2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\left(24\tanh^{-1}(ax)^2\coth\left(\frac{1}{2}\tanh^{-1}(ax)\right)\text{PolyLog}\left(2, e^{\tanh^{-1}(ax)}\right) + 48\tanh^{-1}(ax)\coth\left(\frac{1}{2}\tanh\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a^2*(12*ArcTanh[a*x]^2 - (4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/(a*x) + Pi^4
*Coth[ArcTanh[a*x]/2] - 2*ArcTanh[a*x]^4*Coth[ArcTanh[a*x]/2] - 12*ArcTanh[
a*x]^2*Coth[ArcTanh[a*x]/2]^2 - (a*x*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^4)
/Sqrt[1 - a^2*x^2] + 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*Log[1 - E^(-ArcTa
nh[a*x])] - 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*Log[1 + E^(-ArcTanh[a*x])]
 - 8*ArcTanh[a*x]^3*Coth[ArcTanh[a*x]/2]*Log[1 + E^(-ArcTanh[a*x])] + 8*Arc
Tanh[a*x]^3*Coth[ArcTanh[a*x]/2]*Log[1 - E^ArcTanh[a*x]] + 24*(2 + ArcTanh[
a*x]^2)*Coth[ArcTanh[a*x]/2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 48*Coth[ArcTa
nh[a*x]/2]*PolyLog[2, E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*Coth[ArcTanh[a
*x]/2]*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*Po
lyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2]*PolyLog
[3, E^ArcTanh[a*x]] + 48*Coth[ArcTanh[a*x]/2]*PolyLog[4, -E^(-ArcTanh[a*x])
] + 48*Coth[ArcTanh[a*x]/2]*PolyLog[4, E^ArcTanh[a*x]])*Tanh[ArcTanh[a*x]/2
])/16
```

Maple [A] time = 0.321, size = 386, normalized size = 1.5

$$-\frac{(\text{Artanh}\,(ax))^2\,(3\,ax+\text{Artanh}\,(ax))}{2\,x^2}\sqrt{-(ax-1)\,(ax+1)} - \frac{a^2\,(\text{Artanh}\,(ax))^3}{2}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{3\,a^2\,(\text{Ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2*(3*a*x+arctanh(a*x))/x^2-1/2*a
^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*p
olylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(3,-(a*x+1)
/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*a
rctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2
*x^2+1)^(1/2))+3*a^2*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*
x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(
1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*polylog(2,(
a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^3}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^2*x^5 - x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

3.387 $\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=24

$$\text{Unintegrable}\left(\frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Rubi [A] time = 0.0763378, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx = \int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx$$

Mathematica [A] time = 0.424767, size = 0, normalized size = 0.

$$\int \frac{x^m \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2), x]
```

Maple [A] time = 0.826, size = 0, normalized size = 0.

$$\int x^m \text{Artanh}(ax) \left(-a^2x^2+1\right)^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m \operatorname{artanh}(ax)}{a^4x^4-2a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

3.388 $\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=74

$$-\frac{x}{a^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^4}$$

```
[Out] -(x/(a^3*Sqrt[1 - a^2*x^2])) - ArcSin[a*x]/a^4 + ArcTanh[a*x]/(a^4*Sqrt[1 -
 a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4
```

Rubi [A] time = 0.169943, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6028, 5994, 216, 191}

$$-\frac{x}{a^3\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4}+\frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}}-\frac{\sin^{-1}(ax)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(x/(a^3*Sqrt[1 - a^2*x^2])) - ArcSin[a*x]/a^4 + ArcTanh[a*x]/(a^4*Sqrt[1 -
 a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a^4
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4} - \frac{\int \frac{1}{(1-a^2x^2)^{3/2}}\,dx}{a^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$

$$= -\frac{x}{a^3\sqrt{1-a^2x^2}} - \frac{\sin^{-1}(ax)}{a^4} + \frac{\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^4}$$

Mathematica [A] time = 0.0632908, size = 76, normalized size = 1.03

$$\frac{ax\sqrt{1-a^2x^2} + (1-a^2x^2)\sin^{-1}(ax) + \sqrt{1-a^2x^2}\,(a^2x^2-2)\tanh^{-1}(ax)}{a^4\,(a^2x^2-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] + (1 - a^2*x^2)*ArcSin[a*x] + Sqrt[1 - a^2*x^2]*(-2
+ a^2*x^2)*ArcTanh[a*x])/(a^4*(-1 + a^2*x^2))
```

Maple [C] time = 0.249, size = 144, normalized size = 2.

$$-\frac{\mathrm{Artanh}\,(ax)-1}{2\,a^4\,(ax-1)}\sqrt{-(ax-1)\,(ax+1)} + \frac{\mathrm{Artanh}\,(ax)+1}{2\,a^4\,(ax+1)}\sqrt{-(ax-1)\,(ax+1)} + \frac{\mathrm{Artanh}\,(ax)}{a^4}\sqrt{-(ax-1)\,(ax+1)} + \frac{i}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x-1)+1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x+1)+arctanh(a*x)*(-(a*x-1)*(a*x+1))^(
1/2)/a^4+I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4-I*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)+I)/a^4
```

Maxima [A] time = 1.45301, size = 146, normalized size = 1.97

$$a\left(\frac{\frac{x}{\sqrt{-a^2x^2+1}a^2} - \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}}{a^2} - \frac{2\,x}{\sqrt{-a^2x^2+1}a^4}\right) - \left(\frac{x^2}{\sqrt{-a^2x^2+1}a^2} - \frac{2}{\sqrt{-a^2x^2+1}a^4}\right)\mathrm{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] a*((x/(sqrt(-a^2*x^2 + 1)*a^2) - arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))/a
^2 - 2*x/(sqrt(-a^2*x^2 + 1)*a^4)) - (x^2/(sqrt(-a^2*x^2 + 1)*a^2) - 2/(sqr
```

```
t(-a^2*x^2 + 1)*a^4))*arctanh(a*x)
```

Fricas [A] time = 2.02198, size = 201, normalized size = 2.72

$$\frac{4\left(a^2x^2-1\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+\sqrt{-a^2x^2+1}\left(2\,ax+\left(a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^2*x^2 - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + sqrt(-a^2*x^2
 + 1)*(2*a*x + (a^2*x^2 - 2)*log(-(a*x + 1)/(a*x - 1))))/(a^6*x^2 - a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\operatorname{atanh}(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.22875, size = 120, normalized size = 1.62

$$\frac{\left(\sqrt{-a^2x^2+1}+\frac{1}{\sqrt{-a^2x^2+1}}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{2\,a^4}-\frac{\arcsin(ax)\operatorname{sgn}(a)}{a^3|a|}+\frac{\sqrt{-a^2x^2+1}x}{\left(a^2x^2-1\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1) + 1/sqrt(-a^2*x^2 + 1))*log(-(a*x + 1)/(a*x - 1))/a
^4 - arcsin(a*x)*sgn(a)/(a^3*abs(a)) + sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*
a^3)
```

3.389 $\int \frac{x^2 \tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=137

$$\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{1}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}$$

```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) + (
2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 + (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 - (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a^3
```

Rubi [A] time = 0.105353, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5998, 5950}

$$\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^3}-\frac{1}{a^3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}+\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```


```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) + (
2*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^3 + (I*PolyLog[2, ((-
I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 - (I*PolyLog[2, (I*Sqrt[1 - a*x])/Sqr
t[1 + a*x]])/a^3
```


Rule 5998

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_),
x_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] -
 Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*c^2*d*(q + 1)), x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -5
/2]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{x^2\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx = -\frac{1}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{1}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} + \frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^3} + \frac{i\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3} - \frac{i\mathrm{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a^3}$$

Mathematica [A] time = 0.22646, size = 121, normalized size = 0.88

$$\frac{i\left(\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right) - \mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right) + \frac{i}{\sqrt{1-a^2x^2}} - \frac{iax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-1}(ax)}\right) \ldots\right)}{a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (I*(I/Sqrt[1 - a^2*x^2] - (I*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[
a*x]*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + P
olyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/a^3
```

Maple [A] time = 0.346, size = 190, normalized size = 1.4

$$-\frac{\mathrm{Artanh}(ax)-1}{2a^3(ax-1)}\sqrt{-(ax-1)(ax+1)} - \frac{\mathrm{Artanh}(ax)+1}{2a^3(ax+1)}\sqrt{-(ax-1)(ax+1)} + \frac{i\mathrm{Artanh}(ax)}{a^3}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x-1)-1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^3/(a*x+1)+I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/
2))*arctanh(a*x)/a^3-I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+
I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3-I*dilog(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,\mathrm{artanh}(ax)}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)}{a^4x^4-2a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)/(a^4*x^4 - 2*a^2*x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)/(-a^2*x^2 + 1)^(3/2), x)
```

3.390 $\int \frac{x \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}$$

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0506343, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5994, 191}

$$\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{x}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2])) + ArcTanh[a*x]/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x \tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}} dx &= \frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}} dx}{a}\\ &= -\frac{x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0357683, size = 27, normalized size = 0.63

$$\frac{\tanh^{-1}(ax)-ax}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x])/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-(a*x) + ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.205, size = 66, normalized size = 1.5

$$-\frac{\operatorname{Artanh}(ax)-1}{2a^2(ax-1)}\sqrt{-(ax-1)(ax+1)}+\frac{\operatorname{Artanh}(ax)+1}{2a^2(ax+1)}\sqrt{-(ax-1)(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)+1/2*(arctanh(a*x
)+1)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)
```

Maxima [A] time = 0.955062, size = 53, normalized size = 1.23

$$-\frac{x}{\sqrt{-a^2x^2+1}a}+\frac{\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a^2)
```

Fricas [A] time = 2.0054, size = 103, normalized size = 2.4

$$\frac{\sqrt{-a^2x^2+1}\left(2\,ax-\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*(2*a*x - log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\operatorname{atanh}(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.23861, size = 82, normalized size = 1.91

$$\frac{\sqrt{-a^2x^2+1}x}{(a^2x^2-1)a}+\frac{\log\left(-\frac{ax+1}{ax-1}\right)}{2\sqrt{-a^2x^2+1}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*a) + 1/2*log(-(a*x + 1)/(a*x - 1))/(sqr
t(-a^2*x^2 + 1)*a^2)
```

3.391 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}$$

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.0252848, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5958}

$$\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{1}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2])) + (x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx=-\frac{1}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0312152, size = 27, normalized size = 0.68

$$\frac{ax\tanh^{-1}(ax)-1}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-1 + a*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.22, size = 38, normalized size = 1.

$$-\frac{ax\mathrm{Artanh}\,(ax)-1}{a\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(a*x*arctanh(a*x)-1)/(a^2*x^2-1)
```

Maxima [A] time = 0.957387, size = 49, normalized size = 1.22

$$\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(a*x)/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 1.98146, size = 101, normalized size = 2.52

$$-\frac{\sqrt{-a^2x^2+1}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.22904, size = 80, normalized size = 2.

$$-\frac{\sqrt{-a^2x^2+1}x\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^2x^2-1\right)}-\frac{1}{\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1) - 1/(sqrt
(-a^2*x^2 + 1)*a)
```

3.392 $\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=112

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] - 2*ArcTanh[a*x
]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
+ a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.185961, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6030, 6018, 5994, 191}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{ax}{\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```


```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] - 2*ArcTanh[a*x
]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
+ a*x])] - PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{x\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - a\int \frac{1}{\left(1-a^2\right.}$$

$$= -\frac{ax}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \text{Li}_2\left(-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{Li}_2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right.$$

Mathematica [A] time = 0.178578, size = 97, normalized size = 0.87

$$\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - \text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) - \frac{ax}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right) -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a*x)/Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]*
Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + Poly
Log[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A] time = 0.253, size = 157, normalized size = 1.4

$$-\frac{\text{Artanh}(ax)-1}{2ax-2}\sqrt{-(ax-1)(ax+1)} + \frac{\text{Artanh}(ax)+1}{2ax+2}\sqrt{-(ax-1)(ax+1)} - \text{Artanh}(ax)\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)+1)
*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1
/2))-polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)*ln(1-(a*x+1)/(-a^2
*x^2+1)^(1/2))+polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{a^4x^5-2\,a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{atanh}\,(ax)}{x\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{artanh}\,(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.393 $\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}$$

```
[Out] -(a/Sqrt[1 - a^2*x^2]) + (a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/x - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rubi [A] time = 0.170999, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6030, 6008, 266, 63, 208, 5958}

$$-\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -(a/Sqrt[1 - a^2*x^2]) + (a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x])/x - a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{x^2(1-a^2x^2)^{3/2}}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+\frac{1}{2}a\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a}\\
&= -\frac{a}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.109833, size = 89, normalized size = 1.09

$$\frac{ax\left(\sqrt{1-a^2x^2}\log(x)-\sqrt{1-a^2x^2}\log\left(\sqrt{1-a^2x^2}+1\right)-1\right)+\left(2a^2x^2-1\right)\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] ((-1 + 2*a^2*x^2)*ArcTanh[a*x] + a*x*(-1 + Sqrt[1 - a^2*x^2]*Log[x] - Sqrt[
1 - a^2*x^2]*Log[1 + Sqrt[1 - a^2*x^2]]))/(x*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.253, size = 132, normalized size = 1.6

$$-\frac{a(\mathrm{Artanh}(ax)-1)}{2ax-2}\sqrt{-(ax-1)(ax+1)}-\frac{(\mathrm{Artanh}(ax)+1)a}{2ax+2}\sqrt{-(ax-1)(ax+1)}-\frac{\mathrm{Artanh}(ax)}{x}\sqrt{-(ax-1)(ax+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)-1/2*(arctanh(a*x)+
1)*a*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)
/x+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 0.957291, size = 113, normalized size = 1.38

$$-a\left(\frac{1}{\sqrt{-a^2x^2+1}}+\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)\right)+\left(\frac{2a^2x}{\sqrt{-a^2x^2+1}}-\frac{1}{\sqrt{-a^2x^2+1}x}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(1/sqrt(-a^2*x^2 + 1) + log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))) + (
2*a^2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*x))*arctanh(a*x)
```

Fricas [A] time = 2.06518, size = 223, normalized size = 2.72

$$-\frac{2a^3x^3-2ax-2\left(a^3x^3-ax\right)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-\sqrt{-a^2x^2+1}\left(2ax-\left(2a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{2\left(a^2x^3-x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*x^3 - 2*a*x - 2*(a^3*x^3 - a*x)*log((sqrt(-a^2*x^2 + 1) - 1)/x)
 - sqrt(-a^2*x^2 + 1)*(2*a*x - (2*a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/
(a^2*x^3 - x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}(ax)}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] time = 1.26088, size = 209, normalized size = 2.55

$$-\frac{1}{2}a\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{1}{2}a\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{4}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|}-\frac{2\sqrt{-a^2x^2+1}a^2x}{a^2x^2-1}-\frac{\sqrt{-a^2x^2+1}|a}{x|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*a*log(sqrt(-a^2*x^2 + 1) + 1) + 1/2*a*log(-sqrt(-a^2*x^2 + 1) + 1) + 1
/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - 2*sqrt(-a^2*x^2 + 1)*a
^2*x/(a^2*x^2 - 1) - (sqrt(-a^2*x^2 + 1)*abs(a) + a)/(x*abs(a)))*log(-(a*x
+ 1)/(a*x - 1)) - a/sqrt(-a^2*x^2 + 1)
```

3.394 $\int \frac{\tanh^{-1}(ax)}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=179

$$\frac{3}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\text{ta}}{2x}$$

```
[Out] -((a^3*x)/Sqrt[1 - a^2*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x) + (a^2*ArcTanh[a
*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - 3*a^2*A
rcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (3*a^2*PolyLog[2, -(Sqrt
[1 - a*x]/Sqrt[1 + a*x])])/2 - (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rubi [A] time = 0.402211, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6030, 6026, 264, 6018, 5994, 191}

$$\frac{3}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\text{ta}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] -((a^3*x)/Sqrt[1 - a^2*x^2]) - (a*Sqrt[1 - a^2*x^2])/(2*x) + (a^2*ArcTanh[a
*x])/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) - 3*a^2*A
rcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (3*a^2*PolyLog[2, -(Sqrt
[1 - a*x]/Sqrt[1 + a*x])])/2 - (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx &= a^2\int \frac{\tanh^{-1}(ax)}{x\left(1-a^2x^2\right)^{3/2}}\,dx+\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx+\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx+a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}-3a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\\ &= -\frac{a^3x}{\sqrt{1-a^2x^2}}-\frac{a\sqrt{1-a^2x^2}}{2x}+\frac{a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}-3a^2\tanh^{-1}(ax)\tanh^{-1}\left(\right.\end{aligned}$$

Mathematica [A] time = 1.52742, size = 182, normalized size = 1.02

$$\frac{1}{8}a^2\left(12\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-12\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\frac{8ax}{\sqrt{1-a^2x^2}}+\frac{8\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{ax\,\text{csch}^2\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*((-8*a*x)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*
x*Csch[ArcTanh[a*x]/2]^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]*Csch[ArcTanh[a*x
]/2]^2 + 12*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]*Log[1
 + E^(-ArcTanh[a*x])] + 12*PolyLog[2, -E^(-ArcTanh[a*x])] - 12*PolyLog[2, E
^(-ArcTanh[a*x])] - ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*
x]/2]))/8
```

Maple [A] time = 0.289, size = 205, normalized size = 1.2

$$-\frac{a^2(\text{Artanh}(ax)-1)}{2ax-2}\sqrt{-(ax-1)(ax+1)}+\frac{(\text{Artanh}(ax)+1)a^2}{2ax+2}\sqrt{-(ax-1)(ax+1)}-\frac{ax+\text{Artanh}(ax)}{2x^2}\sqrt{-(ax-1)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a^2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x
)+1)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x
+arctanh(a*x))/x^2-3/2*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/
2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)*ln(1-(a*x
+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{a^4x^7-2\,a^2x^5+x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/x**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.395 $\int \frac{x^m \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Rubi [A] time = 0.10192, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} \, dx = \int \frac{x^m \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} \, dx$$

Mathematica [A] time = 0.490455, size = 0, normalized size = 0.

$$\int \frac{x^m \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2), x]
```

Maple [A] time = 0.759, size = 0, normalized size = 0.

$$\int x^m \left(\text{Artanh}\,(ax)\right)^2 \left(-a^2x^2+1\right)^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m \operatorname{artanh}(ax)^2}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.396 $\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=186

$$\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}-\frac{2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}+\frac{2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^4}+\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh}{a^3\sqrt{1-}}$$

```
[Out] 2/(a^4*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) + (4
*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^4 + ArcTanh[a*x]^2/(a^
4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^4 + ((2*I)*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 - ((2*I)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rubi [A] time = 0.310389, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6028, 5994, 5950, 5958}

$$\frac{2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}-\frac{2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a^4}+\frac{2}{a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^4}+\frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}}-\frac{2x\tanh}{a^3\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 2/(a^4*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) + (4
*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a^4 + ArcTanh[a*x]^2/(a^
4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/a^4 + ((2*I)*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 - ((2*I)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
```

```
0]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{a^4} - \frac{2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^3} - \frac{2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$

$$= \frac{2}{a^4\sqrt{1-a^2x^2}} - \frac{2x\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a^4} + \frac{\tanh^{-1}(ax)^2}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}}{a^4}$$

Mathematica [A] time = 0.371595, size = 165, normalized size = 0.89

$$\frac{\frac{-2i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\left(2-a^2x^2\right)\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)\left(-i\sqrt{1-a^2x^2}\log\left(1-ie^{-\tanh^{-1}(ax)}\right)+i\sqrt{1-a^2x^2}\log\left(1+ie^{-\tanh^{-1}(ax)}\right)+ax\right)+2}{\sqrt{1-a^2x^2}}+2i}{a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (2 + (2 - a^2*x^2)*ArcTanh[a*x]^2
- 2*ArcTanh[a*x]*(a*x - I*Sqrt[1 - a^2*x^2]*Log[1 - I/E^ArcTanh[a*x]] + I*S
qrt[1 - a^2*x^2]*Log[1 + I/E^ArcTanh[a*x]]) - (2*I)*Sqrt[1 - a^2*x^2]*PolyL
og[2, I/E^ArcTanh[a*x]])/Sqrt[1 - a^2*x^2])/a^4
```

Maple [A] time = 0.273, size = 230, normalized size = 1.2

$$-\frac{(\text{Artanh}(ax))^2-2\,\text{Artanh}(ax)+2}{2\,a^4\,(ax-1)}\sqrt{-(ax-1)(ax+1)}+\frac{(\text{Artanh}(ax))^2+2\,\text{Artanh}(ax)+2}{2\,a^4\,(ax+1)}\sqrt{-(ax-1)(ax+1)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^4/(a*x+1)
+arctanh(a*x)^2*(-(a*x-1)*(a*x+1))^(1/2)/a^4+2*I*ln(1+I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^4-2*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*
x)/a^4+2*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^4-2*I*dilog(1-I*(a*x+1)/
(-a^2*x^2+1)^(1/2))/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^3 \operatorname{artanh}(ax)^2}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.397 $\int \frac{x^2 \tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=171

$$\frac{2i \tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i \tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{2i\text{Po}}{}$$

```
[Out] (2*x)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) +
(x*ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcT
anh[a*x]^2)/a^3 + ((2*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3
- ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^3 - ((2*I)*PolyLog[3,
 (-I)*E^ArcTanh[a*x]])/a^3 + ((2*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rubi [A] time = 0.251751, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6028, 5952, 4180, 2531, 2282, 6589, 5962, 191}

$$\frac{2i \tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i \tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{2i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}+\frac{2i\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```


```
[Out] (2*x)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(a^3*Sqrt[1 - a^2*x^2]) +
(x*ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcT
anh[a*x]^2)/a^3 + ((2*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3
- ((2*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^3 - ((2*I)*PolyLog[3,
 (-I)*E^ArcTanh[a*x]])/a^3 + ((2*I)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```


Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int x^2\textbf{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^3} + \frac{2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a^2}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{(2i)\,\text{Sub}}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{2i\tanh}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{2i\tanh}{}$$

$$= \frac{2x}{a^2\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^3} + \frac{2i\tanh}{}$$

Mathematica [A] time = 0.312353, size = 193, normalized size = 1.13

$$\frac{i\left(2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2\text{I}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (I*(((-2*I)*a*x)/Sqrt[1 - a^2*x^2] + ((2*I)*ArcTanh[a*x])/Sqrt[1 - a^2*x^2]
 - (I*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - I/E^Ar
cTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*Pol
yLog[2, (-I)/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]]
+ 2*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/a^3
```

Maple [F] time = 0.242, size = 0, normalized size = 0.

$$\int x^2 (\text{Artanh}(ax))^2 \left(-a^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)^2}{a^4x^4-2a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.398 $\int \frac{x \tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}} dx$

Optimal. Leaf size=68

$$\frac{2}{a^2\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] 2/(a^2*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + ArcT
anh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0983558, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5994, 5958}

$$\frac{2}{a^2\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] 2/(a^2*Sqrt[1 - a^2*x^2]) - (2*x*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + ArcT
anh[a*x]^2/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}\\ &= \frac{2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.055104, size = 34, normalized size = 0.5

$$\frac{\tanh^{-1}(ax)^2-2ax\tanh^{-1}(ax)+2}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^2)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2 - 2*a*x*ArcTanh[a*x] + ArcTanh[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.211, size = 82, normalized size = 1.2

$$-\frac{(\operatorname{Artanh}(ax))^2 - 2\operatorname{Artanh}(ax) + 2}{2a^2(ax-1)}\sqrt{-(ax-1)(ax+1)} + \frac{(\operatorname{Artanh}(ax))^2 + 2\operatorname{Artanh}(ax) + 2}{2a^2(ax+1)}\sqrt{-(ax-1)(ax+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)
```

Maxima [A] time = 0.952745, size = 84, normalized size = 1.24

$$-\frac{2x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}a} + \frac{\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}a^2} + \frac{2}{\sqrt{-a^2x^2+1}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*x*arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^2/(sqrt(-a^2*x^2 +
1)*a^2) + 2/(sqrt(-a^2*x^2 + 1)*a^2)
```

Fricas [A] time = 2.00201, size = 146, normalized size = 2.15

$$\frac{\sqrt{-a^2x^2+1}\left(4\,ax\log\left(-\frac{ax+1}{ax-1}\right) - \log\left(-\frac{ax+1}{ax-1}\right)^2 - 8\right)}{4\left(a^4x^2 - a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-a^2*x^2 + 1)*(4*a*x*log(-(a*x + 1)/(a*x - 1)) - log(-(a*x + 1)/(a
*x - 1))^2 - 8)/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\operatorname{atanh}^2(ax)}{(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.399 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2x}{\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

```
[Out] (2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTa
nh[a*x]^2)/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.0396381, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5962, 191}

$$\frac{2x}{\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*x)/Sqrt[1 - a^2*x^2] - (2*ArcTanh[a*x])/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTa
nh[a*x]^2)/Sqrt[1 - a^2*x^2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx&=-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}+2\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx\\&=\frac{2x}{\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0406033, size = 38, normalized size = 0.6

$$\frac{2ax+ax\tanh^{-1}(ax)^2-2\tanh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (2*a*x - 2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.225, size = 49, normalized size = 0.8

$$-\frac{(\mathrm{Artanh}(ax))^2 ax + 2 ax - 2 \mathrm{Artanh}(ax)}{a(a^2x^2 - 1)}\sqrt{-a^2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^2*a*x+2*a*x-2*arctanh(a*x))/(a^2*x^2-
1)
```

Maxima [A] time = 0.955922, size = 77, normalized size = 1.22

$$\frac{x \operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2 + 1}} + \frac{2x}{\sqrt{-a^2x^2 + 1}} - \frac{2 \operatorname{artanh}(ax)}{\sqrt{-a^2x^2 + 1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(a*x)^2/sqrt(-a^2*x^2 + 1) + 2*x/sqrt(-a^2*x^2 + 1) - 2*arctanh(a*
x)/(sqrt(-a^2*x^2 + 1)*a)
```

Fricas [A] time = 2.28507, size = 150, normalized size = 2.38

$$-\frac{\sqrt{-a^2x^2 + 1}\left(ax \log\left(-\frac{ax+1}{ax-1}\right)^2 + 8 ax - 4 \log\left(-\frac{ax+1}{ax-1}\right)\right)}{4(a^3x^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + 1)*(a*x*log(-(a*x + 1)/(a*x - 1))^2 + 8*a*x - 4*log(-(
a*x + 1)/(a*x - 1)))/(a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{(-(ax - 1)(ax + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(3/2), x)
```

3.400 $\int \frac{\tanh^{-1}(ax)^2}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=127

$$-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-2\text{PolyLo}$$

```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh
[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x
]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.341227, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6030, 6020, 4182, 2531, 2282, 6589, 5994, 5958}

$$-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-2\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```


```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh
[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x
]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^ArcTanh[a*x]]
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - (2a)\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx + \text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$

$$= \frac{2}{\sqrt{1-a^2x^2}} - \frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\,\text{Subst}\left(\right.$$

$$= \frac{2}{\sqrt{1-a^2x^2}} - \frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}($$

$$= \frac{2}{\sqrt{1-a^2x^2}} - \frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}($$

$$= \frac{2}{\sqrt{1-a^2x^2}} - \frac{2ax\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 2\tanh^{-1}($$

Mathematica [A] time = 0.240585, size = 159, normalized size = 1.25

$$2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 2\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) - 2\text{P}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] 2/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]
^2/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[
a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[
a*x])] - 2*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 2*PolyLog[3, -E^(-A
rcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a*x])]
```

Maple [A] time = 0.27, size = 232, normalized size = 1.8

$$-\frac{(\operatorname{Artanh}(ax))^2-2\operatorname{Artanh}(ax)+2}{2ax-2}\sqrt{-(ax-1)(ax+1)}+\frac{(\operatorname{Artanh}(ax))^2+2\operatorname{Artanh}(ax)+2}{2ax+2}\sqrt{-(ax-1)(ax+1)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)+1/2
*(arctanh(a*x)^2+2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-arctanh
(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-2*arctanh(a*x)*polylog(2,-(a*x+1)/
(-a^2*x^2+1)^(1/2))+2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+arctanh(a*x)^2
*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^
2+1)^(1/2))-2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^2}{a^4x^5-2a^2x^3+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{x(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.401 $\int \frac{\tanh^{-1}(ax)^2}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=171

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{2a^2x}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\text{ta}}{x}$$

```
[Out] (2*a^2*x)/Sqrt[1 - a^2*x^2] - (2*a*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (a^2*x
*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x -
 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(S
qrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.312958, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6030, 6008, 6018, 5962, 191}

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{2a^2x}{\sqrt{1-a^2x^2}}+\frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{\sqrt{1-a^2x^2}\,\text{ta}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (2*a^2*x)/Sqrt[1 - a^2*x^2] - (2*a*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (a^2*x
*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x -
 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 2*a*PolyLog[2, -(S
qrt[1 - a*x]/Sqrt[1 + a*x])] - 2*a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} + (2a)\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx + \left(2a\right.$$
$$= \frac{2a^2x}{\sqrt{1-a^2x^2}} - \frac{2a\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} - 4a\tanh^{-1}(ax)\,\mathrm{ta}$$

Mathematica [A] time = 1.07844, size = 215, normalized size = 1.26

$$\frac{a\left(4\sqrt{1-a^2x^2}\mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-4\sqrt{1-a^2x^2}\mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+4\sqrt{1-a^2x^2}\tanh^{-1}(ax)\log\left(1-e^{-}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a*(4*a*x - 4*ArcTanh[a*x] + 2*a*x*ArcTanh[a*x]^2 - (a*x*ArcTanh[a*x]^2*Csc
h[ArcTanh[a*x]/2]^2)/2 + 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-ArcTa
nh[a*x])] - 4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 4
*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 4*Sqrt[1 - a^2*x^2]*Pol
yLog[2, E^(-ArcTanh[a*x])] - (2*(-1 + a^2*x^2)*ArcTanh[a*x]^2*Sinh[ArcTanh[
a*x]/2]^2)/(a*x)))/(2*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.27, size = 207, normalized size = 1.2

$$-\frac{a\left((\mathrm{Artanh}(ax))^2-2\,\mathrm{Artanh}(ax)+2\right)}{2\,ax-2}\sqrt{-(ax-1)(ax+1)}-\frac{\left((\mathrm{Artanh}(ax))^2+2\,\mathrm{Artanh}(ax)+2\right)a}{2\,ax+2}\sqrt{-(ax-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)-1
/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(
a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^2/x-2*a*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*
x^2+1)^(1/2))-2*a*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*arctanh(a*x)*l
```

```
n(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+2*a*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^2}{a^4x^6-2\,a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{x^2(-(ax-1)(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

3.402 $\int \frac{\tanh^{-1}(ax)^2}{x^3(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=221

$$-3a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+3a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-$$

```
[Out] (2*a^2)/Sqrt[1 - a^2*x^2] - (2*a^3*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*S
qrt[1 - a^2*x^2]*ArcTanh[a*x])/x + (a^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] -
 (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]
*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1 - a^2*x^2]] - 3*a^2*ArcTanh[a*x]*PolyL
og[2, -E^ArcTanh[a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 3*
a^2*PolyLog[3, -E^ArcTanh[a*x]] - 3*a^2*PolyLog[3, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.776324, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6030, 6026, 6008, 266, 63, 208, 6020, 4182, 2531, 2282, 6589, 5994, 5958}

$$-3a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+3a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```


```
[Out] (2*a^2)/Sqrt[1 - a^2*x^2] - (2*a^3*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (a*S
qrt[1 - a^2*x^2]*ArcTanh[a*x])/x + (a^2*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] -
 (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]
*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1 - a^2*x^2]] - 3*a^2*ArcTanh[a*x]*PolyL
og[2, -E^ArcTanh[a*x]] + 3*a^2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + 3*
a^2*PolyLog[3, -E^ArcTanh[a*x]] - 3*a^2*PolyLog[3, E^ArcTanh[a*x]]
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
```

```
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^2}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx + a^2\int \frac{\tanh^{-1}(ax}{x\sqrt{1-a^2x}}$$
$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int x^2\mathrm{csc}\right.$$
$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh}{2x^2}$$
$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh}{2x^2}$$
$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh}{2x^2}$$
$$= \frac{2a^2}{\sqrt{1-a^2x^2}} - \frac{2a^3x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} + \frac{a^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh}{2x^2}$$

Mathematica [A] time = 2.7034, size = 266, normalized size = 1.2

$$\frac{1}{8}a^2\left(24\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 24\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 24\mathrm{PolyLog}\left(3,-e^{-\tanh^{-1}(}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^2/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*(16/Sqrt[1 - a^2*x^2] - (16*a*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] + (8*A
rcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (2*a*x*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]
^2)/Sqrt[1 - a^2*x^2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*x]/2]^2 + 12*ArcTanh[
a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 12*ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a
*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] + 24*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 24*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 24*PolyLog[3, -
E^(-ArcTanh[a*x])] - 24*PolyLog[3, E^(-ArcTanh[a*x])] - ArcTanh[a*x]^2*Sech
```

```
[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A] time = 0.299, size = 313, normalized size = 1.4

$$-\frac{a^2\left((\operatorname{Artanh}(ax))^2-2\operatorname{Artanh}(ax)+2\right)}{2ax-2}\sqrt{-(ax-1)(ax+1)}+\frac{\left((\operatorname{Artanh}(ax))^2+2\operatorname{Artanh}(ax)+2\right)a^2}{2ax+2}\sqrt{-(ax-1)(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a^2*(arctanh(a*x)^2-2*arctanh(a*x)+2)*(-(a*x-1)*(a*x+1))^(1/2)/(a*x-1)
+1/2*(arctanh(a*x)^2+2*arctanh(a*x)+2)*a^2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)
-1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2-2*a^2*a
rctanh((a*x+1)/(-a^2*x^2+1)^(1/2))-3/2*a^2*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^
2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3
*a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^2*ln(1-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*polylog(2,(a*x+1)/(-a^2*x^2+1)^
(1/2))-3*a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^2}{a^4x^7-2a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{x^3\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/x**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.403 $\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}},x\right)$$

```
[Out] Unintegrable[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]
```

Rubi [A] time = 0.100265, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Defer[Int][(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx = \int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

Mathematica [A] time = 0.475587, size = 0, normalized size = 0.

$$\int \frac{x^m \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^m*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2), x]
```

Maple [A] time = 0.747, size = 0, normalized size = 0.

$$\int x^m (\text{Artanh}(ax))^3 \left(-a^2x^2+1\right)^{-\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m \operatorname{artanh}(ax)^3}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^m*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.404 $\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} \, dx$

Optimal. Leaf size=220

$$\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{6i\text{Po}}{}$$

```
[Out] (-6*x)/(a^3*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^4*Sqrt[1 - a^2*x^2]) -
 (3*x*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) - (6*ArcTan[E^ArcTanh[a*x]]*A
rcTanh[a*x]^2)/a^4 + ArcTanh[a*x]^3/(a^4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^3)/a^4 + ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a
*x]])/a^4 - ((6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 - ((6*I)*
PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^4 + ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])
/a^4
```

Rubi [A] time = 0.405468, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6028, 5994, 5952, 4180, 2531, 2282, 6589, 5962, 191}

$$\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^4}-\frac{6i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^4}+\frac{6i\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```


```
[Out] (-6*x)/(a^3*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^4*Sqrt[1 - a^2*x^2]) -
 (3*x*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) - (6*ArcTan[E^ArcTanh[a*x]]*A
rcTanh[a*x]^2)/a^4 + ArcTanh[a*x]^3/(a^4*Sqrt[1 - a^2*x^2]) + (Sqrt[1 - a^2
*x^2]*ArcTanh[a*x]^3)/a^4 + ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a
*x]])/a^4 - ((6*I)*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x]])/a^4 - ((6*I)*
PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^4 + ((6*I)*PolyLog[3, I*E^ArcTanh[a*x]])
/a^4
```


Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
```

```
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x^3 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{x\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= \frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^4} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{3/2}}\,dx}{a^3} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{a^3}$$

$$= \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{a^4} - \frac{3\,\mathrm{Subst}\left(\int x^2\mathrm{sech}\right.}{}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} + \frac{\tanh^{-1}(}{a^4\sqrt{1-}}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} + \frac{\tanh^{-1}(}{a^4\sqrt{1-}}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} + \frac{\tanh^{-1}(}{a^4\sqrt{1-}}$$

$$= -\frac{6x}{a^3\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^4\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{6\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2}{a^4} + \frac{\tanh^{-1}(}{a^4\sqrt{1-}}$$

Mathematica [A] time = 0.419736, size = 249, normalized size = 1.13

$$\frac{6i\sqrt{1-a^2x^2}\mathrm{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-6i\sqrt{1-a^2x^2}\mathrm{PolyLog}\left(3,ie^{-\tanh^{-1}(ax)}\right)-a^2x^2\tanh^{-1}(ax)^3+3i\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-3i\sqrt{1-a^2x^2}\,\mathrm{ta}}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((6*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (6*I)*ArcTanh[a*x]*Po
lyLog[2, I/E^ArcTanh[a*x]] + (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^
2 + 2*ArcTanh[a*x]^3 - a^2*x^2*ArcTanh[a*x]^3 + (3*I)*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - (3*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2*Log[1 + I/E^ArcTanh[a*x]] + (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, (-I)/E^A
rcTanh[a*x]] - (6*I)*Sqrt[1 - a^2*x^2]*PolyLog[3, I/E^ArcTanh[a*x]])/Sqrt[1
 - a^2*x^2])/a^4
```

Maple [F] time = 0.291, size = 0, normalized size = 0.

$$\int x^3 (\mathrm{Artanh}(ax))^3 \left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}x^3 \operatorname{artanh}(ax)^3}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**3*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.405 $\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}} dx$

Optimal. Leaf size=246

$$\frac{3i \tanh^{-1}(ax)^2 \text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i \tanh^{-1}(ax)^2 \text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6i \tanh^{-1}(ax) \text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}$$

```
[Out] -6/(a^3*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(a^2*Sqrt[1
- a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 + ((3*I)*ArcTan
h[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]^2*PolyL
og[2, I*E^ArcTanh[a*x]])/a^3 - ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTan
h[a*x]])/a^3 + ((6*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a^3 + ((6*
I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a^3 - ((6*I)*PolyLog[4, I*E^ArcTanh[a*x
]])/a^3
```

Rubi [A] time = 0.317475, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6028, 5952, 4180, 2531, 6609, 2282, 6589, 5962, 5958}

$$\frac{3i \tanh^{-1}(ax)^2 \text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{3i \tanh^{-1}(ax)^2 \text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a^3}-\frac{6i \tanh^{-1}(ax) \text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```


```
[Out] -6/(a^3*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*ArcTanh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(a^2*Sqrt[1
- a^2*x^2]) - (2*ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a^3 + ((3*I)*ArcTan
h[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 - ((3*I)*ArcTanh[a*x]^2*PolyL
og[2, I*E^ArcTanh[a*x]])/a^3 - ((6*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTan
h[a*x]])/a^3 + ((6*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[a*x]])/a^3 + ((6*
I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a^3 - ((6*I)*PolyLog[4, I*E^ArcTanh[a*x
]])/a^3
```


Rule 6028

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTanh[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
h[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Inte
gersQ[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{x^2 \tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx = \frac{\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{3/2}}\,dx}{a^2} - \frac{\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a^2}$$

$$= -\frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{\text{Subst}\left(\int x^3 \mathbf{sech}(x)\,dx, x, \tanh^{-1}(ax)\right)}{a^3} + \frac{6\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{3/2}}\,dx}{a^2}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^-}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^-}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^-}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^-}{a^3}$$

$$= -\frac{6}{a^3\sqrt{1-a^2x^2}} + \frac{6x\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3\tanh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{2\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^-}{a^3}$$

Mathematica [B] time = 0.936025, size = 541, normalized size = 2.2

$$\frac{192i\tanh^{-1}(ax)^2\text{PolyLog}\left(2, -ie^{\tanh^{-1}(ax)}\right) - 192\pi\tanh^{-1}(ax)\text{PolyLog}\left(2, ie^{\tanh^{-1}(ax)}\right) + 384i\tanh^{-1}(ax)\text{PolyLog}\left(3, -\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] ((7*I)*Pi^4 - 384/Sqrt[1 - a^2*x^2] - 8*Pi^3*ArcTanh[a*x] + (384*a*x*ArcTan
h[a*x])/Sqrt[1 - a^2*x^2] + (24*I)*Pi^2*ArcTanh[a*x]^2 - (192*ArcTanh[a*x]^
2)/Sqrt[1 - a^2*x^2] + 32*Pi*ArcTanh[a*x]^3 + (64*a*x*ArcTanh[a*x]^3)/Sqrt[
1 - a^2*x^2] - (16*I)*ArcTanh[a*x]^4 - 8*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + (
48*I)*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 96*Pi*ArcTanh[a*x]^2*Lo
g[1 + I/E^ArcTanh[a*x]] - (64*I)*ArcTanh[a*x]^3*Log[1 + I/E^ArcTanh[a*x]] -
 (48*I)*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[a*x]] - 96*Pi*ArcTanh[a*x]^2*
Log[1 - I*E^ArcTanh[a*x]] + 8*Pi^3*Log[1 + I*E^ArcTanh[a*x]] + (64*I)*ArcTa
nh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] - 8*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*
x])/4]] - (48*I)*(Pi - (2*I)*ArcTanh[a*x])^2*PolyLog[2, (-I)/E^ArcTanh[a*x]
] + (192*I)*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (48*I)*Pi^2*Po
lyLog[2, I*E^ArcTanh[a*x]] - 192*Pi*ArcTanh[a*x]*PolyLog[2, I*E^ArcTanh[a*x
]] - 192*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + (384*I)*ArcTanh[a*x]*PolyLog[
3, (-I)/E^ArcTanh[a*x]] - (384*I)*ArcTanh[a*x]*PolyLog[3, (-I)*E^ArcTanh[a*
x]] + 192*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + (384*I)*PolyLog[4, (-I)/E^ArcTa
nh[a*x]] + (384*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/(64*a^3)
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int x^2 \left(\text{Artanh}(ax)\right)^3 \left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)^3}{a^4x^4-2\,a^2x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^3/(a^4*x^4 - 2*a^2*x^2 + 1), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.406 $\int \frac{x \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=94

$$-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

```
[Out] (-6*x)/(a*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^
2*x^2])
```

Rubi [A] time = 0.124535, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5994, 5962, 191}

$$-\frac{6x}{a\sqrt{1-a^2x^2}}+\frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```


```
[Out] (-6*x)/(a*Sqrt[1 - a^2*x^2]) + (6*ArcTanh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (
3*x*ArcTanh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + ArcTanh[a*x]^3/(a^2*Sqrt[1 - a^
2*x^2])
```


Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{x \tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx = \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{3\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}$$
$$= \frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{6\int \frac{1}{\left(1-a^2x^2\right)^{3/2}}\,dx}{a}$$
$$= -\frac{6x}{a\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.0618625, size = 45, normalized size = 0.48

$$\frac{-6ax + \tanh^{-1}(ax)^3 - 3ax\tanh^{-1}(ax)^2 + 6\tanh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTanh[a*x]^3)/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6*a*x + 6*ArcTanh[a*x] - 3*a*x*ArcTanh[a*x]^2 + ArcTanh[a*x]^3)/(a^2*Sqrt
[1 - a^2*x^2])
```

Maple [A] time = 0.212, size = 98, normalized size = 1.

$$-\frac{(\mathrm{Artanh}(ax))^3 - 3(\mathrm{Artanh}(ax))^2 + 6\,\mathrm{Artanh}(ax) - 6}{2a^2(ax-1)}\sqrt{-(ax-1)(ax+1)} + \frac{(\mathrm{Artanh}(ax))^3 + 3(\mathrm{Artanh}(ax))^2 + 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))^
(1/2)/a^2/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-
(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)
```

Maxima [A] time = 0.960278, size = 119, normalized size = 1.27

$$-\frac{3x\,\mathrm{artanh}(ax)^2}{\sqrt{-a^2x^2+1}a} + \frac{\mathrm{artanh}(ax)^3}{\sqrt{-a^2x^2+1}a^2} - \frac{6\left(\frac{x}{\sqrt{-a^2x^2+1}} - \frac{\mathrm{artanh}(ax)}{\sqrt{-a^2x^2+1}a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*x*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*a) + arctanh(a*x)^3/(sqrt(-a^2*x^2
+ 1)*a^2) - 6*(x/sqrt(-a^2*x^2 + 1) - arctanh(a*x)/(sqrt(-a^2*x^2 + 1)*a))/
a
```

Fricas [A] time = 2.28271, size = 197, normalized size = 2.1

$$\frac{\sqrt{-a^2x^2+1}\left(6\,ax\log\left(-\frac{ax+1}{ax-1}\right)^2-\log\left(-\frac{ax+1}{ax-1}\right)^3+48\,ax-24\,\log\left(-\frac{ax+1}{ax-1}\right)\right)}{8\,\left(a^4x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-a^2*x^2 + 1)*(6*a*x*log(-(a*x + 1)/(a*x - 1))^2 - log(-(a*x + 1)/
(a*x - 1))^3 + 48*a*x - 24*log(-(a*x + 1)/(a*x - 1)))/(a^4*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,\text{atanh}^3\,(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x*atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\,\text{artanh}\,(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.407 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

```
[Out] -6/(a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*ArcTan
h[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2]
```

Rubi [A] time = 0.0707551, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5962, 5958}

$$-\frac{6}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```


```
[Out] -6/(a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*ArcTan
h[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2]
```


Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx &= -\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}+6\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\ &= -\frac{6}{a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}-\frac{3\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0503811, size = 45, normalized size = 0.51

$$\frac{ax\tanh^{-1}(ax)^3-3\tanh^{-1}(ax)^2+6ax\tanh^{-1}(ax)-6}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(3/2),x]
```

```
[Out] (-6 + 6*a*x*ArcTanh[a*x] - 3*ArcTanh[a*x]^2 + a*x*ArcTanh[a*x]^3)/(a*Sqrt[1
 - a^2*x^2])
```

Maple [A] time = 0.224, size = 56, normalized size = 0.6

$$-\frac{(\operatorname{Artanh}(ax))^3 ax + 6\,ax\operatorname{Artanh}(ax) - 3\,(\operatorname{Artanh}(ax))^2 - 6}{a\left(a^2x^2-1\right)}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/a*(-a^2*x^2+1)^(1/2)*(arctanh(a*x)^3*a*x+6*a*x*arctanh(a*x)-3*arctanh(a*
x)^2-6)/(a^2*x^2-1)
```

Maxima [A] time = 0.968363, size = 116, normalized size = 1.32

$$\frac{x\operatorname{artanh}(ax)^3}{\sqrt{-a^2x^2+1}} + 6\,a\left(\frac{x\operatorname{artanh}(ax)}{\sqrt{-a^2x^2+1}a} - \frac{1}{\sqrt{-a^2x^2+1}a^2}\right) - \frac{3\operatorname{artanh}(ax)^2}{\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x*arctanh(a*x)^3/sqrt(-a^2*x^2 + 1) + 6*a*(x*arctanh(a*x)/(sqrt(-a^2*x^2 +
1)*a) - 1/(sqrt(-a^2*x^2 + 1)*a^2)) - 3*arctanh(a*x)^2/(sqrt(-a^2*x^2 + 1)*
a)
```

Fricas [A] time = 2.24759, size = 196, normalized size = 2.23

$$-\frac{\left(ax\log\left(-\frac{ax+1}{ax-1}\right)^3 + 24\,ax\log\left(-\frac{ax+1}{ax-1}\right) - 6\,\log\left(-\frac{ax+1}{ax-1}\right)^2 - 48\right)\sqrt{-a^2x^2+1}}{8\left(a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a*x*log(-(a*x + 1)/(a*x - 1))^3 + 24*a*x*log(-(a*x + 1)/(a*x - 1)) -
6*log(-(a*x + 1)/(a*x - 1))^2 - 48)*sqrt(-a^2*x^2 + 1)/(a^3*x^2 - a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(3/2), x)
```

3.408 $\int \frac{\tanh^{-1}(ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=185

$$-3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right.$$

```
[Out] (-6*a*x)/Sqrt[1 - a^2*x^2] + (6*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*x*Ar
cTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2] - 2*ArcT
anh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^ArcTanh
[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyL
og[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 6*Poly
Log[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.40942, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6030, 6020, 4182, 2531, 6609, 2282, 6589, 5994, 5962, 191}

$$-3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+3\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right.$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a*x)/Sqrt[1 - a^2*x^2] + (6*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*x*Ar
cTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + ArcTanh[a*x]^3/Sqrt[1 - a^2*x^2] - 2*ArcT
anh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 3*ArcTanh[a*x]^2*PolyLog[2, -E^ArcTanh
[a*x]] + 3*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyL
og[3, -E^ArcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 6*Poly
Log[4, -E^ArcTanh[a*x]] + 6*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{x\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - (3a)\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx + \text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, \tanh^{-1}(ax)\right)$$

$$= \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^3 - 3\,\text{Sub}$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

$$= -\frac{6ax}{\sqrt{1-a^2x^2}} + \frac{6\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3ax\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - 2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

Mathematica [A] time = 0.339043, size = 230, normalized size = 1.24

$$\frac{1}{8}\left(24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) + 24\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right) + 48\tanh^{-1}(ax)\text{PolyLog}\left(3,\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (Pi^4 - (48*a*x)/Sqrt[1 - a^2*x^2] + (48*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] -
(24*a*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (8*ArcTanh[a*x]^3)/Sqrt[1 - a^2
*x^2] - 2*ArcTanh[a*x]^4 - 8*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])] + 8*
ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*ArcTanh[a*x]^2*PolyLog[2, -E^(-
ArcTanh[a*x])] + 24*ArcTanh[a*x]^2*PolyLog[2, E^ArcTanh[a*x]] + 48*ArcTanh[
a*x]*PolyLog[3, -E^(-ArcTanh[a*x])] - 48*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[
a*x]] + 48*PolyLog[4, -E^(-ArcTanh[a*x])] + 48*PolyLog[4, E^ArcTanh[a*x]])/
8
```

Maple [A] time = 0.287, size = 305, normalized size = 1.7

$$-\frac{(\text{Artanh}\,(ax))^3 - 3\,(\text{Artanh}\,(ax))^2 + 6\,\text{Artanh}\,(ax) - 6}{2\,ax-2}\sqrt{-(ax-1)(ax+1)} + \frac{(\text{Artanh}\,(ax))^3 + 3\,(\text{Artanh}\,(ax))^2}{2\,ax+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1))^
(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*(-(a*x
-1)*(a*x+1))^(1/2)/(a*x+1)-arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-
3*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*arctanh(a*x)*poly
log(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-6*polylog(4,-(a*x+1)/(-a^2*x^2+1)^(1/2))
+arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*arctanh(a*x)^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*arctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^
```

```
(1/2))+6*polylog(4,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^3}{a^4x^5-2\,a^2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^5 - 2*a^2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x), x)
```

3.409 $\int \frac{\tanh^{-1}(ax)^3}{x^2(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=187

$$-6a\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6a\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-6a$$

```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - 6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x
]^2 + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x
]*PolyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLo
g[3, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.424716, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6030, 6008, 6020, 4182, 2531, 2282, 6589, 5962, 5958}

$$-6a\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+6a\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+6a\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)-6a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```


```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - 6*a*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x
]^2 + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^3)/x - 6*a*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] + 6*a*ArcTanh[a*x
]*PolyLog[2, E^ArcTanh[a*x]] + 6*a*PolyLog[3, -E^ArcTanh[a*x]] - 6*a*PolyLo
g[3, E^ArcTanh[a*x]]
```


Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
```

```
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^2\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{x} + (3a)\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx + \Big($$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} + \frac{a^2x\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(a}{x}$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 + \frac{a}{}$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 + \frac{a}{}$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 + \frac{a}{}$$

$$= -\frac{6a}{\sqrt{1-a^2x^2}} + \frac{6a^2x\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - 6a\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 + \frac{a}{}$$

Mathematica [A] time = 1.88996, size = 270, normalized size = 1.44

$$6a\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) - 6a\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) + 6a\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^2*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a)/Sqrt[1 - a^2*x^2] + (6*a^2*x*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*a*
ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] + (a^2*x*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2
] - (a^2*x*ArcTanh[a*x]^3*Csch[ArcTanh[a*x]/2]^2)/(4*Sqrt[1 - a^2*x^2]) + 3
*a*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] - 3*a*ArcTanh[a*x]^2*Log[1 + E
^(-ArcTanh[a*x])] + 6*a*ArcTanh[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*a*A
rcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])] + 6*a*PolyLog[3, -E^(-ArcTanh[a*x
])] - 6*a*PolyLog[3, E^(-ArcTanh[a*x])] + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3
*Sinh[ArcTanh[a*x]/2]^2)/x
```

Maple [A] time = 0.292, size = 282, normalized size = 1.5

$$-\frac{a\left(\left(\text{Artanh}\,(ax)\right)^3 - 3\,\left(\text{Artanh}\,(ax)\right)^2 + 6\,\text{Artanh}\,(ax) - 6\right)}{2\,ax-2}\sqrt{-\left(ax-1\right)\left(ax+1\right)} - \frac{\left(\left(\text{Artanh}\,(ax)\right)^3 + 3\,\left(\text{Artanh}\,(a\right.\right.}{2\,ax+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+1)
)^(1/2)/(a*x-1)-1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*a*(-
(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)^3/x-3*
a*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*arctanh(a*x)*polylog(
2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3
*a*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+6*a*arctanh(a*x)*polylog
```

```
(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-6*a*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^3}{a^4x^6-2\,a^2x^4+x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^6 - 2*a^2*x^4 + x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{x^2\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**2/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^2/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^2), x)
```

3.410 $\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=360

$$3a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+\frac{9}{2}a^2\tanh^{-1}(a$$

```
[Out] (-6*a^3*x)/Sqrt[1 - a^2*x^2] + (6*a^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*
a^3*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^3)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (9*a^2*ArcTanh[a*x]^
2*PolyLog[2, -E^ArcTanh[a*x]])/2 + (9*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTa
nh[a*x]])/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*Poly
Log[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 9*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcT
anh[a*x]] - 9*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 9*a^2*PolyLog[4
, -E^ArcTanh[a*x]] + 9*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.982383, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {6030, 6026, 6008, 6018, 6020, 4182, 2531, 6609, 2282, 6589, 5994, 5962, 191}

$$3a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-3a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{9}{2}a^2\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+\frac{9}{2}a^2\tanh^{-1}(a$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (-6*a^3*x)/Sqrt[1 - a^2*x^2] + (6*a^2*ArcTanh[a*x])/Sqrt[1 - a^2*x^2] - (3*
a^3*x*ArcTanh[a*x]^2)/Sqrt[1 - a^2*x^2] - (3*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^2)/(2*x) + (a^2*ArcTanh[a*x]^3)/Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x]^3)/(2*x^2) - 3*a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^3 - 6*a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (9*a^2*ArcTanh[a*x]^
2*PolyLog[2, -E^ArcTanh[a*x]])/2 + (9*a^2*ArcTanh[a*x]^2*PolyLog[2, E^ArcTa
nh[a*x]])/2 + 3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 3*a^2*Poly
Log[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] + 9*a^2*ArcTanh[a*x]*PolyLog[3, -E^ArcT
anh[a*x]] - 9*a^2*ArcTanh[a*x]*PolyLog[3, E^ArcTanh[a*x]] - 9*a^2*PolyLog[4
, -E^ArcTanh[a*x]] + 9*a^2*PolyLog[4, E^ArcTanh[a*x]]
```

Rule 6030

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Dist[1/d, Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[
c*x])^p, x], x] - Dist[e/d, Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTanh[c*x]
)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegersQ[
p, 2*q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
```

```
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{x^3\left(1-a^2x^2\right)^{3/2}}\,dx = a^2\int \frac{\tanh^{-1}(ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx + \int \frac{\tanh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a)\int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx + a^2\int \frac{\tanh}{x\sqrt{1}}$$

$$= -\frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}{2x^2} + \frac{1}{2}a^2\,\mathrm{Subst}\left(\int x^3$$

$$= \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a}}{}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}}{\sqrt{1-a^2}}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}}{\sqrt{1-a^2}}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}}{\sqrt{1-a^2}}$$

$$= -\frac{6a^3x}{\sqrt{1-a^2x^2}} + \frac{6a^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}} - \frac{3a^3x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{3a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x} + \frac{a^2\tanh^{-1}}{\sqrt{1-a^2}}$$

Mathematica [A] time = 7.72953, size = 555, normalized size = 1.54

$$a^2\left(72\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)+144\sqrt{1-a^2x^2}\tanh^{-1}(ax)\text{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right)-144\sqrt{1-a$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]^3/(x^3*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-96*a*x + 3*Pi^4*Sqrt[1 - a^2*x^2] + 96*ArcTanh[a*x] - 48*a*x*ArcTanh
[a*x]^2 + 16*ArcTanh[a*x]^3 - 6*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^4 - 12*Sqrt[
1 - a^2*x^2]*ArcTanh[a*x]^2*Coth[ArcTanh[a*x]/2] - 2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]^3*Csch[ArcTanh[a*x]/2]^2 + 48*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1
 - E^(-ArcTanh[a*x])] - 48*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTa
nh[a*x])] - 24*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3*Log[1 + E^(-ArcTanh[a*x])]
+ 24*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3*Log[1 - E^ArcTanh[a*x]] + 24*Sqrt[1 -
 a^2*x^2]*(2 + 3*ArcTanh[a*x]^2)*PolyLog[2, -E^(-ArcTanh[a*x])] - 48*Sqrt[1
 - a^2*x^2]*PolyLog[2, E^(-ArcTanh[a*x])] + 72*Sqrt[1 - a^2*x^2]*ArcTanh[a*
x]^2*PolyLog[2, E^ArcTanh[a*x]] + 144*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*PolyLo
g[3, -E^(-ArcTanh[a*x])] - 144*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*PolyLog[3, E^
ArcTanh[a*x]] + 144*Sqrt[1 - a^2*x^2]*PolyLog[4, -E^(-ArcTanh[a*x])] + 144*
Sqrt[1 - a^2*x^2]*PolyLog[4, E^ArcTanh[a*x]] - 2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]^3*Sech[ArcTanh[a*x]/2]^2 + 12*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2*Tanh[Ar
cTanh[a*x]/2]))/(16*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.346, size = 482, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*a^2*(arctanh(a*x)^3-3*arctanh(a*x)^2+6*arctanh(a*x)-6)*(-(a*x-1)*(a*x+
1))^(1/2)/(a*x-1)+1/2*(arctanh(a*x)^3+3*arctanh(a*x)^2+6*arctanh(a*x)+6)*a^
2*(-(a*x-1)*(a*x+1))^(1/2)/(a*x+1)-1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x
)^2*(3*a*x+arctanh(a*x))/x^2-3/2*a^2*arctanh(a*x)^3*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))-9/2*a^2*arctanh(a*x)^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a
^2*arctanh(a*x)*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*polylog(4,-(a*
x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)^3*ln(1-(a*x+1)/(-a^2*x^2+1)^(
1/2))+9/2*a^2*arctanh(a*x)^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))-9*a^2*ar
ctanh(a*x)*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))+9*a^2*polylog(4,(a*x+1)/(-
a^2*x^2+1)^(1/2))-3*a^2*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3*a^2
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a
^2*x^2+1)^(1/2))+3*a^2*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\,(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^3}{a^4x^7-2\,a^2x^5+x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3/(a^4*x^7 - 2*a^2*x^5 + x^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{atanh}^3\,(ax)}{x^3\,(-\,(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/x**3/(-a**2*x**2+1)**(3/2),x)

[Out] Integral(atanh(a*x)**3/(x**3*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}\,(ax)^3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/x^3/(-a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] integrate(arctanh(a*x)^3/((-a^2*x^2 + 1)^(3/2)*x^3), x)
```

3.411 $\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.123502, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.410781, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.74, size = 0, normalized size = 0.

$$\int \frac{x^m}{\mathrm{Artanh}\,(ax)}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

3.412 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.132471, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 3.53698, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.27, size = 0, normalized size = 0.

$$\int \frac{x^2}{\mathrm{Artanh}\,(ax)}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}x^2}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x
)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}} \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

3.413 $\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

```
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

Rubi [A] time = 0.103655, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}\\ &= \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0688572, size = 9, normalized size = 1.

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] SinhIntegral[ArcTanh[a*x]]/a^2
```

Maple [B] time = 0.21, size = 26, normalized size = 2.9

$$-\frac{\mathrm{Ei}\left(1,-\mathrm{Artanh}\left(ax\right)\right)}{2\,a^{2}}+\frac{\mathrm{Ei}\left(1,\mathrm{Artanh}\left(ax\right)\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] -1/2*Ei(1,-arctanh(a*x))/a^2+1/2*Ei(1,arctanh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\,\mathrm{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^{2}x^{2}+1}x}{\left(a^{4}x^{4}-2\,a^{2}x^{2}+1\right)\mathrm{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\,\mathrm{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

3.414 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rubi [A] time = 0.0622549, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5968, 3301}

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\ &= \frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.0804722, size = 9, normalized size = 1.

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Maple [A] time = 0.219, size = 10, normalized size = 1.1

$$\frac{\mathrm{Chi}\left(\mathrm{Artanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] Chi(arctanh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

3.415 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.118475, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] Defer[Int][1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx=\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.0374, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]

[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]), x]
```

Maple [A] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{1}{x\text{Artanh}\,(ax)}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^5-2\,a^2x^3+x\right)\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} x \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)), x)
```

3.416 $\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.109002, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx=\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.443556, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.738, size = 0, normalized size = 0.

$$\int \frac{x^m}{(\text{Artanh}\,(ax))^2}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,x^m}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

3.417 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2},x\right)}{a^2}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^3}-\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a^3
- Unintegrable[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/a^2
```

Rubi [A] time = 0.266309, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a^3
- Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/a^2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}+\frac{\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx}{a}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}\\
&= -\frac{1}{a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a^3}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{a^2}
\end{aligned}$$

Mathematica [A] time = 3.0621, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.289, size = 0, normalized size = 0.

$$\int \frac{x^2}{(\mathrm{Artanh}(ax))^2}\left(-a^2x^2+1\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}x^2}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2),
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

3.418 $\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=36

$$\frac{\operatorname{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2
```

Rubi [A] time = 0.121882, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6006, 5968, 3301}

$$\frac{\operatorname{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}-\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(x/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]]/a^2
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx = -\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx}{a}$$

$$= -\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a^2}$$

$$= -\frac{x}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a^2}$$

Mathematica [A] time = 0.0323687, size = 34, normalized size = 0.94

$$\frac{\mathrm{Chi}\left(\tanh^{-1}(ax)\right) - \frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + CoshIntegral[ArcTanh[a*x]])/a^
2
```

Maple [B] time = 0.227, size = 90, normalized size = 2.5

$$\frac{1}{2\,a^2\,(ax-1)\,\mathrm{Artanh}\,(ax)}\sqrt{-(ax-1)\,(ax+1)} - \frac{\mathrm{Ei}\,(1,-\mathrm{Artanh}\,(ax))}{2\,a^2} + \frac{1}{2\,a^2\,(ax+1)\,\mathrm{Artanh}\,(ax)}\sqrt{-(ax-1)\,(ax+1)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/2*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)-1/2*Ei(1,-arctanh(a*x
))/a^2+1/2*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x+1)/arctanh(a*x)-1/2*Ei(1,arcta
nh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\mathrm{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}x}{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

3.419 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rubi [A] time = 0.125834, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5966, 6034, 3298}

$$\frac{\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.0843125, size = 32, normalized size = 0.91

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right) - \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

Maple [A] time = 0.22, size = 62, normalized size = 1.8

$$\frac{1}{a\text{Artanh}(ax)\left(a^2x^2-1\right)}\left(\text{Artanh}(ax)\,\text{Shi}(\text{Artanh}(ax))\,x^2a^2 - \text{Shi}(\text{Artanh}(ax))\,\text{Artanh}(ax) + \sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2-Shi(arctanh(a*x))*arctanh(a*x)+
(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

3.420 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)},x\right)}{a}-\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}+\text{Chi}\left(\tanh^{-1}(ax)\right)$$

```
[Out] -((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) - Sqrt[1 - a^2*x^2]/(a*x*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]] - Unintegrable[1/(x^2*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x]), x]/a
```

Rubi [A] time = 0.417318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -((a*x)/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) - Sqrt[1 - a^2*x^2]/(a*x*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]] - Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]), x]/a
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx &= a^2\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx\\
&= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx}{a}+a\int \frac{}{(1-a
\\
&= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx}{a}+\text{Subst}\Bigg(\int
\\
&= -\frac{ax}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{ax\tanh^{-1}(ax)}+\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-}}}{a}
\end{aligned}$$

Mathematica [A] time = 5.89887, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.287, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\operatorname{Artanh}\left(ax\right)\right)^{2}}\left(-a^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x\operatorname{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^{2}x^{2}+1}}{\left(a^{4}x^{5}-2\,a^{2}x^{3}+x\right)\operatorname{artanh}\left(ax\right)^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} x \operatorname{artanh}\left(ax\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^2), x)
```

3.421 $\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi [A] time = 0.106831, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] Defer[Int][x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Rubi steps

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3} \, dx = \int \frac{x^m}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3} \, dx$$

Mathematica [A] time = 0.490247, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[x^m/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.746, size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(\text{Artanh}\,(ax)\right)^3} \left(-a^2x^2+1\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] int(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x^m}{\left(a^4x^4-2\,a^2x^2+1\right)\operatorname{artanh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^m/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

3.422 $\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)^3} \, dx$

Optimal. Leaf size=96

$$-\frac{\text{Unintegrable}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3},x\right)}{a^2}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a^3}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}$$

```
[Out] -1/(2*a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*a^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a^3) - Unintegrable[1/(Sqrt[1 -
 a^2*x^2]*ArcTanh[a*x]^3), x]/a^2
```

Rubi [A] time = 0.303998, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*a^2*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a^3) - Defer[Int][1/(Sqrt[1 - a
^2*x^2]*ArcTanh[a*x]^3), x]/a^2
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3} \, dx &= \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3} \, dx}{a^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} \, dx}{a^2}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} \, dx}{a^2}+\frac{\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2} \, dx}{2a}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} \, dx}{2a^2}-\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x} \, dx,x,\tan\right.}{2a^3}\\
&= -\frac{1}{2a^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a^3}-\frac{\int \frac{}{\sqrt{1}}}{}
\end{aligned}$$

Mathematica [A] time = 6.18917, size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[x^2/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{x^2}{(\mathrm{Artanh}\,(ax))^3}\left(-a^2x^2+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] int(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\,\mathrm{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}x^2}{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3),
 x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\,\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(x**2/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

3.423 $\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=68

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] -x/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - 1/(2*a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]) + SinhIntegral[ArcTanh[a*x]]/(2*a^2)
```

Rubi [A] time = 0.19596, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6006, 5966, 6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}-\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -x/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - 1/(2*a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]) + SinhIntegral[ArcTanh[a*x]]/(2*a^2)
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} + \frac{\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx}{2a}$$

$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{1}{2}\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(}$$

$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh}{2a^2}$$

$$= -\frac{x}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2} - \frac{1}{2a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{2a^2}$$

Mathematica [A] time = 0.122611, size = 43, normalized size = 0.63

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right) - \frac{ax+\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((a*x + ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + SinhIntegral[
ArcTanh[a*x]])/(2*a^2)
```

Maple [B] time = 0.248, size = 154, normalized size = 2.3

$$\frac{1}{4a^2(ax-1)(\text{Artanh}(ax))^2}\sqrt{-(ax-1)(ax+1)} + \frac{1}{4a^2(ax-1)\text{Artanh}(ax)}\sqrt{-(ax-1)(ax+1)} - \frac{\text{Ei}(1,-\text{Artanh}(ax))}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/4*(-(a*x-1)*(a*x+1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)^2+1/4*(-(a*x-1)*(a*x+
1))^(1/2)/a^2/(a*x-1)/arctanh(a*x)-1/4*Ei(1,-arctanh(a*x))/a^2+1/4*(-(a*x-1
)*(a*x+1))^(1/2)/a^2/(a*x+1)/arctanh(a*x)^2-1/4*(-(a*x-1)*(a*x+1))^(1/2)/a^
2/(a*x+1)/arctanh(a*x)+1/4*Ei(1,arctanh(a*x))/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}x}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\text{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(x/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

3.424 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] -1/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rubi [A] time = 0.163092, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5966, 6006, 5968, 3301}

$$-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{1}{2}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\tan\right.}{2a}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.0902135, size = 44, normalized size = 0.68

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{ax\tanh^{-1}(ax)+1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegra
l[ArcTanh[a*x]])/(2*a)
```

Maple [A] time = 0.228, size = 86, normalized size = 1.3

$$\frac{1}{2a\left(\text{Artanh}(ax)\right)^2\left(a^2x^2-1\right)}\left(\left(\text{Artanh}(ax)\right)^2\text{Chi}\left(\text{Artanh}(ax)\right)x^2a^2+\sqrt{-a^2x^2+1}ax\text{Artanh}(ax)-\text{Chi}\left(\text{Artanh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+(-a^2*x^2+1)^(1/2)*a*x*arct
anh(a*x)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x)^
2/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\text{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

3.425 $\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{\text{Unintegrable}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2},x\right)}{2a}-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}+\frac{1}{2}$$

```
[Out] -(a*x)/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - Sqrt[1 - a^2*x^2]/(2*a*x*ArcT
anh[a*x]^2) - 1/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]) + SinhIntegral[ArcTanh[a
*x]]/2 - Unintegrable[1/(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi [A] time = 0.484825, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -(a*x)/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - Sqrt[1 - a^2*x^2]/(2*a*x*ArcT
anh[a*x]^2) - 1/(2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]) + SinhIntegral[ArcTanh[a
*x]]/2 - Defer[Int][1/(x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]/(2*a)
```

Rubi steps

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx = a^2\int \frac{x}{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3}\,dx$$

$$=-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{\int \frac{1}{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx}{2a}+\frac{1}{2}a\int$$

$$=-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{\int \frac{}{x^2\sqrt{}}}{}$$

$$=-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{1}{2}\text{Sub}$$

$$=-\frac{ax}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{\sqrt{1-a^2x^2}}{2ax\tanh^{-1}(ax)^2}-\frac{1}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{1}{2}\text{Shi}$$

Mathematica [A] time = 18.0729, size = 0, normalized size = 0.

$$\int \frac{1}{x(1-a^2x^2)^{3/2}\tanh^{-1}(ax)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.311, size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\mathrm{Artanh}\left(ax\right)\right)^{3}}\left(-a^{2}x^{2}+1\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] int(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}x\,\mathrm{artanh}\left(ax\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^{2}x^{2}+1}}{\left(a^{4}x^{5}-2\,a^{2}x^{3}+x\right)\mathrm{artanh}\left(ax\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^5 - 2*a^2*x^3 + x)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\mathrm{atanh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} x \operatorname{artanh}\left(ax\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*x*arctanh(a*x)^3), x)
```

3.426 $\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=243

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{7\left(1-a^2x^2\right)^{3/2}}{72a^5}+\frac{\sqrt{1-a^2x^2}}{16a^5}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}($$

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^5) - (7*(1 - a^2*x^2)^(3/2))/(72*a^5) + (1 - a^2*x^
2)^(5/2)/(30*a^5) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^4) - (x^3*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/6 - (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^5) - ((I/16)*P
olyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + ((I/16)*PolyLog[2, (I*
Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rubi [A] time = 0.307826, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6010, 6016, 266, 43, 261, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{7\left(1-a^2x^2\right)^{3/2}}{72a^5}+\frac{\sqrt{1-a^2x^2}}{16a^5}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(16*a^5) - (7*(1 - a^2*x^2)^(3/2))/(72*a^5) + (1 - a^2*x^
2)^(5/2)/(30*a^5) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^4) - (x^3*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/6 - (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^5) - ((I/16)*P
olyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + ((I/16)*PolyLog[2, (I*
Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```


Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{6}\int\frac{x^4\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-\frac{1}{6}a\int\frac{x^5}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{\int\frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^2}+\frac{\int\frac{x^3}{\sqrt{1-a^{\cdots}}}}{24\cdots}$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\cdots$$

$$= \frac{5\sqrt{1-a^2x^2}}{48a^5}-\frac{\left(1-a^2x^2\right)^{3/2}}{9a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^{\cdots}}}{24\cdots}$$

$$= \frac{\sqrt{1-a^2x^2}}{16a^5}-\frac{7\left(1-a^2x^2\right)^{3/2}}{72a^5}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a^5}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^{\cdots}}}{2\cdots}$$

Mathematica [A] time = 0.669035, size = 178, normalized size = 0.73

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{45i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+24\left(a^2x^2\cdots\right.\right.}{720a^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(45 + 70*(-1 + a^2*x^2) + 24*(-1 + a^2*x^2)^2 + 45*a*x*A
rcTanh[a*x] + 210*a*x*(-1 + a^2*x^2)*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^
2*ArcTanh[a*x] - ((45*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
 I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcT
```

```
anh[a*x]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

Maple [A] time = 0.282, size = 195, normalized size = 0.8

$$\frac{120\,\mathrm{Artanh}\,(ax)\,x^5a^5+24\,x^4a^4-30\,a^3x^3\mathrm{Artanh}\,(ax)+22\,a^2x^2-45\,ax\mathrm{Artanh}\,(ax)-1}{720\,a^5}\sqrt{-\left(ax-1\right)\left(ax+1\right)}-\frac{\frac{i}{16}\mathrm{Artar}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/720/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*x^5*a^5+24*x^4*a^4-30*
a^3*x^3*arctanh(a*x)+22*a^2*x^2-45*a*x*arctanh(a*x)-1)-1/16*I*ln(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(
1/2))*arctanh(a*x)/a^5-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+1/1
6*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^4\,\mathrm{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-a^2x^2+1}x^4\,\mathrm{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\mathrm{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2 + 1}x^4 \operatorname{artanh}(ax) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x), x)
```

3.427 $\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=136

$$\frac{x^3\sqrt{1-a^2x^2}}{20a}+\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}+\frac{11\sin^{-1}}{120a}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(24*a^3) + (x^3*Sqrt[1 - a^2*x^2])/(20*a) + (11*ArcSi
n[a*x])/(120*a^4) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^2*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/5
```

Rubi [A] time = 0.203294, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6010, 6016, 321, 216, 5994}

$$\frac{x^3\sqrt{1-a^2x^2}}{20a}+\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}+\frac{11\sin^{-1}}{120a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```


```
[Out] (x*Sqrt[1 - a^2*x^2])/(24*a^3) + (x^3*Sqrt[1 - a^2*x^2])/(20*a) + (11*ArcSi
n[a*x])/(120*a^4) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^4) - (x^2*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])
/5
```


Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{5}\int\frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-\frac{1}{5}a\int\frac{x^4}{\sqrt{1-a^2x^2}}\,dx$$

$$=\frac{x^3\sqrt{1-a^2x^2}}{20a}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{2\int\frac{x\tanh^{-1}(a}{\sqrt{1-a^2x^2}}}{15a^2}$$

$$=\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{x^3\sqrt{1-a^2x^2}}{20a}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^2}+$$

$$=\frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{x^3\sqrt{1-a^2x^2}}{20a}+\frac{11\sin^{-1}(ax)}{120a^4}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}-\frac{x^2\sqrt{1-a^2x^2}}{15}$$

Mathematica [A] time = 0.0649956, size = 79, normalized size = 0.58

$$\frac{ax\sqrt{1-a^2x^2}\left(6a^2x^2+5\right)+8\sqrt{1-a^2x^2}\left(3a^4x^4-a^2x^2-2\right)\tanh^{-1}(ax)+11\sin^{-1}(ax)}{120a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(5 + 6*a^2*x^2) + 11*ArcSin[a*x] + 8*Sqrt[1 - a^2*x^
2]*(-2 - a^2*x^2 + 3*a^4*x^4)*ArcTanh[a*x])/(120*a^4)
```

Maple [C] time = 0.259, size = 120, normalized size = 0.9

$$\frac{24\,a^4x^4\mathrm{Artanh}\,(ax)+6\,x^3a^3-8\,a^2x^2\mathrm{Artanh}\,(ax)+5\,ax-16\,\mathrm{Artanh}\,(ax)}{120\,a^4}\sqrt{-(ax-1)(ax+1)}+\frac{\frac{11\,i}{120}}{a^4}\ln\left((ax+1)\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/120/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(24*a^4*x^4*arctanh(a*x)+6*x^3*a^3-8*a^2
*x^2*arctanh(a*x)+5*a*x-16*arctanh(a*x))+11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(
1/2)+I)/a^4-11/120*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^4
```

Maxima [A] time = 1.47476, size = 201, normalized size = 1.48

$$-\frac{1}{120}\,a\left(\frac{3\left(\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{-a^2x^2+1}x}{a^2}-\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)}{a^2}-\frac{8\left(\sqrt{-a^2x^2+1}x+\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}\right)}{a^4}\right)-\frac{1}{15}\left(\frac{3\left(-a^2x^2+1\right)^{\frac{3}{2}}x^2}{a^2}+\frac{2\left(-a^2x^2\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/120*a*(3*(2*(-a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(-a^2*x^2 + 1)*x/a^2 - arcs
in(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))/a^2 - 8*(sqrt(-a^2*x^2 + 1)*x + arcsin
(a^2*x/sqrt(a^2))/sqrt(a^2))/a^4) - 1/15*(3*(-a^2*x^2 + 1)^(3/2)*x^2/a^2 +
2*(-a^2*x^2 + 1)^(3/2)/a^4)*arctanh(a*x)
```

Fricas [A] time = 2.24214, size = 205, normalized size = 1.51

$$\frac{\left(6\,a^3x^3+5\,ax+4\left(3\,a^4x^4-a^2x^2-2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}-22\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{120\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*((6*a^3*x^3 + 5*a*x + 4*(3*a^4*x^4 - a^2*x^2 - 2)*log(-(a*x + 1)/(a*x
 - 1)))*sqrt(-a^2*x^2 + 1) - 22*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [A] time = 1.18705, size = 139, normalized size = 1.02

$$\frac{\left(6\,a^2x^2+5\right)\sqrt{-a^2x^2+1}x+\frac{11\arcsin(ax)\mathrm{sgn}(a)}{|a|}}{120\,a^3}+\frac{\left(3\left(a^2x^2-1\right)^2\sqrt{-a^2x^2+1}-5\left(-a^2x^2+1\right)^{\frac{3}{2}}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{30\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*((6*a^2*x^2 + 5)*sqrt(-a^2*x^2 + 1)*x + 11*arcsin(a*x)*sgn(a)/abs(a))
/a^3 + 1/30*(3*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1) - 5*(-a^2*x^2 + 1)^(3/2))
*log(-(a*x + 1)/(a*x - 1))/a^4
```

3.428 $\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=194

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{12a^3}+\frac{\sqrt{1-a^2x^2}}{8a^3}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x\sqrt{1-a^2x^2}}{8}$$

```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3) - (1 - a^2*x^2)^(3/2)/(12*a^3) - (x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/4 - (ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^3) - ((I/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/8)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```

Rubi [A] time = 0.193429, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6010, 6016, 261, 5950, 266, 43}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{12a^3}+\frac{\sqrt{1-a^2x^2}}{8a^3}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x\sqrt{1-a^2x^2}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(8*a^3) - (1 - a^2*x^2)^(3/2)/(12*a^3) - (x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/4 - (ArcT
an[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a^3) - ((I/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/8)*PolyLog[2, (I*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/a^3
```


Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx = \frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}\int\frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-\frac{1}{4}a\int\frac{x^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^2}+\frac{\int\frac{x}{\sqrt{1-a^2x^2}}}{8a}$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a^3}$$

$$= \frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{\left(1-a^2x^2\right)^{3/2}}{12a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-$$

Mathematica [A] time = 0.437281, size = 160, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{3i\left(\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)\right)}{\sqrt{1-a^2x^2}}+2a^2x^2+6ax\left(a^2x^2-1\right)\tanh^{-1}(ax)-\frac{3i\tanh^{-1}(ax)\left(\log\left(1-ie^{-\right.\right.}}{}\right.}{24a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2 + 3*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^
2)*ArcTanh[a*x] - ((3*I)*ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 +
I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2] - ((3*I)*(PolyLog[2, (-I)/E^ArcTanh[a
*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(24*a^3)
```

Maple [A] time = 0.26, size = 175, normalized size = 0.9

$$\frac{6a^3x^3\mathrm{Artanh}(ax)+2a^2x^2-3ax\mathrm{Artanh}(ax)+1}{24a^3}\sqrt{-(ax-1)(ax+1)}-\frac{\frac{i}{8}\mathrm{Artanh}(ax)}{a^3}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/24/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-3*a*x*a
rctanh(a*x)+1)-1/8*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/
8*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3-1/8*I*dilog(1+I*(a*
x+1)/(-a^2*x^2+1)^(1/2))/a^3+1/8*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^
3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-(ax-1)(ax+1)} \operatorname{atanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x), x)
```

3.429 $\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{1-a^2x^2}}{6a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\sin^{-1}(ax)}{6a^2}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(6*a) + ArcSin[a*x]/(6*a^2) - ((1 - a^2*x^2)^(3/2)*Ar
cTanh[a*x])/(3*a^2)
```

Rubi [A] time = 0.0485783, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5994, 195, 216}

$$\frac{x\sqrt{1-a^2x^2}}{6a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\sin^{-1}(ax)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(6*a) + ArcSin[a*x]/(6*a^2) - ((1 - a^2*x^2)^(3/2)*Ar
cTanh[a*x])/(3*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\int\sqrt{1-a^2x^2}\,dx}{3a}\\ &= \frac{x\sqrt{1-a^2x^2}}{6a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}+\frac{\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{6a}\\ &= \frac{x\sqrt{1-a^2x^2}}{6a}+\frac{\sin^{-1}(ax)}{6a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.0420423, size = 49, normalized size = 0.83

$$\frac{ax\sqrt{1-a^2x^2}-2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\sin^{-1}(ax)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2] + ArcSin[a*x] - 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/
(6*a^2)
```

Maple [C] time = 0.233, size = 99, normalized size = 1.7

$$\frac{2\,a^2x^2\mathrm{Artanh}\,(ax)+ax-2\,\mathrm{Artanh}\,(ax)}{6\,a^2}\sqrt{-(ax-1)\,(ax+1)}+\frac{\frac{i}{6}}{a^2}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}+i\right)-\frac{\frac{i}{6}}{a^2}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/6/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-2*arctanh(a*x)
)+1/6*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^2-1/6*I*ln((a*x+1)/(-a^2*x^2+1)^
(1/2)-I)/a^2
```

Maxima [A] time = 1.43938, size = 80, normalized size = 1.36

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\mathrm{artanh}\,(ax)}{3\,a^2}+\frac{\sqrt{-a^2x^2+1}x+\frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/a^2 + 1/6*(sqrt(-a^2*x^2 + 1)*x + ar
csin(a^2*x/sqrt(a^2))/sqrt(a^2))/a
```

Fricas [A] time = 2.06703, size = 163, normalized size = 2.76

$$\frac{\sqrt{-a^2x^2+1}\left(ax+\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)-2\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{6\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(-a^2*x^2 + 1)*(a*x + (a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))) - 2
*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [A] time = 1.2134, size = 86, normalized size = 1.46

$$-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(-\frac{ax+1}{ax-1}\right)}{6\,a^2}+\frac{\sqrt{-a^2x^2+1}x+\frac{\arcsin(ax)\mathrm{sgn}(a)}{|a|}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(-a^2*x^2 + 1)^(3/2)*log(-(a*x + 1)/(a*x - 1))/a^2 + 1/6*(sqrt(-a^2*x^
2 + 1)*x + arcsin(a*x)*sgn(a)/abs(a))/a
```

3.430 $\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=143

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rubi [A] time = 0.0554435, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```


Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])]]/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx=\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{2}\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$=\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{i\text{L}}{}$$

Mathematica [A] time = 0.246557, size = 117, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+ax\tanh^{-1}(\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^Ar
cTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]]
- PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [A] time = 0.231, size = 152, normalized size = 1.1

$$\frac{ax\text{Artanh}(ax)+1}{2a}\sqrt{-a^2x^2+1}-\frac{\frac{i}{2}\text{Artanh}(ax)}{a}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\frac{i}{2}\text{Artanh}(ax)}{a}\ln\left(1-i(ax+1)\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a-1/2*I/a*arctanh(a*x)*ln(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2x^2+1}\,\text{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,\text{artanh}(ax),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\left(ax-1\right)\left(ax+1\right)}\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\operatorname{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

3.431 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=100

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\sin^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{ax+}}\right.$$

```
[Out] -ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 2*ArcTanh[a*x]*ArcTanh[Sqrt
[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - Pol
yLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.126076, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {6010, 6018, 216}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\sin^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-}}{\sqrt{ax+}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

```
[Out] -ArcSin[a*x] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 2*ArcTanh[a*x]*ArcTanh[Sqrt
[1 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - Pol
yLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx=\sqrt{1-a^2x^2}\tanh^{-1}(ax)-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$$

$$=-\sin^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)-2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{Li}_2\left(-\frac{\sqrt{1-}}{\sqrt{1+}}\right.$$

Mathematica [A] time = 0.115122, size = 91, normalized size = 0.91

$$\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\tanh^{-1}(ax)\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\tan$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x,x]
```

```
[Out] -2*ArcTan[Tanh[ArcTanh[a*x]/2]] + Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + ArcTanh[
a*x]*Log[1 - E^(-ArcTanh[a*x])] - ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] +
 PolyLog[2, -E^(-ArcTanh[a*x])] - PolyLog[2, E^(-ArcTanh[a*x])]
```

Maple [A] time = 0.269, size = 113, normalized size = 1.1

$$\sqrt{-(ax-1)(ax+1)}\text{Artanh}(ax)-2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)-\text{dilog}\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)-\text{dilog}\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] (-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)-2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-
dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arcta
nh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

3.432 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=130

$$ia\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-ia\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a\tan^{-1}\left(\frac{\sqrt{1}}{\sqrt{ax}}\right.$$

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + 2*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a
*x]]*ArcTanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + I*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - I*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]
]
```

Rubi [A] time = 0.162952, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6008, 266, 63, 208, 5950}

$$ia\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-ia\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+2a\tan^{-1}\left(\frac{\sqrt{1}}{\sqrt{ax}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```


```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) + 2*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a
*x]]*ArcTanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + I*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - I*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]
]
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}\,dx &= -\left(a^2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)-ia\mathrm{L}\\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)-ia\mathrm{L}\\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+ia\mathrm{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)-ia\mathrm{L}\\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}+2a\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+i\end{aligned}$$

Mathematica [A] time = 0.345885, size = 121, normalized size = 0.93

$$a\left(i\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-i\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{ax}+i\tanh^{-1}(ax)\log\left(1-ie^{-\tanh^{-}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^2,x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I/E^A
rcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + Log[Tanh[ArcTanh[
a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh[a*x
]])
```

Maple [A] time = 0.289, size = 188, normalized size = 1.5

$$-\frac{\mathrm{Artanh}(ax)}{x}\sqrt{-(ax-1)(ax+1)}+ia\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)\mathrm{Artanh}(ax)-ia\ln\left(1-i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] -(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)/x+I*a*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1
/2))*arctanh(a*x)-I*a*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)+I*a*d
ilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-I*a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/
2))+a*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

3.433 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=136

$$-\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\tanh$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 + (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rubi [A] time = 0.198167, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6010, 6026, 264, 6018}

$$-\frac{1}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+a^2\tanh^{-1}(ax)\tanh$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + a
^2*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (a^2*PolyLog[2, -(Sq
rt[1 - a*x]/Sqrt[1 + a*x])])/2 + (a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x
]])/2
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2} + a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx - \int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} - \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx - \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{x\sqrt{1-a^2x^2}}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2} + a^2\tanh^{-1}(ax)\tanh^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \frac{1}{2}a^2\text{Li}_2\left(\right.$$

Mathematica [A] time = 0.662267, size = 126, normalized size = 0.93

$$\frac{1}{8}a^2\left(-4\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+4\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)-4\tanh^{-1}(ax)\log\left(1-e^{-}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^3,x]
```

```
[Out] (a^2*(-2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 4*Arc
Tanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a
*x])] - 4*PolyLog[2, -E^(-ArcTanh[a*x])] + 4*PolyLog[2, E^(-ArcTanh[a*x])]
- ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A] time = 0.256, size = 141, normalized size = 1.

$$-\frac{ax+\text{Artanh}(ax)}{2x^2}\sqrt{-(ax-1)(ax+1)}+\frac{a^2\text{Artanh}(ax)}{2}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{a^2}{2}\text{polylog}\left(2,-(ax+1)\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x+arctanh(a*x))/x^2+1/2*a^2*arctanh(a*x)*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*a^2*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/
2))-1/2*a^2*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/2*a^2*polylog(2
,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)}{x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}\,(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

3.434 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4} dx$

Optimal. Leaf size=70

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(6*x^2) - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3)
 + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6
```

Rubi [A] time = 0.0816485, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6008, 266, 47, 63, 208}

$$-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(6*x^2) - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3)
 + (a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx \\
&= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-\frac{1}{12}a^3\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right) \\
&= -\frac{a\sqrt{1-a^2x^2}}{6x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0719546, size = 79, normalized size = 1.13

$$-\frac{ax\sqrt{1-a^2x^2}+a^3x^3\log(x)-a^3x^3\log\left(\sqrt{1-a^2x^2}+1\right)+2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a*x*Sqrt[1 - a^2*x^2] + 2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x] + a^3*x^3*Log[
x] - a^3*x^3*Log[1 + Sqrt[1 - a^2*x^2]])/(6*x^3)
```

Maple [A] time = 0.236, size = 96, normalized size = 1.4

$$\frac{2a^2x^2\mathrm{Artanh}(ax)-ax-2\,\mathrm{Artanh}(ax)}{6x^3}\sqrt{-(ax-1)(ax+1)}+\frac{a^3}{6}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)-\frac{a^3}{6}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] 1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x^
3+1/6*a^3*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^
(1/2)-1)
```

Maxima [A] time = 1.43899, size = 122, normalized size = 1.74

$$\frac{1}{6}\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}a^2-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}}{x^2}\right)a-\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\mathrm{artanh}(ax)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x^2 + 1)*a
^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a - 1/3*(-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^
3
```

Fricas [A] time = 2.04219, size = 163, normalized size = 2.33

$$-\frac{a^3x^3\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\sqrt{-a^2x^2+1}\left(ax-\left(a^2x^2-1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(a^3*x^3*log((sqrt(-a^2*x^2 + 1) - 1)/x) + sqrt(-a^2*x^2 + 1)*(a*x - (
a^2*x^2 - 1)*log(-(a*x + 1)/(a*x - 1))))/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**4, x)
```

Giac [B] time = 1.27278, size = 263, normalized size = 3.76

$$\frac{1}{12}\,a^3\log\left(\sqrt{-a^2x^2+1}+1\right)-\frac{1}{12}\,a^3\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{48}\left(\left(\frac{\left(a^4-\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}+\frac{\frac{3\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}-\left(\right.}{a^2|a|}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*a^3*log(sqrt(-a^2*x^2 + 1) + 1) - 1/12*a^3*log(-sqrt(-a^2*x^2 + 1) + 1
) + 1/48*((a^4 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a
^2*x^2 + 1)*abs(a) + a)^3*abs(a)) + (3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/
x - (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*log(-(a*x + 1)/(a*
x - 1)) - 1/6*sqrt(-a^2*x^2 + 1)*a/x^2
```

3.435 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=191

$$-\frac{1}{8}a^4\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^4\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3\sqrt{1-a^2x^2}}{24x}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(12*x^3) - (a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*
x^2) + (a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 - (a^4*Pol
yLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 + (a^4*PolyLog[2, Sqrt[1 - a*x]/
Sqrt[1 + a*x]])/8
```

Rubi [A] time = 0.299966, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6010, 6026, 271, 264, 6018}

$$-\frac{1}{8}a^4\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{8}a^4\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^3\sqrt{1-a^2x^2}}{24x}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(12*x^3) - (a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(8*
x^2) + (a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 - (a^4*Pol
yLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 + (a^4*PolyLog[2, Sqrt[1 - a*x]/
Sqrt[1 + a*x]])/8
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx &= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^4} - \frac{1}{3}\int \frac{\tanh^{-1}(ax)}{x^5\sqrt{1-a^2x^2}}\,dx + \frac{1}{3}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx \\ &= -\frac{a\sqrt{1-a^2x^2}}{9x^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4} - \frac{1}{12}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx - \frac{1}{4}a^2\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}} \\ &= -\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{2a^3\sqrt{1-a^2x^2}}{9x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2} - \\ &= -\frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{a^3\sqrt{1-a^2x^2}}{24x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2} + \frac{1}{4}\end{aligned}$$

Mathematica [A] time = 1.62085, size = 222, normalized size = 1.16

$$\frac{1}{192}a^4\left(-24\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+24\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-\frac{16\left(1-a^2x^2\right)^{3/2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{a^3x^3}-\frac{ax\text{cs}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^5,x]
```

```
[Out] (a^4*(-8*Coth[ArcTanh[a*x]/2] - 6*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (a*
x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*ArcTanh[a*x]*Csch[ArcTanh[a
*x]/2]^4 - 24*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] + 24*ArcTanh[a*x]*Log
[1 + E^(-ArcTanh[a*x])] - 24*PolyLog[2, -E^(-ArcTanh[a*x])] + 24*PolyLog[2,
 E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 3*ArcTanh[a*x
]*Sech[ArcTanh[a*x]/2]^4 - (16*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]^4)/
(a^3*x^3) + 8*Tanh[ArcTanh[a*x]/2]))/192
```

Maple [A] time = 0.313, size = 164, normalized size = 0.9

$$\frac{-x^3a^3+3\,a^2x^2\text{Artanh}\,(ax)-2\,ax-6\,\text{Artanh}\,(ax)}{24\,x^4}\sqrt{-(ax-1)(ax+1)}+\frac{a^4\text{Artanh}\,(ax)}{8}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x)
```

```
[Out] 1/24*(-(a*x-1)*(a*x+1))^(1/2)*(-x^3*a^3+3*a^2*x^2*arctanh(a*x)-2*a*x-6*arct
anh(a*x))/x^4+1/8*a^4*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/8*a^4
*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/8*a^4*arctanh(a*x)*ln(1-(a*x+1)/(
-a^2*x^2+1)^(1/2))-1/8*a^4*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

3.436 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=150

$$-\frac{a^3\sqrt{1-a^2x^2}}{24x^2}-\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{11}{120}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{2a^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3}-$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(20*x^4) - (a^3*Sqrt[1 - a^2*x^2])/(24*x^2) - (Sqrt[
1 - a^2*x^2]*ArcTanh[a*x])/(5*x^5) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(
15*x^3) + (2*a^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*x) + (11*a^5*ArcTanh[S
qrt[1 - a^2*x^2]])/120
```

Rubi [A] time = 0.37311, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6010, 6026, 266, 51, 63, 208, 6008}

$$-\frac{a^3\sqrt{1-a^2x^2}}{24x^2}-\frac{a\sqrt{1-a^2x^2}}{20x^4}+\frac{11}{120}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{2a^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```


```
[Out] -(a*Sqrt[1 - a^2*x^2])/(20*x^4) - (a^3*Sqrt[1 - a^2*x^2])/(24*x^2) - (Sqrt[
1 - a^2*x^2]*ArcTanh[a*x])/(5*x^5) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(
15*x^3) + (2*a^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(15*x) + (11*a^5*ArcTanh[S
qrt[1 - a^2*x^2]])/120
```


Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^6}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^5} - \frac{1}{4}\int \frac{\tanh^{-1}(ax)}{x^6\sqrt{1-a^2x^2}}\,dx + \frac{1}{4}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} - \frac{1}{20}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx + \frac{1}{8}a\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}}\,dx, x, x^2\right)$$

$$= -\frac{a\sqrt{1-a^2x^2}}{16x^4} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} - \frac{1}{40}a\,\text{Subst}\Bigg(\int$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{3a^3\sqrt{1-a^2x^2}}{32x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} + \frac{2a^4}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} + \frac{2a^4\sqrt{}}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} + \frac{2a^4\sqrt{}}{}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{a^3\sqrt{1-a^2x^2}}{24x^2} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^5} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15x^3} + \frac{2a^4\sqrt{}}{}$$

Mathematica [A] time = 0.121911, size = 104, normalized size = 0.69

$$\frac{1}{120}\left(-\frac{a\sqrt{1-a^2x^2}\left(5a^2x^2+6\right)}{x^4} + 11a^5\log\left(\sqrt{1-a^2x^2}+1\right) + \frac{8\sqrt{1-a^2x^2}\left(2a^4x^4+a^2x^2-3\right)\tanh^{-1}(ax)}{x^5} - 11a^5\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^6,x]
```

```
[Out] (-((a*Sqrt[1 - a^2*x^2]*(6 + 5*a^2*x^2))/x^4) + (8*Sqrt[1 - a^2*x^2]*(-3 +
a^2*x^2 + 2*a^4*x^4)*ArcTanh[a*x])/x^5 - 11*a^5*Log[x] + 11*a^5*Log[1 + Sqr
t[1 - a^2*x^2]])/120
```

Maple [A] time = 0.276, size = 116, normalized size = 0.8

$$\frac{16\,a^4x^4\mathrm{Artanh}\,(ax)-5\,x^3a^3+8\,a^2x^2\mathrm{Artanh}\,(ax)-6\,ax-24\,\mathrm{Artanh}\,(ax)}{120\,x^5}\sqrt{-(ax-1)\,(ax+1)}+\frac{11\,a^5}{120}\ln\Bigg(1+(ax+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x)
```

```
[Out] 1/120*(-(a*x-1)*(a*x+1))^(1/2)*(16*a^4*x^4*arctanh(a*x)-5*x^3*a^3+8*a^2*x^2
*arctanh(a*x)-6*a*x-24*arctanh(a*x))/x^5+11/120*a^5*ln(1+(a*x+1)/(-a^2*x^2+
1)^(1/2))-11/120*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)
```

Maxima [A] time = 1.454, size = 275, normalized size = 1.83

$$\frac{1}{120}\left(3\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-3\sqrt{-a^2x^2+1}a^4+8\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\sqrt{-a^2x^2+1}a^2-\frac{\left(-a^2x^2+1\right.}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/120*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - 3*sqrt(-a^2*x^2
+ 1)*a^4 + 8*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-a^2*x
^2 + 1)*a^2 - (-a^2*x^2 + 1)^(3/2)/x^2)*a^2 - 3*(-a^2*x^2 + 1)^(3/2)*a^2/x^
2 - 6*(-a^2*x^2 + 1)^(3/2)/x^4)*a - 1/15*(2*(-a^2*x^2 + 1)^(3/2)*a^2/x^3 +
3*(-a^2*x^2 + 1)^(3/2)/x^5)*arctanh(a*x)
```

Fricas [A] time = 1.98047, size = 208, normalized size = 1.39

$$-\frac{11\,a^5x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(5\,a^3x^3+6\,ax-4\left(2\,a^4x^4+a^2x^2-3\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{120\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/120*(11*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 + 6*a*x - 4
*(2*a^4*x^4 + a^2*x^2 - 3)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x
^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)\,(ax+1)}\,\mathrm{atanh}\,(ax)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**6, x)
```

Giac [B] time = 2.2548, size = 377, normalized size = 2.51

$$\frac{11}{240}a^5\log\left(\sqrt{-a^2x^2+1}+1\right)-\frac{11}{240}a^5\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{960}\left(\frac{\left(3a^6+\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2}{x^2}-\frac{30\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^2x^4}\right)a^{10}x^5}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] 11/240*a^5*log(sqrt(-a^2*x^2 + 1) + 1) - 11/240*a^5*log(-sqrt(-a^2*x^2 + 1)
 + 1) + 1/960*((3*a^6 + 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2/x^2 - 30*(s
qrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^2*x^4))*a^10*x^5/((sqrt(-a^2*x^2 + 1)*ab
s(a) + a)^5*abs(a)) + (30*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^8/x - 5*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^3*a^4/x^3 - 3*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/x^
5)/(a^4*abs(a)))*log(-(a*x + 1)/(a*x - 1)) + 1/120*(5*(-a^2*x^2 + 1)^(3/2)*
a^5 - 11*sqrt(-a^2*x^2 + 1)*a^5)/(a^4*x^4)
```

3.437 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^7}\,dx$

Optimal. Leaf size=243

$$-\frac{1}{16}a^6\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{16}a^6\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{a^5\sqrt{1-a^2x^2}}{720x}-\frac{11a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{a^4\sqrt{1}}{}$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(30*x^5) - (11*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (a
^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6) +
(a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) + (a^4*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/(16*x^2) + (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]
)/8 - (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 + (a^6*PolyLog[2,
 Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rubi [A] time = 0.412757, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6010, 6026, 271, 264, 6018}

$$-\frac{1}{16}a^6\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{16}a^6\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{a^5\sqrt{1-a^2x^2}}{720x}-\frac{11a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{a^4\sqrt{1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```


```
[Out] -(a*Sqrt[1 - a^2*x^2])/(30*x^5) - (11*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (a
^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6) +
(a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) + (a^4*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x])/(16*x^2) + (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]
)/8 - (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 + (a^6*PolyLog[2,
 Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```


Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^7}\,dx = -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^6} - \frac{1}{5}\int \frac{\tanh^{-1}(ax)}{x^7\sqrt{1-a^2x^2}}\,dx + \frac{1}{5}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{25x^5} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} - \frac{1}{30}a\int \frac{1}{x^6\sqrt{1-a^2x^2}}\,dx - \frac{1}{6}a^2\int \frac{\tanh^{-1}(ax)}{x^5\sqrt{1-a^2x^2}}\,dx$$
$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{4a^3\sqrt{1-a^2x^2}}{75x^3} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24x^4} - \frac{1}{75}($$
$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{11a^3\sqrt{1-a^2x^2}}{360x^3} - \frac{8a^5\sqrt{1-a^2x^2}}{75x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{1-a^2x^2}\,\text{t}}{24x^4}$$
$$= -\frac{a\sqrt{1-a^2x^2}}{30x^5} - \frac{11a^3\sqrt{1-a^2x^2}}{360x^3} + \frac{a^5\sqrt{1-a^2x^2}}{720x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6} + \frac{a^2\sqrt{1-a^2x^2}\,\text{ta}}{24x^4}$$

Mathematica [A] time = 3.37897, size = 307, normalized size = 1.26

$$a^6\left(-360\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) + 360\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) - \frac{416\left(1-a^2x^2\right)^{3/2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{a^3x^3} - \frac{3ax\text{csch}^6\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x^7,x]
```

```
[Out] (a^6*(-76*Coth[ArcTanh[a*x]/2] - 90*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - (
26*a*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 90*ArcTanh[a*x]*Csch[Arc
Tanh[a*x]/2]^4 - (3*a*x*Csch[ArcTanh[a*x]/2]^6)/Sqrt[1 - a^2*x^2] - 15*ArcT
anh[a*x]*Csch[ArcTanh[a*x]/2]^6 - 360*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x]
)] + 360*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] - 360*PolyLog[2, -E^(-ArcT
anh[a*x])] + 360*PolyLog[2, E^(-ArcTanh[a*x])] - 90*ArcTanh[a*x]*Sech[ArcTa
nh[a*x]/2]^2 + 90*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - 15*ArcTanh[a*x]*Sec
h[ArcTanh[a*x]/2]^6 - (416*(1 - a^2*x^2)^(3/2)*Sinh[ArcTanh[a*x]/2]^4)/(a^3
*x^3) + 76*Tanh[ArcTanh[a*x]/2] + 6*Sech[ArcTanh[a*x]/2]^4*Tanh[ArcTanh[a*x
]/2]))/5760
```

Maple [A] time = 0.286, size = 183, normalized size = 0.8

$$\frac{x^5a^5+45\,a^4x^4\mathrm{Artanh}\,(ax)-22\,x^3a^3+30\,a^2x^2\mathrm{Artanh}\,(ax)-24\,ax-120\,\mathrm{Artanh}\,(ax)}{720\,x^6}\sqrt{-\,(ax-1)\,(ax+1)}+\frac{a^6\mathrm{Art}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x)
```

```
[Out] 1/720*(-(a*x-1)*(a*x+1))^(1/2)*(x^5*a^5+45*a^4*x^4*arctanh(a*x)-22*x^3*a^3+
30*a^2*x^2*arctanh(a*x)-24*a*x-120*arctanh(a*x))/x^6+1/16*a^6*arctanh(a*x)*
ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(
1/2))-1/16*a^6*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polyl
og(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\,(ax-1)\,(ax+1)}\,\mathrm{atanh}\,(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)*(-a**2*x**2+1)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)*(-a^2*x^2+1)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

3.438 $\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=336

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{i\text{Po}}{}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(18*a^4) + (x^3*Sqrt[1 - a^2*x^2])/(60*a^2) - (19*Arc
Sin[a*x])/(360*a^5) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(360*a^5) + (11*x^2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(180*a^3) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x])/(15*a) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(16*a^4) - (x^3*Sqrt[1
- a^2*x^2]*ArcTanh[a*x]^2)/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/6 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(8*a^5) - ((I/8)*ArcTanh[a*x]
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^5 + ((I/8)*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a^5 + ((I/8)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^5 - ((I/8)*P
olyLog[3, I*E^ArcTanh[a*x]])/a^5
```

Rubi [A] time = 1.40273, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6014, 6016, 321, 216, 5994, 5952, 4180, 2531, 2282, 6589}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{8a^5}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{8a^5}-\frac{i\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```


```
[Out] (x*Sqrt[1 - a^2*x^2])/(18*a^4) + (x^3*Sqrt[1 - a^2*x^2])/(60*a^2) - (19*Arc
Sin[a*x])/(360*a^5) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(360*a^5) + (11*x^2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(180*a^3) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x])/(15*a) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(16*a^4) - (x^3*Sqrt[1
- a^2*x^2]*ArcTanh[a*x]^2)/(24*a^2) + (x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/6 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(8*a^5) - ((I/8)*ArcTanh[a*x]
*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^5 + ((I/8)*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a^5 + ((I/8)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^5 - ((I/8)*P
olyLog[3, I*E^ArcTanh[a*x]])/a^5
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
```

```
m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int \frac{x^6\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{4a^2}+\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-\frac{5}{6}\int \frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a^3}+\frac{x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a}-\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{8a^4}$$

$$= -\frac{x\sqrt{1-a^2x^2}}{12a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{13\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^5}+\frac{11x^2\sqrt{1-a^2x^2}\tanh^{-1}(a}{180a^3}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}+\frac{7\sin^{-1}(ax)}{6a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1-a^2x}{180}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1-a^2}{18}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1-a^2}{18}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1-a^2}{18}$$

$$= \frac{x\sqrt{1-a^2x^2}}{18a^4}+\frac{x^3\sqrt{1-a^2x^2}}{60a^2}-\frac{19\sin^{-1}(ax)}{360a^5}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{360a^5}+\frac{11x^2\sqrt{1-a^2}{18}$$

Mathematica [A] time = 1.56427, size = 268, normalized size = 0.8

$$\sqrt{1-a^2x^2}\left(-\frac{i\left(90\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-90\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+90\mathrm{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-90\mathrm{PolyLog}\left(3,ie^{-t}\right.\right.}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(90*ArcTanh[a*x] + 140*(-1 + a^2*x^2)*ArcTanh[a*x] + 48*
(-1 + a^2*x^2)^2*ArcTanh[a*x] + 120*a*x*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 6
*a*x*(-1 + a^2*x^2)*(2 + 35*ArcTanh[a*x]^2) + a*x*(52 + 45*ArcTanh[a*x]^2)
- (I*((-76*I)*ArcTan[Tanh[ArcTanh[a*x]/2]] + 45*ArcTanh[a*x]^2*Log[1 - I/E^
ArcTanh[a*x]] - 45*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 90*ArcTanh[a*
x]*PolyLog[2, (-I)/E^ArcTanh[a*x]] - 90*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh
[a*x]] + 90*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 90*PolyLog[3, I/E^ArcTanh[a*x
]]))/Sqrt[1 - a^2*x^2]))/(720*a^5)
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int x^4\left(\mathrm{Artanh}\,(ax)\right)^2\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^4 \operatorname{artanh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-a^2x^2+1}x^4 \operatorname{artanh}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{-(ax-1)(ax+1)} \operatorname{atanh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^4 \operatorname{artanh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^4*arctanh(a*x)^2, x)
```

3.439 $\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=281

$$-\frac{11i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}+\frac{11i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{30a^4}+\frac{11\sqrt{1-a^2x^2}}{60a^4}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+$$

```
[Out] (11*Sqrt[1 - a^2*x^2])/(60*a^4) - (1 - a^2*x^2)^(3/2)/(30*a^4) + (x*Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(12*a^3) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(1
0*a) - (11*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(30*a^4) - (2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(15*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^2)/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/5 - (((11*I)/60)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (((11*I)/60)*PolyLog
[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```

Rubi [A] time = 1.07307, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {6014, 6016, 261, 5950, 5994, 266, 43}

$$-\frac{11i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}+\frac{11i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{60a^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{30a^4}+\frac{11\sqrt{1-a^2x^2}}{60a^4}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```


```
[Out] (11*Sqrt[1 - a^2*x^2])/(60*a^4) - (1 - a^2*x^2)^(3/2)/(30*a^4) + (x*Sqrt[1
- a^2*x^2]*ArcTanh[a*x])/(12*a^3) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(1
0*a) - (11*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(30*a^4) - (2*
Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(15*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh
[a*x]^2)/(15*a^2) + (x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/5 - (((11*I)/60)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4 + (((11*I)/60)*PolyLog
[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^4
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int \frac{x^5\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{x^3\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^2}+\frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-\frac{4}{5}\int \frac{x^3\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx+$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{3a^4}-$$

$$= -\frac{\sqrt{1-a^2x^2}}{3a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{10\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{3a^4}$$

$$= \frac{\sqrt{1-a^2x^2}}{12a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-\frac{11\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\text{ta}}{30a^4}$$

$$= \frac{11\sqrt{1-a^2x^2}}{60a^4}-\frac{\left(1-a^2x^2\right)^{3/2}}{30a^4}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3}+\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{10a}-$$

Mathematica [A] time = 0.640527, size = 175, normalized size = 0.62

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{11i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+12\left(a^2x^2-\right.\right.}{60a^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(11 + 11*a*x*ArcTanh[a*x] + 6*a*x*(-1 + a^2*x^2)*ArcTanh
[a*x] + 12*(-1 + a^2*x^2)^2*ArcTanh[a*x]^2 + 2*(-1 + a^2*x^2)*(1 + 10*ArcTa
nh[a*x]^2) - ((11*I)*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E
^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[
a*x]]))/Sqrt[1 - a^2*x^2]))/(60*a^4)
```

Maple [A] time = 0.273, size = 211, normalized size = 0.8

$$\frac{12\,a^4x^4\left(\text{Artanh}\left(ax\right)\right)^2+6\,a^3x^3\text{Artanh}\left(ax\right)-4\,a^2x^2\left(\text{Artanh}\left(ax\right)\right)^2+2\,a^2x^2+5\,ax\text{Artanh}\left(ax\right)-8\,\left(\text{Artanh}\left(ax\right)\right)^2}{60\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/60/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(12*a^4*x^4*arctanh(a*x)^2+6*a^3*x^3*arct
anh(a*x)-4*a^2*x^2*arctanh(a*x)^2+2*a^2*x^2+5*a*x*arctanh(a*x)-8*arctanh(a*
x)^2+9)-11/60*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4+11/60*I
*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^4-11/60*I*dilog(1+I*(a*x
+1)/(-a^2*x^2+1)^(1/2))/a^4+11/60*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a
^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2x^2+1}x^3\,\text{artanh}\left(ax\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}x^3\,\text{artanh}\left(ax\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^3\,\text{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^3*arctanh(a*x)^2, x)
```

3.440 $\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=254

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{4a^3}-\frac{i\text{Po}}{}$$

```
[Out] (x*Sqrt[1 - a^2*x^2])/(12*a^2) - ArcSin[a*x]/(6*a^3) + (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x])/(12*a^3) + (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*a) - (x*Sqr
t[1 - a^2*x^2]*ArcTanh[a*x]^2)/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2)/4 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(4*a^3) - ((I/4)*ArcTanh[a
*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + ((I/4)*ArcTanh[a*x]*PolyLog[2, I
*E^ArcTanh[a*x]])/a^3 + ((I/4)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((I/4
)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```

Rubi [A] time = 0.852707, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6014, 6016, 5994, 216, 5952, 4180, 2531, 2282, 6589, 321}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{4a^3}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{4a^3}-\frac{i\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```


```
[Out] (x*Sqrt[1 - a^2*x^2])/(12*a^2) - ArcSin[a*x]/(6*a^3) + (Sqrt[1 - a^2*x^2]*A
rcTanh[a*x])/(12*a^3) + (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*a) - (x*Sqr
t[1 - a^2*x^2]*ArcTanh[a*x]^2)/(8*a^2) + (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x
]^2)/4 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/(4*a^3) - ((I/4)*ArcTanh[a
*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a^3 + ((I/4)*ArcTanh[a*x]*PolyLog[2, I
*E^ArcTanh[a*x]])/a^3 + ((I/4)*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a^3 - ((I/4
)*PolyLog[3, I*E^ArcTanh[a*x]])/a^3
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\left(a^2\int \frac{x^4\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a^2} + \frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 - \frac{3}{4}\int \frac{x^2\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a^3} + \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a} - \frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{8a^2} + \frac{1}{4}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2} + \frac{\sin^{-1}(ax)}{a^3} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3} + \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a} - x\sqrt{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2} - \frac{\sin^{-1}(ax)}{6a^3} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3} + \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a} - x\sqrt{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2} - \frac{\sin^{-1}(ax)}{6a^3} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3} + \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a} - x\sqrt{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2} - \frac{\sin^{-1}(ax)}{6a^3} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3} + \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a} - x\sqrt{}$$

$$= \frac{x\sqrt{1-a^2x^2}}{12a^2} - \frac{\sin^{-1}(ax)}{6a^3} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{12a^3} + \frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6a} - x\sqrt{}$$

Mathematica [A] time = 1.10596, size = 228, normalized size = 0.9

$$\sqrt{1-a^2x^2}\left(-\frac{i\left(6\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-6\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+6\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(3,ie^{-\tanh}\right.\right.}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6*ArcTanh[a*x] - 4*(1 - a^2*x^2)*ArcTanh[a*x] - 6*a*x*(
1 - a^2*x^2)*ArcTanh[a*x]^2 + a*x*(2 + 3*ArcTanh[a*x]^2) - (I*((-8*I)*ArcTa
n[Tanh[ArcTanh[a*x]/2]] + 3*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - 3*Ar
cTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 6*ArcTanh[a*x]*PolyLog[2, (-I)/E^A
rcTanh[a*x]] - 6*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 6*PolyLog[3, (
-I)/E^ArcTanh[a*x]] - 6*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/
(24*a^3)
```

Maple [F] time = 0.303, size = 0, normalized size = 0.

$$\int x^2\left(\text{Artanh}\left(ax\right)\right)^2\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-(ax-1)(ax+1)} \operatorname{atanh}^2(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x^2 \operatorname{artanh}(ax)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x^2*arctanh(a*x)^2, x)
```

3.441 $\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=175

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{\sqrt{1-a^2x^2}}{3a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a}$$

```
[Out] Sqrt[1 - a^2*x^2]/(3*a^2) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a) - (2*A
rcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^2) - ((1 - a^2*x^2)^(
3/2)*ArcTanh[a*x]^2)/(3*a^2) - ((I/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a^2 + ((I/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```

Rubi [A] time = 0.124585, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5994, 5942, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{3a^2}+\frac{\sqrt{1-a^2x^2}}{3a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```


```
[Out] Sqrt[1 - a^2*x^2]/(3*a^2) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*a) - (2*A
rcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(3*a^2) - ((1 - a^2*x^2)^(
3/2)*ArcTanh[a*x]^2)/(3*a^2) - ((I/3)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[
1 + a*x]])/a^2 + ((I/3)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^2
```


Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{2\int\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx}{3a}$$
$$=\frac{\sqrt{1-a^2x^2}}{3a^2}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3a^2}+\frac{\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}dx}{3a}$$
$$=\frac{\sqrt{1-a^2x^2}}{3a^2}+\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3a}-\frac{2\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{3a^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tan}{3a^2}$$

Mathematica [A] time = 0.393182, size = 135, normalized size = 0.77

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}-\left(1-a^2x^2\right)\tan\right.}{3a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (1 - a^2*x^2)*ArcTanh[a*x]^2 - (
I*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + P
olyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^
2*x^2]))/(3*a^2)
```

Maple [A] time = 0.252, size = 175, normalized size = 1.

$$\frac{a^2x^2\left(\text{Artanh}\,(ax)\right)^2+ax\text{Artanh}\,(ax)-\left(\text{Artanh}\,(ax)\right)^2+1}{3\,a^2}\sqrt{-\left(ax-1\right)\left(ax+1\right)}-\frac{\frac{i}{3}\text{Artanh}\,(ax)}{a^2}\ln\left(1+i\left(ax+1\right)\frac{}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/3/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2+a*x*arctanh(a*x)-a
rctanh(a*x)^2+1)-1/3*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^2+
1/3*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^2-1/3*I*dilog(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))/a^2+1/3*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/
a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2x^2+1}x\,\text{artanh}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}x\,\text{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atanh(a*x)**2*(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}x\,\text{artanh}\,(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctanh(a*x)^2*(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*x*arctanh(a*x)^2, x)
```

3.442 $\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=158

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\text{PolyI}}{}$$

```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```

Rubi [A] time = 0.1462, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5944, 5952, 4180, 2531, 2282, 6589, 216}

$$-\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\text{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```


```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```


Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx - \int \frac{1}{\sqrt{1-}}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\text{Subst}\left(\int x^2\text{sech}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

Mathematica [A] time = 0.714139, size = 187, normalized size = 1.18

$$\sqrt{1-a^2x^2}\left(-\frac{i\left(2\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2\text{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2\text{PolyLog}\left(3,ie^{-\tanh}\right.\right.}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [F] time = 0.206, size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,(\mathrm{Artanh}\,(ax))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}^2\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

3.443 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}\,dx$

Optimal. Leaf size=174

$$2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}$$

```
[Out] 4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x] + Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]^2 - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*Poly
Log[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + (2*I)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] - (2*I)*PolyLog[2, (I*Sqrt[
1 - a*x])/Sqrt[1 + a*x]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^A
rcTanh[a*x]]
```

Rubi [A] time = 0.393152, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6014, 6020, 4182, 2531, 2282, 6589, 5994, 5950}

$$2i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)+2\tanh^{-1}(ax)\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```


```
[Out] 4*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x] + Sqrt[1 - a^2*x^2]*ArcT
anh[a*x]^2 - 2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - 2*ArcTanh[a*x]*Poly
Log[2, -E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, E^ArcTanh[a*x]] + (2*I)
*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] - (2*I)*PolyLog[2, (I*Sqrt[
1 - a*x])/Sqrt[1 + a*x]] + 2*PolyLog[3, -E^ArcTanh[a*x]] - 2*PolyLog[3, E^A
rcTanh[a*x]]
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x}\,dx = -\left(a^2\int \frac{x\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right)+\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$$

$$= \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-(2a)\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx+\text{Subst}\left(\int x^2\mathbf{csch}(x)\,dx,x,\tanh^{-1}\right.$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

$$= 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2-2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh$$

Mathematica [A] time = 0.278736, size = 203, normalized size = 1.17

$$2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+2i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2i\text{Po}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x,x]
```

```
[Out] Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 + ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])
] + (2*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (2*I)*ArcTanh[a*x]*Log[1
 + I/E^ArcTanh[a*x]] - ArcTanh[a*x]^2*Log[1 + E^(-ArcTanh[a*x])] + 2*ArcTan
h[a*x]*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*PolyLog[2, (-I)/E^ArcTanh[a*x
]] - (2*I)*PolyLog[2, I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, E^(-Arc
Tanh[a*x])] + 2*PolyLog[3, -E^(-ArcTanh[a*x])] - 2*PolyLog[3, E^(-ArcTanh[a
*x])]
```

Maple [F] time = 0.262, size = 0, normalized size = 0.

$$\int \frac{(\text{Artanh}(ax))^2}{x}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x)
```

```
[Out] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\operatorname{atanh}^2(ax)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\operatorname{artanh}(ax)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x, x)
```

3.444 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^2}\,dx$

Optimal. Leaf size=197

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+2ia\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)-2ia\tanh^{-1}(ax)\text{Poly}$$

```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 2*a*ArcTan[E^ArcTanh[a*x]]*ArcTan
h[a*x]^2 - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (2*I)*a*
ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (2*I)*a*ArcTanh[a*x]*PolyLog
[2, I*E^ArcTanh[a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*
a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (2*I)*a*PolyLog[3, (-I)*E^ArcTa
nh[a*x]] + (2*I)*a*PolyLog[3, I*E^ArcTanh[a*x]]
```

Rubi [A] time = 0.382518, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6014, 6008, 6018, 5952, 4180, 2531, 2282, 6589}

$$2a\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-2a\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+2ia\tanh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)-2ia\tanh^{-1}(ax)\text{Poly}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```


```
[Out] -((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x) - 2*a*ArcTan[E^ArcTanh[a*x]]*ArcTan
h[a*x]^2 - 4*a*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + (2*I)*a*
ArcTanh[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]] - (2*I)*a*ArcTanh[a*x]*PolyLog
[2, I*E^ArcTanh[a*x]] + 2*a*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - 2*
a*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (2*I)*a*PolyLog[3, (-I)*E^ArcTa
nh[a*x]] + (2*I)*a*PolyLog[3, I*E^ArcTanh[a*x]]
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
```

```
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^2}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{\tanh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} - a\,\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \tanh^{-1}(ax)\right) + (2a)\int \frac{\tanh^{-1}(a}{x\sqrt{1-a^2}}$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)\tanh^{-1}\left(\right.$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)\tanh^{-1}\left(\right.$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)\tanh^{-1}\left(\right.$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x} - 2a\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2 - 4a\tanh^{-1}(ax)\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.726134, size = 223, normalized size = 1.13

$$a\left(2i\tanh^{-1}(ax)\text{PolyLog}\left(2, -ie^{-\tanh^{-1}(ax)}\right) - 2i\tanh^{-1}(ax)\text{PolyLog}\left(2, ie^{-\tanh^{-1}(ax)}\right) + 2\text{PolyLog}\left(2, -e^{-\tanh^{-1}(ax)}\right) - 2\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^2,x]
```

```
[Out] a*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/(a*x)) + 2*ArcTanh[a*x]*Log[1 - E^(
-ArcTanh[a*x])] + I*ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - I*ArcTanh[a*
x]^2*Log[1 + I/E^ArcTanh[a*x]] - 2*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
+ 2*PolyLog[2, -E^(-ArcTanh[a*x])] + (2*I)*ArcTanh[a*x]*PolyLog[2, (-I)/E^A
rcTanh[a*x]] - (2*I)*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] - 2*PolyLog[
2, E^(-ArcTanh[a*x])] + (2*I)*PolyLog[3, (-I)/E^ArcTanh[a*x]] - (2*I)*PolyL
og[3, I/E^ArcTanh[a*x]])
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int \frac{(\text{Artanh}(ax))^2}{x^2}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x)
```

```
[Out] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^2, x)
```

3.445 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^3}\,dx$

Optimal. Leaf size=151

$$a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)-a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)+a^2\text{PolyL}$$

```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) + a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] + a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] - a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] - a^2*PolyLog[3, -E^ArcTanh[a*x]] + a^2*Poly
Log[3, E^ArcTanh[a*x]]
```

Rubi [A] time = 0.544467, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {6014, 6026, 6008, 266, 63, 208, 6020, 4182, 2531, 2282, 6589}

$$a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,-e^{\tanh^{-1}(ax)}\right)-a^2\tanh^{-1}(ax)\text{PolyLog}\left(2,e^{\tanh^{-1}(ax)}\right)-a^2\text{PolyLog}\left(3,-e^{\tanh^{-1}(ax)}\right)+a^2\text{PolyL}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```


```
[Out] -((a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2
)/(2*x^2) + a^2*ArcTanh[E^ArcTanh[a*x]]*ArcTanh[a*x]^2 - a^2*ArcTanh[Sqrt[1
 - a^2*x^2]] + a^2*ArcTanh[a*x]*PolyLog[2, -E^ArcTanh[a*x]] - a^2*ArcTanh[a
*x]*PolyLog[2, E^ArcTanh[a*x]] - a^2*PolyLog[3, -E^ArcTanh[a*x]] + a^2*Poly
Log[3, E^ArcTanh[a*x]]
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
```

```
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6020

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Dist[1/Sqrt[d], Subst[Int[(a + b*x)^p*Csch[x], x], x, ArcT
anh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p,
0] && GtQ[d, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^3}\,dx = -\left(a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx\right) + \int \frac{\tanh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx + \frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx - a^2\,\mathrm{Subst}\Big(\int$$

$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + 2a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax$$

$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

$$= -\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x} - \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2x^2} + a^2\tanh^{-1}\left(e^{\tanh^{-1}(ax)}\right)\tanh^{-1}(ax)^2$$

Mathematica [A] time = 1.2173, size = 188, normalized size = 1.25

$$\frac{1}{8}a^2\left(-8\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right) + 8\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right) - 8\mathrm{PolyLog}\left(3,-e^{-\tanh^{-1}(ax)}\right) +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^3,x]
```

```
[Out] (a^2*(-4*ArcTanh[a*x]*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]^2*Csch[ArcTanh[a*
x]/2]^2 - 4*ArcTanh[a*x]^2*Log[1 - E^(-ArcTanh[a*x])] + 4*ArcTanh[a*x]^2*Lo
g[1 + E^(-ArcTanh[a*x])] + 8*Log[Tanh[ArcTanh[a*x]/2]] - 8*ArcTanh[a*x]*Pol
yLog[2, -E^(-ArcTanh[a*x])] + 8*ArcTanh[a*x]*PolyLog[2, E^(-ArcTanh[a*x])]
- 8*PolyLog[3, -E^(-ArcTanh[a*x])] + 8*PolyLog[3, E^(-ArcTanh[a*x])] - ArcT
anh[a*x]^2*Sech[ArcTanh[a*x]/2]^2 + 4*ArcTanh[a*x]*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A] time = 0.289, size = 231, normalized size = 1.5

$$-\frac{\mathrm{Artanh}\,(ax)\,(2\,ax+\mathrm{Artanh}\,(ax))}{2\,x^2}\sqrt{-(ax-1)\,(ax+1)} + \frac{a^2\,(\mathrm{Artanh}\,(ax))^2}{2}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right) + a^2\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*arctanh(a*x)*(2*a*x+arctanh(a*x))/x^2+1/2*a^2
*arctanh(a*x)^2*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*arctanh(a*x)*polylog(2
,-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*polylog(3,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/
2*a^2*arctanh(a*x)^2*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))-a^2*arctanh(a*x)*poly
log(2,(a*x+1)/(-a^2*x^2+1)^(1/2))+a^2*polylog(3,(a*x+1)/(-a^2*x^2+1)^(1/2))
-2*a^2*arctanh((a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\text{atanh}^2\,(ax)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^3, x)
```

3.446 $\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^4}\,dx$

Optimal. Leaf size=169

$$-\frac{1}{3}a^3\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{3}a^3\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tan}{3x^3}$$

```
[Out] -(a^2*Sqrt[1 - a^2*x^2])/(3*x) - (a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^2)
 - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2)/(3*x^3) + (2*a^3*ArcTanh[a*x]*ArcTa
nh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3 - (a^3*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
 + a*x])])/3 + (a^3*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3
```

Rubi [A] time = 0.297462, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {6008, 6010, 6026, 264, 6018}

$$-\frac{1}{3}a^3\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{3}a^3\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tan}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```


```
[Out] -(a^2*Sqrt[1 - a^2*x^2])/(3*x) - (a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(3*x^2)
 - ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2)/(3*x^3) + (2*a^3*ArcTanh[a*x]*ArcTa
nh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3 - (a^3*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1
 + a*x])])/3 + (a^3*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/3
```


Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
```

```
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{x^4}\,dx = -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -\frac{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3}-\frac{1}{3}(2a)\int \frac{\tanh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx+$$

$$= -\frac{2a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3}-\frac{1}{3}a^2\int \frac{}{x^2\sqrt{}}$$

$$= -\frac{a^2\sqrt{1-a^2x^2}}{3x}-\frac{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^2}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}{3x^3}+\frac{2}{3}a^3\tanh^{-1}($$

Mathematica [A] time = 2.0897, size = 177, normalized size = 1.05

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(-\frac{4a^3x^3\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)}{\left(1-a^2x^2\right)^{3/2}}+\tanh^{-1}(ax)\left(\frac{\left(\log\left(1-e^{-\tanh^{-1}(ax)}\right)-\log\left(e^{-\tanh^{-1}(ax)}+1\right)\right)\left(\sqrt{1-a^2x^2}\sinh\left(3\tanh^{-1}(ax)\right)-3a\right.}{\sqrt{1-a^2x^2}}\right.\right.}{12x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)/x^4,x]
```

```
[Out] -(a^3*PolyLog[2, -E^(-ArcTanh[a*x])])/3 - ((1 - a^2*x^2)^(3/2)*(4*ArcTanh[a
*x]^2 + 2*(-1 + Cosh[2*ArcTanh[a*x]]) - (4*a^3*x^3*PolyLog[2, E^(-ArcTanh[a
*x])])/(1 - a^2*x^2)^(3/2) + ArcTanh[a*x]*(2*Sinh[2*ArcTanh[a*x]] + ((Log[1
 - E^(-ArcTanh[a*x])] - Log[1 + E^(-ArcTanh[a*x])])*(-3*a*x + Sqrt[1 - a^2*
x^2]*Sinh[3*ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2])))/(12*x^3)
```

Maple [A] time = 0.264, size = 171, normalized size = 1.

$$\frac{a^2x^2\left(\text{Artanh}\,(ax)\right)^2-a^2x^2-ax\text{Artanh}\,(ax)-\left(\text{Artanh}\,(ax)\right)^2}{3\,x^3}\sqrt{-(ax-1)\,(ax+1)}+\frac{a^3\text{Artanh}\,(ax)}{3}\ln\left(1+(ax+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x)
```

```
[Out] 1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a^2*x^2*arctanh(a*x)^2-a^2*x^2-a*x*arctanh(a*
x)-arctanh(a*x)^2)/x^3+1/3*a^3*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2)
)+1/3*a^3*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*a^3*arctanh(a*x)*ln(1-
(a*x+1)/(-a^2*x^2+1)^(1/2))-1/3*a^3*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2*(-a**2*x**2+1)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2/x^4, x)
```

3.447 $\int x^4 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=292

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{192a^5}+\frac{3\sqrt{1-a^2x^2}}{128a^5}-$$

```
[Out] (3*Sqrt[1 - a^2*x^2])/(128*a^5) + (1 - a^2*x^2)^(3/2)/(192*a^5) - (3*(1 - a
^2*x^2)^(5/2))/(80*a^5) + (1 - a^2*x^2)^(7/2)/(56*a^5) - (3*x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(128*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(64*a^2
) + (3*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 - (a^2*x^7*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x])/8 - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(64*
a^5) - (((3*I)/128)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (
((3*I)/128)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```

Rubi [A] time = 0.817538, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6010, 6016, 266, 43, 261, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{128a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{192a^5}+\frac{3\sqrt{1-a^2x^2}}{128a^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```


```
[Out] (3*Sqrt[1 - a^2*x^2])/(128*a^5) + (1 - a^2*x^2)^(3/2)/(192*a^5) - (3*(1 - a
^2*x^2)^(5/2))/(80*a^5) + (1 - a^2*x^2)^(7/2)/(56*a^5) - (3*x*Sqrt[1 - a^2*
x^2]*ArcTanh[a*x])/(128*a^4) - (x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(64*a^2
) + (3*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 - (a^2*x^7*Sqrt[1 - a^2*x^2]*
ArcTanh[a*x])/8 - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(64*
a^5) - (((3*I)/128)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5 + (
((3*I)/128)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^5
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
```

```
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int x^4\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx=-\left(a^2\int x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\right)+\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$$

$$=\frac{1}{6}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{6}\int\frac{x^4\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-$$

$$=-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{24a^2}+\frac{3}{16}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{8}a^2x^7\sqrt{1-a^2x^2}\tanh^{-1}(a$$

$$=-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^4}-\frac{x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{64a^2}+\frac{3}{16}x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)-$$

$$=-\frac{\sqrt{1-a^2x^2}}{48a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^5}-\frac{\left(1-a^2x^2\right)^{5/2}}{24a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(a}{128a^4}$$

$$=-\frac{\sqrt{1-a^2x^2}}{384a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}}{128a^4}$$

$$=\frac{3\sqrt{1-a^2x^2}}{128a^5}+\frac{\left(1-a^2x^2\right)^{3/2}}{192a^5}-\frac{3\left(1-a^2x^2\right)^{5/2}}{80a^5}+\frac{\left(1-a^2x^2\right)^{7/2}}{56a^5}-\frac{3x\sqrt{1-a^2x^2}\tanh^{-1}(}{128a^4}$$

Mathematica [A] time = 1.33109, size = 272, normalized size = 0.93

$$\frac{-315i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+315i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-240a^6x^6\sqrt{1-a^2x^2}+216a^4x^4\sqrt{1-a^2x^2}+218a^2x^2\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (121*Sqrt[1 - a^2*x^2] + 218*a^2*x^2*Sqrt[1 - a^2*x^2] + 216*a^4*x^4*Sqrt[1
 - a^2*x^2] - 240*a^6*x^6*Sqrt[1 - a^2*x^2] - 315*a*x*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x] - 210*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 2520*a^5*x^5*Sqrt[
1 - a^2*x^2]*ArcTanh[a*x] - 1680*a^7*x^7*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (
315*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (315*I)*ArcTanh[a*x]*Log[1
+ I/E^ArcTanh[a*x]] - (315*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (315*I)*Pol
yLog[2, I/E^ArcTanh[a*x]])/(13440*a^5)
```

Maple [A] time = 0.213, size = 215, normalized size = 0.7

$$-\frac{1680\,\text{Artanh}(ax)\,x^7a^7+240\,x^6a^6-2520\,\text{Artanh}(ax)\,x^5a^5-216\,x^4a^4+210\,a^3x^3\text{Artanh}(ax)-218\,a^2x^2+315\,ax\cdots}{13440\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/13440/a^5*(-(a*x-1)*(a*x+1))^(1/2)*(1680*arctanh(a*x)*x^7*a^7+240*x^6*a^
6-2520*arctanh(a*x)*x^5*a^5-216*x^4*a^4+210*a^3*x^3*arctanh(a*x)-218*a^2*x^
2+315*a*x*arctanh(a*x)-121)-3/128*I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arct
anh(a*x)/a^5+3/128*I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^5-3/
128*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^5+3/128*I*dilog(1-I*(a*x+1)/(
-a^2*x^2+1)^(1/2))/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2x^2+1\right)^{\frac{3}{2}}x^4\,\text{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2x^6-x^4\right)\sqrt{-a^2x^2+1}\,\text{artanh}(ax),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^6 - x^4)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}x^4\,\mathrm{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^4*arctanh(a*x), x)
```

3.448 $\int x^3 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=186

$$-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}+\frac{3x\sqrt{1-a^2x^2}}{112a^3}-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{8}{35}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x^2}{}$$

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(112*a^3) + (23*x^3*Sqrt[1 - a^2*x^2])/(840*a) - (a
*x^5*Sqrt[1 - a^2*x^2])/42 + (17*ArcSin[a*x])/(560*a^4) - (2*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(35*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(35*a^2)
+ (8*x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/35 - (a^2*x^6*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])/7
```

Rubi [A] time = 0.570737, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6016, 321, 216, 5994}

$$-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}+\frac{3x\sqrt{1-a^2x^2}}{112a^3}-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{8}{35}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```


```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(112*a^3) + (23*x^3*Sqrt[1 - a^2*x^2])/(840*a) - (a
*x^5*Sqrt[1 - a^2*x^2])/42 + (17*ArcSin[a*x])/(560*a^4) - (2*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x])/(35*a^4) - (x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(35*a^2)
+ (8*x^4*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/35 - (a^2*x^6*Sqrt[1 - a^2*x^2]*Ar
cTanh[a*x])/7
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
```

```
m, 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rubi steps

$$\int x^3\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx = -\left(a^2\int x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\right)+\int x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$$

$$= \frac{1}{5}x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{7}a^2x^6\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{5}\int\frac{x^3\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{20a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}-\frac{x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^2}+\frac{8}{35}x^4\sqrt{1-a^2x^2}\tanh^{-1}$$

$$= \frac{x\sqrt{1-a^2x^2}}{24a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}-\frac{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{15a^4}-\frac{x^2\sqrt{1-}}{}$$

$$= \frac{3x\sqrt{1-a^2x^2}}{112a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{11\sin^{-1}(ax)}{120a^4}-\frac{2\sqrt{1-a^2x^2}\tanh}{35a^4}$$

$$= \frac{3x\sqrt{1-a^2x^2}}{112a^3}+\frac{23x^3\sqrt{1-a^2x^2}}{840a}-\frac{1}{42}ax^5\sqrt{1-a^2x^2}+\frac{17\sin^{-1}(ax)}{560a^4}-\frac{2\sqrt{1-a^2x^2}\tanh}{35a^4}$$

Mathematica [A] time = 0.084368, size = 79, normalized size = 0.42

$$\frac{ax\left(-40a^4x^4+46a^2x^2+45\right)\sqrt{1-a^2x^2}-48\left(5a^2x^2+2\right)\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+51\sin^{-1}(ax)}{1680a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (a*x*Sqrt[1 - a^2*x^2]*(45 + 46*a^2*x^2 - 40*a^4*x^4) + 51*ArcSin[a*x] - 48
*(1 - a^2*x^2)^(5/2)*(2 + 5*a^2*x^2)*ArcTanh[a*x])/(1680*a^4)
```

Maple [C] time = 0.18, size = 140, normalized size = 0.8

$$-\frac{240\,\mathrm{Artanh}\,(ax)\,x^6a^6+40\,x^5a^5-384\,a^4x^4\mathrm{Artanh}\,(ax)-46\,x^3a^3+48\,a^2x^2\mathrm{Artanh}\,(ax)-45\,ax+96\,\mathrm{Artanh}\,(ax)}{1680\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/1680/a^4*(-(a*x-1)*(a*x+1))^(1/2)*(240*arctanh(a*x)*x^6*a^6+40*x^5*a^5-3
84*a^4*x^4*arctanh(a*x)-46*x^3*a^3+48*a^2*x^2*arctanh(a*x)-45*a*x+96*arctan
h(a*x))+17/560*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)+I)/a^4-17/560*I*ln((a*x+1)/(
-a^2*x^2+1)^(1/2)-I)/a^4
```

Maxima [A] time = 1.46478, size = 248, normalized size = 1.33

$$-\frac{1}{1680}a\left(\frac{5\left(\frac{8\left(-a^2x^2+1\right)^{\frac{5}{2}}x}{a^2}-\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{3\sqrt{-a^2x^2+1}x}{a^2}-\frac{3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2}\right)}{a^2}-\frac{12\left(2\left(-a^2x^2+1\right)^{\frac{3}{2}}x+3\sqrt{-a^2x^2+1}x+\frac{3\arcsi}{\sqrt{}}\right.}{a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/1680*a*(5*(8*(-a^2*x^2 + 1)^(5/2)*x/a^2 - 2*(-a^2*x^2 + 1)^(3/2)*x/a^2 -
 3*sqrt(-a^2*x^2 + 1)*x/a^2 - 3*arcsin(a^2*x/sqrt(a^2))/(sqrt(a^2)*a^2))/a^
2 - 12*(2*(-a^2*x^2 + 1)^(3/2)*x + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a^2*x/
sqrt(a^2))/sqrt(a^2))/a^4) - 1/35*(5*(-a^2*x^2 + 1)^(5/2)*x^2/a^2 + 2*(-a^2
*x^2 + 1)^(5/2)/a^4)*arctanh(a*x)
```

Fricas [A] time = 1.42565, size = 247, normalized size = 1.33

$$-\frac{\left(40\,a^5x^5-46\,a^3x^3-45\,ax+24\left(5\,a^6x^6-8\,a^4x^4+a^2x^2+2\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+102\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{1680\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/1680*((40*a^5*x^5 - 46*a^3*x^3 - 45*a*x + 24*(5*a^6*x^6 - 8*a^4*x^4 + a^
2*x^2 + 2)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1) + 102*arctan((sqrt
(-a^2*x^2 + 1) - 1)/(a*x)))/a^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27316, size = 255, normalized size = 1.37

$$-\frac{\left(\frac{15\left(a^{2}x^{2}-1\right)^{3}\sqrt{-a^{2}x^{2}+1}+42\left(a^{2}x^{2}-1\right)^{2}\sqrt{-a^{2}x^{2}+1}-35\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}}{a^{2}}-\frac{7\left(3\left(a^{2}x^{2}-1\right)^{2}\sqrt{-a^{2}x^{2}+1}-5\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\right)}{a^{2}}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{210\,a^{2}}-\frac{\sqrt{-a^{2}x^{2}+1}\left(2\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/210*((15*(a^2*x^2 - 1)^3*sqrt(-a^2*x^2 + 1) + 42*(a^2*x^2 - 1)^2*sqrt(-a
^2*x^2 + 1) - 35*(-a^2*x^2 + 1)^(3/2))/a^2 - 7*(3*(a^2*x^2 - 1)^2*sqrt(-a^2
*x^2 + 1) - 5*(-a^2*x^2 + 1)^(3/2))/a^2)*log(-(a*x + 1)/(a*x - 1))/a^2 - 1/
1680*(sqrt(-a^2*x^2 + 1)*(2*(20*a^4*x^2 - 23*a^2)*x^2 - 45)*x - 51*arcsin(a
*x)*sgn(a)/abs(a))/a^3
```

3.449 $\int x^2 \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=243

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^3}+\frac{\sqrt{1-a^2x^2}}{16a^3}-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\tanh$$

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^3) + (1 - a^2*x^2)^(3/2)/(72*a^3) - (1 - a^2*x^2)^(
5/2)/(30*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^2) + (7*x^3*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/24 - (a^2*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/6 -
 (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^3) - ((I/16)*PolyL
og[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^3
```

Rubi [A] time = 0.572297, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6010, 6016, 261, 5950, 266, 43}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^3}+\frac{\sqrt{1-a^2x^2}}{16a^3}-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\tanh$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(16*a^3) + (1 - a^2*x^2)^(3/2)/(72*a^3) - (1 - a^2*x^2)^(
5/2)/(30*a^3) - (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(16*a^2) + (7*x^3*Sqrt[1
 - a^2*x^2]*ArcTanh[a*x])/24 - (a^2*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/6 -
 (ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a^3) - ((I/16)*PolyL
og[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a^3 + ((I/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a^3
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6016

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> -Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a +
b*ArcTanh[c*x])^p)/(c^2*d*m), x] + (Dist[(b*f*p)/(c*m), Int[((f*x)^(m - 1)*
```

```
(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(f^2*(m - 1))/
(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d + e*x^2], x], x]
) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0] && GtQ[
m, 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx = -\left(a^2\int x^4\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\right)+\int x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$$

$$= \frac{1}{4}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{4}\int\frac{x^2\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-$$

$$= -\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8a^2}+\frac{7}{24}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{6}a^2x^5\sqrt{1-a^2x^2}\tanh^{-1}(ax$$

$$= -\frac{\sqrt{1-a^2x^2}}{8a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{6}a^2x^5\sqrt{1-a^2}$$

$$= \frac{\sqrt{1-a^2x^2}}{48a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{36a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3\sqrt{1-a^2}$$

$$= \frac{\sqrt{1-a^2x^2}}{16a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{72a^3}-\frac{\left(1-a^2x^2\right)^{5/2}}{30a^3}-\frac{x\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{16a^2}+\frac{7}{24}x^3\sqrt{1-a^2}$$

Mathematica [A] time = 0.906854, size = 224, normalized size = 0.92

$$-45i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+45i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-24a^4x^4\sqrt{1-a^2x^2}+38a^2x^2\sqrt{1-a^2x^2}+31\sqrt{1-a^2x^2}-$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (31*Sqrt[1 - a^2*x^2] + 38*a^2*x^2*Sqrt[1 - a^2*x^2] - 24*a^4*x^4*Sqrt[1 -
a^2*x^2] - 45*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 210*a^3*x^3*Sqrt[1 - a^2
*x^2]*ArcTanh[a*x] - 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (45*I)*Ar
cTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (45*I)*ArcTanh[a*x]*Log[1 + I/E^ArcT
anh[a*x]] - (45*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (45*I)*PolyLog[2, I/E^
ArcTanh[a*x]])/(720*a^3)
```

Maple [A] time = 0.195, size = 195, normalized size = 0.8

$$-\frac{120\,\mathrm{Artanh}\,(ax)\,x^5a^5+24\,x^4a^4-210\,a^3x^3\mathrm{Artanh}\,(ax)-38\,a^2x^2+45\,ax\mathrm{Artanh}\,(ax)-31}{720\,a^3}\sqrt{-(ax-1)\,(ax+1)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/720/a^3*(-(a*x-1)*(a*x+1))^(1/2)*(120*arctanh(a*x)*x^5*a^5+24*x^4*a^4-21
0*a^3*x^3*arctanh(a*x)-38*a^2*x^2+45*a*x*arctanh(a*x)-31)-1/16*I*ln(1+I*(a*
x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)/a^3+1/16*I*ln(1-I*(a*x+1)/(-a^2*x^2+1
)^(1/2))*arctanh(a*x)/a^3-1/16*I*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3+
1/16*I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}x^2\,\mathrm{artanh}\,(ax)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^2x^4-x^2\right)\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^4 - x^2)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x**2*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}} x^2 \operatorname{artanh}\left(ax\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*x^2*arctanh(a*x), x)
```

3.450 $\int x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=81

$$\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3x\sqrt{1-a^2x^2}}{40a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\sin^{-1}(ax)}{40a^2}$$

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(40*a) + (x*(1 - a^2*x^2)^(3/2))/(20*a) + (3*ArcSin
[a*x])/(40*a^2) - ((1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*a^2)
```

Rubi [A] time = 0.0575997, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5994, 195, 216}

$$\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3x\sqrt{1-a^2x^2}}{40a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\sin^{-1}(ax)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(40*a) + (x*(1 - a^2*x^2)^(3/2))/(20*a) + (3*ArcSin
[a*x])/(40*a^2) - ((1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*a^2)
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx = -\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{\int\left(1-a^2x^2\right)^{3/2}dx}{5a}$$
$$=\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\int\sqrt{1-a^2x^2}\,dx}{20a}$$
$$=\frac{3x\sqrt{1-a^2x^2}}{40a}+\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}+\frac{3\int\frac{1}{\sqrt{1-a^2x^2}}dx}{40a}$$
$$=\frac{3x\sqrt{1-a^2x^2}}{40a}+\frac{x\left(1-a^2x^2\right)^{3/2}}{20a}+\frac{3\sin^{-1}(ax)}{40a^2}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5a^2}$$

Mathematica [A] time = 0.0648409, size = 61, normalized size = 0.75

$$\frac{ax\left(5-2a^2x^2\right)\sqrt{1-a^2x^2}-8\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+3\sin^{-1}(ax)}{40a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (a*x*(5 - 2*a^2*x^2)*Sqrt[1 - a^2*x^2] + 3*ArcSin[a*x] - 8*(1 - a^2*x^2)^(5
/2)*ArcTanh[a*x])/(40*a^2)
```

Maple [C] time = 0.195, size = 120, normalized size = 1.5

$$-\frac{8a^4x^4\text{Artanh}(ax)+2x^3a^3-16a^2x^2\text{Artanh}(ax)-5ax+8\text{Artanh}(ax)}{40a^2}\sqrt{-(ax-1)(ax+1)}+\frac{\frac{3i}{40}}{a^2}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/40/a^2*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)+2*x^3*a^3-16*a^2
*x^2*arctanh(a*x)-5*a*x+8*arctanh(a*x))+3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2
)+I)/a^2-3/40*I*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-I)/a^2
```

Maxima [A] time = 1.43464, size = 103, normalized size = 1.27

$$-\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}\text{artanh}(ax)}{5a^2}+\frac{2\left(-a^2x^2+1\right)^{\frac{3}{2}}x+3\sqrt{-a^2x^2+1}x+\frac{3\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}}}{40a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] -1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/a^2 + 1/40*(2*(-a^2*x^2 + 1)^(3/2)*x
 + 3*sqrt(-a^2*x^2 + 1)*x + 3*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2))/a
```

Fricas [A] time = 1.44603, size = 204, normalized size = 2.52

$$-\frac{\left(2\,a^3x^3-5\,ax+4\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{40\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] -1/40*((2*a^3*x^3 - 5*a*x + 4*(a^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x
 - 1)))*sqrt(-a^2*x^2 + 1) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/a^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral(x*(-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [A] time = 1.181, size = 116, normalized size = 1.43

$$-\frac{\left(a^2x^2-1\right)^2\sqrt{-a^2x^2+1}\log\left(-\frac{ax+1}{ax-1}\right)}{10\,a^2}-\frac{\left(2\,a^2x^2-5\right)\sqrt{-a^2x^2+1}x-\frac{3\arcsin(ax)\mathrm{sgn}(a)}{|a|}}{40\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] -1/10*(a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1))/a^2 - 1/
40*((2*a^2*x^2 - 5)*sqrt(-a^2*x^2 + 1)*x - 3*arcsin(a*x)*sgn(a)/abs(a))/a
```

3.451 $\int \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=189

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x\sqrt{}$$

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rubi [A] time = 0.0859679, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}x\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```


```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```


Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax) + \frac{3}{4}\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$$
$$= \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(a$$
$$= \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(a$$

Mathematica [A] time = 0.584002, size = 176, normalized size = 0.93

$$\frac{-9i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+9i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-2a^2x^2\sqrt{1-a^2x^2}+11\sqrt{1-a^2x^2}-6a^3x^3\sqrt{1-a^2x^2}\tan}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*
x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcTanh[
a*x]])/(24*a)
```

Maple [A] time = 0.253, size = 173, normalized size = 0.9

$$-\frac{6\,a^3x^3\text{Artanh}\,(ax)+2\,a^2x^2-15\,ax\text{Artanh}\,(ax)-11}{24\,a}\sqrt{-a^2x^2+1}-\frac{\frac{3\,i}{8}\text{Artanh}\,(ax)}{a}\ln\left(1+i\,(ax+1)\,\frac{1}{\sqrt{-a^2x^2+1}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+1
)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a*a
rctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\text{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

3.452 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x}\,dx$

Optimal. Leaf size=144

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{6}ax\sqrt{1-a^2x^2}+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}($$

```
[Out] -(a*x*Sqrt[1 - a^2*x^2])/6 - (7*ArcSin[a*x])/6 + Sqrt[1 - a^2*x^2]*ArcTanh[
a*x] + ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/3 - 2*ArcTanh[a*x]*ArcTanh[Sqrt[1
 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.23155, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6018, 216, 5994, 195}

$$\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{6}ax\sqrt{1-a^2x^2}+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

```
[Out] -(a*x*Sqrt[1 - a^2*x^2])/6 - (7*ArcSin[a*x])/6 + Sqrt[1 - a^2*x^2]*ArcTanh[
a*x] + ((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/3 - 2*ArcTanh[a*x]*ArcTanh[Sqrt[1
 - a*x]/Sqrt[1 + a*x]] + PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])] - PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 5994

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(2*e*(q
 + 1)), x] + Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTanh[c*x
])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[c^2*d + e, 0] &&
GtQ[p, 0] && NeQ[q, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x}\,dx &= -\left(a^2\int x\sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx\\ &= \sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)-\frac{1}{3}a\int \sqrt{1-a^2x^2}\,dx-a\int \frac{1}{\sqrt{1-\ldots}}\\ &= -\frac{1}{6}ax\sqrt{1-a^2x^2}-\sin^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)-2\ldots\\ &= -\frac{1}{6}ax\sqrt{1-a^2x^2}-\frac{7}{6}\sin^{-1}(ax)+\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{3}\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)-\ldots\end{aligned}$$

Mathematica [A] time = 0.211454, size = 143, normalized size = 0.99

$$\frac{1}{6}\left(6\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-6\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-ax\sqrt{1-a^2x^2}-2a^2x^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)+8\sqrt{1-a^2x^2}\ldots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x,x]
```

```
[Out] (-(a*x*Sqrt[1 - a^2*x^2]) - 14*ArcTan[Tanh[ArcTanh[a*x]/2]] + 8*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x] - 2*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*ArcTanh[
a*x]*Log[1 - E^(-ArcTanh[a*x])] - 6*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 + 6*PolyLog[2, -E^(-ArcTanh[a*x])] - 6*PolyLog[2, E^(-ArcTanh[a*x])])/6
```

Maple [A] time = 0.2, size = 132, normalized size = 0.9

$$-\frac{2a^2x^2\text{Artanh}(ax)+ax-8\,\text{Artanh}(ax)}{6}\sqrt{-(ax-1)(ax+1)}-\frac{7}{3}\arctan\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)-\text{dilog}\left((ax+1)\frac{\ldots}{\sqrt{-a\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x)
```

```
[Out] -1/6*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x-8*arctanh(a*x))-7
/3*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))-dilog((a*x+1)/(-a^2*x^2+1)^(1/2))-dil
og(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\operatorname{artanh}\left(ax\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}} \operatorname{atanh}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x, x)
```

3.453 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^2}\,dx$

Optimal. Leaf size=179

$$\frac{3}{2}ia\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}ia\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-\right.$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (a^2*x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/2 + 3*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*Arc
Tanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + ((3*I)/2)*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - ((3*I)/2)*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]]
```

Rubi [A] time = 0.284488, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6008, 266, 63, 208, 5950, 5942}

$$\frac{3}{2}ia\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{2}ia\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-a\tanh^{-1}\left(\sqrt{1-\right.$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/2 - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (a^2*x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/2 + 3*a*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*Arc
Tanh[a*x] - a*ArcTanh[Sqrt[1 - a^2*x^2]] + ((3*I)/2)*a*PolyLog[2, ((-I)*Sqr
t[1 - a*x])/Sqrt[1 + a*x]] - ((3*I)/2)*a*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[
1 + a*x]]
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^2}\,dx = -\left(a^2\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}\,dx$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{1}{2}a^2\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx-a^2\int \frac{\tanh^{-1}}{\sqrt{1-}}$$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+3a\tan^{-1}\Bigg($$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+3a\tan^{-1}\Bigg($$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+3a\tan^{-1}\Bigg($$

$$= -\frac{1}{2}a\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{1}{2}a^2x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+3a\tan^{-1}\Bigg($$

Mathematica [A] time = 0.578006, size = 168, normalized size = 0.94

$$\frac{1}{2}\Bigg(3ia\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-3ia\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-a\sqrt{1-a^2x^2}+a^2(-x)\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{2\sqrt{1}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^2,x]
```

```
[Out] (-(a*Sqrt[1 - a^2*x^2]) - (2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - a^2*x*Sqrt
[1 - a^2*x^2]*ArcTanh[a*x] + (3*I)*a*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]]
 - (3*I)*a*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 2*a*Log[Tanh[ArcTanh[a*
x]/2]] + (3*I)*a*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (3*I)*a*PolyLog[2, I/E^A
rcTanh[a*x]])/2
```

Maple [A] time = 0.221, size = 205, normalized size = 1.2

$$-\frac{a^2x^2\text{Artanh}\,(ax)+ax+2\,\text{Artanh}\,(ax)}{2\,x}\sqrt{-(ax-1)\,(ax+1)}+a\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}-1\right)-a\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x)
```

```
[Out] -1/2*(a^2*x^2*arctanh(a*x)+a*x+2*arctanh(a*x))*(-(a*x-1)*(a*x+1))^(1/2)/x+a
*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)-a*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*I*
a*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)-3/2*I*a*ln(1-I*(a*x+1)/(-
a^2*x^2+1)^(1/2))*arctanh(a*x)+3/2*I*a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2)
)-3/2*I*a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\,\text{artanh}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\,(ax)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\text{atanh}\,(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**2,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^2,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^2, x)
```

3.454 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^3}\,dx$

Optimal. Leaf size=168

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}}{2x^2}$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) + a^2*ArcSin[a*x] - a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + 3*a^2*ArcTanh[a*x]*Ar
cTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt
[1 + a*x])])/2 + (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/2
```

Rubi [A] time = 0.387339, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6026, 264, 6018, 216}

$$-\frac{3}{2}a^2\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{3}{2}a^2\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{2x}-a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(2*x) + a^2*ArcSin[a*x] - a^2*Sqrt[1 - a^2*x^2]*ArcT
anh[a*x] - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(2*x^2) + 3*a^2*ArcTanh[a*x]*Ar
cTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - (3*a^2*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt
[1 + a*x])])/2 + (3*a^2*PolyLog[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/2
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
```

```
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^3}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx$$

$$= -a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}+a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx-a^2\int \frac{\text{ta}}{x}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{x}+a^2\sin^{-1}(ax)-a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+2$$

$$= -\frac{a\sqrt{1-a^2x^2}}{2x}+a^2\sin^{-1}(ax)-a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}+3$$

Mathematica [A] time = 0.942964, size = 158, normalized size = 0.94

$$\frac{1}{8}a^2\left(-12\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)+12\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)-8\sqrt{1-a^2x^2}\tanh^{-1}(ax)+2\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^3,x]
```

```
[Out] (a^2*(16*ArcTan[Tanh[ArcTanh[a*x]/2]] - 8*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] -
2*Coth[ArcTanh[a*x]/2] - ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^2 - 12*ArcTanh[a
*x]*Log[1 - E^(-ArcTanh[a*x])] + 12*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])]
 - 12*PolyLog[2, -E^(-ArcTanh[a*x])] + 12*PolyLog[2, E^(-ArcTanh[a*x])] - A
rcTanh[a*x]*Sech[ArcTanh[a*x]/2]^2 + 2*Tanh[ArcTanh[a*x]/2]))/8
```

Maple [A] time = 0.204, size = 145, normalized size = 0.9

$$-\frac{2a^2x^2\text{Artanh}(ax)+ax+\text{Artanh}(ax)}{2x^2}\sqrt{-(ax-1)(ax+1)}+2a^2\arctan\left(\frac{ax+1}{\sqrt{-a^2x^2+1}}\right)+\frac{3a^2}{2}\text{dilog}\left((ax+1)\frac{}{\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x)
```

```
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(2*a^2*x^2*arctanh(a*x)+a*x+arctanh(a*x))/x^2
+2*a^2*arctan((a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*dilog((a*x+1)/(-a^2*x^2+1
)^(1/2))+3/2*a^2*dilog(1+(a*x+1)/(-a^2*x^2+1)^(1/2))+3/2*a^2*arctanh(a*x)*l
n(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\operatorname{artanh}\left(ax\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**3,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^3, x)
```

3.455 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^4}\,dx$

Optimal. Leaf size=189

$$-ia^3\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)+ia^3\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{7}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{a^2\sqrt{1-a^2x^2}\tanh}{x}$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(6*x^2) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (
(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3) - 2*a^3*ArcTan[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]*ArcTanh[a*x] + (7*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6 - I*a^3*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + I*a^3*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]]
```

Rubi [A] time = 0.31227, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6014, 6008, 266, 47, 63, 208, 5950}

$$-ia^3\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)+ia^3\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{7}{6}a^3\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)+\frac{a^2\sqrt{1-a^2x^2}\tanh}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```


```
[Out] -(a*Sqrt[1 - a^2*x^2])/(6*x^2) + (a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/x - (
(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/(3*x^3) - 2*a^3*ArcTan[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]*ArcTanh[a*x] + (7*a^3*ArcTanh[Sqrt[1 - a^2*x^2]])/6 - I*a^3*Poly
Log[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + I*a^3*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]]
```


Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^4}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^4}\,dx$$

$$= -\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}+\frac{1}{3}a\int \frac{\sqrt{1-a^2x^2}}{x^3}\,dx-a^2\int \frac{\tanh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx+a^4\int \frac{\text{ta}}{\sqrt{\ }}$$

$$= \frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan^{-1}\Big(\frac{\sqrt{}}{\sqrt{}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan^{-1}\Big(\frac{\sqrt{}}{\sqrt{}}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{6x^2}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x}-\frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{3x^3}-2a^3\tan^{-1}\Big(\frac{\sqrt{}}{\sqrt{}}$$

Mathematica [A] time = 1.1868, size = 199, normalized size = 1.05

$$-\frac{\left(1-a^2x^2\right)^{3/2}\left(\log\left(\tanh\left(\frac{1}{2}\tanh^{-1}(ax)\right)\right)\left(\sinh\left(3\tanh^{-1}(ax)\right)-\frac{3ax}{\sqrt{1-a^2x^2}}\right)+8\tanh^{-1}(ax)+2\sinh\left(2\tanh^{-1}(ax)\right)\right)}{24x^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^4,x]
```

```
[Out] -(a^3*(-((Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(a*x)) + I*ArcTanh[a*x]*Log[1 - I
/E^ArcTanh[a*x]] - I*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + Log[Tanh[ArcT
anh[a*x]/2]] + I*PolyLog[2, (-I)/E^ArcTanh[a*x]] - I*PolyLog[2, I/E^ArcTanh
[a*x]])) - ((1 - a^2*x^2)^(3/2)*(8*ArcTanh[a*x] + 2*Sinh[2*ArcTanh[a*x]] +
Log[Tanh[ArcTanh[a*x]/2]]*((-3*a*x)/Sqrt[1 - a^2*x^2] + Sinh[3*ArcTanh[a*x]
])))/(24*x^3)
```

Maple [A] time = 0.223, size = 220, normalized size = 1.2

$$\frac{8\,a^2x^2\mathrm{Artanh}\,(ax)-ax-2\,\mathrm{Artanh}\,(ax)}{6\,x^3}\sqrt{-(ax-1)\,(ax+1)}-\frac{7\,a^3}{6}\ln\left((ax+1)\frac{1}{\sqrt{-a^2x^2+1}}-1\right)+\frac{7\,a^3}{6}\ln\left(1+(ax+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x)
```

```
[Out] 1/6*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^2*x^2*arctanh(a*x)-a*x-2*arctanh(a*x))/x^
3-7/6*a^3*ln((a*x+1)/(-a^2*x^2+1)^(1/2)-1)+7/6*a^3*ln(1+(a*x+1)/(-a^2*x^2+1
)^(1/2))-I*a^3*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)+I*a^3*ln(1-I
*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)-I*a^3*dilog(1+I*(a*x+1)/(-a^2*x^2
+1)^(1/2))+I*a^3*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\mathrm{artanh}\,(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(ax-1)(ax+1))^{\frac{3}{2}}\operatorname{atanh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**4,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^4, x)
```

3.456 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^5}\,dx$

Optimal. Leaf size=191

$$\frac{3}{8}a^4\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{8}a^4\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(12*x^3) + (11*a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (5*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]
)/(8*x^2) - (3*a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + (
3*a^4*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 - (3*a^4*PolyLog[2, Sqr
t[1 - a*x]/Sqrt[1 + a*x]])/8
```

Rubi [A] time = 0.56626, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6026, 271, 264, 6018}

$$\frac{3}{8}a^4\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{3}{8}a^4\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(12*x^3) + (11*a^3*Sqrt[1 - a^2*x^2])/(24*x) - (Sqrt
[1 - a^2*x^2]*ArcTanh[a*x])/(4*x^4) + (5*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]
)/(8*x^2) - (3*a^4*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + (
3*a^4*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/8 - (3*a^4*PolyLog[2, Sqr
t[1 - a*x]/Sqrt[1 + a*x]])/8
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
```

```
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^5}\,dx = -\left(a^2\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^3}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx$$

$$= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^4}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^2}-\frac{1}{3}\int \frac{\tanh^{-1}(ax)}{x^5\sqrt{1-a^2x^2}}\,dx+\frac{1}{3}a\int$$

$$= -\frac{a\sqrt{1-a^2x^2}}{9x^3}+\frac{a^3\sqrt{1-a^2x^2}}{x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{2x^2}-$$

$$= -\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{5a^3\sqrt{1-a^2x^2}}{18x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{8x^2}$$

$$= -\frac{a\sqrt{1-a^2x^2}}{12x^3}+\frac{11a^3\sqrt{1-a^2x^2}}{24x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}+\frac{5a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax}{8x^2}$$

Mathematica [A] time = 3.60173, size = 282, normalized size = 1.48

$$\frac{1}{192}a\left(72a^3\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-72a^3\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+\frac{16a^2\sqrt{1-a^2x^2}\sinh^4\left(\frac{1}{2}\tanh^{-1}(ax)\right)}{x}-\frac{16\sqrt{}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^5,x]
```

```
[Out] (a*(40*a^3*Coth[ArcTanh[a*x]/2] + 18*a^3*ArcTanh[a*x]*Csch[ArcTanh[a*x]/2]^
2 - (a^4*x*Csch[ArcTanh[a*x]/2]^4)/Sqrt[1 - a^2*x^2] - 3*a^3*ArcTanh[a*x]*C
sch[ArcTanh[a*x]/2]^4 + 72*a^3*ArcTanh[a*x]*Log[1 - E^(-ArcTanh[a*x])] - 72
*a^3*ArcTanh[a*x]*Log[1 + E^(-ArcTanh[a*x])] + 72*a^3*PolyLog[2, -E^(-ArcTa
nh[a*x])] - 72*a^3*PolyLog[2, E^(-ArcTanh[a*x])] + 18*a^3*ArcTanh[a*x]*Sech
[ArcTanh[a*x]/2]^2 + 3*a^3*ArcTanh[a*x]*Sech[ArcTanh[a*x]/2]^4 - (16*Sqrt[1
```

```
- a^2*x^2]*Sinh[ArcTanh[a*x]/2]^4)/x^3 + (16*a^2*Sqrt[1 - a^2*x^2]*Sinh[Ar
cTanh[a*x]/2]^4)/x - 40*a^3*Tanh[ArcTanh[a*x]/2]))/192
```

Maple [A] time = 0.184, size = 164, normalized size = 0.9

$$\frac{11\,x^3a^3+15\,a^2x^2\mathrm{Artanh}\,(ax)-2\,ax-6\,\mathrm{Artanh}\,(ax)}{24\,x^4}\sqrt{-(ax-1)\,(ax+1)}-\frac{3\,a^4\mathrm{Artanh}\,(ax)}{8}\ln\left(1+(ax+1)\frac{1}{\sqrt{-a^2x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x)
```

```
[Out] 1/24*(-(a*x-1)*(a*x+1))^(1/2)*(11*x^3*a^3+15*a^2*x^2*arctanh(a*x)-2*a*x-6*a
rctanh(a*x))/x^4-3/8*a^4*arctanh(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*
a^4*polylog(2,-(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*a^4*arctanh(a*x)*ln(1-(a*x+1
)/(-a^2*x^2+1)^(1/2))+3/8*a^4*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\mathrm{artanh}\,(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(-(ax-1)\,(ax+1))^{\frac{3}{2}}\,\mathrm{atanh}\,(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**5,x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2+1)^{\frac{3}{2}}\operatorname{artanh}(ax)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^5, x)
```

3.457 $\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^6}\,dx$

Optimal. Leaf size=94

$$\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}$$

```
[Out] (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - (a*(1 - a^2*x^2)^(3/2))/(20*x^4) - ((1
 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]]
)/40
```

Rubi [A] time = 0.100864, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {6008, 266, 47, 63, 208}

$$\frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - (a*(1 - a^2*x^2)^(3/2))/(20*x^4) - ((1
 - a^2*x^2)^(5/2)*ArcTanh[a*x])/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]]
)/40
```

Rule 6008

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a
+ b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(m + 1), Int[(f*x)^(m +
 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d
, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^6}\,dx = -\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{5}a\int \frac{\left(1-a^2x^2\right)^{3/2}}{x^5}\,dx$$

$$= -\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{10}a\,\text{Subst}\left(\int \frac{\left(1-a^2x\right)^{3/2}}{x^3}\,dx,x,x^2\right)$$

$$= -\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{\sqrt{1-a^2x}}{x^2}\,dx,x,x$$

$$= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}+\frac{1}{80}\left(3a^5\right)\text{Subst}\left(\int -\right.$$

$$= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \right.$$

$$= \frac{3a^3\sqrt{1-a^2x^2}}{40x^2}-\frac{a\left(1-a^2x^2\right)^{3/2}}{20x^4}-\frac{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}{5x^5}-\frac{3}{40}a^5\tanh^{-1}\left(\sqrt{1-a^2\right.}$$

Mathematica [A] time = 0.0818989, size = 104, normalized size = 1.11

$$\left(\frac{a^3}{8x^2}-\frac{a}{20x^4}\right)\sqrt{1-a^2x^2}-\frac{3}{40}a^5\log\left(\sqrt{1-a^2x^2}+1\right)-\frac{\sqrt{1-a^2x^2}\left(a^2x^2-1\right)^2\tanh^{-1}(ax)}{5x^5}+\frac{3}{40}a^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^6,x]
```

```
[Out] (-a/(20*x^4) + a^3/(8*x^2))*Sqrt[1 - a^2*x^2] - (Sqrt[1 - a^2*x^2]*(-1 + a^
2*x^2)^2*ArcTanh[a*x])/(5*x^5) + (3*a^5*Log[x])/40 - (3*a^5*Log[1 + Sqrt[1
- a^2*x^2]])/40
```

Maple [A] time = 0.169, size = 116, normalized size = 1.2

$$-\frac{8\,a^4x^4\text{Artanh}\,(ax)-5\,x^3a^3-16\,a^2x^2\text{Artanh}\,(ax)+2\,ax+8\,\text{Artanh}\,(ax)}{40\,x^5}\sqrt{-(ax-1)\,(ax+1)}+\frac{3\,a^5}{40}\ln\left((ax+1)\frac{}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x)
```

```
[Out] -1/40*(-(a*x-1)*(a*x+1))^(1/2)*(8*a^4*x^4*arctanh(a*x)-5*x^3*a^3-16*a^2*x^2
*arctanh(a*x)+2*a*x+8*arctanh(a*x))/x^5+3/40*a^5*ln((a*x+1)/(-a^2*x^2+1)^(1
```

```
/2)-1)-3/40*a^5*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [A] time = 1.45061, size = 170, normalized size = 1.81

$$\frac{1}{40}\left(\left(-a^2x^2+1\right)^{\frac{3}{2}}a^4-3\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+3\sqrt{-a^2x^2+1}a^4+\frac{\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2}{x^2}-\frac{2\left(-a^2x^2+1\right)^{\frac{5}{2}}}{x^4}\right)a-\frac{\left(-a^2x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/40*((-a^2*x^2 + 1)^(3/2)*a^4 - 3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/
abs(x)) + 3*sqrt(-a^2*x^2 + 1)*a^4 + (-a^2*x^2 + 1)^(5/2)*a^2/x^2 - 2*(-a^2
*x^2 + 1)^(5/2)/x^4)*a - 1/5*(-a^2*x^2 + 1)^(5/2)*arctanh(a*x)/x^5
```

Fricas [A] time = 1.32788, size = 204, normalized size = 2.17

$$\frac{3\,a^5x^5\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)+\left(5\,a^3x^3-2\,ax-4\left(a^4x^4-2\,a^2x^2+1\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{40\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/40*(3*a^5*x^5*log((sqrt(-a^2*x^2 + 1) - 1)/x) + (5*a^3*x^3 - 2*a*x - 4*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1))/x^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.82267, size = 374, normalized size = 3.98

$$-\frac{3}{80}\,a^5\log\left(\sqrt{-a^2x^2+1}+1\right)+\frac{3}{80}\,a^5\log\left(-\sqrt{-a^2x^2+1}+1\right)+\frac{1}{320}\left(\frac{\left(a^6-\frac{5\left(\sqrt{-a^2x^2+1}|a|+a\right)^2a^2}{x^2}+\frac{10\left(\sqrt{-a^2x^2+1}|a|+a\right)^4}{a^2x^4}\right)a^{10}x^5}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^5|a|}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -3/80*a^5*log(sqrt(-a^2*x^2 + 1) + 1) + 3/80*a^5*log(-sqrt(-a^2*x^2 + 1) +
1) + 1/320*((a^6 - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2*a^2/x^2 + 10*(sqrt(-
a^2*x^2 + 1)*abs(a) + a)^4/(a^2*x^4))*a^10*x^5/((sqrt(-a^2*x^2 + 1)*abs(a)
+ a)^5*abs(a)) - (10*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^8/x - 5*(sqrt(-a^2*x
^2 + 1)*abs(a) + a)^3*a^4/x^3 + (sqrt(-a^2*x^2 + 1)*abs(a) + a)^5/x^5)/(a^4
*abs(a)))*log(-(a*x + 1)/(a*x - 1)) - 1/40*(5*(-a^2*x^2 + 1)^(3/2)*a^5 - 3*
sqrt(-a^2*x^2 + 1)*a^5)/(a^4*x^4)
```

3.458 $\int \frac{(1-a^2x^2)^{3/2}\tanh^{-1}(ax)}{x^7}\,dx$

Optimal. Leaf size=243

$$\frac{1}{16}a^6\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{16}a^6\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{31a^5\sqrt{1-a^2x^2}}{720x}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{a\sqrt{1-a^2x^2}}{30x^5}-\frac{a^4\sqrt{1-}}{}$$

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(30*x^5) + (19*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (3
1*a^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6)
 + (7*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) - (a^4*Sqrt[1 - a^2*x^2]
*ArcTanh[a*x])/(16*x^2) - (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 +
a*x]])/8 + (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 - (a^6*PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rubi [A] time = 0.772984, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6014, 6010, 6026, 271, 264, 6018}

$$\frac{1}{16}a^6\text{PolyLog}\left(2,-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{16}a^6\text{PolyLog}\left(2,\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{31a^5\sqrt{1-a^2x^2}}{720x}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{a\sqrt{1-a^2x^2}}{30x^5}-\frac{a^4\sqrt{1-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

```
[Out] -(a*Sqrt[1 - a^2*x^2])/(30*x^5) + (19*a^3*Sqrt[1 - a^2*x^2])/(360*x^3) + (3
1*a^5*Sqrt[1 - a^2*x^2])/(720*x) - (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(6*x^6)
 + (7*a^2*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/(24*x^4) - (a^4*Sqrt[1 - a^2*x^2]
*ArcTanh[a*x])/(16*x^2) - (a^6*ArcTanh[a*x]*ArcTanh[Sqrt[1 - a*x]/Sqrt[1 +
a*x]])/8 + (a^6*PolyLog[2, -(Sqrt[1 - a*x]/Sqrt[1 + a*x])])/16 - (a^6*PolyL
og[2, Sqrt[1 - a*x]/Sqrt[1 + a*x]])/16
```

Rule 6014

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTanh[c*x])^p, x], x] - Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTanh[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
 && EqQ[c^2*d + e, 0] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p
, 1] && IntegerQ[q]))
```

Rule 6010

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcTanh[c
*x]))/(f*(m + 2)), x] + (Dist[d/(m + 2), Int[((f*x)^m*(a + b*ArcTanh[c*x]))
/Sqrt[d + e*x^2], x], x] - Dist[(b*c*d)/(f*(m + 2)), Int[(f*x)^(m + 1)/Sqrt
[d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0
] && NeQ[m, -2]
```

Rule 6026

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*A
rcTanh[c*x])^p)/(d*f*(m + 1)), x] + (-Dist[(b*c*p)/(f*(m + 1)), Int[((f*x)^
```

```
(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/Sqrt[d + e*x^2], x], x] + Dist[(c^2*(
m + 2))/(f^2*(m + 1)), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/Sqrt[d +
e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ
[p, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6018

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x
_Symbol] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTanh[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])/Sqrt[d], x] + (Simp[(b*PolyLog[2, -(Sqrt[1 - c*x]/Sqrt[1 + c*x])])/Sqr
t[d], x] - Simp[(b*PolyLog[2, Sqrt[1 - c*x]/Sqrt[1 + c*x]])/Sqrt[d], x]) /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}{x^7}\,dx &= -\left(a^2\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^5}\,dx\right)+\int \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{x^7}\,dx \\
&= -\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{5x^6}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{3x^4}-\frac{1}{5}\int \frac{\tanh^{-1}(ax)}{x^7\sqrt{1-a^2x^2}}\,dx+\frac{1}{5}a\int \\
&= -\frac{a\sqrt{1-a^2x^2}}{25x^5}+\frac{a^3\sqrt{1-a^2x^2}}{9x^3}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}+\frac{a^2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{4x^4}- \\
&= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{3a^3\sqrt{1-a^2x^2}}{100x^3}+\frac{2a^5\sqrt{1-a^2x^2}}{9x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}+\frac{7a^2\sqrt{1-}}{} \\
&= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}-\frac{13a^5\sqrt{1-a^2x^2}}{200x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}+\frac{7a^2\sqrt{1}}{} \\
&= -\frac{a\sqrt{1-a^2x^2}}{30x^5}+\frac{19a^3\sqrt{1-a^2x^2}}{360x^3}+\frac{31a^5\sqrt{1-a^2x^2}}{720x}-\frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{6x^6}+\frac{7a^2\sqrt{1}}{}
\end{aligned}$$

Mathematica [A] time = 6.80202, size = 474, normalized size = 1.95

$$\frac{360a^6x^3\sqrt{1-a^2x^2}\text{PolyLog}\left(2,-e^{-\tanh^{-1}(ax)}\right)-360a^6x^3\sqrt{1-a^2x^2}\text{PolyLog}\left(2,e^{-\tanh^{-1}(ax)}\right)+360a^6x^3\sqrt{1-a^2x^2}\tanh}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/x^7,x]
```

```
[Out] (82*a^7*x^4*Csch[ArcTanh[a*x]/2]^2 + 90*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a
*x]*Csch[ArcTanh[a*x]/2]^2 + 4*a^7*x^4*Csch[ArcTanh[a*x]/2]^4 - 3*a^7*x^4*C
```

```
sch[ArcTanh[a*x]/2]^6 - 15*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Csch[ArcT
anh[a*x]/2]^6 + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 - E^(-ArcT
anh[a*x])] - 360*a^6*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]*Log[1 + E^(-ArcTanh
[a*x])] + 360*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, -E^(-ArcTanh[a*x])] - 36
0*a^6*x^3*Sqrt[1 - a^2*x^2]*PolyLog[2, E^(-ArcTanh[a*x])] + 328*a^5*x^2*(-1
 + a^2*x^2)*Sinh[ArcTanh[a*x]/2]^2 + 360*a^4*x*(1 - a^2*x^2)^(3/2)*ArcTanh[
a*x]*Sinh[ArcTanh[a*x]/2]^2 + 64*a^3*Sinh[ArcTanh[a*x]/2]^4 - 128*a^5*x^2*S
inh[ArcTanh[a*x]/2]^4 + 64*a^7*x^4*Sinh[ArcTanh[a*x]/2]^4 - (192*a*(-1 + a^
2*x^2)^3*Sinh[ArcTanh[a*x]/2]^6)/x^2 - (960*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x
]*Sinh[ArcTanh[a*x]/2]^6)/x^3)/(5760*x^3*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.203, size = 184, normalized size = 0.8

$$-\frac{-31\,x^5a^5+45\,a^4x^4\mathrm{Artanh}\,(ax)-38\,x^3a^3-210\,a^2x^2\mathrm{Artanh}\,(ax)+24\,ax+120\,\mathrm{Artanh}\,(ax)}{720\,x^6}\sqrt{-(ax-1)(ax+1)}-\frac{a^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x)
```

```
[Out] -1/720*(-(a*x-1)*(a*x+1))^(1/2)*(-31*x^5*a^5+45*a^4*x^4*arctanh(a*x)-38*x^3
*a^3-210*a^2*x^2*arctanh(a*x)+24*a*x+120*arctanh(a*x))/x^6-1/16*a^6*arctanh
(a*x)*ln(1+(a*x+1)/(-a^2*x^2+1)^(1/2))-1/16*a^6*polylog(2,-(a*x+1)/(-a^2*x^
2+1)^(1/2))+1/16*a^6*arctanh(a*x)*ln(1-(a*x+1)/(-a^2*x^2+1)^(1/2))+1/16*a^6
*polylog(2,(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}}\,\mathrm{artanh}\,(ax)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x)/x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x)/x**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}\left(ax\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x)/x^7,x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)/x^7, x)
```

3.459 $\int (1 - a^2x^2)^{5/2} \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=233

$$-\frac{5i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{5i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{(1-a^2x^2)^{5/2}}{30a}+\frac{5(1-a^2x^2)^{3/2}}{72a}+\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{1}{6}x(1-a^2x^2)^{5/2}\text{ta}$$

```
[Out] (5*Sqrt[1 - a^2*x^2])/(16*a) + (5*(1 - a^2*x^2)^(3/2))/(72*a) + (1 - a^2*x^
2)^(5/2)/(30*a) + (5*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 + (5*x*(1 - a^2*x
^2)^(3/2)*ArcTanh[a*x])/24 + (x*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/6 - (5*Ar
cTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a) - (((5*I)/16)*PolyLog
[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((5*I)/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a
```

Rubi [A] time = 0.12105, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{5i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{5i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{16a}+\frac{(1-a^2x^2)^{5/2}}{30a}+\frac{5(1-a^2x^2)^{3/2}}{72a}+\frac{5\sqrt{1-a^2x^2}}{16a}+\frac{1}{6}x(1-a^2x^2)^{5/2}\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

```
[Out] (5*Sqrt[1 - a^2*x^2])/(16*a) + (5*(1 - a^2*x^2)^(3/2))/(72*a) + (1 - a^2*x^
2)^(5/2)/(30*a) + (5*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/16 + (5*x*(1 - a^2*x
^2)^(3/2)*ArcTanh[a*x])/24 + (x*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])/6 - (5*Ar
cTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(8*a) - (((5*I)/16)*PolyLog
[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((5*I)/16)*PolyLog[2, (I*Sqrt
[1 - a*x])/Sqrt[1 + a*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)+\frac{5}{6}\int \left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)\,dx$$
$$= \frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5}{24}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{1}{6}x\left(1-a^2x^2\right)^{5/2}\text{ta}$$
$$= \frac{5\sqrt{1-a^2x^2}}{16a}+\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{5}{24}x(1$$
$$= \frac{5\sqrt{1-a^2x^2}}{16a}+\frac{5\left(1-a^2x^2\right)^{3/2}}{72a}+\frac{\left(1-a^2x^2\right)^{5/2}}{30a}+\frac{5}{16}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{5}{24}x(1$$

Mathematica [A] time = 1.09752, size = 224, normalized size = 0.96

$$\frac{-225i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+225i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+24a^4x^4\sqrt{1-a^2x^2}-98a^2x^2\sqrt{1-a^2x^2}+299\sqrt{1-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^(5/2)*ArcTanh[a*x],x]
```

```
[Out] (299*Sqrt[1 - a^2*x^2] - 98*a^2*x^2*Sqrt[1 - a^2*x^2] + 24*a^4*x^4*Sqrt[1 -
 a^2*x^2] + 495*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - 390*a^3*x^3*Sqrt[1 - a
^2*x^2]*ArcTanh[a*x] + 120*a^5*x^5*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (225*I)
*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (225*I)*ArcTanh[a*x]*Log[1 + I/E^
ArcTanh[a*x]] - (225*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (225*I)*PolyLog[2
, I/E^ArcTanh[a*x]])/(720*a)
```

Maple [A] time = 0.281, size = 193, normalized size = 0.8

$$\frac{120\,\text{Artanh}(ax)x^5a^5+24\,x^4a^4-390\,a^3x^3\text{Artanh}(ax)-98\,a^2x^2+495\,ax\text{Artanh}(ax)+299}{720\,a}\sqrt{-a^2x^2+1}-\frac{\frac{5i}{16}\text{Artar}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(5/2)*arctanh(a*x),x)
```

```
[Out] 1/720*(120*arctanh(a*x)*x^5*a^5+24*x^4*a^4-390*a^3*x^3*arctanh(a*x)-98*a^2*
x^2+495*a*x*arctanh(a*x)+299)*(-a^2*x^2+1)^(1/2)/a-5/16*I/a*arctanh(a*x)*ln
(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2
*x^2+1)^(1/2))-5/16*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+5/16*I/a*dilo
g(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{5}{2}}\text{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4x^4-2\,a^2x^2+1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(5/2)*atanh(a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2x^2+1\right)^{\frac{5}{2}}\text{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(5/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(5/2)*arctanh(a*x), x)
```

3.460 $\int \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=189

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}$$

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rubi [A] time = 0.0890607, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a}+\frac{\left(1-a^2x^2\right)^{3/2}}{12a}+\frac{3\sqrt{1-a^2x^2}}{8a}+\frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)+\frac{3}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*Sqrt[1 - a^2*x^2])/(8*a) + (1 - a^2*x^2)^(3/2)/(12*a) + (3*x*Sqrt[1 - a^
2*x^2]*ArcTanh[a*x])/8 + (x*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])/4 - (3*ArcTan
[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(4*a) - (((3*I)/8)*PolyLog[2, (
(-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + (((3*I)/8)*PolyLog[2, (I*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(1-a^2x^2\right)^{3/2} \tanh^{-1}(ax)\,dx = \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax) + \frac{3}{4}\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$$
$$= \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax) -$$
$$= \frac{3\sqrt{1-a^2x^2}}{8a} + \frac{\left(1-a^2x^2\right)^{3/2}}{12a} + \frac{3}{8}x\sqrt{1-a^2x^2}\tanh^{-1}(ax) + \frac{1}{4}x\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax) -$$

Mathematica [A] time = 0.0599153, size = 176, normalized size = 0.93

$$\frac{-9i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)+9i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)-2a^2x^2\sqrt{1-a^2x^2}+11\sqrt{1-a^2x^2}-6a^3x^3\sqrt{1-a^2x^2}\tanh^{-}}{24a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - a^2*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (11*Sqrt[1 - a^2*x^2] - 2*a^2*x^2*Sqrt[1 - a^2*x^2] + 15*a*x*Sqrt[1 - a^2*x
^2]*ArcTanh[a*x] - 6*a^3*x^3*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] - (9*I)*ArcTanh
[a*x]*Log[1 - I/E^ArcTanh[a*x]] + (9*I)*ArcTanh[a*x]*Log[1 + I/E^ArcTanh[a*
x]] - (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, I/E^ArcTanh[
a*x]])/(24*a)
```

Maple [A] time = 0.251, size = 173, normalized size = 0.9

$$-\frac{6a^3x^3\text{Artanh}(ax)+2a^2x^2-15ax\text{Artanh}(ax)-11}{24a}\sqrt{-a^2x^2+1}-\frac{\frac{3i}{8}\text{Artanh}(ax)}{a}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{3i}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/24*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a*x*arctanh(a*x)-11)*(-a^2*x^2+1
)^(1/2)/a-3/8*I/a*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I/a*a
rctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I/a*dilog(1+I*(a*x+1)/(-
a^2*x^2+1)^(1/2))+3/8*I/a*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}\,\text{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\,\text{atanh}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2x^2+1\right)^{\frac{3}{2}}\,\text{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*x^2 + 1)^(3/2)*arctanh(a*x), x)
```

3.461 $\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=143

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rubi [A] time = 0.0540413, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5942, 5950}

$$-\frac{i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}+\frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/2 - (ArcTan[Sq
rt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a - ((I/2)*PolyLog[2, ((-I)*Sqrt[1
 - a*x])/Sqrt[1 + a*x]])/a + ((I/2)*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a
*x]])/a
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)\,dx &= \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)+\frac{1}{2}\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{2a}+\frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)-\frac{\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a}-\frac{i\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{i\text{L}}{}\end{aligned}$$

Mathematica [A] time = 0.066716, size = 117, normalized size = 0.82

$$\frac{\sqrt{1-a^2x^2}\left(-\frac{i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+ax\tanh^{-1}(\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/E^Ar
cTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]]
- PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [A] time = 0.21, size = 152, normalized size = 1.1

$$\frac{ax\text{Artanh}(ax)+1}{2a}\sqrt{-a^2x^2+1}-\frac{\frac{i}{2}\text{Artanh}(ax)}{a}\ln\left(1+i(ax+1)\frac{1}{\sqrt{-a^2x^2+1}}\right)+\frac{\frac{i}{2}\text{Artanh}(ax)}{a}\ln\left(1-i(ax+1)\frac{}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x),x)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-a^2*x^2+1)^(1/2)/a-1/2*I/a*arctanh(a*x)*ln(1+I*(
a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)
^(1/2))-1/2*I/a*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*dilog(1-I*(a*
x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a^2x^2+1}\,\text{artanh}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2x^2+1}\,\text{artanh}(ax),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x), x)
```

3.462 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2}{3a\sqrt{1-a^2x^2}}-\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] -1/(9*a*(1 - a^2*x^2)^(3/2)) - 2/(3*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])
/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0513439, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5960, 5958}

$$-\frac{2}{3a\sqrt{1-a^2x^2}}-\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*(1 - a^2*x^2)^(3/2)) - 2/(3*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])
/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*Sqrt[1 - a^2*x^2])
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2}{3}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,dx\\ &= -\frac{1}{9a\left(1-a^2x^2\right)^{3/2}}-\frac{2}{3a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0512738, size = 49, normalized size = 0.55

$$-\frac{-6a^2x^2+\left(6a^3x^3-9ax\right)\tanh^{-1}(ax)+7}{9a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] -(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x])/(9*a*(1 - a^2*x^2)^(3/
2))
```

Maple [A] time = 0.208, size = 59, normalized size = 0.7

$$-\frac{6\,a^3x^3\text{Artanh}\,(ax)-6\,a^2x^2-9\,ax\text{Artanh}\,(ax)+7}{9\,a\left(a^2x^2-1\right)^2}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] -1/9/a*(-a^2*x^2+1)^(1/2)*(6*a^3*x^3*arctanh(a*x)-6*a^2*x^2-9*a*x*arctanh(a
*x)+7)/(a^2*x^2-1)^2
```

Maxima [A] time = 0.962393, size = 100, normalized size = 1.12

$$-\frac{1}{9}\,a\left(\frac{6}{\sqrt{-a^2x^2+1}a^2}+\frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}\right)+\frac{1}{3}\left(\frac{2\,x}{\sqrt{-a^2x^2+1}}+\frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\right)\text{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(6/(sqrt(-a^2*x^2 + 1)*a^2) + 1/((-a^2*x^2 + 1)^(3/2)*a^2)) + 1/3*(2
*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)
```

Fricas [A] time = 1.45211, size = 161, normalized size = 1.81

$$\frac{\left(12\,a^2x^2-3\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)\sqrt{-a^2x^2+1}}{18\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1)) - 14)*sq
rt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}\,(ax)}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [A] time = 1.32977, size = 122, normalized size = 1.37

$$-\frac{\left(2\,a^2x^2-3\right)\sqrt{-a^2x^2+1}x\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(a^2x^2-1\right)^2}-\frac{6\,a^2x^2-7}{9\left(a^2x^2-1\right)\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*a^2*x^2 - 3)*sqrt(-a^2*x^2 + 1)*x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^
2 - 1)^2 - 1/9*(6*a^2*x^2 - 7)/((a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a)
```

3.463 $\int \frac{\tanh^{-1}(ax)}{(1-a^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{8}{15a\sqrt{1-a^2x^2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}$$

```
[Out] -1/(25*a*(1 - a^2*x^2)^(5/2)) - 4/(45*a*(1 - a^2*x^2)^(3/2)) - 8/(15*a*Sqrt
[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*
x])/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.0806322, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5960, 5958}

$$-\frac{8}{15a\sqrt{1-a^2x^2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] -1/(25*a*(1 - a^2*x^2)^(5/2)) - 4/(45*a*(1 - a^2*x^2)^(3/2)) - 8/(15*a*Sqrt
[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*
x])/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*Sqrt[1 - a^2*x^2])
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx &= -\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}+\frac{4}{5}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx\\ &= -\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{8}{15}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{3/2}}\,d\\ &= -\frac{1}{25a\left(1-a^2x^2\right)^{5/2}}-\frac{4}{45a\left(1-a^2x^2\right)^{3/2}}-\frac{8}{15a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/2}}+\frac{4x\tanh^{-1}(ax)}{15\left(1-a^2x^2\right)^{3/2}}+\frac{8x\,\mathrm{t}}{15}\end{aligned}$$

Mathematica [A] time = 0.0618712, size = 65, normalized size = 0.49

$$\frac{-120a^4x^4+260a^2x^2+15ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)-149}{225a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-149 + 260*a^2*x^2 - 120*a^4*x^4 + 15*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*Ar
cTanh[a*x])/(225*a*(1 - a^2*x^2)^(5/2))
```

Maple [A] time = 0.172, size = 79, normalized size = 0.6

$$-\frac{120\,\mathrm{Artanh}\,(ax)\,x^5a^5-120\,x^4a^4-300\,a^3x^3\mathrm{Artanh}\,(ax)+260\,a^2x^2+225\,ax\mathrm{Artanh}\,(ax)-149}{225\,a\left(a^2x^2-1\right)^3}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -1/225/a*(-a^2*x^2+1)^(1/2)*(120*arctanh(a*x)*x^5*a^5-120*x^4*a^4-300*a^3*x
^3*arctanh(a*x)+260*a^2*x^2+225*a*x*arctanh(a*x)-149)/(a^2*x^2-1)^3
```

Maxima [A] time = 0.977163, size = 146, normalized size = 1.1

$$-\frac{1}{225}a\left(\frac{120}{\sqrt{-a^2x^2+1}a^2}+\frac{20}{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2}+\frac{9}{\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2}\right)+\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}+\frac{3\,x}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\right)\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/225*a*(120/(sqrt(-a^2*x^2 + 1)*a^2) + 20/((-a^2*x^2 + 1)^(3/2)*a^2) + 9/
((-a^2*x^2 + 1)^(5/2)*a^2)) + 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2
+ 1)^(3/2) + 3*x/(-a^2*x^2 + 1)^(5/2))*arctanh(a*x)
```

Fricas [A] time = 1.54921, size = 220, normalized size = 1.65

$$\frac{\left(240\,a^4x^4-520\,a^2x^2-15\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+298\right)\sqrt{-a^2x^2+1}}{450\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*log
(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4 + 3*a
```

```
^3*x^2 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27205, size = 154, normalized size = 1.16

$$-\frac{\sqrt{-a^2x^2+1}\left(4\left(2\,a^4x^2-5\,a^2\right)x^2+15\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^2x^2-1\right)^3}+\frac{20\,a^2x^2-120\left(a^2x^2-1\right)^2-29}{225\left(a^2x^2-1\right)^2\sqrt{-a^2x^2+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a^2*x^2 + 1)*(4*(2*a^4*x^2 - 5*a^2)*x^2 + 15)*x*log(-(a*x + 1)/
(a*x - 1))/(a^2*x^2 - 1)^3 + 1/225*(20*a^2*x^2 - 120*(a^2*x^2 - 1)^2 - 29)/
((a^2*x^2 - 1)^2*sqrt(-a^2*x^2 + 1)*a)
```

3.464 $\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{16}{35a\sqrt{1-a^2x^2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{3/2}}+$$

```
[Out] -1/(49*a*(1 - a^2*x^2)^(7/2)) - 6/(175*a*(1 - a^2*x^2)^(5/2)) - 8/(105*a*(1
 - a^2*x^2)^(3/2)) - 16/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(7*(1 -
 a^2*x^2)^(7/2)) + (6*x*ArcTanh[a*x])/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTa
nh[a*x])/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*Sqrt[1 - a^2*x^
2])
```

Rubi [A] time = 0.113113, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5960, 5958}

$$-\frac{16}{35a\sqrt{1-a^2x^2}}-\frac{8}{105a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{1}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{3/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```


```
[Out] -1/(49*a*(1 - a^2*x^2)^(7/2)) - 6/(175*a*(1 - a^2*x^2)^(5/2)) - 8/(105*a*(1
 - a^2*x^2)^(3/2)) - 16/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x])/(7*(1 -
 a^2*x^2)^(7/2)) + (6*x*ArcTanh[a*x])/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTa
nh[a*x])/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*Sqrt[1 - a^2*x^
2])
```


Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{9/2}}\,dx = -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}} + \frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}} + \frac{6}{7}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}} - \frac{6}{175a\left(1-a^2x^2\right)^{5/2}} + \frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{5/2}} + \frac{24}{35}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}$$

$$= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}} - \frac{6}{175a\left(1-a^2x^2\right)^{5/2}} - \frac{8}{105a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35\left(1-a^2x^2\right)^{5/2}} +$$

$$= -\frac{1}{49a\left(1-a^2x^2\right)^{7/2}} - \frac{6}{175a\left(1-a^2x^2\right)^{5/2}} - \frac{8}{105a\left(1-a^2x^2\right)^{3/2}} - \frac{16}{35a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)}{7\left(1-a^2x^2\right)^{7/2}} + $$

Mathematica [A] time = 0.0762956, size = 81, normalized size = 0.46

$$\frac{1680a^6x^6 - 5320a^4x^4 + 5726a^2x^2 - 105ax\left(16a^6x^6 - 56a^4x^4 + 70a^2x^2 - 35\right)\tanh^{-1}(ax) - 2161}{3675a\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6 - 105*a*x*(-35 + 70*a^2
*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x])/(3675*a*(1 - a^2*x^2)^(7/2))
```

Maple [A] time = 0.174, size = 99, normalized size = 0.6

$$-\frac{1680\,\mathrm{Artanh}(ax)\,x^7a^7 - 1680\,x^6a^6 - 5880\,\mathrm{Artanh}(ax)\,x^5a^5 + 5320\,x^4a^4 + 7350\,a^3x^3\mathrm{Artanh}(ax) - 5726\,a^2x^2 - 3675}{3675\,a\left(a^2x^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] -1/3675/a*(-a^2*x^2+1)^(1/2)*(1680*arctanh(a*x)*x^7*a^7-1680*x^6*a^6-5880*a
rctanh(a*x)*x^5*a^5+5320*x^4*a^4+7350*a^3*x^3*arctanh(a*x)-5726*a^2*x^2-367
5*a*x*arctanh(a*x)+2161)/(a^2*x^2-1)^4
```

Maxima [A] time = 0.987439, size = 189, normalized size = 1.07

$$-\frac{1}{3675}a\left(\frac{1680}{\sqrt{-a^2x^2+1}a^2} + \frac{280}{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2} + \frac{126}{\left(-a^2x^2+1\right)^{\frac{5}{2}}a^2} + \frac{75}{\left(-a^2x^2+1\right)^{\frac{7}{2}}a^2}\right) + \frac{1}{35}\left(\frac{16x}{\sqrt{-a^2x^2+1}} + \frac{8x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3675*a*(1680/(sqrt(-a^2*x^2 + 1)*a^2) + 280/((-a^2*x^2 + 1)^(3/2)*a^2) +
 126/((-a^2*x^2 + 1)^(5/2)*a^2) + 75/((-a^2*x^2 + 1)^(7/2)*a^2)) + 1/35*(16
```

```
*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 + 1)^(5/2)
 + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)
```

Fricas [A] time = 1.75634, size = 285, normalized size = 1.61

$$\frac{\left(3360\,a^{6}x^{6}-10640\,a^{4}x^{4}+11452\,a^{2}x^{2}-105\left(16\,a^{7}x^{7}-56\,a^{5}x^{5}+70\,a^{3}x^{3}-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-4322\right)\sqrt{-a^{2}x^{2}+1}}{7350\left(a^{9}x^{8}-4\,a^{7}x^{6}+6\,a^{5}x^{4}-4\,a^{3}x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7350*(3360*a^6*x^6 - 10640*a^4*x^4 + 11452*a^2*x^2 - 105*(16*a^7*x^7 - 56
*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1)) - 4322)*sqrt(-a^2
*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29118, size = 186, normalized size = 1.05

$$-\frac{\sqrt{-a^{2}x^{2}+1}\left(2\left(4\left(2\,a^{6}x^{2}-7\,a^{4}\right)x^{2}+35\,a^{2}\right)x^{2}-35\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{70\left(a^{2}x^{2}-1\right)^{4}}-\frac{126\,a^{2}x^{2}+1680\left(a^{2}x^{2}-1\right)^{3}-280\left(a^{2}x^{2}-1\right)^{2}-}{3675\left(a^{2}x^{2}-1\right)^{3}\sqrt{-a^{2}x^{2}+1}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] -1/70*sqrt(-a^2*x^2 + 1)*(2*(4*(2*a^6*x^2 - 7*a^4)*x^2 + 35*a^2)*x^2 - 35)*
x*log(-(a*x + 1)/(a*x - 1))/(a^2*x^2 - 1)^4 - 1/3675*(126*a^2*x^2 + 1680*(a
^2*x^2 - 1)^3 - 280*(a^2*x^2 - 1)^2 - 201)/((a^2*x^2 - 1)^3*sqrt(-a^2*x^2 +
 1)*a)
```

3.465 $\int \left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=291

$$-\frac{3ic^2\sqrt{1-a^2x^2}\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}+\frac{3ic^2\sqrt{1-a^2x^2}\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}-\frac{3c^2\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a\sqrt{c-a^2cx^2}}+$$

```
[Out] (3*c*Sqrt[c - a^2*c*x^2])/(8*a) + (c - a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sqr
t[c - a^2*c*x^2]*ArcTanh[a*x])/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x])/4
 - (3*c^2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x
])/(4*a*Sqrt[c - a^2*c*x^2]) - (((3*I)/8)*c^2*Sqrt[1 - a^2*x^2]*PolyLog[2,
((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (((3*I)/8)*c
^2*Sqrt[1 - a^2*x^2]*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c
 - a^2*c*x^2])
```

Rubi [A] time = 0.149635, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5942, 5954, 5950}

$$-\frac{3ic^2\sqrt{1-a^2x^2}\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}+\frac{3ic^2\sqrt{1-a^2x^2}\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{8a\sqrt{c-a^2cx^2}}-\frac{3c^2\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{4a\sqrt{c-a^2cx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] (3*c*Sqrt[c - a^2*c*x^2])/(8*a) + (c - a^2*c*x^2)^(3/2)/(12*a) + (3*c*x*Sqr
t[c - a^2*c*x^2]*ArcTanh[a*x])/8 + (x*(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x])/4
 - (3*c^2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x
])/(4*a*Sqrt[c - a^2*c*x^2]) - (((3*I)/8)*c^2*Sqrt[1 - a^2*x^2]*PolyLog[2,
((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (((3*I)/8)*c
^2*Sqrt[1 - a^2*x^2]*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c
 - a^2*c*x^2])
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
```

```
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)\,dx = \frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\tanh^{-1}(ax)+\frac{1}{4}(3c)\int \sqrt{c-a^2cx^2}\tanh^{-1}(ax)\,dx$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\tan$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\tan$$

$$= \frac{3c\sqrt{c-a^2cx^2}}{8a}+\frac{\left(c-a^2cx^2\right)^{3/2}}{12a}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\tan$$

Mathematica [A] time = 0.653791, size = 206, normalized size = 0.71

$$-\frac{c\sqrt{c-a^2cx^2}\left(9i\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-9i\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2a^2x^2\sqrt{1-a^2x^2}-11\sqrt{1-a^2x^2}+6a^3x^3}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcTanh[a*x],x]
```

```
[Out] -(c*Sqrt[c - a^2*c*x^2]*(-11*Sqrt[1 - a^2*x^2] + 2*a^2*x^2*Sqrt[1 - a^2*x^2
] - 15*a*x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x] + 6*a^3*x^3*Sqrt[1 - a^2*x^2]*Arc
Tanh[a*x] + (9*I)*ArcTanh[a*x]*Log[1 - I/E^ArcTanh[a*x]] - (9*I)*ArcTanh[a*
x]*Log[1 + I/E^ArcTanh[a*x]] + (9*I)*PolyLog[2, (-I)/E^ArcTanh[a*x]] - (9*I
)*PolyLog[2, I/E^ArcTanh[a*x]]))/(24*a*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.319, size = 345, normalized size = 1.2

$$-\frac{c\left(6a^3x^3\text{Artanh}(ax)+2a^2x^2-15ax\text{Artanh}(ax)-11\right)}{24a}\sqrt{-(ax-1)(ax+1)c}+\frac{\frac{3i}{8}c\text{Artanh}(ax)}{a(ax+1)(ax-1)}\sqrt{-a^2x^2+1}\sqrt{-(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x)
```

```
[Out] -1/24*c/a*(-(a*x-1)*(a*x+1)*c)^(1/2)*(6*a^3*x^3*arctanh(a*x)+2*a^2*x^2-15*a
*x*arctanh(a*x)-11)+3/8*I*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*
(a*x+1)*c)^(1/2)*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I*c/a/
(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*arctanh(a*x)*
ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*I*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*
x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-3/8*I
*c/a/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)*dilog(1-
I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\,\text{artanh}\left(ax\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(-c(ax-1)(ax+1))^{\frac{3}{2}}\,\text{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*atanh(a*x),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-a^2cx^2+c\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arctanh(a*x), x)
```

3.466 $\int \sqrt{c-a^2cx^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=235

$$-\frac{ic\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{ic\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)-$$

```
[Out] Sqrt[c - a^2*c*x^2]/(2*a) + (x*Sqrt[c - a^2*c*x^2]*ArcTanh[a*x])/2 - (c*Sqr
t[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*Sqrt[c
- a^2*c*x^2]) - ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLo
g[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.101953, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {5942, 5954, 5950}

$$-\frac{ic\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{ic\sqrt{1-a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a\sqrt{c-a^2cx^2}}+\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```


```
[Out] Sqrt[c - a^2*c*x^2]/(2*a) + (x*Sqrt[c - a^2*c*x^2]*ArcTanh[a*x])/2 - (c*Sqr
t[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*Sqrt[c
- a^2*c*x^2]) - ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])/
Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + ((I/2)*c*Sqrt[1 - a^2*x^2]*PolyLo
g[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```


Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\tanh^{-1}(ax)\,dx = \frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{1}{2}c\int\frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx$$
$$=\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)+\frac{\left(c\sqrt{1-a^2x^2}\right)\int\frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{2\sqrt{c-a^2cx^2}}$$
$$=\frac{\sqrt{c-a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c-a^2cx^2}\tanh^{-1}(ax)-\frac{c\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{ic\sqrt{1}}{}$$

Mathematica [A] time = 0.282841, size = 119, normalized size = 0.51

$$\frac{\sqrt{c\left(1-a^2x^2\right)}\left(-\frac{i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\left(1+ie^{-\tanh^{-1}(ax)}\right)\right)\right)}{\sqrt{1-a^2x^2}}+ax\tanh^{-1}(\right.}{2a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcTanh[a*x],x]
```

```
[Out] (Sqrt[c*(1 - a^2*x^2)]*(1 + a*x*ArcTanh[a*x] - (I*(ArcTanh[a*x]*(Log[1 - I/
E^ArcTanh[a*x]] - Log[1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*
x]] - PolyLog[2, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [A] time = 0.414, size = 319, normalized size = 1.4

$$\frac{ax\text{Artanh}\,(ax)+1}{2\,a}\sqrt{-(ax-1)\,(ax+1)\,c}+\frac{\frac{i}{2}\text{Artanh}\,(ax)}{a\,(ax+1)\,(ax-1)}\sqrt{-(ax-1)\,(ax+1)\,c}\sqrt{-a^2x^2+1}\ln\left(1+i\,(ax+1)\,\frac{1}{\sqrt{-a^2x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x)
```

```
[Out] 1/2*(a*x*arctanh(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a+1/2*I/a*(-(a*x-1)*(a*
x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1+I*(a*x+1
)/(-a^2*x^2+1)^(1/2))-1/2*I/a*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2+
1)^(1/2)/(a*x-1)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+1/2*I/a*(-
(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2+1)^(1/2)/(a*x-1)*dilog(1+I*(a*x+
1)/(-a^2*x^2+1)^(1/2))-1/2*I/a*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)*(-a^2*x^2
+1)^(1/2)/(a*x-1)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\,\text{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-c\,(ax-1)\,(ax+1)}\,\text{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*atanh(a*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2cx^2+c}\,\text{artanh}\,(ax)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arctanh(a*x), x)
```

3.467 $\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=182

$$-\frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}$$

```
[Out] (-2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*
Sqrt[c - a^2*c*x^2]) - (I*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])
/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (I*Sqrt[1 - a^2*x^2]*PolyLog[2,
(I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0622075, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5954, 5950}

$$-\frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a\sqrt{c-a^2cx^2}}-\frac{2\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (-2*Sqrt[1 - a^2*x^2]*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/(a*
Sqrt[c - a^2*c*x^2]) - (I*Sqrt[1 - a^2*x^2]*PolyLog[2, ((-I)*Sqrt[1 - a*x])
/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2]) + (I*Sqrt[1 - a^2*x^2]*PolyLog[2,
(I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\sqrt{1-a^2x^2}\int \frac{\tanh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{1-a^2x^2}\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a\sqrt{c-a^2cx^2}}-\frac{i\sqrt{1-a^2x^2}\,\text{Li}_2\left(-\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}+\frac{i\sqrt{1-a^2x^2}\,\text{Li}_2\left(\frac{i\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.118544, size = 109, normalized size = 0.6

$$-\frac{i\sqrt{c\left(1-a^2x^2\right)}\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+\tanh^{-1}(ax)\left(\log\left(1-ie^{-\tanh^{-1}(ax)}\right)-\log\right.\right.}{ac\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] ((-I)*Sqrt[c*(1 - a^2*x^2)]*(ArcTanh[a*x]*(Log[1 - I/E^ArcTanh[a*x]] - Log[
1 + I/E^ArcTanh[a*x]]) + PolyLog[2, (-I)/E^ArcTanh[a*x]] - PolyLog[2, I/E^A
rcTanh[a*x]]))/(a*c*Sqrt[1 - a^2*x^2])
```

Maple [A] time = 0.362, size = 290, normalized size = 1.6

$$\frac{i\text{Artanh}\,(ax)}{\left(a^2x^2-1\right)ac}\ln\left(1+i\,(ax+1)\,\frac{1}{\sqrt{-a^2x^2+1}}\right)\sqrt{-a^2x^2+1}\sqrt{-\,(ax-1)\,(ax+1)\,c}-\frac{i\text{Artanh}\,(ax)}{\left(a^2x^2-1\right)ac}\ln\left(1-i\,(ax+1)\,\frac{}{\sqrt{-a^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] I*ln(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*arctanh(a*x)*(-a^2*x^2+1)^(1/2)*(-(a*x
-1)*(a*x+1)*c)^(1/2)/(a^2*x^2-1)/a/c-I*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*a
rctanh(a*x)*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a^2*x^2-1)/a/c+I
*dilog(1+I*(a*x+1)/(-a^2*x^2+1)^(1/2))*(-a^2*x^2+1)^(1/2)*(-(a*x-1)*(a*x+1)
*c)^(1/2)/(a^2*x^2-1)/a/c-I*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))*(-a^2*x^2
+1)^(1/2)*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a^2*x^2-1)/a/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\text{artanh}\,(ax)}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arctanh(a*x)/(a^2*c*x^2 - c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(-a^2*c*x^2 + c), x)
```

3.468 $\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}$$

```
[Out] -(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a*x])/(c*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.0289554, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5958}

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}-\frac{1}{ac\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] -(1/(a*c*Sqrt[c - a^2*c*x^2])) + (x*ArcTanh[a*x])/(c*Sqrt[c - a^2*c*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{1}{ac\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.0553198, size = 43, normalized size = 0.9

$$\frac{\sqrt{c-a^2cx^2}\left(1-ax\tanh^{-1}(ax)\right)}{ac^2\left(a^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(1 - a*x*ArcTanh[a*x]))/(a*c^2*(-1 + a^2*x^2))
```

Maple [A] time = 0.25, size = 74, normalized size = 1.5

$$-\frac{\operatorname{Artanh}(ax)-1}{2a(ax-1)c^2}\sqrt{-(ax-1)(ax+1)c}-\frac{\operatorname{Artanh}(ax)+1}{2a(ax+1)c^2}\sqrt{-(ax-1)(ax+1)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -1/2*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^2-1/2*(arctanh
(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6681, size = 115, normalized size = 2.4

$$-\frac{\sqrt{-a^2cx^2+c}\left(ax\log\left(-\frac{ax+1}{ax-1}\right)-2\right)}{2\left(a^3c^2x^2-ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*(a*x*log(-(a*x + 1)/(a*x - 1)) - 2)/(a^3*c^2*x^2
- a*c^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}(ax)}{\left(-c(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

Giac [A] time = 1.27115, size = 95, normalized size = 1.98

$$-\frac{\sqrt{-a^2cx^2+c}x\log\left(-\frac{ax+1}{ax-1}\right)}{2\left(a^2cx^2-c\right)c}-\frac{1}{\sqrt{-a^2cx^2+c}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*c*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1))/((a^2*c*x^2 - c)*c) -
 1/(sqrt(-a^2*c*x^2 + c)*a*c)
```

3.469 $\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}-\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}$$

```
[Out] -1/(9*a*c*(c - a^2*c*x^2)^(3/2)) - 2/(3*a*c^2*Sqrt[c - a^2*c*x^2]) + (x*Arc
Tanh[a*x])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c -
 a^2*c*x^2])
```

Rubi [A] time = 0.0656546, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5960, 5958}

$$-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}-\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*c*(c - a^2*c*x^2)^(3/2)) - 2/(3*a*c^2*Sqrt[c - a^2*c*x^2]) + (x*Arc
Tanh[a*x])/(3*c*(c - a^2*c*x^2)^(3/2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c -
 a^2*c*x^2])
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{5/2}}\,dx &= -\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{3/2}}\,dx}{3c}\\ &= -\frac{1}{9ac\left(c-a^2cx^2\right)^{3/2}}-\frac{2}{3ac^2\sqrt{c-a^2cx^2}}+\frac{x\tanh^{-1}(ax)}{3c\left(c-a^2cx^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.0685026, size = 64, normalized size = 0.61

$$-\frac{\sqrt{c-a^2cx^2}\left(-6a^2x^2+\left(6a^3x^3-9ax\right)\tanh^{-1}(ax)+7\right)}{9ac^3\left(a^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[c - a^2*c*x^2]*(7 - 6*a^2*x^2 + (-9*a*x + 6*a^3*x^3)*ArcTanh[a*x]))/
(9*a*c^3*(-1 + a^2*x^2)^2)
```

Maple [A] time = 0.256, size = 160, normalized size = 1.5

$$\frac{(ax+1)(-1+3\operatorname{Artanh}(ax))}{72\,a\,(ax-1)^2\,c^3}\sqrt{-(ax-1)(ax+1)c}-\frac{3\operatorname{Artanh}(ax)-3}{8\,a\,(ax-1)\,c^3}\sqrt{-(ax-1)(ax+1)c}-\frac{3\operatorname{Artanh}(ax)+3}{8\,a\,(ax+1)\,c^3}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] 1/72*(a*x+1)*(-1+3*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)^2/c^3
-3/8*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^3-3/8*(arctanh
(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^3+1/72*(a*x-1)*(1+3*arctanh
(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)^2/c^3
```

Maxima [A] time = 0.989851, size = 122, normalized size = 1.16

$$-\frac{1}{9}a\left(\frac{6}{\sqrt{-a^2cx^2+c}a^2c^2}+\frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}a^2c}\right)+\frac{1}{3}\left(\frac{2x}{\sqrt{-a^2cx^2+c}c^2}+\frac{x}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}c}\right)\operatorname{artanh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/9*a*(6/(sqrt(-a^2*c*x^2 + c)*a^2*c^2) + 1/((-a^2*c*x^2 + c)^(3/2)*a^2*c)
) + 1/3*(2*x/(sqrt(-a^2*c*x^2 + c)*c^2) + x/((-a^2*c*x^2 + c)^(3/2)*c))*arc
tanh(a*x)
```

Fricas [A] time = 1.5965, size = 180, normalized size = 1.71

$$\frac{\sqrt{-a^2cx^2+c}\left(12\,a^2x^2-3\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-14\right)}{18\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a^2*c*x^2 + c)*(12*a^2*x^2 - 3*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1
)/(a*x - 1)) - 14)/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{(-c(ax-1)(ax+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [A] time = 1.29052, size = 150, normalized size = 1.43

$$-\frac{\sqrt{-a^2cx^2+c}\left(\frac{2\,a^2x^2}{c}-\frac{3}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{6\left(a^2cx^2-c\right)^2}-\frac{6\,a^2cx^2-7\,c}{9\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a^2*c*x^2 + c)*(2*a^2*x^2/c - 3/c)*x*log(-(a*x + 1)/(a*x - 1))/(
a^2*c*x^2 - c)^2 - 1/9*(6*a^2*c*x^2 - 7*c)/((a^2*c*x^2 - c)*sqrt(-a^2*c*x^2
 + c)*a*c^2)
```

3.470 $\int \frac{\tanh^{-1}(ax)}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}$$

```
[Out] -1/(25*a*c*(c - a^2*c*x^2)^(5/2)) - 4/(45*a*c^2*(c - a^2*c*x^2)^(3/2)) - 8/
(15*a*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcTanh[a*x])/(5*c*(c - a^2*c*x^2)^(5/2
)) + (4*x*ArcTanh[a*x])/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcTanh[a*x])
/(15*c^3*Sqrt[c - a^2*c*x^2])
```

Rubi [A] time = 0.101478, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {5960, 5958}

$$-\frac{8}{15ac^3\sqrt{c-a^2cx^2}}-\frac{4}{45ac^2\left(c-a^2cx^2\right)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}+\frac{4x\tanh^{-1}(ax)}{15c^2\left(c-a^2cx^2\right)^{3/2}}-\frac{1}{25ac\left(c-a^2cx^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)}{5c\left(c-a^2cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/(25*a*c*(c - a^2*c*x^2)^(5/2)) - 4/(45*a*c^2*(c - a^2*c*x^2)^(3/2)) - 8/
(15*a*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcTanh[a*x])/(5*c*(c - a^2*c*x^2)^(5/2
)) + (4*x*ArcTanh[a*x])/(15*c^2*(c - a^2*c*x^2)^(3/2)) + (8*x*ArcTanh[a*x])
/(15*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{7/2}}\,dx = -\frac{1}{25ac(c-a^2cx^2)^{5/2}} + \frac{x\tanh^{-1}(ax)}{5c(c-a^2cx^2)^{5/2}} + \frac{4\int \frac{\tanh^{-1}(ax)}{(c-a^2cx^2)^{5/2}}dx}{5c}$$

$$= -\frac{1}{25ac(c-a^2cx^2)^{5/2}} - \frac{4}{45ac^2(c-a^2cx^2)^{3/2}} + \frac{x\tanh^{-1}(ax)}{5c(c-a^2cx^2)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2(c-a^2cx^2)^{3/2}} + \frac{8\int \frac{\tanh^{-1}}{(c-a^2cx}}{15c^2}$$

$$= -\frac{1}{25ac(c-a^2cx^2)^{5/2}} - \frac{4}{45ac^2(c-a^2cx^2)^{3/2}} - \frac{8}{15ac^3\sqrt{c-a^2cx^2}} + \frac{x\tanh^{-1}(ax)}{5c(c-a^2cx^2)^{5/2}} + \frac{4x\tanh^{-1}}{15c^2(c-a^2c}$$

Mathematica [A] time = 0.0820581, size = 80, normalized size = 0.51

$$\frac{\sqrt{c-a^2cx^2}\left(120a^4x^4-260a^2x^2-15ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)+149\right)}{225ac^4\left(a^2x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(149 - 260*a^2*x^2 + 120*a^4*x^4 - 15*a*x*(15 - 20*a^2
*x^2 + 8*a^4*x^4)*ArcTanh[a*x]))/(225*a*c^4*(-1 + a^2*x^2)^3)
```

Maple [A] time = 0.268, size = 250, normalized size = 1.6

$$-\frac{(ax+1)^2(-1+5\,\mathrm{Artanh}\,(ax))}{800\,a\,(ax-1)^3\,c^4}\sqrt{-(ax-1)(ax+1)c} + \frac{(5\,ax+5)(-1+3\,\mathrm{Artanh}\,(ax))}{288\,a\,(ax-1)^2\,c^4}\sqrt{-(ax-1)(ax+1)c} - \frac{5\,\mathrm{Arta}}{16\,a\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/800*(a*x+1)^2*(-1+5*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)^3
/c^4+5/288*(a*x+1)*(-1+3*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)
^2/c^4-5/16*(arctanh(a*x)-1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x-1)/c^4-5/16*
(arctanh(a*x)+1)*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)/c^4+5/288*(a*x-1)*(1+
3*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/a/(a*x+1)^2/c^4-1/800*(a*x-1)^2*
(1+5*arctanh(a*x))*(-(a*x-1)*(a*x+1)*c)^(1/2)/(a*x+1)^3/a/c^4
```

Maxima [A] time = 1.02541, size = 178, normalized size = 1.13

$$-\frac{1}{225}a\left(\frac{120}{\sqrt{-a^2cx^2+ca^2c^3}} + \frac{20}{(-a^2cx^2+c)^{\frac{3}{2}}a^2c^2} + \frac{9}{(-a^2cx^2+c)^{\frac{5}{2}}a^2c}\right) + \frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2cx^2+cc^3}} + \frac{4\,x}{(-a^2cx^2+c)^{\frac{3}{2}}c^2} + \frac{3}{(-a^2cx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/225*a*(120/(sqrt(-a^2*c*x^2 + c)*a^2*c^3) + 20/((-a^2*c*x^2 + c)^(3/2)*a
^2*c^2) + 9/((-a^2*c*x^2 + c)^(5/2)*a^2*c)) + 1/15*(8*x/(sqrt(-a^2*c*x^2 +
c)*c^3) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)*c)
)*arctanh(a*x)
```

Fricas [A] time = 1.59179, size = 244, normalized size = 1.55

$$\frac{\left(240\,a^{4}x^{4}-520\,a^{2}x^{2}-15\left(8\,a^{5}x^{5}-20\,a^{3}x^{3}+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)+298\right)\sqrt{-a^{2}cx^{2}+c}}{450\left(a^{7}c^{4}x^{6}-3\,a^{5}c^{4}x^{4}+3\,a^{3}c^{4}x^{2}-ac^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*a^4*x^4 - 520*a^2*x^2 - 15*(8*a^5*x^5 - 20*a^3*x^3 + 15*a*x)*log
(-(a*x + 1)/(a*x - 1)) + 298)*sqrt(-a^2*c*x^2 + c)/(a^7*c^4*x^6 - 3*a^5*c^4
*x^4 + 3*a^3*c^4*x^2 - a*c^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31219, size = 201, normalized size = 1.28

$$-\frac{\sqrt{-a^{2}cx^{2}+c}\left(4\left(\frac{2\,a^{4}x^{2}}{c}-\frac{5\,a^{2}}{c}\right)x^{2}+\frac{15}{c}\right)x\log\left(-\frac{ax+1}{ax-1}\right)}{30\left(a^{2}cx^{2}-c\right)^{3}}-\frac{120\left(a^{2}cx^{2}-c\right)^{2}-20\left(a^{2}cx^{2}-c\right)c+9\,c^{2}}{225\left(a^{2}cx^{2}-c\right)^{2}\sqrt{-a^{2}cx^{2}+c}ac^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c - 5*a^2/c)*x^2 + 15/c)*x*log(-(a
*x + 1)/(a*x - 1))/(a^2*c*x^2 - c)^3 - 1/225*(120*(a^2*c*x^2 - c)^2 - 20*(a
^2*c*x^2 - c)*c + 9*c^2)/((a^2*c*x^2 - c)^2*sqrt(-a^2*c*x^2 + c)*a*c^3)
```

3.471 $\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx$

Optimal. Leaf size=158

$$-\frac{i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\mathrm{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\mathrm{PolyI}}{}$$

```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```

Rubi [A] time = 0.143049, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5944, 5952, 4180, 2531, 2282, 6589, 216}

$$-\frac{i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right)}{a}+\frac{i\mathrm{PolyLog}\left(3,-ie^{\tanh^{-1}(ax)}\right)}{a}-\frac{i\mathrm{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```


```
[Out] -(ArcSin[a*x]/a) + (Sqrt[1 - a^2*x^2]*ArcTanh[a*x])/a + (x*Sqrt[1 - a^2*x^2
]*ArcTanh[a*x]^2)/2 + (ArcTan[E^ArcTanh[a*x]]*ArcTanh[a*x]^2)/a - (I*ArcTan
h[a*x]*PolyLog[2, (-I)*E^ArcTanh[a*x]])/a + (I*ArcTanh[a*x]*PolyLog[2, I*E^
ArcTanh[a*x]])/a + (I*PolyLog[3, (-I)*E^ArcTanh[a*x]])/a - (I*PolyLog[3, I*
E^ArcTanh[a*x]])/a
```


Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^2\,dx = \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx - \int \frac{1}{\sqrt{1-}}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\mathrm{Subst}\left(\int x^2\mathrm{sech}\right.}{}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

$$= -\frac{\sin^{-1}(ax)}{a} + \frac{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}{a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2 + \frac{\tan^{-1}\left(e^{\tanh^{-1}(ax)}\right.}{a}$$

Mathematica [A] time = 0.0998037, size = 187, normalized size = 1.18

$$\sqrt{1-a^2x^2}\left(-\frac{i\left(2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{-\tanh^{-1}(ax)}\right)-2\tanh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{-\tanh^{-1}(ax)}\right)+2\mathrm{PolyLog}\left(3,-ie^{-\tanh^{-1}(ax)}\right)-2\mathrm{PolyLog}\left(3,ie^{-\tanh}\right.\right.}{\sqrt{1-a^2x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2,x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*ArcTanh[a*x] + a*x*ArcTanh[a*x]^2 - (I*((-4*I)*ArcTan
[Tanh[ArcTanh[a*x]/2]] + ArcTanh[a*x]^2*Log[1 - I/E^ArcTanh[a*x]] - ArcTanh
[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] + 2*ArcTanh[a*x]*PolyLog[2, (-I)/E^ArcTan
h[a*x]] - 2*ArcTanh[a*x]*PolyLog[2, I/E^ArcTanh[a*x]] + 2*PolyLog[3, (-I)/E
^ArcTanh[a*x]] - 2*PolyLog[3, I/E^ArcTanh[a*x]]))/Sqrt[1 - a^2*x^2]))/(2*a)
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\left(\operatorname{Artanh}(ax)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2, x)
```

3.472 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{5/2}}\,dx$

Optimal. Leaf size=139

$$\frac{40x}{27\sqrt{1-a^2x^2}}+\frac{2x}{27\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}}$$

```
[Out] (2*x)/(27*(1 - a^2*x^2)^(3/2)) + (40*x)/(27*Sqrt[1 - a^2*x^2]) - (2*ArcTanh
[a*x])/(9*a*(1 - a^2*x^2)^(3/2)) - (4*ArcTanh[a*x])/(3*a*Sqrt[1 - a^2*x^2])
 + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^2)/(3*Sqr
t[1 - a^2*x^2])
```

Rubi [A] time = 0.0928308, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5964, 5962, 191, 192}

$$\frac{40x}{27\sqrt{1-a^2x^2}}+\frac{2x}{27\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}}-\frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```


```
[Out] (2*x)/(27*(1 - a^2*x^2)^(3/2)) + (40*x)/(27*Sqrt[1 - a^2*x^2]) - (2*ArcTanh
[a*x])/(9*a*(1 - a^2*x^2)^(3/2)) - (4*ArcTanh[a*x])/(3*a*Sqrt[1 - a^2*x^2])
 + (x*ArcTanh[a*x]^2)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^2)/(3*Sqr
t[1 - a^2*x^2])
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx = -\frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}} + \frac{2}{9}\int \frac{1}{\left(1-a^2x^2\right)^{5/2}}\,dx + \frac{2}{3}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{3/2}}\,dx$$

$$= \frac{2x}{27\left(1-a^2x^2\right)^{3/2}} - \frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}} - \frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)^2}{3\sqrt{1-a^2x^2}} + \frac{4}{27}\int$$

$$= \frac{2x}{27\left(1-a^2x^2\right)^{3/2}} + \frac{40x}{27\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{9a\left(1-a^2x^2\right)^{3/2}} - \frac{4\tanh^{-1}(ax)}{3a\sqrt{1-a^2x^2}} + \frac{x\tanh^{-1}(ax)^2}{3\left(1-a^2x^2\right)^{3/2}} + \frac{2x\tanh}{3\sqrt{1-}}$$

Mathematica [A] time = 0.073697, size = 70, normalized size = 0.5

$$\frac{-40a^3x^3 - 9ax\left(2a^2x^2-3\right)\tanh^{-1}(ax)^2 + 6\left(6a^2x^2-7\right)\tanh^{-1}(ax) + 42ax}{27a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (42*a*x - 40*a^3*x^3 + 6*(-7 + 6*a^2*x^2)*ArcTanh[a*x] - 9*a*x*(-3 + 2*a^2*
x^2)*ArcTanh[a*x]^2)/(27*a*(1 - a^2*x^2)^(3/2))
```

Maple [A] time = 0.17, size = 84, normalized size = 0.6

$$-\frac{18\left(\mathrm{Artanh}\left(ax\right)\right)^2x^3a^3 + 40x^3a^3 - 36a^2x^2\mathrm{Artanh}\left(ax\right) - 27\left(\mathrm{Artanh}\left(ax\right)\right)^2xa - 42ax + 42\,\mathrm{Artanh}\left(ax\right)}{27a\left(a^2x^2-1\right)^2}\sqrt{-a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] -1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^2*x^3*a^3+40*x^3*a^3-36*a^2*x^2
*arctanh(a*x)-27*arctanh(a*x)^2*x*a-42*a*x+42*arctanh(a*x))/(a^2*x^2-1)^2
```

Maxima [B] time = 1.6954, size = 410, normalized size = 2.95

$$\frac{1}{3}\left(\frac{2x}{\sqrt{-a^2x^2+1}} + \frac{x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}\right)\mathrm{artanh}\left(ax\right)^2 + \frac{1}{27}a\left(\frac{\frac{2x}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}a^2x+\sqrt{-a^2x^2+1}a}}{a} + \frac{\frac{2x}{\sqrt{-a^2x^2+1}} - \frac{1}{\sqrt{-a^2x^2+1}a^2x-\sqrt{-a}}}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(-a^2*x^2 + 1) + x/(-a^2*x^2 + 1)^(3/2))*arctanh(a*x)^2 + 1/27
*a*((2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 +
 1)*a))/a + (2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a
^2*x^2 + 1)*a))/a - 18*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 18*sqrt(-a^2*x^
2 + 1)/((a^2*x - a)*a) - 18*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 18*log(
-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 3*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a
^2) + 3*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2))
```

Fricas [A] time = 1.67599, size = 238, normalized size = 1.71

$$-\frac{\left(160\,a^3x^3+9\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-168\,ax-12\left(6\,a^2x^2-7\right)\log\left(-\frac{ax+1}{ax-1}\right)\right)\sqrt{-a^2x^2+1}}{108\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/108*(160*a^3*x^3 + 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^2 - 1
68*a*x - 12*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(
a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^2(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)**2/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(5/2), x)
```

3.473 $\int \frac{\tanh^{-1}(ax)^2}{(1-a^2x^2)^{7/2}}\,dx$

Optimal. Leaf size=208

$$\frac{4144x}{3375\sqrt{1-a^2x^2}}+\frac{272x}{3375\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{125\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^2}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^2}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}}-\frac{16}{15}$$

```
[Out] (2*x)/(125*(1 - a^2*x^2)^(5/2)) + (272*x)/(3375*(1 - a^2*x^2)^(3/2)) + (414
4*x)/(3375*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(25*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x])/(45*a*(1 - a^2*x^2)^(3/2)) - (16*ArcTanh[a*x])/(15*a*Sq
rt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTan
h[a*x]^2)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^2)/(15*Sqrt[1 - a^2*
x^2])
```

Rubi [A] time = 0.152059, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5964, 5962, 191, 192}

$$\frac{4144x}{3375\sqrt{1-a^2x^2}}+\frac{272x}{3375\left(1-a^2x^2\right)^{3/2}}+\frac{2x}{125\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^2}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^2}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}}-\frac{16}{15}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```


```
[Out] (2*x)/(125*(1 - a^2*x^2)^(5/2)) + (272*x)/(3375*(1 - a^2*x^2)^(3/2)) + (414
4*x)/(3375*Sqrt[1 - a^2*x^2]) - (2*ArcTanh[a*x])/(25*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x])/(45*a*(1 - a^2*x^2)^(3/2)) - (16*ArcTanh[a*x])/(15*a*Sq
rt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTan
h[a*x]^2)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^2)/(15*Sqrt[1 - a^2*
x^2])
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx = -\frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} + \frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}} + \frac{2}{25}\int \frac{1}{\left(1-a^2x^2\right)^{7/2}}\,dx + \frac{4}{5}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{5/2}}\,dx$$
$$= \frac{2x}{125\left(1-a^2x^2\right)^{5/2}} - \frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}} + \frac{x\tanh^{-1}(ax)^2}{5\left(1-a^2x^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)^2}{15\left(1-a^2x^2\right)^{3/2}} + \frac{8}{12\ldots}$$
$$= \frac{2x}{125\left(1-a^2x^2\right)^{5/2}} + \frac{272x}{3375\left(1-a^2x^2\right)^{3/2}} - \frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}} - \frac{16\tanh^{-1}(ax)}{15a\sqrt{1-a^2x^2}} + \ldots$$
$$= \frac{2x}{125\left(1-a^2x^2\right)^{5/2}} + \frac{272x}{3375\left(1-a^2x^2\right)^{3/2}} + \frac{4144x}{3375\sqrt{1-a^2x^2}} - \frac{2\tanh^{-1}(ax)}{25a\left(1-a^2x^2\right)^{5/2}} - \frac{8\tanh^{-1}(ax)}{45a\left(1-a^2x^2\right)^{3/2}} - \ldots$$

Mathematica [A] time = 0.0849286, size = 94, normalized size = 0.45

$$\frac{4144a^5x^5 - 8560a^3x^3 + 225ax\left(8a^4x^4 - 20a^2x^2 + 15\right)\tanh^{-1}(ax)^2 - 30\left(120a^4x^4 - 260a^2x^2 + 149\right)\tanh^{-1}(ax) + 4470ax}{3375a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (4470*a*x - 8560*a^3*x^3 + 4144*a^5*x^5 - 30*(149 - 260*a^2*x^2 + 120*a^4*x
^4)*ArcTanh[a*x] + 225*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^2)/(3
375*a*(1 - a^2*x^2)^(5/2))
```

Maple [A] time = 0.178, size = 118, normalized size = 0.6

$$-\frac{1800\left(\mathrm{Artanh}(ax)\right)^2x^5a^5 + 4144x^5a^5 - 3600a^4x^4\mathrm{Artanh}(ax) - 4500\left(\mathrm{Artanh}(ax)\right)^2x^3a^3 - 8560x^3a^3 + 7800a^2x^2\mathrm{A}\ldots}{3375a\left(a^2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -1/3375/a*(-a^2*x^2+1)^(1/2)*(1800*arctanh(a*x)^2*x^5*a^5+4144*x^5*a^5-3600
*a^4*x^4*arctanh(a*x)-4500*arctanh(a*x)^2*x^3*a^3-8560*x^3*a^3+7800*a^2*x^2
*arctanh(a*x)+3375*arctanh(a*x)^2*x*a+4470*a*x-4470*arctanh(a*x))/(a^2*x^2-
1)^3
```

Maxima [B] time = 1.7301, size = 694, normalized size = 3.34

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}+\frac{3\,x}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\right)\operatorname{artanh}\left(ax\right)^2+\frac{1}{3375}\,a\left(\frac{9\left(\frac{8\,x}{\sqrt{-a^2x^2+1}}+\frac{4\,x}{\left(-a^2x^2+1\right)^{\frac{3}{2}}}-\frac{3}{\left(-a^2x^2+1\right)^{\frac{3}{2}}a^2x+(-\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) + 3*x/(-a^2*x^2 + 1
)^(5/2))*arctanh(a*x)^2 + 1/3375*a*(9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x
^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a^2*x^2 + 1)^(3/2)*a))/a
+ 9*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^
(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 100*(2*x/sqrt(-a^2*x^2 + 1) - 1/
(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a))/a + 100*(2*x/sqrt(-a^2*x
^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a^2*x^2 + 1)*a))/a - 1800*sqr
t(-a^2*x^2 + 1)/((a^2*x + a)*a) - 1800*sqrt(-a^2*x^2 + 1)/((a^2*x - a)*a) -
 1800*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) + 1800*log(-a*x + 1)/(sqrt(-a^2
*x^2 + 1)*a^2) - 300*log(a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) + 300*log(-a*x
 + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) - 135*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a
^2) + 135*log(-a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2))
```

Fricas [A] time = 1.72755, size = 329, normalized size = 1.58

$$-\frac{\left(16576\,a^5x^5-34240\,a^3x^3+225\left(8\,a^5x^5-20\,a^3x^3+15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2+17880\,ax-60\left(120\,a^4x^4-260\,a^2x^2+149\right.\right.}{13500\left(a^7x^6-3\,a^5x^4+3\,a^3x^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/13500*(16576*a^5*x^5 - 34240*a^3*x^3 + 225*(8*a^5*x^5 - 20*a^3*x^3 + 15*
a*x)*log(-(a*x + 1)/(a*x - 1))^2 + 17880*a*x - 60*(120*a^4*x^4 - 260*a^2*x^
2 + 149)*log(-(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x^4
 + 3*a^3*x^2 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{\left(-a^2x^2+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(7/2), x)
```

3.474 $\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=277

$$\frac{413312x}{385875\sqrt{1-a^2x^2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^2}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}}{35\left(1-a^2\right.}$$

```
[Out] (2*x)/(343*(1 - a^2*x^2)^(7/2)) + (888*x)/(42875*(1 - a^2*x^2)^(5/2)) + (30
256*x)/(385875*(1 - a^2*x^2)^(3/2)) + (413312*x)/(385875*Sqrt[1 - a^2*x^2])
 - (2*ArcTanh[a*x])/(49*a*(1 - a^2*x^2)^(7/2)) - (12*ArcTanh[a*x])/(175*a*(
1 - a^2*x^2)^(5/2)) - (16*ArcTanh[a*x])/(105*a*(1 - a^2*x^2)^(3/2)) - (32*A
rcTanh[a*x])/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(7*(1 - a^2*x^2)
^(7/2)) + (6*x*ArcTanh[a*x]^2)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]
^2)/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x]^2)/(35*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.228388, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5964, 5962, 191, 192}

$$\frac{413312x}{385875\sqrt{1-a^2x^2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^2}{35\sqrt{1-a^2x^2}}+\frac{8x\tanh^{-1}}{35\left(1-a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (2*x)/(343*(1 - a^2*x^2)^(7/2)) + (888*x)/(42875*(1 - a^2*x^2)^(5/2)) + (30
256*x)/(385875*(1 - a^2*x^2)^(3/2)) + (413312*x)/(385875*Sqrt[1 - a^2*x^2])
 - (2*ArcTanh[a*x])/(49*a*(1 - a^2*x^2)^(7/2)) - (12*ArcTanh[a*x])/(175*a*(
1 - a^2*x^2)^(5/2)) - (16*ArcTanh[a*x])/(105*a*(1 - a^2*x^2)^(3/2)) - (32*A
rcTanh[a*x])/(35*a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^2)/(7*(1 - a^2*x^2)
^(7/2)) + (6*x*ArcTanh[a*x]^2)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]
^2)/(35*(1 - a^2*x^2)^(3/2)) + (16*x*ArcTanh[a*x]^2)/(35*Sqrt[1 - a^2*x^2])
```

Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{9/2}}\,dx &= -\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{x\tanh^{-1}(ax)^2}{7\left(1-a^2x^2\right)^{7/2}}+\frac{2}{49}\int \frac{1}{\left(1-a^2x^2\right)^{9/2}}\,dx+\frac{6}{7}\int \frac{\tanh^{-1}(ax)^2}{\left(1-a^2x^2\right)^{7/2}}\,dx\\
&= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{12\tanh^{-1}(ax)}{175a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)^2}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tanh^{-1}(ax)^2}{35\left(1-a^2x^2\right)^{5/2}}+\frac{1}{3}\\
&= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{12\tanh^{-1}(ax)}{175a\left(1-a^2x^2\right)^{5/2}}-\frac{16\tanh^{-1}(ax}{105a\left(1-a^2x^2\right)}\\
&= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{12\tanh^{-1}(}{175a\left(1-a^2\right.}\\
&= \frac{2x}{343\left(1-a^2x^2\right)^{7/2}}+\frac{888x}{42875\left(1-a^2x^2\right)^{5/2}}+\frac{30256x}{385875\left(1-a^2x^2\right)^{3/2}}+\frac{413312x}{385875\sqrt{1-a^2x^2}}-\frac{2\tanh^{-1}(}{49a\left(1-a^2\right.}
\end{aligned}$$

Mathematica [A] time = 0.110049, size = 120, normalized size = 0.43

$$\frac{2ax\left(-206656a^6x^6+635096a^4x^4-654220a^2x^2+226905\right)-11025ax\left(16a^6x^6-56a^4x^4+70a^2x^2-35\right)\tanh^{-1}(ax)^2+2}{385875a\left(1-a^2x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^2/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (2*a*x*(226905 - 654220*a^2*x^2 + 635096*a^4*x^4 - 206656*a^6*x^6) + 210*(-
2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*x] - 11025*a*x
*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]^2)/(385875*a*(1
- a^2*x^2)^(7/2))
```

Maple [A] time = 0.191, size = 152, normalized size = 0.6

$$-\frac{176400\left(\mathrm{Artanh}(ax)\right)^2x^7a^7+413312a^7x^7-352800\,\mathrm{Artanh}(ax)\,x^6a^6-617400\left(\mathrm{Artanh}(ax)\right)^2x^5a^5-1270192x^5a^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] -1/385875/a*(-a^2*x^2+1)^(1/2)*(176400*arctanh(a*x)^2*x^7*a^7+413312*a^7*x^
7-352800*arctanh(a*x)*x^6*a^6-617400*arctanh(a*x)^2*x^5*a^5-1270192*x^5*a^5
+1117200*a^4*x^4*arctanh(a*x)+771750*arctanh(a*x)^2*x^3*a^3+1308440*x^3*a^3
-1202460*a^2*x^2*arctanh(a*x)-385875*arctanh(a*x)^2*x*a-453810*a*x+453810*a
rctanh(a*x))/(a^2*x^2-1)^4
```

Maxima [B] time = 1.79396, size = 1014, normalized size = 3.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/35*(16*x/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) + 6*x/(-a^2*x^2 +
1)^(5/2) + 5*x/(-a^2*x^2 + 1)^(7/2))*arctanh(a*x)^2 + 1/385875*a*(225*(16*x
/sqrt(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a^
2*x + (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 225*(16*x/sqr
t(-a^2*x^2 + 1) + 8*x/(-a^2*x^2 + 1)^(3/2) - 5/((-a^2*x^2 + 1)^(5/2)*a^2*x
- (-a^2*x^2 + 1)^(5/2)*a) + 6*x/(-a^2*x^2 + 1)^(5/2))/a + 882*(8*x/sqrt(-a^
2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x + (-a
^2*x^2 + 1)^(3/2)*a))/a + 882*(8*x/sqrt(-a^2*x^2 + 1) + 4*x/(-a^2*x^2 + 1)^
(3/2) - 3/((-a^2*x^2 + 1)^(3/2)*a^2*x - (-a^2*x^2 + 1)^(3/2)*a))/a + 9800*(
2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x + sqrt(-a^2*x^2 + 1)*a
))/a + 9800*(2*x/sqrt(-a^2*x^2 + 1) - 1/(sqrt(-a^2*x^2 + 1)*a^2*x - sqrt(-a
^2*x^2 + 1)*a))/a - 176400*sqrt(-a^2*x^2 + 1)/((a^2*x + a)*a) - 176400*sqrt
(-a^2*x^2 + 1)/((a^2*x - a)*a) - 176400*log(a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^
2) + 176400*log(-a*x + 1)/(sqrt(-a^2*x^2 + 1)*a^2) - 29400*log(a*x + 1)/((-
a^2*x^2 + 1)^(3/2)*a^2) + 29400*log(-a*x + 1)/((-a^2*x^2 + 1)^(3/2)*a^2) -
13230*log(a*x + 1)/((-a^2*x^2 + 1)^(5/2)*a^2) + 13230*log(-a*x + 1)/((-a^2*
x^2 + 1)^(5/2)*a^2) - 7875*log(a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2) + 7875*l
og(-a*x + 1)/((-a^2*x^2 + 1)^(7/2)*a^2))
```

Fricas [A] time = 1.70749, size = 428, normalized size = 1.55

$$-\frac{\left(1653248\,a^7x^7 - 5080768\,a^5x^5 + 5233760\,a^3x^3 + 11025\left(16\,a^7x^7 - 56\,a^5x^5 + 70\,a^3x^3 - 35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^2 - 181524\cdots}{1543500\left(a^9x^8 - 4\,a^7x^6 + 6\,a^5x^4 - 4\,a^3x^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/1543500*(1653248*a^7*x^7 - 5080768*a^5*x^5 + 5233760*a^3*x^3 + 11025*(16
*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x - 1))^2 -
1815240*a*x - 420*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(-
(a*x + 1)/(a*x - 1)))*sqrt(-a^2*x^2 + 1)/(a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 -
 4*a^3*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**2/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^2}{(-a^2x^2+1)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^2/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^2/(-a^2*x^2 + 1)^(9/2), x)
```

3.475 $\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^3\,dx$

Optimal. Leaf size=302

$$\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{2a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyL}}{2a}$$

```
[Out] (6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a + (3*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/2 + (ArcTan
[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, (
-I)*E^ArcTanh[a*x]])/a + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[a
*x]])/a + ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a - ((3*I)
*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + ((3*I)*ArcTanh[a*x]*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a - ((3*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[
a*x]])/a - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*PolyLog[4, I*
E^ArcTanh[a*x]])/a
```

Rubi [A] time = 0.197346, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5944, 5952, 4180, 2531, 6609, 2282, 6589, 5950}

$$\frac{3i\text{PolyLog}\left(2,-\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\text{PolyLog}\left(2,\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}-\frac{3i\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right)}{2a}+\frac{3i\tanh^{-1}(ax)^2\text{PolyL}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

```
[Out] (6*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]]*ArcTanh[a*x])/a + (3*Sqrt[1 - a^2*x^
2]*ArcTanh[a*x]^2)/(2*a) + (x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3)/2 + (ArcTan
[E^ArcTanh[a*x]]*ArcTanh[a*x]^3)/a - (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, (
-I)*E^ArcTanh[a*x]])/a + (((3*I)/2)*ArcTanh[a*x]^2*PolyLog[2, I*E^ArcTanh[a
*x]])/a + ((3*I)*PolyLog[2, ((-I)*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a - ((3*I)
*PolyLog[2, (I*Sqrt[1 - a*x])/Sqrt[1 + a*x]])/a + ((3*I)*ArcTanh[a*x]*PolyL
og[3, (-I)*E^ArcTanh[a*x]])/a - ((3*I)*ArcTanh[a*x]*PolyLog[3, I*E^ArcTanh[
a*x]])/a - ((3*I)*PolyLog[4, (-I)*E^ArcTanh[a*x]])/a + ((3*I)*PolyLog[4, I*
E^ArcTanh[a*x]])/a
```

Rule 5944

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 1))/(2*c*q*(2
*q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcT
anh[c*x])^p, x], x] - Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^
2)^(q - 1)*(a + b*ArcTanh[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a
+ b*ArcTanh[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2
*d + e, 0] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 5952

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sech[x], x], x, ArcTan
h[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
 && GtQ[d, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{1-a^2x^2}\tanh^{-1}(ax)^3\,dx = \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3 + \frac{1}{2}\int \frac{\tanh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx - 3\int$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3$$

$$= \frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)\tanh^{-1}(ax)}{a} + \frac{3\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}{2a} + \frac{1}{2}x\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3$$

Mathematica [A] time = 3.70685, size = 569, normalized size = 1.88

$$-\frac{i\left(192\tanh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\tanh^{-1}(ax)}\right) + 192i\pi\tanh^{-1}(ax)\text{PolyLog}\left(2,ie^{\tanh^{-1}(ax)}\right) + 384\tanh^{-1}(ax)\text{PolyLo}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3,x]
```

```
[Out] ((-I/128)*(7*Pi^4 + (8*I)*Pi^3*ArcTanh[a*x] + 24*Pi^2*ArcTanh[a*x]^2 + (192
*I)*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2 - (32*I)*Pi*ArcTanh[a*x]^3 + (64*I)*a*
x*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3 - 16*ArcTanh[a*x]^4 - 384*ArcTanh[a*x]*L
og[1 - I/E^ArcTanh[a*x]] + (8*I)*Pi^3*Log[1 + I/E^ArcTanh[a*x]] + 384*ArcTa
nh[a*x]*Log[1 + I/E^ArcTanh[a*x]] + 48*Pi^2*ArcTanh[a*x]*Log[1 + I/E^ArcTan
h[a*x]] - (96*I)*Pi*ArcTanh[a*x]^2*Log[1 + I/E^ArcTanh[a*x]] - 64*ArcTanh[a
*x]^3*Log[1 + I/E^ArcTanh[a*x]] - 48*Pi^2*ArcTanh[a*x]*Log[1 - I*E^ArcTanh[
a*x]] + (96*I)*Pi*ArcTanh[a*x]^2*Log[1 - I*E^ArcTanh[a*x]] - (8*I)*Pi^3*Log
[1 + I*E^ArcTanh[a*x]] + 64*ArcTanh[a*x]^3*Log[1 + I*E^ArcTanh[a*x]] + (8*I
)*Pi^3*Log[Tan[(Pi + (2*I)*ArcTanh[a*x])/4]] - 48*(8 + Pi^2 - (4*I)*Pi*ArcT
anh[a*x] - 4*ArcTanh[a*x]^2)*PolyLog[2, (-I)/E^ArcTanh[a*x]] + 384*PolyLog[
2, I/E^ArcTanh[a*x]] + 192*ArcTanh[a*x]^2*PolyLog[2, (-I)*E^ArcTanh[a*x]] -
 48*Pi^2*PolyLog[2, I*E^ArcTanh[a*x]] + (192*I)*Pi*ArcTanh[a*x]*PolyLog[2,
I*E^ArcTanh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcTanh[a*x]] + 384*ArcTan
h[a*x]*PolyLog[3, (-I)/E^ArcTanh[a*x]] - 384*ArcTanh[a*x]*PolyLog[3, (-I)*E
^ArcTanh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcTanh[a*x]] + 384*PolyLog[4, (
-I)/E^ArcTanh[a*x]] + 384*PolyLog[4, (-I)*E^ArcTanh[a*x]]))/a
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,(\text{Artanh}\,(ax))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^3(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)*atanh(a*x)**3,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)*arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3, x)
```

3.476 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{40}{9a\sqrt{1-a^2x^2}}-\frac{2}{27a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^3}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}+\frac{40x\tan}{9\sqrt{1-}}$$

```
[Out] -2/(27*a*(1 - a^2*x^2)^(3/2)) - 40/(9*a*Sqrt[1 - a^2*x^2]) + (2*x*ArcTanh[a
*x])/(9*(1 - a^2*x^2)^(3/2)) + (40*x*ArcTanh[a*x])/(9*Sqrt[1 - a^2*x^2]) -
ArcTanh[a*x]^2/(3*a*(1 - a^2*x^2)^(3/2)) - (2*ArcTanh[a*x]^2)/(a*Sqrt[1 - a
^2*x^2]) + (x*ArcTanh[a*x]^3)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^3
)/(3*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.164848, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5964, 5962, 5958, 5960}

$$-\frac{40}{9a\sqrt{1-a^2x^2}}-\frac{2}{27a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)^3}{3\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}+\frac{40x\tan}{9\sqrt{1-}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```


```
[Out] -2/(27*a*(1 - a^2*x^2)^(3/2)) - 40/(9*a*Sqrt[1 - a^2*x^2]) + (2*x*ArcTanh[a
*x])/(9*(1 - a^2*x^2)^(3/2)) + (40*x*ArcTanh[a*x])/(9*Sqrt[1 - a^2*x^2]) -
ArcTanh[a*x]^2/(3*a*(1 - a^2*x^2)^(3/2)) - (2*ArcTanh[a*x]^2)/(a*Sqrt[1 - a
^2*x^2]) + (x*ArcTanh[a*x]^3)/(3*(1 - a^2*x^2)^(3/2)) + (2*x*ArcTanh[a*x]^3
)/(3*Sqrt[1 - a^2*x^2])
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx = -\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2}{3}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{5/2}}\,dx+\frac{2}{3}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{3/2}}\,dx$$
$$= -\frac{2}{27a\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tanh^{-1}(ax)}{9\left(1-a^2x^2\right)^{3/2}}-\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}+\frac{x\tanh^{-1}(ax)^3}{3\left(1-a^2x^2\right)^{3/2}}+\frac{2x\tan}{3\sqrt{1}}$$
$$= -\frac{2}{27a\left(1-a^2x^2\right)^{3/2}}-\frac{40}{9a\sqrt{1-a^2x^2}}+\frac{2x\tanh^{-1}(ax)}{9\left(1-a^2x^2\right)^{3/2}}+\frac{40x\tanh^{-1}(ax)}{9\sqrt{1-a^2x^2}}-\frac{\tanh^{-1}(ax)^2}{3a\left(1-a^2x^2\right)^{3/2}}-\frac{2\tanh}{a\sqrt{1}}$$

Mathematica [A] time = 0.0927984, size = 87, normalized size = 0.46

$$\frac{120a^2x^2-9ax\left(2a^2x^2-3\right)\tanh^{-1}(ax)^3+9\left(6a^2x^2-7\right)\tanh^{-1}(ax)^2-6ax\left(20a^2x^2-21\right)\tanh^{-1}(ax)-122}{27a\left(1-a^2x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(5/2),x]
```

```
[Out] (-122 + 120*a^2*x^2 - 6*a*x*(-21 + 20*a^2*x^2)*ArcTanh[a*x] + 9*(-7 + 6*a^2
*x^2)*ArcTanh[a*x]^2 - 9*a*x*(-3 + 2*a^2*x^2)*ArcTanh[a*x]^3)/(27*a*(1 - a^
2*x^2)^(3/2))
```

Maple [A] time = 0.178, size = 105, normalized size = 0.6

$$-\frac{18\,(\mathrm{Artanh}\,(ax))^3\,x^3a^3+120\,a^3x^3\mathrm{Artanh}\,(ax)-54\,a^2x^2\,(\mathrm{Artanh}\,(ax))^2-27\,(\mathrm{Artanh}\,(ax))^3\,ax-120\,a^2x^2-126\,ax\mathrm{A}}{27\,a\left(a^2x^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x)
```

```
[Out] -1/27/a*(-a^2*x^2+1)^(1/2)*(18*arctanh(a*x)^3*x^3*a^3+120*a^3*x^3*arctanh(a
*x)-54*a^2*x^2*arctanh(a*x)^2-27*arctanh(a*x)^3*a*x-120*a^2*x^2-126*a*x*arc
tanh(a*x)+63*arctanh(a*x)^2+122)/(a^2*x^2-1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\mathrm{artanh}\,(ax)^3}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

Fricas [A] time = 1.6996, size = 305, normalized size = 1.6

$$\frac{\left(960\,a^2x^2-9\left(2\,a^3x^3-3\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3+18\left(6\,a^2x^2-7\right)\log\left(-\frac{ax+1}{ax-1}\right)^2-24\left(20\,a^3x^3-21\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)-976\right)\sqrt{}}{216\left(a^5x^4-2\,a^3x^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/216*(960*a^2*x^2 - 9*(2*a^3*x^3 - 3*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 18
*(6*a^2*x^2 - 7)*log(-(a*x + 1)/(a*x - 1))^2 - 24*(20*a^3*x^3 - 21*a*x)*log
(-(a*x + 1)/(a*x - 1)) - 976)*sqrt(-a^2*x^2 + 1)/(a^5*x^4 - 2*a^3*x^2 + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}^3(ax)}{\left(-(ax-1)(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)**3/(-(a*x - 1)*(a*x + 1))**(5/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{\left(-a^2x^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(5/2), x)
```

3.477 $\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=289

$$-\frac{4144}{1125a\sqrt{1-a^2x^2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^3}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^3}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}}-\frac{8\cdots}{5\cdots}$$

```
[Out] -6/(625*a*(1 - a^2*x^2)^(5/2)) - 272/(3375*a*(1 - a^2*x^2)^(3/2)) - 4144/(1
125*a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(125*(1 - a^2*x^2)^(5/2)) + (
272*x*ArcTanh[a*x])/(1125*(1 - a^2*x^2)^(3/2)) + (4144*x*ArcTanh[a*x])/(112
5*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(25*a*(1 - a^2*x^2)^(5/2)) - (4*A
rcTanh[a*x]^2)/(15*a*(1 - a^2*x^2)^(3/2)) - (8*ArcTanh[a*x]^2)/(5*a*Sqrt[1
- a^2*x^2]) + (x*ArcTanh[a*x]^3)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*x
]^3)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^3)/(15*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.304252, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5964, 5962, 5958, 5960}

$$-\frac{4144}{1125a\sqrt{1-a^2x^2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}+\frac{8x\tanh^{-1}(ax)^3}{15\sqrt{1-a^2x^2}}+\frac{4x\tanh^{-1}(ax)^3}{15\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}}-\frac{8\cdots}{5\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```


```
[Out] -6/(625*a*(1 - a^2*x^2)^(5/2)) - 272/(3375*a*(1 - a^2*x^2)^(3/2)) - 4144/(1
125*a*Sqrt[1 - a^2*x^2]) + (6*x*ArcTanh[a*x])/(125*(1 - a^2*x^2)^(5/2)) + (
272*x*ArcTanh[a*x])/(1125*(1 - a^2*x^2)^(3/2)) + (4144*x*ArcTanh[a*x])/(112
5*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(25*a*(1 - a^2*x^2)^(5/2)) - (4*A
rcTanh[a*x]^2)/(15*a*(1 - a^2*x^2)^(3/2)) - (8*ArcTanh[a*x]^2)/(5*a*Sqrt[1
- a^2*x^2]) + (x*ArcTanh[a*x]^3)/(5*(1 - a^2*x^2)^(5/2)) + (4*x*ArcTanh[a*x
]^3)/(15*(1 - a^2*x^2)^(3/2)) + (8*x*ArcTanh[a*x]^3)/(15*Sqrt[1 - a^2*x^2])
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx = -\frac{3\tanh^{-1}(ax)^2}{25a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)^3}{5\left(1-a^2x^2\right)^{5/2}}+\frac{6}{25}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{7/2}}\,dx+\frac{4}{5}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{5/2}}\,dx$$

$$= -\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}+\frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}}-\frac{3\tanh^{-1}(ax)^2}{25a\left(1-a^2x^2\right)^{5/2}}-\frac{4\tanh^{-1}(ax)^2}{15a\left(1-a^2x^2\right)^{3/2}}+\frac{x\tanh^{-1}(ax)}{5\left(1-a^2x^2\right)^{5/}}$$

$$= -\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}}+\frac{272x\tanh^{-1}(ax)}{1125\left(1-a^2x^2\right)^{3/2}}-\frac{3\tanh^{-}}{25a\left(1-a\right.}$$

$$= -\frac{6}{625a\left(1-a^2x^2\right)^{5/2}}-\frac{272}{3375a\left(1-a^2x^2\right)^{3/2}}-\frac{4144}{1125a\sqrt{1-a^2x^2}}+\frac{6x\tanh^{-1}(ax)}{125\left(1-a^2x^2\right)^{5/2}}+\frac{272x\tanh}{1125\left(1-a\right.}$$

Mathematica [A] time = 0.109655, size = 119, normalized size = 0.41

$$\frac{-62160a^4x^4+125680a^2x^2+1125ax\left(8a^4x^4-20a^2x^2+15\right)\tanh^{-1}(ax)^3+30ax\left(2072a^4x^4-4280a^2x^2+2235\right)\tanh}{16875a\left(1-a^2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(7/2),x]
```

```
[Out] (-63682 + 125680*a^2*x^2 - 62160*a^4*x^4 + 30*a*x*(2235 - 4280*a^2*x^2 + 20
72*a^4*x^4)*ArcTanh[a*x] - 225*(149 - 260*a^2*x^2 + 120*a^4*x^4)*ArcTanh[a*
x]^2 + 1125*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcTanh[a*x]^3)/(16875*a*(1 -
 a^2*x^2)^(5/2))
```

Maple [A] time = 0.211, size = 153, normalized size = 0.5

$$-\frac{9000\left(\mathrm{Artanh}(ax)\right)^3x^5a^5+62160\,\mathrm{Artanh}(ax)\,x^5a^5-27000\,a^4x^4\left(\mathrm{Artanh}(ax)\right)^2-22500\left(\mathrm{Artanh}(ax)\right)^3x^3a^3-6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x)
```

```
[Out] -1/16875/a*(-a^2*x^2+1)^(1/2)*(9000*arctanh(a*x)^3*x^5*a^5+62160*arctanh(a*
x)*x^5*a^5-27000*a^4*x^4*arctanh(a*x)^2-22500*arctanh(a*x)^3*x^3*a^3-62160*
x^4*a^4-128400*a^3*x^3*arctanh(a*x)+58500*a^2*x^2*arctanh(a*x)^2+16875*arct
```

```
anh(a*x)^3*a*x+125680*a^2*x^2+67050*a*x*arctanh(a*x)-33525*arctanh(a*x)^2-6
3682)/(a^2*x^2-1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)
```

Fricas [A] time = 1.71556, size = 433, normalized size = 1.5

$$\frac{\left(497280\,a^4x^4 - 1005440\,a^2x^2 - 1125\left(8\,a^5x^5 - 20\,a^3x^3 + 15\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^3 + 450\left(120\,a^4x^4 - 260\,a^2x^2 + 149\right)\log\left(-\frac{ax}{ax}\right.\right.}{135000\left(a^7x^6 - 3\,a^5x^4 + 3\,a^3x^2 - a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/135000*(497280*a^4*x^4 - 1005440*a^2*x^2 - 1125*(8*a^5*x^5 - 20*a^3*x^3 +
 15*a*x)*log(-(a*x + 1)/(a*x - 1))^3 + 450*(120*a^4*x^4 - 260*a^2*x^2 + 149
)*log(-(a*x + 1)/(a*x - 1))^2 - 120*(2072*a^5*x^5 - 4280*a^3*x^3 + 2235*a*x
)*log(-(a*x + 1)/(a*x - 1)) + 509456)*sqrt(-a^2*x^2 + 1)/(a^7*x^6 - 3*a^5*x
^4 + 3*a^3*x^2 - a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(7/2), x)
```

3.478 $\int \frac{\tanh^{-1}(ax)^3}{(1-a^2x^2)^{9/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{413312}{128625a\sqrt{1-a^2x^2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^3}{35\sqrt{1-a^2x^2}}+\frac{8x}{35}$$

```
[Out] -6/(2401*a*(1 - a^2*x^2)^(7/2)) - 2664/(214375*a*(1 - a^2*x^2)^(5/2)) - 302
56/(385875*a*(1 - a^2*x^2)^(3/2)) - 413312/(128625*a*Sqrt[1 - a^2*x^2]) + (
6*x*ArcTanh[a*x])/(343*(1 - a^2*x^2)^(7/2)) + (2664*x*ArcTanh[a*x])/(42875*
(1 - a^2*x^2)^(5/2)) + (30256*x*ArcTanh[a*x])/(128625*(1 - a^2*x^2)^(3/2))
+ (413312*x*ArcTanh[a*x])/(128625*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(
49*a*(1 - a^2*x^2)^(7/2)) - (18*ArcTanh[a*x]^2)/(175*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x]^2)/(35*a*(1 - a^2*x^2)^(3/2)) - (48*ArcTanh[a*x]^2)/(35*
a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(7*(1 - a^2*x^2)^(7/2)) + (6*x*Ar
cTanh[a*x]^3)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]^3)/(35*(1 - a^2*
x^2)^(3/2)) + (16*x*ArcTanh[a*x]^3)/(35*Sqrt[1 - a^2*x^2])
```

Rubi [A] time = 0.483868, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5964, 5962, 5958, 5960}

$$-\frac{413312}{128625a\sqrt{1-a^2x^2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{16x\tanh^{-1}(ax)^3}{35\sqrt{1-a^2x^2}}+\frac{8x}{35}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```



```
[Out] -6/(2401*a*(1 - a^2*x^2)^(7/2)) - 2664/(214375*a*(1 - a^2*x^2)^(5/2)) - 302
56/(385875*a*(1 - a^2*x^2)^(3/2)) - 413312/(128625*a*Sqrt[1 - a^2*x^2]) + (
6*x*ArcTanh[a*x])/(343*(1 - a^2*x^2)^(7/2)) + (2664*x*ArcTanh[a*x])/(42875*
(1 - a^2*x^2)^(5/2)) + (30256*x*ArcTanh[a*x])/(128625*(1 - a^2*x^2)^(3/2))
+ (413312*x*ArcTanh[a*x])/(128625*Sqrt[1 - a^2*x^2]) - (3*ArcTanh[a*x]^2)/(
49*a*(1 - a^2*x^2)^(7/2)) - (18*ArcTanh[a*x]^2)/(175*a*(1 - a^2*x^2)^(5/2))
 - (8*ArcTanh[a*x]^2)/(35*a*(1 - a^2*x^2)^(3/2)) - (48*ArcTanh[a*x]^2)/(35*
a*Sqrt[1 - a^2*x^2]) + (x*ArcTanh[a*x]^3)/(7*(1 - a^2*x^2)^(7/2)) + (6*x*Ar
cTanh[a*x]^3)/(35*(1 - a^2*x^2)^(5/2)) + (8*x*ArcTanh[a*x]^3)/(35*(1 - a^2*
x^2)^(3/2)) + (16*x*ArcTanh[a*x]^3)/(35*Sqrt[1 - a^2*x^2])
```


Rule 5964

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(4*
c*d*(q + 1)^2), x] + (Dist[(2*q + 3)/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*(a + b*ArcTanh[c*x])^p, x], x] + Dist[(b^2*p*(p - 1))/(4*(q + 1)^2), Int[(
d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p - 2), x], x] - Simp[(x*(d + e*x^2)^(q
+ 1)*(a + b*ArcTanh[c*x])^p)/(2*d*(q + 1)), x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

Rule 5962

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> -Simp[(b*p*(a + b*ArcTanh[c*x])^(p - 1))/(c*d*Sqrt[d + e*x^2]),
```

```
 x] + (Dist[b^2*p*(p - 1), Int[(a + b*ArcTanh[c*x])^(p - 2)/(d + e*x^2)^(3/
2), x], x] + Simp[(x*(a + b*ArcTanh[c*x])^p)/(d*Sqrt[d + e*x^2]), x]) /; Fr
eeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 1]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{9/2}}\,dx = -\frac{3\tanh^{-1}(ax)^2}{49a\left(1-a^2x^2\right)^{7/2}}+\frac{x\tanh^{-1}(ax)^3}{7\left(1-a^2x^2\right)^{7/2}}+\frac{6}{49}\int \frac{\tanh^{-1}(ax)}{\left(1-a^2x^2\right)^{9/2}}\,dx+\frac{6}{7}\int \frac{\tanh^{-1}(ax)^3}{\left(1-a^2x^2\right)^{7/2}}\,dx$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}+\frac{6x\tanh^{-1}(ax)}{343\left(1-a^2x^2\right)^{7/2}}-\frac{3\tanh^{-1}(ax)^2}{49a\left(1-a^2x^2\right)^{7/2}}-\frac{18\tanh^{-1}(ax)^2}{175a\left(1-a^2x^2\right)^{5/2}}+\frac{x\tanh^{-1}(ax)^3}{7\left(1-a^2x^2\right)^{7/2}}$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}+\frac{6x\tanh^{-1}(ax)}{343\left(1-a^2x^2\right)^{7/2}}+\frac{2664x\tanh^{-1}(ax)}{42875\left(1-a^2x^2\right)^{5/2}}-\frac{3\tanh}{49a\left(1-\right.}$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}+\frac{6x\tanh^{-1}(ax)}{343\left(1-a^2x^2\right)^{7/2}}+\frac{2664}{4287}$$

$$= -\frac{6}{2401a\left(1-a^2x^2\right)^{7/2}}-\frac{2664}{214375a\left(1-a^2x^2\right)^{5/2}}-\frac{30256}{385875a\left(1-a^2x^2\right)^{3/2}}-\frac{413312}{128625a\sqrt{1-a^2x^2}}+\frac{6x}{343}$$

Mathematica [A] time = 0.132237, size = 151, normalized size = 0.39

$$\frac{43397760a^6x^6-131252240a^4x^4+132479032a^2x^2-385875ax\left(16a^6x^6-56a^4x^4+70a^2x^2-35\right)\tanh^{-1}(ax)^3-210ax\left(2\right.}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]^3/(1 - a^2*x^2)^(9/2),x]
```

```
[Out] (-44658302 + 132479032*a^2*x^2 - 131252240*a^4*x^4 + 43397760*a^6*x^6 - 210
*a*x*(-226905 + 654220*a^2*x^2 - 635096*a^4*x^4 + 206656*a^6*x^6)*ArcTanh[a
*x] + 11025*(-2161 + 5726*a^2*x^2 - 5320*a^4*x^4 + 1680*a^6*x^6)*ArcTanh[a*
x]^2 - 385875*a*x*(-35 + 70*a^2*x^2 - 56*a^4*x^4 + 16*a^6*x^6)*ArcTanh[a*x]
^3)/(13505625*a*(1 - a^2*x^2)^(7/2))
```

Maple [A] time = 0.202, size = 201, normalized size = 0.5

$$-\frac{6174000\left(\operatorname{Artanh}(ax)\right)^{3}x^{7}a^{7}+43397760\operatorname{Artanh}(ax)x^{7}a^{7}-18522000\left(\operatorname{Artanh}(ax)\right)^{2}x^{6}a^{6}-21609000\left(\operatorname{Artanh}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x)
```

```
[Out] -1/13505625/a*(-a^2*x^2+1)^(1/2)*(6174000*arctanh(a*x)^3*x^7*a^7+43397760*a
rctanh(a*x)*x^7*a^7-18522000*arctanh(a*x)^2*x^6*a^6-21609000*arctanh(a*x)^3
*x^5*a^5-43397760*x^6*a^6-133370160*arctanh(a*x)*x^5*a^5+58653000*a^4*x^4*a
rctanh(a*x)^2+27011250*arctanh(a*x)^3*x^3*a^3+131252240*x^4*a^4+137386200*a
^3*x^3*arctanh(a*x)-63129150*a^2*x^2*arctanh(a*x)^2-13505625*arctanh(a*x)^3
*a*x-132479032*a^2*x^2-47650050*a*x*arctanh(a*x)+23825025*arctanh(a*x)^2+44
658302)/(a^2*x^2-1)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^{3}}{\left(-a^{2}x^{2}+1\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

Fricas [A] time = 1.71167, size = 574, normalized size = 1.49

$$\frac{\left(347182080\,a^{6}x^{6}-1050017920\,a^{4}x^{4}+1059832256\,a^{2}x^{2}-385875\left(16\,a^{7}x^{7}-56\,a^{5}x^{5}+70\,a^{3}x^{3}-35\,ax\right)\log\left(-\frac{ax+1}{ax-1}\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/108045000*(347182080*a^6*x^6 - 1050017920*a^4*x^4 + 1059832256*a^2*x^2 -
385875*(16*a^7*x^7 - 56*a^5*x^5 + 70*a^3*x^3 - 35*a*x)*log(-(a*x + 1)/(a*x
- 1))^3 + 22050*(1680*a^6*x^6 - 5320*a^4*x^4 + 5726*a^2*x^2 - 2161)*log(-(a
*x + 1)/(a*x - 1))^2 - 840*(206656*a^7*x^7 - 635096*a^5*x^5 + 654220*a^3*x^
3 - 226905*a*x)*log(-(a*x + 1)/(a*x - 1)) - 357266416)*sqrt(-a^2*x^2 + 1)/(
a^9*x^8 - 4*a^7*x^6 + 6*a^5*x^4 - 4*a^3*x^2 + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)**3/(-a**2*x**2+1)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)^3}{(-a^2x^2+1)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)^3/(-a^2*x^2+1)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)^3/(-a^2*x^2 + 1)^(9/2), x)
```

3.479 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Rubi [A] time = 0.0343793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]

[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx=\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.14772, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x],x]

[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x], x]
```

Maple [A] time = 0.238, size = 0, normalized size = 0.

$$\int \frac{1}{\text{Artanh}\,(ax)}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)

[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\operatorname{atanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x),x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x), x)
```

3.480 $\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Rubi [A] time = 0.0345926, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx = \int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$$

Mathematica [A] time = 0.161795, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]),x]
```

```
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]), x]
```

Maple [A] time = 0.37, size = 0, normalized size = 0.

$$\int \frac{1}{\text{Artanh}(ax)}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

```
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^2-1\right)\mathrm{artanh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)), x)
```

3.481 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=9

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rubi [A] time = 0.0529276, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5968, 3301}

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}\\ &= \frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}\end{aligned}$$

Mathematica [A] time = 0.106657, size = 9, normalized size = 1.

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]),x]
```

```
[Out] CoshIntegral[ArcTanh[a*x]]/a
```

Maple [A] time = 0.233, size = 10, normalized size = 1.1

$$\frac{\mathrm{Chi}\left(\mathrm{Artanh}\left(ax\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x)
```

```
[Out] Chi(arctanh(a*x))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\,\mathrm{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\mathrm{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\,\mathrm{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}} \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)), x)
```

3.482 $\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

```
[Out] (3*CoshIntegral[ArcTanh[a*x]])/(4*a) + CoshIntegral[3*ArcTanh[a*x]]/(4*a)
```

Rubi [A] time = 0.0930111, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5968, 3312, 3301}

$$\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

```
[Out] (3*CoshIntegral[ArcTanh[a*x]])/(4*a) + CoshIntegral[3*ArcTanh[a*x]]/(4*a)
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}\,dx &= \frac{\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx,x,\tanh^{-1}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{4a}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\tanh^{-1}(ax)\right)}{4a}\\
&= \frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{\text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0602268, size = 22, normalized size = 0.81

$$\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right) + \text{Chi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]),x]
```

```
[Out] (3*CoshIntegral[ArcTanh[a*x]] + CoshIntegral[3*ArcTanh[a*x]])/(4*a)
```

Maple [A] time = 0.155, size = 21, normalized size = 0.8

$$\frac{3\,\text{Chi}\left(\text{Artanh}\left(ax\right)\right) + \text{Chi}\left(3\,\text{Artanh}\left(ax\right)\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x)
```

```
[Out] 1/4*(3*Chi(arctanh(a*x))+Chi(3*arctanh(a*x)))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2 + 1\right)^{\frac{5}{2}}\,\text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2 + 1}}{\left(a^6x^6 - 3\,a^4x^4 + 3\,a^2x^2 - 1\right)\text{artanh}\left(ax\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(ax - 1\right)\left(ax + 1\right)\right)^{\frac{5}{2}}\,\text{atanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x),x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}} \operatorname{artanh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)), x)
```

3.483 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=41

$$\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] (5*CoshIntegral[ArcTanh[a*x]])/(8*a) + (5*CoshIntegral[3*ArcTanh[a*x]])/(16
*a) + CoshIntegral[5*ArcTanh[a*x]]/(16*a)
```

Rubi [A] time = 0.108834, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5968, 3312, 3301}

$$\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

```
[Out] (5*CoshIntegral[ArcTanh[a*x]])/(8*a) + (5*CoshIntegral[3*ArcTanh[a*x]])/(16
*a) + CoshIntegral[5*ArcTanh[a*x]]/(16*a)
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^5(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{5\cosh(x)}{8x} + \frac{5\cosh(3x)}{16x} + \frac{\cosh(5x)}{16x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{5\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{5\,\text{Sub}}{}$$
$$= \frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a} + \frac{5\text{Chi}\left(3\tanh^{-1}(ax)\right)}{16a} + \frac{\text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Mathematica [A] time = 0.0627063, size = 31, normalized size = 0.76

$$\frac{10\text{Chi}\left(\tanh^{-1}(ax)\right) + 5\text{Chi}\left(3\tanh^{-1}(ax)\right) + \text{Chi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]),x]
```

```
[Out] (10*CoshIntegral[ArcTanh[a*x]] + 5*CoshIntegral[3*ArcTanh[a*x]] + CoshInteg
ral[5*ArcTanh[a*x]])/(16*a)
```

Maple [A] time = 0.154, size = 30, normalized size = 0.7

$$\frac{10\,\text{Chi}\left(\text{Artanh}\,(ax)\right) + 5\,\text{Chi}\left(3\,\text{Artanh}\,(ax)\right) + \text{Chi}\left(5\,\text{Artanh}\,(ax)\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x)
```

```
[Out] 1/16*(10*Chi(arctanh(a*x))+5*Chi(3*arctanh(a*x))+Chi(5*arctanh(a*x)))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}}\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^8x^8 - 4\,a^6x^6 + 6\,a^4x^4 - 4\,a^2x^2 + 1\right)\text{artanh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}} \operatorname{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)), x)
```

3.484 $\int \frac{1}{(1-a^2x^2)^{9/2}\tanh^{-1}(ax)}\,dx$

Optimal. Leaf size=55

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

```
[Out] (35*CoshIntegral[ArcTanh[a*x]])/(64*a) + (21*CoshIntegral[3*ArcTanh[a*x]])/
(64*a) + (7*CoshIntegral[5*ArcTanh[a*x]])/(64*a) + CoshIntegral[7*ArcTanh[a
*x]]/(64*a)
```

Rubi [A] time = 0.122995, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5968, 3312, 3301}

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

```
[Out] (35*CoshIntegral[ArcTanh[a*x]])/(64*a) + (21*CoshIntegral[3*ArcTanh[a*x]])/
(64*a) + (7*CoshIntegral[5*ArcTanh[a*x]])/(64*a) + CoshIntegral[7*ArcTanh[a
*x]]/(64*a)
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)}\,dx = \frac{\text{Subst}\left(\int \frac{\cosh^7(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{35\cosh(x)}{64x}+\frac{21\cosh(3x)}{64x}+\frac{7\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a} + \frac{7\,\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a} + \frac{21}{}$$

$$= \frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{64a} + \frac{21\text{Chi}\left(3\tanh^{-1}(ax)\right)}{64a} + \frac{7\text{Chi}\left(5\tanh^{-1}(ax)\right)}{64a} + \frac{\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Mathematica [A] time = 0.0702491, size = 40, normalized size = 0.73

$$\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)+21\text{Chi}\left(3\tanh^{-1}(ax)\right)+7\text{Chi}\left(5\tanh^{-1}(ax)\right)+\text{Chi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]),x]
```

```
[Out] (35*CoshIntegral[ArcTanh[a*x]] + 21*CoshIntegral[3*ArcTanh[a*x]] + 7*CoshIn
tegral[5*ArcTanh[a*x]] + CoshIntegral[7*ArcTanh[a*x]])/(64*a)
```

Maple [A] time = 0.168, size = 39, normalized size = 0.7

$$\frac{35\,\text{Chi}\,(\text{Artanh}\,(ax))+21\,\text{Chi}\,(3\,\text{Artanh}\,(ax))+7\,\text{Chi}\,(5\,\text{Artanh}\,(ax))+\text{Chi}\,(7\,\text{Artanh}\,(ax))}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x)
```

```
[Out] 1/64*(35*Chi(arctanh(a*x))+21*Chi(3*arctanh(a*x))+7*Chi(5*arctanh(a*x))+Chi
(7*arctanh(a*x)))/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\,\text{artanh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^{10}x^{10}-5\,a^8x^8+10\,a^6x^6-10\,a^4x^4+5\,a^2x^2-1\right)\text{artanh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\text{artanh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)), x)
```

3.485 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Rubi [A] time = 0.0302258, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
```

```
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2} dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2} dx$$

Mathematica [A] time = 1.46264, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2,x]
```

```
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^2, x]
```

Maple [A] time = 0.247, size = 0, normalized size = 0.

$$\int \frac{1}{\left(\text{Artanh}\,(ax)\right)^2}\sqrt{-a^2x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^2, x)
```

3.486 $\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Rubi [A] time = 0.0332581, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx=\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

Mathematica [A] time = 0.893669, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2),x]
```

```
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2), x]
```

Maple [A] time = 0.385, size = 0, normalized size = 0.

$$\int \frac{1}{(\text{Artanh}(ax))^2}\frac{1}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^2-1\right)\operatorname{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{atanh}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

3.487 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rubi [A] time = 0.123095, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5966, 6034, 3298}

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}-\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]]/a
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\sqrt{1-a^2x^2}\tanh^{-1}(ax)} + \frac{\text{Shi}\left(\tanh^{-1}(ax)\right)}{a}$$

Mathematica [A] time = 0.110464, size = 32, normalized size = 0.91

$$\frac{\text{Shi}\left(\tanh^{-1}(ax)\right) - \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-(1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x])) + SinhIntegral[ArcTanh[a*x]])/a
```

Maple [A] time = 0.24, size = 62, normalized size = 1.8

$$\frac{1}{a\text{Artanh}(ax)\left(a^2x^2-1\right)}\left(\text{Artanh}(ax)\,\text{Shi}(\text{Artanh}(ax))\,x^2a^2 - \text{Shi}(\text{Artanh}(ax))\,\text{Artanh}(ax) + \sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/a*(arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2-Shi(arctanh(a*x))*arctanh(a*x)+
(-a^2*x^2+1)^(1/2))/arctanh(a*x)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{atanh}^{2}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^{2}x^{2}+1\right)^{\frac{3}{2}}\operatorname{artanh}\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^2), x)
```

3.488 $\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\mathrm{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

```
[Out] -(1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])) + (3*SinhIntegral[ArcTanh[a*x]])/
(4*a) + (3*SinhIntegral[3*ArcTanh[a*x]])/(4*a)
```

Rubi [A] time = 0.159893, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\mathrm{Shi}\left(\tanh^{-1}(ax)\right)}{4a}+\frac{3\mathrm{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x])) + (3*SinhIntegral[ArcTanh[a*x]])/
(4*a) + (3*SinhIntegral[3*ArcTanh[a*x]])/(4*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + (3a)\int \frac{x}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}\,dx$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4x} + \frac{\sinh(3x)}{4x}\right)dx, x, \tanh^{-1}(ax)\right)}{a}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{4a} + \frac{3\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{1}{a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)} + \frac{3\text{Shi}\left(\tanh^{-1}(ax)\right)}{4a} + \frac{3\text{Shi}\left(3\tanh^{-1}(ax)\right)}{4a}$$

Mathematica [A] time = 0.13313, size = 45, normalized size = 0.87

$$\frac{3\left(\text{Shi}\left(\tanh^{-1}(ax)\right) + \text{Shi}\left(3\tanh^{-1}(ax)\right)\right) - \frac{4}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-4/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]) + 3*(SinhIntegral[ArcTanh[a*x]] + Si
nhIntegral[3*ArcTanh[a*x]]))/(4*a)
```

Maple [B] time = 0.157, size = 120, normalized size = 2.3

$$\frac{1}{4\,a\text{Artanh}\,(ax)\left(a^2x^2-1\right)}\left(3\,\text{Artanh}\,(ax)\,\text{Shi}\,(\text{Artanh}\,(ax))\,x^2a^2 + 3\,\text{Artanh}\,(ax)\,\text{Shi}\,(3\,\text{Artanh}\,(ax))\,x^2a^2 - \cosh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/4/a*(3*arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2+3*arctanh(a*x)*Shi(3*arctan
h(a*x))*x^2*a^2-cosh(3*arctanh(a*x))*x^2*a^2-3*Shi(arctanh(a*x))*arctanh(a*
x)-3*Shi(3*arctanh(a*x))*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a
*x)))/arctanh(a*x)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}}\text{artanh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\text{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}\text{atanh}^2\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**2,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(5/2)*atanh(a*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}}\text{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^2), x)
```

3.489 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{5\text{Shi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

```
[Out] -(1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])) + (5*SinhIntegral[ArcTanh[a*x]])/
(8*a) + (15*SinhIntegral[3*ArcTanh[a*x]])/(16*a) + (5*SinhIntegral[5*ArcTan
h[a*x]])/(16*a)
```

Rubi [A] time = 0.176986, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a}+\frac{5\text{Shi}\left(5\tanh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x])) + (5*SinhIntegral[ArcTanh[a*x]])/
(8*a) + (15*SinhIntegral[3*ArcTanh[a*x]])/(16*a) + (5*SinhIntegral[5*ArcTan
h[a*x]])/(16*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + (5a)\int \frac{x}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\,\text{Subst}\left(\int \frac{\cosh^4(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x} + \frac{3\sinh(3x)}{16x} + \frac{\sinh(5x)}{16x}\right)dx, x, \tanh^{-1}\right.}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\,\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{16a} + \frac{5\,\text{Subst}\left(\int \frac{\sin}{}\right.}{}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)} + \frac{5\text{Shi}\left(\tanh^{-1}(ax)\right)}{8a} + \frac{15\text{Shi}\left(3\tanh^{-1}(ax)\right)}{16a} + \frac{5\text{Shi}\left(5\right.}{}$$

Mathematica [A] time = 0.166656, size = 56, normalized size = 0.85

$$\frac{5\left(2\text{Shi}\left(\tanh^{-1}(ax)\right)+3\text{Shi}\left(3\tanh^{-1}(ax)\right)+\text{Shi}\left(5\tanh^{-1}(ax)\right)\right)-\frac{16}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]) + 5*(2*SinhIntegral[ArcTanh[a*x]] +
 3*SinhIntegral[3*ArcTanh[a*x]] + SinhIntegral[5*ArcTanh[a*x]]))/(16*a)
```

Maple [B] time = 0.171, size = 176, normalized size = 2.7

$$\frac{1}{16\,a\text{Artanh}\,(ax)\left(a^2x^2-1\right)}\left(5\,\text{Artanh}\,(ax)\,\text{Shi}\,(5\,\text{Artanh}\,(ax))\,x^2a^2+10\,\text{Artanh}\,(ax)\,\text{Shi}\,(\text{Artanh}\,(ax))\,x^2a^2+15\,\text{Art}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/16/a*(5*arctanh(a*x)*Shi(5*arctanh(a*x))*x^2*a^2+10*arctanh(a*x)*Shi(arct
anh(a*x))*x^2*a^2+15*arctanh(a*x)*Shi(3*arctanh(a*x))*x^2*a^2-5*cosh(3*arct
anh(a*x))*x^2*a^2-cosh(5*arctanh(a*x))*x^2*a^2-5*Shi(5*arctanh(a*x))*arctan
h(a*x)-10*Shi(arctanh(a*x))*arctanh(a*x)-15*Shi(3*arctanh(a*x))*arctanh(a*x
)+10*(-a^2*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x))+cosh(5*arctanh(a*x)))/arctan
h(a*x)/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}} \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^8x^8-4\,a^6x^6+6\,a^4x^4-4\,a^2x^2+1\right)\operatorname{artanh}\left(ax\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}} \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^2), x)
```

3.490 $\int \frac{1}{\left(1-a^2x^2\right)^{9/2} \tanh^{-1}(ax)^2} dx$

Optimal. Leaf size=80

$$-\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{63\text{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{35\text{Shi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Shi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

```
[Out] -(1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x])) + (35*SinhIntegral[ArcTanh[a*x]])
/(64*a) + (63*SinhIntegral[3*ArcTanh[a*x]])/(64*a) + (35*SinhIntegral[5*Arc
Tanh[a*x]])/(64*a) + (7*SinhIntegral[7*ArcTanh[a*x]])/(64*a)
```

Rubi [A] time = 0.190509, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5966, 6034, 5448, 3298}

$$-\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a}+\frac{63\text{Shi}\left(3\tanh^{-1}(ax)\right)}{64a}+\frac{35\text{Shi}\left(5\tanh^{-1}(ax)\right)}{64a}+\frac{7\text{Shi}\left(7\tanh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

```
[Out] -(1/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x])) + (35*SinhIntegral[ArcTanh[a*x]])
/(64*a) + (63*SinhIntegral[3*ArcTanh[a*x]])/(64*a) + (35*SinhIntegral[5*Arc
Tanh[a*x]])/(64*a) + (7*SinhIntegral[7*ArcTanh[a*x]])/(64*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^2}\,dx = -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + (7a)\int \frac{x}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)}\,dx$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{7\,\text{Subst}\left(\int \frac{\cosh^6(x)\sinh(x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{7\,\text{Subst}\left(\int \left(\frac{5\sinh(x)}{64x} + \frac{9\sinh(3x)}{64x} + \frac{5\sinh(5x)}{64x} + \frac{\sinh(7x}{64x}\right.\right.}{a}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{7\,\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \tanh^{-1}(ax)\right)}{64a} + \frac{35\,\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)} + \frac{35\text{Shi}\left(\tanh^{-1}(ax)\right)}{64a} + \frac{63\text{Shi}\left(3\tanh^{-1}(ax)\right)}{64a} + \frac{35\text{S}}{}$$

Mathematica [A] time = 0.209039, size = 65, normalized size = 0.81

$$\frac{7\left(5\text{Shi}\left(\tanh^{-1}(ax)\right) + 9\text{Shi}\left(3\tanh^{-1}(ax)\right) + 5\text{Shi}\left(5\tanh^{-1}(ax)\right) + \text{Shi}\left(7\tanh^{-1}(ax)\right)\right) - \frac{64}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^2),x]
```

```
[Out] (-64/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]) + 7*(5*SinhIntegral[ArcTanh[a*x]] +
 9*SinhIntegral[3*ArcTanh[a*x]] + 5*SinhIntegral[5*ArcTanh[a*x]] + SinhInte
gral[7*ArcTanh[a*x]]))/(64*a)
```

Maple [B] time = 0.159, size = 232, normalized size = 2.9

$$\frac{1}{64\,a\text{Artanh}\,(ax)\left(a^2x^2-1\right)}\left(35\,\text{Artanh}\,(ax)\,\text{Shi}\,(\text{Artanh}\,(ax))\,x^2a^2 + 63\,\text{Artanh}\,(ax)\,\text{Shi}\,(3\,\text{Artanh}\,(ax))\,x^2a^2 + 35\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x)
```

```
[Out] 1/64/a*(35*arctanh(a*x)*Shi(arctanh(a*x))*x^2*a^2+63*arctanh(a*x)*Shi(3*arc
tanh(a*x))*x^2*a^2+35*arctanh(a*x)*Shi(5*arctanh(a*x))*x^2*a^2+7*arctanh(a*
x)*Shi(7*arctanh(a*x))*x^2*a^2-21*cosh(3*arctanh(a*x))*x^2*a^2-7*cosh(5*arc
tanh(a*x))*x^2*a^2-cosh(7*arctanh(a*x))*x^2*a^2-35*Shi(arctanh(a*x))*arctan
h(a*x)-63*Shi(3*arctanh(a*x))*arctanh(a*x)-35*Shi(5*arctanh(a*x))*arctanh(a
*x)-7*Shi(7*arctanh(a*x))*arctanh(a*x)+35*(-a^2*x^2+1)^(1/2)+21*cosh(3*arct
anh(a*x))+7*cosh(5*arctanh(a*x))+cosh(7*arctanh(a*x)))/arctanh(a*x)/(a^2*x^
2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}} \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^{10}x^{10}-5\,a^8x^8+10\,a^6x^6-10\,a^4x^4+5\,a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}} \operatorname{artanh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^2), x)
```

3.491 $\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Rubi [A] time = 0.0311558, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
```

```
[Out] Defer[Int][Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Rubi steps

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx = \int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Mathematica [A] time = 1.7143, size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-a^2x^2}}{\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3,x]
```

```
[Out] Integrate[Sqrt[1 - a^2*x^2]/ArcTanh[a*x]^3, x]
```

Maple [A] time = 0.27, size = 0, normalized size = 0.

$$\int \frac{1}{(\text{Artanh}(ax))^3} \sqrt{-a^2x^2+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

```
[Out] int((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")

[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")

[Out] integral(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\operatorname{atanh}^3(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)

[Out] Integral(sqrt(-(a*x - 1)*(a*x + 1))/atanh(a*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2+1}}{\operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")

[Out] integrate(sqrt(-a^2*x^2 + 1)/arctanh(a*x)^3, x)
```

3.492 $\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3},x\right)$$

```
[Out] Unintegrable[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Rubi [A] time = 0.03318, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} dx = \int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} dx$$

Mathematica [A] time = 1.0724, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3),x]
```

```
[Out] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^3), x]
```

Maple [A] time = 0.559, size = 0, normalized size = 0.

$$\int \frac{1}{(\text{Artanh}(ax))^3} \frac{1}{\sqrt{-a^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

```
[Out] int(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^2-1\right)\mathrm{artanh}\,(ax)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)}\,\mathrm{atanh}^3\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(1/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*atanh(a*x)**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\mathrm{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(1/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

3.493 $\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=65

$$-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{\operatorname{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

```
[Out] -1/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rubi [A] time = 0.167022, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {5966, 6006, 5968, 3301}

$$-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}-\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{\operatorname{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2) - x/(2*Sqrt[1 - a^2*x^2]*ArcTanh[
a*x]) + CoshIntegral[ArcTanh[a*x]]/(2*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6006

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^m*(d + e*x^2)^(q + 1)*(a + b*Arc
Tanh[c*x])^(p + 1))/(b*c*d*(p + 1)), x] - Dist[(f*m)/(b*c*(p + 1)), Int[(f*
x)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e, f, m, q}, x] && EqQ[c^2*d + e, 0] && EqQ[m + 2*q + 2, 0] && LtQ
[p, -1]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}+\frac{1}{2}a\int \frac{x}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}\,dx$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{1}{2}\int \frac{1}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx,x,\tanh^{-1}\right.}{2a}$$
$$= -\frac{1}{2a\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}-\frac{x}{2\sqrt{1-a^2x^2}\tanh^{-1}(ax)}+\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)}{2a}$$

Mathematica [A] time = 0.114615, size = 44, normalized size = 0.68

$$\frac{\text{Chi}\left(\tanh^{-1}(ax)\right)-\frac{ax\tanh^{-1}(ax)+1}{\sqrt{1-a^2x^2}\tanh^{-1}(ax)^2}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-((1 + a*x*ArcTanh[a*x])/(Sqrt[1 - a^2*x^2]*ArcTanh[a*x]^2)) + CoshIntegra
l[ArcTanh[a*x]])/(2*a)
```

Maple [A] time = 0.253, size = 86, normalized size = 1.3

$$\frac{1}{2a(\text{Artanh}(ax))^2\left(a^2x^2-1\right)}\left((\text{Artanh}(ax))^2\,\text{Chi}(\text{Artanh}(ax))\,x^2a^2+\sqrt{-a^2x^2+1}ax\text{Artanh}(ax)-\text{Chi}(\text{Artanh}(ax)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/2/a*(arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+(-a^2*x^2+1)^(1/2)*a*x*arct
anh(a*x)-Chi(arctanh(a*x))*arctanh(a*x)^2+(-a^2*x^2+1)^(1/2))/arctanh(a*x)^
2/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-a^2x^2+1}}{\left(a^4x^4-2\,a^2x^2+1\right)\text{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^4*x^4 - 2*a^2*x^2 + 1)*arctanh(a*x)^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\text{atanh}^3\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(3/2)/atanh(a*x)**3,x)
```

```
[Out] Integral(1/((-(a*x - 1)*(a*x + 1))**(3/2)*atanh(a*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-a^2x^2+1\right)^{\frac{3}{2}}\text{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(3/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(3/2)*arctanh(a*x)^3), x)
```

3.494 $\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=79

$$-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{9\text{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

```
[Out] -1/(2*a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) - (3*x)/(2*(1 - a^2*x^2)^(3/2)*
ArcTanh[a*x]) + (3*CoshIntegral[ArcTanh[a*x]])/(8*a) + (9*CoshIntegral[3*Ar
cTanh[a*x]])/(8*a)
```

Rubi [A] time = 0.361985, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}+\frac{9\text{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```


```
[Out] -1/(2*a*(1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) - (3*x)/(2*(1 - a^2*x^2)^(3/2)*
ArcTanh[a*x]) + (3*CoshIntegral[ArcTanh[a*x]])/(8*a) + (9*CoshIntegral[3*Ar
cTanh[a*x]])/(8*a)
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^3}\,dx &= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+\frac{1}{2}(3a)\int \frac{x}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}\,dx\\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3}{2}\int \frac{1}{\left(1-a^2x^2\right)^{5/2}\text{ta}\\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\,\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,d\right.}{2a}\\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}\right.\right.}{}\\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,d\right.}{8a}\\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}-\frac{3x}{2\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)}+\frac{3\text{Chi}\left(\tanh^{-1}(ax)\right)}{8a}
\end{aligned}$$

Mathematica [A] time = 0.206677, size = 56, normalized size = 0.71

$$\frac{-\frac{4\left(3ax\tanh^{-1}(ax)+1\right)}{\left(1-a^2x^2\right)^{3/2}\tanh^{-1}(ax)^2}+3\text{Chi}\left(\tanh^{-1}(ax)\right)+9\text{Chi}\left(3\tanh^{-1}(ax)\right)}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^3),x]
```

```
[Out] ((-4*(1 + 3*a*x*ArcTanh[a*x]))/((1 - a^2*x^2)^(3/2)*ArcTanh[a*x]^2) + 3*Cos
hIntegral[ArcTanh[a*x]] + 9*CoshIntegral[3*ArcTanh[a*x]])/(8*a)
```

Maple [B] time = 0.163, size = 180, normalized size = 2.3

$$\frac{1}{8\,a\,(\mathrm{Artanh}\,(ax))^2\,(a^2x^2-1)}\Big(9\,(\mathrm{Artanh}\,(ax))^2\,\mathrm{Chi}\,(3\,\mathrm{Artanh}\,(ax))\,x^2a^2+3\,(\mathrm{Artanh}\,(ax))^2\,\mathrm{Chi}\,(\mathrm{Artanh}\,(ax))\,x^2a^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/8/a*(9*arctanh(a*x)^2*Chi(3*arctanh(a*x))*x^2*a^2+3*arctanh(a*x)^2*Chi(ar
ctanh(a*x))*x^2*a^2-3*arctanh(a*x)*sinh(3*arctanh(a*x))*x^2*a^2-cosh(3*arct
anh(a*x))*x^2*a^2+3*(-a^2*x^2+1)^(1/2)*a*x*arctanh(a*x)-9*Chi(3*arctanh(a*x
))*arctanh(a*x)^2-3*Chi(arctanh(a*x))*arctanh(a*x)^2+3*sinh(3*arctanh(a*x))
*arctanh(a*x)+3*(-a^2*x^2+1)^(1/2)+cosh(3*arctanh(a*x)))/arctanh(a*x)^2/(a^
2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}}\,\mathrm{artanh}\,(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^6x^6-3\,a^4x^4+3\,a^2x^2-1\right)\mathrm{artanh}\,(ax)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*arctanh
(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(5/2)/atanh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{5}{2}} \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(5/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(5/2)*arctanh(a*x)^3), x)
```

3.495 $\int \frac{1}{(1-a^2x^2)^{7/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{16a}+\frac{45\text{Chi}\left(3\tanh^{-1}(ax)\right)}{32a}+\frac{25\text{Chi}\left(5\tanh^{-1}(ax)\right)}{32a}$$

```
[Out] -1/(2*a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (5*x)/(2*(1 - a^2*x^2)^(5/2)*
ArcTanh[a*x]) + (5*CoshIntegral[ArcTanh[a*x]])/(16*a) + (45*CoshIntegral[3*
ArcTanh[a*x]])/(32*a) + (25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

Rubi [A] time = 0.404697, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+\frac{5\text{Chi}\left(\tanh^{-1}(ax)\right)}{16a}+\frac{45\text{Chi}\left(3\tanh^{-1}(ax)\right)}{32a}+\frac{25\text{Chi}\left(5\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```


```
[Out] -1/(2*a*(1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (5*x)/(2*(1 - a^2*x^2)^(5/2)*
ArcTanh[a*x]) + (5*CoshIntegral[ArcTanh[a*x]])/(16*a) + (45*CoshIntegral[3*
ArcTanh[a*x]])/(32*a) + (25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```


Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^3}\,dx = -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+\frac{1}{2}(5a)\int \frac{x}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}\,dx$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5}{2}\int \frac{1}{\left(1-a^2x^2\right)^{7/2}\,\mathrm{ta}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\,\mathrm{Subst}\left(\int \frac{\cosh^5(x)}{x}\,\mathit{d}\right.}{2a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\,\mathrm{Subst}\left(\int\left(\frac{5\cosh(x)}{8x}\right.\right.}{}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\,\mathrm{Subst}\left(\int \frac{\cosh(5x)}{x}\,\mathit{d}\right.}{32a}$$

$$= -\frac{1}{2a\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}-\frac{5x}{2\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}+\frac{5\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{16a}$$

Mathematica [A] time = 0.229125, size = 79, normalized size = 0.85

$$\frac{-\frac{80ax}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)}-\frac{16}{\left(1-a^2x^2\right)^{5/2}\tanh^{-1}(ax)^2}+10\mathrm{Chi}\left(\tanh^{-1}(ax)\right)+45\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)+25\mathrm{Chi}\left(5\tanh^{-1}(ax)\right)}{32a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-16/((1 - a^2*x^2)^(5/2)*ArcTanh[a*x]^2) - (80*a*x)/((1 - a^2*x^2)^(5/2)*A
rcTanh[a*x]) + 10*CoshIntegral[ArcTanh[a*x]] + 45*CoshIntegral[3*ArcTanh[a*
```

```
x]] + 25*CoshIntegral[5*ArcTanh[a*x]])/(32*a)
```

Maple [B] time = 0.174, size = 272, normalized size = 2.9

$$\frac{1}{32\,a\,(\mathrm{Artanh}\,(ax))^{2}\,(a^{2}x^{2}-1)}\left(10\,(\mathrm{Artanh}\,(ax))^{2}\,\mathrm{Chi}\,(\mathrm{Artanh}\,(ax))\,x^{2}a^{2}+45\,(\mathrm{Artanh}\,(ax))^{2}\,\mathrm{Chi}\,(3\,\mathrm{Artanh}\,(ax))\,x^{2}a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/32/a*(10*arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+45*arctanh(a*x)^2*Chi(3
*arctanh(a*x))*x^2*a^2+25*arctanh(a*x)^2*Chi(5*arctanh(a*x))*x^2*a^2-5*arct
anh(a*x)*sinh(5*arctanh(a*x))*x^2*a^2-15*arctanh(a*x)*sinh(3*arctanh(a*x))*
x^2*a^2-5*cosh(3*arctanh(a*x))*x^2*a^2-cosh(5*arctanh(a*x))*x^2*a^2+10*(-a^
2*x^2+1)^(1/2)*a*x*arctanh(a*x)-10*Chi(arctanh(a*x))*arctanh(a*x)^2-45*Chi(
3*arctanh(a*x))*arctanh(a*x)^2-25*Chi(5*arctanh(a*x))*arctanh(a*x)^2+5*sinh
(5*arctanh(a*x))*arctanh(a*x)+15*sinh(3*arctanh(a*x))*arctanh(a*x)+10*(-a^2
*x^2+1)^(1/2)+5*cosh(3*arctanh(a*x))+cosh(5*arctanh(a*x)))/arctanh(a*x)^2/(
a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-a^{2}x^{2}+1)^{\frac{7}{2}}\,\mathrm{artanh}\,(ax)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-a^{2}x^{2}+1}}{(a^{8}x^{8}-4\,a^{6}x^{6}+6\,a^{4}x^{4}-4\,a^{2}x^{2}+1)\,\mathrm{artanh}\,(ax)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*x^2 + 1)/((a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 +
 1)*arctanh(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(7/2)/atanh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{7}{2}} \operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(7/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(7/2)*arctanh(a*x)^3), x)
```

3.496 $\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=107

$$-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{128a}+\frac{189\text{Chi}\left(3\tanh^{-1}(ax)\right)}{128a}+\frac{175\text{Chi}}{}$$

```
[Out] -1/(2*a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (7*x)/(2*(1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/(128*a) + (189*CoshIntegral
[3*ArcTanh[a*x]])/(128*a) + (175*CoshIntegral[5*ArcTanh[a*x]])/(128*a) + (4
9*CoshIntegral[7*ArcTanh[a*x]])/(128*a)
```

Rubi [A] time = 0.43919, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5966, 6032, 6034, 5448, 3301, 5968, 3312}

$$-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\text{Chi}\left(\tanh^{-1}(ax)\right)}{128a}+\frac{189\text{Chi}\left(3\tanh^{-1}(ax)\right)}{128a}+\frac{175\text{Chi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

```
[Out] -1/(2*a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (7*x)/(2*(1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/(128*a) + (189*CoshIntegral
[3*ArcTanh[a*x]])/(128*a) + (175*CoshIntegral[5*ArcTanh[a*x]])/(128*a) + (4
9*CoshIntegral[7*ArcTanh[a*x]])/(128*a)
```

Rule 5966

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(p + 1))/(b*c*d*(
p + 1)), x] + Dist[(2*c*(q + 1))/(b*(p + 1)), Int[x*(d + e*x^2)^q*(a + b*Ar
cTanh[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
0] && LtQ[q, -1] && LtQ[p, -1]
```

Rule 6032

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x])^(
p + 1))/(b*c*d*(p + 1)), x] + (Dist[(c*(m + 2*q + 2))/(b*(p + 1)), Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x] - Dist[m/(b*c*(p +
 1)), Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTanh[c*x])^(p + 1), x], x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && LtQ[q, -1]
 && LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

Rule 6034

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Dist[d^q/c^(m + 1), Subst[Int[((a + b*x)^p*Sinh[x]^m
)/Cosh[x]^(m + 2*(q + 1)), x], x, ArcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e
, p}, x] && EqQ[c^2*d + e, 0] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (Int
egerQ[q] || GtQ[d, 0])
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5968

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Dist[d^q/c, Subst[Int[(a + b*x)^p/Cosh[x]^(2*(q + 1)), x], x, A
rcTanh[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && IL
tQ[2*(q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^3}\,dx &= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{1}{2}(7a)\int \frac{x}{\left(1-a^2x^2\right)^{9/2}\tanh^{-1}(ax)^2}\,dx \\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7}{2}\int \frac{1}{\left(1-a^2x^2\right)^{9/2}\,\mathrm{ta}} \\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7\,\mathrm{Subst}\left(\int \frac{\cosh^7(x)}{x}\,d\right.}{2a} \\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7\,\mathrm{Subst}\left(\int \left(\frac{35\cosh(}{64x}\right.\right.}{} \\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{7\,\mathrm{Subst}\left(\int \frac{\cosh(7x)}{x}\,d\right.}{128} \\
&= -\frac{1}{2a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}-\frac{7x}{2\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}+\frac{35\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{128a}
\end{aligned}
$$

Mathematica [A] time = 0.21183, size = 99, normalized size = 0.93

$$
\frac{1}{128}\left(-\frac{448x}{\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)}-\frac{64}{a\left(1-a^2x^2\right)^{7/2}\tanh^{-1}(ax)^2}+\frac{35\mathrm{Chi}\left(\tanh^{-1}(ax)\right)}{a}+\frac{189\mathrm{Chi}\left(3\tanh^{-1}(ax)\right)}{a}+\frac{17}{}\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a^2*x^2)^(9/2)*ArcTanh[a*x]^3),x]
```

```
[Out] (-64/(a*(1 - a^2*x^2)^(7/2)*ArcTanh[a*x]^2) - (448*x)/((1 - a^2*x^2)^(7/2)*
ArcTanh[a*x]) + (35*CoshIntegral[ArcTanh[a*x]])/a + (189*CoshIntegral[3*Arc
Tanh[a*x]])/a + (175*CoshIntegral[5*ArcTanh[a*x]])/a + (49*CoshIntegral[7*A
rcTanh[a*x]])/a)/128
```

Maple [B] time = 0.186, size = 364, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x)
```

```
[Out] 1/128/a*(35*arctanh(a*x)^2*Chi(arctanh(a*x))*x^2*a^2+189*arctanh(a*x)^2*Chi
(3*arctanh(a*x))*x^2*a^2+175*arctanh(a*x)^2*Chi(5*arctanh(a*x))*x^2*a^2+49*
arctanh(a*x)^2*Chi(7*arctanh(a*x))*x^2*a^2-63*arctanh(a*x)*sinh(3*arctanh(a
*x))*x^2*a^2-35*arctanh(a*x)*sinh(5*arctanh(a*x))*x^2*a^2-7*arctanh(a*x)*si
nh(7*arctanh(a*x))*x^2*a^2-21*cosh(3*arctanh(a*x))*x^2*a^2-7*cosh(5*arctanh
(a*x))*x^2*a^2-cosh(7*arctanh(a*x))*x^2*a^2+35*(-a^2*x^2+1)^(1/2)*a*x*arcta
nh(a*x)-35*Chi(arctanh(a*x))*arctanh(a*x)^2-189*Chi(3*arctanh(a*x))*arctanh
(a*x)^2-175*Chi(5*arctanh(a*x))*arctanh(a*x)^2-49*Chi(7*arctanh(a*x))*arcta
nh(a*x)^2+63*sinh(3*arctanh(a*x))*arctanh(a*x)+35*sinh(5*arctanh(a*x))*arct
anh(a*x)+7*sinh(7*arctanh(a*x))*arctanh(a*x)+35*(-a^2*x^2+1)^(1/2)+21*cosh(
3*arctanh(a*x))+7*cosh(5*arctanh(a*x))+cosh(7*arctanh(a*x)))/arctanh(a*x)^2
/(a^2*x^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}}\operatorname{artanh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^{10}x^{10}-5\,a^8x^8+10\,a^6x^6-10\,a^4x^4+5\,a^2x^2-1\right)\operatorname{artanh}\left(ax\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integrate(-sqrt(-a^2*x^2 + 1)/((a^10*x^10 - 5*a^8*x^8 + 10*a^6*x^6 - 10*a^4*
x^4 + 5*a^2*x^2 - 1)*arctanh(a*x)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*x**2+1)**(9/2)/atanh(a*x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-a^2x^2+1\right)^{\frac{9}{2}} \operatorname{artanh}(ax)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*x^2+1)^(9/2)/arctanh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*x^2 + 1)^(9/2)*arctanh(a*x)^3), x)
```

3.497 $\int \frac{(d+ex)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$

Optimal. Leaf size=122

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}-\frac{b^2e\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^2}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\log\left(\frac{2}{1-cx}\right)\left(a+b\tan\right.}{c^2}$$

```
[Out] (d*(a + b*ArcTanh[c*x])^3)/(3*b*c) - (e*(a + b*ArcTanh[c*x])^3)/(3*b*c^2) +
 (e*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^2 + (b*e*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 - c*x)])/c^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2
*c^2)
```

Rubi [A] time = 0.318412, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6048, 5948, 5984, 5918, 6058, 6610}

$$\frac{be\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)\left(a+b\tanh^{-1}(cx)\right)}{c^2}-\frac{b^2e\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^2}-\frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2}+\frac{e\log\left(\frac{2}{1-cx}\right)\left(a+b\tan\right.}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```


```
[Out] (d*(a + b*ArcTanh[c*x])^3)/(3*b*c) - (e*(a + b*ArcTanh[c*x])^3)/(3*b*c^2) +
 (e*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c^2 + (b*e*(a + b*ArcTanh[c*x]
)*PolyLog[2, 1 - 2/(1 - c*x)])/c^2 - (b^2*e*PolyLog[3, 1 - 2/(1 - c*x)])/(2
*c^2)
```


Rule 6048

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(m_.))/(
(d_) + (e_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTanh[c*x])
^p/(d + e*x^2), (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && I
GtQ[p, 0] && EqQ[c^2*d + e, 0] && IGtQ[m, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
```

```
]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(d+ex)\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx = \int \left(\frac{d\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}+\frac{ex\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\right)dx$$
$$= d\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx + e\int \frac{x\left(a+b\tanh^{-1}(cx)\right)^2}{1-c^2x^2}\,dx$$
$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\int \frac{\left(a+b\tanh^{-1}(cx)\right)^2}{1-cx}\,dx}{c}$$
$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right.}{c^2}$$
$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right.}{c^2}$$
$$= \frac{d\left(a+b\tanh^{-1}(cx)\right)^3}{3bc} - \frac{e\left(a+b\tanh^{-1}(cx)\right)^3}{3bc^2} + \frac{e\left(a+b\tanh^{-1}(cx)\right)^2\log\left(\frac{2}{1-cx}\right.}{c^2}$$

Mathematica [A] time = 0.386973, size = 193, normalized size = 1.58

$$\frac{-6be\text{PolyLog}\left(2,-e^{-2\tanh^{-1}(cx)}\right)\left(a+b\tanh^{-1}(cx)\right)-3b^2e\text{PolyLog}\left(3,-e^{-2\tanh^{-1}(cx)}\right)-3a^2cd\log(1-cx)+3a^2cd\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)*(a + b*ArcTanh[c*x])^2)/(1 - c^2*x^2),x]
```

```
[Out] (6*a*b*c*d*ArcTanh[c*x]^2 + 6*a*b*e*ArcTanh[c*x]^2 + 2*b^2*c*d*ArcTanh[c*x]
^3 + 2*b^2*e*ArcTanh[c*x]^3 + 12*a*b*e*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c
*x])] + 6*b^2*e*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] - 3*a^2*c*d*Log
[1 - c*x] - 3*a^2*e*Log[1 - c*x] + 3*a^2*c*d*Log[1 + c*x] - 3*a^2*e*Log[1 +
 c*x] - 6*b*e*(a + b*ArcTanh[c*x])*PolyLog[2, -E^(-2*ArcTanh[c*x])] - 3*b^2
*e*PolyLog[3, -E^(-2*ArcTanh[c*x])])/(6*c^2)
```

Maple [C] time = 0.444, size = 1871, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x)
```

```
[Out] 1/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*
x^2+1)+1))^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi*e-1/2*I/c*b^2*arctanh(c*
x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi*
d-1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*
x^2+1)+1))^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi*d-1/4*I/c*b^2*arctanh(c*
x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*
d+1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2
/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*d+1/4*I/c^2*b^2*arctanh(c*x)^
2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e+1
/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e+1/2*I/c^2*
b^2*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi*e-1/2*I/c^2*b^2*
arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e+1/4*I/c^2*b^2*arct
anh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi*e-
1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*d-1/4*I/c*b^2
*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*
Pi*d+1/2*I/c*b^2*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*Pi*d-1
/2*I/c*b^2*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*d+1/2*I/c
^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(
c^2*x^2-1))^2*Pi*e-1/4*I/c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1
))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e+1/2/c*a*
b*ln(1/2+1/2*c*x)*ln(c*x-1)*d-1/c^2*a*b*arctanh(c*x)*ln(c*x+1)*e+1/2/c^2*a*
b*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2)*e-1/2/c^2*a*b*ln(-1/2*c*x+1/2)*ln(c*x+1)
*e+1/2/c^2*a*b*ln(1/2+1/2*c*x)*ln(c*x-1)*e-1/c*a*b*arctanh(c*x)*ln(c*x-1)*d
+1/c*a*b*arctanh(c*x)*ln(c*x+1)*d-1/2/c*a*b*ln(1/2+1/2*c*x)*ln(-1/2*c*x+1/2
)*d+1/2/c*a*b*ln(-1/2*c*x+1/2)*ln(c*x+1)*d-1/c^2*a*b*arctanh(c*x)*ln(c*x-1)
*e+1/2*I/c*b^2*arctanh(c*x)^2*Pi*d+1/2*I/c^2*b^2*arctanh(c*x)^2*Pi*e-1/4*I/
c^2*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*Pi*e+
1/4*I/c*b^2*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(
c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*P
i*d+1/3/c*b^2*d*arctanh(c*x)^3+1/2/c*a^2*ln(c*x+1)*d-1/2/c^2*a^2*ln(c*x-1)*
e-1/2/c^2*a^2*ln(c*x+1)*e-1/2/c^2*b^2*e*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-
1/3/c^2*b^2*e*arctanh(c*x)^3-1/2/c*a^2*ln(c*x-1)*d-1/4/c*a*b*ln(c*x+1)^2*d-
1/4/c*a*b*ln(c*x-1)^2*d-1/c*b^2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2
))*d-1/2/c*b^2*arctanh(c*x)^2*ln(c*x-1)*d+1/2/c*b^2*arctanh(c*x)^2*ln(c*x+1
)*d-1/2/c^2*b^2*arctanh(c*x)^2*ln(c*x-1)*e+1/c^2*b^2*arctanh(c*x)^2*ln(2)*e
+1/4/c^2*a*b*ln(c*x+1)^2*e+1/c^2*a*b*dilog(1/2+1/2*c*x)*e-1/4/c^2*a*b*ln(c*
x-1)^2*e-1/2/c^2*b^2*arctanh(c*x)^2*ln(c*x+1)*e+1/c^2*b^2*arctanh(c*x)^2*ln
((c*x+1)/(-c^2*x^2+1)^(1/2))*e+1/c^2*b^2*e*arctanh(c*x)*polylog(2,-(c*x+1)^
2/(-c^2*x^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$abd\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right)\operatorname{artanh}(cx) + \frac{1}{2}a^2d\left(\frac{\log(cx+1)}{c} - \frac{\log(cx-1)}{c}\right) - \frac{\left(\log(cx+1)^2 - 2\log(cx+1)\log(c\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="maxima")
```

```
[Out] a*b*d*(log(c*x + 1)/c - log(c*x - 1)/c)*arctanh(c*x) + 1/2*a^2*d*(log(c*x +
 1)/c - log(c*x - 1)/c) - 1/4*(log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1)
```

```
 + log(c*x - 1)^2)*a*b*d/c - 1/2*a^2*e*log(c^2*x^2 - 1)/c^2 + 1/24*(3*(c*d
- e)*b^2*log(c*x + 1)*log(-c*x + 1)^2 - (c*d + e)*b^2*log(-c*x + 1)^3)/c^2
- integrate(1/4*(4*a*b*c*e*x*log(c*x + 1) + (b^2*c*e*x + b^2*c*d)*log(c*x +
 1)^2 - (4*a*b*c*e*x - ((c^2*d - 3*c*e)*b^2*x - (c*d + e)*b^2)*log(c*x + 1)
)*log(-c*x + 1))/(c^3*x^2 - c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{a^2ex + a^2d + \left(b^2ex + b^2d\right)\text{artanh}\left(cx\right)^2 + 2\left(abex + abd\right)\text{artanh}\left(cx\right)}{c^2x^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e*x + a^2*d + (b^2*e*x + b^2*d)*arctanh(c*x)^2 + 2*(a*b*e*x
+ a*b*d)*arctanh(c*x))/(c^2*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{a^2d}{c^2x^2-1}\,dx - \int\frac{a^2ex}{c^2x^2-1}\,dx - \int\frac{b^2d\,\text{atanh}^2\left(cx\right)}{c^2x^2-1}\,dx - \int\frac{2abd\,\text{atanh}\left(cx\right)}{c^2x^2-1}\,dx - \int\frac{b^2ex\,\text{atanh}^2\left(cx\right)}{c^2x^2-1}\,dx - \int\frac{2ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*atanh(c*x))**2/(-c**2*x**2+1),x)
```

```
[Out] -Integral(a**2*d/(c**2*x**2 - 1), x) - Integral(a**2*e*x/(c**2*x**2 - 1), x
) - Integral(b**2*d*atanh(c*x)**2/(c**2*x**2 - 1), x) - Integral(2*a*b*d*at
anh(c*x)/(c**2*x**2 - 1), x) - Integral(b**2*e*x*atanh(c*x)**2/(c**2*x**2 -
 1), x) - Integral(2*a*b*e*x*atanh(c*x)/(c**2*x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(ex + d)(b\,\text{artanh}\left(cx\right) + a)^2}{c^2x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arctanh(c*x))^2/(-c^2*x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(e*x + d)*(b*arctanh(c*x) + a)^2/(c^2*x^2 - 1), x)
```

3.498 $\int (c + dx^2)^4 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=245

$$\frac{d^2x^4\left(378a^4c^2+180a^2cd+35d^2\right)}{1260a^5}+\frac{dx^2\left(378a^4c^2d+420a^6c^3+180a^2cd^2+35d^3\right)}{630a^7}+\frac{\left(378a^4c^2d^2+420a^6c^3d+315a^8c^4+\right.}{630a}$$

```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcTanh[a*x] + (4*c^3*d*x^
3*ArcTanh[a*x])/3 + (6*c^2*d^2*x^5*ArcTanh[a*x])/5 + (4*c*d^3*x^7*ArcTanh[a
*x])/7 + (d^4*x^9*ArcTanh[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```

Rubi [A] time = 0.179877, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {194, 5976, 1810, 260}

$$\frac{d^2x^4\left(378a^4c^2+180a^2cd+35d^2\right)}{1260a^5}+\frac{dx^2\left(378a^4c^2d+420a^6c^3+180a^2cd^2+35d^3\right)}{630a^7}+\frac{\left(378a^4c^2d^2+420a^6c^3d+315a^8c^4+\right.}{630a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^4*ArcTanh[a*x],x]
```

```
[Out] (d*(420*a^6*c^3 + 378*a^4*c^2*d + 180*a^2*c*d^2 + 35*d^3)*x^2)/(630*a^7) +
(d^2*(378*a^4*c^2 + 180*a^2*c*d + 35*d^2)*x^4)/(1260*a^5) + (d^3*(36*a^2*c
+ 7*d)*x^6)/(378*a^3) + (d^4*x^8)/(72*a) + c^4*x*ArcTanh[a*x] + (4*c^3*d*x^
3*ArcTanh[a*x])/3 + (6*c^2*d^2*x^5*ArcTanh[a*x])/5 + (4*c*d^3*x^7*ArcTanh[a
*x])/7 + (d^4*x^9*ArcTanh[a*x])/9 + ((315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(630*a^9)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+dx^2\right)^4 \tanh^{-1}(ax)\,dx = c^4x\tanh^{-1}(ax)+\frac{4}{3}c^3dx^3\tanh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tanh^{-1}(ax)+\frac{4}{7}cd^3x^7\tanh^{-1}(ax)+\frac{1}{9}$$

$$= c^4x\tanh^{-1}(ax)+\frac{4}{3}c^3dx^3\tanh^{-1}(ax)+\frac{6}{5}c^2d^2x^5\tanh^{-1}(ax)+\frac{4}{7}cd^3x^7\tanh^{-1}(ax)+\frac{1}{9}$$

$$= \frac{d\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)x^2}{630a^7}+\frac{d^2\left(378a^4c^2+180a^2cd+35d^2\right)x^4}{1260a^5}+$$

$$= \frac{d\left(420a^6c^3+378a^4c^2d+180a^2cd^2+35d^3\right)x^2}{630a^7}+\frac{d^2\left(378a^4c^2+180a^2cd+35d^2\right)x^4}{1260a^5}+$$

Mathematica [A] time = 0.111614, size = 213, normalized size = 0.87

$$\frac{a^2dx^2\left(3a^6\left(756c^2dx^2+1680c^3+240cd^2x^4+35d^3x^6\right)+4a^4d\left(1134c^2+270cdx^2+35d^2x^4\right)+30a^2d^2\left(72c+7dx^2\right)+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^4*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) + 4*a^4*d*(1134*c^2 + 270
*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c*d^2*x^4 +
35*d^3*x^6)) + 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*x^4 + 180*c*
d^3*x^6 + 35*d^4*x^8)*ArcTanh[a*x] + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 378*
a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*Log[1 - a^2*x^2])/(7560*a^9)
```

Maple [A] time = 0.035, size = 334, normalized size = 1.4

$$\frac{d^4x^8}{72\,a}+\frac{x^6d^4}{54\,a^3}+\frac{x^2d^4}{18\,a^7}+\frac{x^4d^4}{36\,a^5}+\frac{\ln(ax-1)\,c^4}{2\,a}+\frac{\ln(ax-1)\,d^4}{18\,a^9}+\frac{\ln(ax+1)\,c^4}{2\,a}+\frac{\ln(ax+1)\,d^4}{18\,a^9}+\frac{d^4x^9\,\mathrm{Artanh}\,(ax)}{9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^4*arctanh(a*x),x)
```

```
[Out] 4/3*c^3*d*x^3*arctanh(a*x)+6/5*c^2*d^2*x^5*arctanh(a*x)+4/7*c*d^3*x^7*arcta
nh(a*x)+1/72*d^4*x^8/a+1/9*d^4*x^9*arctanh(a*x)+1/54/a^3*x^6*d^4+1/18/a^7*x
^2*d^4+1/36/a^5*x^4*d^4+1/2/a*ln(a*x-1)*c^4+1/18/a^9*ln(a*x-1)*d^4+1/2/a*ln
(a*x+1)*c^4+1/18/a^9*ln(a*x+1)*d^4+1/7/a^3*x^4*c*d^3+2/7/a^5*x^2*c*d^3+2/3/
a^3*ln(a*x-1)*c^3*d+3/5/a^5*ln(a*x-1)*c^2*d^2+2/7/a^7*ln(a*x-1)*c*d^3+2/3/a
^3*ln(a*x+1)*c^3*d+3/5/a^5*ln(a*x+1)*c^2*d^2+2/7/a^7*ln(a*x+1)*c*d^3+3/5/a^
3*c^2*d^2*x^2+2/21/a*c*d^3*x^6+3/10/a*c^2*d^2*x^4+2/3/a*c^3*d*x^2+c^4*x*arc
tanh(a*x)
```

Maxima [A] time = 0.966531, size = 373, normalized size = 1.52

$$\frac{1}{7560}\,a\left(\frac{105\,a^6d^4x^8+20\left(36\,a^6cd^3+7\,a^4d^4\right)x^6+6\left(378\,a^6c^2d^2+180\,a^4cd^3+35\,a^2d^4\right)x^4+12\left(420\,a^6c^3d+378\,a^4c^2d^2\cdots\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 + 7*a^4*d^4)*x^6 + 6*(378*a^6
*c^2*d^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d + 378*a^4*c^
2*d^2 + 180*a^2*c*d^3 + 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 + 420*a^6*c^3*d
+ 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x + 1)/a^10 + 12*(315*a^8
*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a*x -
1)/a^10) + 1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*
x^3 + 315*c^4*x)*arctanh(a*x)
```

Fricas [A] time = 1.94526, size = 559, normalized size = 2.28

$$\frac{105\,a^8d^4x^8 + 20\left(36\,a^8cd^3 + 7\,a^6d^4\right)x^6 + 6\left(378\,a^8c^2d^2 + 180\,a^6cd^3 + 35\,a^4d^4\right)x^4 + 12\left(420\,a^8c^3d + 378\,a^6c^2d^2 + 180\,a^4cd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 + 7*a^6*d^4)*x^6 + 6*(378*a^8*c^
2*d^2 + 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d + 378*a^6*c^2*d
^2 + 180*a^4*c*d^3 + 35*a^2*d^4)*x^2 + 12*(315*a^8*c^4 + 420*a^6*c^3*d + 37
8*a^4*c^2*d^2 + 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 - 1) + 12*(35*a^9*d^4*x
^9 + 180*a^9*c*d^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*
c^4*x)*log(-(a*x + 1)/(a*x - 1)))/a^9
```

Sympy [A] time = 15.0584, size = 372, normalized size = 1.52

$$\begin{cases} c^4x\,\mathrm{atanh}\,(ax) + \frac{4c^3dx^3\,\mathrm{atanh}\,(ax)}{3} + \frac{6c^2d^2x^5\,\mathrm{atanh}\,(ax)}{5} + \frac{4cd^3x^7\,\mathrm{atanh}\,(ax)}{7} + \frac{d^4x^9\,\mathrm{atanh}\,(ax)}{9} + \frac{c^4\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^4\,\mathrm{atanh}\,(ax)}{a} + \frac{2c^3dx^2}{3a} + \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**4*atanh(a*x),x)
```

```
[Out] Piecewise((c**4*x*atanh(a*x) + 4*c**3*d*x**3*atanh(a*x)/3 + 6*c**2*d**2*x**
5*atanh(a*x)/5 + 4*c*d**3*x**7*atanh(a*x)/7 + d**4*x**9*atanh(a*x)/9 + c**4
*log(x - 1/a)/a + c**4*atanh(a*x)/a + 2*c**3*d*x**2/(3*a) + 3*c**2*d**2*x**
4/(10*a) + 2*c*d**3*x**6/(21*a) + d**4*x**8/(72*a) + 4*c**3*d*log(x - 1/a)/
(3*a**3) + 4*c**3*d*atanh(a*x)/(3*a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**
3*x**4/(7*a**3) + d**4*x**6/(54*a**3) + 6*c**2*d**2*log(x - 1/a)/(5*a**5) +
 6*c**2*d**2*atanh(a*x)/(5*a**5) + 2*c*d**3*x**2/(7*a**5) + d**4*x**4/(36*a
**5) + 4*c*d**3*log(x - 1/a)/(7*a**7) + 4*c*d**3*atanh(a*x)/(7*a**7) + d**4
*x**2/(18*a**7) + d**4*log(x - 1/a)/(9*a**9) + d**4*atanh(a*x)/(9*a**9), Ne
(a, 0)), (0, True))
```

Giac [A] time = 1.1733, size = 331, normalized size = 1.35

$$\frac{1}{630}\left(35\,d^4x^9 + 180\,cd^3x^7 + 378\,c^2d^2x^5 + 420\,c^3dx^3 + 315\,c^4x\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{105\,a^7d^4x^8 + 720\,a^7cd^3x^6 + 2268\,a^7c^2d^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^4*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/630*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*log(-(a*x + 1)/(a*x - 1)) + 1/7560*(105*a^7*d^4*x^8 + 720*a^7*c*d^3*x
^6 + 2268*a^7*c^2*d^2*x^4 + 140*a^5*d^4*x^6 + 5040*a^7*c^3*d*x^2 + 1080*a^5
*c*d^3*x^4 + 4536*a^5*c^2*d^2*x^2 + 210*a^3*d^4*x^4 + 2160*a^3*c*d^3*x^2 +
420*a*d^4*x^2)/a^8 + 1/630*(315*a^8*c^4 + 420*a^6*c^3*d + 378*a^4*c^2*d^2 +
 180*a^2*c*d^3 + 35*d^4)*log(abs(a^2*x^2 - 1))/a^9
```

3.499 $\int (c+dx^2)^3 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=169

$$\frac{dx^2\left(35a^4c^2+21a^2cd+5d^2\right)}{70a^5}+\frac{\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a^2x^2\right)}{70a^7}+\frac{d^2x^4\left(21a^2c+5d\right)}{140a^3}+c^2dx^3\tanh^{-1}($$

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcTanh[a*x] + c^2*d*x^3*ArcTanh[
a*x] + (3*c*d^2*x^5*ArcTanh[a*x])/5 + (d^3*x^7*ArcTanh[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rubi [A] time = 0.129409, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {194, 5976, 1810, 260}

$$\frac{dx^2\left(35a^4c^2+21a^2cd+5d^2\right)}{70a^5}+\frac{\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a^2x^2\right)}{70a^7}+\frac{d^2x^4\left(21a^2c+5d\right)}{140a^3}+c^2dx^3\tanh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (d*(35*a^4*c^2 + 21*a^2*c*d + 5*d^2)*x^2)/(70*a^5) + (d^2*(21*a^2*c + 5*d)*
x^4)/(140*a^3) + (d^3*x^6)/(42*a) + c^3*x*ArcTanh[a*x] + c^2*d*x^3*ArcTanh[
a*x] + (3*c*d^2*x^5*ArcTanh[a*x])/5 + (d^3*x^7*ArcTanh[a*x])/7 + ((35*a^6*c
^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1 - a^2*x^2])/(70*a^7)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(c+dx^2\right)^3 \tanh^{-1}(ax)\,dx = c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(ax)-a\int$$

$$= c^3x\tanh^{-1}(ax)+c^2dx^3\tanh^{-1}(ax)+\frac{3}{5}cd^2x^5\tanh^{-1}(ax)+\frac{1}{7}d^3x^7\tanh^{-1}(ax)-a\int$$

$$= \frac{d\left(35a^4c^2+21a^2cd+5d^2\right)x^2}{70a^5}+\frac{d^2\left(21a^2c+5d\right)x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)+c^2dx^3$$

$$= \frac{d\left(35a^4c^2+21a^2cd+5d^2\right)x^2}{70a^5}+\frac{d^2\left(21a^2c+5d\right)x^4}{140a^3}+\frac{d^3x^6}{42a}+c^3x\tanh^{-1}(ax)+c^2dx^3$$

Mathematica [A] time = 0.0755131, size = 150, normalized size = 0.89

$$\frac{a^2dx^2\left(a^4\left(210c^2+63cdx^2+10d^2x^4\right)+3a^2d\left(42c+5dx^2\right)+30d^2\right)+6\left(35a^4c^2d+35a^6c^3+21a^2cd^2+5d^3\right)\log\left(1-a\right.}{420a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^3*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(30*d^2 + 3*a^2*d*(42*c + 5*d*x^2) + a^4*(210*c^2 + 63*c*d*x^2 +
 10*d^2*x^4)) + 12*a^7*x*(35*c^3 + 35*c^2*d*x^2 + 21*c*d^2*x^4 + 5*d^3*x^6)
*ArcTanh[a*x] + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*Log[1
- a^2*x^2])/(420*a^7)
```

Maple [A] time = 0.035, size = 233, normalized size = 1.4

$$\frac{d^3x^7\mathrm{Artanh}\,(ax)}{7}+\frac{3\,cd^2x^5\mathrm{Artanh}\,(ax)}{5}+c^2dx^3\mathrm{Artanh}\,(ax)+c^3x\mathrm{Artanh}\,(ax)+\frac{3\,cd^2x^2}{10\,a^3}+\frac{3\,cd^2x^4}{20\,a}+\frac{c^2x^2d}{2\,a}+\frac{d^3x^6}{42\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^3*arctanh(a*x),x)
```

```
[Out] 1/7*d^3*x^7*arctanh(a*x)+3/5*c*d^2*x^5*arctanh(a*x)+c^2*d*x^3*arctanh(a*x)+
c^3*x*arctanh(a*x)+3/10/a^3*c*d^2*x^2+3/20/a*c*d^2*x^4+1/2/a*x^2*c^2*d+1/42
*d^3*x^6/a+1/14/a^5*d^3*x^2+1/2/a*ln(a*x-1)*c^3+1/2/a^3*ln(a*x-1)*c^2*d+3/1
0/a^5*ln(a*x-1)*c*d^2+1/14/a^7*ln(a*x-1)*d^3+1/28/a^3*x^4*d^3+1/2/a*ln(a*x+
1)*c^3+1/2/a^3*ln(a*x+1)*c^2*d+3/10/a^5*ln(a*x+1)*c*d^2+1/14/a^7*ln(a*x+1)*
d^3
```

Maxima [A] time = 0.96193, size = 267, normalized size = 1.58

$$\frac{1}{420}\,a\left(\frac{10\,a^4d^3x^6+3\left(21\,a^4cd^2+5\,a^2d^3\right)x^4+6\left(35\,a^4c^2d+21\,a^2cd^2+5\,d^3\right)x^2}{a^6}+\frac{6\left(35\,a^6c^3+35\,a^4c^2d+21\,a^2cd^2+5\,d\right.}{a^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/420*a*((10*a^4*d^3*x^6 + 3*(21*a^4*c*d^2 + 5*a^2*d^3)*x^4 + 6*(35*a^4*c^2
*d + 21*a^2*c*d^2 + 5*d^3)*x^2)/a^6 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2
```

```
*c*d^2 + 5*d^3)*log(a*x + 1)/a^8 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*
d^2 + 5*d^3)*log(a*x - 1)/a^8) + 1/35*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*
x^3 + 35*c^3*x)*arctanh(a*x)
```

Fricas [A] time = 1.94719, size = 383, normalized size = 2.27

$$\frac{10\,a^6d^3x^6 + 3\left(21\,a^6cd^2 + 5\,a^4d^3\right)x^4 + 6\left(35\,a^6c^2d + 21\,a^4cd^2 + 5\,a^2d^3\right)x^2 + 6\left(35\,a^6c^3 + 35\,a^4c^2d + 21\,a^2cd^2 + 5\,d^3\right)\log\left(a\right.}{420\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/420*(10*a^6*d^3*x^6 + 3*(21*a^6*c*d^2 + 5*a^4*d^3)*x^4 + 6*(35*a^6*c^2*d
+ 21*a^4*c*d^2 + 5*a^2*d^3)*x^2 + 6*(35*a^6*c^3 + 35*a^4*c^2*d + 21*a^2*c*d
^2 + 5*d^3)*log(a^2*x^2 - 1) + 6*(5*a^7*d^3*x^7 + 21*a^7*c*d^2*x^5 + 35*a^7
*c^2*d*x^3 + 35*a^7*c^3*x)*log(-(a*x + 1)/(a*x - 1)))/a^7
```

Sympy [A] time = 7.83885, size = 245, normalized size = 1.45

$$\begin{cases} c^3x\,\mathrm{atanh}\,(ax) + c^2dx^3\,\mathrm{atanh}\,(ax) + \frac{3cd^2x^5\,\mathrm{atanh}\,(ax)}{5} + \frac{d^3x^7\,\mathrm{atanh}\,(ax)}{7} + \frac{c^3\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^3\,\mathrm{atanh}\,(ax)}{a} + \frac{c^2dx^2}{2a} + \frac{3cd^2x^4}{20a} + \frac{d^3x^6}{42a} + \cdots \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**3*atanh(a*x),x)
```

```
[Out] Piecewise((c**3*x*atanh(a*x) + c**2*d*x**3*atanh(a*x) + 3*c*d**2*x**5*atanh
(a*x)/5 + d**3*x**7*atanh(a*x)/7 + c**3*log(x - 1/a)/a + c**3*atanh(a*x)/a
+ c**2*d*x**2/(2*a) + 3*c*d**2*x**4/(20*a) + d**3*x**6/(42*a) + c**2*d*log(
x - 1/a)/a**3 + c**2*d*atanh(a*x)/a**3 + 3*c*d**2*x**2/(10*a**3) + d**3*x**
4/(28*a**3) + 3*c*d**2*log(x - 1/a)/(5*a**5) + 3*c*d**2*atanh(a*x)/(5*a**5)
 + d**3*x**2/(14*a**5) + d**3*log(x - 1/a)/(7*a**7) + d**3*atanh(a*x)/(7*a*
*7), Ne(a, 0)), (0, True))
```

Giac [A] time = 1.2196, size = 232, normalized size = 1.37

$$\frac{1}{70}\left(5\,d^3x^7 + 21\,cd^2x^5 + 35\,c^2dx^3 + 35\,c^3x\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{10\,a^5d^3x^6 + 63\,a^5cd^2x^4 + 210\,a^5c^2dx^2 + 15\,a^3d^3x^4 + 126\,a^3c\cdots}{420\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^3*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/70*(5*d^3*x^7 + 21*c*d^2*x^5 + 35*c^2*d*x^3 + 35*c^3*x)*log(-(a*x + 1)/(a
*x - 1)) + 1/420*(10*a^5*d^3*x^6 + 63*a^5*c*d^2*x^4 + 210*a^5*c^2*d*x^2 + 1
5*a^3*d^3*x^4 + 126*a^3*c*d^2*x^2 + 30*a*d^3*x^2)/a^6 + 1/70*(35*a^6*c^3 +
35*a^4*c^2*d + 21*a^2*c*d^2 + 5*d^3)*log(abs(a^2*x^2 - 1))/a^7
```

3.500 $\int \left(c+dx^2\right)^2 \tanh^{-1}(ax)\,dx$

Optimal. Leaf size=110

$$\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\log\left(1-a^2x^2\right)}{30a^5}+\frac{dx^2\left(10a^2c+3d\right)}{30a^3}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{d^2x^4}{20a}+\frac{1}{5}d^2x^5\,\mathrm{ta}$$

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcTanh[a*x] +
 (2*c*d*x^3*ArcTanh[a*x])/3 + (d^2*x^5*ArcTanh[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rubi [A] time = 0.135195, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {194, 5976, 1594, 1247, 698}

$$\frac{\left(15a^4c^2+10a^2cd+3d^2\right)\log\left(1-a^2x^2\right)}{30a^5}+\frac{dx^2\left(10a^2c+3d\right)}{30a^3}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{d^2x^4}{20a}+\frac{1}{5}d^2x^5\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (d*(10*a^2*c + 3*d)*x^2)/(30*a^3) + (d^2*x^4)/(20*a) + c^2*x*ArcTanh[a*x] +
 (2*c*d*x^3*ArcTanh[a*x])/3 + (d^2*x^5*ArcTanh[a*x])/5 + ((15*a^4*c^2 + 10*
a^2*c*d + 3*d^2)*Log[1 - a^2*x^2])/(30*a^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
```

```
&& IntegerQ[m]))
```

Rubi steps

$$\int \left(c+dx^2\right)^2 \tanh^{-1}(ax)\,dx = c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-a\int \frac{c^2x+\frac{2}{3}cdx^3+\frac{d^2x^5}{5}}{1-a^2x^2}\,dx$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-a\int \frac{x\left(c^2+\frac{2}{3}cdx^2+\frac{d^2x^4}{5}\right)}{1-a^2x^2}\,dx$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int \frac{c^2+\frac{2cdx}{3}+\frac{d^2x^2}{5}}{1-a^2x}\right.$$

$$= c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\left(-\frac{d\left(10a^2c+3d\right.}{15a^4}\right.\right.$$

$$= \frac{d\left(10a^2c+3d\right)x^2}{30a^3}+\frac{d^2x^4}{20a}+c^2x\tanh^{-1}(ax)+\frac{2}{3}cdx^3\tanh^{-1}(ax)+\frac{1}{5}d^2x^5\tanh^{-1}(ax)+\ldots$$

Mathematica [A] time = 0.0510372, size = 98, normalized size = 0.89

$$\frac{\left(30a^4c^2+20a^2cd+6d^2\right)\log\left(1-a^2x^2\right)+4a^5x\tanh^{-1}(ax)\left(15c^2+10cdx^2+3d^2x^4\right)+a^2dx^2\left(a^2\left(20c+3dx^2\right)+6d\right)}{60a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)^2*ArcTanh[a*x],x]
```

```
[Out] (a^2*d*x^2*(6*d + a^2*(20*c + 3*d*x^2)) + 4*a^5*x*(15*c^2 + 10*c*d*x^2 + 3*
d^2*x^4)*ArcTanh[a*x] + (30*a^4*c^2 + 20*a^2*c*d + 6*d^2)*Log[1 - a^2*x^2])
/(60*a^5)
```

Maple [A] time = 0.031, size = 148, normalized size = 1.4

$$\frac{d^2x^5\text{Artanh}(ax)}{5}+\frac{2\,cdx^3\text{Artanh}(ax)}{3}+c^2x\text{Artanh}(ax)+\frac{d^2x^4}{20\,a}+\frac{cx^2d}{3\,a}+\frac{d^2x^2}{10\,a^3}+\frac{\ln(ax-1)\,c^2}{2\,a}+\frac{\ln(ax-1)\,cd}{3\,a^3}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^2*arctanh(a*x),x)
```

```
[Out] 1/5*d^2*x^5*arctanh(a*x)+2/3*c*d*x^3*arctanh(a*x)+c^2*x*arctanh(a*x)+1/20*d
^2*x^4/a+1/3/a*x^2*c*d+1/10/a^3*x^2*d^2+1/2/a*ln(a*x-1)*c^2+1/3/a^3*ln(a*x-
1)*c*d+1/10/a^5*ln(a*x-1)*d^2+1/2/a*ln(a*x+1)*c^2+1/3/a^3*ln(a*x+1)*c*d+1/1
0/a^5*ln(a*x+1)*d^2
```

Maxima [A] time = 0.983814, size = 177, normalized size = 1.61

$$\frac{1}{60}a\left(\frac{3\,a^2d^2x^4+2\left(10\,a^2cd+3\,d^2\right)x^2}{a^4}+\frac{2\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log(ax+1)}{a^6}+\frac{2\left(15\,a^4c^2+10\,a^2cd+3\,d^2\right)\log(ax-\ldots}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/60*a*((3*a^2*d^2*x^4 + 2*(10*a^2*c*d + 3*d^2)*x^2)/a^4 + 2*(15*a^4*c^2 +
10*a^2*c*d + 3*d^2)*log(a*x + 1)/a^6 + 2*(15*a^4*c^2 + 10*a^2*c*d + 3*d^2)*
log(a*x - 1)/a^6) + 1/15*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*arctanh(a*x)
```

Fricas [A] time = 1.89839, size = 259, normalized size = 2.35

$$\frac{3\,a^4d^2x^4 + 2\left(10\,a^4cd + 3\,a^2d^2\right)x^2 + 2\left(15\,a^4c^2 + 10\,a^2cd + 3\,d^2\right)\log\left(a^2x^2 - 1\right) + 2\left(3\,a^5d^2x^5 + 10\,a^5cdx^3 + 15\,a^5c^2x\right)\mathrm{l}}{60\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/60*(3*a^4*d^2*x^4 + 2*(10*a^4*c*d + 3*a^2*d^2)*x^2 + 2*(15*a^4*c^2 + 10*a
^2*c*d + 3*d^2)*log(a^2*x^2 - 1) + 2*(3*a^5*d^2*x^5 + 10*a^5*c*d*x^3 + 15*a
^5*c^2*x)*log(-(a*x + 1)/(a*x - 1)))/a^5
```

Sympy [A] time = 3.33042, size = 155, normalized size = 1.41

$$\begin{cases} c^2x\operatorname{atanh}(ax) + \frac{2cdx^3\operatorname{atanh}(ax)}{3} + \frac{d^2x^5\operatorname{atanh}(ax)}{5} + \frac{c^2\log\left(x-\frac{1}{a}\right)}{a} + \frac{c^2\operatorname{atanh}(ax)}{a} + \frac{cdx^2}{3a} + \frac{d^2x^4}{20a} + \frac{2cd\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{2cd\operatorname{atanh}(ax)}{3a^3} + \\ 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**2*atanh(a*x),x)
```

```
[Out] Piecewise((c**2*x*atanh(a*x) + 2*c*d*x**3*atanh(a*x)/3 + d**2*x**5*atanh(a*
x)/5 + c**2*log(x - 1/a)/a + c**2*atanh(a*x)/a + c*d*x**2/(3*a) + d**2*x**4
/(20*a) + 2*c*d*log(x - 1/a)/(3*a**3) + 2*c*d*atanh(a*x)/(3*a**3) + d**2*x*
*2/(10*a**3) + d**2*log(x - 1/a)/(5*a**5) + d**2*atanh(a*x)/(5*a**5), Ne(a,
 0)), (0, True))
```

Giac [A] time = 1.17649, size = 153, normalized size = 1.39

$$\frac{1}{30}\left(3\,d^2x^5 + 10\,cdx^3 + 15\,c^2x\right)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{3\,a^3d^2x^4 + 20\,a^3cdx^2 + 6\,ad^2x^2}{60\,a^4} + \frac{\left(15\,a^4c^2 + 10\,a^2cd + 3\,d^2\right)\log\left(\left|a^2x^2\right.\right.}{30\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^2*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/30*(3*d^2*x^5 + 10*c*d*x^3 + 15*c^2*x)*log(-(a*x + 1)/(a*x - 1)) + 1/60*(
3*a^3*d^2*x^4 + 20*a^3*c*d*x^2 + 6*a*d^2*x^2)/a^4 + 1/30*(15*a^4*c^2 + 10*a
^2*c*d + 3*d^2)*log(abs(a^2*x^2 - 1))/a^5
```

3.501 $\int (c + dx^2) \tanh^{-1}(ax)\, dx$

Optimal. Leaf size=57

$$\frac{(3a^2c + d)\log(1 - a^2x^2)}{6a^3} + cx\tanh^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3}dx^3\tanh^{-1}(ax)$$

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rubi [A] time = 0.066788, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5976, 1593, 444, 43}

$$\frac{(3a^2c + d)\log(1 - a^2x^2)}{6a^3} + cx\tanh^{-1}(ax) + \frac{dx^2}{6a} + \frac{1}{3}dx^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)*ArcTanh[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + ((3*a^2*c + d)*
Log[1 - a^2*x^2])/(6*a^3)
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(c+dx^2\right)\tanh^{-1}(ax)\,dx = cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-a\int\frac{cx+\frac{dx^3}{3}}{1-a^2x^2}\,dx$$
$$= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-a\int\frac{x\left(c+\frac{dx^2}{3}\right)}{1-a^2x^2}\,dx$$
$$= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\frac{c+\frac{dx}{3}}{1-a^2x}\,dx,x,x^2\right)$$
$$= cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)-\frac{1}{2}a\,\text{Subst}\left(\int\left(-\frac{d}{3a^2}+\frac{-3a^2c-d}{3a^2\left(-1+a^2x\right)}\right)dx,x,x^2\right)$$
$$= \frac{dx^2}{6a}+cx\tanh^{-1}(ax)+\frac{1}{3}dx^3\tanh^{-1}(ax)+\frac{\left(3a^2c+d\right)\log\left(1-a^2x^2\right)}{6a^3}$$

Mathematica [A] time = 0.0113418, size = 69, normalized size = 1.21

$$\frac{c\log\left(1-a^2x^2\right)}{2a}+\frac{d\log\left(1-a^2x^2\right)}{6a^3}+cx\tanh^{-1}(ax)+\frac{dx^2}{6a}+\frac{1}{3}dx^3\tanh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)*ArcTanh[a*x],x]
```

```
[Out] (d*x^2)/(6*a) + c*x*ArcTanh[a*x] + (d*x^3*ArcTanh[a*x])/3 + (c*Log[1 - a^2*
x^2])/(2*a) + (d*Log[1 - a^2*x^2])/(6*a^3)
```

Maple [A] time = 0.031, size = 76, normalized size = 1.3

$$\frac{dx^3\text{Artanh}\,(ax)}{3}+cx\text{Artanh}\,(ax)+\frac{dx^2}{6\,a}+\frac{\ln(ax-1)\,c}{2\,a}+\frac{\ln(ax-1)\,d}{6\,a^3}+\frac{\ln(ax+1)\,c}{2\,a}+\frac{\ln(ax+1)\,d}{6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)*arctanh(a*x),x)
```

```
[Out] 1/3*d*x^3*arctanh(a*x)+c*x*arctanh(a*x)+1/6*d*x^2/a+1/2/a*ln(a*x-1)*c+1/6/a
^3*ln(a*x-1)*d+1/2/a*ln(a*x+1)*c+1/6/a^3*ln(a*x+1)*d
```

Maxima [A] time = 0.953481, size = 88, normalized size = 1.54

$$\frac{1}{6}\,a\left(\frac{dx^2}{a^2}+\frac{\left(3\,a^2c+d\right)\log(ax+1)}{a^4}+\frac{\left(3\,a^2c+d\right)\log(ax-1)}{a^4}\right)+\frac{1}{3}\left(dx^3+3\,cx\right)\text{artanh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] 1/6*a*(d*x^2/a^2 + (3*a^2*c + d)*log(a*x + 1)/a^4 + (3*a^2*c + d)*log(a*x -
 1)/a^4) + 1/3*(d*x^3 + 3*c*x)*arctanh(a*x)
```

Fricas [A] time = 1.96813, size = 143, normalized size = 2.51

$$\frac{a^2dx^2 + (3a^2c + d)\log(a^2x^2 - 1) + (a^3dx^3 + 3a^3cx)\log\left(-\frac{ax+1}{ax-1}\right)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d*x^2 + (3*a^2*c + d)*log(a^2*x^2 - 1) + (a^3*d*x^3 + 3*a^3*c*x)*l
og(-(a*x + 1)/(a*x - 1)))/a^3
```

Sympy [A] time = 1.51762, size = 73, normalized size = 1.28

$$\begin{cases} cx\operatorname{atanh}(ax) + \frac{dx^3\operatorname{atanh}(ax)}{3} + \frac{c\log\left(x-\frac{1}{a}\right)}{a} + \frac{c\operatorname{atanh}(ax)}{a} + \frac{dx^2}{6a} + \frac{d\log\left(x-\frac{1}{a}\right)}{3a^3} + \frac{d\operatorname{atanh}(ax)}{3a^3} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)*atanh(a*x),x)
```

```
[Out] Piecewise((c*x*atanh(a*x) + d*x**3*atanh(a*x)/3 + c*log(x - 1/a)/a + c*atan
h(a*x)/a + d*x**2/(6*a) + d*log(x - 1/a)/(3*a**3) + d*atanh(a*x)/(3*a**3),
Ne(a, 0)), (0, True))
```

Giac [A] time = 1.20345, size = 82, normalized size = 1.44

$$\frac{dx^2}{6a} + \frac{1}{6}(dx^3 + 3cx)\log\left(-\frac{ax+1}{ax-1}\right) + \frac{(3a^2c + d)\log(|a^2x^2 - 1|)}{6a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] 1/6*d*x^2/a + 1/6*(d*x^3 + 3*c*x)*log(-(a*x + 1)/(a*x - 1)) + 1/6*(3*a^2*c
+ d)*log(abs(a^2*x^2 - 1))/a^3
```

3.502 $\int \frac{\tanh^{-1}(ax)}{c+dx^2}\,dx$

Optimal. Leaf size=429

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1-ax)}{4}$$

```
[Out] -(Log[1 - a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*S
qrt[-c]*Sqrt[d]) + (Log[1 + a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c]
 + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - (Log[1 + a*x]*Log[(a*(Sqrt[-c] + Sqrt[
d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 - a*x]*Log[(a
*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - Po
lyLog[2, -((Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d]
) + PolyLog[2, (Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt[-c]*Sqrt
[d]) - PolyLog[2, -((Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c
]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt
[-c]*Sqrt[d])
```

Rubi [A] time = 0.441711, antiderivative size = 429, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5972, 2409, 2394, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(1-ax)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{d}(ax+1)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1-ax)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2),x]
```

```
[Out] -(Log[1 - a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*S
qrt[-c]*Sqrt[d]) + (Log[1 + a*x]*Log[(a*(Sqrt[-c] - Sqrt[d]*x))/(a*Sqrt[-c]
 + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - (Log[1 + a*x]*Log[(a*(Sqrt[-c] + Sqrt[
d]*x))/(a*Sqrt[-c] - Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) + (Log[1 - a*x]*Log[(a
*(Sqrt[-c] + Sqrt[d]*x))/(a*Sqrt[-c] + Sqrt[d])])/(4*Sqrt[-c]*Sqrt[d]) - Po
lyLog[2, -((Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c]*Sqrt[d]
) + PolyLog[2, (Sqrt[d]*(1 - a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt[-c]*Sqrt
[d]) - PolyLog[2, -((Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] - Sqrt[d]))]/(4*Sqrt[-c
]*Sqrt[d]) + PolyLog[2, (Sqrt[d]*(1 + a*x))/(a*Sqrt[-c] + Sqrt[d])]/(4*Sqrt
[-c]*Sqrt[d])
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{c+dx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-ax)}{c+dx^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+ax)}{c+dx^2}\,dx$$

$$= -\left(\frac{1}{2}\int \left(\frac{\sqrt{-c}\log(1-ax)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(1-ax)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\sqrt{-c}\log(1+ax)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\log(1+ax)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$= \frac{\int \frac{\log(1-ax)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4\sqrt{-c}}+\frac{\int \frac{\log(1-ax)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4\sqrt{-c}}-\frac{\int \frac{\log(1+ax)}{\sqrt{-c}-\sqrt{d}x}\,dx}{4\sqrt{-c}}-\frac{\int \frac{\log(1+ax)}{\sqrt{-c}+\sqrt{d}x}\,dx}{4\sqrt{-c}}$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\ldots$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\ldots$$

$$= -\frac{\log(1-ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}-\sqrt{d}x\right)}{a\sqrt{-c}+\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}-\frac{\log(1+ax)\log\left(\frac{a\left(\sqrt{-c}+\sqrt{d}x\right)}{a\sqrt{-c}-\sqrt{d}}\right)}{4\sqrt{-c}\sqrt{d}}+\ldots$$

Mathematica [C] time = 1.34531, size = 662, normalized size = 1.54

$$-\frac{a\left(i\left(\text{PolyLog}\left(2,\frac{\left(2i\sqrt{a^2cd}+a^2(-c)+d\right)\left(x\sqrt{a^2cd}+iac\right)}{\left(a^2c+d\right)\left(x\sqrt{a^2cd}-iac\right)}\right)-\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{a^2cd}+a^2(-c)+d\right)\left(x\sqrt{a^2cd}+iac\right)}{\left(a^2c+d\right)\left(x\sqrt{a^2cd}-iac\right)}\right)\right)-2i\cos^{-1}\left(\frac{d-a^2c}{a^2c+d}\right)\tan^{-1}\left(\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2),x]
```

```
[Out] -(a*((-2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]
] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[(-(a^2*c) + d)
/(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sqrt
[a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (ArcCos[(
```

```
-(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*c*(d
 + I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (
ArcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + A
rcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]
*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + (Arc
Cos[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcT
an[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqr
t[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(-Pol
yLog[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((
a^2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] + PolyLog[2, ((-(a^2*c) + d + (2*
I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[
a^2*c*d]*x))])))/(4*Sqrt[a^2*c*d])
```

Maple [B] time = 0.197, size = 833, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c),x)
```

```
[Out] 1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a
^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)*c+a/(a^4*c^2+2*a^
2*c*d+d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)
+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)-1/2*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*arctan
h(a*x)^2*(-a^2*c*d)^(1/2)*c-a/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*
c*d)^(1/2)+1/4*a^3/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/
(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*c+1/2*a/(a^4*c
^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^
2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)+1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2
*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-
a^2*c*d)^(1/2)*d-1/2/a/c/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*c*d)^
(1/2)*d+1/4/a/c/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2
*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*d-1/2/a*(-a^2*c*d)^
(1/2)/c/d*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^
2*c*d)^(1/2)+d))+1/2/a*(-a^2*c*d)^(1/2)/c/d*arctanh(a*x)^2-1/4/a*(-a^2*c*d)
^(1/2)/c/d*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^
(1/2)+d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}\,(ax)}{dx^2 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d*x^2 + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{c + dx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{dx^2 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c), x)
```

3.503 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^2}\,dx$

Optimal. Leaf size=590

$$\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{a\log}{4c(}$$

```
[Out] (x*ArcTanh[a*x])/(2*c*(c + d*x^2)) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTanh[a
*x])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2
)*Sqrt[d])
```

Rubi [A] time = 0.910465, antiderivative size = 590, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 13, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {199, 205, 5976, 6725, 517, 444, 36, 31, 4908, 2409, 2394, 2393, 2391}

$$\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}-\frac{i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{8c^{3/2}\sqrt{d}}+\frac{a\log}{4c(}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (x*ArcTanh[a*x])/(2*c*(c + d*x^2)) + (ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTanh[a
*x])/(2*c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] + Sq
rt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((S
qrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])
/(c^(3/2)*Sqrt[d]) - ((I/8)*Log[-((Sqrt[d]*(1 - a*x))/(I*a*Sqrt[c] - Sqrt[d
]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*Sqrt[d]) + ((I/8)*Log[(Sqrt[d
]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(3
/2)*Sqrt[d]) + (a*Log[1 - a^2*x^2])/(4*c*(a^2*c + d)) - (a*Log[c + d*x^2])/
(4*c*(a^2*c + d)) + ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c
] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/8)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt
[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) + ((I/8)*PolyLog[2, (a*
(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])]])/(c^(3/2)*Sqrt[d]) - ((I/
8)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d])]])/(c^(3/2
)*Sqrt[d])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
```

```
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
```

```
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^2}\,dx = \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - a\int \frac{\frac{x}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - a\int \left(-\frac{x}{2c(-1+ax)(1+ax)\left(c+dx^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}\left(-1+a^2\right.}\right.$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{(-1+ax)(1+ax)\left(c+dx^2\right)}\,dx}{2c} + \frac{a\int \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2}\,dx}{2c^{3/2}\sqrt{d}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\int \frac{x}{\left(-1+a^2x^2\right)\left(c+dx^2\right)}\,dx}{2c} + \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{-1+a^2x^2}\,dx}{4c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log}{-}}{4c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\left(-1+a^2x\right)(c+dx)}\,dx, x, x^2\right)}{4c} + \frac{(ia)\int \left(-\frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{2(1-ax)}\right.}{4c^{3/2}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1-ax}\,dx}{8c^{3/2}\sqrt{d}} - \frac{(ia)\int \frac{\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{1+ax}\,dx}{8c^{3/2}\sqrt{d}} + \frac{(ia)\int \frac{\log(1\cdot}{1-}}{8c^{3/2}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1\cdot\right.}{8c^{3/2}\sqrt{d}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1\cdot\right.}{8c^{3/2}\sqrt{d}}$$

$$= \frac{x\tanh^{-1}(ax)}{2c\left(c+dx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{2c^{3/2}\sqrt{d}} + \frac{i\log\left(\frac{\sqrt{d}(1-ax)}{ia\sqrt{c}+\sqrt{d}}\right)\log\left(1-\frac{i\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}} - \frac{i\log\left(-\frac{\sqrt{d}(1+ax)}{ia\sqrt{c}-\sqrt{d}}\right)\log\left(1\cdot\right.}{8c^{3/2}\sqrt{d}}$$

Mathematica [A] time = 7.17112, size = 746, normalized size = 1.26

$$a\left(\frac{i\left(\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{a^2cd}+a^2(-c)+d\right)\left(x\sqrt{a^2cd}+iac\right)}{\left(a^2c+d\right)\left(x\sqrt{a^2cd}-iac\right)}\right)-\text{PolyLog}\left(2,\frac{\left(2i\sqrt{a^2cd}+a^2(-c)+d\right)\left(x\sqrt{a^2cd}+iac\right)}{\left(a^2c+d\right)\left(x\sqrt{a^2cd}-iac\right)}\right)\right)+2i\cos^{-1}\left(\frac{d-a^2c}{a^2c+d}\right)\tan^{-1}\left(\frac{adx}{\sqrt{a^2cd}}\right)-4\tanh^{-1}(ax)\tan^{-1}\left(\frac{ac}{x\sqrt{a^2cd}}\right)+\text{l}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^2,x]
```

```
[Out] (a*((-2*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)])/(a^2*c + d
) + ((2*I)*ArcCos[(-(a^2*c) + d)/(a^2*c + d)]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]
```

```
 - 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] + (ArcCos[(-(a^2*c) + d)/
(a^2*c + d)] + 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[((2*I)*a*c*(I*d + Sqrt[
a^2*c*d])*(-1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] + (ArcCos[(-
(a^2*c) + d)/(a^2*c + d)] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[(2*a*c*(d
+ I*Sqrt[a^2*c*d])*(1 + a*x))/((a^2*c + d)*(a*c + I*Sqrt[a^2*c*d]*x))] - (A
rcCos[(-(a^2*c) + d)/(a^2*c + d)] + 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + Ar
cTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*
E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] - (ArcC
os[(-(a^2*c) + d)/(a^2*c + d)] - 2*(ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] + ArcTa
n[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt
[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyL
og[2, ((-(a^2*c) + d - (2*I)*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^
2*c + d)*((-I)*a*c + Sqrt[a^2*c*d]*x))] - PolyLog[2, ((-(a^2*c) + d + (2*I)
*Sqrt[a^2*c*d])*(I*a*c + Sqrt[a^2*c*d]*x))/((a^2*c + d)*((-I)*a*c + Sqrt[a^
2*c*d]*x))]))/Sqrt[a^2*c*d] + (4*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]])/(a^2*c
- d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]])))/(8*c)
```

Maple [B] time = 0.555, size = 2346, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^2,x)
```

```
[Out] -1/4*(c*d)^(1/2)/c^2/(a^2*c+d)*arctan(a/d*(c*d)^(1/2))-1/2*a^3*arctanh(a*x)
/(a^2*c+d)/(a^2*d*x^2+a^2*c)-1/4*(c*d)^(1/2)/c^2/(a^2*c+d)*arctan(1/(a^2*c+
d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2
)*x-a/(a^2*c+d)*(c*d)^(1/2))+1/4/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2
)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arct
anh(a*x)*(-a^2*c*d)^(1/2)+3/4*a/c*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-
(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x
)*(-a^2*c*d)^(1/2)+1/4*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*ln(1-(a^2*c+
d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2
*c*d)^(1/2)*c-1/4*a^4*(c*d)^(1/2)/d/(a^2*c+d)^2*arctan(a/d*(c*d)^(1/2))+1/4
*(c*d)^(1/2)/c^2*d/(a^2*c+d)^2*arctan(a/d*(c*d)^(1/2))+1/4*a^2*(c*d)^(1/2)/
c/d/(a^2*c+d)*arctan(a/d*(c*d)^(1/2))+3/4*a^3/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+
d^2)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*a
rctanh(a*x)*(-a^2*c*d)^(1/2)+1/4*a*(-a^2*c*d)^(1/2)/c/d/(a^2*c+d)*arctanh(a
*x)^2-1/4/a*(-a^2*c*d)^(1/2)/c^2/(a^2*c+d)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x
+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))-1/8*a*(-a^2*c*d)^(1/2)/c/
d/(a^2*c+d)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)
^(1/2)+d))+1/4*a^2*(c*d)^(1/2)/c/d/(a^2*c+d)*arctan(1/(a^2*c+d)*d^2/(c*d)^(
1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d
)*(c*d)^(1/2))+a^3/(a^2*c+d)^2*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*a^3/(a^2*
c+d)^2*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(
a*x+1)^4/(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)+1/4/a*(-a^2*c*d
)^(1/2)/c^2/(a^2*c+d)*arctanh(a*x)^2-1/8/a*(-a^2*c*d)^(1/2)/c^2/(a^2*c+d)*p
olylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+a/
c/(a^2*c+d)^2*d*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-1/4*a^4*(c*d)^(1/2)/d/(a^2*c
+d)^2*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^
2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-3/4*a^3/(a^4*c^2+2*a^2*c
*d+d^2)/(a^2*c+d)*arctanh(a*x)^2*(-a^2*c*d)^(1/2)+1/2*a^4*arctanh(a*x)/(a^2
*c+d)/(a^2*d*x^2+a^2*c)*x+3/8*a^3/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*polylog
(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*
d)^(1/2)-1/4*a/c/(a^2*c+d)^2*d*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a
*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2
```

```
+1)*d+d)+1/4*(c*d)^(1/2)/c^2*d/(a^2*c+d)^2*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/
2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*
(c*d)^(1/2))-1/4*a*(-a^2*c*d)^(1/2)/c/d/(a^2*c+d)*arctanh(a*x)*ln(1-(a^2*c+
d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))-1/2*a^3*arctanh(a*
x)/c/(a^2*c+d)/(a^2*d*x^2+a^2*c)*d*x^2+1/2*a^2*arctanh(a*x)/c/(a^2*c+d)/(a^
2*d*x^2+a^2*c)*x*d+1/8/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(
2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d
)^(1/2)-1/4/a/c^2*d^2/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^
2*c*d)^(1/2)-1/4*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a
^2*c*d)^(1/2)*c+1/8*a^5/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*
c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)
*c+3/8*a/c*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^
2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)-3/4*a/c*d/(a
^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2*(-a^2*c*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(ax)}{d^2x^4 + 2\,cdx^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax)}{(dx^2 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c)^2, x)
```

3.504 $\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^3}\,dx$

Optimal. Leaf size=657

$$\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{a\,(5}{}$$

```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcTanh[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcTanh[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTa
nh[a*x])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rubi [A] time = 0.966296, antiderivative size = 657, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {199, 205, 5976, 6725, 571, 77, 4908, 2409, 2394, 2393, 2391}

$$\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}-i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}-i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}-\frac{3i\text{PolyLog}\left(2,\frac{a\left(\sqrt{c}+i\sqrt{d}x\right)}{a\sqrt{c}+i\sqrt{d}}\right)}{32c^{5/2}\sqrt{d}}+\frac{a\,(5}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^3,x]
```


```
[Out] a/(8*c*(a^2*c + d)*(c + d*x^2)) + (x*ArcTanh[a*x])/(4*c*(c + d*x^2)^2) + (3
*x*ArcTanh[a*x])/(8*c^2*(c + d*x^2)) + (3*ArcTan[(Sqrt[d]*x)/Sqrt[c]]*ArcTa
nh[a*x])/(8*c^(5/2)*Sqrt[d]) + (((3*I)/32)*Log[(Sqrt[d]*(1 - a*x))/(I*a*Sqr
t[c] + Sqrt[d])]*Log[1 - (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I
)/32)*Log[-((Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] - Sqrt[d]))]*Log[1 - (I*Sqrt[d
]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*Log[-((Sqrt[d]*(1 - a*x))/(I
*a*Sqrt[c] - Sqrt[d]))]*Log[1 + (I*Sqrt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) +
 (((3*I)/32)*Log[(Sqrt[d]*(1 + a*x))/(I*a*Sqrt[c] + Sqrt[d])]*Log[1 + (I*Sq
rt[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d]) + (a*(5*a^2*c + 3*d)*Log[1 - a^2*x^2])
/(16*c^2*(a^2*c + d)^2) - (a*(5*a^2*c + 3*d)*Log[c + d*x^2])/(16*c^2*(a^2*c
 + d)^2) + (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqrt[d]*x))/(a*Sqrt[c] -
I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) - (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] - I*Sqr
t[d]*x))/(a*Sqrt[c] + I*Sqrt[d])])/(c^(5/2)*Sqrt[d]) + (((3*I)/32)*PolyLog[
2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] - I*Sqrt[d])])/(c^(5/2)*Sqrt[d])
- (((3*I)/32)*PolyLog[2, (a*(Sqrt[c] + I*Sqrt[d]*x))/(a*Sqrt[c] + I*Sqrt[d]
)])/(c^(5/2)*Sqrt[d])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 4908

```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[I/2, Int[L
og[1 - I*c*x]/(d + e*x^2), x], x] - Dist[I/2, Int[Log[1 + I*c*x]/(d + e*x^2
), x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^3}\,dx = \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} - a\int \frac{\frac{x}{4c\left(c+dx^2\right)^2} + \frac{3x}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-}}{8c^{5/}}}{1-a^2x^2}$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} - a\int\left(-\frac{x\left(5c+3dx^2\right)}{8c^2\left(-1+a^2x^2\right)\left(c+dx^2\right)^2}\right.$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\int \frac{x\left(5c+3dx^2\right)}{\left(-1+a^2x^2\right)\left(c+dx^2\right)^2}\,dx}{8c^2} + \frac{(3a)\int -}{8c}$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\,\mathrm{Subst}\left(\int \frac{5c+3dx}{\left(-1+a^2x\right)(c+dx)^2}\,dx, x, x^2\right.}{16c^2}$$

$$= \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\,\mathrm{Subst}\left(\int\left(\frac{a^2\left(5a^2c+3d\right)}{\left(a^2c+d\right)^2\left(-1+a^2x\right)} - \frac{}{\left(a^2c\right.}\right.\right.}{16}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{a\left(5a^2c+\right.}{16}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{3i\log\left(\right.}{}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{3i\log\left(\right.}{}$$

$$= \frac{a}{8c\left(a^2c+d\right)\left(c+dx^2\right)} + \frac{x\tanh^{-1}(ax)}{4c\left(c+dx^2\right)^2} + \frac{3x\tanh^{-1}(ax)}{8c^2\left(c+dx^2\right)} + \frac{3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\tanh^{-1}(ax)}{8c^{5/2}\sqrt{d}} + \frac{3i\log\left(\right.}{}$$

Mathematica [B] time = 12.9022, size = 1840, normalized size = 2.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^3,x]
```

```
[Out] a^5*((-5*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c - d)])/(16*a^2*c
*(a^2*c + d)^2) - (3*d*Log[1 + ((a^2*c + d)*Cosh[2*ArcTanh[a*x]])/(a^2*c -
d)])/(16*a^4*c^2*(a^2*c + d)^2) - (3*((-2*I)*ArcCos[-((a^2*c - d)/(a^2*c +
d))]*ArcTan[(a*d*x)/Sqrt[a^2*c*d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcT
anh[a*x] - (ArcCos[-((a^2*c - d)/(a^2*c + d))] - 2*ArcTan[(a*d*x)/Sqrt[a^2*
```

```
c*d]])*Log[1 - ((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a
^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] + (-ArcCos[-
((a^2*c - d)/(a^2*c + d))] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[1 - ((a^2
*c - d + (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c
+ d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] + (ArcCos[-((a^2*c - d)/(a^2*c +
 d))] + (2*I)*((-I)*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt
[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcTanh[a*x]*Sq
rt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + (ArcCos[-((a^2*c - d)/
(a^2*c + d))] - (2*I)*((-I)*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d
*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt[a^2*c
 + d]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyLog[2,
((a^2*c - d - (2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a
^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))] - PolyLog[2, ((a^2*c - d +
(2*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a
^2*c + (2*I)*a*Sqrt[a^2*c*d]*x))])))/(32*a^2*c*Sqrt[a^2*c*d]*(a^2*c + d)) -
 (3*d*((-2*I)*ArcCos[-((a^2*c - d)/(a^2*c + d))]*ArcTan[(a*d*x)/Sqrt[a^2*c*
d]] + 4*ArcTan[(a*c)/(Sqrt[a^2*c*d]*x)]*ArcTanh[a*x] - (ArcCos[-((a^2*c - d
)/(a^2*c + d))] - 2*ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[1 - ((a^2*c - d - (2
*I)*Sqrt[a^2*c*d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2
*c + (2*I)*a*Sqrt[a^2*c*d]*x))] + (-ArcCos[-((a^2*c - d)/(a^2*c + d))] - 2*
ArcTan[(a*d*x)/Sqrt[a^2*c*d]])*Log[1 - ((a^2*c - d + (2*I)*Sqrt[a^2*c*d])*(
2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^
2*c*d]*x))] + (ArcCos[-((a^2*c - d)/(a^2*c + d))] + (2*I)*((-I)*ArcTan[(a*c
)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[2]*Sqrt[
a^2*c*d])/(Sqrt[a^2*c + d]*E^ArcTanh[a*x]*Sqrt[a^2*c - d + (a^2*c + d)*Cosh
[2*ArcTanh[a*x]]])] + (ArcCos[-((a^2*c - d)/(a^2*c + d))] - (2*I)*((-I)*Arc
Tan[(a*c)/(Sqrt[a^2*c*d]*x)] - I*ArcTan[(a*d*x)/Sqrt[a^2*c*d]]))*Log[(Sqrt[
2]*Sqrt[a^2*c*d]*E^ArcTanh[a*x])/(Sqrt[a^2*c + d]*Sqrt[a^2*c - d + (a^2*c +
 d)*Cosh[2*ArcTanh[a*x]]])] + I*(PolyLog[2, ((a^2*c - d - (2*I)*Sqrt[a^2*c*
d])*(2*a^2*c - (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sq
rt[a^2*c*d]*x))] - PolyLog[2, ((a^2*c - d + (2*I)*Sqrt[a^2*c*d])*(2*a^2*c -
 (2*I)*a*Sqrt[a^2*c*d]*x))/((a^2*c + d)*(2*a^2*c + (2*I)*a*Sqrt[a^2*c*d]*x)
)])))/(32*a^4*c^2*Sqrt[a^2*c*d]*(a^2*c + d)) + (d*ArcTanh[a*x]*Sinh[2*ArcTa
nh[a*x]])/(2*a^2*c*(a^2*c + d)*(a^2*c - d + a^2*c*Cosh[2*ArcTanh[a*x]] + d*
Cosh[2*ArcTanh[a*x]])^2) + (2*a^2*c*d + 5*a^4*c^2*ArcTanh[a*x]*Sinh[2*ArcTa
nh[a*x]] + 8*a^2*c*d*ArcTanh[a*x]*Sinh[2*ArcTanh[a*x]] + 3*d^2*ArcTanh[a*x]
*Sinh[2*ArcTanh[a*x]])/(8*a^4*c^2*(a^2*c + d)^2*(a^2*c - d + a^2*c*Cosh[2*A
rcTanh[a*x]] + d*Cosh[2*ArcTanh[a*x]]))
```

Maple [B] time = 0.707, size = 4311, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^3,x)
```

```
[Out] 1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d-5/16*a^5/(a^4*c^2+2*a
^2*c*d+d^2)/(a^2*c+d)*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(
-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)+
5/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*c+d)*ln((a*x+1)/(-a^2*x^2+1)^(1/2))-3/
4*a^5/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2*(-a^2*c*d)^(1/2)+3/8*a^5/(a^
4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2
*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)-5/16*a^6*(c*d)^(1/2)/d/(a^2*c+d)/(a^
4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))+5/16*a^4*(c*d)^(1/2)/c/d/(a^4*
c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-3/16*a^4*(c*d)^(1/2)/c/(a^2*c+d)
/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))+3/16*(c*d)^(1/2)/c^3*d^2/(
```

a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-1/8*a^7/(a^4*c^2+2
*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/c*d^2*x^4+1/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)
/(a^2*d*x^2+a^2*c)^2/c*d^2*x^2+5/16*a^2*(c*d)^(1/2)/c^2*d/(a^2*c+d)/(a^4*c^
2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))+9/8*a^3/c/(a^4*c^2+2*a^2*c*d+d^2)^
2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arct
anh(a*x)*(-a^2*c*d)^(1/2)*d-5/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*
c)^2/c*arctanh(a*x)*x^4*d^2-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*
c)^2/c^2*arctanh(a*x)*x^4*d^3+5/8*a^8/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^
2*c)^2*c*arctanh(a*x)*x-3/8*a*(-a^2*c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*
arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/
2)+d))+9/16*a^3/c/(a^4*c^2+2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(
-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*d-3/4*a/c^2*d^2
/(a^4*c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2*(-a^2*c*d)^(1/2)-3/16*a^7/d/(a^4*
c^2+2*a^2*c*d+d^2)^2*arctanh(a*x)^2*(-a^2*c*d)^(1/2)*c+3/32*a^7/d/(a^4*c^2+
2*a^2*c*d+d^2)^2*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2
*c*d)^(1/2)+d))*(-a^2*c*d)^(1/2)*c+3/16/a*(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2+2
*a^2*c*d+d^2)*arctanh(a*x)^2+3/32/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2*polyl
og(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*(-a^2*
c*d)^(1/2)+3/16*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*
x)^2+3/4*a/(a^4*c^2+2*a^2*c*d+d^2)/c^2*d^2/(a^2*c+d)*ln((a*x+1)/(-a^2*x^2+1
)^(1/2))-1/2*a^3/(a^4*c^2+2*a^2*c*d+d^2)/c*d/(a^2*c+d)*ln(a^2*c*(a*x+1)^4/(
-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(a*x+1)^4/(-a^2*x^2+1)^2+a^2
c-2(a*x+1)^2/(-a^2*x^2+1)*d+d)-3/16*a/(a^4*c^2+2*a^2*c*d+d^2)/c^2*d^2/(a^
2*c+d)*ln(a^2*c*(a*x+1)^4/(-a^2*x^2+1)^2+2*a^2*c*(a*x+1)^2/(-a^2*x^2+1)+d*(
a*x+1)^4/(-a^2*x^2+1)^2+a^2*c-2*(a*x+1)^2/(-a^2*x^2+1)*d+d)-3/32/a*(-a^2*c*
d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*
x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))-3/16/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2
)^2*arctanh(a*x)^2*(-a^2*c*d)^(1/2)+2*a^3/(a^4*c^2+2*a^2*c*d+d^2)/c*d/(a^2*
c+d)*ln((a*x+1)/(-a^2*x^2+1)^(1/2))+3/8*a/c^2*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2
*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*
(-a^2*c*d)^(1/2)-5/16*a^6*(c*d)^(1/2)/d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*a
rctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*
c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-9/8*a^3/c/(a^4*c^2+2*a^2*c*d+d^
2)^2*arctanh(a*x)^2*(-a^2*c*d)^(1/2)*d+5/16*a^4*(c*d)^(1/2)/c/d/(a^4*c^2+2*
a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)
*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-3/16*a^4*(c*d)^(1
/2)/c/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*
x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*
d)^(1/2))-3/32*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(
a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(1/2)+d))+3/8*a^8/(a^4
*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arctanh(a*x)*x^3*d-5/4*a^7/(a^4*c^2
+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arctanh(a*x)*x^2*d+5/4*a^6/(a^4*c^2+2*a
^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*arctanh(a*x)*x*d+3/16*(c*d)^(1/2)/c^3*d^2/(
a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^
2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2
))-3/16*(c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-1
/8*a^2*(c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctan(a/d*(c*d)^(1/2))-1/8*
a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*x^2-3/16*(c*d)^(1/2)/c^3*
d/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*
d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)*(c*d)^(1/2))-1/8*
a^2*(c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c+d)*d^2/(c*d)^(1
/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/2)*x-a/(a^2*c+d)
*(c*d)^(1/2))-3/8*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2*d*arctanh
(a*x)+3/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+
1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)+3/8*a*(-a^2
*c*d)^(1/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*arctanh(a*x)^2-3/16*a*(-a^2*c*d)^(1
/2)/c^2/(a^4*c^2+2*a^2*c*d+d^2)*polylog(2,(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/
(-a^2*c+2*(-a^2*c*d)^(1/2)+d))-5/8*a^7/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a
^2*c)^2*c*arctanh(a*x)+3/4*a^6/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/

```
c*arctanh(a*x)*x^3*d^2+3/8*a^4/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^2/
c^2*arctanh(a*x)*x^3*d^3-3/4*a^5/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^
2/c*arctanh(a*x)*x^2*d^2+5/8*a^4/(a^4*c^2+2*a^2*c*d+d^2)/(a^2*d*x^2+a^2*c)^
2/c*arctanh(a*x)*x*d^2-3/16/a*(-a^2*c*d)^(1/2)/c^3*d/(a^4*c^2+2*a^2*c*d+d^2
)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2*c*d)^(
1/2)+d))+3/16/a/c^3*d^3/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/
(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)+5
/16*a^2*(c*d)^(1/2)/c^2*d/(a^2*c+d)/(a^4*c^2+2*a^2*c*d+d^2)*arctan(1/(a^2*c
+d)*d^2/(c*d)^(1/2)*x+1/(a^2*c+d)*d/(c*d)^(1/2)*a*c+a^2/(a^2*c+d)*(c*d)^(1/
2)*x-a/(a^2*c+d)*(c*d)^(1/2))-3/16*a^3*(-a^2*c*d)^(1/2)/c/d/(a^4*c^2+2*a^2*
c*d+d^2)*arctanh(a*x)*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*x^2+1)/(-a^2*c+2*(-a^2
*c*d)^(1/2)+d))+3/16*a^7/d/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)
^2/(-a^2*x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2
)*c+3/4*a/c^2*d^2/(a^4*c^2+2*a^2*c*d+d^2)^2*ln(1-(a^2*c+d)*(a*x+1)^2/(-a^2*
x^2+1)/(-a^2*c-2*(-a^2*c*d)^(1/2)+d))*arctanh(a*x)*(-a^2*c*d)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{artanh}(ax)}{d^3x^6 + 3\,cd^2x^4 + 3\,c^2dx^2 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\left(dx^2 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/(d*x^2 + c)^3, x)
```

3.505 $\int \frac{1}{(a-ax^2)\left(b-2b\tanh^{-1}(x)\right)}\,dx$

Optimal. Leaf size=17

$$-\frac{\log\left(1-2\tanh^{-1}(x)\right)}{2ab}$$

```
[Out] -Log[1 - 2*ArcTanh[x]]/(2*a*b)
```

Rubi [A] time = 0.0444294, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {5946}

$$-\frac{\log\left(1-2\tanh^{-1}(x)\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

```
[Out] -Log[1 - 2*ArcTanh[x]]/(2*a*b)
```

Rule 5946

```
Int[1/(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol
] :> Simp[Log[RemoveContent[a + b*ArcTanh[c*x], x]]/(b*c*d), x] /; FreeQ[{a
, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{1}{(a-ax^2)\left(b-2b\tanh^{-1}(x)\right)}\,dx=-\frac{\log\left(1-2\tanh^{-1}(x)\right)}{2ab}$$

Mathematica [A] time = 0.0546647, size = 17, normalized size = 1.

$$-\frac{\log\left(2\tanh^{-1}(x)-1\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - a*x^2)*(b - 2*b*ArcTanh[x])),x]
```

```
[Out] -Log[-1 + 2*ArcTanh[x]]/(2*a*b)
```

Maple [A] time = 0.059, size = 19, normalized size = 1.1

$$-\frac{\ln\left(2\,b\mathrm{Artanh}\,(x)-b\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x)
```

```
[Out] -1/2/a*ln(2*b*arctanh(x)-b)/b
```

Maxima [A] time = 0.970007, size = 31, normalized size = 1.82

$$-\frac{\log\left(-\log\left(x+1\right)+\log\left(-x+1\right)+1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(-log(x + 1) + log(-x + 1) + 1)/(a*b)
```

Fricas [A] time = 1.91186, size = 58, normalized size = 3.41

$$-\frac{\log\left(\log\left(-\frac{x+1}{x-1}\right)-1\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(log(-(x + 1)/(x - 1)) - 1)/(a*b)
```

Sympy [A] time = 1.57957, size = 14, normalized size = 0.82

$$-\frac{\log\left(\operatorname{atanh}\left(x\right)-\frac{1}{2}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x**2+a)/(b-2*b*atanh(x)),x)
```

```
[Out] -log(atanh(x) - 1/2)/(2*a*b)
```

Giac [B] time = 1.16613, size = 107, normalized size = 6.29

$$-\frac{\log\left(\frac{1}{4}\left(\pi(\operatorname{sgn}\left(x+1\right)-1)-\pi(\operatorname{sgn}\left(x-1\right)+1)+4\,\pi\left\lfloor-\frac{\pi(\operatorname{sgn}(x+1)-1)-\pi(\operatorname{sgn}(x-1)+1)}{4\,\pi}+\frac{1}{2}\right\rfloor\right)^{2}+\left(\log\left(\frac{|x+1|}{|-x+1|}\right)-1\right)^{2}\right)}{4\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x^2+a)/(b-2*b*arctanh(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(1/4*(pi*(sgn(x + 1) - 1) - pi*(sgn(x - 1) + 1) + 4*pi*floor(-1/4*(
pi*(sgn(x + 1) - 1) - pi*(sgn(x - 1) + 1))/pi + 1/2))^2 + (log(abs(x + 1)/a
bs(-x + 1)) - 1)^2)/(a*b)
```

3.506 $\int \frac{\tanh^{-1}(bx)}{1-x^2} \, dx$

Optimal. Leaf size=171

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{1-bx}{1-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1-bx}{b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{bx+1}{1-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{bx+1}{b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\text{l}$$

```
[Out] (Log[-((b*(1 - x))/(1 - b))]*Log[1 - b*x])/4 - (Log[(b*(1 + x))/(1 + b)]*Lo
g[1 - b*x])/4 - (Log[(b*(1 - x))/(1 + b)]*Log[1 + b*x])/4 + (Log[-((b*(1 +
x))/(1 - b))]*Log[1 + b*x])/4 + PolyLog[2, (1 - b*x)/(1 - b)]/4 - PolyLog[2
, (1 - b*x)/(1 + b)]/4 + PolyLog[2, (1 + b*x)/(1 - b)]/4 - PolyLog[2, (1 +
b*x)/(1 + b)]/4
```

Rubi [A] time = 0.240118, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5972, 2409, 2394, 2393, 2391}

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{1-bx}{1-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{1-bx}{b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{bx+1}{1-b}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{bx+1}{b+1}\right)+\frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\text{l}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - b))]*Log[1 - b*x])/4 - (Log[(b*(1 + x))/(1 + b)]*Lo
g[1 - b*x])/4 - (Log[(b*(1 - x))/(1 + b)]*Log[1 + b*x])/4 + (Log[-((b*(1 +
x))/(1 - b))]*Log[1 + b*x])/4 + PolyLog[2, (1 - b*x)/(1 - b)]/4 - PolyLog[2
, (1 - b*x)/(1 + b)]/4 + PolyLog[2, (1 + b*x)/(1 - b)]/4 - PolyLog[2, (1 +
b*x)/(1 + b)]/4
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
```

```
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(bx)}{1-x^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-bx)}{1-x^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+bx)}{1-x^2}\,dx$$
$$= -\left(\frac{1}{2}\int \left(\frac{\log(1-bx)}{2(1-x)}+\frac{\log(1-bx)}{2(1+x)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+bx)}{2(1-x)}+\frac{\log(1+bx)}{2(1+x)}\right)dx$$
$$= -\left(\frac{1}{4}\int \frac{\log(1-bx)}{1-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-bx)}{1+x}\,dx+\frac{1}{4}\int \frac{\log(1+bx)}{1-x}\,dx+\frac{1}{4}\int \frac{\log(1+bx)}{1+x}\,dx$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)+\frac{1}{4}\mathrm{l}$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)+\frac{1}{4}\mathrm{l}$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1+b}\right)\log(1-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+b}\right)\log(1+bx)+\frac{1}{4}\mathrm{l}$$

Mathematica [C] time = 0.885032, size = 576, normalized size = 3.37

$$-\frac{b\left(i\left(\mathrm{PolyLog}\left(2,\frac{\left(b^2-2i\sqrt{-b^2}+1\right)\left(b-i\sqrt{-b^2}x\right)}{\left(b^2-1\right)\left(b+i\sqrt{-b^2}x\right)}\right)-\mathrm{PolyLog}\left(2,\frac{\left(b^2+2i\sqrt{-b^2}+1\right)\left(b-i\sqrt{-b^2}x\right)}{\left(b^2-1\right)\left(b+i\sqrt{-b^2}x\right)}\right)\right)+2i\cos^{-1}\left(\frac{b^2+1}{1-b^2}\right)\tan^{-1}\left(\frac{bx}{\sqrt{-b^2}}\right)-4\,\mathrm{t}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[b*x]/(1 - x^2),x]
```

```
[Out] -(b*((2*I)*ArcCos[(1 + b^2)/(1 - b^2)]*ArcTan[(b*x)/Sqrt[-b^2]] - 4*ArcTan[
Sqrt[-b^2]/(b*x)]*ArcTanh[b*x] - (ArcCos[(1 + b^2)/(1 - b^2)] - 2*ArcTan[(b
*x)/Sqrt[-b^2]])*Log[(2*b*(-I + Sqrt[-b^2])*(-1 + b*x))/((-1 + b^2)*((-I)*b
 + Sqrt[-b^2]*x))] - (ArcCos[(1 + b^2)/(1 - b^2)] + 2*ArcTan[(b*x)/Sqrt[-b^
2]])*Log[(2*b*(I + Sqrt[-b^2])*(1 + b*x))/((-1 + b^2)*((-I)*b + Sqrt[-b^2]*
x))] + (ArcCos[(1 + b^2)/(1 - b^2)] - 2*(ArcTan[Sqrt[-b^2]/(b*x)] + ArcTan[
(b*x)/Sqrt[-b^2]]))*Log[(Sqrt[2]*Sqrt[-b^2])/(Sqrt[-1 + b^2]*E^ArcTanh[b*x]
*Sqrt[1 + b^2 + (-1 + b^2)*Cosh[2*ArcTanh[b*x]]])] + (ArcCos[(1 + b^2)/(1 -
 b^2)] + 2*(ArcTan[Sqrt[-b^2]/(b*x)] + ArcTan[(b*x)/Sqrt[-b^2]]))*Log[(Sqrt
[2]*Sqrt[-b^2]*E^ArcTanh[b*x])/(Sqrt[-1 + b^2]*Sqrt[1 + b^2 + (-1 + b^2)*Co
sh[2*ArcTanh[b*x]]])] + I*(PolyLog[2, ((1 + b^2 - (2*I)*Sqrt[-b^2])*(b - I*
Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x))] - PolyLog[2, ((1 + b^2 +
(2*I)*Sqrt[-b^2])*(b - I*Sqrt[-b^2]*x))/((-1 + b^2)*(b + I*Sqrt[-b^2]*x))])
))/(4*Sqrt[-b^2])
```

Maple [A] time = 0.106, size = 176, normalized size = 1.

$$\frac{\mathrm{Artanh}\,(bx)\ln(bx+b)}{2}-\frac{\mathrm{Artanh}\,(bx)\ln(bx-b)}{2}-\frac{1}{4}\mathrm{dilog}\left(\frac{bx+1}{1-b}\right)-\frac{\ln(bx+b)}{4}\ln\left(\frac{bx+1}{1-b}\right)+\frac{1}{4}\mathrm{dilog}\left(\frac{bx-1}{-b-1}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x)/(-x^2+1),x)
```

```
[Out] 1/2*arctanh(b*x)*ln(b*x+b)-1/2*arctanh(b*x)*ln(b*x-b)-1/4*dilog((b*x+1)/(1-
b))-1/4*ln(b*x+b)*ln((b*x+1)/(1-b))+1/4*dilog((b*x-1)/(-b-1))+1/4*ln(b*x+b)
*ln((b*x-1)/(-b-1))+1/4*dilog((b*x+1)/(1+b))+1/4*ln(b*x-b)*ln((b*x+1)/(1+b)
)-1/4*dilog((b*x-1)/(-1+b))-1/4*ln(b*x-b)*ln((b*x-1)/(-1+b))
```

Maxima [A] time = 0.959717, size = 243, normalized size = 1.42

$$\frac{1}{4}b\left(\frac{\log(x+1)\log\left(-\frac{bx+b}{b+1}+1\right)+\text{Li}_2\left(\frac{bx+b}{b+1}\right)}{b}+\frac{\log(x-1)\log\left(\frac{bx-b}{b+1}+1\right)+\text{Li}_2\left(-\frac{bx-b}{b+1}\right)}{b}-\frac{\log(x+1)\log\left(-\frac{bx+b}{b-1}+1\right)+\text{L}}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(x + 1)*log(-(b*x + b)/(b + 1) + 1) + dilog((b*x + b)/(b + 1)))/
b + (log(x - 1)*log((b*x - b)/(b + 1) + 1) + dilog(-(b*x - b)/(b + 1)))/b -
 (log(x + 1)*log(-(b*x + b)/(b - 1) + 1) + dilog((b*x + b)/(b - 1)))/b - (l
og(x - 1)*log((b*x - b)/(b - 1) + 1) + dilog(-(b*x - b)/(b - 1)))/b) + 1/2*
(log(x + 1) - log(x - 1))*arctanh(b*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\text{artanh}(bx)}{x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(b*x)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\text{atanh}(bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x)/(-x**2+1),x)
```

```
[Out] -Integral(atanh(b*x)/(x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\text{artanh}(bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(b*x)/(x^2 - 1), x)
```

3.507 $\int \frac{\tanh^{-1}(a+bx)}{1-x^2}\,dx$

Optimal. Leaf size=203

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a+b+1}\right)+$$

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

Rubi [A] time = 0.257796, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6115, 2409, 2394, 2393, 2391}

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a+b+1}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a + b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

Rule 6115

```
Int[ArcTanh[(c_) + (d_.)*(x_)]/((e_) + (f_.)*(x_)^(n_.)), x_Symbol] :> Dist
[1/2, Int[Log[1 + c + d*x]/(e + f*x^n), x], x] - Dist[1/2, Int[Log[1 - c -
d*x]/(e + f*x^n), x], x] /; FreeQ[{c, d, e, f}, x] && RationalQ[n]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

```
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(a+bx)}{1-x^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-a-bx)}{1-x^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+a+bx)}{1-x^2}\,dx$$
$$= -\left(\frac{1}{2}\int \left(\frac{\log(1-a-bx)}{2(1-x)}+\frac{\log(1-a-bx)}{2(1+x)}\right)dx\right)+\frac{1}{2}\int \left(\frac{\log(1+a+bx)}{2(1-x)}+\frac{\log(1+a+bx)}{2(1+x)}\right)$$
$$= -\left(\frac{1}{4}\int \frac{\log(1-a-bx)}{1-x}\,dx\right)-\frac{1}{4}\int \frac{\log(1-a-bx)}{1+x}\,dx+\frac{1}{4}\int \frac{\log(1+a+bx)}{1-x}\,dx+\frac{1}{4}\int \frac{\log(}{}$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+a+b}\right)\log$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+a+b}\right)\log$$
$$= \frac{1}{4}\log\left(-\frac{b(1-x)}{1-a-b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1+x)}{1-a+b}\right)\log(1-a-bx)-\frac{1}{4}\log\left(\frac{b(1-x)}{1+a+b}\right)\log$$

Mathematica [A] time = 0.0392179, size = 203, normalized size = 1.

$$\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{-a-bx+1}{-a+b+1}\right)+\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a-b+1}\right)-\frac{1}{4}\text{PolyLog}\left(2,\frac{a+bx+1}{a+b+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a + b*x]/(1 - x^2),x]
```

```
[Out] (Log[-((b*(1 - x))/(1 - a - b))]*Log[1 - a - b*x])/4 - (Log[(b*(1 + x))/(1
- a + b)]*Log[1 - a - b*x])/4 - (Log[(b*(1 - x))/(1 + a + b)]*Log[1 + a + b
*x])/4 + (Log[-((b*(1 + x))/(1 + a - b))]*Log[1 + a + b*x])/4 + PolyLog[2,
(1 - a - b*x)/(1 - a - b)]/4 - PolyLog[2, (1 - a - b*x)/(1 - a + b)]/4 + Po
lyLog[2, (1 + a + b*x)/(1 + a - b)]/4 - PolyLog[2, (1 + a + b*x)/(1 + a + b
)]/4
```

Maple [A] time = 0.161, size = 196, normalized size = 1.

$$-\frac{\text{Artanh}(bx+a)\ln(bx-b)}{2}+\frac{\text{Artanh}(bx+a)\ln(bx+b)}{2}-\frac{1}{4}\text{dilog}\left(\frac{bx+a+1}{1+a-b}\right)-\frac{\ln(bx+b)}{4}\ln\left(\frac{bx+a+1}{1+a-b}\right)+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(b*x+a)/(-x^2+1),x)
```

```
[Out] -1/2*arctanh(b*x+a)*ln(b*x-b)+1/2*arctanh(b*x+a)*ln(b*x+b)-1/4*dilog((b*x+a
+1)/(1+a-b))-1/4*ln(b*x+b)*ln((b*x+a+1)/(1+a-b))+1/4*dilog((b*x+a-1)/(-b-1+
a))+1/4*ln(b*x+b)*ln((b*x+a-1)/(-b-1+a))-1/4*dilog((b*x+a-1)/(b-1+a))-1/4*l
```

```
n(b*x-b)*ln((b*x+a-1)/(b-1+a))+1/4*dilog((b*x+a+1)/(1+a+b))+1/4*ln(b*x-b)*l
n((b*x+a+1)/(1+a+b))
```

Maxima [A] time = 0.972205, size = 267, normalized size = 1.32

$$\frac{1}{4}b\left(\frac{\log(x-1)\log\left(\frac{bx-b}{a+b+1}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{a+b+1}\right)}{b}-\frac{\log(x-1)\log\left(\frac{bx-b}{a+b-1}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{a+b-1}\right)}{b}-\frac{\log(x+1)\log\left(\frac{bx+b}{a-b+1}+1\right)}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/4*b*((log(x - 1)*log((b*x - b)/(a + b + 1) + 1) + dilog(-(b*x - b)/(a + b
 + 1)))/b - (log(x - 1)*log((b*x - b)/(a + b - 1) + 1) + dilog(-(b*x - b)/(
a + b - 1)))/b - (log(x + 1)*log((b*x + b)/(a - b + 1) + 1) + dilog(-(b*x +
 b)/(a - b + 1)))/b + (log(x + 1)*log((b*x + b)/(a - b - 1) + 1) + dilog(-(
b*x + b)/(a - b - 1)))/b) + 1/2*(log(x + 1) - log(x - 1))*arctanh(b*x + a)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\operatorname{artanh}(bx+a)}{x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-arctanh(b*x + a)/(x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\operatorname{atanh}(a+bx)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(b*x+a)/(-x**2+1),x)
```

```
[Out] -Integral(atanh(a + b*x)/(x**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{\operatorname{artanh}(bx+a)}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(b*x+a)/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-arctanh(b*x + a)/(x^2 - 1), x)
```

3.508 $\int \frac{\tanh^{-1}(x)}{a+bx}\,dx$

Optimal. Leaf size=86

$$-\frac{\text{PolyLog}\left(2,1-\frac{2(a+bx)}{(x+1)(a+b)}\right)}{2b}+\frac{\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)}{2b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(x+1)(a+b)}\right)}{b}-\frac{\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)}{b}$$

```
[Out] -((ArcTanh[x]*Log[2/(1 + x)])/b) + (ArcTanh[x]*Log[(2*(a + b*x))/((a + b)*(
1 + x))])/b + PolyLog[2, 1 - 2/(1 + x)]/(2*b) - PolyLog[2, 1 - (2*(a + b*x)
)/((a + b)*(1 + x))]/(2*b)
```

Rubi [A] time = 0.0608797, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {5920, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2(a+bx)}{(x+1)(a+b)}\right)}{2b}+\frac{\text{PolyLog}\left(2,1-\frac{2}{x+1}\right)}{2b}+\frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(x+1)(a+b)}\right)}{b}-\frac{\log\left(\frac{2}{x+1}\right)\tanh^{-1}(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x),x]
```

```
[Out] -((ArcTanh[x]*Log[2/(1 + x)])/b) + (ArcTanh[x]*Log[(2*(a + b*x))/((a + b)*(
1 + x))])/b + PolyLog[2, 1 - 2/(1 + x)]/(2*b) - PolyLog[2, 1 - (2*(a + b*x)
)/((a + b)*(1 + x))]/(2*b)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx}\,dx = -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b} + \frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b} + \frac{\int \frac{\log\left(\frac{2}{1+x}\right)}{1-x^2}dx}{b} - \frac{\int \frac{\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{1-x^2}dx}{b}$$
$$= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b} + \frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b} - \frac{\mathrm{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(1+x)}\right)}{2b} + \frac{\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}dx, x, \frac{1}{1+x}\right)}{b}$$
$$= -\frac{\tanh^{-1}(x)\log\left(\frac{2}{1+x}\right)}{b} + \frac{\tanh^{-1}(x)\log\left(\frac{2(a+bx)}{(a+b)(1+x)}\right)}{b} + \frac{\mathrm{Li}_2\left(1-\frac{2}{1+x}\right)}{2b} - \frac{\mathrm{Li}_2\left(1-\frac{2(a+bx)}{(a+b)(1+x)}\right)}{2b}$$

Mathematica [C] time = 0.0845367, size = 260, normalized size = 3.02

$$\frac{-4\mathrm{PolyLog}\left(2, e^{-2\left(\tanh^{-1}\left(\frac{a}{b}\right)+\tanh^{-1}(x)\right)}\right) - 4\mathrm{PolyLog}\left(2, -e^{2\tanh^{-1}(x)}\right) + 8\tanh^{-1}(x)\tanh^{-1}\left(\frac{a}{b}\right) + 8\tanh^{-1}\left(\frac{a}{b}\right)\log\left(1-e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/(a + b*x),x]
```

```
[Out] (-Pi^2 + 4*ArcTanh[a/b]^2 + (4*I)*Pi*ArcTanh[x] + 8*ArcTanh[a/b]*ArcTanh[x]
 + 8*ArcTanh[x]^2 - (4*I)*Pi*Log[1 + E^(2*ArcTanh[x])] - 8*ArcTanh[x]*Log[1
 + E^(2*ArcTanh[x])] + 8*ArcTanh[a/b]*Log[1 - E^(-2*(ArcTanh[a/b] + ArcTanh
[x]))] + 8*ArcTanh[x]*Log[1 - E^(-2*(ArcTanh[a/b] + ArcTanh[x]))] + (4*I)*P
i*Log[2/Sqrt[1 - x^2]] + 8*ArcTanh[x]*Log[2/Sqrt[1 - x^2]] + 4*ArcTanh[x]*L
og[1 - x^2] + 8*ArcTanh[x]*Log[I*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 8*ArcTa
nh[a/b]*Log[(2*I)*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 8*ArcTanh[x]*Log[(2*I)
*Sinh[ArcTanh[a/b] + ArcTanh[x]]] - 4*PolyLog[2, -E^(2*ArcTanh[x])] - 4*Pol
yLog[2, E^(-2*(ArcTanh[a/b] + ArcTanh[x]))])/(8*b)
```

Maple [A] time = 0.11, size = 110, normalized size = 1.3

$$\frac{\ln(bx+a)\,\mathrm{Artanh}(x)}{b} + \frac{\ln(bx+a)}{2b}\ln\left(\frac{bx-b}{-a-b}\right) + \frac{1}{2b}\mathrm{dilog}\left(\frac{bx-b}{-a-b}\right) - \frac{\ln(bx+a)}{2b}\ln\left(\frac{bx+b}{-a+b}\right) - \frac{1}{2b}\mathrm{dilog}\left(\frac{bx+b}{-a+b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b*arctanh(x)+1/2/b*ln(b*x+a)*ln((b*x-b)/(-a-b))+1/2/b*dilog((b*x-
b)/(-a-b))-1/2/b*ln(b*x+a)*ln((b*x+b)/(-a+b))-1/2/b*dilog((b*x+b)/(-a+b))
```

Maxima [A] time = 0.974155, size = 161, normalized size = 1.87

$$-\frac{(\log(x+1)-\log(x-1))\log(bx+a)}{2b} + \frac{\mathrm{artanh}(x)\log(bx+a)}{b} - \frac{\log(x-1)\log\left(\frac{bx-b}{a+b}+1\right)+\mathrm{Li}_2\left(-\frac{bx-b}{a+b}\right)}{2b} + \frac{\log(x+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(x + 1) - log(x - 1))*log(b*x + a)/b + arctanh(x)*log(b*x + a)/b -
 1/2*(log(x - 1)*log((b*x - b)/(a + b) + 1) + dilog(-(b*x - b)/(a + b)))/b
+ 1/2*(log(x + 1)*log((b*x + b)/(a - b) + 1) + dilog(-(b*x + b)/(a - b)))/b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}(x)}{bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(x)}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(b*x+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(x)}{bx + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(b*x + a), x)
```

3.509 $\int \frac{\tanh^{-1}(x)}{a+bx^2}\,dx$

Optimal. Leaf size=397

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{b}(1-x)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(1-x)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{b}(x+1)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(x+1)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1-x)\log\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{4\sqrt{-a}\sqrt{b}}$$

```
[Out] -(Log[1 - x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] - Sqrt[b])])/(4*Sqrt[-a]*
Sqrt[b]) + (Log[1 + x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] + Sqrt[b])])/(4
*Sqrt[-a]*Sqrt[b]) - (Log[1 + x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt[-a] - Sqr
t[b])])/(4*Sqrt[-a]*Sqrt[b]) + (Log[1 - x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt
[-a] + Sqrt[b])])/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 - x))/(Sq
rt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 - x))/(Sq
rt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 + x))/(S
qrt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 + x))/(S
qrt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b])
```

Rubi [A] time = 0.372074, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5972, 2409, 2394, 2393, 2391}

$$-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{b}(1-x)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(1-x)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\text{PolyLog}\left(2,-\frac{\sqrt{b}(x+1)}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{b}(x+1)}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1-x)\log\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{4\sqrt{-a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x^2),x]
```


```
[Out] -(Log[1 - x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] - Sqrt[b])])/(4*Sqrt[-a]*
Sqrt[b]) + (Log[1 + x]*Log[(Sqrt[-a] - Sqrt[b]*x)/(Sqrt[-a] + Sqrt[b])])/(4
*Sqrt[-a]*Sqrt[b]) - (Log[1 + x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt[-a] - Sqr
t[b])])/(4*Sqrt[-a]*Sqrt[b]) + (Log[1 - x]*Log[(Sqrt[-a] + Sqrt[b]*x)/(Sqrt
[-a] + Sqrt[b])])/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 - x))/(Sq
rt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 - x))/(Sq
rt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b]) - PolyLog[2, -((Sqrt[b]*(1 + x))/(S
qrt[-a] - Sqrt[b]))]/(4*Sqrt[-a]*Sqrt[b]) + PolyLog[2, (Sqrt[b]*(1 + x))/(S
qrt[-a] + Sqrt[b])]/(4*Sqrt[-a]*Sqrt[b])
```


Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
```

```
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx^2}\,dx = -\left(\frac{1}{2}\int \frac{\log(1-x)}{a+bx^2}\,dx\right)+\frac{1}{2}\int \frac{\log(1+x)}{a+bx^2}\,dx$$

$$= -\left(\frac{1}{2}\int\left(\frac{\sqrt{-a}\log(1-x)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\log(1-x)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx\right)+\frac{1}{2}\int\left(\frac{\sqrt{-a}\log(1+x)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\log(1+x)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)$$

$$= \frac{\int \frac{\log(1-x)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}}+\frac{\int \frac{\log(1-x)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}}-\frac{\int \frac{\log(1+x)}{\sqrt{-a}-\sqrt{b}x}\,dx}{4\sqrt{-a}}-\frac{\int \frac{\log(1+x)}{\sqrt{-a}+\sqrt{b}x}\,dx}{4\sqrt{-a}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(1-x)\log}{4\sqrt{-}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(1-x)\log}{4\sqrt{-}}$$

$$= -\frac{\log(1-x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(1+x)\log\left(\frac{\sqrt{-a}-\sqrt{b}x}{\sqrt{-a}+\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}-\frac{\log(1+x)\log\left(\frac{\sqrt{-a}+\sqrt{b}x}{\sqrt{-a}-\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}}+\frac{\log(1-x)\log}{4\sqrt{-}}$$

Mathematica [C] time = 1.0085, size = 485, normalized size = 1.22

$$-\frac{i\left(\text{PolyLog}\left(2,\frac{\left(2i\sqrt{ab}-a+b\right)\left(x\sqrt{ab}+ia\right)}{(a+b)\left(x\sqrt{ab}-ia\right)}\right)-\text{PolyLog}\left(2,\frac{\left(-2i\sqrt{ab}-a+b\right)\left(x\sqrt{ab}+ia\right)}{(a+b)\left(x\sqrt{ab}-ia\right)}\right)\right)-2i\cos^{-1}\left(\frac{b-a}{a+b}\right)\tan^{-1}\left(\frac{bx}{\sqrt{ab}}\right)+4\tanh^{-1}(x)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a + b*x^2),x]
```

```
[Out] -((-2*I)*ArcCos[(-a + b)/(a + b)]*ArcTan[(b*x)/Sqrt[a*b]] + 4*ArcTan[a/(Sqr
t[a*b]*x)]*ArcTanh[x] - (ArcCos[(-a + b)/(a + b)] + 2*ArcTan[(b*x)/Sqrt[a*b
]])*Log[((2*I)*a*(I*b + Sqrt[a*b])*(-1 + x))/((a + b)*(a + I*Sqrt[a*b]*x))]
 - (ArcCos[(-a + b)/(a + b)] - 2*ArcTan[(b*x)/Sqrt[a*b]])*Log[(2*a*(b + I*S
qrt[a*b])*(1 + x))/((a + b)*(a + I*Sqrt[a*b]*x))] + (ArcCos[(-a + b)/(a + b
)] + 2*(ArcTan[a/(Sqrt[a*b]*x)] + ArcTan[(b*x)/Sqrt[a*b]]))*Log[(Sqrt[2]*Sq
rt[a*b])/(Sqrt[a + b]*E^ArcTanh[x]*Sqrt[a - b + (a + b)*Cosh[2*ArcTanh[x]]]
)] + (ArcCos[(-a + b)/(a + b)] - 2*(ArcTan[a/(Sqrt[a*b]*x)] + ArcTan[(b*x)/
```

```
Sqrt[a*b]]))*Log[(Sqrt[2]*Sqrt[a*b]*E^ArcTanh[x])/(Sqrt[a + b]*Sqrt[a - b +
 (a + b)*Cosh[2*ArcTanh[x]]])] + I*(-PolyLog[2, ((-a + b - (2*I)*Sqrt[a*b])
*(I*a + Sqrt[a*b]*x))/((a + b)*((-I)*a + Sqrt[a*b]*x))] + PolyLog[2, ((-a +
 b + (2*I)*Sqrt[a*b])*(I*a + Sqrt[a*b]*x))/((a + b)*((-I)*a + Sqrt[a*b]*x))
]))/(4*Sqrt[a*b])
```

Maple [B] time = 0.186, size = 606, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(b*x^2+a),x)
```

```
[Out] -(-2*(-a*b)^(1/2)+a-b)/(a^2+2*a*b+b^2)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*
b)^(1/2)-a+b))*arctanh(x)+1/2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/b/(a^2+
2*a*b+b^2)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*arctanh(x)-1/
2*(2*a*b+(-a*b)^(1/2)*a-(-a*b)^(1/2)*b)/a/(a^2+2*a*b+b^2)*ln(1-(a+b)*(1+x)^
2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*arctanh(x)-1/(a^2+2*a*b+b^2)*arctanh(x)^2
*(-a*b)^(1/2)-1/2/b/(a^2+2*a*b+b^2)*arctanh(x)^2*a*(-a*b)^(1/2)-1/2/a/(a^2+
2*a*b+b^2)*arctanh(x)^2*b*(-a*b)^(1/2)+1/2/(a^2+2*a*b+b^2)*polylog(2,(a+b)*
(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*(-a*b)^(1/2)+1/4/b/(a^2+2*a*b+b^2)*
polylog(2,(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*a*(-a*b)^(1/2)+1/4/
a/(a^2+2*a*b+b^2)*polylog(2,(a+b)*(1+x)^2/(-x^2+1)/(-2*(-a*b)^(1/2)-a+b))*b
*(-a*b)^(1/2)-1/2*(-a*b)^(1/2)/a/b*arctanh(x)*ln(1-(a+b)*(1+x)^2/(-x^2+1)/(
2*(-a*b)^(1/2)-a+b))+1/2*(-a*b)^(1/2)/a/b*arctanh(x)^2-1/4*(-a*b)^(1/2)/a/b
*polylog(2,(a+b)*(1+x)^2/(-x^2+1)/(2*(-a*b)^(1/2)-a+b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}(x)}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(x)}{a+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(b*x**2+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(x)}{bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(b*x^2 + a), x)
```

3.510 $\int \frac{\tanh^{-1}(x)}{a+bx+cx^2} dx$

Optimal. Leaf size=258

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(\sqrt{b^2-4ac}+b+2c\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tanh^{-1}(x)\log\left(\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}-$$

```
[Out] (ArcTanh[x]*Log[(2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2 -
4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c] - (ArcTanh[x]*Log[(2*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c]
 - PolyLog[2, 1 - (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2
- 4*a*c])*(1 + x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 - (2*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))]/(2*Sqrt[b^2 -
4*a*c])
```

Rubi [A] time = 0.328003, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {618, 206, 6728, 5920, 2402, 2315, 2447}

$$-\frac{\text{PolyLog}\left(2,1-\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\text{PolyLog}\left(2,1-\frac{2\left(\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(\sqrt{b^2-4ac}+b+2c\right)}\right)}{2\sqrt{b^2-4ac}}+\frac{\tanh^{-1}(x)\log\left(\frac{2\left(-\sqrt{b^2-4ac}+b+2cx\right)}{(x+1)\left(-\sqrt{b^2-4ac}+b+2c\right)}\right)}{\sqrt{b^2-4ac}}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a + b*x + c*x^2),x]
```


```
[Out] (ArcTanh[x]*Log[(2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2 -
4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c] - (ArcTanh[x]*Log[(2*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))])/Sqrt[b^2 - 4*a*c]
 - PolyLog[2, 1 - (2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/((b + 2*c - Sqrt[b^2
- 4*a*c])*(1 + x))]/(2*Sqrt[b^2 - 4*a*c]) + PolyLog[2, 1 - (2*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x))/((b + 2*c + Sqrt[b^2 - 4*a*c])*(1 + x))]/(2*Sqrt[b^2 -
4*a*c])
```


Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{a+bx+cx^2}\,dx = \int \left(\frac{2c\tanh^{-1}(x)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c\tanh^{-1}(x)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx$$

$$= \frac{(2c)\int \frac{\tanh^{-1}(x)}{b-\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\tanh^{-1}(x)}{b+\sqrt{b^2-4ac}+2cx}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\int \frac{\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{1-x^2}}{\sqrt{b^2-4ac}}$$

$$= \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b-\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c-\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\tanh^{-1}(x)\log\left(\frac{2\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(b+2c+\sqrt{b^2-4ac}\right)(1+x)}\right)}{\sqrt{b^2-4ac}} - \frac{\mathrm{Li}_2\left(1-\frac{2\left(b-\sqrt{b^2-4ac}\right.}{\left(b+2c-\sqrt{b^2-4a\right.}}\right.}{2\sqrt{b^2-4ac}}$$

Mathematica [C] time = 18.6729, size = 874, normalized size = 3.39

$$\frac{2\sqrt{4ac-b^2}\left(b\left(\sqrt{\frac{c(a+b+c)}{4ac-b^2}}e^{i\tan^{-1}\left(\frac{-b-2c}{\sqrt{4ac-b^2}}\right)}-\sqrt{\frac{c(a-b+c)}{4ac-b^2}}e^{i\tan^{-1}\left(\frac{2c-b}{\sqrt{4ac-b^2}}\right)}\right)-2c\left(e^{i\tan^{-1}\left(\frac{2c-b}{\sqrt{4ac-b^2}}\right)}\sqrt{\frac{c(a-b+c)}{4ac-b^2}}+\sqrt{\frac{c(a+b+c)}{4ac-b^2}}e^{i\tan^{-1}\left(\frac{-b-2c}{\sqrt{4ac-b^2}}\right)}-1\right)\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right.}{b^2-4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/(a + b*x + c*x^2),x]
```

```
[Out] ((2*Sqrt[-b^2 + 4*a*c]*(b*(Sqrt[(c*(a + b + c))/(-b^2 + 4*a*c)]*E^(I*ArcTan
[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]) - Sqrt[(c*(a - b + c))/(-b^2 + 4*a*c)]*E^(
I*ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]])) - 2*c*(-1 + Sqrt[(c*(a + b + c))/
(-b^2 + 4*a*c)]*E^(I*ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]) + Sqrt[(c*(a -
b + c))/(-b^2 + 4*a*c)]*E^(I*ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]])))*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]^2)/(b^2 - 4*c^2) + 2*ArcTan[(b + 2*c*x)/S
qrt[-b^2 + 4*a*c]]*((-I)*ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + I*ArcTan[(
-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + 2*ArcTanh[x] + Log[1 - E^((2*I)*(ArcTan[(-b
 - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]))] - L
og[1 - E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]]))]) + 2*(ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]]*(Log[1
- E^((2*I)*(ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]]))] - Log[Sin[ArcTan[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan
[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]]]) + ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]]
*(-Log[1 - E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*
c*x)/Sqrt[-b^2 + 4*a*c]]))] + Log[Sin[ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]]
 + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]]])) - I*PolyLog[2, E^((2*I)*(ArcT
an[(-b - 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
)] + I*PolyLog[2, E^((2*I)*(ArcTan[(-b + 2*c)/Sqrt[-b^2 + 4*a*c]] + ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]))])/(2*Sqrt[-b^2 + 4*a*c])
```

Maple [B] time = 0.337, size = 1599, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(c*x^2+b*x+a),x)
```

```
[Out] 1/2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-
4*a*c+b^2)^(1/2)-a+c))*c^2*(-4*a*c+b^2)^(1/2)-2/(4*a*c-b^2)/(a^2+2*a*c-b^2+
c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*c^2*a+1/
2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*
a*c+b^2)^(1/2)-a+c))*c*b^2-1/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a
^2*(-4*a*c+b^2)^(1/2)-4/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a^2*c+
1/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a*b^2-1/(4*a*c-b^2)/(a^2+2*a
*c-b^2+c^2)*arctanh(x)^2*c^2*(-4*a*c+b^2)^(1/2)+4/(4*a*c-b^2)/(a^2+2*a*c-b^
2+c^2)*arctanh(x)^2*c^2*a-1/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*c*
b^2+1/2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/
(-(-4*a*c+b^2)^(1/2)-a+c))*a^2*(-4*a*c+b^2)^(1/2)+2/(4*a*c-b^2)/(a^2+2*a*c-
b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a^2*
c+(4*a*c-b^2+(-4*a*c+b^2)^(1/2)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*
a*c-b^2+c^2)*ln(1-(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*arcta
nh(x)*a+2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a*c*(-4*a*c+b^2)^(1/
2)-1/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(
-4*a*c+b^2)^(1/2)-a+c))*a*c*(-4*a*c+b^2)^(1/2)-(4*a*c-b^2+(-4*a*c+b^2)^(1/2
)*a-(-4*a*c+b^2)^(1/2)*c)/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)*ln(1-(a+b+c)*(1+x
)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*arctanh(x)*c-(-4*a*c+b^2)^(1/2)/(4*
a*c-b^2)*arctanh(x)*ln(1-(a+b+c)*(1+x)^2/(-x^2+1)/((-4*a*c+b^2)^(1/2)-a+c))
-(-(-4*a*c+b^2)^(1/2)+a-c)/(a^2+2*a*c-b^2+c^2)*ln(1-(a+b+c)*(1+x)^2/(-x^2+1
)/(-(-4*a*c+b^2)^(1/2)-a+c))*arctanh(x)-1/2/(4*a*c-b^2)/(a^2+2*a*c-b^2+c^2)
*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*a*b^2-1/2*(-
4*a*c+b^2)^(1/2)/(4*a*c-b^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/((-4*a*c+b^
2)^(1/2)-a+c))+(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*arctanh(x)^2-1/(a^2+2*a*c-b^2
+c^2)*arctanh(x)^2*(-4*a*c+b^2)^(1/2)+1/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*a-
1/(a^2+2*a*c-b^2+c^2)*arctanh(x)^2*c+1/2/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b
+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c))*(-4*a*c+b^2)^(1/2)-1/2/(a^2
```

```
+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b^2)^(1/2)-a+c
))*a+1/2/(a^2+2*a*c-b^2+c^2)*polylog(2,(a+b+c)*(1+x)^2/(-x^2+1)/(-(-4*a*c+b
^2)^(1/2)-a+c))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\operatorname{artanh}(x)}{cx^2+bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] integral(arctanh(x)/(c*x^2 + b*x + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(x)}{a+bx+cx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(c*x**2+b*x+a),x)
```

```
[Out] Integral(atanh(x)/(a + b*x + c*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(x)}{cx^2+bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/(c*x^2 + b*x + a), x)
```

3.511 $\int \sqrt{c+dx^2}\tanh^{-1}(ax)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\tanh^{-1}(ax)\sqrt{c+dx^2},x\right)$$

```
[Out] Unintegrable[Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Rubi [A] time = 0.0218606, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{c+dx^2}\tanh^{-1}(ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Rubi steps

$$\int \sqrt{c+dx^2}\tanh^{-1}(ax)\,dx=\int \sqrt{c+dx^2}\tanh^{-1}(ax)\,dx$$

Mathematica [A] time = 3.93921, size = 0, normalized size = 0.

$$\int \sqrt{c+dx^2}\tanh^{-1}(ax)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x],x]
```

```
[Out] Integrate[Sqrt[c + d*x^2]*ArcTanh[a*x], x]
```

Maple [A] time = 0.772, size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\,\text{Artanh}\,(ax)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

```
[Out] int((d*x^2+c)^(1/2)*arctanh(a*x),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx^2 + c}\,\text{artanh}\,(ax)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx^2}\,\text{atanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*atanh(a*x),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*atanh(a*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + c}\,\text{artanh}\,(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*arctanh(a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)*arctanh(a*x), x)
```

3.512 $\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}},x\right)$$

```
[Out] Unintegrable[ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi [A] time = 0.0239079, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Defer[Int][ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx = \int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

Mathematica [A] time = 3.85779, size = 0, normalized size = 0.

$$\int \frac{\tanh^{-1}(ax)}{\sqrt{c+dx^2}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2],x]
```

```
[Out] Integrate[ArcTanh[a*x]/Sqrt[c + d*x^2], x]
```

Maple [A] time = 0.632, size = 0, normalized size = 0.

$$\int \text{Artanh}(ax) \frac{1}{\sqrt{dx^2+c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{artanh}(ax)}{\sqrt{dx^2+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax)}{\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(atanh(a*x)/sqrt(c + d*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax)}{\sqrt{dx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(a*x)/sqrt(d*x^2 + c), x)
```

3.513 $\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

```
[Out] (x*ArcTanh[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rubi [A] time = 0.121636, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {191, 5976, 12, 444, 63, 208}

$$\frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}}-\frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] (x*ArcTanh[a*x])/(c*Sqrt[c + d*x^2]) - ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2
*c + d]]/(c*Sqrt[a^2*c + d])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{3/2}}\,dx &= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - a\int \frac{x}{c\left(1-a^2x^2\right)\sqrt{c+dx^2}}\,dx \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\int \frac{x}{\left(1-a^2x^2\right)\sqrt{c+dx^2}}\,dx}{c} \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\left(1-a^2x\right)\sqrt{c+dx}}\,dx, x, x^2\right)}{2c} \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\,dx, x, \sqrt{c+dx^2}\right)}{cd} \\
&= \frac{x\tanh^{-1}(ax)}{c\sqrt{c+dx^2}} - \frac{\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{c\sqrt{a^2c+d}}
\end{aligned}$$

Mathematica [A] time = 0.110718, size = 119, normalized size = 1.92

$$\frac{\frac{-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx\right)-\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx\right)+\log(1-ax)+\log(ax+1)}{\sqrt{a^2c+d}}+\frac{2x\tanh^{-1}(ax)}{\sqrt{c+dx^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*x*ArcTanh[a*x])/Sqrt[c + d*x^2] + (Log[1 - a*x] + Log[1 + a*x] - Log[a*
c - d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]] - Log[a*c + d*x + Sqrt[a^2*c + d
]*Sqrt[c + d*x^2]])/Sqrt[a^2*c + d])/(2*c)
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \mathrm{Artanh}\,(ax)\left(dx^2+c\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(3/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13399, size = 763, normalized size = 12.31

$$\left[\frac{2\left(a^2c+d\right)\sqrt{dx^2+c}x\log\left(-\frac{ax+1}{ax-1}\right)+\sqrt{a^2c+d}\left(dx^2+c\right)\log\left(\frac{a^4d^2x^4+8a^4c^2+8a^2cd+2\left(4a^4cd+3a^2d^2\right)x^2-4\left(a^3dx^2+2a^3c+ad\right)\sqrt{a^2c+d}\sqrt{dx^2+\cdots}}{a^4x^4-2a^2x^2+1}\right.\cdots}{4\left(a^2c^3+c^2d+\left(a^2c^2d+cd^2\right)x^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + sqrt(a^2*
c + d)*(d*x^2 + c)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d
+ 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2
 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)))/(a^2*c^3 + c^2*d + (a^2*c^2*d + c*
d^2)*x^2), 1/2*((a^2*c + d)*sqrt(d*x^2 + c)*x*log(-(a*x + 1)/(a*x - 1)) + s
qrt(-a^2*c - d)*(d*x^2 + c)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*
c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)))/(a^2*c^3
 + c^2*d + (a^2*c^2*d + c*d^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{atanh}(ax)}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(3/2), x)
```

Giac [A] time = 1.2462, size = 96, normalized size = 1.55

$$\frac{x\log\left(-\frac{ax+1}{ax-1}\right)}{2\sqrt{dx^2+c}c}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{\sqrt{-a^2c-d}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*x*log(-(a*x + 1)/(a*x - 1))/(sqrt(d*x^2 + c)*c) + arctan(sqrt(d*x^2 + c
)*a/sqrt(-a^2*c - d))/(sqrt(-a^2*c - d)*c)
```

3.514 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}+\frac{a}{3c(a^2c+d)\sqrt{c+dx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rubi [A] time = 0.374972, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5976, 6688, 12, 571, 78, 63, 208}

$$-\frac{(3a^2c+2d)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2(a^2c+d)^{3/2}}+\frac{a}{3c(a^2c+d)\sqrt{c+dx^2}}+\frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}}+\frac{x\tanh^{-1}(ax)}{3c(c+dx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] a/(3*c*(a^2*c + d)*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(3*c*(c + d*x^2)^(3/
2)) + (2*x*ArcTanh[a*x])/(3*c^2*Sqrt[c + d*x^2]) - ((3*a^2*c + 2*d)*ArcTanh
[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c + d]])/(3*c^2*(a^2*c + d)^(3/2))
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{\frac{x}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - a\int \frac{x\left(3c+2dx^2\right)}{3c^2\left(1-a^2x^2\right)\left(c+dx^2\right)^{3/2}}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(3c+2dx^2\right)}{\left(1-a^2x^2\right)\left(c+dx^2\right)^{3/2}}\,dx}{3c^2}$$

$$= \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{3c+2dx}{\left(1-a^2x\right)(c+dx)^{3/2}}\,dx, x, x^2\right)}{6c^2}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\left(3a^2c+2d\right)\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-a^2x\right)\sqrt{c+d}\cdots}\right.}{6c^2\left(a^2c+d\right)}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(a\left(3a^2c+2d\right)\right)\mathrm{Subst}\left(\int \frac{1}{1+\frac{a^2c}{d}-\frac{a^2x^2}{d}}\cdots\right.}{3c^2d\left(a^2c+d\right)}$$

$$= \frac{a}{3c\left(a^2c+d\right)\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{3c\left(c+dx^2\right)^{3/2}} + \frac{2x\tanh^{-1}(ax)}{3c^2\sqrt{c+dx^2}} - \frac{\left(3a^2c+2d\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{3c^2\left(a^2c+d\right)^{3/2}}$$

Mathematica [A] time = 0.305437, size = 226, normalized size = 1.77

$$\frac{\frac{2ac}{\left(a^2c+d\right)\sqrt{c+dx^2}} - \frac{\left(3a^2c+2d\right)\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac-dx\right)}{\left(a^2c+d\right)^{3/2}} - \frac{\left(3a^2c+2d\right)\log\left(\sqrt{a^2c+d}\sqrt{c+dx^2}+ac+dx\right)}{\left(a^2c+d\right)^{3/2}} + \frac{\left(3a^2c+2d\right)\log(1-ax)}{\left(a^2c+d\right)^{3/2}} + \frac{\left(3a^2c+2d\right)\log(ax+1)}{\left(a^2c+d\right)^{3/2}}\cdots}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c)/((a^2*c + d)*Sqrt[c + d*x^2]) + (2*x*(3*c + 2*d*x^2)*ArcTanh[a*x])
/(c + d*x^2)^(3/2) + ((3*a^2*c + 2*d)*Log[1 - a*x])/(a^2*c + d)^(3/2) + ((3
*a^2*c + 2*d)*Log[1 + a*x])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d)*Log[a*c -
d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2) - ((3*a^2*c + 2*d
)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(a^2*c + d)^(3/2))/(6*c
^2)
```

Maple [F] time = 0.485, size = 0, normalized size = 0.

$$\int \mathrm{Artanh}\,(ax)\left(dx^2+c\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(5/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.50753, size = 1499, normalized size = 11.71

$$\left[\frac{\left(3\,a^{2}c^{3}+\left(3\,a^{2}cd^{2}+2\,d^{3}\right)x^{4}+2\,c^{2}d+2\left(3\,a^{2}c^{2}d+2\,cd^{2}\right)x^{2}\right)\sqrt{a^{2}c+d}\,\log\left(\frac{a^{4}d^{2}x^{4}+8\,a^{4}c^{2}+8\,a^{2}cd+2\left(4\,a^{4}cd+3\,a^{2}d^{2}\right)x^{2}-4\left(a^{3}dx^{2}+2\,a^{3}c\right.}{a^{4}x^{4}-2\,a^{2}x^{2}+1}\right.}{12\left(a^{4}c^{6}+2\,a^{2}c^{5}d+c^{4}d^{2}+\left(a^{4}c^{4}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((3*a^2*c^3 + (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d +
2*c*d^2)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*
(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 + 2*a^3*c + a*d)*sqrt(a^2*c + d)
*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2 + 1)) + 2*(2*a^3*c^3 + 2*a*c^2
*d + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 +
 3*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2
 + c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^
2*d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2), 1/6*((3*a^2*c^3
+ (3*a^2*c*d^2 + 2*d^3)*x^4 + 2*c^2*d + 2*(3*a^2*c^2*d + 2*c*d^2)*x^2)*sqrt
(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x
^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (2*a^3*c^3 + 2*a*c^2*d
 + 2*(a^3*c^2*d + a*c*d^2)*x^2 + (2*(a^4*c^2*d + 2*a^2*c*d^2 + d^3)*x^3 + 3
*(a^4*c^3 + 2*a^2*c^2*d + c*d^2)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 +
 c))/(a^4*c^6 + 2*a^2*c^5*d + c^4*d^2 + (a^4*c^4*d^2 + 2*a^2*c^3*d^3 + c^2*
d^4)*x^4 + 2*(a^4*c^5*d + 2*a^2*c^4*d^2 + c^3*d^3)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}{\left(ax \right)}}{\left(c+dx^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(5/2),x)
```

```
[Out] Integral(atanh(a*x)/(c + d*x**2)**(5/2), x)
```

Giac [A] time = 1.22498, size = 182, normalized size = 1.42

$$\frac{1}{3}a\left(\frac{(3a^2c+2d)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{(a^2c^3+c^2d)\sqrt{-a^2c-d}a}+\frac{1}{(a^2c^2+cd)\sqrt{dx^2+c}}\right)+\frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)\log\left(-\frac{ax+1}{ax-1}\right)}{6\,(dx^2+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*a*((3*a^2*c + 2*d)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^2*c^3
 + c^2*d)*sqrt(-a^2*c - d)*a) + 1/((a^2*c^2 + c*d)*sqrt(d*x^2 + c))) + 1/6*
x*(2*d*x^2/c^2 + 3/c)*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(3/2)
```

3.515 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{7/2}}\,dx$

Optimal. Leaf size=200

$$-\frac{(15a^4c^2+20a^2cd+8d^2)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c+d)^{5/2}}+\frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}}+\frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+$$

```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcTanh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```

Rubi [A] time = 1.08157, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5976, 6688, 12, 6715, 897, 1261, 208}

$$-\frac{(15a^4c^2+20a^2cd+8d^2)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{15c^3(a^2c+d)^{5/2}}+\frac{a(7a^2c+4d)}{15c^2(a^2c+d)^2\sqrt{c+dx^2}}+\frac{a}{15c(a^2c+d)(c+dx^2)^{3/2}}+\frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] a/(15*c*(a^2*c + d)*(c + d*x^2)^(3/2)) + (a*(7*a^2*c + 4*d))/(15*c^2*(a^2*c
 + d)^2*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*
ArcTanh[a*x])/(15*c^2*(c + d*x^2)^(3/2)) + (8*x*ArcTanh[a*x])/(15*c^3*Sqrt[
c + d*x^2]) - ((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(15*c^3*(a^2*c + d)^(5/2))
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{7/2}}\,dx = \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{\frac{x}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x}{15c^3\sqrt{c+dx^2}}}{1-a^2x^2}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{15c^3\left(1-a^2x^2\right)\left(c+dx^2\right)^{5/2}}\,dx$$

$$= \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(15c^2+20cdx^2+8d^2x^4\right)}{\left(1-a^2x^2\right)\left(c+dx^2\right)^{5/2}}\,dx}{15c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{15c^2+20cdx+8d^2x^2}{\left(1-a^2x\right)(c+dx)^{5/2}}\,dx, x, x^2\right)}{30c^3}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \frac{3c^2+4cx^2+8x^4}{x^4\left(\frac{a^2c+d}{d}-\frac{a^2x^2}{d}\right)}\,dx, x, \sqrt{c+dx^2}\right)}{15c^3d}$$

$$= \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}(ax)}{15c^3\sqrt{c+dx^2}} - \frac{a\,\text{Subst}\left(\int \left(\frac{3c^2d}{\left(a^2c+d\right)x^4} + \frac{cd\left(7a^2c+4d\right)}{\left(a^2c+d\right)^2x^2} + \frac{d\left(15a^4c^2+\right.}{\left(a^2c+d\right)^2\left(\right.}\right.\right.}{15c^3d}$$

$$= \frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}} + \frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}}{15c^3}$$

$$= \frac{a}{15c\left(a^2c+d\right)\left(c+dx^2\right)^{3/2}} + \frac{a\left(7a^2c+4d\right)}{15c^2\left(a^2c+d\right)^2\sqrt{c+dx^2}} + \frac{x\tanh^{-1}(ax)}{5c\left(c+dx^2\right)^{5/2}} + \frac{4x\tanh^{-1}(ax)}{15c^2\left(c+dx^2\right)^{3/2}} + \frac{8x\tanh^{-1}}{15c^3}$$

Mathematica [A] time = 0.582117, size = 329, normalized size = 1.64

$$\left(15a^4c^2+20a^2cd+8d^2\right)\log(1-ax)\left(c+dx^2\right)^{5/2} + \left(15a^4c^2+20a^2cd+8d^2\right)\log(ax+1)\left(c+dx^2\right)^{5/2} - \left(15a^4c^2+20a^2cd\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(7/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(d*(5*c + 4*d*x^2) + a^2*c*(8*c + 7*d*x^
2)) + 2*(a^2*c + d)^(5/2)*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTanh[a*x]
+ (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 - a*x] + (15*a^
4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[1 + a*x] - (15*a^4*c^2 +
20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sqrt[
c + d*x^2]] - (15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*(c + d*x^2)^(5/2)*Log[a*c +
 d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(30*c^3*(a^2*c + d)^(5/2)*(c + d*x
^2)^(5/2))
```

Maple [F] time = 0.483, size = 0, normalized size = 0.

$$\int \text{Artanh}\,(ax)\left(dx^2+c\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(7/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.88935, size = 2612, normalized size = 13.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/60*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 20*a^2*c*d^4 + 8*d^5)
*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*(1
5*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2
*x^4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2
 + 2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x
^2 + 1)) + 2*(16*a^5*c^5 + 26*a^3*c^4*d + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 +
 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2
*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(
a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^
4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2
 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*
a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3
 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*
d^3 + c^5*d^4)*x^2), 1/30*((15*a^4*c^5 + 20*a^2*c^4*d + (15*a^4*c^2*d^3 + 2
0*a^2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 + 20*a^2*c^2*d^3 +
 8*c*d^4)*x^4 + 3*(15*a^4*c^4*d + 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^
2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 +
 c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*d^2)*x^2)) + (16*a^5*c^5 + 26*a^3*c^4*d
 + 10*a*c^3*d^2 + 2*(7*a^5*c^3*d^2 + 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 6*(5
*a^5*c^4*d + 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 + (8*(a^6*c^3*d^2 + 3*a^4*c^2
*d^3 + 3*a^2*c*d^4 + d^5)*x^5 + 20*(a^6*c^4*d + 3*a^4*c^3*d^2 + 3*a^2*c^2*d
^3 + c*d^4)*x^3 + 15*(a^6*c^5 + 3*a^4*c^4*d + 3*a^2*c^3*d^2 + c^2*d^3)*x)*l
og(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^6*c^9 + 3*a^4*c^8*d + 3*a^2*c
^7*d^2 + c^6*d^3 + (a^6*c^6*d^3 + 3*a^4*c^5*d^4 + 3*a^2*c^4*d^5 + c^3*d^6)*
x^6 + 3*(a^6*c^7*d^2 + 3*a^4*c^6*d^3 + 3*a^2*c^5*d^4 + c^4*d^5)*x^4 + 3*(a^
6*c^8*d + 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 + c^5*d^4)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24505, size = 294, normalized size = 1.47

$$\frac{1}{15}a\left(\frac{\left(15\,a^4c^2+20\,a^2cd+8\,d^2\right)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c-d}}\right)}{\left(a^4c^5+2\,a^2c^4d+c^3d^2\right)\sqrt{-a^2c-d}a}+\frac{7\left(dx^2+c\right)a^2c+a^2c^2+4\left(dx^2+c\right)d+cd}{\left(a^4c^4+2\,a^2c^3d+c^2d^2\right)\left(dx^2+c\right)^{\frac{3}{2}}}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*a*((15*a^4*c^2 + 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-a^
2*c - d))/((a^4*c^5 + 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c - d)*a) + (7*(d*x^
2 + c)*a^2*c + a^2*c^2 + 4*(d*x^2 + c)*d + c*d)/((a^4*c^4 + 2*a^2*c^3*d + c
^2*d^2)*(d*x^2 + c)^(3/2))) + 1/30*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) + 15/c)
*x*log(-(a*x + 1)/(a*x - 1))/(d*x^2 + c)^(5/2)
```

3.516 $\int \frac{\tanh^{-1}(ax)}{(c+dx^2)^{9/2}}\,dx$

Optimal. Leaf size=283

$$\frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}}-\frac{\left(70a^4c^2d+35a^6c^3+56a^2cd^2+16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\left(a^2c+d\right)^{7/2}}+\frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}$$

```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcTanh[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTanh[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```

Rubi [A] time = 1.5111, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {192, 191, 5976, 6688, 12, 6715, 1619, 63, 208}

$$\frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}}-\frac{\left(70a^4c^2d+35a^6c^3+56a^2cd^2+16d^3\right)\tanh^{-1}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c+d}}\right)}{35c^4\left(a^2c+d\right)^{7/2}}+\frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] a/(35*c*(a^2*c + d)*(c + d*x^2)^(5/2)) + (a*(11*a^2*c + 6*d))/(105*c^2*(a^2
*c + d)^2*(c + d*x^2)^(3/2)) + (a*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2))/(35*c^
3*(a^2*c + d)^3*Sqrt[c + d*x^2]) + (x*ArcTanh[a*x])/(7*c*(c + d*x^2)^(7/2))
 + (6*x*ArcTanh[a*x])/(35*c^2*(c + d*x^2)^(5/2)) + (8*x*ArcTanh[a*x])/(35*c
^3*(c + d*x^2)^(3/2)) + (16*x*ArcTanh[a*x])/(35*c^4*Sqrt[c + d*x^2]) - ((35
*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^2]
)/Sqrt[a^2*c + d]])/(35*c^4*(a^2*c + d)^(7/2))
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5976

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Dist[a + b*ArcTanh[c*x], u, x
] - Dist[b*c, Int[u/(1 - c^2*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
(IntegerQ[q] || ILtQ[q + 1/2, 0])
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^{-1}(ax)}{\left(c+dx^2\right)^{9/2}}\,dx = \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{\frac{x}{7c\left(c+dx^2\right)^{7/2}} + \cdots}{\cdots}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - a\int \frac{x\left(35c^3+70c^2d\cdots\right)}{35c^4\left(1-\cdots\right)}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\int \frac{x\left(35c^3+70c^2dx^2+56c\cdots\right)}{\left(1-a^2x^2\right)\left(c+\cdots\right)}}{35c^4}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int \frac{35c^3+70c^2\cdots}{(1-\cdots}\right.}{\cdots}$$

$$= \frac{x\tanh^{-1}(ax)}{7c\left(c+dx^2\right)^{7/2}} + \frac{6x\tanh^{-1}(ax)}{35c^2\left(c+dx^2\right)^{5/2}} + \frac{8x\tanh^{-1}(ax)}{35c^3\left(c+dx^2\right)^{3/2}} + \frac{16x\tanh^{-1}(ax)}{35c^4\sqrt{c+dx^2}} - \frac{a\,\mathrm{Subst}\left(\int\left(\frac{5c^3\cdots}{\left(a^2c+d\right)(\cdots}\right.\right.}{\cdots}$$

$$= \frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}} + \frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}} + \frac{x\tanh\cdots}{7c\left(c+\cdots\right.}$$

$$= \frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}} + \frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}} + \frac{x\tanh\cdots}{7c\left(c+\cdots\right.}$$

$$= \frac{a}{35c\left(a^2c+d\right)\left(c+dx^2\right)^{5/2}} + \frac{a\left(11a^2c+6d\right)}{105c^2\left(a^2c+d\right)^2\left(c+dx^2\right)^{3/2}} + \frac{a\left(19a^4c^2+22a^2cd+8d^2\right)}{35c^3\left(a^2c+d\right)^3\sqrt{c+dx^2}} + \frac{x\tanh\cdots}{7c\left(c+\cdots\right.}$$

Mathematica [A] time = 0.933125, size = 431, normalized size = 1.52

$$\frac{2ac\sqrt{a^2c+d}\left(c+dx^2\right)\left(3\left(19a^4c^2+22a^2cd+8d^2\right)\left(c+dx^2\right)^2+3c^2\left(a^2c+d\right)^2+c\left(11a^2c+6d\right)\left(a^2c+d\right)\left(c+dx^2\right)\right)+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[a*x]/(c + d*x^2)^(9/2),x]
```

```
[Out] (2*a*c*Sqrt[a^2*c + d]*(c + d*x^2)*(3*c^2*(a^2*c + d)^2 + c*(a^2*c + d)*(11
*a^2*c + 6*d)*(c + d*x^2) + 3*(19*a^4*c^2 + 22*a^2*c*d + 8*d^2)*(c + d*x^2)
^2) + 6*(a^2*c + d)^(7/2)*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*
x^6)*ArcTanh[a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(
c + d*x^2)^(7/2)*Log[1 - a*x] + 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2
 + 16*d^3)*(c + d*x^2)^(7/2)*Log[1 + a*x] - 3*(35*a^6*c^3 + 70*a^4*c^2*d +
56*a^2*c*d^2 + 16*d^3)*(c + d*x^2)^(7/2)*Log[a*c - d*x + Sqrt[a^2*c + d]*Sq
rt[c + d*x^2]] - 3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*(c +
 d*x^2)^(7/2)*Log[a*c + d*x + Sqrt[a^2*c + d]*Sqrt[c + d*x^2]])/(210*c^4*(a
^2*c + d)^(7/2)*(c + d*x^2)^(7/2))
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \mathrm{Artanh}\left(ax\right)\left(dx^2+c\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

```
[Out] int(arctanh(a*x)/(d*x^2+c)^(9/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.69069, size = 4143, normalized size = 14.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(3*(35*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 7
0*a^4*c^2*d^5 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3
 + 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70
*a^4*c^4*d^3 + 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*
c^5*d^2 + 56*a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(a^2*c + d)*log((a^4*d^2*x^
4 + 8*a^4*c^2 + 8*a^2*c*d + 2*(4*a^4*c*d + 3*a^2*d^2)*x^2 - 4*(a^3*d*x^2 +
2*a^3*c + a*d)*sqrt(a^2*c + d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 - 2*a^2*x^2
+ 1)) + 2*(142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3 + 6
*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*(18
2*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 + 2*(
196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 + 3*(
16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7 +
56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)*x^
5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*d^5
)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*d^4
)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*d +
 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5 +
6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^8*d
^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*a^6
*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d + 4
*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2), 1/210*(3*(35
*a^6*c^7 + 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 + 70*a^4*c^2*d^5
 + 56*a^2*c*d^6 + 16*d^7)*x^8 + 16*c^4*d^3 + 4*(35*a^6*c^4*d^3 + 70*a^4*c^3
*d^4 + 56*a^2*c^2*d^5 + 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 + 70*a^4*c^4*d^3
+ 56*a^2*c^3*d^4 + 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d + 70*a^4*c^5*d^2 + 56*
a^2*c^4*d^3 + 16*c^3*d^4)*x^2)*sqrt(-a^2*c - d)*arctan(1/2*(a^2*d*x^2 + 2*a
^2*c + d)*sqrt(-a^2*c - d)*sqrt(d*x^2 + c)/(a^3*c^2 + a*c*d + (a^3*c*d + a*
d^2)*x^2)) + (142*a^7*c^7 + 320*a^5*c^6*d + 244*a^3*c^5*d^2 + 66*a*c^4*d^3
+ 6*(19*a^7*c^4*d^3 + 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 + 8*a*c*d^6)*x^6 + 2*
(182*a^7*c^5*d^2 + 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 + 78*a*c^2*d^5)*x^4 +
```

```
2*(196*a^7*c^6*d + 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 + 87*a*c^3*d^4)*x^2 +
3*(16*(a^8*c^4*d^3 + 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 + 4*a^2*c*d^6 + d^7)*x^7
 + 56*(a^8*c^5*d^2 + 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 + 4*a^2*c^2*d^5 + c*d^6)
*x^5 + 70*(a^8*c^6*d + 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 + 4*a^2*c^3*d^4 + c^2*
d^5)*x^3 + 35*(a^8*c^7 + 4*a^6*c^6*d + 6*a^4*c^5*d^2 + 4*a^2*c^4*d^3 + c^3*
d^4)*x)*log(-(a*x + 1)/(a*x - 1)))*sqrt(d*x^2 + c))/(a^8*c^12 + 4*a^6*c^11*
d + 6*a^4*c^10*d^2 + 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 + 4*a^6*c^7*d^5
 + 6*a^4*c^6*d^6 + 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 + 4*a^6*c^
8*d^4 + 6*a^4*c^7*d^5 + 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 + 4*
a^6*c^9*d^3 + 6*a^4*c^8*d^4 + 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d
+ 4*a^6*c^10*d^2 + 6*a^4*c^9*d^3 + 4*a^2*c^8*d^4 + c^7*d^5)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(a*x)/(d*x**2+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30643, size = 471, normalized size = 1.66

$$\frac{1}{105}a\left(\frac{3\left(35\,a^{6}c^{3}+70\,a^{4}c^{2}d+56\,a^{2}cd^{2}+16\,d^{3}\right)\arctan\left(\frac{\sqrt{dx^{2}+c}a}{\sqrt{-a^{2}c-d}}\right)}{\left(a^{6}c^{7}+3\,a^{4}c^{6}d+3\,a^{2}c^{5}d^{2}+c^{4}d^{3}\right)\sqrt{-a^{2}c-d}a}+\frac{57\left(dx^{2}+c\right)^{2}a^{4}c^{2}+11\left(dx^{2}+c\right)a^{4}c^{3}+3\,a^{4}c^{4}+6}{(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*a*(3*(35*a^6*c^3 + 70*a^4*c^2*d + 56*a^2*c*d^2 + 16*d^3)*arctan(sqrt(
d*x^2 + c)*a/sqrt(-a^2*c - d))/((a^6*c^7 + 3*a^4*c^6*d + 3*a^2*c^5*d^2 + c^
4*d^3)*sqrt(-a^2*c - d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)*a^4
*c^3 + 3*a^4*c^4 + 66*(d*x^2 + c)^2*a^2*c*d + 17*(d*x^2 + c)*a^2*c^2*d + 6*
a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/((a^6*c
^6 + 3*a^4*c^5*d + 3*a^2*c^4*d^2 + c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/70*(2*(
4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*log(-(a*x + 1)/
(a*x - 1))/(d*x^2 + c)^(7/2)
```

3.517 $\int \sqrt{a-ax^2}\tanh^{-1}(x)\,dx$

Optimal. Leaf size=186

$$-\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\tan}{\sqrt{}}$$

```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcTanh[x])/2 - (a*Sqrt[1 - x^2]*Arc
Tan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```

Rubi [A] time = 0.0880305, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {5942, 5954, 5950}

$$-\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{ia\sqrt{1-x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{2\sqrt{a-ax^2}}+\frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\tan}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

```
[Out] Sqrt[a - a*x^2]/2 + (x*Sqrt[a - a*x^2]*ArcTanh[x])/2 - (a*Sqrt[1 - x^2]*Arc
Tan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^2] - ((I/2)*a*Sqrt[1
- x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2] + ((I/2)
*a*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a - a*x^2]
```

Rule 5942

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> Simp[(b*(d + e*x^2)^q)/(2*c*q*(2*q + 1)), x] + (Dist[(2*d*q)/(2*q +
1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTanh[c*x]), x], x] + Simp[(x*(d + e*x
^2)^q*(a + b*ArcTanh[c*x]))/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[c^2*d + e, 0] && GtQ[q, 0]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\int \sqrt{a-ax^2}\tanh^{-1}(x)\,dx = \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)+\frac{1}{2}a\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx$$
$$= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)+\frac{\left(a\sqrt{1-x^2}\right)\int \frac{\tanh^{-1}(x)}{\sqrt{1-x^2}}\,dx}{2\sqrt{a-ax^2}}$$
$$= \frac{1}{2}\sqrt{a-ax^2}+\frac{1}{2}x\sqrt{a-ax^2}\tanh^{-1}(x)-\frac{a\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{ia\sqrt{1-x^2}\text{Li}_2}{2\sqrt{a-}}$$

Mathematica [A] time = 0.324162, size = 97, normalized size = 0.52

$$\frac{1}{2}\sqrt{a\left(1-x^2\right)}\left(-\frac{i\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(x)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(x)}\right)+\tanh^{-1}(x)\left(\log\left(1-ie^{-\tanh^{-1}(x)}\right)-\log\left(1\right.\right.\right.}{\sqrt{1-x^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - a*x^2]*ArcTanh[x],x]
```

```
[Out] (Sqrt[a*(1 - x^2)]*(1 + x*ArcTanh[x] - (I*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[
x]] - Log[1 + I/E^ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2,
 I/E^ArcTanh[x]]))/Sqrt[1 - x^2]))/2
```

Maple [A] time = 0.434, size = 229, normalized size = 1.2

$$\frac{\text{Artanh}\,(x)\,x+1}{2}\sqrt{-(-1+x)\,(1+x)\,a}+\frac{\frac{i}{2}\text{Artanh}\,(x)}{(-1+x)\,(1+x)}\sqrt{-(-1+x)\,(1+x)\,a}\sqrt{-x^2+1}\ln\left(1+i\,(1+x)\,\frac{1}{\sqrt{-x^2+1}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+a)^(1/2)*arctanh(x),x)
```

```
[Out] 1/2*(arctanh(x)*x+1)*(-(-1+x)*(1+x)*a)^(1/2)+1/2*I*(-(-1+x)*(1+x)*a)^(1/2)/
(1+x)*(-x^2+1)^(1/2)/(-1+x)*arctanh(x)*ln(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(
-(-1+x)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(-1+x)*arctanh(x)*ln(1-I*(1+x)/
(-x^2+1)^(1/2))+1/2*I*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)^(1/2)/(-1+x)*d
ilog(1+I*(1+x)/(-x^2+1)^(1/2))-1/2*I*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)*(-x^2+1)
^(1/2)/(-1+x)*dilog(1-I*(1+x)/(-x^2+1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-ax^2 + a}\,\text{artanh}\,(x)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a*x^2 + a)*arctanh(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-a\,(x-1)\,(x+1)}\,\text{atanh}\,(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+a)**(1/2)*atanh(x),x)
```

```
[Out] Integral(sqrt(-a*(x - 1)*(x + 1))*atanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-ax^2 + a}\,\text{artanh}\,(x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+a)^(1/2)*arctanh(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + a)*arctanh(x), x)
```

3.518 $\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx$

Optimal. Leaf size=144

$$-\frac{i\sqrt{1-x^2}\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}-\frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}$$

```
[Out] (-2*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rubi [A] time = 0.0543704, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5954, 5950}

$$-\frac{i\sqrt{1-x^2}\text{PolyLog}\left(2,-\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\text{PolyLog}\left(2,\frac{i\sqrt{1-x}}{\sqrt{x+1}}\right)}{\sqrt{a-ax^2}}-\frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

```
[Out] (-2*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]*ArcTanh[x])/Sqrt[a - a*x^
2] - (I*Sqrt[1 - x^2]*PolyLog[2, ((-I)*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
a*x^2] + (I*Sqrt[1 - x^2]*PolyLog[2, (I*Sqrt[1 - x])/Sqrt[1 + x]])/Sqrt[a -
 a*x^2]
```

Rule 5954

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcTanh[c*x])
^p/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e,
 0] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

Rule 5950

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol
] :> Simp[(-2*(a + b*ArcTanh[c*x])*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(c*
Sqrt[d]), x] + (-Simp[(I*b*PolyLog[2, -((I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/
(c*Sqrt[d]), x] + Simp[(I*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]])/(c
*Sqrt[d]), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[d,
0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{\sqrt{a-ax^2}}\,dx &= \frac{\sqrt{1-x^2}\int \frac{\tanh^{-1}(x)}{\sqrt{1-x^2}}\,dx}{\sqrt{a-ax^2}}\\ &= -\frac{2\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)\tanh^{-1}(x)}{\sqrt{a-ax^2}}-\frac{i\sqrt{1-x^2}\text{Li}_2\left(-\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}+\frac{i\sqrt{1-x^2}\text{Li}_2\left(\frac{i\sqrt{1-x}}{\sqrt{1+x}}\right)}{\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A] time = 0.0990153, size = 90, normalized size = 0.62

$$-\frac{i\sqrt{a\left(1-x^2\right)}\left(\text{PolyLog}\left(2,-ie^{-\tanh^{-1}(x)}\right)-\text{PolyLog}\left(2,ie^{-\tanh^{-1}(x)}\right)+\tanh^{-1}(x)\left(\log\left(1-ie^{-\tanh^{-1}(x)}\right)-\log\left(1+ie^{-\text{ta}}\right.\right.\right.}{a\sqrt{1-x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcTanh[x]/Sqrt[a - a*x^2],x]
```

```
[Out] ((-I)*Sqrt[a*(1 - x^2)]*(ArcTanh[x]*(Log[1 - I/E^ArcTanh[x]] - Log[1 + I/E^
ArcTanh[x]]) + PolyLog[2, (-I)/E^ArcTanh[x]] - PolyLog[2, I/E^ArcTanh[x]]))
/(a*Sqrt[1 - x^2])
```

Maple [A] time = 0.354, size = 210, normalized size = 1.5

$$\frac{i\text{Artanh}\,(x)}{a\left(x^2-1\right)}\ln\left(1+i\left(1+x\right)\frac{1}{\sqrt{-x^2+1}}\right)\sqrt{-x^2+1}\sqrt{-\left(-1+x\right)\left(1+x\right)a}-\frac{i\text{Artanh}\,(x)}{a\left(x^2-1\right)}\ln\left(1-i\left(1+x\right)\frac{1}{\sqrt{-x^2+1}}\right)\sqrt{-x^2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(1/2),x)
```

```
[Out] I*ln(1+I*(1+x)/(-x^2+1)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)*(-(-1+x)*(1+x)*a)^
(1/2)/a/(x^2-1)-I*ln(1-I*(1+x)/(-x^2+1)^(1/2))*arctanh(x)*(-x^2+1)^(1/2)*(-
(-1+x)*(1+x)*a)^(1/2)/a/(x^2-1)+I*dilog(1+I*(1+x)/(-x^2+1)^(1/2))*(-x^2+1)^
(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/a/(x^2-1)-I*dilog(1-I*(1+x)/(-x^2+1)^(1/2))*(
-x^2+1)^(1/2)*(-(-1+x)*(1+x)*a)^(1/2)/a/(x^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-ax^2+a}\,\text{artanh}\,(x)}{ax^2-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a*x^2 + a)*arctanh(x)/(a*x^2 - a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(x)}{\sqrt{-a(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(1/2),x)
```

```
[Out] Integral(atanh(x)/sqrt(-a*(x - 1)*(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(x)}{\sqrt{-ax^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arctanh(x)/sqrt(-a*x^2 + a), x)
```

3.519 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

```
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])
```

Rubi [A] time = 0.0245407, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5958}

$$\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}-\frac{1}{a\sqrt{a-ax^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(3/2),x]
```

```
[Out] -(1/(a*Sqrt[a - a*x^2])) + (x*ArcTanh[x])/(a*Sqrt[a - a*x^2])
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{3/2}}\,dx=-\frac{1}{a\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{a\sqrt{a-ax^2}}$$

Mathematica [A] time = 0.0420467, size = 30, normalized size = 0.81

$$\frac{\sqrt{a-ax^2}\left(1-x\tanh^{-1}(x)\right)}{a^2\left(x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a - a*x^2]*(1 - x*ArcTanh[x]))/(a^2*(-1 + x^2))
```

Maple [A] time = 0.217, size = 52, normalized size = 1.4

$$-\frac{\mathrm{Artanh}\,(x)-1}{(-2+2\,x)\,a^2}\sqrt{-(-1+x)\,(1+x)\,a}-\frac{1+\mathrm{Artanh}\,(x)}{(2+2\,x)\,a^2}\sqrt{-(-1+x)\,(1+x)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(3/2),x)
```

```
[Out] -1/2*(arctanh(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^2-1/2*(1+arctanh(x))*(
-(-1+x)*(1+x)*a)^(1/2)/(1+x)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01893, size = 93, normalized size = 2.51

$$-\frac{\sqrt{-ax^2+a}\left(x\log\left(-\frac{x+1}{x-1}\right)-2\right)}{2\left(a^2x^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*(x*log(-(x + 1)/(x - 1)) - 2)/(a^2*x^2 - a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(x)}{\left(-a\,(x-1)\,(x+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(3/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(3/2), x)
```

Giac [A] time = 1.19783, size = 73, normalized size = 1.97

$$-\frac{\sqrt{-ax^2+a}\,x\log\left(-\frac{x+1}{x-1}\right)}{2\left(ax^2-a\right)a}-\frac{1}{\sqrt{-ax^2+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a*x^2 + a)*x*log(-(x + 1)/(x - 1))/((a*x^2 - a)*a) - 1/(sqrt(-a*
x^2 + a)*a)
```

3.520 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

```
[Out] -1/(9*a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcTanh[x])/(3*
a*(a - a*x^2)^(3/2)) + (2*x*ArcTanh[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rubi [A] time = 0.0526752, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5960, 5958}

$$-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}-\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -1/(9*a*(a - a*x^2)^(3/2)) - 2/(3*a^2*Sqrt[a - a*x^2]) + (x*ArcTanh[x])/(3*
a*(a - a*x^2)^(3/2)) + (2*x*ArcTanh[x])/(3*a^2*Sqrt[a - a*x^2])
```

Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{5/2}}\,dx &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx}{3a}\\ &= -\frac{1}{9a\left(a-ax^2\right)^{3/2}}-\frac{2}{3a^2\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{3a\left(a-ax^2\right)^{3/2}}+\frac{2x\tanh^{-1}(x)}{3a^2\sqrt{a-ax^2}}\end{aligned}$$

Mathematica [A] time = 0.0486671, size = 45, normalized size = 0.54

$$-\frac{\sqrt{a-ax^2}\left(-6x^2+\left(6x^3-9x\right)\tanh^{-1}(x)+7\right)}{9a^3\left(x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(5/2),x]
```

```
[Out] -(Sqrt[a - a*x^2]*(7 - 6*x^2 + (-9*x + 6*x^3)*ArcTanh[x]))/(9*a^3*(-1 + x^2
)^2)
```

Maple [A] time = 0.237, size = 112, normalized size = 1.4

$$\frac{(1+x)(-1+3\operatorname{Artanh}(x))}{72(-1+x)^2a^3}\sqrt{-(-1+x)(1+x)a}-\frac{3\operatorname{Artanh}(x)-3}{(-8+8x)a^3}\sqrt{-(-1+x)(1+x)a}-\frac{3+3\operatorname{Artanh}(x)}{(8+8x)a^3}\sqrt{-(-1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(5/2),x)
```

```
[Out] 1/72*(1+x)*(-1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^2/a^3-3/8*(arct
anh(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^3-3/8*(1+arctanh(x))*(-(-1+x)*(1
+x)*a)^(1/2)/(1+x)/a^3+1/72*(-1+x)*(1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)
/(1+x)^2/a^3
```

Maxima [A] time = 0.975357, size = 90, normalized size = 1.08

$$\frac{1}{3}\left(\frac{2x}{\sqrt{-ax^2+a}a^2}+\frac{x}{\left(-ax^2+a\right)^{\frac{3}{2}}a}\right)\operatorname{artanh}(x)-\frac{2}{3\sqrt{-ax^2+a}a^2}-\frac{1}{9\left(-ax^2+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/(sqrt(-a*x^2 + a)*a^2) + x/((-a*x^2 + a)^(3/2)*a))*arctanh(x) - 2/
3/(sqrt(-a*x^2 + a)*a^2) - 1/9/((-a*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 1.9577, size = 142, normalized size = 1.71

$$\frac{\sqrt{-ax^2+a}\left(12x^2-3\left(2x^3-3x\right)\log\left(-\frac{x+1}{x-1}\right)-14\right)}{18\left(a^3x^4-2a^3x^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(-a*x^2 + a)*(12*x^2 - 3*(2*x^3 - 3*x)*log(-(x + 1)/(x - 1)) - 14)
/(a^3*x^4 - 2*a^3*x^2 + a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(x)}{(-a(x-1)(x+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(5/2),x)
```

```
[Out] Integral(atanh(x)/(-a*(x - 1)*(x + 1))**(5/2), x)
```

Giac [A] time = 1.21212, size = 116, normalized size = 1.4

$$-\frac{\sqrt{-ax^2+a}\,x\left(\frac{2x^2}{a}-\frac{3}{a}\right)\log\left(-\frac{x+1}{x-1}\right)}{6\left(ax^2-a\right)^2}-\frac{6\,ax^2-7\,a}{9\left(ax^2-a\right)\sqrt{-ax^2+a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(-a*x^2 + a)*x*(2*x^2/a - 3/a)*log(-(x + 1)/(x - 1))/(a*x^2 - a)^2
 - 1/9*(6*a*x^2 - 7*a)/((a*x^2 - a)*sqrt(-a*x^2 + a)*a^2)
```

3.521 $\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{7/2}}\,dx$

Optimal. Leaf size=124

$$-\frac{8}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\tanh^{-1}(x)}{15a^3\sqrt{a-ax^2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

```
[Out] -1/(25*a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sqrt
[a - a*x^2]) + (x*ArcTanh[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcTanh[x])/(1
5*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcTanh[x])/(15*a^3*Sqrt[a - a*x^2])
```

Rubi [A] time = 0.0816819, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {5960, 5958}

$$-\frac{8}{15a^3\sqrt{a-ax^2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\tanh^{-1}(x)}{15a^3\sqrt{a-ax^2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}-\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```


```
[Out] -1/(25*a*(a - a*x^2)^(5/2)) - 4/(45*a^2*(a - a*x^2)^(3/2)) - 8/(15*a^3*Sqrt
[a - a*x^2]) + (x*ArcTanh[x])/(5*a*(a - a*x^2)^(5/2)) + (4*x*ArcTanh[x])/(1
5*a^2*(a - a*x^2)^(3/2)) + (8*x*ArcTanh[x])/(15*a^3*Sqrt[a - a*x^2])
```


Rule 5960

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbo
l] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c*d*(q + 1)^2), x] + (Dist[(2*q + 3)
/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]), x], x] - Simp
[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTanh[c*x]))/(2*d*(q + 1)), x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && LtQ[q, -1] && NeQ[q, -3/2]
```

Rule 5958

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[(x*(a + b*ArcTanh[c*x]))/(d
*Sqrt[d + e*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^{-1}(x)}{\left(a-ax^2\right)^{7/2}}\,dx &= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{5/2}}\,dx}{5a}\\ &= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}+\frac{8\int \frac{\tanh^{-1}(x)}{(a-ax^2)^{3/2}}\,dx}{15a^2}\\ &= -\frac{1}{25a\left(a-ax^2\right)^{5/2}}-\frac{4}{45a^2\left(a-ax^2\right)^{3/2}}-\frac{8}{15a^3\sqrt{a-ax^2}}+\frac{x\tanh^{-1}(x)}{5a\left(a-ax^2\right)^{5/2}}+\frac{4x\tanh^{-1}(x)}{15a^2\left(a-ax^2\right)^{3/2}}+\frac{8x\,\mathrm{t}}{15a^3}\end{aligned}$$

Mathematica [A] time = 0.0588397, size = 55, normalized size = 0.44

$$\frac{\sqrt{a-ax^2}\left(120x^4-260x^2-15\left(8x^4-20x^2+15\right)x\tanh^{-1}(x)+149\right)}{225a^4\left(x^2-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTanh[x]/(a - a*x^2)^(7/2),x]
```

```
[Out] (Sqrt[a - a*x^2]*(149 - 260*x^2 + 120*x^4 - 15*x*(15 - 20*x^2 + 8*x^4)*ArcT
anh[x]))/(225*a^4*(-1 + x^2)^3)
```

Maple [A] time = 0.26, size = 176, normalized size = 1.4

$$-\frac{(1+x)^2(-1+5\,\mathrm{Artanh}\,(x))}{800\,(-1+x)^3\,a^4}\sqrt{-(-1+x)(1+x)\,a}+\frac{(5+5\,x)(-1+3\,\mathrm{Artanh}\,(x))}{288\,(-1+x)^2\,a^4}\sqrt{-(-1+x)(1+x)\,a}-\frac{5\,\mathrm{Artanh}}{(-16+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(x)/(-a*x^2+a)^(7/2),x)
```

```
[Out] -1/800*(1+x)^2*(-1+5*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^3/a^4+5/288
*(1+x)*(-1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)^2/a^4-5/16*(arctanh
(x)-1)*(-(-1+x)*(1+x)*a)^(1/2)/(-1+x)/a^4-5/16*(1+arctanh(x))*(-(-1+x)*(1+x
)*a)^(1/2)/(1+x)/a^4+5/288*(-1+x)*(1+3*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/
(1+x)^2/a^4-1/800*(-1+x)^2*(1+5*arctanh(x))*(-(-1+x)*(1+x)*a)^(1/2)/(1+x)^3
/a^4
```

Maxima [A] time = 0.991639, size = 134, normalized size = 1.08

$$\frac{1}{15}\left(\frac{8\,x}{\sqrt{-ax^2+a}a^3}+\frac{4\,x}{\left(-ax^2+a\right)^{\frac{3}{2}}a^2}+\frac{3\,x}{\left(-ax^2+a\right)^{\frac{5}{2}}a}\right)\mathrm{artanh}\,(x)-\frac{8}{15\sqrt{-ax^2+a}a^3}-\frac{4}{45\left(-ax^2+a\right)^{\frac{3}{2}}a^2}-\frac{1}{25\left(-ax^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*(8*x/(sqrt(-a*x^2 + a)*a^3) + 4*x/((-a*x^2 + a)^(3/2)*a^2) + 3*x/((-a*
x^2 + a)^(5/2)*a))*arctanh(x) - 8/15/(sqrt(-a*x^2 + a)*a^3) - 4/45/((-a*x^2
 + a)^(3/2)*a^2) - 1/25/((-a*x^2 + a)^(5/2)*a)
```

Fricas [A] time = 1.96659, size = 190, normalized size = 1.53

$$\frac{\left(240\,x^4-520\,x^2-15\left(8\,x^5-20\,x^3+15\,x\right)\log\left(-\frac{x+1}{x-1}\right)+298\right)\sqrt{-ax^2+a}}{450\left(a^4x^6-3\,a^4x^4+3\,a^4x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/450*(240*x^4 - 520*x^2 - 15*(8*x^5 - 20*x^3 + 15*x)*log(-(x + 1)/(x - 1))
 + 298)*sqrt(-a*x^2 + a)/(a^4*x^6 - 3*a^4*x^4 + 3*a^4*x^2 - a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(x)/(-a*x**2+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21064, size = 159, normalized size = 1.28

$$-\frac{\sqrt{-ax^2+a}\left(4x^2\left(\frac{2x^2}{a}-\frac{5}{a}\right)+\frac{15}{a}\right)x\log\left(-\frac{x+1}{x-1}\right)}{30\left(ax^2-a\right)^3}-\frac{120\left(ax^2-a\right)^2-20\left(ax^2-a\right)a+9a^2}{225\left(ax^2-a\right)^2\sqrt{-ax^2+a}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x)/(-a*x^2+a)^(7/2),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(-a*x^2 + a)*(4*x^2*(2*x^2/a - 5/a) + 15/a)*x*log(-(x + 1)/(x - 1
))/(a*x^2 - a)^3 - 1/225*(120*(a*x^2 - a)^2 - 20*(a*x^2 - a)*a + 9*a^2)/((a
*x^2 - a)^2*sqrt(-a*x^2 + a)*a^3)
```

3.522 $\int x^4 \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx$

Optimal. Leaf size=315

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}-\frac{2}{25}a$$

```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcTanh[c*x])/(5*c^4) - (2*b*e*x^
3*ArcTanh[c*x])/(15*c^2) - (2*b*e*x^5*ArcTanh[c*x])/25 + (b*e*ArcTanh[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```

Rubi [A] time = 0.77612, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5916, 266, 43, 6085, 6725, 1802, 633, 31, 5980, 5910, 260, 5948, 2475, 2390, 2301}

$$\frac{1}{5}x^5\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{e(4a+3b)\log(1-cx)}{20c^5}+\frac{e(4a-3b)\log(cx+1)}{20c^5}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}-\frac{2}{25}a$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```


```
[Out] (-2*a*e*x)/(5*c^4) - (77*b*e*x^2)/(300*c^3) - (2*a*e*x^3)/(15*c^2) - (9*b*e
*x^4)/(200*c) - (2*a*e*x^5)/25 - (2*b*e*x*ArcTanh[c*x])/(5*c^4) - (2*b*e*x^
3*ArcTanh[c*x])/(15*c^2) - (2*b*e*x^5*ArcTanh[c*x])/25 + (b*e*ArcTanh[c*x]^
2)/(5*c^5) - ((4*a + 3*b)*e*Log[1 - c*x])/(20*c^5) + ((4*a - 3*b)*e*Log[1 +
 c*x])/(20*c^5) - (23*b*e*Log[1 - c^2*x^2])/(75*c^5) - (b*e*Log[1 - c^2*x^2
]^2)/(20*c^5) + (b*x^2*(d + e*Log[1 - c^2*x^2]))/(10*c^3) + (b*x^4*(d + e*L
og[1 - c^2*x^2]))/(20*c) + (x^5*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/5 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(10*c^5)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^4\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(1-c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5\left(a+\right.$$

$$=-\frac{be\log^2\left(1-c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{10c^3}+\frac{bx^4\left(d+e\log\left(\right.\right.}{20c}$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2}{25}bex^5\tanh^{-1}(cx)-$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex^3\tanh^{-1}(cx)}{15c^2}-$$

$$=-\frac{2aex}{5c^4}-\frac{3bex^2}{20c^3}-\frac{2aex^3}{15c^2}-\frac{bex^4}{40c}-\frac{2}{25}aex^5-\frac{2bex\tanh^{-1}(cx)}{5c^4}-2$$

$$=-\frac{2aex}{5c^4}-\frac{19bex^2}{100c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\tanh^{-1}(cx)}{5c^4}-$$

$$=-\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}-\frac{2aex^3}{15c^2}-\frac{9bex^4}{200c}-\frac{2}{25}aex^5-\frac{2bex\tanh^{-1}(cx)}{5c^4}-$$

Mathematica [A] time = 0.157128, size = 236, normalized size = 0.75

$$\frac{30c^2ex^2\log\left(1-c^2x^2\right)\left(4ac^3x^3+b\left(c^2x^2+2\right)+4bc^3x^3\tanh^{-1}(cx)\right)+2\log(1-cx)(-60ae+30bd-137be)+2\log(cx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-240*a*c*e*x + 2*b*c^2*(30*d - 77*e)*x^2 - 80*a*c^3*e*x^3 + 3*b*c^4*(10*d
- 9*e)*x^4 + 24*a*c^5*(5*d - 2*e)*x^5 - 8*b*c*x*(-15*c^4*d*x^4 + 2*e*(15 +
5*c^2*x^2 + 3*c^4*x^4))*ArcTanh[c*x] + 120*b*e*ArcTanh[c*x]^2 + 2*(30*b*d -
 60*a*e - 137*b*e)*Log[1 - c*x] + 2*(30*b*d + 60*a*e - 137*b*e)*Log[1 + c*x
] + 30*c^2*e*x^2*(4*a*c^3*x^3 + b*(2 + c^2*x^2) + 4*b*c^3*x^3*ArcTanh[c*x])
*Log[1 - c^2*x^2] + 30*b*e*Log[1 - c^2*x^2]^2)/(600*c^5)
```

Maple [C] time = 1.915, size = 4757, normalized size = 15.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] 181/600/c^5*b*e-3/10/c^5*b*e*ln(2)-2/25*b*e*x^5*arctanh(c*x)-2/25*a*e*x^5-2
/5*b*e*x*arctanh(c*x)/c^4-2/15*b*e*x^3*arctanh(c*x)/c^2+1/10/c^5*b*e*(4*arc
tanh(c*x)*x^5*c^5+c^4*x^4+2*c^2*x^2+4*arctanh(c*x)-4*ln((c*x+1)^2/(-c^2*x^2
+1)+1)-3)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3-2
/15*a*e*x^3/c^2+1/5*x^5*a*d-9/200*b*e*x^4/c+1/10*I*b*arctanh(c*x)*Pi*csgn(I
*(c*x+1)^2/(c^2*x^2-1))^3*x^5*e+1/10*I*b*arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)+1)^2)^3*x^5*e+1/10*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^5*e+3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)
^2+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-3/
40*I/c^5*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)+1))^2*e*Pi-3/40*I/c^5*b*Pi*e*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(
I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-3/40*I/c^5*b*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*e*Pi+1/10*I/c^
5*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+
1)^2)^3+3/20*I/c^5*b*Pi*e*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)+1)^2)^2-3/20*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(
I*(c*x+1)/(-c^2*x^2+1)^(1/2))*e*Pi-3/20/c^5*b*d+1/5*b*arctanh(c*x)*x^5*d-46
/75/c^5*b*arctanh(c*x)*e+137/150/c^5*b*e*ln((c*x+1)^2/(-c^2*x^2+1)+1)-1/5/c
^5*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*d+1/5/c^5*b*e*ln((c*x+1)^2/(-c^2*x^2+1)+1
)^2+1/5*a*e*x^5*ln(-c^2*x^2+1)-1/5*a*e/c^5*ln(c*x-1)+1/5*a*e/c^5*ln(c*x+1)-
1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^4*e-1/20
*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^2*e+1/10*I
/c^5*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-
c^2*x^2+1)+1)^2)-1/10*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(
-c^2*x^2+1)+1)^2)*x^5*e-1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/((c*x+1)^2/(-c^2*x^2+1)+1)^2)+1/20/c*b*x^4*d+1/5/c^5*b*arctanh(c*x)*d-3/40
*I/c^5*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-
3/40*I/c^5*b*e*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-3/40*I/c^5*b*e*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+3/40*I/c^5*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2
-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1)^2)+1/10*I/c^5*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)
*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e*arctanh(c*x)+1/10*I/c^5*b*csgn(I
*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*e*arctanh(c
*x)+1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csg
```

```
n(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2+1/10*I/c^5*b*Pi*l
n((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/10*I/c^5*b*Pi*ln((c*x+1)^2/
(-c^2*x^2+1)+1)*e*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)+1)^2)+1/5*I/c^5*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*((c*x
+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/10*I/c^5*b*
Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn
(I*(c*x+1)^2/(c^2*x^2-1))+1/40*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^
2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^4*e+1/20*I/c^3*b*Pi*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^2*e+1/40*I/c*
b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((
c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^
2*e+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)
^(1/2))^2*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1
)/(-c^2*x^2+1)^(1/2))^2*x^2*e-1/5*I/c^5*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e
*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/10*I/
c^5*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2
)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2+1/10*I*b*arc
tanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)+1)^2)*x^5*e+1/10*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^5*e+1/10*I*b*arctanh(c*x)*Pi*csgn(I*(c
*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2
*x^2+1)+1)^2)*x^5*e-1/5*I*b*arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+
1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*x^5*e+1/5*I*b*arctanh(c*x)*Pi*cs
gn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^5*e-1/10
*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1
)^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*x^5*e+1/20*I/c^3*b*Pi*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)+1)^2)^3*x^2*e+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*
x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2*e-1/10*I/c^5*b*Pi
*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/10*I/c^5*
b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2
/(-c^2*x^2+1)+1)^2)^3-1/10*I/c^5*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3+1/10*I/c^5*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))
^3*Pi*e*arctanh(c*x)+1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/
(-c^2*x^2+1)+1)^2)^3*x^4*e+1/20*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c
*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^2*e+1/40*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)+1)^2)^3*x^4*e+1/10/c^3*b*x^2*d-1/20*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x
^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*x^4*e-1/10*I/c^3*b*Pi*csgn
(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*x^2*e
+1/20*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^
(1/2))*x^4*e+1/10*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)
/(-c^2*x^2+1)^(1/2))*x^2*e-1/40*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x
+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*x^4*e-1/20*I/c^3*b
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*x^2*e-1/5*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2+1/5*I/c^5*b*arctanh(
c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
2-1/10*I/c^5*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/10/c*b*ln((c*x+1)^2/(-c^
2*x^2+1)+1)*x^4*e-1/5/c^3*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*x^2*e+1/10/c*b*ln(
2)*x^4*e+1/5/c^3*b*ln(2)*x^2*e+2/5/c^5*b*arctanh(c*x)*ln(2)*e-2/5/c^5*b*ln(
(c*x+1)^2/(-c^2*x^2+1)+1)*ln(2)*e-2/5*b*arctanh(c*x)*ln((c*x+1)^2/(-c^2*x^2
+1)+1)*x^5*e+2/5*b*arctanh(c*x)*ln(2)*x^5*e
```

Maxima [C] time = 0.983154, size = 428, normalized size = 1.36

$$\frac{1}{5}adx^5+\frac{1}{75}\left(15x^5\log\left(-c^2x^2+1\right)-c^2\left(\frac{2\left(3c^4x^5+5c^2x^3+15x\right)}{c^6}-\frac{15\log\left(cx+1\right)}{c^7}+\frac{15\log\left(cx-1\right)}{c^7}\right)\right)be\operatorname{artanh}\left(cx\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/5*a*d*x^5 + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b*e*arctanh(c*x
) + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1
)/c^6))*b*d + 1/75*(15*x^5*log(-c^2*x^2 + 1) - c^2*(2*(3*c^4*x^5 + 5*c^2*x^
3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*a*e - 1/600*(3*
(-10*I*pi*c^4 + 9*c^4)*x^4 + 2*(-30*I*pi*c^2 + 77*c^2)*x^2 - (60*I*pi + 30*
c^4*x^4 + 60*c^2*x^2 + 120*log(c*x - 1) - 274)*log(c*x + 1) - (60*I*pi + 30
*c^4*x^4 + 60*c^2*x^2 - 274)*log(c*x - 1))*b*e/c^5
```

Fricas [A] time = 1.93138, size = 590, normalized size = 1.87

$$-\frac{80\,ac^3ex^3-24\left(5\,ac^5d-2\,ac^5e\right)x^5-3\left(10\,bc^4d-9\,bc^4e\right)x^4+240\,acex-30\,be\log\left(-c^2x^2+1\right)^2-30\,be\log\left(-\frac{cx+1}{cx-1}\right)^2-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/600*(80*a*c^3*e*x^3 - 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9
*b*c^4*e)*x^4 + 240*a*c*e*x - 30*b*e*log(-c^2*x^2 + 1)^2 - 30*b*e*log(-(c*x
 + 1)/(c*x - 1))^2 - 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 - 2*(60*a*c^5*e*x^5 +
15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 + 30*b*d - 137*b*e)*log(-c^2*x^2 + 1) - 4*(
15*b*c^5*e*x^5*log(-c^2*x^2 + 1) - 10*b*c^3*e*x^3 + 3*(5*b*c^5*d - 2*b*c^5*
e)*x^5 - 30*b*c*e*x + 30*a*e)*log(-(c*x + 1)/(c*x - 1)))/c^5
```

Sympy [A] time = 31.7729, size = 338, normalized size = 1.07

$$\begin{cases}\frac{adx^5}{5}+\frac{aex^5\log\left(-c^2x^2+1\right)}{5}-\frac{2aex^5}{25}-\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\operatorname{atanh}(cx)}{5c^5}+\frac{bdx^5\operatorname{atanh}(cx)}{5}+\frac{bex^5\log\left(-c^2x^2+1\right)\operatorname{atanh}(cx)}{5}-\frac{2bex^5\operatorname{atanh}(cx)}{25}+\frac{bd}{20} \\ \frac{adx^5}{5}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**5*log(-c**2*x**2 + 1)/5 - 2*a*e*x**5/25 - 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atanh(c*x)/(5*c**5) + b*d*x**
5*atanh(c*x)/5 + b*e*x**5*log(-c**2*x**2 + 1)*atanh(c*x)/5 - 2*b*e*x**5*ata
nh(c*x)/25 + b*d*x**4/(20*c) + b*e*x**4*log(-c**2*x**2 + 1)/(20*c) - 9*b*e*
x**4/(200*c) - 2*b*e*x**3*atanh(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x
**2*log(-c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atanh(
c*x)/(5*c**4) + b*d*log(-c**2*x**2 + 1)/(10*c**5) + b*e*log(-c**2*x**2 + 1)
```

```
**2/(20*c**5) - 137*b*e*log(-c**2*x**2 + 1)/(300*c**5) + b*e*atanh(c*x)**2/
(5*c**5), Ne(c, 0)), (a*d*x**5/5, True))
```

Giac [A] time = 1.50791, size = 614, normalized size = 1.95

$$\frac{60\,bc^5x^5e\log(cx+1)^2 - 60\,bc^5x^5e\log(-cx+1)^2 + 120\,ac^5x^5e\log(cx+1) - 24\,bc^5x^5e\log(cx+1) + 120\,ac^5x^5e\log(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/600*(60*b*c^5*x^5*e*log(c*x + 1)^2 - 60*b*c^5*x^5*e*log(-c*x + 1)^2 + 120
*a*c^5*x^5*e*log(c*x + 1) - 24*b*c^5*x^5*e*log(c*x + 1) + 120*a*c^5*x^5*e*l
og(-c*x + 1) + 24*b*c^5*x^5*e*log(-c*x + 1) + 60*b*c^5*d*x^5*log(-(c*x + 1)
/(c*x - 1)) + 120*a*c^5*d*x^5 - 48*a*c^5*x^5*e + 30*b*c^4*x^4*e*log(c*x + 1
) + 30*b*c^4*x^4*e*log(-c*x + 1) + 30*b*c^4*d*x^4 - 27*b*c^4*x^4*e - 40*b*c
^3*x^3*e*log(c*x + 1) + 40*b*c^3*x^3*e*log(-c*x + 1) - 80*a*c^3*x^3*e + 60*
b*c^2*x^2*e*log(c*x + 1) + 60*b*c^2*x^2*e*log(-c*x + 1) + 60*b*c^2*d*x^2 -
154*b*c^2*x^2*e - 120*b*c*x*e*log(c*x + 1) + 120*b*c*x*e*log(-c*x + 1) - 24
0*a*c*x*e + 60*b*e*log(c*x + 1)^2 - 60*b*e*log(c*x - 1)^2 + 120*b*e*log(c*x
 - 1)*log(-c*x + 1) + 60*b*d*log(c^2*x^2 - 1) + 120*a*e*log(c*x + 1) - 274*
b*e*log(c*x + 1) - 120*a*e*log(c*x - 1) - 274*b*e*log(c*x - 1))/c^5
```

3.523 $\int x^3 \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=225

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+\right.$$

```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*
x])/(3*c^4) - (e*x^2*(a + b*ArcTanh[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTanh[
c*x]))/8 + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcTanh[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcT
anh[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```

Rubi [A] time = 0.269321, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2454, 2395, 43, 6083, 459, 321, 206, 2471, 2448, 2455, 302}

$$\frac{1}{4}x^4\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)-\frac{ex^2\left(a+b\tanh^{-1}(cx)\right)}{4c^2}-\frac{e\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{4c^4}-\frac{1}{8}ex^4\left(a+\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```


```
[Out] (b*(2*d - 3*e)*x)/(8*c^3) - (2*b*e*x)/(3*c^3) + (b*(2*d - e)*x^3)/(24*c) -
(b*e*x^3)/(18*c) - (b*(2*d - 3*e)*ArcTanh[c*x])/(8*c^4) + (2*b*e*ArcTanh[c*
x])/(3*c^4) - (e*x^2*(a + b*ArcTanh[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTanh[
c*x]))/8 + (b*e*x*Log[1 - c^2*x^2])/(4*c^3) + (b*e*x^3*Log[1 - c^2*x^2])/(1
2*c) - (e*(a + b*ArcTanh[c*x])*Log[1 - c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcT
anh[c*x])*(d + e*Log[1 - c^2*x^2]))/4
```


Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2471

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_) +
 (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> With[{t = ExpandIntegrand[(a + b*Log[
c*(d + e*x^n)^p])^q, (f + g*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a,
 b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && Integ
erQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s,
0] && LtQ[r, 0]))
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$\int x^3 \left(a + b \tanh^{-1}(cx)\right) \left(d + e \log\left(1 - c^2x^2\right)\right) dx = -\frac{ex^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4 \left(a + b \tanh^{-1}(cx)\right) - \frac{e \left(a + b \tanh^{-1}\right.}{}$$

$$= -\frac{ex^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4 \left(a + b \tanh^{-1}(cx)\right) - \frac{e \left(a + b \tanh^{-1}\right.}{}$$

$$= \frac{b(2d - e)x^3}{24c} - \frac{ex^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4 \left(a + b \tanh^{-1}(cx)\right) - \frac{e}{}$$

$$= \frac{b(2d - 3e)x}{8c^3} + \frac{b(2d - e)x^3}{24c} - \frac{ex^2 \left(a + b \tanh^{-1}(cx)\right)}{4c^2} - \frac{1}{8}ex^4 \left(a + b \tanh\right.$$

$$= \frac{b(2d - 3e)x}{8c^3} + \frac{b(2d - e)x^3}{24c} - \frac{b(2d - 3e) \tanh^{-1}(cx)}{8c^4} - \frac{ex^2 \left(a + b \tanh\right.}{4c^2}$$

$$= \frac{b(2d - 3e)x}{8c^3} - \frac{bex}{2c^3} + \frac{b(2d - e)x^3}{24c} - \frac{b(2d - 3e) \tanh^{-1}(cx)}{8c^4} - \frac{ex^2 \left(a + \right.}{}$$

$$= \frac{b(2d - 3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d - e)x^3}{24c} - \frac{bex^3}{18c} - \frac{b(2d - 3e) \tanh^{-1}(cx)}{8c^4} +$$

$$= \frac{b(2d - 3e)x}{8c^3} - \frac{2bex}{3c^3} + \frac{b(2d - e)x^3}{24c} - \frac{bex^3}{18c} - \frac{b(2d - 3e) \tanh^{-1}(cx)}{8c^4} +$$

Mathematica [A] time = 0.148819, size = 192, normalized size = 0.85

$$\frac{12e \log\left(1 - c^2x^2\right) \left(3ac^4x^4 + bcx \left(c^2x^2 + 3\right) + 3b \left(c^4x^4 - 1\right) \tanh^{-1}(cx)\right) + 3 \log(1 - cx)(-12ae + 6bd - 25be) - 3 \log(cx +}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (6*b*c*(6*d - 25*e)*x - 36*a*c^2*e*x^2 + 2*b*c^3*(6*d - 7*e)*x^3 + 18*a*c^4
*(2*d - e)*x^4 - 18*b*c^2*x^2*(-2*c^2*d*x^2 + e*(2 + c^2*x^2))*ArcTanh[c*x]
 + 3*(6*b*d - 12*a*e - 25*b*e)*Log[1 - c*x] - 3*(6*b*d + 12*a*e - 25*b*e)*L
og[1 + c*x] + 12*e*(3*a*c^4*x^4 + b*c*x*(3 + c^2*x^2) + 3*b*(-1 + c^4*x^4)*
ArcTanh[c*x])*Log[1 - c^2*x^2])/(144*c^4)
```

Maple [C] time = 1.247, size = 3739, normalized size = 16.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/8*a*x^4*e+1/4*b*d*x/c^3+1/12*b*d*x^3/c+1/6/c^4*b*e*(3*arctanh(c*x)*x^3*c
^3+3*arctanh(c*x)*x^2*c^2+c^2*x^2+3*arctanh(c*x)*x*c+c*x+3*arctanh(c*x)+4)*
(c*x-1)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-1/6*I/c^4*b*e*Pi*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)+1)^2)^3-1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/8
*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*cs
gn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x*e+1/8*I/c^3*b*Pi*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x*e-1/4*I/c^3*b*Pi*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*x*e+
```

1/4*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^
(1/2))*x*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^
2*x^2+1)^(1/2))^2*x*e+1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/8*I/c
^4*b*arctanh(c*x)*Pi*e*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/8*I/c^4*b*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)+1)^2)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e*arctanh(c*x)+
1/4*I/c^4*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)+1))*Pi*e*arctanh(c*x)-1/4*I/c^4*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*Pi*e*arctanh(c*x)-1/8*I/c^4*b*arctanh(c*x)
*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/
6*I/c^4*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)
+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)-1/8*I*b*a
rctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2
csgn(I(c*x+1)^2/(c^2*x^2-1))*x^4*e+1/8*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+
1)+1)^2)*x^4*e+1/8*I*b*arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2
csgn(I((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^4*e-1/4*I*b*arctanh(c*x)*Pi*csgn(I*
((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*x^4*e+1/
4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))*x^4*e+1/8*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*c
sgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^4*e-1/4*a*e/c^2*x^2-1/4*a*e/c^4*ln(c^
2*x^2-1)-1/4/c^4*b*d*arctanh(c*x)+1/4*b*arctanh(c*x)*x^4*d-1/8*b*arctanh(c*
x)*x^4*e-25/24*b*e*x/c^3-7/72*b*e*x^3/c+41/24*b*e*arctanh(c*x)/c^4-1/24*I/c
*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(
c*x+1)^2/(c^2*x^2-1))*x^3*e+1/24*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^3*e+1/24
*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x^2+
1)+1)^2)*x^3*e-1/12*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)+1)^2)^2*x^3*e+1/12*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1
))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^3*e+1/24*I/c*b*Pi*csgn(I*(c*x+1)^
2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^3*e-1/8*I/c^3*b*Pi*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*x+1)^2
/(c^2*x^2-1))*x*e+41/36/c^4*b*e-1/24*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)
^2/(-c^2*x^2+1)+1)^2)*x^3*e-1/8*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c
*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/
(-c^2*x^2+1)+1)^2)*x*e+1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
(c*x+1)^2/(-c^2*x^2+1)+1)^2)-1/8*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/(
(c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^4*e-1/3/c^4*b*d+1/4*x^4*a*e*ln(-c^2*x^2+1)-2
/3/c^4*b*e*ln(2)+1/4*x^4*a*d+1/8*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^4*e+1/8*I*b*arctanh(c*x)*Pi*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^4*e+1/8*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+
1)^2/(c^2*x^2-1))^3*x^4*e+1/6*I/c^4*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/6*I/c^4*b*e*Pi
*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)
^2/(-c^2*x^2+1)+1)^2)^2-1/6*I/c^4*b*e*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))
^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)+1/3*I/c^4*b*e*Pi*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/3*I/c^4*b*e*Pi*cs
gn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/6*I/c^4*
b*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1
/24*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x
^3*e+1/24*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^3*e+1/24*I/c*b*
Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^3*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x*e+1/8*I/c^3*b*Pi*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)+1)^2)^3*x*e+1/8*I/c^3*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3
*x*e-1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/

```
(-c^2*x^2+1)+1)^2)^3-1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*((c*x+1)^2/(-c^2*
x^2+1)+1)^2)^3-1/8*I/c^4*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
3+1/6/c*b*ln(2)*x^3*e-1/6/c*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*x^3*e-1/4/c^2*b*
arctanh(c*x)*x^2*e+1/2/c^3*b*ln(2)*x*e-1/2/c^3*b*ln((c*x+1)^2/(-c^2*x^2+1)+
1)*x*e-1/2/c^4*b*arctanh(c*x)*ln(2)*e+1/2/c^4*b*ln((c*x+1)^2/(-c^2*x^2+1)+1
)*e*arctanh(c*x)+1/2*b*arctanh(c*x)*ln(2)*x^4*e-1/2*b*arctanh(c*x)*ln((c*x+
1)^2/(-c^2*x^2+1)+1)*x^4*e
```

Maxima [C] time = 0.987996, size = 366, normalized size = 1.63

$$\frac{1}{4}adx^4 + \frac{1}{8}\left(2x^4\log\left(-c^2x^2+1\right) - c^2\left(\frac{c^2x^4+2x^2}{c^4} + \frac{2\log\left(c^2x^2-1\right)}{c^6}\right)\right)be\,\text{artanh}(cx) + \frac{1}{24}\left(6x^4\,\text{artanh}(cx) + c\left(\frac{2\left(c^2x^3\right.}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/4*a*d*x^4 + 1/8*(2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2
*log(c^2*x^2 - 1)/c^6))*b*e*arctanh(c*x) + 1/24*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*d + 1/8*(
2*x^4*log(-c^2*x^2 + 1) - c^2*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^2*x^2 - 1)/c
^6))*a*e - 1/144*(2*(-6*I*pi*c^3 + 7*c^3)*x^3 + 6*(-6*I*pi*c + 25*c)*x - (-
18*I*pi + 12*c^3*x^3 + 36*c*x + 75)*log(c*x + 1) - (18*I*pi + 12*c^3*x^3 +
36*c*x - 75)*log(c*x - 1))*b*e/c^4
```

Fricas [A] time = 1.95728, size = 441, normalized size = 1.96

$$-\frac{36\,ac^2ex^2 - 18\left(2\,ac^4d - ac^4e\right)x^4 - 2\left(6\,bc^3d - 7\,bc^3e\right)x^3 - 6\left(6\,bcd - 25\,bce\right)x - 12\left(3\,ac^4ex^4 + bc^3ex^3 + 3\,bcex - 3\,ae\right)\log}{144\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/144*(36*a*c^2*e*x^2 - 18*(2*a*c^4*d - a*c^4*e)*x^4 - 2*(6*b*c^3*d - 7*b*
c^3*e)*x^3 - 6*(6*b*c*d - 25*b*c*e)*x - 12*(3*a*c^4*e*x^4 + b*c^3*e*x^3 + 3
*b*c*e*x - 3*a*e)*log(-c^2*x^2 + 1) + 3*(6*b*c^2*e*x^2 - 3*(2*b*c^4*d - b*c
^4*e)*x^4 + 6*b*d - 25*b*e - 6*(b*c^4*e*x^4 - b*e)*log(-c^2*x^2 + 1))*log(-
(c*x + 1)/(c*x - 1)))/c^4
```

Sympy [A] time = 19.7373, size = 279, normalized size = 1.24

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^4\log\left(-c^2x^2+1\right)}{4} - \frac{aex^4}{8} - \frac{aex^2}{4c^2} - \frac{ae\log\left(-c^2x^2+1\right)}{4c^4} + \frac{bdx^4\,\text{atanh}(cx)}{4} + \frac{bex^4\log\left(-c^2x^2+1\right)\text{atanh}(cx)}{4} - \frac{bex^4\,\text{atanh}(cx)}{8} + \frac{bdx^3}{12c} + \frac{bex}{} \\ \frac{adx^4}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**4*log(-c**2*x**2 + 1)/4 - a*e*x**4/8 - a*e*x
**2/(4*c**2) - a*e*log(-c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atanh(c*x)/4 + b
*e*x**4*log(-c**2*x**2 + 1)*atanh(c*x)/4 - b*e*x**4*atanh(c*x)/8 + b*d*x**3
/(12*c) + b*e*x**3*log(-c**2*x**2 + 1)/(12*c) - 7*b*e*x**3/(72*c) - b*e*x**
2*atanh(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(-c**2*x**2 + 1)/(4*c**3)
 - 25*b*e*x/(24*c**3) - b*d*atanh(c*x)/(4*c**4) - b*e*log(-c**2*x**2 + 1)*a
tanh(c*x)/(4*c**4) + 25*b*e*atanh(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, T
rue))
```

Giac [B] time = 1.45557, size = 555, normalized size = 2.47

$$\frac{18\, bc^4x^4e\log\left(cx+1\right)^2 - 18\, bc^4x^4e\log\left(-cx+1\right)^2 + 36\, ac^4x^4e\log\left(cx+1\right) - 9\, bc^4x^4e\log\left(cx+1\right) + 36\, ac^4x^4e\log\left(-cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/144*(18*b*c^4*x^4*e*log(c*x + 1)^2 - 18*b*c^4*x^4*e*log(-c*x + 1)^2 + 36*
a*c^4*x^4*e*log(c*x + 1) - 9*b*c^4*x^4*e*log(c*x + 1) + 36*a*c^4*x^4*e*log(
-c*x + 1) + 9*b*c^4*x^4*e*log(-c*x + 1) + 18*b*c^4*d*x^4*log(-(c*x + 1)/(c*
x - 1)) + 36*a*c^4*d*x^4 - 18*a*c^4*x^4*e + 12*b*c^3*x^3*e*log(c*x + 1) + 1
2*b*c^3*x^3*e*log(-c*x + 1) + 12*b*c^3*d*x^3 - 14*b*c^3*x^3*e - 18*b*c^2*x^
2*e*log(c*x + 1) + 18*b*c^2*x^2*e*log(-c*x + 1) - 36*a*c^2*x^2*e + 36*b*c*x
*e*log(c*x + 1) + 36*b*c*x*e*log(-c*x + 1) + 36*b*c*d*x - 150*b*c*x*e - 18*
b*e*log(c*x + 1)^2 - 18*b*e*log(c*x - 1)^2 + 36*b*e*log(c*x - 1)*log(-c*x +
 1) - 18*b*d*log(c*x + 1) - 36*a*e*log(c*x + 1) + 75*b*e*log(c*x + 1) + 18*
b*d*log(c*x - 1) - 36*a*e*log(c*x - 1) - 75*b*e*log(c*x - 1))/c^4
```

3.524 $\int x^2 \left(a + b \tanh^{-1}(cx)\right) \left(d + e \log\left(1 - c^2x^2\right)\right) dx$

Optimal. Leaf size=247

$$\frac{1}{3}x^3 \left(a + b \tanh^{-1}(cx)\right) \left(e \log\left(1 - c^2x^2\right) + d\right) - \frac{e(2a + b) \log(1 - cx)}{6c^3} + \frac{e(2a - b) \log(cx + 1)}{6c^3} - \frac{2aex}{3c^2} - \frac{2}{9}aex^3 + \frac{bx^2 \left(e \log\right.}{}$$

```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcTanh[
c*x])/(3*c^2) - (2*b*e*x^3*ArcTanh[c*x])/9 + (b*e*ArcTanh[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```

Rubi [A] time = 0.633544, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5916, 266, 43, 6085, 6725, 801, 633, 31, 5980, 5910, 260, 5948, 2475, 2390, 2301}

$$\frac{1}{3}x^3 \left(a + b \tanh^{-1}(cx)\right) \left(e \log\left(1 - c^2x^2\right) + d\right) - \frac{e(2a + b) \log(1 - cx)}{6c^3} + \frac{e(2a - b) \log(cx + 1)}{6c^3} - \frac{2aex}{3c^2} - \frac{2}{9}aex^3 + \frac{bx^2 \left(e \log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```


```
[Out] (-2*a*e*x)/(3*c^2) - (5*b*e*x^2)/(18*c) - (2*a*e*x^3)/9 - (2*b*e*x*ArcTanh[
c*x])/(3*c^2) - (2*b*e*x^3*ArcTanh[c*x])/9 + (b*e*ArcTanh[c*x]^2)/(3*c^3) -
 ((2*a + b)*e*Log[1 - c*x])/(6*c^3) + ((2*a - b)*e*Log[1 + c*x])/(6*c^3) -
(4*b*e*Log[1 - c^2*x^2])/(9*c^3) - (b*e*Log[1 - c^2*x^2]^2)/(12*c^3) + (b*x
^2*(d + e*Log[1 - c^2*x^2]))/(6*c) + (x^3*(a + b*ArcTanh[c*x])*(d + e*Log[1
 - c^2*x^2]))/3 + (b*Log[1 - c^2*x^2]*(d + e*Log[1 - c^2*x^2]))/(6*c^3)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
```

```
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int x^2\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$

$$=-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}+\frac{bx^2\left(d+e\log\left(1-c^2x^2\right)\right)}{6c}+\frac{1}{3}x^3\left(a+b\tanh^{-1}\right.$$

$$=-\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2}{9}bex^3\tanh^{-1}(cx)-\frac{be\log^2\left(1-c^2x^2\right)}{12c^3}+\frac{b}{}$$

$$=-\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}(cx)+\frac{be}{}$$

$$=-\frac{2aex}{3c^2}-\frac{bex^2}{6c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}(cx)+\frac{be}{}$$

$$=-\frac{2aex}{3c^2}-\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2bex\tanh^{-1}(cx)}{3c^2}-\frac{2}{9}bex^3\tanh^{-1}(cx)+\frac{b}{}$$

Mathematica [A] time = 0.123965, size = 183, normalized size = 0.74

$$\frac{6c^2ex^2\log\left(1-c^2x^2\right)\left(2acx+2bcx\tanh^{-1}(cx)+b\right)+2\log(1-cx)(-6ae+3bd-11be)+2\log(cx+1)(6ae+3bd-11be)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (-24*a*c*e*x + 2*b*c^2*(3*d - 5*e)*x^2 + 4*a*c^3*(3*d - 2*e)*x^3 + 4*b*c*x*
(3*c^2*d*x^2 - 2*e*(3 + c^2*x^2))*ArcTanh[c*x] + 12*b*e*ArcTanh[c*x]^2 + 2*
(3*b*d - 6*a*e - 11*b*e)*Log[1 - c*x] + 2*(3*b*d + 6*a*e - 11*b*e)*Log[1 +
```

```
c*x] + 6*c^2*e*x^2*(b + 2*a*c*x + 2*b*c*x*ArcTanh[c*x])*Log[1 - c^2*x^2] +
3*b*e*Log[1 - c^2*x^2]^2)/(36*c^3)
```

Maple [C] time = 1.142, size = 3994, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] 1/3*x^3*a*d+1/3/c^3*b*e*ln((c*x+1)^2/(-c^2*x^2+1)+1)^2+1/3/c^3*b*arctanh(c*
x)*d-8/9/c^3*b*arctanh(c*x)*e-1/3/c^3*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*d+11/9
/c^3*b*e*ln((c*x+1)^2/(-c^2*x^2+1)+1)+1/3*b*arctanh(c*x)*x^3*d-2/9*a*e*x^3+
1/6*b*d*x^2/c-1/6/c^3*b*d+5/18/c^3*b*e-2/3*b*e*x*arctanh(c*x)/c^2+1/3*x^3*a
*e*ln(-c^2*x^2+1)-1/3*a*e/c^3*ln(c*x-1)+1/3*a*e/c^3*ln(c*x+1)-2/3*a*e*x/c^2
-1/3/c^3*b*e*ln(2)-1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^
2/(-c^2*x^2+1)+1)^2)^3-1/12*I/c^3*b*e*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^
2)^3-1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/3/c^3*b*e*(2*arcta
nh(c*x)*x^3*c^3+c^2*x^2+2*arctanh(c*x)-2*ln((c*x+1)^2/(-c^2*x^2+1)+1)-1)*ln
((c*x+1)/(-c^2*x^2+1)^(1/2))+1/3/c*b*ln(2)*x^2*e-1/3/c*b*ln((c*x+1)^2/(-c^2
*x^2+1)+1)*x^2*e+2/3/c^3*b*arctanh(c*x)*ln(2)*e-2/3/c^3*b*ln((c*x+1)^2/(-c^
2*x^2+1)+1)*ln(2)*e+2/3*b*arctanh(c*x)*ln(2)*x^3*e-2/3*b*arctanh(c*x)*ln((c
*x+1)^2/(-c^2*x^2+1)+1)*x^3*e-1/6*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*
csgn(I*(c*x+1)^2/(c^2*x^2-1))*x^3*e-1/12*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2
-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*x^2*e-1/6*I/c^3*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1
)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)+1/12*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x
^2-1))^3*x^2*e+1/6*I/c^3*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(
(c*x+1)^2/(-c^2*x^2+1)+1)^2)^3+1/6*I/c^3*b*arctanh(c*x)*Pi*e*csgn(I*((c*x+1
)^2/(-c^2*x^2+1)+1)^2)^3+1/6*I/c^3*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e*a
rctanh(c*x)-1/6*I/c^3*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-1/6*I/c^3*b*Pi*ln((c*x+1)^2/(-c
^2*x^2+1)+1)*e*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-1/6*I/c^3*b*ln((c*x+1
)^2/(-c^2*x^2+1)+1)*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/12*I/c^3*b*e*Pi*
csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^
2/(-c^2*x^2+1)+1)^2)^2+1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2+1/6*I/c^3*b*e*Pi*c
sgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/
12*I/c^3*b*e*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)+1)^2)-1/6*I/c^3*b*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(
c*x+1)^2/(c^2*x^2-1))^2-1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-5/18*b*e*x^2/c-2/9*b*e*x^3*arctanh(c*x)+1
/12*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*c
sgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^2*e-1/12*I/c*b*Pi*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*x^2
*e-1/6*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)+1))*x^2*e+1/12*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*x^2*e+1/6*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^2*e+1/12*I/c*b*Pi*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2*e+1/6*I/c^3*b*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*x+1
)^2/(-c^2*x^2+1)+1)^2)*Pi*e*arctanh(c*x)-1/6*I/c^3*b*arctanh(c*x)*Pi*e*csgn
(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2
+1)+1)^2)^2-1/3*I/c^3*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*((c*x
```

```
+1)^2/(-c^2*x^2+1)+1))*Pi*e*arctanh(c*x)+1/6*I/c^3*b*csgn(I*((c*x+1)^2/(-c^
2*x^2+1)+1)^2)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e*arctanh(c*x)+1/3*I
/c^3*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^
2/(c^2*x^2-1))^2+1/6*I/c^3*b*arctanh(c*x)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^
(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/6*I/c^3*b*Pi*ln((c*x+1)^2/(-c^2*x^
2+1)+1)*e*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2+1/6*I/c^3*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(
-c^2*x^2+1)+1)^2)^2+1/3*I/c^3*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*Pi*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/6*I/c^3*
b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*cs
gn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)-1/3*I/c^3*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)
+1)*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2-1/
6*I/c^3*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/
2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))+1/12*I/c^3*b*e*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/
((c*x+1)^2/(-c^2*x^2+1)+1)^2)+1/6*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2
)*x^3*e-1/6*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c
^2*x^2+1)+1)^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*x^3*e-1/3*I*b*arctanh(c*x)*
Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)
)*x^3*e+1/6*I*b*arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)+1))^2*x^3*e+1/3*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x^3*e+1/6*I*b*arctanh(
c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*
x^3*e+1/6*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2
*x^2+1)+1)^2)^3*x^3*e+1/6*I*b*arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1
)+1)^2)^3*x^3*e+1/6*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^3
*e+1/12*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)
^3*x^2*e+1/12*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^2*e+1/6*I/c
^3*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I
/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^
2*x^2+1)+1)^2)
```

Maxima [C] time = 0.997595, size = 340, normalized size = 1.38

$$\frac{1}{3}adx^3 + \frac{1}{9}\left(3x^3\log\left(-c^2x^2+1\right) - c^2\left(\frac{2\left(c^2x^3+3x\right)}{c^4} - \frac{3\log\left(cx+1\right)}{c^5} + \frac{3\log\left(cx-1\right)}{c^5}\right)\right)be\,\operatorname{artanh}\left(cx\right) + \frac{1}{6}\left(2x^3\operatorname{artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima
")
```

```
[Out] 1/3*a*d*x^3 + 1/9*(3*x^3*log(-c^2*x^2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3
*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b*e*arctanh(c*x) + 1/6*(2*x^3*arct
anh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*b*d + 1/9*(3*x^3*log(-c^2*x^
2 + 1) - c^2*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c
^5))*a*e + 1/18*((3*I*pi*c^2 - 5*c^2)*x^2 + (3*I*pi + 3*c^2*x^2 + 6*log(c*x
 - 1) - 11)*log(c*x + 1) + (3*I*pi + 3*c^2*x^2 - 11)*log(c*x - 1))*b*e/c^3
```

Fricas [A] time = 1.95706, size = 455, normalized size = 1.84

$$-\frac{24\,acex - 4\left(3\,ac^3d - 2\,ac^3e\right)x^3 - 3\,be\log\left(-c^2x^2+1\right)^2 - 3\,be\log\left(-\frac{cx+1}{cx-1}\right)^2 - 2\left(3\,bc^2d - 5\,bc^2e\right)x^2 - 2\left(6\,ac^3ex^3 + 3\,bc^2ex\right.}{36\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas
")
```

```
[Out] -1/36*(24*a*c*e*x - 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 3*b*e*log(-c^2*x^2 + 1)
^2 - 3*b*e*log(-(c*x + 1)/(c*x - 1))^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 - 2*
(6*a*c^3*e*x^3 + 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(-c^2*x^2 + 1) - 2*(3*b
*c^3*e*x^3*log(-c^2*x^2 + 1) - 6*b*c*e*x + (3*b*c^3*d - 2*b*c^3*e)*x^3 + 6*
a*e)*log(-(c*x + 1)/(c*x - 1)))/c^3
```

Sympy [A] time = 12.8324, size = 258, normalized size = 1.04

$$\begin{cases} \frac{adx^3}{3} + \frac{aex^3\log\left(-c^2x^2+1\right)}{3} - \frac{2aex^3}{9} - \frac{2aex}{3c^2} + \frac{2ae\operatorname{atanh}(cx)}{3c^3} + \frac{bdx^3\operatorname{atanh}(cx)}{3} + \frac{bex^3\log\left(-c^2x^2+1\right)\operatorname{atanh}(cx)}{3} - \frac{2bex^3\operatorname{atanh}(cx)}{9} + \frac{bdx^2}{6c} + \\ \frac{adx^3}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**3*log(-c**2*x**2 + 1)/3 - 2*a*e*x**3/9 - 2*a
*e*x/(3*c**2) + 2*a*e*atanh(c*x)/(3*c**3) + b*d*x**3*atanh(c*x)/3 + b*e*x**
3*log(-c**2*x**2 + 1)*atanh(c*x)/3 - 2*b*e*x**3*atanh(c*x)/9 + b*d*x**2/(6*
c) + b*e*x**2*log(-c**2*x**2 + 1)/(6*c) - 5*b*e*x**2/(18*c) - 2*b*e*x*atanh
(c*x)/(3*c**2) + b*d*log(-c**2*x**2 + 1)/(6*c**3) + b*e*log(-c**2*x**2 + 1)
**2/(12*c**3) - 11*b*e*log(-c**2*x**2 + 1)/(18*c**3) + b*e*atanh(c*x)**2/(3
*c**3), Ne(c, 0)), (a*d*x**3/3, True))
```

Giac [A] time = 1.30633, size = 477, normalized size = 1.93

$$\frac{3\,bc^3x^3e\log(cx+1)^2 - 3\,bc^3x^3e\log(-cx+1)^2 + 6\,ac^3x^3e\log(cx+1) - 2\,bc^3x^3e\log(cx+1) + 6\,ac^3x^3e\log(-cx+1) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/18*(3*b*c^3*x^3*e*log(c*x + 1)^2 - 3*b*c^3*x^3*e*log(-c*x + 1)^2 + 6*a*c^
3*x^3*e*log(c*x + 1) - 2*b*c^3*x^3*e*log(c*x + 1) + 6*a*c^3*x^3*e*log(-c*x
+ 1) + 2*b*c^3*x^3*e*log(-c*x + 1) + 3*b*c^3*d*x^3*log(-(c*x + 1)/(c*x - 1)
) + 6*a*c^3*d*x^3 - 4*a*c^3*x^3*e + 3*b*c^2*x^2*e*log(c*x + 1) + 3*b*c^2*x^
2*e*log(-c*x + 1) + 3*b*c^2*d*x^2 - 5*b*c^2*x^2*e - 6*b*c*x*e*log(c*x + 1)
+ 6*b*c*x*e*log(-c*x + 1) - 12*a*c*x*e + 3*b*e*log(c*x + 1)^2 - 3*b*e*log(c
*x - 1)^2 + 6*b*e*log(c*x - 1)*log(-c*x + 1) + 3*b*d*log(c^2*x^2 - 1) + 6*a
*e*log(c*x + 1) - 11*b*e*log(c*x + 1) - 6*a*e*log(c*x - 1) - 11*b*e*log(c*x
 - 1))/c^3
```

3.525 $\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)\,dx$

Optimal. Leaf size=140

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-1}(c}{2c^2}$$

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*A
rcTanh[c*x])/c^2 + (d*x^2*(a + b*ArcTanh[c*x]))/2 - (e*x^2*(a + b*ArcTanh[c
*x]))/2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcTanh[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rubi [A] time = 0.118822, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2454, 2389, 2295, 6083, 321, 207, 2448, 206}

$$-\frac{e\left(1-c^2x^2\right)\log\left(1-c^2x^2\right)\left(a+b\tanh^{-1}(cx)\right)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{b(d-e)\tanh^{-1}(c}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (b*e*A
rcTanh[c*x])/c^2 + (d*x^2*(a + b*ArcTanh[c*x]))/2 - (e*x^2*(a + b*ArcTanh[c
*x]))/2 + (b*e*x*Log[1 - c^2*x^2])/(2*c) - (e*(1 - c^2*x^2)*(a + b*ArcTanh[
c*x])*Log[1 - c^2*x^2])/(2*c^2)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{e\left(1-c^2x^2\right)\left(\right.}{}$$

$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)-\frac{e\left(1-c^2x^2\right)\left(\right.}{}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{be\tanh^{-1}(cx)}{c^2}+\frac{1}{2}dx^2\left(a\right.$$

Mathematica [A] time = 0.104307, size = 129, normalized size = 0.92

$$\frac{2e\log\left(1-c^2x^2\right)\left(cx(acx+b)+b\left(c^2x^2-1\right)\tanh^{-1}(cx)\right)+\log(1-cx)(b(d-3e)-2ae)-\log(cx+1)(2ae+b(d-3e))}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] (2*b*c*(d - 3*e)*x + 2*a*c^2*(d - e)*x^2 + 2*b*c^2*(d - e)*x^2*ArcTanh[c*x]
 + (b*(d - 3*e) - 2*a*e)*Log[1 - c*x] - (b*(d - 3*e) + 2*a*e)*Log[1 + c*x]
+ 2*e*(c*x*(b + a*c*x) + b*(-1 + c^2*x^2)*ArcTanh[c*x])*Log[1 - c^2*x^2])/(
4*c^2)
```

Maple [C] time = 0.639, size = 2951, normalized size = 21.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] -1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e*arctanh(c*x)-1/4*I/c^2*b*
e*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*
x+1)^2/(-c^2*x^2+1)+1)^2)^2+1/4*I/c^2*b*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/4*I/c^2*b*e*
Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2
)+1/2*I/c^2*b*e*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c
^2*x^2+1)+1)^2)^2-1/2*I/c^2*b*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2-1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/
2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/2*a*e/c^2*ln(-c^2*x^2+1)+1/2*x^2*a*e*
ln(-c^2*x^2+1)-1/2/c^2*b*d+5/2*b*e*arctanh(c*x)/c^2+1/2*b*d*x/c+1/2*d*a*x^2
-1/2/c^2*b*d*arctanh(c*x)+1/2*b*arctanh(c*x)*x^2*d-1/2*b*arctanh(c*x)*x^2*e
-3/2*b*e*x/c-1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^
2/(-c^2*x^2+1)+1)^2)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/
(c^2*x^2-1))*x^2*e-1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c
^2*x^2+1)+1)^2)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*x*e+1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1
)+1)^2)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
Pi*e*arctanh(c*x)+1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^
2*x^2+1)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x*e-1/4*I/c*b*Pi*csgn
(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*x+1)^2/(
c^2*x^2-1))*x*e+1/4*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((
c*x+1)^2/(-c^2*x^2+1)+1)^2)*x*e-1/2*I/c*b*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)
+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*x*e+1/2*I/c*b*Pi*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*x*e+1/4*I/c*b*Pi*csgn(
I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x*e-1/4*I/c^2
*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I/((c*
x+1)^2/(-c^2*x^2+1)+1)^2)*Pi*e*arctanh(c*x)+1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*
e*arctanh(c*x)-1/4*I/c^2*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e*arctanh(c*x)+1/2*I/c^2*b*csgn(I*((c*x+1)^2/(
-c^2*x^2+1)+1)^2)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*Pi*e*arctanh(c*x)-1/
2*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2)
)*Pi*e*arctanh(c*x)-1/c^2*b*e*ln(2)+3/2/c^2*b*e-1/2*a*e*x^2-b*arctanh(c*x)*
ln((c*x+1)^2/(-c^2*x^2+1)+1)*x^2*e+b*arctanh(c*x)*ln(2)*x^2*e-1/c*b*ln((c*x
+1)^2/(-c^2*x^2+1)+1)*x*e+1/c*b*ln(2)*x*e+1/c^2*b*ln((c*x+1)^2/(-c^2*x^2+1)
+1)*e*arctanh(c*x)-1/c^2*b*ln(2)*e*arctanh(c*x)+1/2*a*e/c^2-1/4*I/c^2*b*csg
n(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*e*arctan
h(c*x)+1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c
^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)+
1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1
)+1)^2)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^2*e-1/4*I*b*arctanh(c*x)*P
i*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*(c*x+
1)^2/(c^2*x^2-1))*x^2*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*x^2*e-1/2*I*b*arctanh(c*x)*P
i*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2
*x^2*e+1/2*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+
1)/(-c^2*x^2+1)^(1/2))*x^2*e+1/4*I*b*arctanh(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*
x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*x^2*e+1/4*I*b*arctanh(c*x)*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^2*e+1/4*I*b*
arctanh(c*x)*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x^2*e+1/4*I*b*arctan
h(c*x)*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*x^2*e+1/4*I/c*b*Pi*csgn(I*(c*x+1)
```

```
^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*x*e+1/4*I/c*b*Pi*csgn(I*((c*
x+1)^2/(-c^2*x^2+1)+1)^2)^3*x*e+1/4*I/c*b*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^
3*x*e-1/4*I/c^2*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2
)^3*Pi*e*arctanh(c*x)-1/4*I/c^2*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*Pi
*e*arctanh(c*x)-1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-
c^2*x^2+1)+1)^2)^3-1/4*I/c^2*b*e*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3-
1/4*I/c^2*b*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3+1/c^2*b*e*(arctanh(c*x)*x*
c+arctanh(c*x)+1)*(c*x-1)*ln((c*x+1)/(-c^2*x^2+1)^(1/2))
```

Maxima [A] time = 0.977084, size = 231, normalized size = 1.65

$$\frac{1}{2}adx^2 + \frac{1}{4}\left(2x^2 \operatorname{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)bd - \frac{\left(c^2x^2 - \left(c^2x^2-1\right)\log\left(-c^2x^2+1\right) - 1\right)be \operatorname{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log
(c*x - 1)/c^3))*b*d - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*b
*e*arctanh(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*x^2 - 1)*log(-c^2*x^2 + 1) - 1)*a
*e/c^2 - 1/2*(3*c*x - (c*x + 1)*log(c*x + 1) - (c*x - 1)*log(-c*x + 1))*b*e
/c^2
```

Fricas [A] time = 2.11355, size = 298, normalized size = 2.13

$$\frac{2\left(ac^2d - ac^2e\right)x^2 + 2\,(bcd - 3\,bce)x + 2\left(ac^2ex^2 + bcex - ae\right)\log\left(-c^2x^2+1\right) + \left(\left(bc^2d - bc^2e\right)x^2 - bd + 3\,be + \left(bc^2ex^2\right.\right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*c^2*d - a*c^2*e)*x^2 + 2*(b*c*d - 3*b*c*e)*x + 2*(a*c^2*e*x^2 + b
*c*e*x - a*e)*log(-c^2*x^2 + 1) + ((b*c^2*d - b*c^2*e)*x^2 - b*d + 3*b*e +
(b*c^2*e*x^2 - b*e)*log(-c^2*x^2 + 1))*log(-(c*x + 1)/(c*x - 1)))/c^2
```

Sympy [A] time = 6.93077, size = 202, normalized size = 1.44

$$\begin{cases} \frac{adx^2}{2} + \frac{aex^2\log\left(-c^2x^2+1\right)}{2} - \frac{aex^2}{2} - \frac{ae\log\left(-c^2x^2+1\right)}{2c^2} + \frac{bdx^2\operatorname{atanh}(cx)}{2} + \frac{bex^2\log\left(-c^2x^2+1\right)\operatorname{atanh}(cx)}{2} - \frac{bex^2\operatorname{atanh}(cx)}{2} + \frac{bdx}{2c} + \frac{bex\log(\cdot}{2} \\ \frac{adx^2}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**2*log(-c**2*x**2 + 1)/2 - a*e*x**2/2 - a*e*l
og(-c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atanh(c*x)/2 + b*e*x**2*log(-c**2*x*
*2 + 1)*atanh(c*x)/2 - b*e*x**2*atanh(c*x)/2 + b*d*x/(2*c) + b*e*x*log(-c**
2*x**2 + 1)/(2*c) - 3*b*e*x/(2*c) - b*d*atanh(c*x)/(2*c**2) - b*e*log(-c**2
```

```
*x**2 + 1)*atanh(c*x)/(2*c**2) + 3*b*e*atanh(c*x)/(2*c**2), Ne(c, 0)), (a*d
*x**2/2, True))
```

Giac [B] time = 1.25823, size = 412, normalized size = 2.94

$$\frac{bc^2x^2e\log(cx+1)^2 - bc^2x^2e\log(-cx+1)^2 + 2\,ac^2x^2e\log(cx+1) - bc^2x^2e\log(cx+1) + 2\,ac^2x^2e\log(-cx+1) + bc^2x^2e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/4*(b*c^2*x^2*e*log(c*x + 1)^2 - b*c^2*x^2*e*log(-c*x + 1)^2 + 2*a*c^2*x^2
*e*log(c*x + 1) - b*c^2*x^2*e*log(c*x + 1) + 2*a*c^2*x^2*e*log(-c*x + 1) +
b*c^2*x^2*e*log(-c*x + 1) + b*c^2*d*x^2*log(-(c*x + 1)/(c*x - 1)) + 2*a*c^2
*d*x^2 - 2*a*c^2*x^2*e + 2*b*c*x*e*log(c*x + 1) + 2*b*c*x*e*log(-c*x + 1) +
 2*b*c*d*x - 6*b*c*x*e - b*e*log(c*x + 1)^2 - b*e*log(c*x - 1)^2 + 2*b*e*lo
g(c*x - 1)*log(-c*x + 1) - b*d*log(c*x + 1) - 2*a*e*log(c*x + 1) + 3*b*e*lo
g(c*x + 1) + b*d*log(c*x - 1) - 2*a*e*log(c*x - 1) - 3*b*e*log(c*x - 1))/c^
2
```

3.526 $\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx$

Optimal. Leaf size=104

$$x\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}-2aex+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be\log\left(1-c^2x\right.}{c}$$

```
[Out] -2*a*e*x - 2*b*e*x*ArcTanh[c*x] + (e*(a + b*ArcTanh[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rubi [A] time = 0.195038, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6073, 2475, 2390, 2301, 5980, 5910, 260, 5948}

$$x\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}-2aex+\frac{b\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4ce}-\frac{be\log\left(1-c^2x\right.}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] -2*a*e*x - 2*b*e*x*ArcTanh[c*x] + (e*(a + b*ArcTanh[c*x])^2)/(b*c) - (b*e*L
og[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]) + (b*(
d + e*Log[1 - c^2*x^2])^2)/(4*c*e)
```

Rule 6073

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[
2*e*g, Int[(x^2*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b,
c, d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)dx = x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-(bc)\int \frac{x\left(d+e\log\left(1-c^2x\right.\right.}{1-c^2x^2}$$
$$= x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{d+e\log}{1-}\right.$$
$$= -2aex+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2\right.\right.$$
$$= -2aex-2bex\tanh^{-1}(cx)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}+x\left(a+b\tanh^{-1}(cx\right.$$
$$= -2aex-2bex\tanh^{-1}(cx)+\frac{e\left(a+b\tanh^{-1}(cx)\right)^2}{bc}-\frac{be\log\left(1-c^2x^2\right)}{c}+$$

Mathematica [A] time = 0.0170782, size = 144, normalized size = 1.38

$$aex\log\left(1-c^2x^2\right)+\frac{2ae\tanh^{-1}(cx)}{c}+adx-2aex+\frac{bd\log\left(1-c^2x^2\right)}{2c}+\frac{be\log^2\left(1-c^2x^2\right)}{4c}-\frac{be\log\left(1-c^2x^2\right)}{c}+bex\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*ArcTanh[c*x])/c + b*d*x*ArcTanh[c*x] - 2*b*e*x*Arc
Tanh[c*x] + (b*e*ArcTanh[c*x]^2)/c + (b*d*Log[1 - c^2*x^2])/(2*c) - (b*e*Lo
g[1 - c^2*x^2])/c + a*e*x*Log[1 - c^2*x^2] + b*e*x*ArcTanh[c*x]*Log[1 - c^2
*x^2] + (b*e*Log[1 - c^2*x^2]^2)/(4*c)
```

Maple [C] time = 0.611, size = 2529, normalized size = 24.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1)),x)
```

```
[Out] 1/2*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*x*e+1/2*I*b*arctanh
(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*Pi*x*e+1
/2*I*b*arctanh(c*x)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*Pi*x*e+1/2*I/c*b
*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^3*Pi*e*arctanh(c*x)-1/2*I/c*b*ln((c*x
+1)^2/(-c^2*x^2+1)+1)*Pi*e*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3-1/2*I/c*b*Pi*ln(
(c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x
^2+1)+1)^2)^3-1/2*I/c*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)+1)^2)^3+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*Pi*e*arctan
h(c*x)+1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)
^3*Pi*e*arctanh(c*x)-1/2*I/c*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*Pi*e*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-1/2*I/c*b*ln((c*x+
1)^2/(-c^2*x^2+1)+1)*Pi*e*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+
1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/2*I/c*b*ln((c*x+1)^2/(-c
^2*x^2+1)+1)*Pi*e*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-
c^2*x^2+1)+1)^2)-a*e/c*ln(c*x-1)+a*e/c*ln(c*x+1)-2*a*x*e+1/2*I/c*b*ln((c*x+
1)^2/(-c^2*x^2+1)+1)*e*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c
^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2+2*b*arctanh(c*x)*ln(2)*x*e-2*b*ar
ctanh(c*x)*ln((c*x+1)^2/(-c^2*x^2+1)+1)*x*e+2*ln((c*x+1)/(-c^2*x^2+1)^(1/2)
)*(arctanh(c*x)*x*c+arctanh(c*x)-ln((c*x+1)^2/(-c^2*x^2+1)+1))*b*e/c+a*x*d+
2/c*b*ln(2)*e*arctanh(c*x)-2/c*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*ln(2)*e-I/c*b
*ln((c*x+1)^2/(-c^2*x^2+1)+1)*Pi*e*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(
I*(c*x+1)^2/(c^2*x^2-1))^2+1/c*b*e*ln((c*x+1)^2/(-c^2*x^2+1)+1)^2+1/c*b*d*a
rctanh(c*x)-2/c*b*e*arctanh(c*x)-1/c*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*d+2/c*b
*e*ln((c*x+1)^2/(-c^2*x^2+1)+1)+b*arctanh(c*x)*x*d+a*x*e*ln(-c^2*x^2+1)-I/c
*b*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)
)*Pi*e*arctanh(c*x)-1/2*I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-
c^2*x^2+1)+1)^2)*Pi*x*e-1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(
-c^2*x^2+1)+1)^2)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^
2*x^2-1))*Pi*e*arctanh(c*x)+1/2*I/c*b*Pi*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*csg
n(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x
+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)+1/2*I/c*b*csgn(I*((c*x+1)^2
/(-c^2*x^2+1)+1)^2)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi*e*arctanh(c*x)+
I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*x^2+1
)^(1/2))*Pi*x*e+I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))*Pi*e*arctanh(c*x)+I/c*b*ln((c*x+1)^2/(-c^2*x^2+1)+1)*e*Pi*csg
n(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2-1/2*
I*b*arctanh(c*x)*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)
/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*Pi*x*e+1/2*I*b*arctanh(c*x)*csgn(I*(c*x+1)
^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*x*e+1/2*I*b*arctanh
(c*x)*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c
*x+1)^2/(-c^2*x^2+1)+1)^2)^2*Pi*x*e+1/2*I*b*arctanh(c*x)*csgn(I*((c*x+1)^2/
(-c^2*x^2+1)+1))^2*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2)*Pi*x*e-I*b*arctanh(
c*x)*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)+1)^2
)^2*Pi*x*e-1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1
)^2)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi*e*arctanh(c*x)+1/2*I/c*b*csgn(I*(c*
x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*Pi*e*arctanh(c*x)+
1/2*I/c*b*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)^2)^2*csgn
(I/((c*x+1)^2/(-c^2*x^2+1)+1)^2)*Pi*e*arctanh(c*x)-2*b*e*x*arctanh(c*x)
```

Maxima [C] time = 0.971981, size = 240, normalized size = 2.31

$$-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\log\left(-c^2x^2+1\right)\right)be\,\mathrm{artanh}(cx)-\left(c^2\left(\frac{2x}{c^2}-\frac{\log(cx+1)}{c^3}+\frac{\log(cx-1)}{c^3}\right)-x\,\mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="maxima")
```

```
[Out] -(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - x*log(-c^2*x^2 + 1)
)*b*e*arctanh(c*x) - (c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) -
 x*log(-c^2*x^2 + 1))*a*e + a*d*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2
+ 1))*b*d/c + 1/2*((I*pi + 2*log(c*x - 1) - 2)*log(c*x + 1) + (I*pi - 2)*lo
g(c*x - 1))*b*e/c
```

Fricas [A] time = 2.24459, size = 306, normalized size = 2.94

$$\frac{be\log\left(-c^2x^2+1\right)^2+be\log\left(-\frac{cx+1}{cx-1}\right)^2+4\,(acd-2\,ace)x+2\,(2\,acex+bd-2\,be)\log\left(-c^2x^2+1\right)+2\,\left(bcex\log\left(-c^2x^2+1\right)\right.}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="fricas")
```

```
[Out] 1/4*(b*e*log(-c^2*x^2 + 1)^2 + b*e*log(-(c*x + 1)/(c*x - 1))^2 + 4*(a*c*d -
 2*a*c*e)*x + 2*(2*a*c*e*x + b*d - 2*b*e)*log(-c^2*x^2 + 1) + 2*(b*c*e*x*lo
g(-c^2*x^2 + 1) + 2*a*e + (b*c*d - 2*b*c*e)*x)*log(-(c*x + 1)/(c*x - 1)))/c
```

Sympy [A] time = 4.48352, size = 148, normalized size = 1.42

$$\begin{cases} adx+aex\log\left(-c^2x^2+1\right)-2aex+\frac{2ae\,\mathrm{atanh}(cx)}{c}+bdx\,\mathrm{atanh}(cx)+bex\log\left(-c^2x^2+1\right)\mathrm{atanh}(cx)-2bex\,\mathrm{atanh}(cx)+ \\ adx \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x*log(-c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atanh(c*x)/c
 + b*d*x*atanh(c*x) + b*e*x*log(-c**2*x**2 + 1)*atanh(c*x) - 2*b*e*x*atanh(
c*x) + b*d*log(-c**2*x**2 + 1)/(2*c) + b*e*log(-c**2*x**2 + 1)**2/(4*c) - b
*e*log(-c**2*x**2 + 1)/c + b*e*atanh(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

Giac [B] time = 1.19316, size = 301, normalized size = 2.89

$$\frac{bcxe\log(cx+1)^2-bcxe\log(-cx+1)^2+2\,acxe\log(cx+1)-2\,bcxe\log(cx+1)+2\,acxe\log(-cx+1)+2\,bcxe\log(-cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1)),x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x*e*log(c*x + 1)^2 - b*c*x*e*log(-c*x + 1)^2 + 2*a*c*x*e*log(c*x +
 1) - 2*b*c*x*e*log(c*x + 1) + 2*a*c*x*e*log(-c*x + 1) + 2*b*c*x*e*log(-c*x
 + 1) + b*c*d*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*d*x - 4*a*c*x*e + b*e*log
(c*x + 1)^2 - b*e*log(c*x - 1)^2 + 2*b*e*log(c*x - 1)*log(-c*x + 1) + b*d*l
og(c^2*x^2 - 1) + 2*a*e*log(c*x + 1) - 2*b*e*log(c*x + 1) - 2*a*e*log(c*x -
 1) - 2*b*e*log(c*x - 1))/c
```

3.527 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$

Optimal. Leaf size=216

$-\frac{1}{2}ae\text{PolyLog}\left(2,c^2x^2\right)+\frac{1}{2}be\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\text{PolyLog}(2,-cx)-\frac{1}{2}be\left(-\log\left(1-c^2x^2\right)+\text{l}$

```
[Out] a*d*Log[x] - (b*e*Log[c*x]*Log[1 - c*x]^2)/2 + (b*e*Log[-(c*x)]*Log[1 + c*x
]^2)/2 - (b*d*PolyLog[2, -(c*x)])/2 + (b*e*(Log[1 - c*x] + Log[1 + c*x] - L
og[1 - c^2*x^2])*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2 - (b*e*(Lo
g[1 - c*x] + Log[1 + c*x] - Log[1 - c^2*x^2])*PolyLog[2, c*x])/2 - (a*e*Pol
yLog[2, c^2*x^2])/2 - b*e*Log[1 - c*x]*PolyLog[2, 1 - c*x] + b*e*Log[1 + c*
x]*PolyLog[2, 1 + c*x] + b*e*PolyLog[3, 1 - c*x] - b*e*PolyLog[3, 1 + c*x]
```

Rubi [A] time = 0.280181, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6079, 5912, 6077, 2391, 6075, 2396, 2433, 2374, 6589}

$-\frac{1}{2}ae\text{PolyLog}\left(2,c^2x^2\right)+\frac{1}{2}be\left(-\log\left(1-c^2x^2\right)+\log(1-cx)+\log(cx+1)\right)\text{PolyLog}(2,-cx)-\frac{1}{2}be\left(-\log\left(1-c^2x^2\right)+\text{l}$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```


```
[Out] a*d*Log[x] - (b*e*Log[c*x]*Log[1 - c*x]^2)/2 + (b*e*Log[-(c*x)]*Log[1 + c*x
]^2)/2 - (b*d*PolyLog[2, -(c*x)])/2 + (b*e*(Log[1 - c*x] + Log[1 + c*x] - L
og[1 - c^2*x^2])*PolyLog[2, -(c*x)])/2 + (b*d*PolyLog[2, c*x])/2 - (b*e*(Lo
g[1 - c*x] + Log[1 + c*x] - Log[1 - c^2*x^2])*PolyLog[2, c*x])/2 - (a*e*Pol
yLog[2, c^2*x^2])/2 - b*e*Log[1 - c*x]*PolyLog[2, 1 - c*x] + b*e*Log[1 + c*
x]*PolyLog[2, 1 + c*x] + b*e*PolyLog[3, 1 - c*x] - b*e*PolyLog[3, 1 + c*x]
```


Rule 6079

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
 (d_)))/(x_), x_Symbol] :> Dist[d, Int[(a + b*ArcTanh[c*x])/x, x], x] + Dis
t[e, Int[(Log[f + g*x^2]*(a + b*ArcTanh[c*x]))/x, x], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 6077

```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTanh[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_
Symbol] :> Dist[a, Int[Log[f + g*x^2]/x, x], x] + Dist[b, Int[(Log[f + g*x^
2]*ArcTanh[c*x])/x, x], x] /; FreeQ[{a, b, c, f, g}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6075

```
Int[(ArcTanh[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Dist
[Log[f + g*x^2] - Log[1 - c*x] - Log[1 + c*x], Int[ArcTanh[c*x]/x, x], x] +
 (-Dist[1/2, Int[Log[1 - c*x]^2/x, x], x] + Dist[1/2, Int[Log[1 + c*x]^2/x,
 x], x]) /; FreeQ[{c, f, g}, x] && EqQ[c^2*f + g, 0]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx = d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx + e\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(1-c^2x^2\right)}{x}\,dx$$

$$= ad\log(x) - \frac{1}{2}bd\mathrm{Li}_2(-cx) + \frac{1}{2}bd\mathrm{Li}_2(cx) + (ae)\int \frac{\log\left(1-c^2x^2\right)}{x}\,dx +$$

$$= ad\log(x) - \frac{1}{2}bd\mathrm{Li}_2(-cx) + \frac{1}{2}bd\mathrm{Li}_2(cx) - \frac{1}{2}ae\mathrm{Li}_2\left(c^2x^2\right) - \frac{1}{2}(be)\int$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1+cx) -$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1+cx) -$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1+cx) -$$

$$= ad\log(x) - \frac{1}{2}be\log(cx)\log^2(1-cx) + \frac{1}{2}be\log(-cx)\log^2(1+cx) -$$

Mathematica [F] time = 0.237389, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x, x]
```

Maple [C] time = 1.191, size = 1638, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x,x)
```

```
[Out] -(1/2*I*Pi*b*e*csgn(I*(c*x-1))^2-1/2*I*Pi*b*e*csgn(I*(c*x-1)*(c*x+1))^2-1/2
*I*Pi*b*e*csgn(I*(c*x-1))^3-1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*cs
gn(I*(c*x-1)*(c*x+1))+1/4*I*Pi*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^
2+1/4*I*Pi*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*b*e*csgn(
I*(c*x-1)*(c*x+1))^3+a*e+1/2*b*d)*dilog(c*x+1)-I*Pi*ln(c*x)*ln(c*x-1)*b*e+d
ilog(c*x)*a*e-1/2*dilog(c*x)*b*d+ln(c*x)*a*d+1/2*b*e*ln(-c*x)*ln(c*x+1)^2+b
*e*ln(c*x+1)*polylog(2,c*x+1)-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1)
)*csgn(I*(c*x-1)*(c*x+1))^2-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x+1))*
csgn(I*(c*x-1)*(c*x+1))^2+1/4*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))*csgn(I*(c
*x+1))*csgn(I*(c*x-1)*(c*x+1))+b*e*polylog(3,-c*x+1)-b*e*polylog(3,c*x+1)-1
/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(c*x+1))^3+1/4*Pi^2*ln
(c*x)*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x-1)*(c*x+1))^3+1/2*Pi^2*ln(c*x)*b*e*
csgn(I*(c*x-1))^2*csgn(I*(c*x-1)*(c*x+1))^2-3/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*
x-1))^3*csgn(I*(c*x-1)*(c*x+1))^2-1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^2*cs
gn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/2*Pi^2*ln(c*x)*b*e+1/4*Pi^2*ln(c*
x)*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+ln(c*x)*
ln(c*x-1)*a*e-1/2*ln(c*x)*ln(c*x-1)*b*d-1/2*ln(c*x)*ln(c*x-1)^2*b*e-ln(c*x-
1)*polylog(2,-c*x+1)*b*e-1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1))*csgn(I*(c*x+1
))*csgn(I*(c*x-1)*(c*x+1))-1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1)*(c
*x+1))^3+1/2*I*Pi*ln(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^3-1/4*I*Pi*dilog(c*x)
*b*e*csgn(I*(c*x-1)*(c*x+1))^3+1/4*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1
))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))-1/2*I*Pi*ln(c*x)*b*d-I*Pi*dilog(
c*x)*b*e-1/2*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^2-1/2*Pi^2*ln(c*x)*b*e*csgn(I
*(c*x-1)*(c*x+1))^2+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1)*(c*x+1))^3-1/4*Pi^2
*ln(c*x)*b*e*csgn(I*(c*x-1))*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))+1/2*I*
Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1))^2+1/2*I*Pi*ln(c*x)*ln(c*x-1)*b*e*c
sgn(I*(c*x-1)*(c*x+1))^2-1/2*I*Pi*ln(c*x)*ln(c*x-1)*b*e*csgn(I*(c*x-1))^3+1
/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^3*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1
))-1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^4*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c
*x+1))-1/4*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2-1/
4*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*ln
(c*x)*a*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*ln(c*x)*a*e*cs
gn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/2*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1)
)^3+I*Pi*ln(c*x)*a*e+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))*csgn(I*(c*x-1)*(c
*x+1))^2+1/4*Pi^2*ln(c*x)*b*e*csgn(I*(c*x+1))*csgn(I*(c*x-1)*(c*x+1))^2+1/4
*Pi^2*ln(c*x)*b*e*csgn(I*(c*x-1))^4*csgn(I*(c*x-1)*(c*x+1))^2-1/2*I*Pi*ln(c
*x)*b*d*csgn(I*(c*x-1))^3-1/2*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))^3-I*Pi*ln
```

```
(c*x)*a*e*csgn(I*(c*x-1)*(c*x+1))^2+1/2*I*Pi*ln(c*x)*b*d*csgn(I*(c*x-1))^2+
1/2*I*Pi*dilog(c*x)*b*e*csgn(I*(c*x-1))^2+1/2*I*Pi*dilog(c*x)*b*e*csgn(I*(c
*x-1)*(c*x+1))^2
```

Maxima [A] time = 1.46246, size = 205, normalized size = 0.95

$$-\frac{1}{2}\left(\log(cx)\log(-cx+1)^2+2\,\mathrm{Li}_2(-cx+1)\log(-cx+1)-2\,\mathrm{Li}_3(-cx+1)\right)be+\frac{1}{2}\left(\log(cx+1)^2\log(-cx)+2\,\mathrm{Li}_2(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(log(c*x)*log(-c*x + 1)^2 + 2*dilog(-c*x + 1)*log(-c*x + 1) - 2*polylo
g(3, -c*x + 1))*b*e + 1/2*(log(c*x + 1)^2*log(-c*x) + 2*dilog(c*x + 1)*log(
c*x + 1) - 2*polylog(3, c*x + 1))*b*e + a*d*log(x) - 1/2*(b*d - 2*a*e)*(log
(c*x)*log(-c*x + 1) + dilog(-c*x + 1)) + 1/2*(b*d + 2*a*e)*(log(c*x + 1)*lo
g(-c*x) + dilog(c*x + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{artanh}(cx)+ad+(be\,\mathrm{artanh}(cx)+ae)\log\left(-c^2x^2+1\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(a+b\,\mathrm{atanh}(cx))\left(d+e\log\left(-c^2x^2+1\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b\,\mathrm{artanh}(cx)+a)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x, x)
```

3.528 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}+\frac{1}{2}bc\log\left(1-\frac{1}{1-c^2x\right.}$$

```
[Out] -((c*e*(a + b*ArcTanh[c*x])^2)/b) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^
2*x^2]))/x + (b*c*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/2 -
 (b*c*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/2
```

Rubi [A] time = 0.255908, antiderivative size = 94, normalized size of antiderivative = 0.9, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6081, 2475, 2411, 2344, 2301, 2316, 2315, 5948}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{x}-\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{bc\left(e\log\left(1-c^2x^2\right)+d\right)^2}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -((c*e*(a + b*ArcTanh[c*x])^2)/b) + b*c*d*Log[x] - ((a + b*ArcTanh[c*x])*(d
 + e*Log[1 - c^2*x^2]))/x - (b*c*(d + e*Log[1 - c^2*x^2])^2)/(4*e) - (b*c*e
*PolyLog[2, c^2*x^2])/2
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}+(bc)\int \frac{d+e\log\left(1-c^{2}\right.}{x\left(1-c^2x^2\right)}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}+bcd\log(x)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.}{x}$$

$$= -\frac{ce\left(a+b\tanh^{-1}(cx)\right)^2}{b}+bcd\log(x)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\right.}{x}$$

Mathematica [B] time = 0.208058, size = 332, normalized size = 3.16

$$-\frac{4bcex\text{PolyLog}(2,-cx)+4bcex\text{PolyLog}(2,cx)-2bcex\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)-2bcex\text{PolyLog}\left(2,\frac{1}{2}(cx+1)\right)+4ae\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^2,x]
```

```
[Out] -(4*a*d + 4*b*d*ArcTanh[c*x] + 8*a*c*e*x*ArcTanh[c*x] + 4*b*c*e*x*ArcTanh[c
*x]^2 - 4*b*c*d*x*Log[x] - b*c*e*x*Log[-c^(-1) + x]^2 - b*c*e*x*Log[c^(-1)
+ x]^2 - 2*b*c*e*x*Log[c^(-1) + x]*Log[(1 - c*x)/2] + 4*b*c*e*x*Log[x]*Log[
1 - c*x] - 2*b*c*e*x*Log[-c^(-1) + x]*Log[(1 + c*x)/2] + 4*b*c*e*x*Log[x]*L
og[1 + c*x] + 4*a*e*Log[1 - c^2*x^2] + 2*b*c*d*x*Log[1 - c^2*x^2] + 4*b*e*A
rcTanh[c*x]*Log[1 - c^2*x^2] - 4*b*c*e*x*Log[x]*Log[1 - c^2*x^2] + 2*b*c*e*
x*Log[-c^(-1) + x]*Log[1 - c^2*x^2] + 2*b*c*e*x*Log[c^(-1) + x]*Log[1 - c^2
*x^2] + 4*b*c*e*x*PolyLog[2, -(c*x)] + 4*b*c*e*x*PolyLog[2, c*x] - 2*b*c*e*
x*PolyLog[2, 1/2 - (c*x)/2] - 2*b*c*e*x*PolyLog[2, (1 + c*x)/2])/(4*x)
```

Maple [F] time = 2.427, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{Artanh}(cx))\left(d + e\ln\left(-c^2x^2 + 1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2 - 1\right) - \log\left(x^2\right)\right) + \frac{2\operatorname{artanh}(cx)}{x}\right)bd - \left(c^2\left(\frac{\log(cx + 1)}{c} - \frac{\log(cx - 1)}{c}\right) + \frac{\log\left(-c^2x^2 + 1\right)}{x}\right)ae + \frac{1}{2}be\Big(\frac{\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b*d - (c^2*(log(c
*x + 1)/c - log(c*x - 1)/c) + log(-c^2*x^2 + 1)/x)*a*e + 1/2*b*e*(log(-c*x
+ 1)^2/x - integrate(-((c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x
^3 - x^2), x)) - a*d/x
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bd\operatorname{artanh}(cx) + ad + (be\operatorname{artanh}(cx) + ae)\log\left(-c^2x^2 + 1\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{atanh}(cx))\left(d + e\log\left(-c^2x^2 + 1\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\operatorname{artanh}(cx) + a)\left(e\log\left(-c^2x^2 + 1\right) + d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^2, x)
```

3.529 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=157

$$\frac{1}{2}bc^2e\text{PolyLog}(2,-cx)-\frac{1}{2}bc^2e\text{PolyLog}(2,cx)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}c^2e(a+b)\log(1-cx)+\frac{1}{2}$$

```
[Out] -(a*c^2*e*Log[x]) + ((a + b)*c^2*e*Log[1 - c*x])/2 + ((a - b)*c^2*e*Log[1 +
 c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d +
e*Log[1 - c^2*x^2]))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(2
*x^2) + (b*c^2*e*PolyLog[2, -(c*x)])/2 - (b*c^2*e*PolyLog[2, c*x])/2
```

Rubi [A] time = 0.140202, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5916, 325, 206, 6085, 801, 5912}

$$\frac{1}{2}bc^2e\text{PolyLog}(2,-cx)-\frac{1}{2}bc^2e\text{PolyLog}(2,cx)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{2x^2}+\frac{1}{2}c^2e(a+b)\log(1-cx)+\frac{1}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] -(a*c^2*e*Log[x]) + ((a + b)*c^2*e*Log[1 - c*x])/2 + ((a - b)*c^2*e*Log[1 +
 c*x])/2 - (b*c*(d + e*Log[1 - c^2*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d +
e*Log[1 - c^2*x^2]))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(2
*x^2) + (b*c^2*e*PolyLog[2, -(c*x)])/2 - (b*c^2*e*PolyLog[2, c*x])/2
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
```

```
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(1-c^2x^2\right)\right)$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(1-c^2x^2\right)\right)$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(1-c^2x^2\right)\right)$$

$$= -ac^2e\log(x)+\frac{1}{2}(a+b)c^2e\log(1-cx)+\frac{1}{2}(a-b)c^2e\log(1+cx)-$$

Mathematica [A] time = 0.156598, size = 152, normalized size = 0.97

$$\frac{1}{2}\left(bc^2e(\text{PolyLog}(2,-cx)-\text{PolyLog}(2,cx))-\frac{e\log\left(1-c^2x^2\right)\left(a+\left(b-bc^2x^2\right)\tanh^{-1}(cx)+bcx\right)}{x^2}+c^2e(a+b)\log(1-\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^3,x]
```

```
[Out] (-((a*d)/x^2) - 2*a*c^2*e*Log[x] + (a + b)*c^2*e*Log[1 - c*x] + (a - b)*c^2
*e*Log[1 + c*x] - (b*d*(2*ArcTanh[c*x] + c*x*(2 + c*x*Log[1 - c*x] - c*x*Lo
g[1 + c*x])))/(2*x^2) - (e*(a + b*c*x + (b - b*c^2*x^2)*ArcTanh[c*x])*Log[1
 - c^2*x^2])/x^2 + b*c^2*e*(PolyLog[2, -(c*x)] - PolyLog[2, c*x]))/2
```

Maple [F] time = 3.545, size = 0, normalized size = 0.

$$\int \frac{(a+b\text{Artanh}\,(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}\left(\left(c\log\left(cx+1\right)-c\log\left(cx-1\right)-\frac{2}{x}\right)c-\frac{2\operatorname{artanh}\left(cx\right)}{x^{2}}\right)bd+\frac{1}{2}\left(c^{2}\left(\log\left(c^{2}x^{2}-1\right)-\log\left(x^{2}\right)\right)-\frac{\log\left(-c^{2}x^{2}+1\right)}{x^{2}}\right)ae+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b*d +
1/2*(c^2*(log(c^2*x^2 - 1) - log(x^2)) - log(-c^2*x^2 + 1)/x^2)*a*e + 1/4*b
*e*(log(-c*x + 1)^2/x^2 - 2*integrate(-((c*x - 1)*log(c*x + 1)^2 - c*x*log(
-c*x + 1))/(c*x^4 - x^3), x)) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\operatorname{artanh}\left(cx\right)+ad+\left(be\operatorname{artanh}\left(cx\right)+ae\right)\log\left(-c^{2}x^{2}+1\right)}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\operatorname{atanh}\left(cx\right)\right)\left(d+e\log\left(-c^{2}x^{2}+1\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\operatorname{artanh}\left(cx\right)+a\right)\left(e\log\left(-c^{2}x^{2}+1\right)+d\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^3, x)
```

3.530 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^4}\,dx$

Optimal. Leaf size=197

$$-\frac{1}{6}bc^3e\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)-\frac{(a+b\tanh^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{2c^2e\left(a+b\tan\right.}{3x}$$

```
[Out] (2*c^2*e*(a + b*ArcTanh[c*x]))/(3*x) - (c^3*e*(a + b*ArcTanh[c*x])^2)/(3*b)
 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*c*(1 - c^2*x^2)*(d +
e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/(3*x^3) + (b*c^3*(d + e*Log[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/
6 - (b*c^3*e*PolyLog[2, (1 - c^2*x^2)^(-1)])/6
```

Rubi [A] time = 0.46461, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.593, Rules used = {6081, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 5982, 5916, 266, 36, 29, 5948}

$$-\frac{1}{6}bc^3e\text{PolyLog}\left(2,c^2x^2\right)-\frac{(a+b\tanh^{-1}(cx))\left(e\log\left(1-c^2x^2\right)+d\right)}{3x^3}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{2c^2e\left(a+b\tanh^{-1}\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] (2*c^2*e*(a + b*ArcTanh[c*x]))/(3*x) - (c^3*e*(a + b*ArcTanh[c*x])^2)/(3*b)
 + (b*c^3*d*Log[x])/3 - b*c^3*e*Log[x] + (b*c^3*e*Log[1 - c^2*x^2])/3 - (b*
c*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(6*x^2) - ((a + b*ArcTanh[c*x])*(
d + e*Log[1 - c^2*x^2]))/(3*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2])^2)/(12*e
) - (b*c^3*e*PolyLog[2, c^2*x^2])/6
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

```
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^4}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{3}(bc)\int\frac{d+e\log\left(1-\right.}{x^3\left(1-c^2x\right.}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{3x^3}+\frac{1}{6}(bc)\,\text{Subst}\left(\int\frac{d+e}{x^2}\right.$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\tanh^{-1}(\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{\left(a+b\tanh^{-1}(\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}-\frac{bc\left(1-c^2x^2\right)\left(\right.}{}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3d\log(x)-$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{3x}-\frac{c^3e\left(a+b\tanh^{-1}(cx)\right)^2}{3b}+\frac{1}{3}bc^3d\log(x)-$$

Mathematica [B] time = 0.384417, size = 460, normalized size = 2.34

$$\frac{1}{6}\left(-2bc^3e\text{PolyLog}(2,-cx)-2bc^3e\text{PolyLog}(2,cx)+bc^3e\text{PolyLog}\left(2,\frac{1}{2}-\frac{cx}{2}\right)+bc^3e\text{PolyLog}\left(2,\frac{1}{2}(cx+1)\right)-\frac{2ae\,\mathrm{l}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^4,x]
```

```
[Out] ((-2*a*d)/x^3 - (b*c*d)/x^2 + (4*a*c^2*e)/x - 4*a*c^3*e*ArcTanh[c*x] - (2*b
*d*ArcTanh[c*x])/x^3 + (4*b*c^2*e*ArcTanh[c*x])/x - 2*b*c^3*e*ArcTanh[c*x]^
2 + 2*b*c^3*d*Log[x] - 2*b*c^3*e*Log[x] + (b*c^3*e*Log[-c^(-1) + x]^2)/2 +
(b*c^3*e*Log[c^(-1) + x]^2)/2 + b*c^3*e*Log[c^(-1) + x]*Log[(1 - c*x)/2] -
2*b*c^3*e*Log[x]*Log[1 - c*x] + b*c^3*e*Log[-c^(-1) + x]*Log[(1 + c*x)/2] -
 2*b*c^3*e*Log[x]*Log[1 + c*x] - 4*b*c^3*e*Log[(c*x)/Sqrt[1 - c^2*x^2]] - b
*c^3*d*Log[1 - c^2*x^2] + b*c^3*e*Log[1 - c^2*x^2] - (2*a*e*Log[1 - c^2*x^2
])/x^3 - (b*c*e*Log[1 - c^2*x^2])/x^2 - (2*b*e*ArcTanh[c*x]*Log[1 - c^2*x^2
])/x^3 + 2*b*c^3*e*Log[x]*Log[1 - c^2*x^2] - b*c^3*e*Log[-c^(-1) + x]*Log[1
 - c^2*x^2] - b*c^3*e*Log[c^(-1) + x]*Log[1 - c^2*x^2] - 2*b*c^3*e*PolyLog[
2, -(c*x)] - 2*b*c^3*e*PolyLog[2, c*x] + b*c^3*e*PolyLog[2, 1/2 - (c*x)/2]
+ b*c^3*e*PolyLog[2, (1 + c*x)/2])/6
```

Maple [F] time = 5.205, size = 0, normalized size = 0.

$$\int \frac{(a + b\operatorname{Artanh}(cx))\left(d + e\ln\left(-c^2x^2 + 1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\left(\left(c^2\log\left(c^2x^2-1\right)-c^2\log\left(x^2\right)+\frac{1}{x^2}\right)c+\frac{2\operatorname{artanh}(cx)}{x^3}\right)bd-\frac{1}{3}\left(\left(c\log(cx+1)-c\log(cx-1)-\frac{2}{x}\right)c^2+\frac{\log\left(-c^2x^2\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="maxima
")
```

```
[Out] -1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*b*d - 1/3*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c^2 + log(-c^2*x^2 + 1)
/x^3)*a*e + 1/6*b*e*(log(-c*x + 1)^2/x^3 - 3*integrate(-1/3*(3*(c*x - 1)*lo
g(c*x + 1)^2 - 2*c*x*log(-c*x + 1))/(c*x^5 - x^4), x)) - 1/3*a*d/x^3
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bd\operatorname{artanh}(cx)+ad+(be\operatorname{artanh}(cx)+ae)\log\left(-c^2x^2+1\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))\left(d + e\log\left(-c^2x^2 + 1\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**4,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\mathrm{artanh}\,(cx) + a)\left(e\log\left(-c^2x^2 + 1\right) + d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^4, x)
```

3.531 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx$

Optimal. Leaf size=244

$$\frac{1}{4}bc^4e\text{PolyLog}(2,-cx)-\frac{1}{4}bc^4e\text{PolyLog}(2,cx)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{12}c^4e(3a+4b)\log(1-cx)$$

```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) - (b*c^4*e*ArcTanh[c*x])/4 + (b*c^2*
e*ArcTanh[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + ((3*a + 4*b)*c^4*e*Log[1 - c
*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 - (b*c*(d + e*Log[1 - c^2*x^2
]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/(4*x) + (b*c^4*ArcTanh[c*x]
*(d + e*Log[1 - c^2*x^2]))/4 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/(4*x^4) + (b*c^4*e*PolyLog[2, -(c*x)])/4 - (b*c^4*e*PolyLog[2, c*x])/4
```

Rubi [A] time = 0.260507, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5916, 325, 206, 6085, 1802, 6044, 5912}

$$\frac{1}{4}bc^4e\text{PolyLog}(2,-cx)-\frac{1}{4}bc^4e\text{PolyLog}(2,cx)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{4x^4}+\frac{1}{12}c^4e(3a+4b)\log(1-cx)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```


```
[Out] (a*c^2*e)/(4*x^2) + (5*b*c^3*e)/(12*x) - (b*c^4*e*ArcTanh[c*x])/4 + (b*c^2*
e*ArcTanh[c*x])/(4*x^2) - (a*c^4*e*Log[x])/2 + ((3*a + 4*b)*c^4*e*Log[1 - c
*x])/12 + ((3*a - 4*b)*c^4*e*Log[1 + c*x])/12 - (b*c*(d + e*Log[1 - c^2*x^2
]))/(12*x^3) - (b*c^3*(d + e*Log[1 - c^2*x^2]))/(4*x) + (b*c^4*ArcTanh[c*x]
*(d + e*Log[1 - c^2*x^2]))/4 - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2
]))/(4*x^4) + (b*c^4*e*PolyLog[2, -(c*x)])/4 - (b*c^4*e*PolyLog[2, c*x])/4
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6044

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTanh[c*
x])^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && (GtQ[q, 0] || IntegerQ[m])
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^5}\,dx = -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3} - \frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x} + \frac{1}{4}bc^4\tanh^{-}$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3} - \frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x} + \frac{1}{4}bc^4\tanh^{-}$$

$$= -\frac{bc\left(d+e\log\left(1-c^2x^2\right)\right)}{12x^3} - \frac{bc^3\left(d+e\log\left(1-c^2x^2\right)\right)}{4x} + \frac{1}{4}bc^4\tanh^{-}$$

$$= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3a+4b)c^4e\log(1-cx) + \frac{1}{12}(3a-$$

$$= \frac{ac^2e}{4x^2} + \frac{bc^3e}{6x} + \frac{bc^2e\tanh^{-1}(cx)}{4x^2} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3a+4b)c^4e\log$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} + \frac{bc^2e\tanh^{-1}(cx)}{4x^2} - \frac{1}{2}ac^4e\log(x) + \frac{1}{12}(3a+4b)c^4e\,\mathrm{lo}$$

$$= \frac{ac^2e}{4x^2} + \frac{5bc^3e}{12x} - \frac{1}{4}bc^4e\tanh^{-1}(cx) + \frac{bc^2e\tanh^{-1}(cx)}{4x^2} - \frac{1}{2}ac^4e\log(x)$$

Mathematica [A] time = 0.15077, size = 299, normalized size = 1.23

$$-\frac{1}{4}bc^4e(\mathrm{PolyLog}(2,cx)-\mathrm{PolyLog}(2,-cx)) + \frac{e\log\left(1-c^2x^2\right)\left(-3a-3bc^3x^3+3bc^4x^4\tanh^{-1}(cx)-bcx-3b\tanh^{-1}(\right.}{12x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^5,x]
```

```
[Out] -(a*d)/(4*x^4) + (a*c^2*e)/(4*x^2) + (b*c^3*e)/(6*x) - (a*c^4*e*Log[x])/2 +
 ((3*a*c^4*e + 4*b*c^4*e)*Log[1 - c*x])/12 - (b*c^4*e*(-ArcTanh[c*x]/(2*c^2
```

```
*x^2) + (-(1/(c*x)) - Log[1 - c*x]/2 + Log[1 + c*x]/2)/2))/2 + b*c^4*d*(-Ar
cTanh[c*x]/(4*c^4*x^4) + (-1/(3*c^3*x^3) - 1/(c*x) - Log[1 - c*x]/2 + Log[1
 + c*x]/2)/4) + ((3*a*c^4*e - 4*b*c^4*e)*Log[1 + c*x])/12 + (e*(-3*a - b*c*
x - 3*b*c^3*x^3 - 3*b*ArcTanh[c*x] + 3*b*c^4*x^4*ArcTanh[c*x])*Log[1 - c^2*
x^2])/(12*x^4) - (b*c^4*e*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/4
```

Maple [F] time = 14.305, size = 0, normalized size = 0.

$$\int \frac{(a + b\text{Artanh}\,(cx))\left(d + e\ln\left(-c^2x^2 + 1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{24}\left(\left(3\,c^3\log\left(cx+1\right) - 3\,c^3\log\left(cx-1\right) - \frac{2\left(3\,c^2x^2+1\right)}{x^3}\right)c - \frac{6\,\text{artanh}\,(cx)}{x^4}\right)bd + \frac{1}{4}\left(\left(c^2\log\left(c^2x^2-1\right) - c^2\log\left(x^2\right) + \frac{1}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="maxima
")
```

```
[Out] 1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*b*d + 1/4*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^
2)*c^2 - log(-c^2*x^2 + 1)/x^4)*a*e + 1/8*b*e*(log(-c*x + 1)^2/x^4 - 4*inte
grate(-1/2*(2*(c*x - 1)*log(c*x + 1)^2 - c*x*log(-c*x + 1))/(c*x^6 - x^5),
x)) - 1/4*a*d/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx) + ad + (be\,\text{artanh}\,(cx) + ae)\log\left(-c^2x^2 + 1\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\text{atanh}\,(cx))\left(d + e\log\left(-c^2x^2 + 1\right)\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)\left(e \log\left(-c^2x^2 + 1\right) + d\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^5, x)
```

3.532 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(1-c^2x^2))}{x^6}\,dx$

Optimal. Leaf size=256

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,\frac{1}{1-c^2x^2}\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}-\frac{c^5e\left(a+b\tanh^{-}\right.}{5b}$$

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcTanh[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTanh[c*x]))/(5*x) - (c^5*e*(a + b*ArcTanh[c*x])^2)/(5*b) - (5*b*c^
5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*c*(d + e*Log[1 - c^2*
x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 - c^2*x^2]))/(10*x^2) -
 ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^5) + (b*c^5*(d + e*Lo
g[1 - c^2*x^2])*Log[1 - (1 - c^2*x^2)^(-1)])/10 - (b*c^5*e*PolyLog[2, (1 -
c^2*x^2)^(-1)])/10
```

Rubi [A] time = 0.664424, antiderivative size = 250, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 18, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6081, 2475, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 5982, 5916, 266, 36, 29, 5948}

$$-\frac{1}{10}bc^5e\text{PolyLog}\left(2,c^2x^2\right)-\frac{\left(a+b\tanh^{-1}(cx)\right)\left(e\log\left(1-c^2x^2\right)+d\right)}{5x^5}+\frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3}-\frac{c^5e\left(a+b\tanh^{-1}(cx\right.}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] (7*b*c^3*e)/(60*x^2) + (2*c^2*e*(a + b*ArcTanh[c*x]))/(15*x^3) + (2*c^4*e*(
a + b*ArcTanh[c*x]))/(5*x) - (c^5*e*(a + b*ArcTanh[c*x])^2)/(5*b) + (b*c^5*
d*Log[x])/5 - (5*b*c^5*e*Log[x])/6 + (19*b*c^5*e*Log[1 - c^2*x^2])/60 - (b*
c*(d + e*Log[1 - c^2*x^2]))/(20*x^4) - (b*c^3*(1 - c^2*x^2)*(d + e*Log[1 -
c^2*x^2]))/(10*x^2) - ((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/(5*x^
5) - (b*c^5*(d + e*Log[1 - c^2*x^2])^2)/(20*e) - (b*c^5*e*PolyLog[2, c^2*x^
2])/10
```

Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
 b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e
*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m +
 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f,
g}, x] && ILtQ[m/2, 0]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x
, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[
{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))),
 x_Symbol] :> Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[
(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I
GtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5} + \frac{1}{5}(bc)\int \frac{d+e\log\left(1-\right.}{x^5\left(1-c^2x\right.}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5} + \frac{1}{10}(bc)\,\text{Subst}\left(\int \frac{d+e}{x}\right.$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{5x^5}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3} + \frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x} - \frac{c^5e\left(a+b\tan\right.}{5b}$$

$$= \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3} + \frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x} - \frac{c^5e\left(a+b\tan\right.}{5b}$$

$$= \frac{bc^3e}{15x^2} + \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3} + \frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x} - \frac{c^5e\left(a\right.}{}$$

$$= \frac{7bc^3e}{60x^2} + \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3} + \frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x} - \frac{c^5e\left(a\right.}{}$$

$$= \frac{7bc^3e}{60x^2} + \frac{2c^2e\left(a+b\tanh^{-1}(cx)\right)}{15x^3} + \frac{2c^4e\left(a+b\tanh^{-1}(cx)\right)}{5x} - \frac{c^5e\left(a\right.}{}$$

Mathematica [F] time = 0.285797, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(1-c^2x^2\right)\right)}{x^6}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[1 - c^2*x^2]))/x^6, x]
```

Maple [F] time = 13.339, size = 0, normalized size = 0.

$$\int \frac{(a+b\text{Artanh}\,(cx))\left(d+e\ln\left(-c^2x^2+1\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(-c^2*x^2+1))/x^6,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{20}\left(\left(2c^4\log\left(c^2x^2-1\right)-2c^4\log\left(x^2\right)+\frac{2c^2x^2+1}{x^4}\right)c+\frac{4\,\mathrm{artanh}\left(cx\right)}{x^5}\right)bd-\frac{1}{15}\left(\left(3c^3\log\left(cx+1\right)-3c^3\log\left(cx-1\right)-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="maxima
")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
4*arctanh(c*x)/x^5)*b*d - 1/15*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) -
2*(3*c^2*x^2 + 1)/x^3)*c^2 + 3*log(-c^2*x^2 + 1)/x^5)*a*e + 1/10*b*e*(log(-
c*x + 1)^2/x^5 - 5*integrate(-1/5*(5*(c*x - 1)*log(c*x + 1)^2 - 2*c*x*log(-
c*x + 1))/(c*x^7 - x^6), x)) - 1/5*a*d/x^5
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{artanh}\left(cx\right)+ad+\left(be\,\mathrm{artanh}\left(cx\right)+ae\right)\log\left(-c^2x^2+1\right)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="fricas
")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(-c^2*x^2 +
1))/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+b\,\mathrm{atanh}\left(cx\right)\right)\left(d+e\log\left(-c^2x^2+1\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(-c**2*x**2+1))/x**6,x)
```

```
[Out] Integral((a + b*atanh(c*x))*(d + e*log(-c**2*x**2 + 1))/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b\,\mathrm{artanh}\left(cx\right)+a\right)\left(e\log\left(-c^2x^2+1\right)+d\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(-c^2*x^2+1))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(-c^2*x^2 + 1) + d)/x^6, x)
```

3.533 $\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx$

Optimal. Leaf size=512

$$\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\cdots\right.}{4c^2g}$$

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])
/(c*Sqrt[g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (d*x^2*(a + b*ArcTanh[c*x
]))/2 - (e*x^2*(a + b*ArcTanh[c*x]))/2 - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))]])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))]])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f + g)*ArcT
anh[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTanh[c*x])*Lo
g[f + g*x^2])/(2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))]])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))]])/(4*c^2*g)
```

Rubi [A] time = 0.762715, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 17, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.773, Rules used = {2454, 2389, 2295, 6083, 321, 207, 517, 2528, 2448, 205, 2470, 12, 5992, 5920, 2402, 2315, 2447}

$$\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4c^2g}-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\cdots\right.}{4c^2g}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] (b*(d - e)*x)/(2*c) - (b*e*x)/c + (b*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])
/(c*Sqrt[g]) - (b*(d - e)*ArcTanh[c*x])/(2*c^2) + (d*x^2*(a + b*ArcTanh[c*x
]))/2 - (e*x^2*(a + b*ArcTanh[c*x]))/2 - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[
2/(1 + c*x)])/(c^2*g) + (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] -
Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))]])/(2*c^2*g) + (b*e*(c^2*f +
g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1
 + c*x))]])/(2*c^2*g) + (b*e*x*Log[f + g*x^2])/(2*c) - (b*e*(c^2*f + g)*ArcT
anh[c*x]*Log[f + g*x^2])/(2*c^2*g) + (e*(f + g*x^2)*(a + b*ArcTanh[c*x])*Lo
g[f + g*x^2])/(2*g) + (b*e*(c^2*f + g)*PolyLog[2, 1 - 2/(1 + c*x)])/(2*c^2*
g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[
-f] - Sqrt[g])*(1 + c*x))]])/(4*c^2*g) - (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*
c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*(1 + c*x))]])/(4*c^2*g)
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 6083

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2]
), x]}, Dist[a + b*ArcTanh[c*x], u, x] - Dist[b*c, Int[ExpandIntegrand[u/(1
 - c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m + 1)
/2, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\left(a+\cdots\right.}{\cdots}$$

$$=\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e\left(f+gx^2\right)\left(a+\cdots\right.}{\cdots}$$

$$=\frac{b(d-e)x}{2c}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a+b\tanh^{-1}(cx)\right)+\frac{e\left(f\cdots\right.}{\cdots}$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}ex^2\left(a\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(a+b\tanh^{-1}(cx)\right)-\frac{1}{2}\cdots$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(\cdots\right.$$

$$=\frac{b(d-e)x}{2c}-\frac{bex}{c}+\frac{be\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{b(d-e)\tanh^{-1}(cx)}{2c^2}+\frac{1}{2}dx^2\left(\cdots\right.$$

Mathematica [C] time = 5.14817, size = 1601, normalized size = 3.13

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -(-2*b*c*d*g*x + 6*b*c*e*g*x - 2*a*c^2*d*g*x^2 + 2*a*c^2*e*g*x^2 - 4*b*c*e*
Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + 2*b*d*g*ArcTanh[c*x] - 2*b*e*
g*ArcTanh[c*x] - 2*b*c^2*d*g*x^2*ArcTanh[c*x] + 2*b*c^2*e*g*x^2*ArcTanh[c*x
] - 4*b*c^2*d*f*ArcTanh[c*x]^2 + 4*b*c^2*e*f*ArcTanh[c*x]^2 + (4*I)*b*c^2*d
*f*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] + (4
*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)
]] + 4*b*c^2*e*f*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + 4*b*e*g*ArcTan
h[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] + (2*I)*b*c^2*d*f*ArcSin[Sqrt[(c^2*f)/(
c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x])
```

```
)*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + (2*I)*b*e*g*A
rcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1
+ E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f +
g))] - 2*b*c^2*d*f*ArcTanh[c*x]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 +
 E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g
))] - 2*b*e*g*ArcTanh[c*x]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2
*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] -
 (2*I)*b*c^2*d*f*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcTa
nh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTa
nh[c*x])*(c^2*f + g))] - (2*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[
(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^
2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - 2*b*c^2*d*f*ArcTanh[c*x]*Log[(
c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2
*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] - 2*b*e*g*ArcTanh[c*x]*Log[(c^2*(
1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g + 2*Sqrt[-(c^2*f*g)
])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*d*f*ArcTanh[c*x]*Log[1 + (E^
(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g)] - 2*b*c^2
*e*f*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqr
t[-f]*Sqrt[g] - g)] + 2*b*c^2*d*f*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*
(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)] - 2*b*c^2*e*f*ArcTanh[c*x]
*Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g
)] - 2*a*c^2*e*f*Log[f + g*x^2] - 2*b*c*e*g*x*Log[f + g*x^2] - 2*a*c^2*e*g*
x^2*Log[f + g*x^2] + 2*b*e*g*ArcTanh[c*x]*Log[f + g*x^2] - 2*b*c^2*e*g*x^2*
ArcTanh[c*x]*Log[f + g*x^2] - 2*b*e*(c^2*f + g)*PolyLog[2, -E^(-2*ArcTanh[c
*x])] + b*c^2*(d - e)*f*PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*
f - 2*c*Sqrt[-f]*Sqrt[g] - g))] + b*c^2*d*f*PolyLog[2, -((E^(2*ArcTanh[c*x]
)*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))] - b*c^2*e*f*PolyLog[2,
-((E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))] + b
*c^2*d*f*PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])
*(c^2*f + g))] + b*e*g*PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2
*ArcTanh[c*x])*(c^2*f + g))] + b*c^2*d*f*PolyLog[2, (-(c^2*f) + g + 2*Sqrt[
-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + b*e*g*PolyLog[2, (-(c^2*f)
 + g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))])/(4*c^2*g)
```

Maple [C] time = 1.908, size = 10161, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bdx\operatorname{artanh}(cx)+adx+(bex\operatorname{artanh}(cx)+aex)\log\left(gx^2+f\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*x*arctanh(c*x) + a*d*x + (b*e*x*arctanh(c*x) + a*e*x)*log(g*x^
2 + f), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\operatorname{artanh}(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

3.534 $\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)\,dx$

Optimal. Leaf size=599

$$\frac{be\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)}{2c}+\frac{be\sqrt{-f}\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\text{PolyLog}\left(2,\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\text{PolyLog}\left(2,-\right.}{2\sqrt{g}}$$

```
[Out] -2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - 2*b*e*x*Ar
cTanh[c*x] + (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*S
qrt[-f] - Sqrt[g])])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*Log[1 + c*x]*Log[(c*(Sqrt[
-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*Log[
1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[g]
) - (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] +
 Sqrt[g])])/(2*Sqrt[g]) - (b*e*Log[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])
*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[
f + g*x^2]))/(2*c) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt
[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 - c*x
))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[
g]*(1 + c*x))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog
[2, (Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*PolyLog
[2, (c^2*(f + g*x^2))/(c^2*f + g)])/(2*c)
```

Rubi [A] time = 0.797311, antiderivative size = 599, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6073, 2475, 2394, 2393, 2391, 5980, 5910, 260, 5974, 205, 5972, 2409}

$$\frac{be\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)}{2c}+\frac{be\sqrt{-f}\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\sqrt{-f}\text{PolyLog}\left(2,\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{be\sqrt{-f}\text{PolyLog}\left(2,-\right.}{2\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] -2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] - 2*b*e*x*Ar
cTanh[c*x] + (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*S
qrt[-f] - Sqrt[g])])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*Log[1 + c*x]*Log[(c*(Sqrt[
-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*Log[
1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[g]
) - (b*e*Sqrt[-f]*Log[1 - c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] +
 Sqrt[g])])/(2*Sqrt[g]) - (b*e*Log[1 - c^2*x^2])/c + x*(a + b*ArcTanh[c*x])
*(d + e*Log[f + g*x^2]) + (b*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[
f + g*x^2]))/(2*c) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt
[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog[2, (Sqrt[g]*(1 - c*x
))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*Sqrt[-f]*PolyLog[2, -((Sqrt[
g]*(1 + c*x))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[g]) - (b*e*Sqrt[-f]*PolyLog
[2, (Sqrt[g]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[g]) + (b*e*PolyLog
[2, (c^2*(f + g*x^2))/(c^2*f + g)])/(2*c)
```

Rule 6073

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTanh[c*x]), x]
 + (-Dist[b*c, Int[(x*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[
2*e*g, Int[(x^2*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b,
```

```
c, d, e, f, g}, x]
```

Rule 2475

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Sim
plify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x
, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ
[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0]
 || IGtQ[q, 0])
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5974

```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTanh[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\int \left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)dx = x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-(bc)\int \frac{x\left(d+e\log\left(f+\right.\right.}{1-c^2x^2}$$

$$= x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}(bc)\,\mathrm{Subst}\left(\int \frac{d+e\,\mathrm{l}}{1}\right.$$

$$= -2aex+x\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)+\frac{b\log\left(\frac{g\left(1-c^2x^2\right)}{c^2f+g}\right.}{}$$

$$= -2aex+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+x\left(a+b\tanh^{-1}(c\right.$$

$$= -2aex+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)-\frac{be\log\left(1-c^2x^2\right)}{c}$$

$$= -2aex+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)-\frac{be\log\left(1-c^2x^2\right)}{c}$$

$$= -2aex+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\log(1-cx}{2}$$

$$= -2aex+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\log(1-cx}{2}$$

$$= -2aex+\frac{2ae\sqrt{f}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}-2bex\tanh^{-1}(cx)+\frac{be\sqrt{-f}\log(1-cx}{2}$$

Mathematica [C] time = 2.98357, size = 1251, normalized size = 2.09

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]),x]
```

```
[Out] a*d*x - 2*a*e*x + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[g] + b*d
*x*ArcTanh[c*x] + (b*d*Log[1 - c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*e
*(x*ArcTanh[c*x] + Log[1 - c^2*x^2]/(2*c))*Log[f + g*x^2] - (b*e*g*(((-Log[
-c^(-1) + x] - Log[c^(-1) + x] + Log[1 - c^2*x^2])*Log[f + g*x^2])/(2*g) +
(Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c
)] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/((-I)*Sqrt[f] - Sqrt[g]/c)])/(2*g)
+ (Log[-c^(-1) + x]*Log[1 - (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)
] + PolyLog[2, (Sqrt[g]*(-c^(-1) + x))/(I*Sqrt[f] - Sqrt[g]/c)])/(2*g) + (L
og[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)] +
 PolyLog[2, (Sqrt[g]*(c^(-1) + x))/((-I)*Sqrt[f] + Sqrt[g]/c)])/(2*g) + (Lo
g[c^(-1) + x]*Log[1 - (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)] + Pol
yLog[2, (Sqrt[g]*(c^(-1) + x))/(I*Sqrt[f] + Sqrt[g]/c)])/(2*g)))/c - (b*e*(
4*c*x*ArcTanh[c*x] - 4*Log[1/Sqrt[1 - c^2*x^2]] + (Sqrt[c^2*f*g]*((-2*I)*Ar
cCos[(-(c^2*f) + g)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] + 4*ArcTan[S
qrt[c^2*f*g]/(c*g*x)]*ArcTanh[c*x] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] -
2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(g + I*Sqrt[c^2*f*g])*(1 + c*
x))/((c^2*f + g)*(c^2*f + I*c*Sqrt[c^2*f*g]*x))] - (ArcCos[(-(c^2*f) + g)/(
c^2*f + g)] + 2*ArcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c^2*f*(I*g + Sqrt[c^2
*f*g])*(-1 + c*x))/((c^2*f + g)*((-I)*c^2*f + c*Sqrt[c^2*f*g]*x))] + (ArcCo
s[(-(c^2*f) + g)/(c^2*f + g)] + 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(
c*g*x)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcTanh[c*x]*Sqrt[c^
2*f + g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + (ArcCos[(-(
c^2*f) + g)/(c^2*f + g)] - 2*(ArcTan[Sqrt[c^2*f*g]/(c*g*x)] + ArcTan[(c*g*x
)/Sqrt[c^2*f*g]]))*Log[(Sqrt[2]*E^ArcTanh[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f +
 g]*Sqrt[c^2*f - g + (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + I*(-PolyLog[2, (
(-(c^2*f) + g - (2*I)*Sqrt[c^2*f*g])*(I*c^2*f + c*Sqrt[c^2*f*g]*x))/((c^2*f
 + g)*((-I)*c^2*f + c*Sqrt[c^2*f*g]*x))] + PolyLog[2, ((-(c^2*f) + g + (2*I
)*Sqrt[c^2*f*g])*(I*c^2*f + c*Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c^2*f +
c*Sqrt[c^2*f*g]*x))])))/g))/(2*c)
```

Maple [F] time = 1.73, size = 0, normalized size = 0.

$$\int (a+b\text{Artanh}(cx))\left(d+e\ln\left(gx^2+f\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f)),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(bd\,\text{artanh}(cx)+ad+(be\,\text{artanh}(cx)+ae)\log\left(gx^2+f\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

```
[Out] integral(b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)\left(e \log\left(gx^2 + f\right) + d\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

3.535 $\int \frac{(a+b\tanh^{-1}(cx))(d+e\log(f+gx^2))}{x}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}ae\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+be\text{CannotIntegrate}\left(\frac{\tanh^{-1}(cx)\log\left(f+gx^2\right)}{x},x\right)-\frac{1}{2}bd\text{PolyLog}(2,-cx)+\frac{1}{2}bd\text{PolyLog}($$

```
[Out] b*e*CannotIntegrate[(ArcTanh[c*x]*Log[f + g*x^2])/x, x] + a*d*Log[x] + (a*e
*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 - (b*d*PolyLog[2, -(c*x)])/2 + (b*d*Po
lyLog[2, c*x])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rubi [A] time = 0.252387, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] a*d*Log[x] + (a*e*Log[-((g*x^2)/f)]*Log[f + g*x^2])/2 - (b*d*PolyLog[2, -(c
*x)])/2 + (b*d*PolyLog[2, c*x])/2 + (a*e*PolyLog[2, 1 + (g*x^2)/f])/2 + b*e
*Defer[Int][(ArcTanh[c*x]*Log[f + g*x^2])/x, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx &= d\int \frac{a+b\tanh^{-1}(cx)}{x}\,dx+e\int \frac{\left(a+b\tanh^{-1}(cx)\right)\log\left(f+gx^2\right)}{x}\,dx\\
&= ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+(ae)\int \frac{\log\left(f+gx^2\right)}{x}\,dx+(be\\
&= ad\log(x)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)+\frac{1}{2}(ae)\,\text{Subst}\left(\int \frac{\log(f+gx)}{x}\right.\\
&= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)\\
&= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-\frac{1}{2}bd\text{Li}_2(-cx)+\frac{1}{2}bd\text{Li}_2(cx)
\end{aligned}$$

Mathematica [A] time = 0.213927, size = 0, normalized size = 0.

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

```
[Out] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

Maple [A] time = 0.759, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{Artanh}\,(cx))\left(d + e\ln\left(gx^2 + f\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$ad\log(x) + \int \frac{be(\log(cx+1) - \log(-cx+1))\log\left(gx^2 + f\right)}{2x} + \frac{bd(\log(cx+1) - \log(-cx+1))}{2x} + \frac{ae\log\left(gx^2 + f\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

```
[Out] a*d*log(x) + integrate(1/2*b*e*(log(c*x + 1) - log(-c*x + 1))*log(g*x^2 + f
)/x + 1/2*b*d*(log(c*x + 1) - log(-c*x + 1))/x + a*e*log(g*x^2 + f)/x, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bd\,\mathrm{artanh}\,(cx) + ad + (be\,\mathrm{artanh}\,(cx) + ae)\log\left(gx^2 + f\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f))
/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)\left(e \log\left(gx^2 + f\right) + d\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

3.536 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$

Optimal. Leaf size=613

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)-\frac{be\sqrt{g}\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\text{PolyLog}\left(2,\right.}{2\sqrt{-f}}$$

```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*Log[1 -
c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) +
 (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqr
t[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]
*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*Log[1 - c*x]*Log[
(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - ((a + b*
ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log
[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*
x^2]))/2 - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt
[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[
-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 + c*x
))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g
]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*c*e*PolyLog[2, (c^2
*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```

Rubi [A] time = 0.738756, antiderivative size = 613, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {6081, 2475, 36, 29, 31, 2416, 2394, 2315, 2393, 2391, 5974, 205, 5972, 2409}

$$-\frac{1}{2}bce\text{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f+g}\right)+\frac{1}{2}bce\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)-\frac{be\sqrt{g}\text{PolyLog}\left(2,-\frac{\sqrt{g}(1-cx)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}}+\frac{be\sqrt{g}\text{PolyLog}\left(2,\right.}{2\sqrt{-f}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```


```
[Out] (2*a*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - (b*e*Sqrt[g]*Log[1 -
c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) +
 (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + Sqr
t[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*Log[1 + c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]
*x))/(c*Sqrt[-f] - Sqrt[g])])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*Log[1 - c*x]*Log[
(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - ((a + b*
ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x + (b*c*Log[-((g*x^2)/f)]*(d + e*Log
[f + g*x^2]))/2 - (b*c*Log[(g*(1 - c^2*x^2))/(c^2*f + g)]*(d + e*Log[f + g*
x^2]))/2 - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 - c*x))/(c*Sqrt[-f] - Sqrt
[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g]*(1 - c*x))/(c*Sqrt[
-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*e*Sqrt[g]*PolyLog[2, -((Sqrt[g]*(1 + c*x
))/(c*Sqrt[-f] - Sqrt[g]))])/(2*Sqrt[-f]) + (b*e*Sqrt[g]*PolyLog[2, (Sqrt[g
]*(1 + c*x))/(c*Sqrt[-f] + Sqrt[g])])/(2*Sqrt[-f]) - (b*c*e*PolyLog[2, (c^2
*(f + g*x^2))/(c^2*f + g)])/2 + (b*c*e*PolyLog[2, 1 + (g*x^2)/f])/2
```


Rule 6081

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> Simp[(x^(m + 1)*(d + e*Log[f + g*x^2])*(a +
```

b*ArcTanh[c*x]))/(m + 1), x] + (-Dist[(b*c)/(m + 1), Int[(x^(m + 1)*(d + e*Log[f + g*x^2]))/(1 - c^2*x^2), x], x] - Dist[(2*e*g)/(m + 1), Int[(x^(m + 2)*(a + b*ArcTanh[c*x]))/(f + g*x^2), x], x]) /; FreeQ[{a, b, c, d, e, f, g}, x] && ILtQ[m/2, 0]

Rule 2475

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IntegerQ[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0])

Rule 36

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2416

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2394

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]

Rule 2315

Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]

Rule 2393

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 5974

```
Int[(ArcTanh[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
 Dist[a, Int[1/(d + e*x^2), x], x] + Dist[b, Int[ArcTanh[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5972

```
Int[ArcTanh[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/2, Int[
Log[1 + c*x]/(d + e*x^2), x], x] - Dist[1/2, Int[Log[1 - c*x]/(d + e*x^2),
x], x] /; FreeQ[{c, d, e}, x]
```

Rule 2409

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + (bc)\int \frac{d+e\log\left(f+g\right.}{x\left(1-c^2x^2\right)}$$

$$= -\frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}(bc)\,\text{Subst}\left(\int \frac{d+e\,l}{x\,(}\right.$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}($$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}($$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{1}{2}b$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{be\sqrt{g}\log}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{be\sqrt{g}\log}{}$$

$$= \frac{2ae\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{be\sqrt{g}\log(1-cx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{be\sqrt{g}\log}{}$$

Mathematica [C] time = 3.23778, size = 1226, normalized size = 2.

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

```
[Out] -((a*d)/x) - (b*d*ArcTanh[c*x])/x + b*c*d*Log[x] - (b*c*d*Log[1 - c^2*x^2])
/2 + a*e*((2*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] - Log[f + g*x^2]/
x) + (b*e*(-(((2*ArcTanh[c*x] + c*x*(-2*Log[x] + Log[1 - c^2*x^2]))*Log[f +
 g*x^2])/x) - 2*c*(Log[x]*(Log[1 - (I*Sqrt[g]*x)/Sqrt[f]] + Log[1 + (I*Sqrt
[g]*x)/Sqrt[f]]) + PolyLog[2, ((-I)*Sqrt[g]*x)/Sqrt[f]] + PolyLog[2, (I*Sqr
t[g]*x)/Sqrt[f]]) + c*(Log[-c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] - I*Sqrt[g])] + Log[c^(-1) + x]*Log[(c*(Sqrt[f] - I*Sqrt[g]*x))/(c*
Sqrt[f] + I*Sqrt[g])] + Log[-c^(-1) + x]*Log[(c*(Sqrt[f] + I*Sqrt[g]*x))/(c
*Sqrt[f] + I*Sqrt[g])] - (Log[-c^(-1) + x] + Log[c^(-1) + x] - Log[1 - c^2*
x^2])*Log[f + g*x^2] + Log[c^(-1) + x]*Log[1 - (Sqrt[g]*(1 + c*x))/(I*c*Sqr
t[f] + Sqrt[g])] + PolyLog[2, (c*Sqrt[g]*(c^(-1) + x))/(I*c*Sqrt[f] + Sqrt[
g])] + PolyLog[2, (I*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] - I*Sqrt[g])] + PolyLog
[2, ((-I)*Sqrt[g]*(-1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])] + PolyLog[2, (I*Sqrt
[g]*(1 + c*x))/(c*Sqrt[f] + I*Sqrt[g])]) + (c*g*((2*I)*ArcCos[(-(c^2*f) + g
)/(c^2*f + g)]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]] - 4*ArcTan[(c*f)/(Sqrt[c^2*f*g
]*x)]*ArcTanh[c*x] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] + 2*ArcTan[(c*g*x)
/Sqrt[c^2*f*g]])*Log[((2*I)*c*f*(I*g + Sqrt[c^2*f*g])*(-1 + c*x))/((c^2*f +
 g)*(c*f + I*Sqrt[c^2*f*g]*x))] + (ArcCos[(-(c^2*f) + g)/(c^2*f + g)] - 2*A
rcTan[(c*g*x)/Sqrt[c^2*f*g]])*Log[(2*c*f*(g + I*Sqrt[c^2*f*g])*(1 + c*x))/(
(c^2*f + g)*(c*f + I*Sqrt[c^2*f*g]*x))] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g
)] + 2*(ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*L
og[(Sqrt[2]*Sqrt[c^2*f*g])/(E^ArcTanh[c*x]*Sqrt[c^2*f + g]*Sqrt[c^2*f - g +
 (c^2*f + g)*Cosh[2*ArcTanh[c*x]]])] - (ArcCos[(-(c^2*f) + g)/(c^2*f + g)]
- 2*(ArcTan[(c*f)/(Sqrt[c^2*f*g]*x)] + ArcTan[(c*g*x)/Sqrt[c^2*f*g]]))*Log[
(Sqrt[2]*E^ArcTanh[c*x]*Sqrt[c^2*f*g])/(Sqrt[c^2*f + g]*Sqrt[c^2*f - g + (c
^2*f + g)*Cosh[2*ArcTanh[c*x]]])] + I*(PolyLog[2, ((-(c^2*f) + g - (2*I)*Sq
rt[c^2*f*g])*(I*c*f + Sqrt[c^2*f*g]*x))/((c^2*f + g)*((-I)*c*f + Sqrt[c^2*f
*g]*x))] - PolyLog[2, ((-(c^2*f) + g + (2*I)*Sqrt[c^2*f*g])*(I*c*f + Sqrt[c
^2*f*g]*x))/((c^2*f + g)*((-I)*c*f + Sqrt[c^2*f*g]*x))])))/Sqrt[c^2*f*g]))/
2
```

Maple [F] time = 0.793, size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{Artanh}\,(cx))\left(d + e\ln\left(gx^2 + f\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

```
[Out] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx) + ad + (be\,\text{artanh}\,(cx) + ae)\log\left(gx^2 + f\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f))
/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{artanh}\,(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

3.537 $\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$

Optimal. Leaf size=470

$$-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2f}+\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4f}+\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c}{(cx+}\right.}{4f}$$

```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*x)])/f - (b*e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/
(2*f) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*
Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f) - (b*
c*(d + e*Log[f + g*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]
))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - (b*e*g*PolyL
og[2, -(c*x)])/(2*f) + (b*e*g*PolyLog[2, c*x])/(2*f) - (b*e*(c^2*f + g)*Pol
yLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqr
t[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(4*f) + (b*e*(c^2*
f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*
(1 + c*x))])/(4*f)
```

Rubi [A] time = 0.746135, antiderivative size = 470, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 17, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5916, 325, 206, 6085, 801, 635, 205, 260, 446, 72, 6725, 5912, 5992, 5920, 2402, 2315, 2447}

$$-\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2}{cx+1}\right)}{2f}+\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-f}-\sqrt{g}x\right)}{(cx+1)\left(c\sqrt{-f}-\sqrt{g}\right)}\right)}{4f}+\frac{be\left(c^2f+g\right)\text{PolyLog}\left(2,1-\frac{2c}{(cx+}\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```


```
[Out] (b*c*e*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/Sqrt[f] + (a*e*g*Log[x])/f + (b
*e*(c^2*f + g)*ArcTanh[c*x]*Log[2/(1 + c*x)])/f - (b*e*(c^2*f + g)*ArcTanh[
c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/
(2*f) - (b*e*(c^2*f + g)*ArcTanh[c*x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*
Sqrt[-f] + Sqrt[g])*(1 + c*x))])/(2*f) - (a*e*g*Log[f + g*x^2])/(2*f) - (b*
c*(d + e*Log[f + g*x^2]))/(2*x) + (b*c^2*ArcTanh[c*x]*(d + e*Log[f + g*x^2]
))/2 - ((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - (b*e*g*PolyL
og[2, -(c*x)])/(2*f) + (b*e*g*PolyLog[2, c*x])/(2*f) - (b*e*(c^2*f + g)*Pol
yLog[2, 1 - 2/(1 + c*x)])/(2*f) + (b*e*(c^2*f + g)*PolyLog[2, 1 - (2*c*(Sqr
t[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - Sqrt[g])*(1 + c*x))])/(4*f) + (b*e*(c^2*
f + g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + Sqrt[g])*
(1 + c*x))])/(4*f)
```


Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6085

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*
(e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTanh[c*x]),
 x]}, Dist[d + e*Log[f + g*x^2], u, x] - Dist[2*e*g, Int[ExpandIntegrand[(x
*u)/(f + g*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ
[m] && NeQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int \frac{\left(a+b\tanh^{-1}(cx)\right)\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx = -\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx^2\right)\right)-$$

$$=-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx^2\right)\right)-$$

$$=-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(f+gx^2\right)\right)-$$

$$=\frac{aeg\log(x)}{f}-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(\right.\right.$$

$$=\frac{aeg\log(x)}{f}-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}+\frac{1}{2}bc^2\tanh^{-1}(cx)\left(d+e\log\left(\right.\right.$$

$$=\frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}-\frac{aeg\log\left(f+gx^2\right)}{2f}-\frac{bc\left(d+e\log\right.}{2}$$

$$=\frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log\left(\frac{2}{1+c}\right.}{f}$$

$$=\frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log\left(\frac{2}{1+c}\right.}{f}$$

$$=\frac{bce\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}+\frac{be\left(c^2f+g\right)\tanh^{-1}(cx)\log\left(\frac{2}{1+c}\right.}{f}$$

Mathematica [C] time = 4.54415, size = 982, normalized size = 2.09

$$-\frac{-4beg\tanh^{-1}(cx)^2x^2-4bce\sqrt{f}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)x^2-2bc^2df\tanh^{-1}(cx)x^2-4ibc^2ef\sin^{-1}\left(\sqrt{\frac{c^2f}{fc^2+g}}\right)\tanh^{-1}\left(\frac{cgx}{\sqrt{-c^2fg}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*ArcTanh[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

```
[Out] -(2*a*d*f + 2*b*c*d*f*x - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sq
rt[f]] + 2*b*d*f*ArcTanh[c*x] - 2*b*c^2*d*f*x^2*ArcTanh[c*x] - 4*b*e*g*x^2*
ArcTanh[c*x]^2 - (4*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*ArcT
anh[(c*g*x)/Sqrt[-(c^2*f*g)]] - 4*b*e*g*x^2*ArcTanh[c*x]*Log[1 - E^(-2*ArcT
anh[c*x])] - 4*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[1 + E^(-2*ArcTanh[c*x])] - (2
*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f + g)]]*Log[(c^2*(1 + E^(2*ArcT
anh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcT
anh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[(c^2*(1 + E^(2*A
rcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g - 2*Sqrt[-(c^2*f*g)])/(E^(2*A
rcTanh[c*x])*(c^2*f + g))] + (2*I)*b*c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f
 + g)]]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*g +
 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*c^2*e*f*x^2*Ar
cTanh[c*x]*Log[(c^2*(1 + E^(2*ArcTanh[c*x]))*f + (-1 + E^(2*ArcTanh[c*x]))*
g + 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(c^2*f + g))] + 2*b*e*g*x^2*Arc
Tanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sq
rt[g] - g)] + 2*b*e*g*x^2*ArcTanh[c*x]*Log[1 + (E^(2*ArcTanh[c*x])*(c^2*f +
```

```
g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g)] - 4*a*e*g*x^2*Log[x] + 2*a*e*f*Log
[f + g*x^2] + 2*b*c*e*f*x*Log[f + g*x^2] + 2*a*e*g*x^2*Log[f + g*x^2] + 2*b
*e*f*ArcTanh[c*x]*Log[f + g*x^2] - 2*b*c^2*e*f*x^2*ArcTanh[c*x]*Log[f + g*x
^2] + 2*b*c^2*e*f*x^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 2*b*e*g*x^2*PolyLo
g[2, E^(-2*ArcTanh[c*x])] + b*e*g*x^2*PolyLog[2, -((E^(2*ArcTanh[c*x])*(c^2
*f + g))/(c^2*f - 2*c*Sqrt[-f]*Sqrt[g] - g))] + b*e*g*x^2*PolyLog[2, -((E^(
2*ArcTanh[c*x])*(c^2*f + g))/(c^2*f + 2*c*Sqrt[-f]*Sqrt[g] - g))] - b*c^2*e
*f*x^2*PolyLog[2, (-(c^2*f) + g - 2*Sqrt[-(c^2*f*g)])/(E^(2*ArcTanh[c*x])*(
c^2*f + g))] - b*c^2*e*f*x^2*PolyLog[2, (-(c^2*f) + g + 2*Sqrt[-(c^2*f*g)])
/(E^(2*ArcTanh[c*x])*(c^2*f + g))])/(4*f*x^2)
```

Maple [B] time = 2.897, size = 961, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

```
[Out] -1/2*b*c*d/x+a*e*g*ln(x)/f-1/2*a*e*g*ln(g*x^2+f)/f-1/4*b*c^2*d*ln(-c*x+1)-1
/2/x^2*a*d-1/2*g*b*e/f*dilog(c*x+1)-1/4*b*e*dilog((c*(-f*g)^(1/2)-(c*x+1)*g
+g)/(c*(-f*g)^(1/2)+g))*c^2-1/4*b*e*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(
-f*g)^(1/2)-g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(
1/2)+g))*c^2+1/4*b*e*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g)
)*c^2-1/4*d*c^2*b*ln(c*x)+1/4*d*c^2*b*ln(c*x+1)-1/4*d*b*ln(c*x+1)/x^2+1/4*d
*c^2*b*ln(-c*x)+1/4*d*b*ln(-c*x+1)/x^2-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(
1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e/f*ln(c*x+1)*ln((c*(-f*g)^(1
/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+g*e*b*c/(f*g)^(1/2)*arctan(x*g/(f*g)^(
1/2))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/
2)+g))+1/4*g*b*e/f*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1
/2)-g))+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))
-1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))*c^2-
1/4*b*e*ln(c*x+1)*ln((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))*c^2-1
/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(c*x+1)*g+g)/(c*(-f*g)^(1/2)+g))-1/4*g*b*e
/f*dilog((c*(-f*g)^(1/2)+(c*x+1)*g-g)/(c*(-f*g)^(1/2)-g))+1/2*g*b*e/f*dilog
(-c*x+1)+1/4*b*e*ln(-c*x+1)*ln((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2
)+g))*c^2+1/4*b*e*ln(-c*x+1)*ln((c*(-f*g)^(1/2)+(-c*x+1)*g-g)/(c*(-f*g)^(1/
2)-g))*c^2+1/4*g*b*e/f*dilog((c*(-f*g)^(1/2)-(-c*x+1)*g+g)/(c*(-f*g)^(1/2)+
g))+(-1/4*b*e/x^2*ln(c*x+1)-1/4*e*(c^2*b*ln(-c*x+1)*x^2-c^2*b*ln(c*x+1)*x^2
+2*x*b*c-b*ln(-c*x+1)+2*a)/x^2)*ln(g*x^2+f)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bd\,\text{artanh}\,(cx)+ad+(be\,\text{artanh}\,(cx)+ae)\log\left(gx^2+f\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

```
[Out] integral((b*d*arctanh(c*x) + a*d + (b*e*arctanh(c*x) + a*e)*log(g*x^2 + f))
/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\,\text{artanh}\,(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

3.538 $\int \frac{\tanh^{-1}(cx)\left(a+b\tanh^{-1}(cx)\right)}{(1+cx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{a+b}{2c(cx+1)}+\frac{(a+b)\tanh^{-1}(cx)}{2c}-\frac{(a+b)\tanh^{-1}(cx)}{c(cx+1)}-\frac{b(1-cx)\tanh^{-1}(cx)^2}{2c(cx+1)}$$

```
[Out] -(a + b)/(2*c*(1 + c*x)) + ((a + b)*ArcTanh[c*x])/(2*c) - ((a + b)*ArcTanh[
c*x])/(c*(1 + c*x)) - (b*(1 - c*x)*ArcTanh[c*x]^2)/(2*c*(1 + c*x))
```

Rubi [A] time = 0.293258, antiderivative size = 122, normalized size of antiderivative = 1.56, number of steps used = 16, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {5926, 627, 44, 207, 6742, 5928, 5948}

$$-\frac{a}{2c(cx+1)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(cx+1)}-\frac{b}{2c(cx+1)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh^{-1}(cx)^2}{c(cx+1)}+\frac{b\tanh^{-1}(cx)}{2c}-\frac{b\tanh^{-1}(c}{c(cx+1}$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

```
[Out] -a/(2*c*(1 + c*x)) - b/(2*c*(1 + c*x)) + (a*ArcTanh[c*x])/(2*c) + (b*ArcTan
h[c*x])/(2*c) - (a*ArcTanh[c*x])/(c*(1 + c*x)) - (b*ArcTanh[c*x])/(c*(1 + c
*x)) + (b*ArcTanh[c*x]^2)/(2*c) - (b*ArcTanh[c*x]^2)/(c*(1 + c*x))
```

Rule 5926

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol
] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x]))/(e*(q + 1)), x] - Dist[(
b*c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 - c^2*x^2), x], x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[q, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5928

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_S
ymbol] :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTanh[c*x])^p)/(e*(q + 1)), x] -
 Dist[(b*c*p)/(e*(q + 1)), Int[ExpandIntegrand[(a + b*ArcTanh[c*x])^(p - 1)
, (d + e*x)^(q + 1)/(1 - c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x]
&& IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^{-1}(cx)\left(a+b\tanh^{-1}(cx)\right)}{(1+cx)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(x)\left(a+b\tanh^{-1}(x)\right)}{(1+x)^2}\,dx,x,cx\right)}{c}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a\tanh^{-1}(x)}{(1+x)^2}+\frac{b\tanh^{-1}(x)^2}{(1+x)^2}\right)dx,x,cx\right)}{c}\\
&= \frac{a\,\text{Subst}\left(\int \frac{\tanh^{-1}(x)}{(1+x)^2}\,dx,x,cx\right)}{c}+\frac{b\,\text{Subst}\left(\int \frac{\tanh^{-1}(x)^2}{(1+x)^2}\,dx,x,cx\right)}{c}\\
&= -\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}+\frac{a\,\text{Subst}\left(\int \frac{1}{(1+x)\left(1-x^2\right)}\,dx,x,cx\right)}{c}+\frac{(2b)\,\text{Sub}}{}\\
&= -\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}+\frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)(1+x)^2}\,dx,x,cx\right)}{c}+\frac{b\,\text{Subst}}{}\\
&= -\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}+\frac{a\,\text{Subst}\left(\right.}{}\\
&= -\frac{a}{2c(1+cx)}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b\tanh^{-1}(cx)^2}{2c}-\frac{b\tanh^{-1}(cx)^2}{c(1+cx)}\\
&= -\frac{a}{2c(1+cx)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b\tanh^{-1}(cx)^2}{2c}\\
&= -\frac{a}{2c(1+cx)}-\frac{b}{2c(1+cx)}+\frac{a\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b\tanh^{-1}(cx)}{c(1+cx)}+\frac{b}{}\\
&= -\frac{a}{2c(1+cx)}-\frac{b}{2c(1+cx)}+\frac{a\tanh^{-1}(cx)}{2c}+\frac{b\tanh^{-1}(cx)}{2c}-\frac{a\tanh^{-1}(cx)}{c(1+cx)}-\frac{b}{}
\end{aligned}$$

Mathematica [A] time = 0.0815359, size = 70, normalized size = 0.9

$$-\frac{(a+b)((cx+1)\log(1-cx)-(cx+1)\log(cx+1)+2)+4(a+b)\tanh^{-1}(cx)-2b(cx-1)\tanh^{-1}(cx)^2}{4c(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTanh[c*x]*(a + b*ArcTanh[c*x]))/(1 + c*x)^2,x]
```

```
[Out] -(4*(a + b)*ArcTanh[c*x] - 2*b*(-1 + c*x)*ArcTanh[c*x]^2 + (a + b)*(2 + (1
+ c*x)*Log[1 - c*x] - (1 + c*x)*Log[1 + c*x]))/(4*c*(1 + c*x))
```

Maple [B] time = 0.063, size = 247, normalized size = 3.2

$$-\frac{a\text{Artanh}\,(cx)}{c\,(cx+1)}-\frac{a\ln\,(cx-1)}{4\,c}-\frac{a}{2\,c\,(cx+1)}+\frac{a\ln\,(cx+1)}{4\,c}-\frac{b\,(\text{Artanh}\,(cx))^2}{c\,(cx+1)}-\frac{b\text{Artanh}\,(cx)\ln\,(cx-1)}{2\,c}-\frac{b\text{Artanh}\,(}{c\,(cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x)
```

```
[Out] -a*arctanh(c*x)/c/(c*x+1)-1/4*a/c*ln(c*x-1)-1/2*a/c/(c*x+1)+1/4*a/c*ln(c*x+
1)-b*arctanh(c*x)^2/c/(c*x+1)-1/2/c*b*arctanh(c*x)*ln(c*x-1)-b*arctanh(c*x)
/c/(c*x+1)+1/2/c*b*arctanh(c*x)*ln(c*x+1)-1/8/c*b*ln(c*x-1)^2+1/4/c*b*ln(c*
x-1)*ln(1/2+1/2*c*x)-1/4/c*b*ln(c*x-1)-1/2*b/c/(c*x+1)+1/4/c*b*ln(c*x+1)-1/
4/c*b*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+1/4/c*b*ln(-1/2*c*x+1/2)*ln(c*x+1)-1
/8/c*b*ln(c*x+1)^2
```

Maxima [C] time = 1.21555, size = 305, normalized size = 3.91

$$-\frac{1}{8}\left(bc\left(\frac{2}{c^4x+c^3}-\frac{\log\,(cx+1)}{c^3}+\frac{\log\,(cx-1)}{c^3}\right)+2\,a\left(\frac{2}{c^3x+c^2}-\frac{\log\,(cx+1)}{c^2}+\frac{\log\,(cx-1)}{c^2}\right)+\frac{-2i\,\pi b+(i\,\pi b+(i\,\pi bc-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b*c*(2/(c^4*x + c^3) - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) + 2*a*(2/
(c^3*x + c^2) - log(c*x + 1)/c^2 + log(c*x - 1)/c^2) + (-2*I*pi*b + (I*pi*b
 + (I*pi*b*c - b*c)*x + b)*log(c*x + 1) + (-I*pi*b + (-I*pi*b*c + b*c)*x -
b)*log(c*x - 1) + 2*b)/(c^3*x + c^2))*c - 1/4*((c*(2/(c^3*x + c^2) - log(c*
x + 1)/c^2 + log(c*x - 1)/c^2) + 4*arctanh(c*x)/(c^2*x + c))*b + 4*a/(c^2*x
 + c))*arctanh(c*x)
```

Fricas [A] time = 1.92193, size = 166, normalized size = 2.13

$$\frac{(bcx-b)\log\left(-\frac{cx+1}{cx-1}\right)^2+2\left((a+b)cx-a-b\right)\log\left(-\frac{cx+1}{cx-1}\right)-4\,a-4\,b}{8\left(c^2x+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((b*c*x - b)*log(-(c*x + 1)/(c*x - 1))^2 + 2*((a + b)*c*x - a - b)*log(
-(c*x + 1)/(c*x - 1)) - 4*a - 4*b)/(c^2*x + c)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\,\mathrm{atanh}\,(cx))\,\mathrm{atanh}\,(cx)}{(cx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atanh(c*x)*(a+b*atanh(c*x))/(c*x+1)**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))*atanh(c*x)/(c*x + 1)**2, x)
```

Giac [A] time = 1.18259, size = 138, normalized size = 1.77

$$\frac{1}{8}\left(\frac{b}{c} - \frac{2\,b}{(cx+1)c}\right)\log\left(\frac{1}{\frac{2}{cx+1}-1}\right)^2 - \frac{(a+b)\log\left(-\frac{2}{cx+1}+1\right)}{4\,c} - \frac{(a+b)\log\left(\frac{1}{\frac{2}{cx+1}-1}\right)}{2\,(cx+1)c} - \frac{a+b}{2\,(cx+1)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(c*x)*(a+b*arctanh(c*x))/(c*x+1)^2,x, algorithm="giac")
```

```
[Out] 1/8*(b/c - 2*b/((c*x + 1)*c))*log(1/(2/(c*x + 1) - 1))^2 - 1/4*(a + b)*log(
-2/(c*x + 1) + 1)/c - 1/2*(a + b)*log(1/(2/(c*x + 1) - 1))/((c*x + 1)*c) -
1/2*(a + b)/((c*x + 1)*c)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```